C.P. No. 1169

C.P. No. 1169

MINISTRY OF AVIATION SUPPLY

*AERONAUTICAL RESEARCH COUNCIL*

*CURRENT PAPERS*

# Theoretical Assessment of a Method for the Flight Measurement of Net Engine Thrust Using Towed Drag Devices

*by*

*W. J. Pinsker*

*Aero/Flight Division R.A.E., Bedford*

LONDON: HER MAJESTY'S STATIONERY OFFICE

1971

PRICE 40*p* NET

UDC 621.438.018.7 : 629.13.078.2 : 629.135.17

C.P. No.1169
December 1969*

# THEORETICAL ASSESSMENT OF A METHOD FOR THE FLIGHT MEASUREMENT OF NET ENGINE THRUST USING TOWED DRAG DEVICES

by

W. J. G. Pinsker

SUMMARY

It is shown that in theory at least engine net-thrust can be determined from flight tests utilising towed drag devices e.g. parachutes. The thrust evaluation is based on the measurement of the pull exerted by the device on the aircraft and the speed change it produces in level flight. A knowledge of the variation of thrust and drag with speed is required, however, and the accuracy of the technique is assessed to be at best 3-5 per cent. There are, moreover, flight conditions and configurations where the methods are of little practical value and these are indicated.

A potential accuracy of nearer 1 per cent is obtainable, if the technique is used to measure the increment of thrust obtained from change of throttle at a fixed speed. This could be useful as a check on thrust measurements by other methods.

Apart from this particular application, the investigated method does not appear to offer a clear advantage in accuracy over existing procedures, but it might be used where simplicity is more important than high accuracy or where other methods are impracticable for some specific reason.

---

* Replaces RAE Technical Report 69281 - ARC 32173.

CONTENTS

## 1 INTRODUCTION

The case for the accurate flight determination of engine thrust has been pleaded by many authors[1,2] and need not be reiterated here. Equally well known are the difficulties which so far have limited the perfect realization of this requirement. The methods currently used are essentially indirect, as the measurement of the actual propulsive force exerted by the engine on the aircraft has been found impracticable with modern engine installations. The most promising indirect technique developed so far appears to be the measurement of the momentum of the flow leaving the engine nozzle by means of a 'swinging probe'. Although this method has given very encouraging results[3], these are still not entirely satisfactory and there are serious doubts if it can be usefully employed with the fan engines where the energy imparted by the powerplant is no longer concentrated into a well defined narrow jet of high velocity air.

The present paper examines the feasibility of a method for in-flight thrust measurements which, although not directly measuring the engine thrust at its origin, exploits the measurement of the effect on performance of a discrete drag force acting on the aircraft, which is used as a basis to assess the nett thrust of the engine or engines. The method makes use of the well known principle by which an unknown quantity is determined by observing the effect of adding a known increment to this quantity. In the proposed technique the known increment in axial force would be provided by a parachute or other drag device towed by the aircraft.

If such a device is towed and its pull on the aircraft is measured at the attachment point, one obtains a known increment in drag, which it is possible to use for the flight determination of thrust and also of aircraft drag, if these measurements are compared with the results of comparative tests without the parachute. This method may be used in two distinct ways.

(i) If the aircraft is stabilised in steady level flight at a selected altitude and speed, alternatively with and without the parachute deployed, the measured drag of the parachute can be equated with the thrust increment required to achieve the same speed in the two tests. Although this technique does not permit the measurement of total thrust, the knowledge of the increment of thrust produced by a measured increment in throttle position, engine rev/min etc. might be useful information in its own right or serve at least as a means of calibrating or checking some other thrust measuring technique.

(ii) The same basic technique can be modified to permit the determination of the total thrust, or for that matter of aircraft drag, if one maintains, for the tests with and without parachute, throttle or some other engine parameter constant, and allows the aircraft to stabilise at the appropriate level flight speed in the two conditions. This technique requires, however, prior knowledge of the change of both aircraft $C_D$ and engine thrust over the speed range covered in the experiment, as these terms enter into the analysis. The accuracy of the results depends then amongst other things on the validity of the assumptions made for these terms and is likely to be best if both $\partial C_D/\partial V$ and $\partial T/\partial V$ are relatively small at the chosen flight condition. There may be many cases where this is true and there the method is potentially capable of giving satisfactory answers.

We have used the term 'thrust' here so far without precise definition. In fact the definition of the 'thrust' measured by this technique is, as with any other method, largely defined by the technique itself. This definition will become apparent in the formal analysis e.g. in equation (12).

The following analysis establishes the basic mathematical framework for the proposed technique and this is followed by an assessment of its potential accuracy.

## 2 THE EQUILIBRIUM LEVEL FLIGHT EQUATIONS

As shown in Fig.1 a drag device towed in the plane of symmetry of an aircraft will generally react on the aircraft with a drag force $D_p$, a force normal to the flight path $Z_p$, and a pitching moment,

$$M_p = -Z_p x_p - D_p z_p . \qquad (1)$$

All three contributions will affect the flight equilibrium; the Z component will change the aerodynamic lift required to sustain level flight and the pitching moment component will require a change in elevator to trim. Both these terms will in turn alter the aircraft drag $C_D$. All these effects must be properly accounted for in the analysis of the flight results. The pull of the parachute (F) on the aircraft must be resolved into a component parallel to the flight direction $D_p$ as indicated in Fig.1.

