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EXTERIOR BALLISTICS OF A PROJECTILE IN VERTICAL FLIGHT

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ABSTRACT

The motion of a projectile fired vertically upward is examined in some detail and the equations which express the instantaneous velocity and time of flight as a function of projectile diameter, weight, configuration, muzzle velocity, and air density are determined.

Based on these equations a computer program has been developed which allows the velocity, time of flight, and vertical range to be calculated for any projectile weight, diameter, and drag coefficient. The FORTRAN listing of the program is given in the report.

The results of a sample problem which considers the effect of drag variation on the time of flight is presented. (Author)

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INTRODUCTION

The Materials Application Division at AMMRC is currently engaged in the development of experimental artillery shell. The projectile features new components, assemblies, materials and methods of manufacture. After the prototype rounds are fabricated, a number of models are test fired to determine the adequacy of the design. At launch, high-speed motion pictures are taken as the round emerges from the muzzle and during the early stages of free flight. Also, the time of flight is recorded for several selected ranges. This information is then used to assess the sufficiency of the experimental rounds.

If the coefficient of drag and its variation with velocity is known for the projectile configuration, then the time of flight is readily calculated as a function of the range. The magnitude of the deviation between the measured time of flight and the calculated time of flight — consistent with the accuracy of the drag coefficient data — serves as a reliable index of projectile performance. Thus, substantial increases of the measured time of flight over the calculated allowable time band would suggest excessive yaw, probably induced by some major component malfunction or projectile break-up, whereas modest deviations would indicate less severe problems.

The program presented in this report, in conjunction with the experimentally determined coefficient of drag, permits the time of flight to be calculated at any range for vertical firings as a function of muzzle velocity and projectile weight.

UPWARD PROJECTILE MOTION

The forces acting on a projectile fired straight up in the air are depicted in Figure 1a.

The equation of motion

$$m\dot{u} = D-mg \tag{1}$$

and from dimensional analysis

$$D = -\rho d^2 u^2 K_D. \tag{2}$$

The density variation in the atmosphere for the standard structure is[1]

$$\rho = \rho_0 e^{-hz}. \tag{3}$$

It is assumed that the speed of sound is constant and

$$K_D = a+bu \tag{4}$$

1. HAYES, T. J. *Elements of Ordnance.* John Wiley & Sons, Inc., 1938.

where

m = projectile mass

$\dot{u}$ = instantaneous projectile deceleration

D = projectile drag

g = acceleration due to gravity

ρ = air density at altitude z

d = projectile diameter

u = instantaneous projectile velocity

K_D = ballistic coefficient of drag

ρ_0 = air density at ground level

h = constant

z = altitude

a = constant

b = constant

In order to simplify the analysis and permit a closed solution to the differential equation of motion, a small increment of z will be chosen such that K_D and ρ are essentially constant over the increment.

Substituting (2) into (1) gives

$$m\dot{u} = -\rho d^2 u^2 K_D - mg$$

or

$$\dot{u} + Ku^2 = -g \tag{5}$$

where

$$K = \frac{\rho d^2}{m} K_D. \tag{6}$$

Velocity-Range

Now

$$u = dz/dt$$

where t = time

and

$$\dot{u} = \frac{du}{dt} = u \frac{du}{dz}.$$

Substituting this result into (5) gives

$$u \frac{du}{dz} + Ku^2 = -g. \tag{7}$$

This is a first-order linear differential equation and is readily solved by means of the following transformation

$$p = u^2, \tag{8a}$$

$$\frac{dp}{dz} = 2u \frac{du}{dz}, \tag{8b}$$

$$\frac{dp}{dz} + 2Kp = -2g. \tag{9}$$

The solution to (9) is

$$p = e^{-\int 2Kdz} \int (-2g) e^{\int 2Kdz} \, dz + Ce^{-\int 2Kdz} \tag{10}$$

where

C = constant.

Now

$$p = (Ce^{-2Kz} - g/K). \tag{11}$$

Substituting (8a) into (11) gives

$$u^2 = (Ce^{-2Kz} - g/K)$$

or

$$u = (Ce^{-2Kz} - g/K)^{1/2}. \tag{12}$$

The constant C is evaluated from the boundary condition

$$u = u_s \text{ at } z = z_s.$$

Then

$$u_s^2 = [Ce^{-2Kz_s} - g/K]$$

and

$$C = (u_s^2 + g/K)e^{2Kz_s}. \tag{13}$$

Substituting (13) into (12) gives

$$u_F = [(u_s^2 + g/K)e^{-2K(z_F - z_s)} - g/K]^{1/2}. \tag{14}$$

The velocity-range is calculated from (14) for the case of a projectile fired vertically upward.

Time of Flight versus Velocity

The relationship between the velocity and time of flight is established from (5).

Then, separating the variables and integrating between the limits

$u = u_s$ at $t = t_s$

$u = u_F$ at $t = t_F$

$$\int_{t_s}^{t_F} dt = \left(-\frac{1}{K}\right) \int_{u_s}^{u_F} \frac{du}{[u^2+(\sqrt{g/K})^2]}$$

which gives

$$(t_F - t_s) = \frac{1}{\sqrt{gK}} \left\{ \tan^{-1}\left(\frac{K}{g}\right) u_s - \tan^{-1}\left(\frac{K}{g}\right) u_F \right\} \qquad (15)$$

DOWNWARD PROJECTILE MOTION

Velocity-Range

The forces acting on the projectile in downward flight are depicted in Figure 1b.

