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Calculation of the Response  
of a Transport Aircraft to  
Continuous Turbulence and  
Discrete Gusts and a Comparison  
with Flight Measurements

*by*

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CALCULATION OF **THE** RESPONSE OF A **TRANSPORT AIRCRAFT** TO  
CONTINUOUS **TURBULENCE AND** DISCRETE GUSTS AND  
COMPARISON **WITH FLIGHT MEASUREMENTS**

by

C. G. B. Mitchell

SUMMARY

The symmetric response of a tri-jet transport aircraft to continuous atmospheric turbulence and to discrete **ramp** gusts has been calculated and compared with the results of flight measurements. The aircraft was represented by two rigid and six elastic modes, and a lifting surface theory was used to calculate airforces. Cockpit and wingtip rms accelerations relative to the cg acceleration were overestimated by the calculations, but wing end tailplane rms bending moments per g agreed with measurements to better than **12%** accuracy.

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\* Replaces R.A.E. Technical Report **68083** - A.R.C.30407.

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1     **INTRODUCTION**

In the spring of 1966 a specially instrumented **HS 121** Trident 1 was flown in turbulence and continuous recordings made of a number of structural accelerations and bending strains'. These recordings were analysed by **Hawker** Siddeley Aviation Ltd. to provide the spectral density, rms and number of zero crossings ( $N_0$ ) for each of the response quantities over a frequency range of 0 **Hz** (c/s) to 12 **Hz**. This frequency range included, for symmetric motion, the short period oscillation and four elastic modes.

This Report describes calculations of the symmetric response of the Trident 1 to continuous atmospheric turbulence, and compares the calculated and measured values of both **accelerations** and loads. The aircraft is shown in **Fig.1**, and data on the **aircraft** and flight condition are given in Table 1. The positions of transducers that responded to symmetric motion and whose outputs were selected for analysis are shown in Fig.2. The calculations included the pitch end heave rigid body modes, and the first six calculated elastic normal modes, which **had natural** frequencies between 2.8 **Hz** and 15.5 **Hz**. Because the gust input **was** not measured, a spectrum shape for atmospheric turbulence has had to be assumed, and it has not been possible to compare absolute values of the measured response quantities with the calculated values.

There are relatively few published comparisons of this type. Ref.2 describes the comparison of symmetric accelerations and wing loads on a **swept-wing** bomber for the frequency range 0 **Hz** to 2 **Hz**, which included the short period mode and the first elastic mode. Ref.3 describes the comparison of both symmetric and antisymmetric accelerations on a fighter aircraft over the frequency range 0.45 **Hz** to 30 **Hz**, which included eight elastic modes. Other unpublished comparisons have been made for a number of aircraft, including the Trident, by their **respective** manufacturers.

This present Report extends the calculation of loads to higher **frequencies** than Ref.2, and includes the calculation of a tail load as well as wing loads. Calculations are also made for the response to **discrete** ramp gusts, to determine how this compares with the response to continuous turbulence.

2     **CALCULATION OF THE RESPONSE TO TURBULENCE**

The method used to calculate the response of the aircraft to symmetric turbulence was similar to that of Ref.3. The equation of motion was derived from **Lagrange's** equation, and the axes used were those of most flutter

calculations. The origin of the axes coincides with the wing apex of the undisturbed aircraft **and** the axes translate without rotation along the undisturbed flight path at the steady flight speed. The aircraft was treated **as** a linear system, all displacements were assumed small, and the forward speed was assumed constant. Structural loads were calculated by summing the separate contributions from the airforces and inertia loads, as is described in Ref.4.

The infinity of degrees of freedom of the aircraft was approximately represented in the calculation by the rigid body modes heave and pitch, and the first six elastic normal modes calculated for the appropriate aircraft weight and cg position. The elastic mode shapes are shown in Fig.4; it was assumed that the structural damping **in** each mode was 0.02 of critical. **It is** believed that sufficient modes have been included to represent adequately both static **and** dynamic aeroelastic effects **in** the frequency range 0 Hz to 12 Hz, with the exception of resonances of the **control** surfaces against their jacks. The controls were taken to be fixed **and** control movements due to the **autopilot** were not included in the **analysis**. The autostabiliser consists of yaw and roll dampers and so did not influence the symmetric motion of the aircraft.

**Davies** lifting surface theory was used to calculate the airforces on the wing end **tail**<sup>5</sup>. The airforces were calculated at six frequencies on the two surfaces separately, and were added after the tailplane airforces due to the heave **and** pitch modes had been multiplied by  $(1 - \partial \epsilon / \partial \alpha)$  to allow for **downwash** from the wing modifying the flow at the tailplane. No allowance was made for the time taken for the **downwash** to convect from the wing to the tail.

The summed airforces at zero frequency were compared with wind tunnel measurements of the total lift **and** moment on the aircraft, both tail on and tail off, to assess the airforces due to the fuselage. It was found that the fuselage did not increase the lift curve slope but did move the aerodynamic centre forward by 0.11 smc (standard mean chord), and the calculated pitching moment on the aircraft was modified to fit the tunnel results. The calculated airforces due to the gust included penetration effects but did not allow for **any** variation of the gust velocity across the span of the aircraft.

The lifting surface computer programme used to calculate the airforces did not automatically yield the bending moments on the wing **and** tail. Therefore a special procedure was needed to find the aerodynamic contributions to these moments. This was done by including in the **airforce** calculations

synthetic modes that had displacements that were zero inboard of the line about which the moment acted, and increased linearly with distance normal to the line outboard from it. Because the computer programme uses polynomials to fit the mode shapes these synthetic modes, which contain **discontinuities** of slope, are not well represented in the calculation. In particular, over the inner portion of the wing where the modal displacements should be uniformly zero they are not. because of the **waviness** of the polynomial approximations to the modes. Thus when, **in the calculation** of the generalised forces, the integral over the whole wing of the product of the local pressure and displacement is evaluated it will **contain an** unwanted **contribution** from the inner wing. **This** will cause errors in the calculated generalised forces, particularly for moments at the more outboard **stations**.

The equation of motion for the modal response of the aircraft to continuous **harmonic** gusts was solved at approximately eighty values of the frequency parameter, and from these **solutions** the transfer **functions** for structural accelerations **and** loads derived. The spectral densities of the responses to continuous atmospheric turbulence were calculated assuming that the turbulence had a spectral **density**  $\Phi_{WW}(\Omega)$  given by

$$\Phi_{WW}(\Omega) = \frac{L}{\pi} \frac{1 + 8/3 (1.339 L \Omega)^2}{[1 + (1.339 L \Omega)^2]^{11/6}} \quad (1)$$

where  $\Omega$  is the wave number in **rad/ft**, and L is the scale length, which in this calculation was chosen as 2500 ft (762m). Further calculations were also made in which L was varied from 250 ft (76 m) to 5000 ft (1525 m). Spectra of various scale lengths are shown in Fig.5.

In addition to the spectral densities the values of the response **rms**,  $N_o$ , and "dynamic response factor" (defined as response **rms/rms** of the structural **acceleration** near the cg) were calculated for **comparison** with the **flight** measurements. The response rms for unit rms excitation is given the symbol A, so the dynamic response factor **can** be defined as  $A_{local}/A_{cg}$  structural **for accelerations** and  $A_{load}/A_{cg}$  structural **acceleration for loads**. A **and**  $N_o$  were obtained from the calculated spectral **densities** by integration over the frequency range 0.2 **Hz** to 12 **Hz**, assuming that the response and its first **derivative** were Independently random and had Gaussian pmbability distributions.

Because the measurements of wing bending moments in flight consisted of measuring the **ratio** of the **bending strains** per g **in** turbulence to that **in** slow

**manoeuvres**<sup>1</sup>, the low frequency response of the aircraft to control movements was calculated, and the results of this calculation used to predict the ratio of wing loads in gusts and manoeuvres. The **tailplane** bending moment per **g** in turbulence was calculated directly. The ratio of **rms** load per **rms g** in turbulence to the load per **g** in a slow manoeuvre is called the turbulence response factor.

Finally, calculations of the response of the aircraft to step gusts and to ramp gusts of lengths between 0 **ft** and 300 **ft** (91 m) were made, using the Fourier transform method of **Ref.4**. These are intended to check the method for the calculation of transient loads, and to show how the **dynamic** and turbulence response factors for discrete gusts and continuous turbulence compare.

### 3 FLIGHT MEASUREMENTS

For the flight measurements a series 1 Trident (**G-ARPB**) was used, as is fully described in **Ref.1**. The aircraft was fitted with twelve accelerometers, of which seven were sensitive to symmetric motion; the positions of these in the airframe are shown in Fig.2. Strain gauges were fitted to measure bending strains at three stations on the port wing, one on the starboard wing, and at the tailplane root. All these measurements were recorded as analogue signals on magnetic tape. Other recorder channels were used for control angles, other strain gauges, aircraft speed, aircraft attitude **and** angular rates, and auto-pilot monitoring signals. It has been assumed that the measured bending strains were directly proportional to the bending moments, whatever the load distribution.

The tailplane bending moment strain gages were calibrated in flight by applying known tail loads through movements of the flaps and spoilers. The wing bending moment gauges were also calibrated in flight, by manoeuvres in which the normal acceleration was increased and then decreased. These **manoeuvres** were made at a constant Mach number and various altitudes to find the variation of wing strain per **g** with dynamic pressure. Thus the wing structural loads were not measured as absolute quantities, but in terms of the strains per **g** in manoeuvres.

The aircraft was flown in turbulence on three flights. The measurements analysed were those made during runs **18, 19** and **20** of flight 738 (see **Ref.1**). All these runs were at approximately 15300 ft (4663 m) altitude and 268 kt (497 **km/hr**) **eas**. For **run** 18 the autopilot and dampers were engaged, for run **19** the dampers were engaged, and **for** run 20 neither the autopilot nor the



dampers were in use. The autopilot and dampers can be expected to modify the response of the aircraft at frequencies below 2 **Hz**.

For spectral analysis the measurements were digitised at 100 points per second and filtered digitally to reduce the effect of drift during digitising. This filter had a time constant of 3.2 seconds. For runs 18 and 20 the sample length was 35 seconds, while for run 19 it was 55 seconds. Spectra were calculated digitally through the autocorrelation function, using a maximum lag of 5 seconds. All values of the response rms and  $N_0$  quoted in this Report are obtained by integration of these spectra over the frequency range 0.2 **Hz** to 12 **Hz**. In Ref.1 the rms were obtained by the direct summation of the squares of the measurements, digitised at 5 points per second. Because of this the values of rms and  $N_0$  given here and in Ref.1 do differ by a few per cent.

To increase the statistical reliability of the measured spectra the author of this Report has averaged the spectral densities from runs 18, 19 and 20 for frequencies greater than 2.0 **Hz**, giving the spectra from each run equal weight. Below 2.0 **Hz** the spectra may be modified by the action of the autopilot, and possibly also the dampers, and the spectra from run 20 alone are used for comparison with the calculation. The spectra that result from this process were then smoothed by eye, ripples less than ~~±25%~~ of the local spectral density being considered insignificant. The smoothed and unsmoothed spectra were made to coincide at major peaks and troughs.

#### 4 RESULTS

Figs.5 and 6 show the calculated and measured spectra for structural accelerations. These are for symmetric motion only, with the exception of the measured spectrum for the tailplane tip acceleration, which contains both symmetric and antisymmetric motion. The calculated spectra are for a 1 **ft/s** tas (0.305 m/s) gust velocity, while the measured are for an unknown gust velocity, which can be deduced to have been about 7.5 **ft/s** (2.3 m/s) tas. This velocity has been derived by comparing the power at the short period peak of the measured and calculated responses, assuming the excitation spectrum has a scale length of 2500 **ft** (762 **m**). In Fig.5 the variation in height of the short period peak at 0.4 **Hz** with fore-end-aft position on the aircraft can be clearly seen. The variation is greater for the measured spectra than for the calculated spectra, showing that the aircraft pitches more at the short period frequency than the calculation predicts.

Fig.7 shows the calculated and measured spectra for wing bending moments, expressed in terms of the equivalent acceleration at the centre of gravity during a slow manoeuvre. It will be noticed that the experimental short period peak for the moment at rib 13 is not as high as the peaks for the moments at the wing root and at rib 8. The peak at 10.2 Hz on the experimental results is attributed by gawker Siddeley Aviation Ltd. to a mode involving predominantly symmetric rotation of the elevators.

Fig.8 shows the calculated and measured spectra for the bending moment at the tailplane root. Fig.9 plots the variation with position on the wing of the turbulence response factor and  $N_0$  for the wing bending moment. It can be seen that the calculation predicts a greater high frequency content at the outboard stations than occurs in practice.

Fig.10 shows how the overall rms and  $N_0$  of the accelerations at the cockpit, the structure near the cg, and the wingtip increase as the upper cut-off frequency for integration of the spectra is raised. Fig.11 shows how these same rms vary if the scale length of the turbulence exciting the aircraft, L in equation (I), ranges from 250 ft (76m) to 5000 ft (1525 m). The integration frequency range for Fig.11 is 0 Hz to 12 Hz.