If the aircraft is maintained in steady level flight at the same altitude during the test without and with the drag source deployed, the equilibrium of forces in the flight path direction is described for these two cases by

$$\frac{\rho}{2} V_1^2 S C_{D_1} - T_1 = 0 , \qquad (2)$$

$$\frac{\rho}{2} V_2^2 S C_{D_2} - T_2 + D_p = 0 . \qquad (3)$$

The evaluation of the desired values of engine thrust or aircraft drag depends on the manipulation of these two equations.

Since generally all the terms involved will differ between the two cases being compared, it will be necessary to make allowance for these differences. Provided the changes involved are not too large, it may be sufficient for the purpose of general analysis rather than actual flight test work to treat these by linear expansion, i.e. we write

$$C_{D_2} = C_{D_1} + \frac{\partial C_D}{\partial \alpha} (\alpha_2 - \alpha_1) + \frac{\partial C_D}{\partial \eta} (\eta_2 - \eta_1) + \ldots . \qquad (4)$$

for the aircraft drag, and

$$T_2 = T_1 + \frac{\partial T}{\partial V} (V_2 - V_1) + \frac{\partial T}{\partial \alpha} (\alpha_2 - \alpha_1) + \ldots . \qquad (5)$$

The partial derivatives must be introduced as assumptions into the analysis and known from independent sources such as wind tunnel data, estimates, engine test bed results, etc., or themselves determined from additional flight tests. Their values must be so chosen that they apply to the relevant range of the associated aircraft parameter, e.g. incidence $\alpha$, elevator angle $\eta$, speed or Mach number. These will generally only apply to one particular test and cannot be treated as constant coefficients for a whole series of tests. Alternatively one can simply treat the terms in the form

$$C_D = f(\alpha_1\ \eta_1\ V_1 \text{ etc.}) \qquad (6)$$

$$T = f(\alpha_1\ \eta_1\ V_1 \text{ etc.}) \qquad (7)$$

and derive the differences $(C_{D_2} - C_{D_1})$ and $(T_2 - T_1)$ from an appropriate carpet of data. We shall use for the present treatment the first of these methods, because it allows analytical deductions to be made more readily, but this should not suggest a preference for this technique.

There are two different flight tests by which a measure of thrust can be derived from the deployment of an externally towed drag source and the measurement of its pull on the aircraft.

## 2.1 Tests at constant speed with throttle adjustment

Initially the aircraft is stabilised in level flight at the desired speed and height and a record taken. A drag parachute is then released and thrust is increased so that the performance deficiency is made up and the aircraft again stabilises at the original speed and height. It may not be possible in a practical flight test to achieve this condition with absolute precision and appropriate corrections have to be made in the data analysis. The appropriate method will be developed in section 2.2 when discussing the second of the two proposed flight techniques. For simplicity we assume here that speed and height are precisely maintained. However, thrust will be changed by a substantial amount so that $T_2 \neq T_1$. If we ignore any other minor changes in flight conditions and assume $C_{D_2} = C_{D_1}$, equations (2) and (3) give:

$$\Delta T = T_2 - T_1 = D_p, \qquad (8)$$

i.e the measured parachute drag equals the increment in thrust applied by the pilot to retain the original flight condition. This method therefore only permits the determination of an increment in thrust but not its absolute value.

The order of the test can of course be reversed, i.e. the aircraft is first flown with the parachute deployed with full thrust, and this is then reduced after the parachute is retrieved or jettisoned.

In order to allow for changes in aircraft trim resulting from the thrust change and the out of line pull of the drag parachute we introduce a drag correction according to equation (4) and obtain

$$T_2 - T_1 = D_p + \frac{\rho}{2} V^2 S \left\{ \frac{\partial C_D}{\partial \alpha} (\alpha_2 - \alpha_1) + \frac{\partial C_D}{\partial \eta} (\eta_2 - \eta_1) \right\}. \qquad (9)$$

It is obvious that this technique can only be used in flight conditions where maintenance of level flight requires significantly less than full thrust.

## 2.2 Tests at constant throttle with speed adjustment

If the aircraft is initially stabilised in level flight and then the parachute deployed without changing throttle, speed will change if the pilot maintains the original height. By measuring both the difference between these two speeds and the parachute drag, it is possible to evaluate the total thrust or drag of the aircraft in the initial flight condition. The general principle and also some important limitations of this method are illustrated in Fig.2. The figure represents the well known variation of steady trimmed aircraft drag with speed and also of the corresponding thrust. In level flight the intersection of these two curves defines the equilibrium speed $V_1$. If aircraft drag is increased by an additional external contribution - a new equilibrium speed is defined by the intersection of these curves and indicated as $V_2$. Fig.2 considers three distinct situations.

In (a) the initial equilibrium speed is well above minimum drag speed and the external drag is sufficiently small for $V_2$ to be also well above minimum drag speed. The appropriate drag and thrust curves intersect each other at an acute angle and as a result, the two equilibrium speeds are well defined and should be relatively easy to establish in flight.