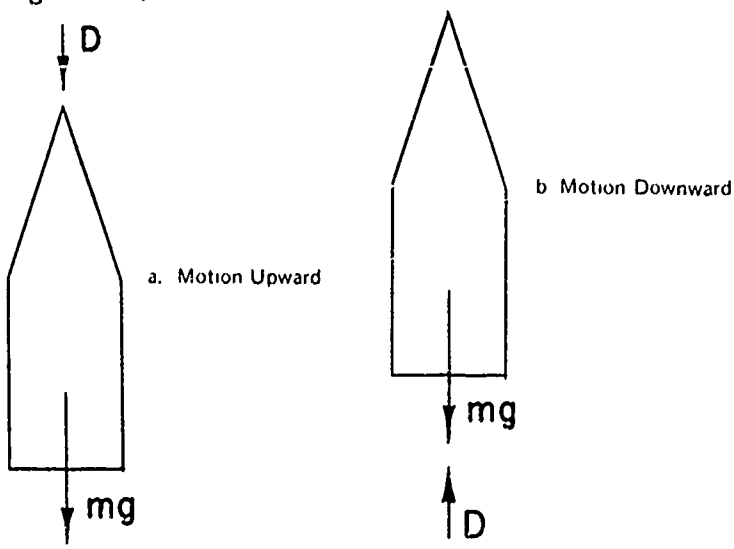

Figure 1. Forces Acting on Projectile in Vertical Flight

The equation of motion is

$$m\dot{u} = D + mg \tag{16}$$

which in the same fashion as before reduces to

$$\dot{u} + Ku^2 = g. \tag{17}$$

The solution follows the identical procedure as with (5) and is written immediately by changing the sign of g in (12) thus

$$u = \left(Ce^{-2Kz} + g/K\right)^{1/2}. \tag{18}$$

For convenience, the center of the coordinate axis is transferred to the maximum point of upward travel, and the constant C is evaluated as before.

Then

$$u_F = \left[(u_s^2 - g/K)e^{-2K(z_F - z_s)} + e/K\right]^{1/2}. \tag{19}$$

Time of Flight versus Velocity

The time of downward flight is determined from (17).

Separating the variables and integrating between the limits

$u = u_s$ at $t = t_s$

$u = u_F$ at $t = t_F$,

$$\int_{t_s}^{t_F} dt = -\left(\frac{1}{K}\right)\int_{u_s}^{u_F} \frac{du}{[u^2 - (\sqrt{g/K})^2]} \tag{20}$$

and

$$(t_F - t_s) = \frac{1}{2\sqrt{gK}}\left\{\ln \frac{(u_F + \sqrt{g/K})}{(u_F - \sqrt{g/K})} - \ln \frac{(u_s + \sqrt{g/K})}{(u_s - \sqrt{g/K})}\right\} \tag{21}$$

RESULTS AND CALCULATIONS

Projectile Parameter

The parameters of a given projectile design which determine the elements of the projectile motion (velocity, range, and time of flight to the target) are the weight, diameter, and general projectile configuration. For the experimental artillery shell under development at AMMRC the weight W is 200 lb and diameter d is 8.00 inches.

The projectile configuration largely determines the aerodynamic forces and moments acting on the projectile in free flight. For the point-mass analysis performed herein the only aerodynamic force required is the drag force. This force is dependent on the coefficient of drag which in turn is a function of the projectile shape and the instantaneous velocity of the projectile. The coefficient of drag is readily estimated from the large amount of data which has been compiled.[2] Based on the available data and the projectile contour illustrated in Figure 2, the drag coefficient shown in Figure 3 has been selected for the motion calculations.

When the projectile reaches its maximum vertical range from a 90° launch condition, it returns to the earth base first. In this attitude, the projectile presents a cylindrical contour to the air stream. The drag coefficient for this case[2] is practically constant and