Table 2 lists the calculated values of A and  $N_0$  for all the response quantities, for excitation by turbulence with a scale length of 2500 ft (762 m). Table 3 gives the calculated wing bending moments in a slow manoeuvre. Tables 4 and 5 compare the calculated and measured values of the dynamic and turbulence response factors and  $N_0$  for all the response quantities. Results from runs 19 and 20 are given to show the scatter between runs, on the assumption that the dampers did not affect the rms of the symmetric response quantities.

Fig.12 shows the calculated transient response of the aircraft to a 1 ft/s (0.305 m/s) step gust. From this the response to discrete gusts of any shape can be synthesised by superposition. In Fig.13 are given the peak responses to ramp gusts with a range of ramp lengths, and the variation with gust ramp length of the wing end tail bending moments per unit acceleration of the structure near the cg. It will be seen that the load per g is not very sensitive to the gust ramp length.

## 5 DISCUSSION

The measurements show that the aircraft pitches about 10% less at the short period frequency than is predicted by the calculation. Also, the

measured dynamic response factors for accelerations at the cockpit and wingtip are less than the calculated factors, **indicating** that, in addition to the **pitching discrepancy**, the **aircraft** does not vibrate structurally as much as the **calculation** would suggest. This latter result is **similar** to the results of **Ref.3**, although on the **fighter** aircraft of that study the effect was even more marked.

The turbulence response factors for the **wing** bending moments (Fig.9) agree with the measurements to **within 10%** at the root, **12%** at rib 8, and **6%** at rib 13, but the calculated and measured trends along the wing are rather **different**. The calculated **tailplane** root bending moment agrees with the measured moment per g to **9%**. The calculated and measured  $N_0$  agree reasonably well, the largest **difference** occurring in the results for the tailplane root **bending** moment, where the measured  $N_0$  is **18%** to **35%** larger than the calculated value. However, since the **elastic** modes **contribute** only slightly to the structural loads, this **aircraft** does not provide a very **rigorous** test of the **calculation**.

The turbulence response factors for the **wing** bending moments reflect at least two effects. The **first** of these is that in turbulence the wing vibrates at the frequency of the structural modes, and that for a given wing **lift** this generally **increases** the **bending** moment relative to that in a slow manoeuvre. The second is that, at the forward cg position used for the flight tests, in a manoeuvre the incremental **tail** load to cause a **positive** Incremental normal **acceleration** is downwards and is equal to **7%** of the Incremental wing lift, while on **entering** an up gust the tail load is upwards and is equal to **12%** of the wing lift. Therefore some **20%** more **wing lift** is required to cause a **given** normal **acceleration** Increment in a manoeuvre than in a gust. correspondingly, the turbulence response factor for **wing** bending moments, in the absence of structural **vibration**, would be about **0.83**.

The shapes of the calculated and measured response spectra agree well, except that the calculated structural resonance **frequencies** are a little lower (about **10%**) than the measured **frequencies**, and some discrepancies occur at the **higher** frequencies. The modes used for the calculation were calculated, and may have been based on stiffnesses a little lower than were achieved in **practice**.

The calculations of the response to discrete **ramp** gusts predict that the structural loads per unit acceleration of the structure near the cg do not vary much with gust **ramp** length. These calculations lead to response factors **for** wing loads that are larger than the factors from the spectral calculations by ~~1%~~ to ~~5%~~, and which differ from the measured factors by ~~7%~~ to ~~14%~~. The **tailplane** root bending moment calculated for discrete gusts is some ~~17%~~ lower than that calculated for continuous turbulence, and ~~7%~~ lower than the measured moment.

The quality of agreement noted here occurs when the calculations are compared with the quantities that were measured directly. On the wing the agreement for the turbulence response factors is better than that for the bending moments per g themselves, probably because the calculated **airforce** contributions to the moments are not very accurate, particularly at the more outboard stations.

The calculations described in this Report have been made using only information that would be available at the design stage of an aircraft. The results suggest that this standard of calculation overestimates the contribution of the elastic modes to the accelerations at the extremities of the aircraft by perhaps ~~20%~~, but that the turbulence response factors for the major structural loads can be calculated to an accuracy that is approaching that required for design purposes. It is likely that this accuracy, when the response calculations are used in conjunction with normal stressing methods to calculate the load distribution in **manoeuvres**, ~~is~~ as good as that of the flight measurements. It must be remembered, however, that the elastic modes are contributing little to the structural loads measured on this particular aircraft.

On the aircraft considered here the wing loads per unit acceleration of the structure near the cg predicted from discrete gust and continuous turbulence calculations are not appreciably different, and although the tail loads g differ by ~~17%~~ the measured tail load is almost mid-way between them. The discrete gust used in this Report is of ramp form, while civil aircraft are usually designed to requirements that define a gust of **(1 - cosine)** shape.

The author of this Report does not believe that the calculation techniques described here are any more sophisticated than those in use in the aircraft industry, so that the results given here indicate the confidence that can be placed on such calculations.

## 6 CONCLUSIONS

Calculations of the response of a transport aircraft to continuous turbulence overestimate the degree of excitation of the elastic modes, but predict the tail root bending moment, and the turbulence response factors for the major wing loads, to accuracies better than **12%**. Calculations of the response to discrete ramp gusts predict the tail root bending moment and the wing load response factors to the same accuracy as the spectral calculations.

The calculations described here are typical of those performed in the aircraft industry, and comparison of the calculated and measured responses of the aircraft to turbulence indicates that the accuracy of the calculation of major structural loads **is** approaching that required for design purposes. It **is** likely that the overall accuracies of the flight measurements **and** the calculations are similar. On the particular aircraft considered the elastic modes contributed only slightly to the structural loads.

## ACKNOWLEDGEMENTS

The author wishes to **thank Hawker Siddeley** Aviation Ltd., Hatfield, for supplying data that made these calculations possible, and for allowing the reproduction of flight test results that were additional to those reported in **Ref.1**.

The author would also like to thank British European Airways for permission to use the photograph printed as Fig.1 in this Report,

Table 1

AIRCRAFT PRINCIPAL DATA

Aircraft: **HSA 121** Trident series 1, **G-ARPB**

wing span	89.84 ft	27.38 m
length	114.75 ft	34.93 m
<b>standard mean</b> chord	15.13 ft	4.61 m
<b>wing</b> area	1358 ft <sup>2</sup>	126.2 m <sup>2</sup>
sweepback at $\frac{1}{4}$ chord	35"	35"

Flight 738

weight	90820 lb	41188 kg
<b>cg</b> position at	0.113 smc	0.113 smc
altitude	15300 ft	4663 m
speed (tas)	586 ft/sec	178 m/sec
Mach number	0.55	0.55
$\frac{\partial C_L}{\partial \alpha}$	4.94 per rad	4.94 per rad
$\frac{\rho V S \partial C_L / \partial \alpha}{2w}$	0.0323 g ft <sup>-1</sup> sec	0.1060 g m <sup>-1</sup> sec

Table 2

**CALCULATED SYMMETRIC RESPONSE m RANDOM TURBULENCE**

	A		N <sub>o</sub> Hz (c/s)
	per 1 m/sec gust	per 1 ft/sec gust	
accelerations (normal)			
centre of gravity			
structure near cg	0.0649 g	0.0193 g	1.75
cockpit	<b>0.0590 g</b>	0.0180 g	4.05
tailplane centreline	<b>0.0971 g</b>	0.0296 g	2.54
wing tip	0.3336 g	0.1017 g	4.49
tailplane tip	0.2201 g	0.0671 g	7.41
<u>bending moments</u>			
wing root rib	34930 Nm	7850 lb ft	1.34
wing rib 8	15600 Nm	3505 lb ft	<b>1.53</b>
wing rib 13	<b>3290 Nm</b>	739 lb ft	1.96
<b>tailplane</b> root	2034 Nm	457 lb ft	<b>1.90</b>

Table 3

**CALCULATED RESPONSE TO A SLOW HARMONIC ELEVATOR MOVEMENT**

	bending moment Nm/g	bending moment lb ft/g
wing root	616970	455000
rib 8	264419	195000
rib 13	53965	<b>39800</b>

Table 4

**CALCULATED AND MEASURED DYNAMIC AND  
 TURBULENCE RESPONSE FACTORS**

	calculated	measured (run 19)	measured (run 20)
<u>accelerations</u>			
cockpit*	0.91	0.70	0.77
tailplane centreline*	1.50	1.50	1.52
wing tip*	5.14	5.07	4.10
tailplane tip*	3.39	3.47 <sup>x</sup>	4.03 <sup>x</sup>
<u>bending moments</u>			
wing root rib**	0.87	0.96	0.95
wing rib 8**	0.91	1.03	1.01
wing rib 13**	0.93	0.87	0.89
tailplane root †	23100 lb ft/g (31310 Nm/g)	21300 lb ft/g (28880 Nm/g)	21200 lb ft/g (28743 Nm/g)

\*dynamic response factor defined as  $A_{local} / A_{structure}$  near cg

\*\*turbulence response factor defined as  $\frac{A_{moment}}{moment \text{ g due to manoeuvre}} / A_{structure}$  near cg

†dynamic response factor defined as  $A_{moment} / A_{structure}$  near cg

<sup>x</sup>includes antisymmetric motion



Table 5

**CALCULATED AND MEASURED FREQUENCIES OF ZERO CROSSINGS**

	calculated, Hz	measured (run <b>19</b> )	measured (run 20)
<u>accelerations</u>			
structure near cg	1.75	1.61	1.63
cockpit	4.05	4.08	4.45
<b>tailplane</b> centreline	2.54	2.80	3.20
wing tip	4.49	4.57	5.77
<b>tailplane</b> tip	7.41	<b>5.83*</b>	5.75'
<u>bending moments</u>			
wing root rib	1.34	1.46	1.37
wing rib 8	1.53	1.52	1.45
wing rib 13	<b>1.96</b>	1.65	<b>1.71</b>
<b>tailplane</b> root	<b>1.90</b>	2.25	2.56

\*includes **antisymmetric** motion

SYMBOLS

A	response rms/excitation rms
L	scale length of turbulence
$N_0$	frequency of zero crossings
W	vertical gust velocity
$\alpha$	incidence
$\phi_{ww}(\Omega)$	spectral density of the vertical component of atmospheric turbulence
$\Omega$	reduced frequency radians/foot
$\epsilon$	downwash angle at tailplane

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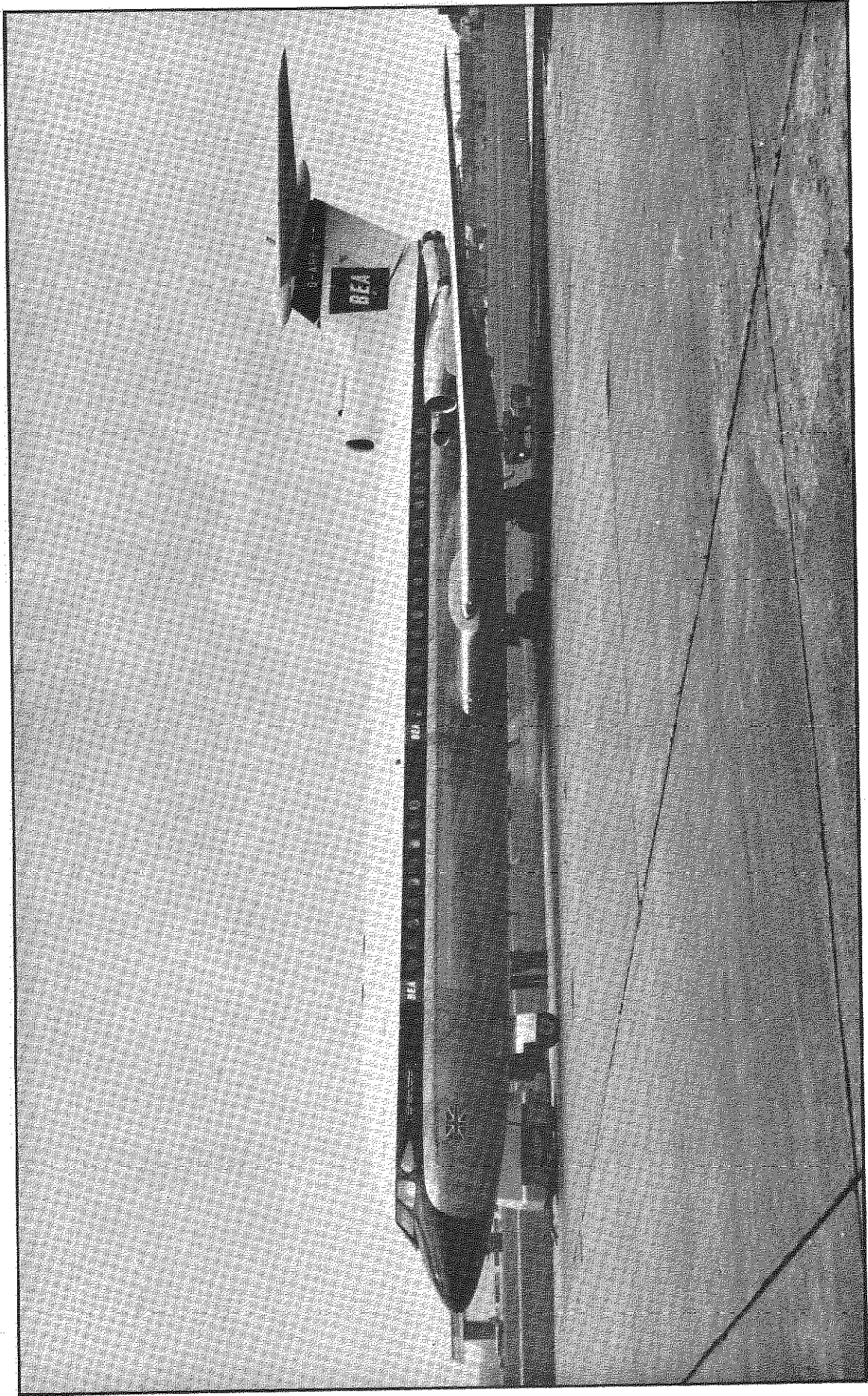