In (b) the initial speed $V_1$ is closer to minimum drag speed and the addition of the parachute results in an intersection close to the bottom of the drag curve. It should be noted that the addition of an increment in $C_{D_o}$ has reduced the minimum drag speed of the whole assembly, aircraft + parachute, by comparison with that of the clean aircraft. There are now two intersections and hence two possible equilibrium conditions within the plausible speed range and the intersections are less acute and presumably less well defined. This will have the consequence that $V_2$ will be more difficult to stabilise and to measure accurately. The lower of the two possible $V_2$ speeds is in fact below the minimum drag speed of the assembly and does therefore not constitute stable equilibrium in a flight condition in which height is tightly constrained. The second consequence of this situation is that one would expect the mathematical drag analysis from this flight case to become ill defined. This is certainly true but perhaps not to the extent that this illustration suggests. This will become clearer when this analysis is considered in detail.

In (c) the original flight condition is so close to minimum drag speed, that the thrust minus drag balance becomes negative when the parachute drag is added and level flight cannot be maintained. The proposed technique is clearly not feasible in this case.

One may summarise this to suggest that the proposed technique is more suitable for measurements at high speed well above minimum drag speed and that it certainly is inapplicable at, or of course below, minimum drag speed.

Before considering detailed analysis of the flight technique there is another important consideration limiting this technique. It is necessary and indeed a primary condition that during the test, throttle is kept constant. In some engines, an automatic control system is provided which makes adjustments to some engine operating parameter in response to speed variations. In such cases it will be difficult, if not impossible, to utilise the present technique which requires that over the speed range covered in the experiment, thrust is not drastically altered.

Thrust is determined in this flight technique again by considering thrust-drag equilibrium in the two steady flight conditions according to equations (2) and (3). There will be a substantial difference in airspeed and hence in the associated aircraft trim condition. It may not be prudent to attempt simplifications of the computation by linear expansions or other mathematical approximations. Instead we write the appropriate increments as

$$\left.\begin{aligned} C_{D_2} &= C_{D_1} + \Delta C_D \\ T_2 &= T_1 + \Delta T \end{aligned}\right\} \qquad (10)$$

and evaluate these difference terms from appropriate carpets of data. It is also convenient to write

$$V_2 = V_1 + \Delta V \qquad (11)$$

where $\Delta V$ is measured during the test.

Equations (2), (3), (10) and (11) can be combined and solved to give

$$T_1 = \frac{D_p - \Delta T + \frac{\rho}{2} S V_2^2 \Delta C_D}{1 - \left(\frac{V_2}{V_1}\right)^2} \tag{12}$$

For purposes of error analysis it may however, be expedient to expand the terms $\Delta T$ and $\Delta C_D$ in a series retaining at present only linear terms. If appropriate, higher order terms can be readily included. Further, introducing the identity $1 - \left(\frac{V_2}{V_1}\right)^2 = - 2 \frac{\Delta V}{V_1} - \left(\frac{\Delta V}{V_1}\right)^2$, equation (12) becomes:

$$T_1 = - \frac{D_p + \frac{\partial T}{\partial V} \Delta V - \frac{\rho}{2} V_2^2 S \left\{ \frac{\partial C_D}{\partial \alpha} \Delta\alpha + \frac{\partial C_D}{\partial \eta} \Delta\eta + \frac{\partial C_D}{\partial V} \Delta V \right\}}{2 \frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} \tag{13}$$

Replacing $\frac{\partial C_D}{\partial \alpha}$ by $\frac{\partial C_D}{\partial C_L}$ and following procedures to be detailed in section 3, (equation (21)) this expression can be transformed into

$$T_1 = - \frac{D_p + \frac{\rho}{2} V_2^2 S \frac{\partial C_D}{\partial \eta} \Delta\eta}{2 \frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} + \frac{\frac{\partial T}{\partial (V/V_1)}}{2 + \left(\frac{\Delta V}{V_1}\right)} + W \frac{\partial C_D}{\partial C_L} \tag{14}$$

$$- \frac{\rho}{2} V_1^2 S \frac{\partial C_D}{\partial V} \left( \frac{\Delta V}{V_1} + \frac{1}{2 + \frac{\Delta V}{V_1}} \right) .$$

This form is particularly attractive as it does not require a knowledge of incidence $\alpha$ which is difficult to measure accurately. This thrust is a net thrust and can therefore be directly related to aircraft drag (equation (2)) by

$$C_{D_1} = \frac{T_1}{\frac{\rho}{2} V_1^2 S} .$$

The proposed flight technique is therefore theoretically capable of determining net engine thrust or aircraft drag from measurements of a speed difference $\Delta V$, of $V_1$, and of the parachute drag $D_p$. Such quantities as incidence $\alpha$, elevator angle $\eta$, and perhaps Mach number are only required to define completely the two compared steady flight conditions. Furthermore the analysis requires a knowledge of the effect of changes in flight condition ($V$, $\alpha$, etc.) on thrust and aircraft drag. The reliability of the result depends therefore both on the accuracy of the flight measurement and on the validity of these external assumptions. This question is clearly of vital importance in judging the practical value of this technique and it will be discussed in some detail in section 3.