$$K_D = 0.55$$

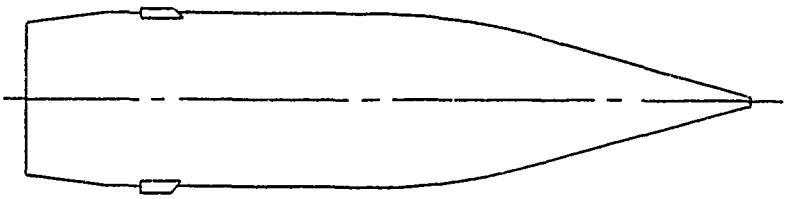

Figure 2. Experimental Projectile Configuration

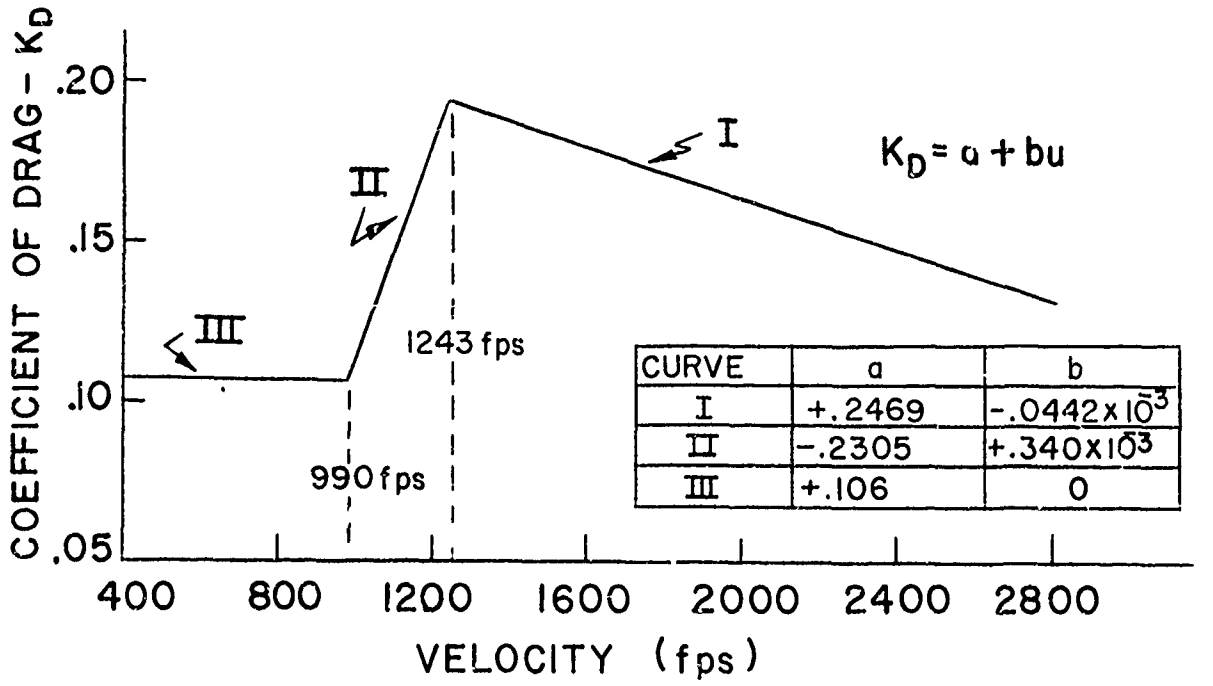

Figure 3. Coefficient of Drag Versus Velocity

2. HITCHCOCK, H. P. *Aerodynamic Data for Spinning Projectiles.* BRL Report No. 620, October 1947.

Launch Conditions

The experimental projectiles were launched from a M110E2 self-propelled howitzer oriented at 90° from the horizontal. The test projectiles were launched at various muzzle velocities up to a maximum of 2800 fps.

Projectile Motion

The velocity and time of flight versus range were calculated for the experimental shell for both the upward and downward trajectories using the applicable equations developed earlier. The calculations were performed for increments of 100 feet. Over each increment, the drag coefficient and air density were assumed to be constant, corresponding to the velocity and range values at the beginning of the increment. At the start of each new increment, the values were recomputed. Thus the terminal conditions for one increment form the initial conditions for the succeeding increment. The equations were programmed for the 1108 UNIVAC computer.

The altitude, velocity, time of flight, air density, and drag coefficient are calculated for an assumed muzzle velocity of 1800 fps. The computer results for both the upward and downward motion are given in Appendix A. The FORTRAN listing of the program is given in Appendix B.

In order to estimate the effect of variations in the experimentally determined drag coefficient K_D on the time of flight, computer runs were made in which the intercept of the drag coefficient for two of the linear fits shown in Figure 3 were altered +10% and then -10% while all the other parameters were maintained constant. The resulting time of flights for the assumed extremes and median drag coefficient at various altitudes are compiled in Table 1. It is noted that for the lower values of the drag coefficient, the time of flights for the same upward range are smaller. However, because the muzzle velocities are the same the lower drag allows the projectile to reach a higher altitude and thus the relationships in downward time of flights is reversed with the lower drag values requiring a larger amount of time to reach the same downward altitude.

Table 1. ALTITUDE VERSUS TIME OF FLIGHT FOR VARIATIONS IN PROJECTILE DRAG*

Altitude (ft)	Time of Flight (sec) A	B	C
0	0	0	0
5,000	3.033	3.065	3.098
10,000	6.675	6.824	6.984
15,000	11.123	11.500	11.904
20,000	16.624	17.451	18.406
25,000	24.081	26.251	29.609
20,000	65.246	60.414	56.730
15,000	72.651	68.211	64.999
10,000	79.715	75.458	27.447
5,000	86.862	82.701	79.793
0	94.361	90.250	87.393

*"a" Values In Drag Coefficient Equation (see Figure 3)

	A (minimum)	B (median)	C (maximum)
I	0.223	0.247	0.272
II	-0.254	-0.230	-0.207
III	0.083	0.106	0.130

The tabulated deviations in the time of flight effectively define the sensitivity limits for the assumed drag scatter. Thus, no inference concerning projectile performance can be made for any range-time of flight measurement that falls within the calculated limits.

APPENDIX A. SAMPLE PROBLEM

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
1	0	1800	.000	.0751	.167
2	100	1793	.056	.0749	.168
3	200	1786	.112	.0747	.168
4	300	1779	.168	.0744	.168
5	400	1772	.224	.0742	.169
6	500	1766	.280	.0740	.169
7	600	1759	.337	.0737	.169
8	700	1752	.394	.0735	.170
9	800	1746	.451	.0733	.170
10	900	1739	.508	.0730	.170
11	1000	1732	.565	.0728	.170
12	1100	1726	.624	.0726	.171
13	1200	1719	.682	.0723	.171
14	1300	1712	.740	.0721	.171
15	1400	1706	.799	.0719	.172
16	1500	1699	.858	.0717	.172
17	1600	1693	.917	.0714	.172
18	1700	1686	.976	.0712	.172
19	1800	1680	1.035	.0710	.173
20	1900	1673	1.095	.0708	.173
21	2000	1667	1.155	.0705	.173
22	2100	1660	1.215	.0703	.174
23	2200	1654	1.275	.0701	.174
24	2300	1647	1.336	.0699	.174
25	2400	1641	1.396	.0696	.174
26	2500	1635	1.457	.0694	.175
27	2600	1628	1.519	.0692	.175
28	2700	1622	1.580	.0690	.175
29	2800	1615	1.642	.0688	.176
30	2900	1609	1.704	.0686	.176
31	3000	1603	1.766	.0683	.176
32	3100	1597	1.829	.0681	.176
33	3200	1590	1.892	.0679	.177
34	3300	1584	1.955	.0677	.177
35	3400	1578	2.018	.0675	.177
36	3500	1572	2.081	.0673	.178
37	3600	1565	2.145	.0671	.178
38	3700	1559	2.209	.0668	.178
39	3800	1553	2.273	.0666	.179
40	3900	1547	2.338	.0664	.179