Fig.1. The est aircraft

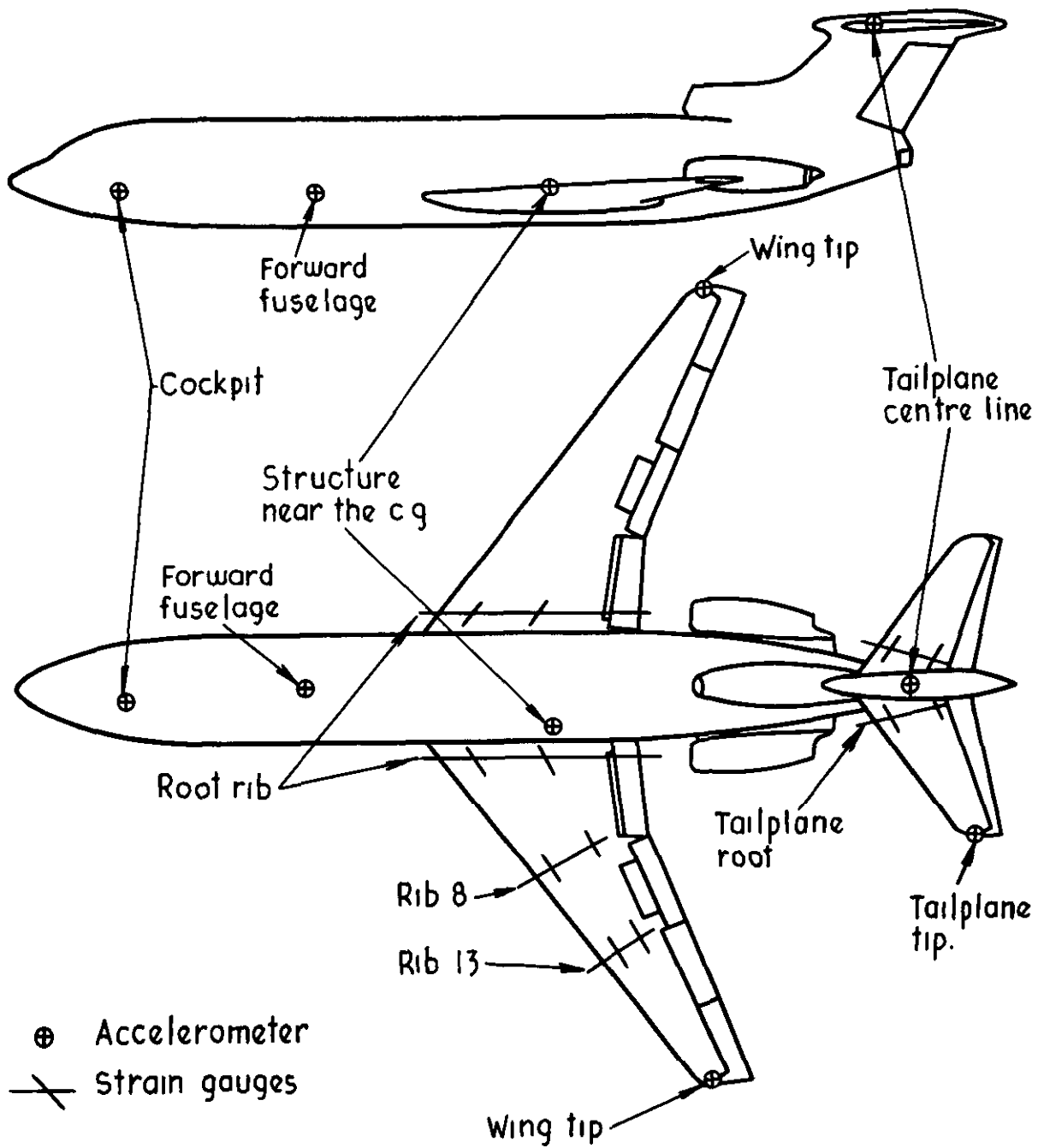
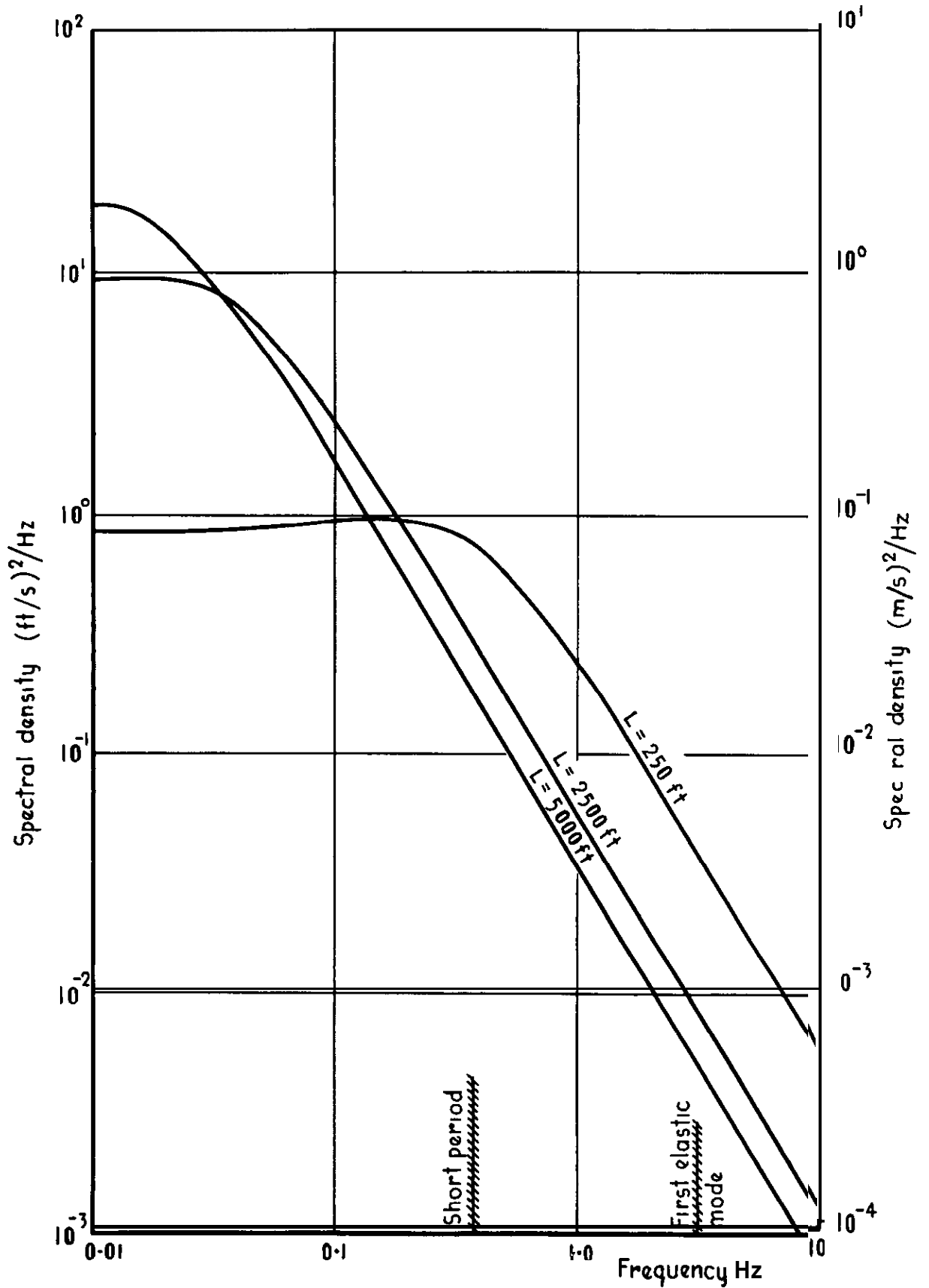
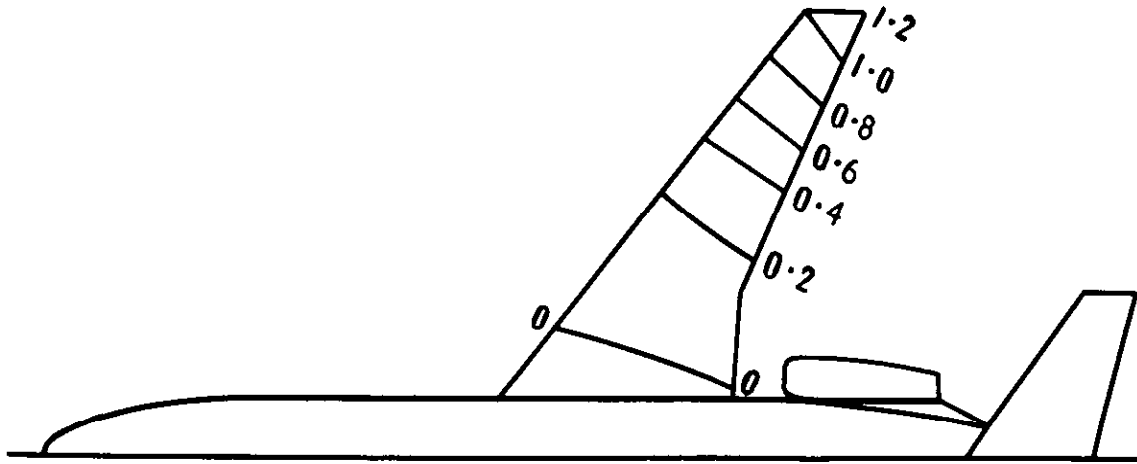


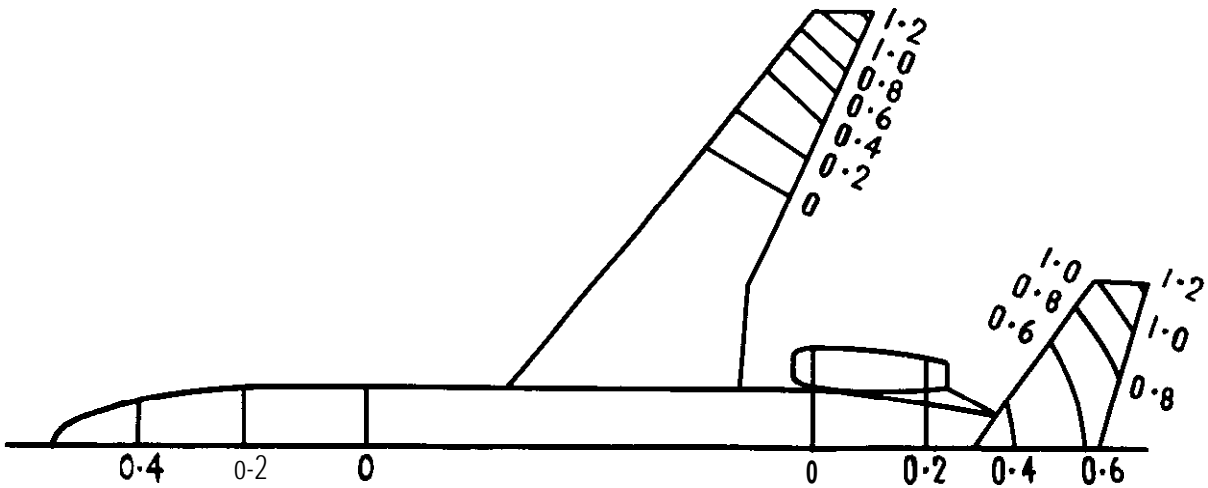
Fig. 2 Location of selected transducers.