It should be noted, however, that it may be possible to eliminate some of the assumptions such as that for $\partial T/\partial V$ or $\partial C_D/\partial V$ and hence the attendant error sources from the analysis by repeating the test with different sizes of parachute, or by partial climb techniques. The aircraft will then stabilize at different speeds and each such new data point will produce an additional solution for equations (14). As long as it can be assumed that over the whole range of speeds covered by such a test series the partial derivatives $\partial T/\partial V$ and/or $\partial C_D/\partial C_L$ are constant, the redundant information provided by these tests can be utilized to eliminate the appropriate quantity as an assumption and derive it instead directly as a result of the analysis. In this way one additonal test can for instance remove $\partial T/\partial V$ as an assumption and with a further test $\partial C_D/\partial C_L$ can also be eliminated. This procedure is straightforward and will not be derived here in detail. However, it may well be possible - in the case of $\partial C_D/\partial C_L$ it is almost certain - that the partial derivatives involved cannot be assumed to be invariant to $\Delta V$, in fact the way in which they have been expressed as linear derivatives should strictly be taken as a shorthand for a difference notation i.e. $\partial T/\partial V$ should be interpreted to read $\Delta T/\Delta V = \frac{T_1 - T_2}{V_1 - V_2}$. In this case the above argument collapses and one may have to introduce higher order terms in the expansion. For instance if we have to write $T_2 = T_1 + \frac{\partial T}{\partial V} \Delta V + \frac{\partial T}{\partial V^2} \Delta V^2$ two additional tests will be required to obtain the two unknowns $\partial T/\partial V$ and $\partial T/\partial V^2$ and hence to dispense with the need to rely on prior knowledge of the change of T with speed. This situation makes this procedure much less attractive than it might appear at first sight.

There is of course another way in which the data acquired by additional tests can be used to improve confidence in the result. By performing separate analysis on the different tests and using the procedure defined by equation (14) one derives independent results for $T_1$ and one can treat these then as scattering round the true value but of course this does not remove systematic errors. It may be necessary, however, to assess first the relative confidence in the various values so obtained, using the general ideas on error analysis developed in section 3 and weight the data correspondingly.

## 3 ACCURACY CONSIDERATIONS

### 3.1 General analysis

In the first of the two techniques discussed above the situation is fairly straightforward. The desired value of the thrust increment $(T_2 - T_1)$, equation (9), is largely determined by the drag $D_p$, and the remaining terms should contribute only minor corrections. Hence one can expect the accuracy of the measured change in thrust to be in the first place directly proportional to the accuracy with which $D_p$ can be measured. Experience with parachutes used at present for other forms of flight tests suggest a resolution to 1% to be quite feasible.

The method discussed in section 2.2 permits the determination of an absolute value of net thrust and is therefore of much greater interest. $D_p$ is still a major term but as equation (13) shows, the answer depends also on the accuracy with which the terms in the denominator, i.e. $\Delta V$ and $V_1$ can be measured. Speed is measured with a certain absolute error $\varepsilon$ (knots) and this effects both $V_1$ and $\Delta V$. In fact since $\Delta V = V_2 - V_1$, and both $V_2$ and $V_1$ are subject to inaccuracy one might think that $\Delta V$ is subject to an error $2\varepsilon$. However, this is not necessarily true, as some part of $\varepsilon$ is systematic in nature, a true error source as far as absolute speed measurement is concerned, but not with respect to a difference between two relatively close speeds. We allow for this rather arbitrarily by assuming the error in $\Delta V$ to be only $\varepsilon$. We now consider first the $D_p$ contribution as

$$T_{1P} = -\frac{D_P}{2\frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} = -\frac{D_P}{2}\,\frac{1}{\frac{\Delta V}{V_1}\left(1 + \frac{\Delta V}{V_1}\,\frac{1}{2}\right)} .$$

For error consideration we can ignore the second order term in the bracket and get

$$T_{1P} \approx -\frac{D_P}{2\left(\frac{\Delta V}{V_1}\right)} \quad . \qquad (15)$$

We already made the assumption that both $\Delta V$ and $V_1$ are subject to an error $\pm\varepsilon$. It is convenient to express this error as a fraction of $V_1$, i.e. as $\varepsilon/V_1$. The consequences of such an error on the derived value of $T_1$ is then readily computed with the result illustrated in Fig.3. It should be noted that in calculating these results, the effects of $\varepsilon$ on $\Delta V$ and on $V_1$ are assumed to be cumulative in the most unfavourable sense, giving the most pessimistic answer. It is seen that the effect of errors in speed measurement diminish with increasing $\Delta V$, this implies then that one should aim at a fairly large speed reduction, i.e. a large parachute. If $\varepsilon = \pm0.5$ knots, a typical figure for a well conducted modern flight experiment, and $V_1 = 500$ knots, i.e. $\varepsilon/V_1 = \pm0.001$, the effect of this inaccuracy is an error (from this source only) of better than ±2% if $\Delta V/V_1 \geqslant 0.1$ and better than ±1% if $\Delta V/V_1 \geqslant 0.17$. As this level of speed resolution is well within present instrumentation capability, this aspect offers an attractive potential.