PROJECTILE-XP1C1 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
41	4000	1541	2.462	.0662	.179
42	4100	1534	2.467	.0660	.179
43	4200	1528	2.533	.0658	.179
44	4300	1522	2.598	.0656	.180
45	4400	1516	2.664	.0654	.180
46	4500	1510	2.730	.0652	.180
47	4600	1504	2.797	.0650	.181
48	4700	1498	2.863	.0648	.181
49	4800	1492	2.930	.0646	.181
50	4900	1486	2.997	.0644	.181
51	5000	1480	3.065	.0641	.182
52	5100	1474	3.132	.0639	.182
53	5200	1468	3.200	.0637	.182
54	5300	1462	3.268	.0635	.182
55	5400	1456	3.337	.0633	.183
56	5500	1450	3.406	.0631	.183
57	5600	1444	3.475	.0629	.183
58	5700	1438	3.544	.0627	.183
59	5800	1432	3.614	.0625	.184
60	5900	1426	3.684	.0624	.184
61	6000	1420	3.754	.0622	.184
62	6100	1414	3.825	.0620	.184
63	6200	1409	3.895	.0618	.185
64	6300	1403	3.967	.0616	.185
65	6400	1397	4.038	.0614	.185
66	6500	1391	4.110	.0612	.186
67	6600	1385	4.182	.0610	.186
68	6700	1379	4.254	.0608	.186
69	6800	1374	4.327	.0606	.186
70	6900	1368	4.400	.0604	.187
71	7000	1362	4.473	.0602	.187
72	7100	1356	4.546	.0600	.187
73	7200	1351	4.620	.0598	.187
74	7300	1345	4.694	.0597	.188
75	7400	1339	4.769	.0595	.188
76	7500	1333	4.844	.0593	.188
77	7600	1328	4.919	.0591	.188
78	7700	1322	4.994	.0589	.189
79	7800	1316	5.070	.0587	.189
80	7900	1311	5.146	.0585	.189

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
81	8000	1305	5.223	.0583	.189
82	8100	1299	5.299	.0582	.190
83	8200	1294	5.376	.0580	.190
84	8300	1288	5.454	.0578	.190
85	8400	1282	5.532	.0576	.190
86	8500	1277	5.610	.0574	.191
87	8600	1271	5.688	.0573	.191
88	8700	1265	5.767	.0571	.191
89	8800	1260	5.846	.0569	.191
90	8900	1254	5.926	.0567	.192
91	9000	1249	6.006	.0565	.192
92	9100	1243	6.086	.0564	.192
93	9200	1237	6.166	.0562	.190
94	9300	1232	6.247	.0560	.188
95	9400	1226	6.329	.0558	.187
96	9500	1221	6.410	.0556	.185
97	9600	1215	6.493	.0555	.183
98	9700	1210	6.575	.0553	.181
99	9800	1205	6.658	.0551	.179
100	9900	1199	6.741	.0549	.177
101	10000	1194	6.824	.0548	.176
102	10100	1189	6.908	.0546	.174
103	10200	1184	6.993	.0544	.172
104	10300	1179	7.077	.0543	.170
105	10400	1173	7.162	.0541	.168
106	10500	1169	7.248	.0539	.167
107	10600	1163	7.333	.0537	.165
108	10700	1158	7.419	.0536	.163
109	10800	1153	7.506	.0534	.162
110	10900	1148	7.593	.0532	.160
111	11000	1143	7.680	.0531	.158
112	11100	1138	7.768	.0529	.157
113	11200	1133	7.856	.0527	.155
114	11300	1128	7.944	.0526	.153
115	11400	1123	8.033	.0524	.152
116	11500	1119	8.122	.0522	.150
117	11600	1114	8.212	.0521	.148
118	11700	1109	8.302	.0519	.147
119	11800	1104	8.392	.0517	.145
120	11900	1099	8.483	.0516	.143

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
121	12000	1095	8.574	.0514	.142
122	12100	1090	8.665	.0513	.140
123	12200	1085	8.757	.0511	.139
124	12300	1081	8.850	.0509	.137
125	12400	1076	8.942	.0508	.135
126	12500	1071	9.035	.0506	.134
127	12600	1067	9.129	.0505	.132
128	12700	1062	9.223	.0503	.131
129	12800	1057	9.317	.0501	.129
130	12900	1053	9.412	.0500	.128
131	13000	1048	9.507	.0498	.126
132	13100	1044	9.602	.0497	.124
133	13200	1039	9.698	.0495	.123
134	13300	1035	9.795	.0494	.121
135	13400	1030	9.892	.0492	.120
136	13500	1026	9.989	.0490	.118
137	13600	1021	10.087	.0489	.117
138	13700	1017	10.185	.0487	.115
139	13800	1012	10.283	.0486	.114
140	13900	1008	10.382	.0484	.112
141	14000	1004	10.481	.0483	.111
142	14100	999	10.581	.0481	.109
143	14200	995	10.682	.0480	.108
144	14300	990	10.782	.0478	.106
145	14400	986	10.883	.0477	.106
146	14500	982	10.985	.0475	.106
147	14600	977	11.087	.0474	.106
148	14700	973	11.189	.0472	.106
149	14800	968	11.292	.0471	.106
150	14900	964	11.396	.0469	.106
151	15000	960	11.500	.0468	.106
152	15100	955	11.604	.0466	.106
153	15200	951	11.709	.0465	.106
154	15300	946	11.814	.0463	.106
155	15400	942	11.920	.0462	.106
156	15500	938	12.027	.0460	.106
157	15600	933	12.134	.0459	.106
158	15700	929	12.241	.0457	.106
159	15800	924	12.349	.0456	.106
160	15900	920	12.457	.0455	.106