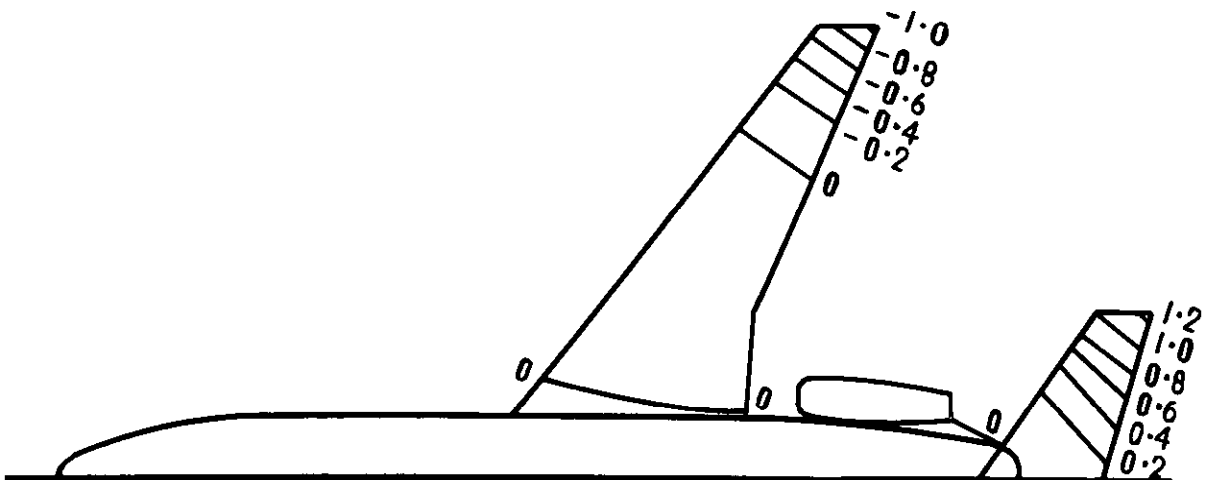
**Fig.3.** The von Kármán spectrum of turbulence for a flight speed of 586ft /sec. and unit gust velocity



Elastic mode 1 2.82 Hz



Elastic mode 2 4.85 Hz

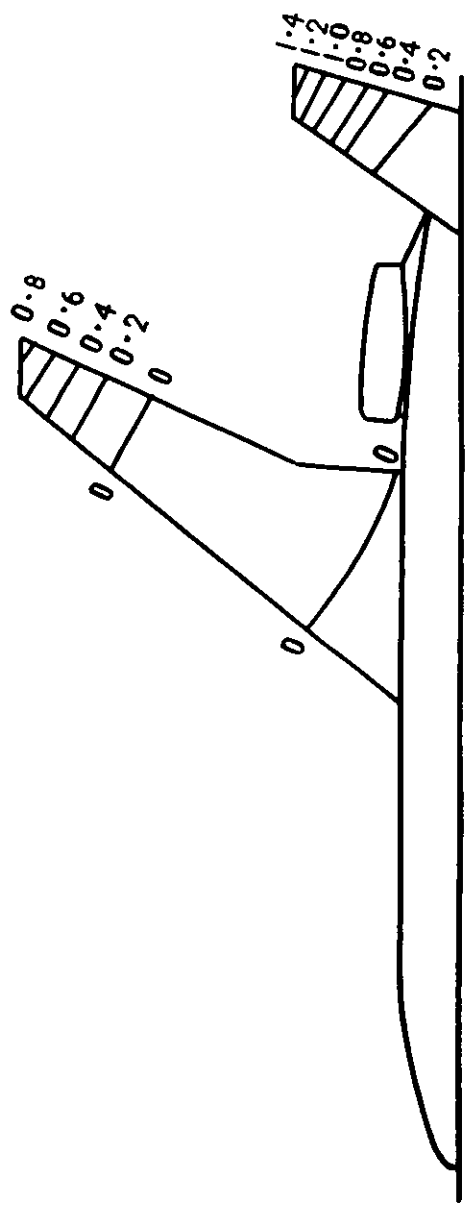


Elastic mode 3 7.69 Hz

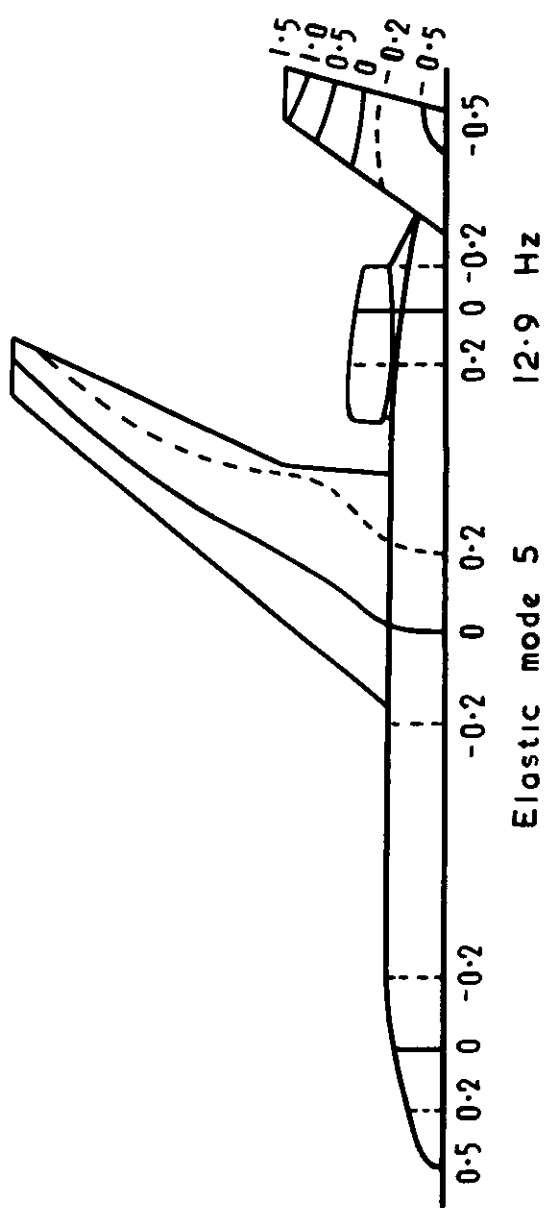
Fig. 4a Calculated normal modes.



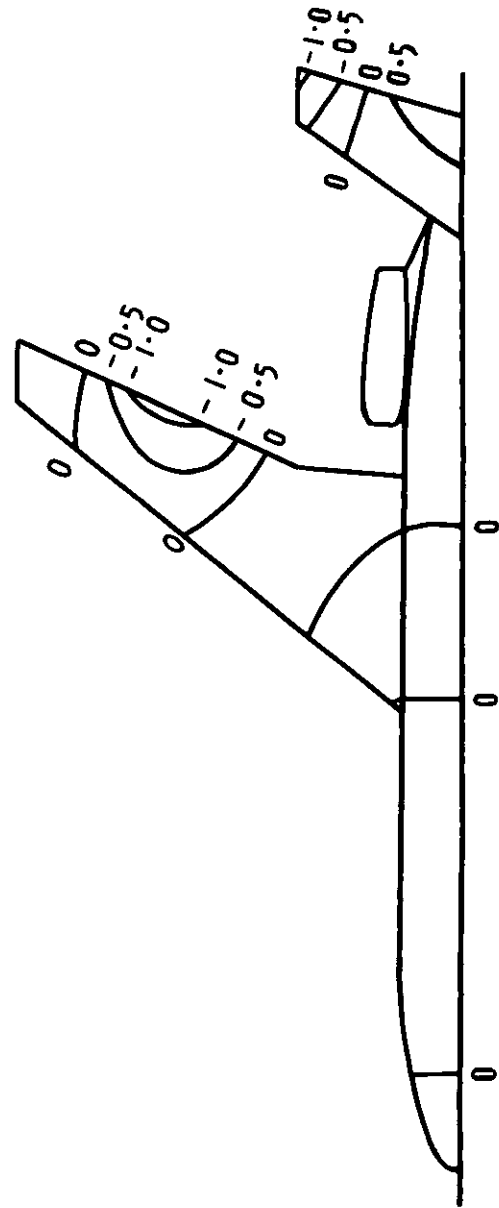
Fig 4 b



Elastic mode 4 8.10 Hz



Elastic mode 5 12.9 Hz



Elastic mode 6 15.5 Hz

Fig 4 b Calculated normal modes.