However, since this method depends essentially on a deliberate change in flight condition, there are a number of other contributions which can seriously affect the result, namely the correction one has to make to allow for changes in engine thrust and aircraft drag. The thrust correction term appears in equation (13) as:

$$T_{1T} \approx \frac{\frac{\partial T}{\partial V}\,\Delta V}{2\,\frac{\Delta V}{V_1}} = \frac{\frac{dT}{dV}}{2}\,V_1 \quad . \qquad (16)$$

It is interesting to note that errors in $\Delta V$ do not affect this term. The accuracy of this correction term is definied mainly by the accuracy with which $dT/dV$ is known. In Fig.4 the resulting error sensitivity of $T_1$ is plotted against the nondimensionalised thrust-versus-speed derivative $\partial(T/T_1)/\partial(V/V_1)$ and the accuracy by which this term can be assumed to be known. This contribution can be seen to have a rather detrimental effect on the result unless $\partial T/\partial V$ itself is rather small. The proposed technique will therefore not be useful in a flight regime where thrust is suspected to change strongly with speed.

The last contribution one has to consider is the correction for aircraft drag variations. The major element will normally be the induced drag contribution which equation (13) gives as

$$T_{1D} = -\frac{\rho}{2} V_2^2 S \frac{\frac{\partial C_D}{\partial \alpha} \Delta\alpha}{2 \frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} .$$

It is more convenient to express this as an equivalent variation with $C_L$, i.e.

$$T_{1D} = -\frac{\rho}{2} V_2^2 S \frac{\frac{\partial C_D}{\partial C_L}\left(C_{L_2} - C_{L_1}\right)}{2 \frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} \qquad (17)$$

with

$$C_{L_2} = \frac{W/S}{\frac{\rho}{2} V_2^2} \quad \text{and} \quad C_{L_1} = \frac{W/S}{\frac{\rho}{2} V_1^2 S} .$$

This can be reduced to

$$T_{1D} = W V_2^2 \frac{\frac{\partial C_D}{\partial C_L}\left(\frac{1}{V_2^2} - \frac{1}{V_1^2}\right)}{2 \frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} \qquad (18)$$

and with

$$\frac{W}{T_1} = \left(\frac{L}{D}\right)_1 \quad \text{and} \quad V_2 = V_1 + \Delta V$$

we get finally

$$\frac{T_{1D}}{T_1} = \left(\frac{L}{D}\right)_1 \frac{\partial C_D}{\partial C_L} . \qquad (19)$$

The result of an error analysis on this term is shown in Fig.5 based on the well known approximation

$$\frac{\partial C_D}{\partial C_L} \approx \frac{2C_L}{AR\pi} \quad . \tag{20}$$

The value of $C_L$ in this expression ought to be taken as the mean between those applicable to $V_1$ and $V_2$. The calculations have been made for two values of aspect ratio AR = 3 and 6. As is to be expected the error in $T_1$ introduced by this correction term increases with increasing $C_L$ and is potentially larger for the smaller aspect ratio. This would suggest that the method is more promising at relatively high speed.

Furthermore this analysis has shown that it is possible to simplify the representation of the $\Delta\alpha$ term by teating it as a $\Delta C_L$ contribution and to reduce it to

$$\frac{\frac{\rho}{2} V_2^2 \frac{\partial C_D}{\partial C_L} \Delta C_L}{2\frac{\Delta V}{V_1} + \left(\frac{\Delta V}{V_1}\right)^2} = W \frac{\partial C_D}{\partial C_L} \tag{21}$$

In this form the terms become insensitive to measurements of speed and incidence, but a knowledge of the aircraft weight W is now required.

The analysis presented here assumes perfectly steady trimmed flight conditions. In practice, one must expect this to be not achieved with absolute precision. If speed and/or height are varying somewhat whilst the measurements are taken, and if these variations are recorded, appropriate corrections can be made in the analysis. This will introduce a further error source which is not treated in detail here. It would appear however, that this will not be a significant factor. The method does, however, require a perfectly calm atmosphere, only steady wind is acceptable since any wind speed variations (unless known) will invalidate the above mentioned corrections.

It is practically impossible to summarise the results of this error analysis into an answer of general validity. However from the numerical data presented, it would seem possible to obtain measurements of net thrust $T_1$ within 3-5% accuracy if $\Delta V$ is reasonably large by comparison with $V_1$ (Fig.3) and if also $C_L$ is small, i.e. if both $V_1$ and $V_2$ are relatively large (Fig.5). The process of error analysis discussed here should permit realistic estimates to be made of the expected accuracy of any specific test under consideration, and, equally important, should allow the test conditions to be so chosen as to minimise the error potential. Numerical examples are considered in the next section.

### 3.2 Numerical examples

A really meaningful assessment of the potential accuracy of the method of thrust measurement discussed in this paper requires careful and detailed consideration of all the factors particular to a given aircraft, engine etc. Broad generalizations are not recommended in such a delicate field. Nevertheless it would be desirable to attempt to make some numerical predictions so as to get the results of the analysis of section 3.1 into perspective.

For this we assume an aircraft with a wing of aspect ratio 6 and assume that in the datum condition it flies at a $C_L$ of 0.15 with a true speed of 500 knots. We shall now try to evaluate the accuracy one might obtain for measuring the nett thrust at this condition by deploying a drag device which reduces speed to 450, 400 and 350 knots respectively.

Further assumptions are that speed can be measured to ±1 knot, the parachute drag to ±1%, that the variation of thrust with speed, given from e.g. test bed data, is $\partial(T/T_1)/(\partial(V/V_1) = 0.1$, i.e. over the speed range of interest the slope of T with speed is such as to correspond to a 10% change in thrust from $V = 0$ to $V = V_1$. We assume that the value of this slope is uncertain to ±20% and further that the assumed change in induced drag with $C_L$ (here simply taken as that given by classical airfoil theory) is uncertain to ±15%.