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
161	16000	915	12.566	.0453	.106
162	16100	911	12.676	.0452	.106
163	16200	906	12.786	.0450	.106
164	16300	902	12.896	.0449	.106
165	16400	897	13.007	.0447	.106
166	16500	893	13.113	.0446	.106
167	16600	888	13.231	.0445	.106
168	16700	883	13.344	.0443	.106
169	16800	879	13.458	.0442	.106
170	16900	874	13.572	.0440	.106
171	17000	870	13.686	.0439	.106
172	17100	865	13.801	.0438	.106
173	17200	860	13.917	.0436	.106
174	17300	856	14.034	.0435	.106
175	17400	851	14.151	.0434	.106
176	17500	847	14.269	.0432	.106
177	17600	842	14.387	.0431	.106
178	17700	837	14.506	.0429	.106
179	17800	832	14.626	.0428	.106
180	17900	828	14.746	.0427	.106
181	18000	823	14.867	.0425	.106
182	18100	818	14.988	.0424	.106
183	18200	814	15.111	.0423	.106
184	18300	809	15.235	.0421	.106
185	18400	804	15.359	.0420	.106
186	18500	799	15.483	.0419	.106
187	18600	794	15.609	.0417	.106
188	18700	789	15.735	.0416	.106
189	18800	785	15.862	.0415	.106
190	18900	780	15.990	.0413	.106
191	19000	775	16.118	.0412	.106
192	19100	770	16.248	.0411	.106
193	19200	765	16.378	.0410	.106
194	19300	760	16.509	.0408	.106
195	19400	755	16.641	.0407	.106
196	19500	750	16.774	.0406	.106
197	19600	745	16.907	.0404	.106
198	19700	740	17.042	.0403	.106
199	19800	735	17.177	.0402	.106
200	19900	730	17.314	.0401	.106

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
201	20000	725	17.451	.0399	.106
202	20100	720	17.590	.0398	.106
203	20200	714	17.729	.0397	.106
204	20300	709	17.869	.0396	.106
205	20400	704	18.011	.0394	.106
206	20500	699	18.153	.0393	.106
207	20600	694	18.297	.0392	.106
208	20700	688	18.442	.0391	.106
209	20800	683	18.587	.0389	.106
210	20900	678	18.734	.0388	.106
211	21000	672	18.882	.0387	.106
212	21100	667	19.032	.0386	.106
213	21200	661	19.182	.0384	.106
214	21300	656	19.334	.0383	.106
215	21400	650	19.487	.0382	.106
216	21500	645	19.641	.0381	.106
217	21600	639	19.797	.0380	.106
218	21700	634	19.954	.0378	.106
219	21800	628	20.112	.0377	.106
220	21900	622	20.272	.0376	.106
221	22000	616	20.433	.0375	.106
222	22100	611	20.596	.0374	.106
223	22200	605	20.761	.0373	.106
224	22300	599	20.927	.0371	.106
225	22400	593	21.094	.0370	.106
226	22500	587	21.264	.0369	.106
227	22600	581	21.435	.0368	.106
228	22700	575	21.608	.0367	.106
229	22800	569	21.782	.0366	.106
230	22900	563	21.959	.0364	.106
231	23000	556	22.138	.0363	.106
232	23100	550	22.318	.0362	.106
233	23200	544	22.501	.0361	.106
234	23300	537	22.686	.0360	.106
235	23400	531	22.873	.0359	.106
236	23500	524	23.062	.0358	.106
237	23600	518	23.254	.0356	.106
238	23700	511	23.448	.0355	.106
239	23800	504	23.645	.0354	.106
240	23900	497	23.844	.0353	.106

PROJECTILE-XP1C1 MUZZLE VELOCITY-18CC(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
241	24000	491	24.046	.0352	.106
242	24100	484	24.252	.0351	.106
243	24200	476	24.460	.0350	.106
244	24300	469	24.671	.0349	.106
245	24400	462	24.885	.0348	.106
246	24500	455	25.103	.0346	.106
247	24600	447	25.325	.0345	.106
248	24700	439	25.550	.0344	.106
249	24800	432	25.780	.0343	.106
250	24900	424	26.013	.0342	.106
251	25000	416	26.251	.0341	.106
252	25100	408	26.494	.0340	.106
253	25200	399	26.741	.0339	.106
254	25300	391	26.994	.0338	.106
255	25400	382	27.252	.0337	.106
256	25500	373	27.517	.0336	.106
257	25600	364	27.787	.0335	.106
258	25700	355	28.065	.0334	.106
259	25800	346	28.350	.0332	.106
260	25900	336	28.642	.0331	.106
261	26000	326	28.944	.0330	.106
262	26100	316	29.255	.0329	.106
263	26200	305	29.576	.0328	.106
264	26300	294	29.909	.0327	.106
265	26400	283	30.255	.0326	.106
266	26500	271	30.615	.0325	.106
267	26600	259	30.992	.0324	.106
268	26700	246	31.388	.0323	.106
269	26800	232	31.805	.0322	.106
270	26900	218	32.243	.0321	.106
271	27000	202	32.723	.0320	.106
272	27100	186	33.237	.0319	.106
273	27200	167	33.802	.0318	.106
274	27300	147	34.436	.0317	.106
275	27400	123	35.175	.0316	.106
276	27500	93	36.096	.0315	.106
277	27600	48	37.505	.0314	.106

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
1	27800	0	37.505	.0314	.550
2	27500	90	39.939	.0315	.550
3	27400	113	41.034	.0316	.550
4	27300	133	41.830	.0317	.550
5	27200	159	42.502	.0318	.550
6	27100	177	43.036	.0319	.550
7	27000	194	43.633	.0320	.550
8	26900	209	44.128	.0321	.550
9	26800	223	44.590	.0322	.550
10	26700	236	45.025	.0323	.550
11	26600	248	45.437	.0324	.550
12	26500	260	45.830	.0325	.550
13	26400	271	46.205	.0326	.550
14	26300	282	46.567	.0327	.550
15	26200	292	46.915	.0328	.550
16	26100	301	47.252	.0329	.550
17	26000	311	47.578	.0330	.550
18	25900	320	47.895	.0331	.550
19	25800	328	48.203	.0332	.550
20	25700	336	48.504	.0334	.550
21	25600	345	48.797	.0335	.550
22	25500	352	49.084	.0336	.550
23	25400	360	49.364	.0337	.550
24	25300	367	49.633	.0338	.550
25	25200	374	49.908	.0339	.550
26	25100	381	50.172	.0340	.550
27	25000	388	50.432	.0341	.550
28	24900	395	50.687	.0342	.550
29	24800	401	50.938	.0343	.550
30	24700	407	51.185	.0344	.550
31	24600	413	51.429	.0345	.550
32	24500	419	51.669	.0346	.550
33	24400	425	51.905	.0348	.550
34	24300	431	52.138	.0349	.550
35	24200	436	52.369	.0350	.550
36	24100	442	52.596	.0351	.550
37	24000	447	52.821	.0352	.550
38	23900	452	53.043	.0353	.550
39	23800	458	53.262	.0354	.550
40	23700	463	53.479	.0355	.550