Fig 5a&b-2

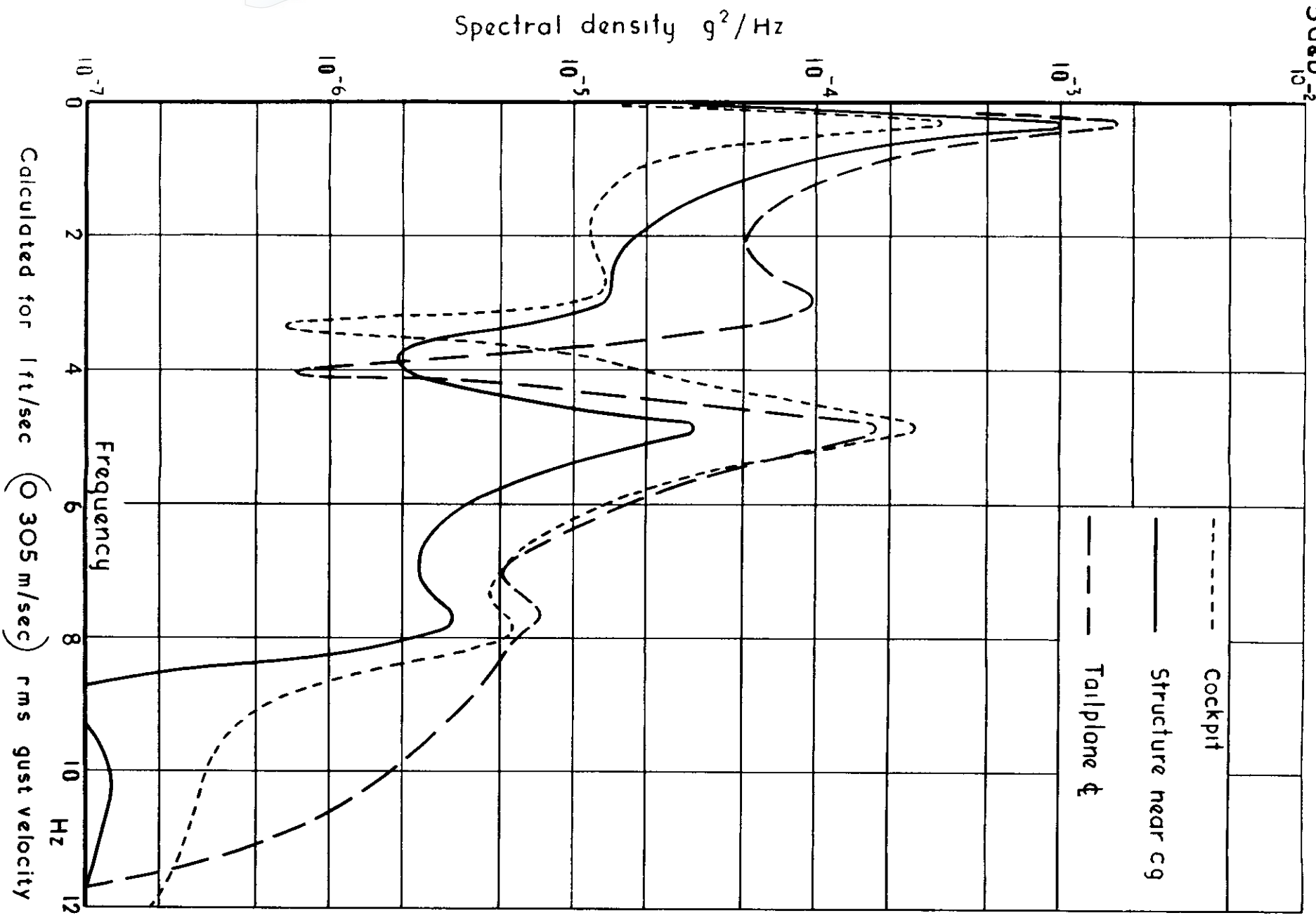


Fig 5a Calculated fuselage normal accelerations

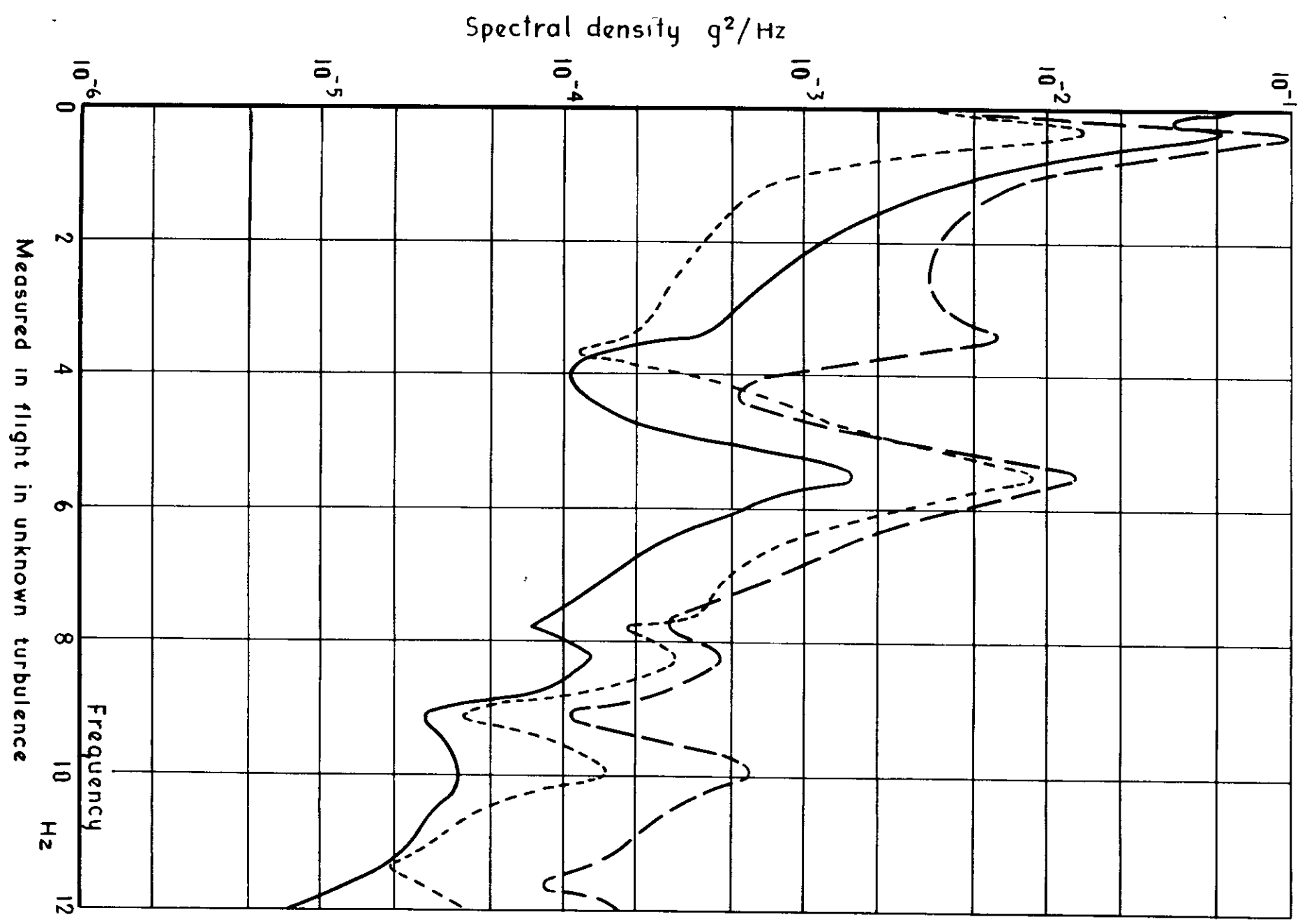
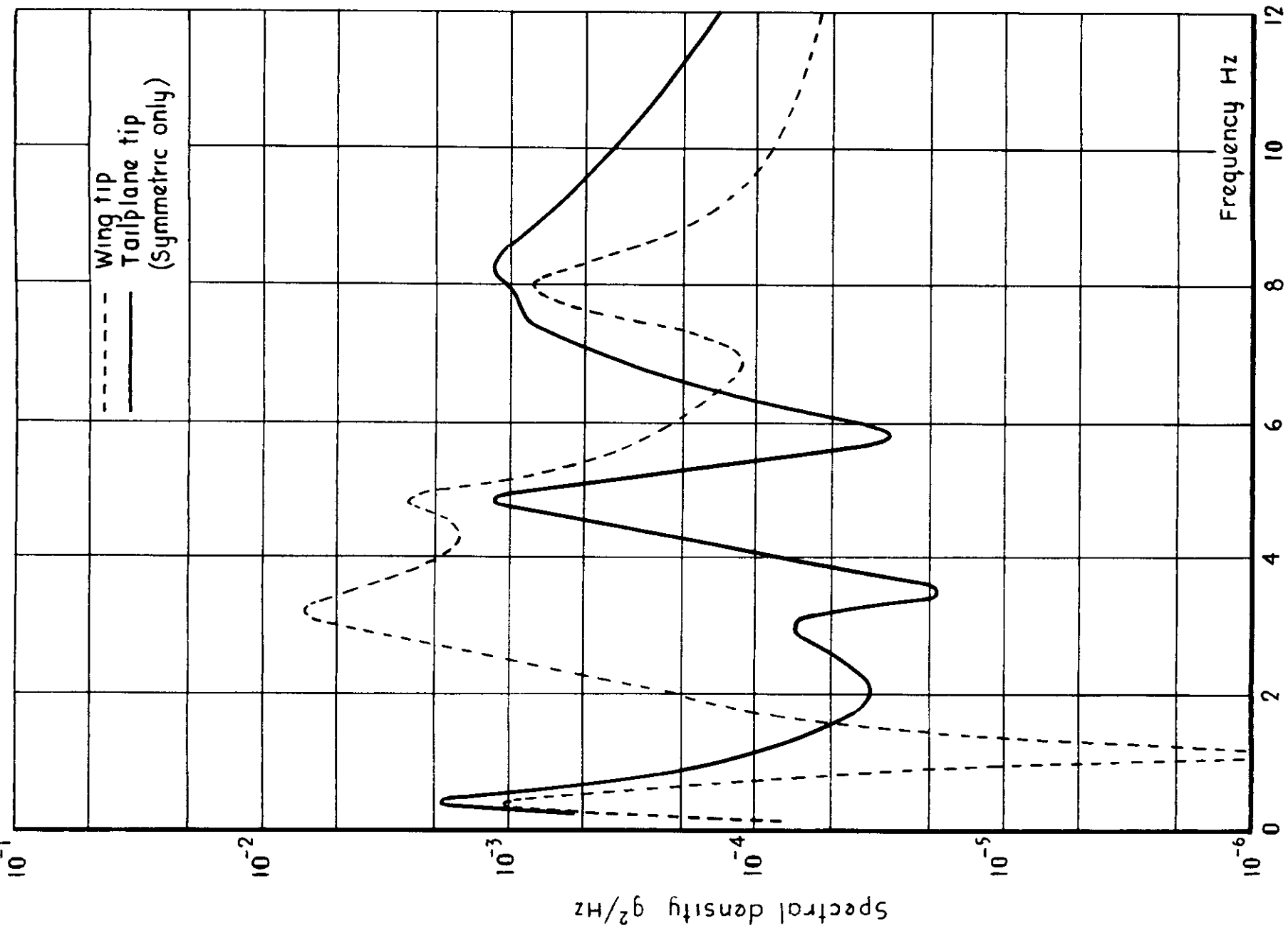


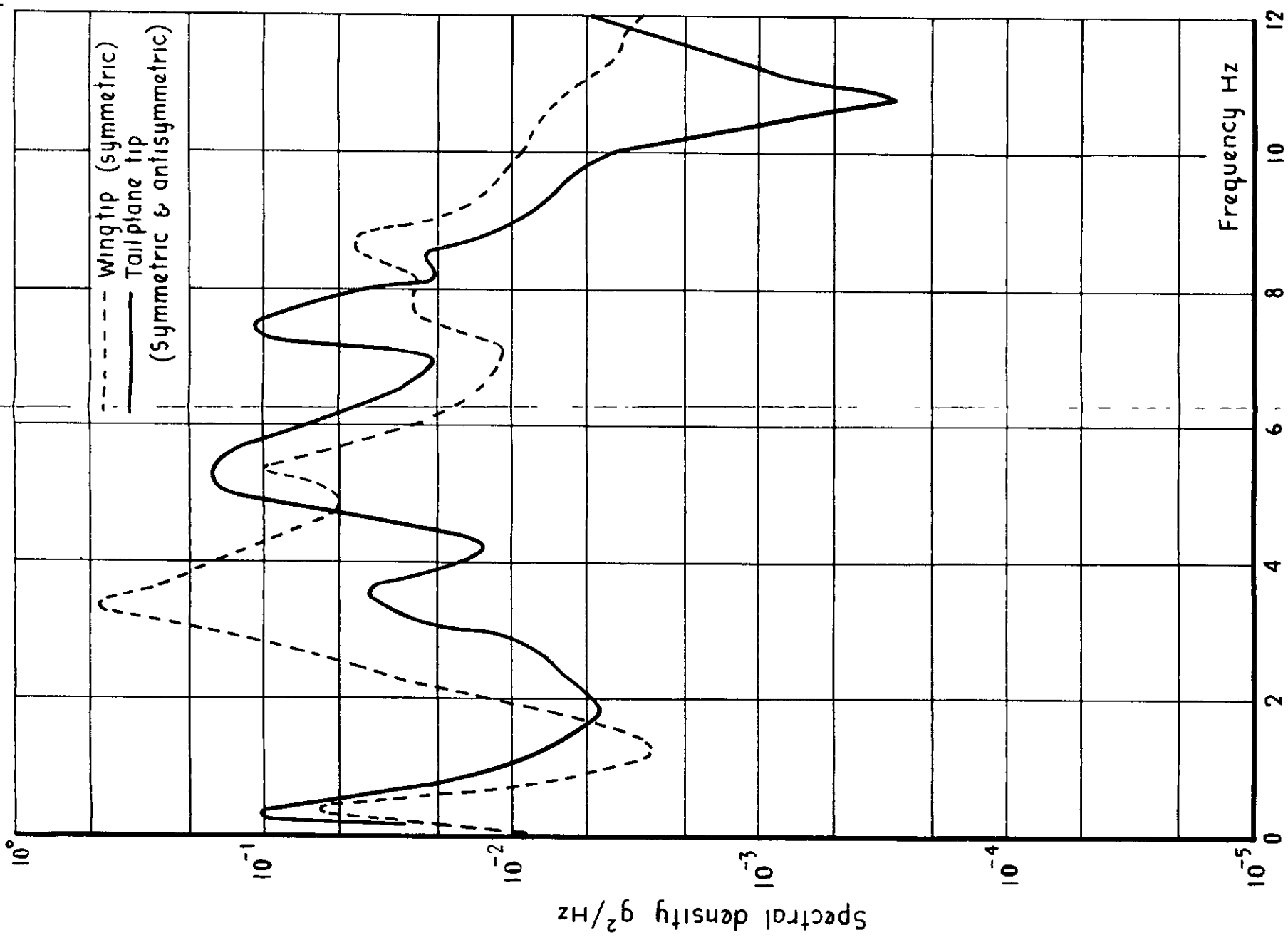
Fig. 5b Measured fuselage normal accelerations

Fig. 6a&b



Calculated for 1ft/sec (0.305 m/sec) rms gust velocity.

Fig 6a Calculated wing and tail tip normal accelerations.