To complete the definition of our hypothetical aircraft it is assumed to have a lift-drag ratio as shown in the insert in Fig.5.

With these assumptions we can now calculate the contributions from the four principal sources of error discussed earlier, i.e. those associated with measurement of parachute drag, airspeed and also those associated with variation with speed of thrust and aircraft $C_D$. These calculations are straightforward, using the precalculated results given in Figs.3, 4 and 5.

The results are tabulated below:

| $V_2$ (knots) | 450 | 400 | 350 |
|---|---|---|---|
| Error in $D_p$ | 1.0 | 1.0 | 1.0 |
| Error in speed measurment | 3.9 | 1.7 | 1.1 |
| Error in $T/\partial V$ assumption | 1.2 | 1.45 | 1.74 |
| Error in $\partial C_D/\partial C_L$ assumption | 2.0 | 2.0 | 2.0 |
| Arithmetic total | 8.1% | 6.15% | 5.84% |

Although simple arithmetic summation of the individual error contributions cannot be dismissed as indicating a physically possible total error, the probability of all components to be a maximum and all acting in the same sense simultaneosuly is clearly rather remote. Statistical theory provides a more sophisticated approach to this problem, operating on the assumption that errors have a Gaussian distribution and are defined by their individual rms values. Although the accuracies quoted in the type of test situation considered here are not defined in this way, but rather as absolute maxima, it would nevertheless seem appropriate to employ a method which makes some allowance for the probability of errors accumulating in assessing a plausible total error. The 'expected error' so defined is calculated as the root of the sum of the squares of the individual contributions and for the three cases considered would give values of

| $V_2$ (knots) | 450 | 400 | 350 |
|---|---|---|---|
| Expected error | 4.65% | 3.14% | 3.03% |

These values are substantially lower than those derived from arithmetic addition and it is suggested that they are likely to be nearer the truth than the former.

Similar calculations have been performed for the same aircraft, but now using 400 knots true speed as the datum condition $V_1$. Corresponding results are

| $V_2$ (knots) | 360 | 320 | 280 |
|---|---|---|---|
| Total error | 9.85% | 8.1% | 8.3% |
| Expected error | 4.2% | 4.3% | 4.85% |

It is interesting to note that in this example the trends with choice of $V_2$ given by the two error criteria contradict one another, i.e. total error is minimized for $V_2$ = 320 knots, when 'expected error' has a minimum for $V_2$ = 360 knots or at an even higher value of $V_2$.

Another case considered was an aircraft with a wing of aspect ratio 3; otherwise the same assumptions have been used as in the first example. The results are as follows

| (a) $V_1$ = 500 knots true speed | | | |
|---|---|---|---|
| $V_2$ (knots) | 450 | 400 | 350 |
| Total error | 9.2% | 7.5% | 7.45% |
| Expected error | 5.05% | 3.95% | 4.6% |
| (b) $V_1$ = 400 knots | | | |
| $V_2$ (knots) | 360 | 320 | 280 |
| Total error | 12.2% | 10.9% | 11.9% |
| Expected error | 6.95% | 6.8% | 8.2% |

These estimates are of course entirely at the mercy of a number of assumptions, they have been made as far as possible to reflect the best in present state of the art in the various disciplines involved. It would appear that only in some conditions can accuracy better than 4% be expected, if one ignores the more severe answers given by simply totting up contributions to 'total error'. This degree of resolution is approximately that offered by other currently available methods. As was expected there are many cases where this target is, however, entirely outside the practical scope of the technique investigated here.

One might conclude that this technique does not offer the promise of the desired breakthrough in accuracy of measuring engine thrust in flight, but that there are perhaps cases where a carefully conducted and planned flight experiment using towed drag parachutes could by just superior to conventional techniques.

## 4 CONCLUSIONS

The possibility was investigated of measuring net thrust in flight by a technique which relates thrust to a known increment in drag deliberately applied to the aircraft. The measurement of thrust is thereby reduced essentially to the measurement of the pull exerted on the aircraft by a towed parachute or some other drag body. Two distinct types of test and therefore thrust measurement have been shown to be potentially possible.

One variant of the technique requires the establishment of a selected level flight speed first with, and then without, the parachute deployed, and the measured parachute drag is then a direct measure of the thrust increment required by throttle adjustment to maintain speed against the additional pull of the parachute. This method is of course only capable of determining an increment and not an absolute value of thrust but it promises very high accuracy which is practically only limited by the accuracy with which the parachute pull at the attachment point on an aircraft can be measured. Such a test could be used as an accurate check on thrust measurements made by another method.

In the second variation of this technique the engine throttle will be maintained constant and after deployment of the drag device, the aircraft is allowed to settle to a new and reduced level flight speed at the original height and by measuring the two equilibrium speeds and the externally applied drag, the total drag or net thrust of the aircraft can be evaluated.

The potential accuracy of the two techniques is assessed. In the first case, this is essentially determined by the accuracy by which the pull of the drag device can be measured and this is likely to be not much worse than 1%.

The second technique is more indirect and depends on the validity of a number of corrections which have to be made but in favourable conditions an accuracy of about 4-5% appears feasible.