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
41	23600	467	53.694	.0356	.550
42	23500	472	53.907	.0358	.550
43	23400	477	54.117	.0359	.550
44	23300	482	54.326	.0360	.550
45	23200	486	54.532	.0361	.550
46	23100	491	54.737	.0362	.550
47	23000	495	54.940	.0363	.550
48	22900	499	55.141	.0364	.550
49	22800	503	55.340	.0366	.550
50	22700	507	55.538	.0367	.550
51	22600	511	55.734	.0368	.550
52	22500	515	55.928	.0369	.550
53	22400	519	56.121	.0370	.550
54	22300	523	56.313	.0371	.550
55	22200	527	56.503	.0373	.550
56	22100	531	56.692	.0374	.550
57	22000	534	56.880	.0375	.550
58	21900	538	57.067	.0376	.550
59	21800	541	57.252	.0377	.550
60	21700	545	57.436	.0378	.550
61	21600	548	57.619	.0380	.550
62	21500	551	57.800	.0381	.550
63	21400	554	57.981	.0382	.550
64	21300	558	58.161	.0383	.550
65	21200	561	58.339	.0384	.550
66	21100	564	58.517	.0396	.550
67	21000	567	58.694	.0387	.550
68	20900	570	58.870	.0388	.550
69	20800	573	59.045	.0389	.550
70	20700	576	59.219	.0391	.550
71	20600	578	59.392	.0392	.550
72	20500	581	59.564	.0393	.550
73	20400	584	59.735	.0394	.550
74	20300	587	59.906	.0396	.550
75	20200	589	60.076	.0397	.550
76	20100	592	60.245	.0398	.550
77	20000	594	60.414	.0399	.550
78	19900	597	60.582	.0401	.550
79	19800	599	60.749	.0402	.550
80	19700	602	60.915	.0403	.550

PROJECTILE-XP1C1 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
81	19600	604	61.081	.0404	.550
82	19500	606	61.246	.0406	.550
83	19400	609	61.410	.0407	.550
84	19300	611	61.574	.0408	.550
85	19200	613	61.737	.0410	.550
86	19100	615	61.900	.0411	.550
87	19000	617	62.062	.0412	.550
88	18900	619	62.224	.0413	.550
89	18800	622	62.385	.0415	.550
90	18700	624	62.545	.0416	.550
91	18600	626	62.705	.0417	.550
92	18500	627	62.865	.0419	.550
93	18400	629	63.024	.0420	.550
94	18300	631	63.182	.0421	.550
95	18200	633	63.340	.0423	.550
96	18100	635	63.498	.0424	.550
97	18000	637	63.655	.0425	.550
98	17900	638	63.812	.0427	.550
99	17800	640	63.968	.0428	.550
100	17700	642	64.124	.0429	.550
101	17600	643	64.280	.0431	.550
102	17500	645	64.435	.0432	.550
103	17400	647	64.589	.0434	.550
104	17300	648	64.744	.0435	.550
105	17200	650	64.898	.0436	.550
106	17100	651	65.052	.0438	.550
107	17000	652	65.205	.0439	.550
108	16900	654	65.358	.0440	.550
109	16800	655	65.510	.0442	.550
110	16700	657	65.663	.0443	.550
111	16600	658	65.815	.0445	.550
112	16500	659	65.967	.0446	.550
113	16400	661	66.118	.0447	.550
114	16300	662	66.269	.0449	.550
115	16200	663	66.420	.0450	.550
116	16100	664	66.570	.0452	.550
117	16000	665	66.721	.0453	.550
118	15900	667	66.871	.0455	.550
119	15800	668	67.021	.0456	.550
120	15700	669	67.170	.0457	.550

PROJECTILE-XP1C1 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
121	15600	670	67.319	.0459	.550
122	15500	671	67.463	.0460	.550
123	15400	672	67.617	.0462	.550
124	15300	673	67.760	.0463	.550
125	15200	674	67.914	.0465	.550
126	15100	675	68.063	.0466	.550
127	15000	676	68.211	.0468	.550
128	14900	677	68.358	.0469	.550
129	14800	677	68.506	.0471	.550
130	14700	678	68.653	.0472	.550
131	14600	679	68.801	.0474	.550
132	14500	680	68.948	.0475	.550
133	14400	681	69.095	.0477	.550
134	14300	681	69.241	.0478	.550
135	14200	682	69.388	.0480	.550
136	14100	683	69.534	.0481	.550
137	14000	684	69.681	.0483	.550
138	13900	684	69.827	.0484	.550
139	13800	685	69.973	.0486	.550
140	13700	685	70.119	.0487	.550
141	13600	686	70.264	.0489	.550
142	13500	687	70.410	.0490	.550
143	13400	687	70.555	.0492	.550
144	13300	688	70.701	.0494	.550
145	13200	688	70.846	.0495	.550
146	13100	689	70.991	.0497	.550
147	13000	689	71.136	.0498	.550
148	12900	690	71.281	.0500	.550
149	12800	690	71.426	.0501	.550
150	12700	691	71.571	.0503	.550
151	12600	691	71.715	.0505	.550
152	12500	691	71.860	.0506	.550
153	12400	692	72.004	.0508	.550
154	12300	692	72.149	.0509	.550
155	12200	692	72.293	.0511	.550
156	12100	693	72.437	.0513	.550
157	12000	693	72.581	.0514	.550
158	11900	693	72.726	.0516	.550
159	11800	694	72.870	.0517	.550
160	11700	694	73.014	.0519	.550