Measured in flight in unknown turbulence

Fig 6b Measured wing and tail tip normal accelerations

Fig 7a&b-3

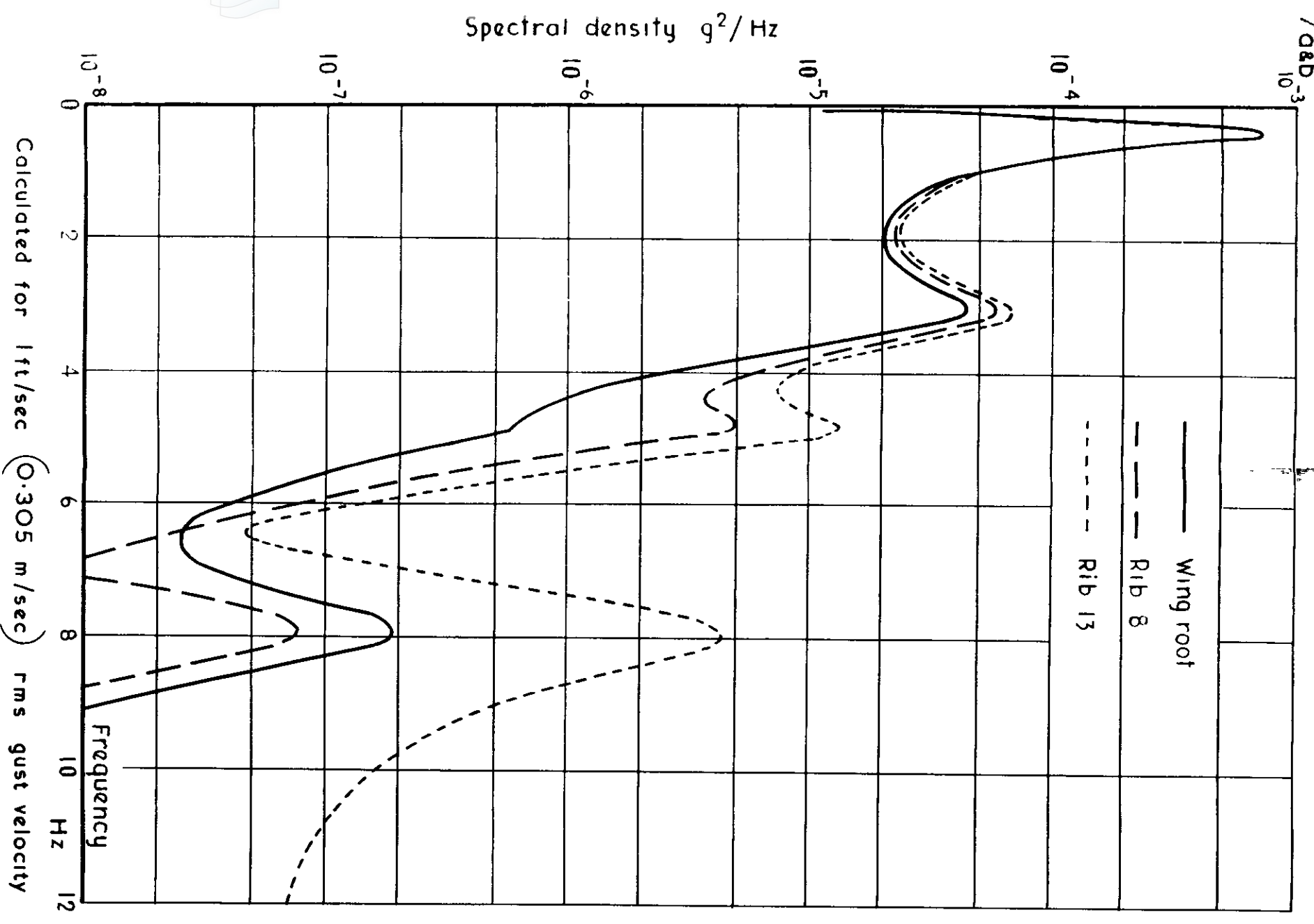


Fig 7a Calculated wing bending moments in equivalent g units

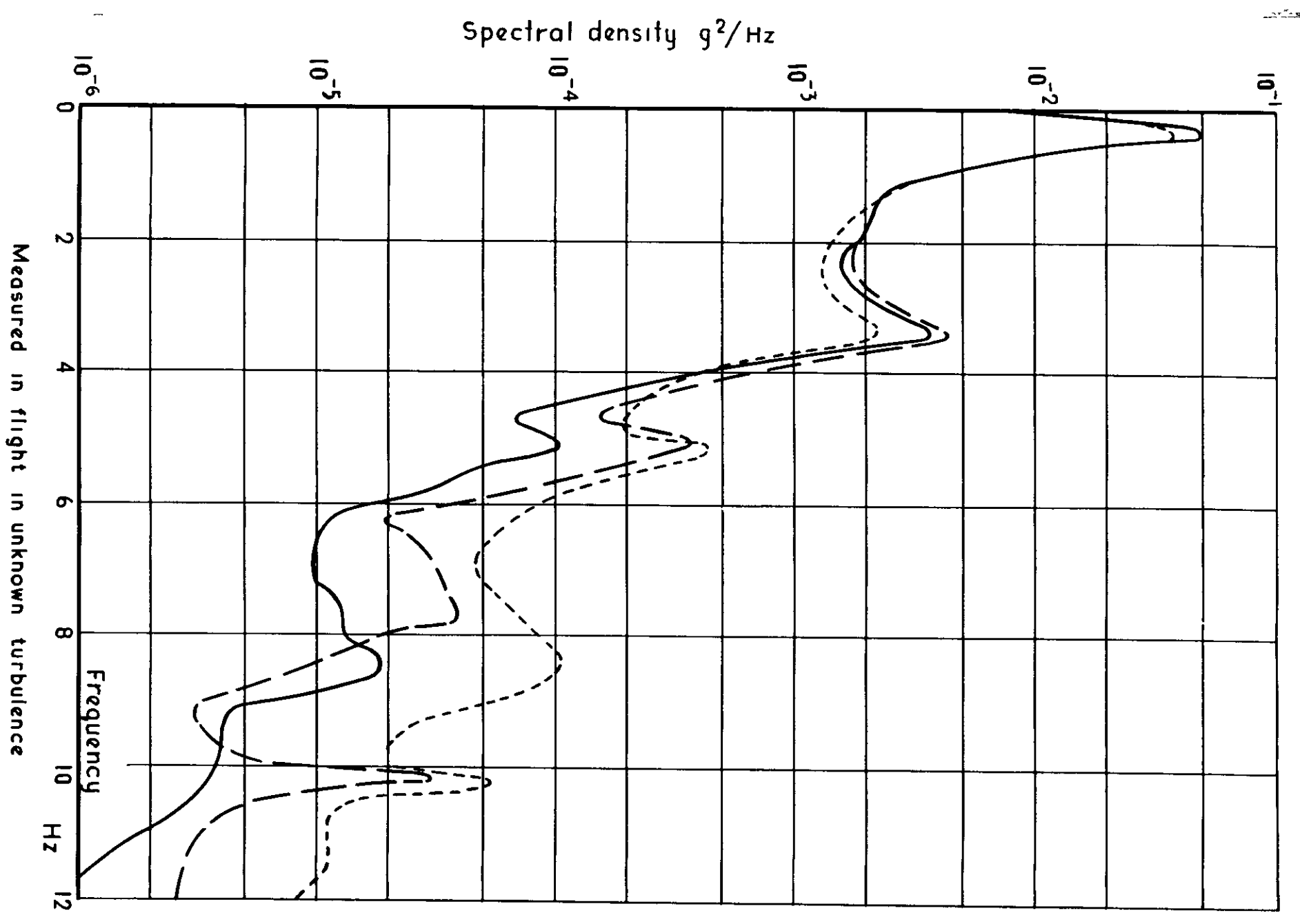


Fig 7b Measured wing bending moments in equivalent g units

Fig 8a&b

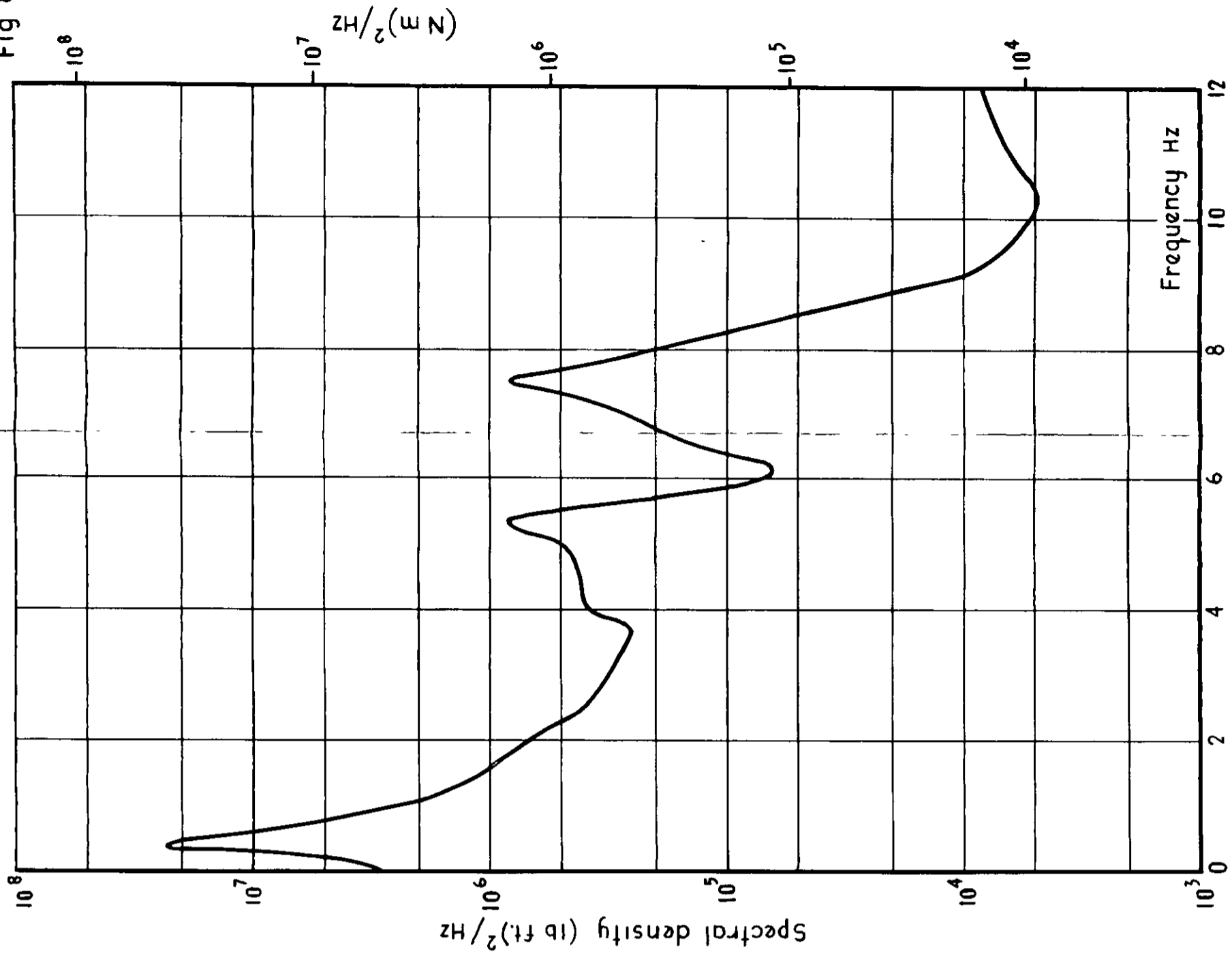


Fig 8b Measured tailplane bending moment

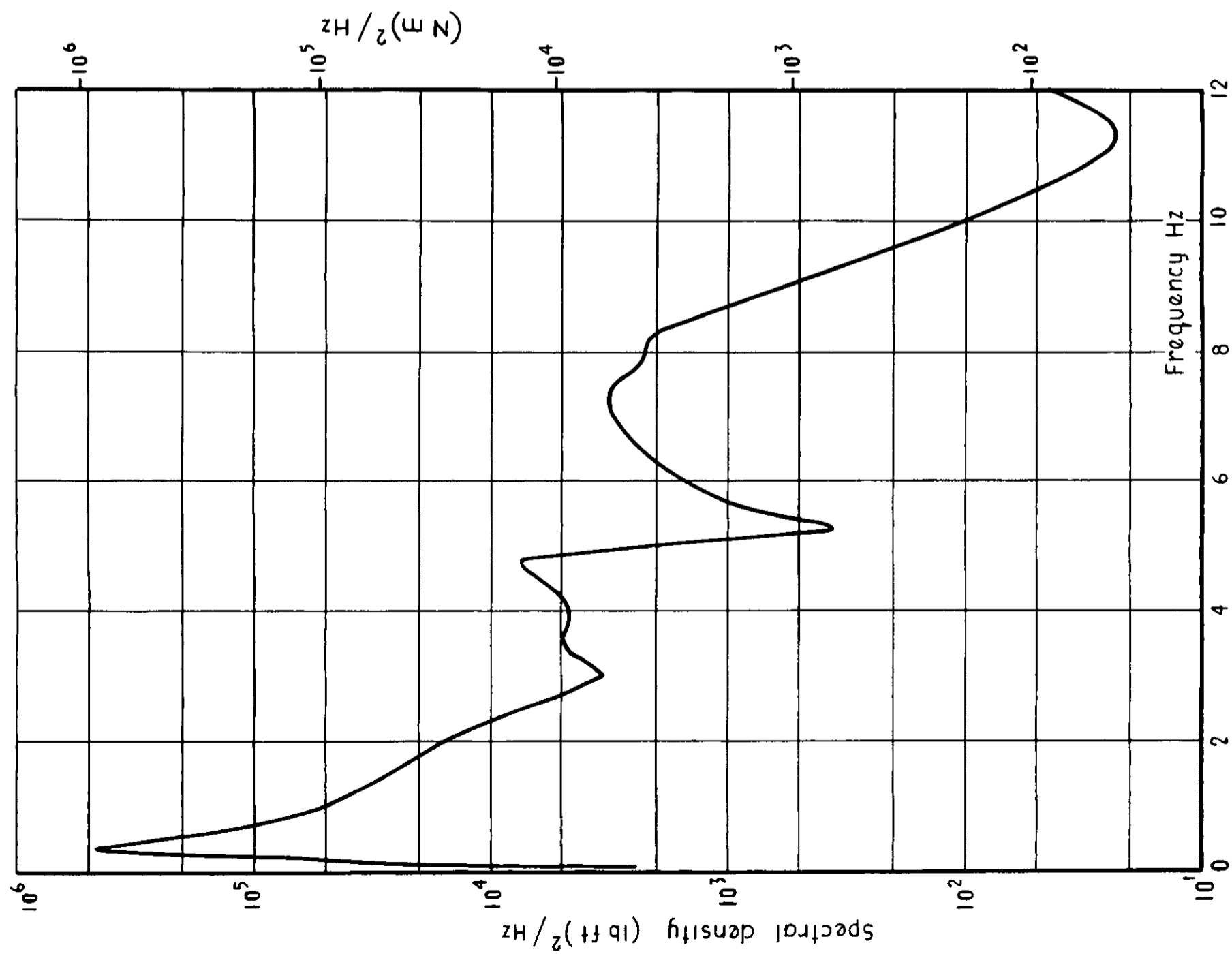


Fig 8a Calculated tailplane bending moment.