There are, however, flight conditions in which the proposed technique is unlikely to give satisfactory results. These are flights at high values of lift coefficient and also regimes where thrust and/or $C_D$ vary rapidly with speed. This would rule out the transonic region. A method of error analysis is presented which will allow an assessment of the potential accuracy for specific cases and this will also allow conditions to be determined which optimise the accuracy potential.

Although in potential accuracy the proposed technique does not promise to improve on or even compete with more sophisticated methods, it has the merit simplicity and might be useful where a quick answer is desired and not necessarily the utmost in precision.

## SYMBOLS

| | | |
|---|---|---|
| $C_D$ | $= \dfrac{D}{\frac{\rho}{2} V^2 S}$ | drag coefficient |
| $C_L$ | $= \dfrac{L}{\frac{\rho}{2} V^2 S}$ | lift coefficient |
| D | | drag |
| $D_p$ | | drag of externally towed device |
| L | | lift |
| $M_p$ | | pitching moment applied by external drag device |
| S | | wing area |
| T | | net-thrust |
| V | | airspeed |
| W | | aircraft weight |
| $x_p$, $z_p$ | | coordinates of parachute attachment (Fig.1) |
| $Z_p$ | | vertical component of parachute pull |
| $\alpha$ | | incidence |
| $\eta$ | | elevator angle |
| $\rho$ | | air density |
| Suffix 1 | | refers to initial condition without parachute |
| Suffix 2 | | refers to final condition with parachute |

## REFERENCES


| No. | Author | Title, etc. |
|---|---|---|
| 1 | R. Rose | The need for accurate thrust measurements in flight.<br>R.A.E. Technical Report 67066<br>(A.R.C. 29260)(1966) |
| 2 | Theron W. Davidson | Measurement of net thrust in flight.<br>Journal of Aircraft Vol.1 No.3 (May-June 1964) |
| 3 | A. A. Woodfield<br>Ann Cronin<br>Glynis Vorley | Measurements of the performance of a turbo-jet engine with reheat (Rolls Royce RA 28R Avon) in high speed flight and under simulated conditions in test beds.<br>A.R.C. R & M 3538 (1966) |