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
161	11600	694	73.158	.0521	.550
162	11500	694	73.302	.0522	.550
163	11400	695	73.446	.0524	.550
164	11300	695	73.589	.0526	.550
165	11200	695	73.733	.0527	.550
166	11100	695	73.877	.0529	.550
167	11000	695	74.021	.0531	.550
168	10900	695	74.165	.0532	.550
169	10800	695	74.308	.0534	.550
170	10700	695	74.452	.0536	.550
171	10600	696	74.596	.0537	.550
172	10500	696	74.739	.0539	.550
173	10400	696	74.883	.0541	.550
174	10300	696	75.027	.0543	.550
175	10200	696	75.170	.0544	.550
176	10100	696	75.314	.0546	.550
177	10000	696	75.458	.0548	.550
178	9900	696	75.601	.0549	.550
179	9800	696	75.745	.0551	.550
180	9700	695	75.889	.0553	.550
181	9600	695	76.032	.0555	.550
182	9500	695	76.176	.0556	.550
183	9400	695	76.320	.0558	.550
184	9300	695	76.463	.0560	.550
185	9200	695	76.607	.0562	.550
186	9100	695	76.751	.0564	.550
187	9000	695	76.895	.0565	.550
188	8900	694	77.039	.0567	.550
189	8800	694	77.183	.0569	.550
190	8700	694	77.326	.0571	.550
191	8600	694	77.470	.0573	.550
192	8500	694	77.614	.0574	.550
193	8400	693	77.758	.0576	.550
194	8300	693	77.903	.0578	.550
195	8200	693	78.047	.0580	.550
196	8100	693	78.191	.0582	.550
197	8000	692	78.335	.0583	.550
198	7900	692	78.480	.0585	.550
199	7800	692	78.624	.0587	.550
200	7700	691	78.768	.0589	.550

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
201	7600	691	78.913	.0591	.550
202	7500	691	79.057	.0593	.550
203	7400	690	79.202	.0595	.550
204	7300	690	79.347	.0597	.550
205	7200	690	79.492	.0598	.550
206	7100	689	79.636	.0600	.550
207	7000	689	79.781	.0602	.550
208	6900	689	79.926	.0604	.550
209	6800	688	80.072	.0606	.550
210	6700	688	80.217	.0608	.550
211	6600	687	80.362	.0610	.550
212	6500	687	80.507	.0612	.550
213	6400	686	80.653	.0614	.550
214	6300	686	80.799	.0616	.550
215	6200	685	80.944	.0618	.550
216	6100	685	81.090	.0620	.550
217	6000	685	81.235	.0622	.550
218	5900	684	81.382	.0624	.550
219	5800	683	81.528	.0625	.550
220	5700	683	81.674	.0627	.550
221	5600	682	81.820	.0629	.550
222	5500	682	81.967	.0631	.550
223	5400	681	82.113	.0633	.550
224	5300	681	82.260	.0635	.550
225	5200	680	82.407	.0637	.550
226	5100	680	82.554	.0639	.550
227	5000	679	82.701	.0641	.550
228	4900	679	82.848	.0644	.550
229	4800	678	82.995	.0646	.550
230	4700	677	83.143	.0648	.550
231	4600	677	83.290	.0650	.550
232	4500	676	83.438	.0652	.550
233	4400	675	83.586	.0654	.550
234	4300	675	83.734	.0656	.550
235	4200	674	83.882	.0658	.550
236	4100	674	84.030	.0660	.550
237	4000	673	84.178	.0662	.550
238	3900	672	84.327	.0664	.550
239	3800	672	84.475	.0666	.550
240	3700	671	84.624	.0668	.550

PROJECTILE-XP101 MUZZLE VELOCITY-1800(F/S)

COUNT	ALTITUDE (FT)	VELOCITY (F/S)	TIME OF FLIGHT (SECS)	AIR DENSITY (LBS/FT3)	DRAG COEFFICIENT
241	3600	670	84.773	.0671	.550
242	3500	670	84.922	.0673	.550
243	3400	669	85.071	.0675	.550
244	3300	668	85.221	.0677	.550
245	3200	667	85.370	.0679	.550
246	3100	667	85.520	.0681	.550
247	3000	666	85.670	.0683	.550
248	2900	665	85.820	.0686	.550
249	2800	665	85.970	.0688	.550
250	2700	664	86.121	.0690	.550
251	2600	663	86.271	.0692	.550
252	2500	662	86.422	.0694	.550
253	2400	661	86.573	.0696	.550
254	2300	661	86.724	.0699	.550
255	2200	660	86.875	.0701	.550
256	2100	659	87.026	.0703	.550
257	2000	658	87.178	.0705	.550
258	1900	658	87.330	.0708	.550
259	1800	657	87.482	.0710	.550
260	1700	656	87.634	.0712	.550
261	1600	655	87.786	.0714	.550
262	1500	654	87.939	.0717	.550
263	1400	654	88.091	.0719	.550
264	1300	653	88.244	.0721	.550
265	1200	652	88.397	.0723	.550
266	1100	651	88.551	.0726	.550
267	1000	650	88.704	.0728	.550
268	900	649	88.858	.0730	.550
269	800	649	89.012	.0733	.550
270	700	648	89.166	.0735	.550
271	600	647	89.320	.0737	.550
272	500	646	89.474	.0740	.550
273	400	645	89.629	.0742	.550
274	300	644	89.784	.0744	.550
275	200	644	89.939	.0747	.550
276	100	643	90.094	.0749	.550
277	0	642	90.250	.0751	.550