Fig. 9

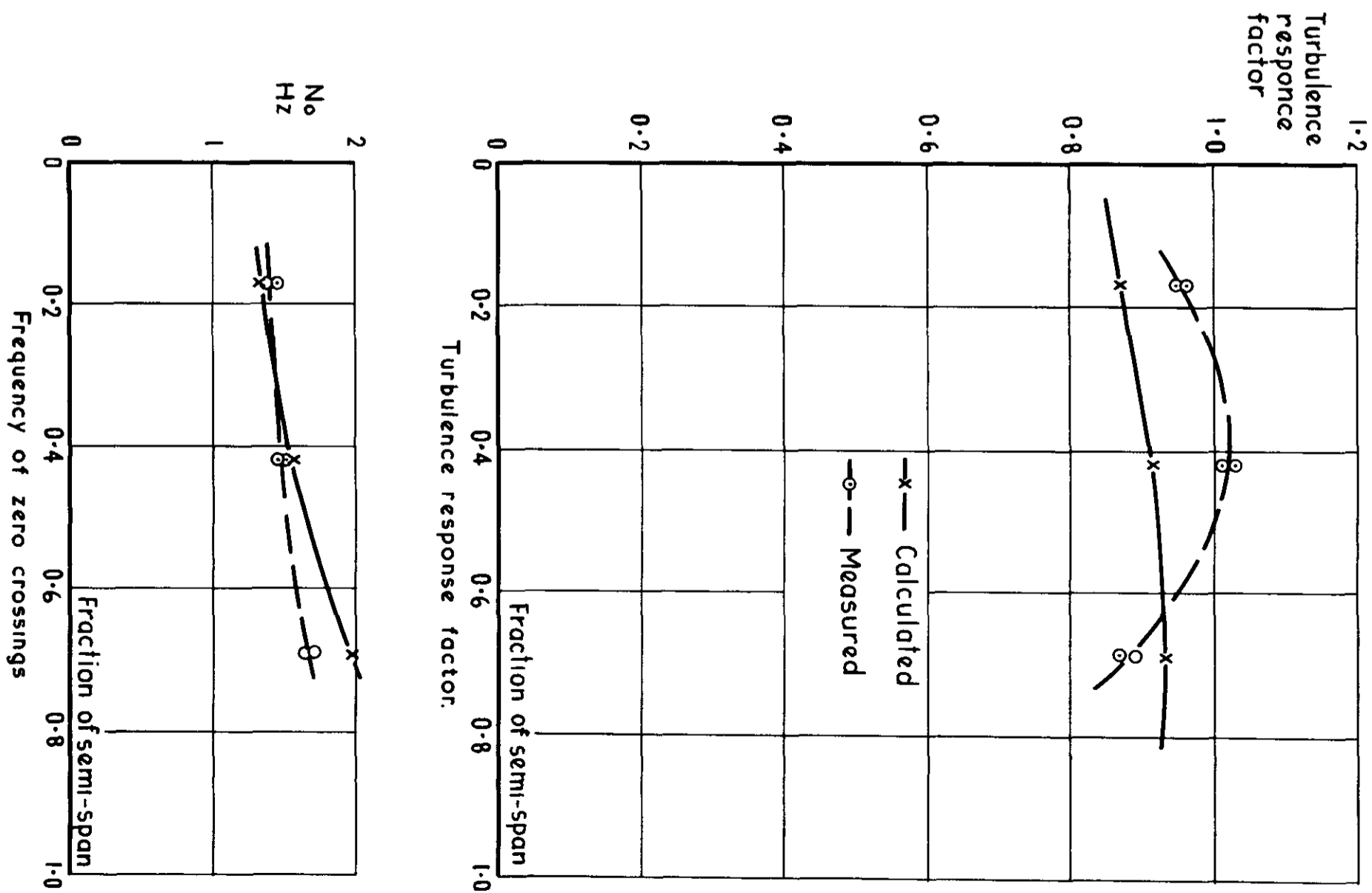


Fig. 9 Calculated and measured wing bending moments

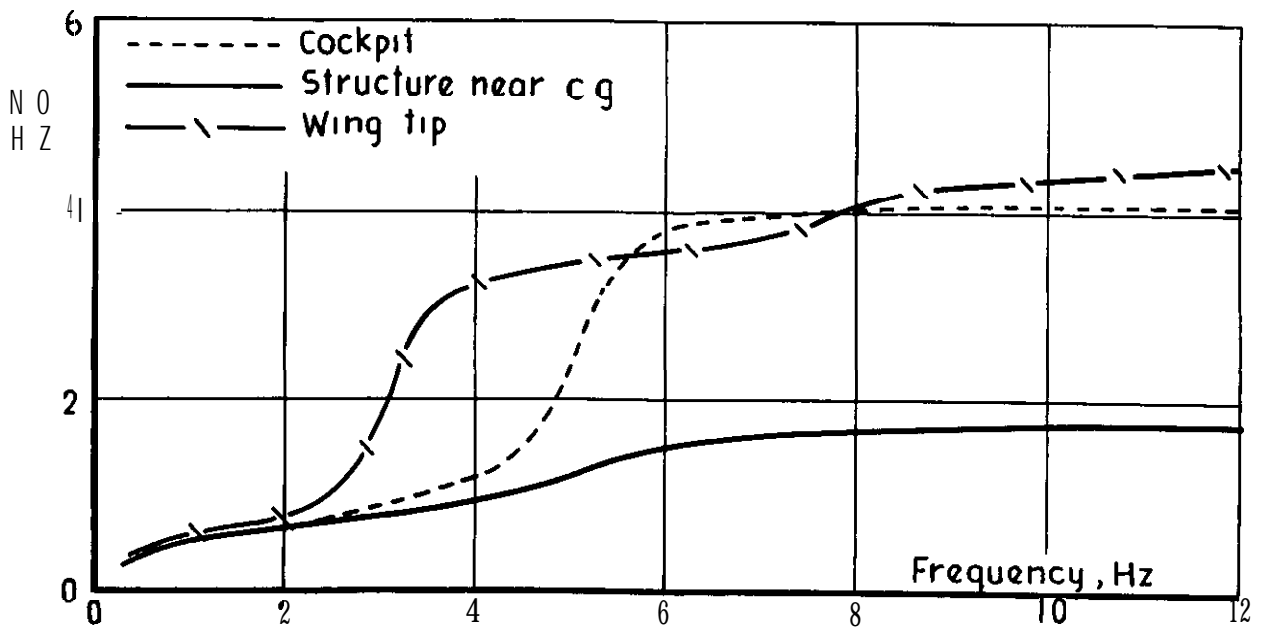
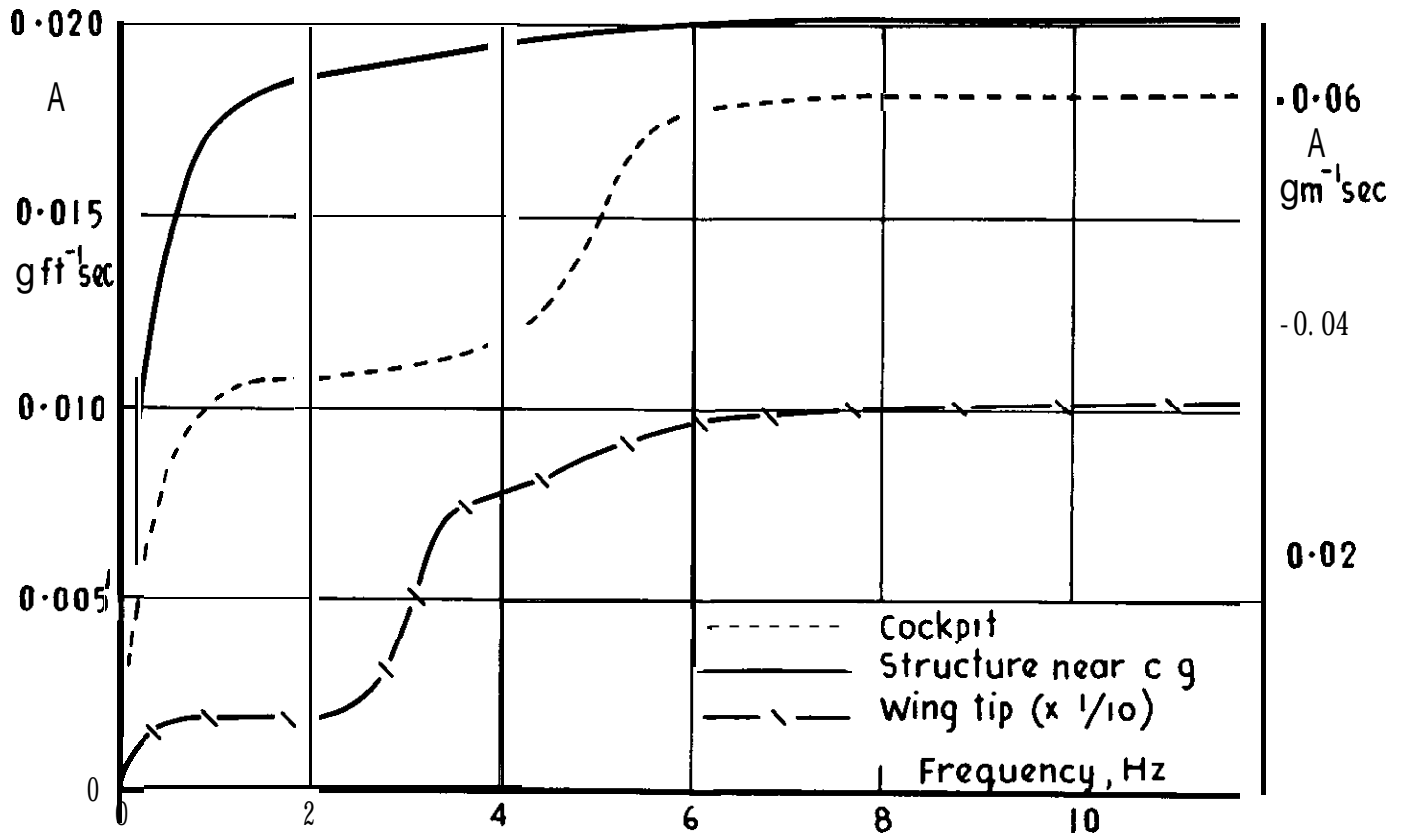


Fig. 10 Variation of  $A$  and  $N_0$  for structural accelerations with integration upper cutoff frequency.

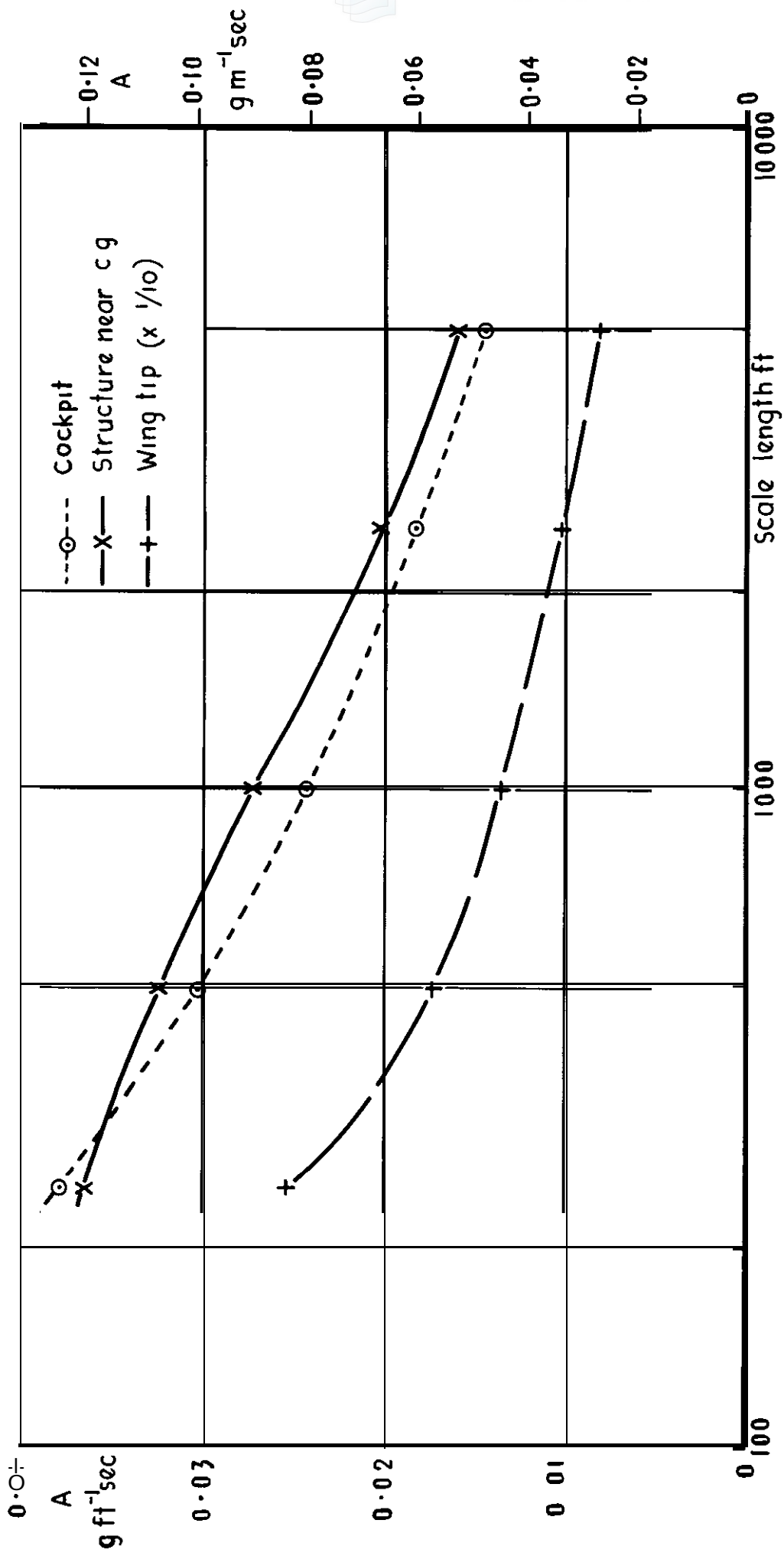
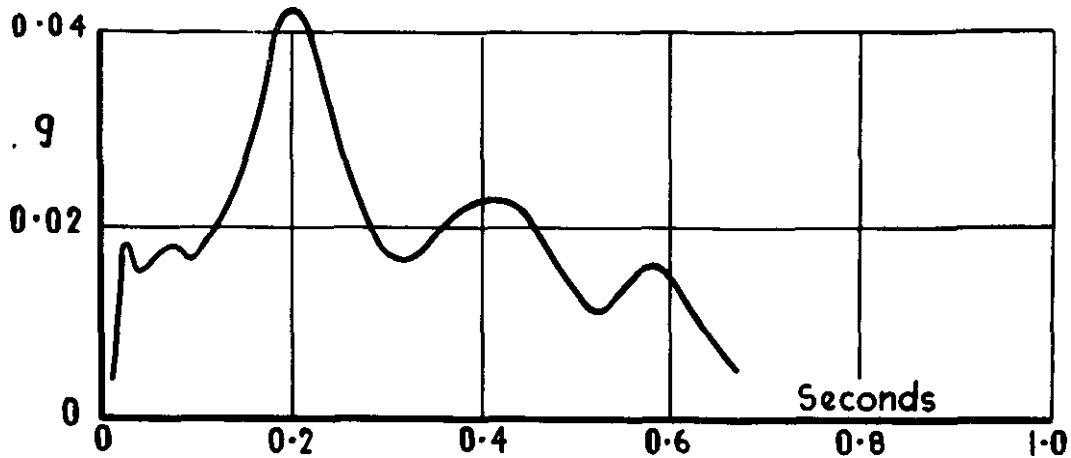
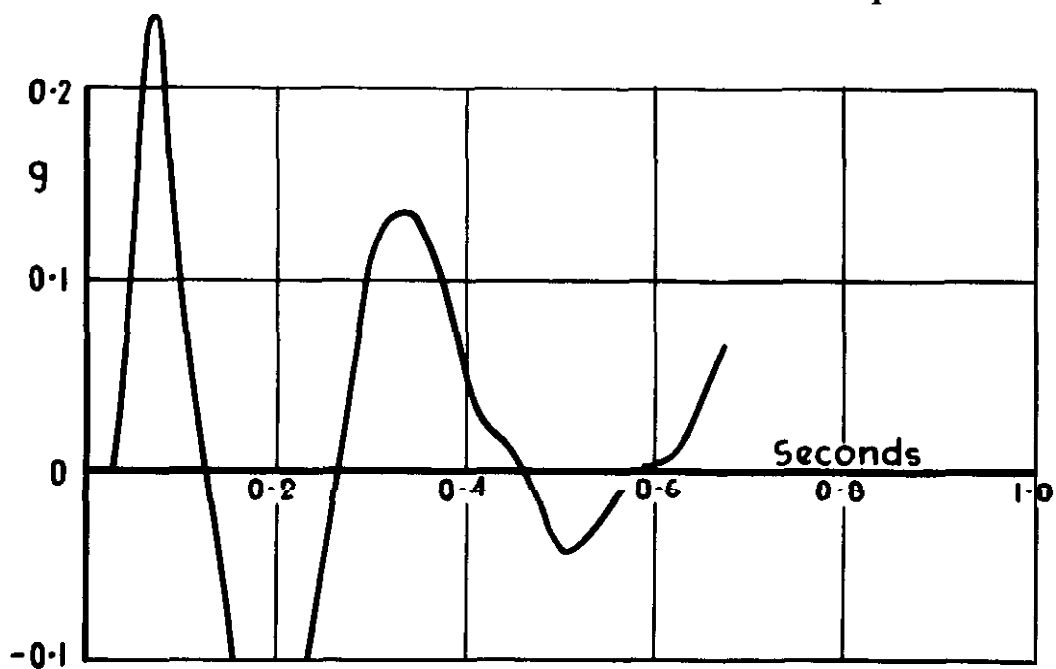


Fig. 11 Variation with turbulence scale length of A for structural accelerations.

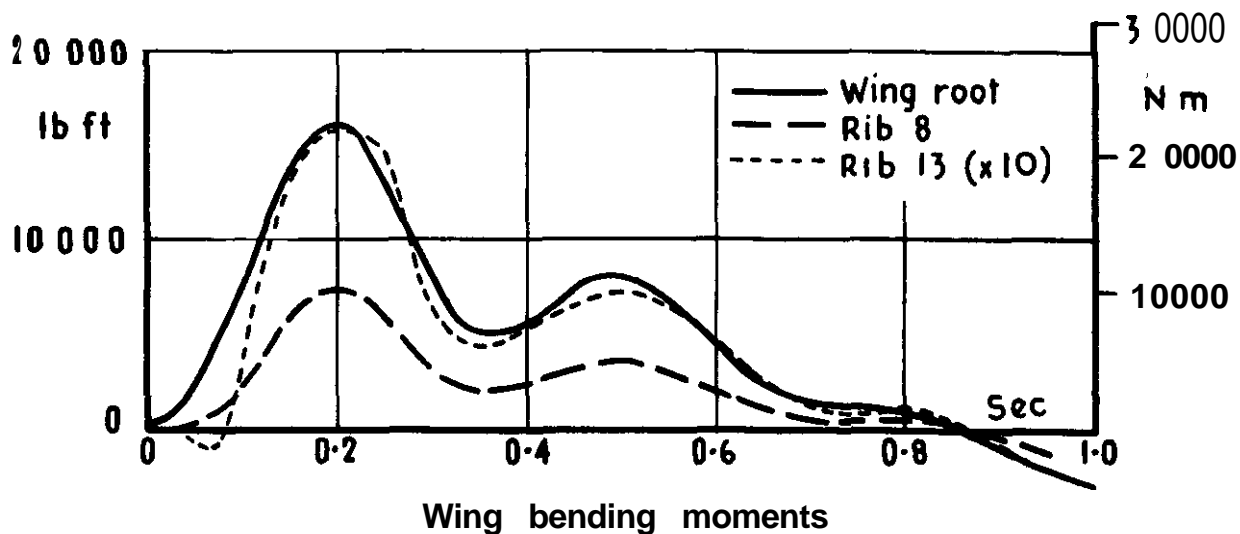




Acceleration of the structure near  $c_q$

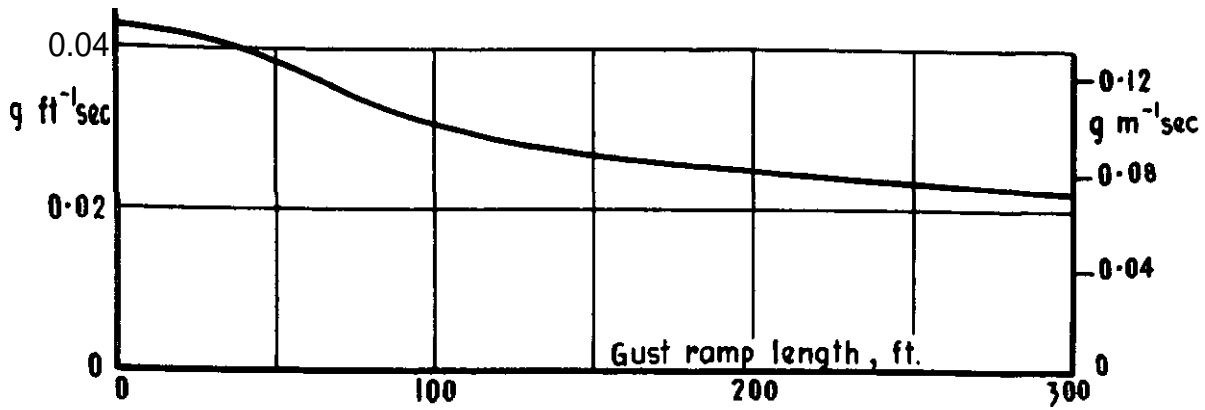


Acceleration at the wing tip.

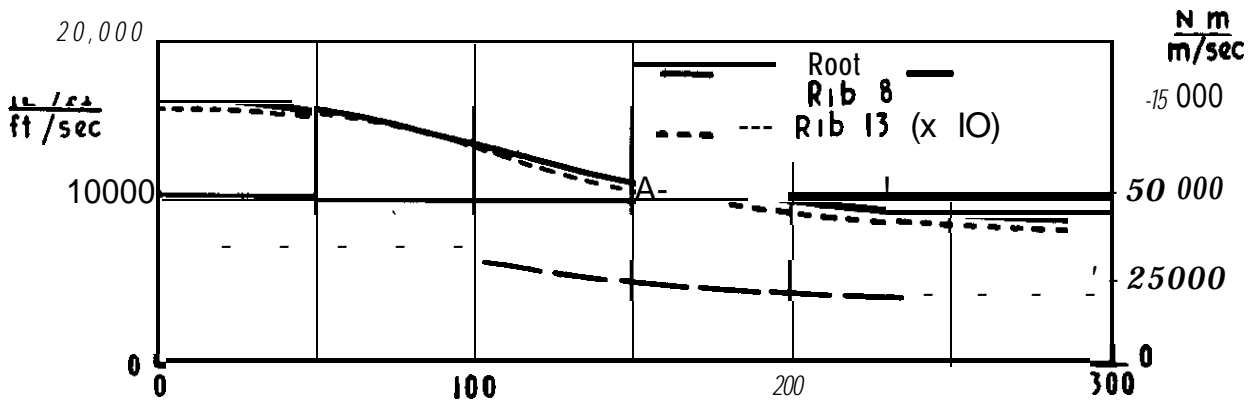


Wing bending moments

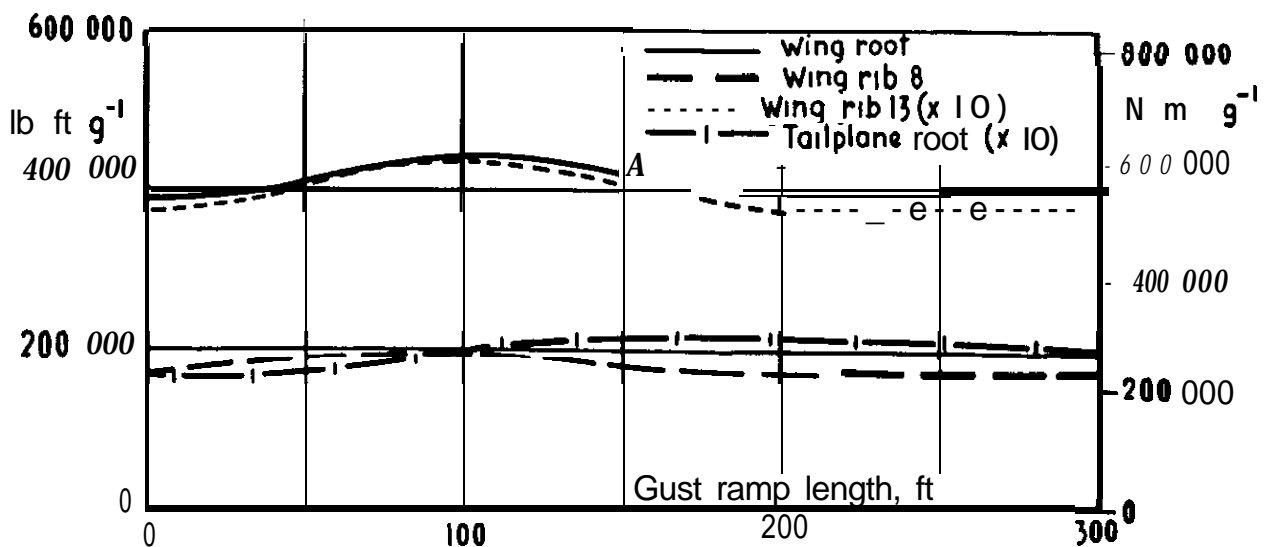
Fig.12 Transient response to a 1 ft/sec (0.305 m/sec) step gust,



Acceleration of the structure near the cg



Wing bending moments



Bending moments per Unit acceleration of the structure near the cg

Fig.13 Variation with gust ramp length of the peak **cg** acceleration and wing bending moments, and the wing and tailplane bending moments per g

A. R. C. C.P.1035  
April 1968

Mitchell, C.O.B.

55X6.013.47 :  
629.137.1 :  
551.551 :  
533.6.048.5

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