APPENDIX B. FORTRAN LISTING OF VERTICAL FLIGHT COMPUTER PROGRAM

```
.S FILEA,,STOP
-RLIB67-1C  07/24-08:44:43
           DIMENSION  ADRG(3), BDRG(7),UCHG(3), ITYPE(2)
           REAL KD,KOVC, MACHOL
      C
      C THIS PROGRAM CALCULATES PROJECTILE VELOCITY AND TIME OF FLIGHT AC
      C   OF RANGE FOR A PROJECTILE FIRED VERTICALLY UPWARDS
      C
           READ (5,106)  (ITYPE(K),K=1,2), WGT, DIAM, UMUZL
           READ (5,107)  (ADRG(K),BDRG(K),UCHG(K), K=1,3 )
      C
           DIAM=DIAM/12.
           UI1=UMUZL
           KUMUZL=UMUZL
      C
      C  PROGRAM CONSTANTS FOLLOW
      C
           G=32.2
           RHOGRD=0.07513
           H=0.0000316
      C
      C  INITIALIZE ALL VARIABLES
           DELZ=100.
           JUMP=0
           KOUNT=0
           NUMLIN=40
           Z=0
           T=0
      C
      C  CALCULATE MAXIMUM ALTITUDE (IN 100'S -FT) THAT PROJECTILE WILL T
      C       UPWARD. ASSUMES DRAG=0
      C
           ASTOP = (UMUZL**2)/(200.*G)
           ISTOP=ASTOP
      C
      C  DETERMINE WHICH SECTION OF SEPARATE THREE LINEAR FITS TO DRAG EQ
      C       APPLIES
      C
           DO 8 K=1,3
           IF(UMUZL .GT. UCHG(K)  ) GO TO 9
         8 CONTINUE
      C
         9 CONTINUE
           JDRG = K
      C
      C  CALCULATE TIME AND VELOCITY AT POINT I USING (I-1) VALUES FOR UP
      C  DO 10 LOOP VELOCITY,ALTITUDE AND TIME OF FLIGHT FOR MOTION UPWAR
      C  IF VELOCITY IS LESS THAN ZERO APEX HAS BEEN REACHED
      C
```

```
C
      DO 10 I=1,TSTOP
      UOLD=UI1
      RHO=RHOSRD*EXP(-H*Z)
      IF(UI1 .LE. UCHC(JDRC)) JDRG=JDRG + 1
      MACHOL= UOLD/1100.
      KD=ADRC(JDRG) + BDRG(JDRG)*MACHOL
      CONST=RHO *DIAM**2 * KD/WGT
      E=EXP(-2.*CONST*DELZ)
      COVK= G/CONST
      DENOM=SQRT(G*CONST)
      KOVG =SQRT(CONST/G)
      IF(KOUNT .GE. NUMLIN )  KOUNT=0
      IF(KOUNT .EQ. 0) WRITE(6,101) (ITYPE(K),K=1,2), KUMUZL
      KOUNT=KOUNT +1
      KUI1=UI1
      KZ=Z
      WRITE (6,102) I,KZ, KUI1,T,RHO,KD
      JUMP=JUMP + 1
      IF(JUMP .NE. 10) GO TO 15
      WRITE (6,104)
      JUMP=0
   15 CONTINUE
      CHECK = (UOLD**2 + COVK)*E-COVK
      IF (CHECK .LT. 0) GO TO 20
      UI1= SQRT ((UOLD**2 + COVK )*E - COVK )
C
C   NOW THE TIME OF FLIGHT CALCULATIONS FOLLOW
C
      DELT=(ATAN(KOVG*UOLD) - ATAN(KOVG*UI1))/DENOM
      T=T+DELT
      Z=Z+DELZ
   10 CONTINUE
C
C
C   NEXT CALCULATE TIME AND VELOCITY ON DECENDING FLIGHT
C
   20 CONTINUE
      JUMP=0
      KD=0.55
      UI1=0.0
      KSTOP=I
      KOUNT=0
C
```

```
C
      DO 30 I=1,KSTOP
      UOLD=UI1
      RHO=RHOGRD * EXP(-H*Z)
      CONST=RHO * DIAM**2 * KD/WGT
      E= EXP(-2.*CONST*DELZ)
      GOVK =G/CONST
      GOVKHF=SQRT(GOVK)
      DENOM= 2. * SQRT(G *CONST)
      IF(KOUNT .GE. NUMLIN )  KOUNT=0
      IF(KOUNT .EQ. 0) WRITE(6,101) (ITYPE(K),K=1,2), KUMUZL
      KOUNT=KOUNT +1
      KUI1=UI1
      KZ=Z
      WRITE (6,102) I,KZ, KUI1,T,RHO,KD
      JUMP=JUMP + 1
      IF(JUMP .NE. 10) GO TO 35
      WRITE (6,104)
      JUMP=0
   35 CONTINUE
      UI1=SQRT ((UOLD**2 - GOVK )*E + GOVK )
      IF(UOLD .NE. 0 ) DELT=DELZ/UOLD
      FIRST = (GOVKHF + UI1)/ (GOVKHF - UI1)
      SECOND= (GOVKHF + UOLD)/(GOVKHF - UOLD)
      IF (FIRST .GT. 0 .AND. SECOND .GT. 0)
     1DELT= (ALOG(FIRST) - ALOG(SECOND))/DENOM
      T= T +DELT
      Z=Z - DELZ
      IF (Z .LT. 0 ) GO TO 40
   30 CONTINUE
C
   40 CONTINUE
C
C********          *        *  FORMATS ******
C
  101 FORMAT ( 1H1,     19X, ' PROJECTILE-', A2,I3, 12X, ' MUZZLE VEL
     1Y-', I4,'(F/S),', ///
     2    20X, ' ALTITUDE', 2X, ' VELOCITY', 2X,'   TIME OF', 5X, '
     3DENSITY', 4X, ' DRAG', /,  13X, ' COUNT', 3X,
     4       ' (FT)', 6X, ' (F/S)', 3X, ' FLIGHT (SECS)', 2X,' (LBS/F
     5   ,  2X, ' COEFFICIENT', / )
C
C                    COUNT     ALT      VEL       TIME      DENSITY   DRAG
  102 FORMAT (  14X, I4, 3X, I6, 7X, I4, 5X, F8.3, 5X, F9.4, 8X,F5.3
C
  104 FORMAT ( /  )
C
  106 FORMAT (5X, A2,I3,3F10.0  )
C
  107 FORMAT ( 3( 2F10.0 , F5.0 ) )
C
C
      END
```