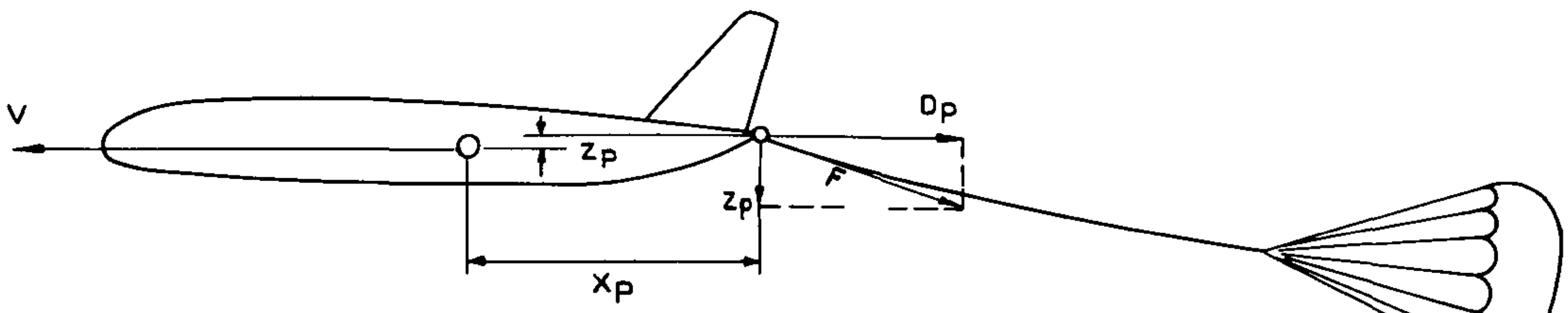


Fig. 1 Pull of a towed parachute as a means of thrust measurement

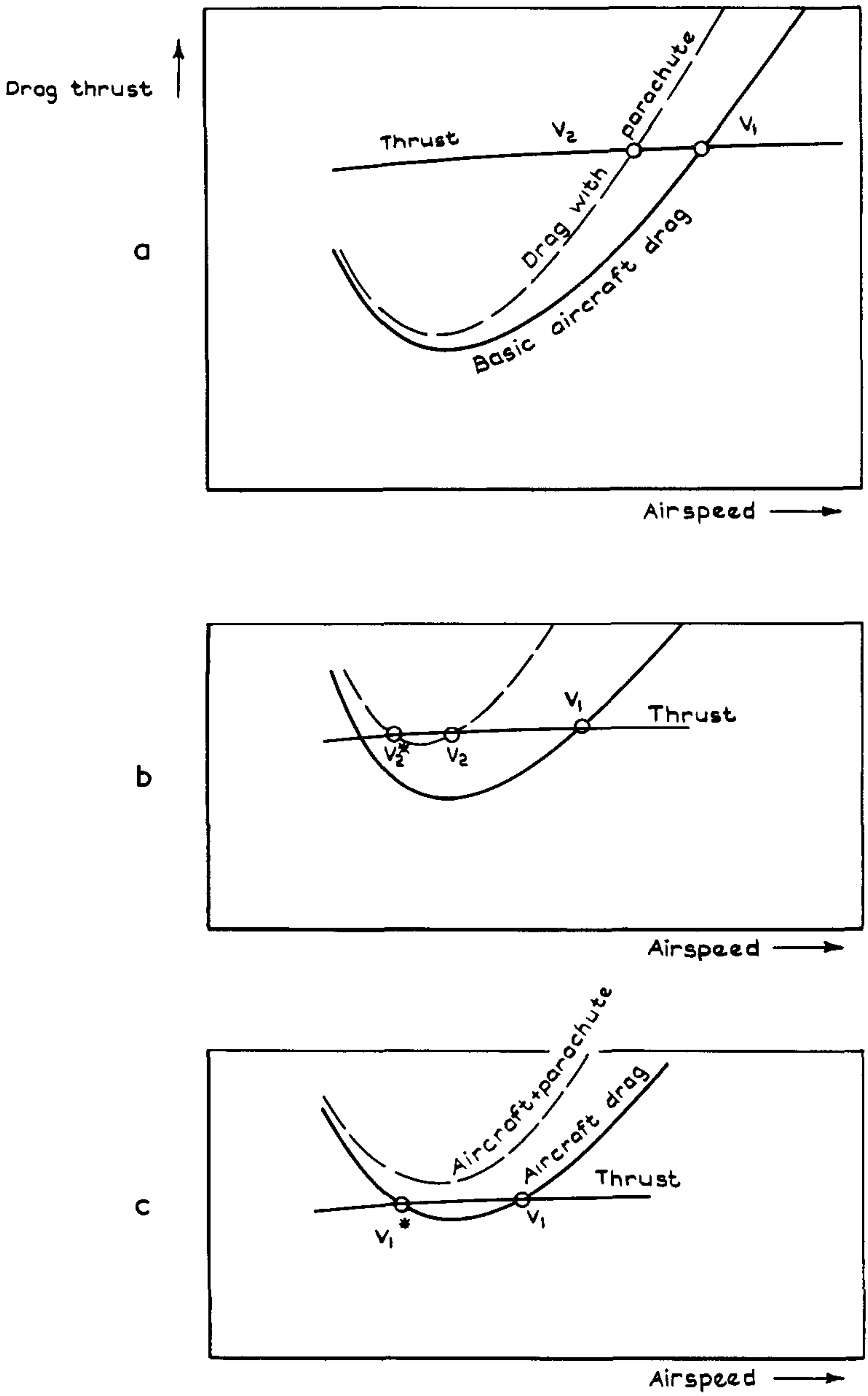


Fig. 2 a–c Effect of externally applied drag on equilibrium level flight speed

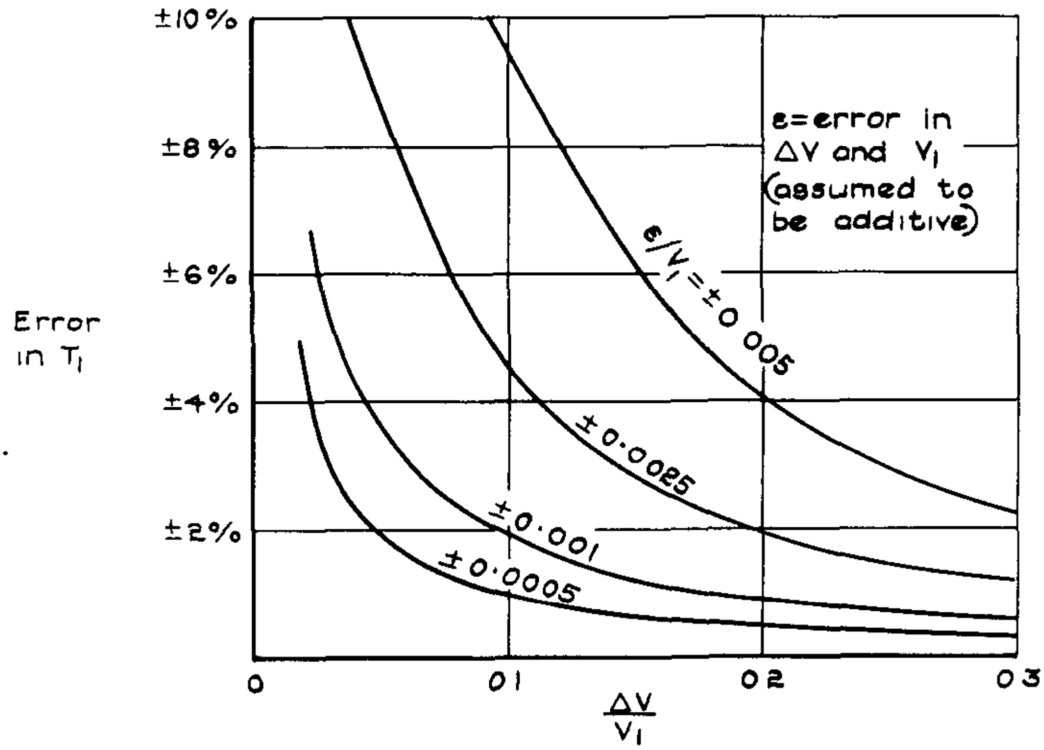


Fig. 3 Effect of error in measuring $V_1$ and speed difference $\Delta V$ on thrust evaluation according to equation 13

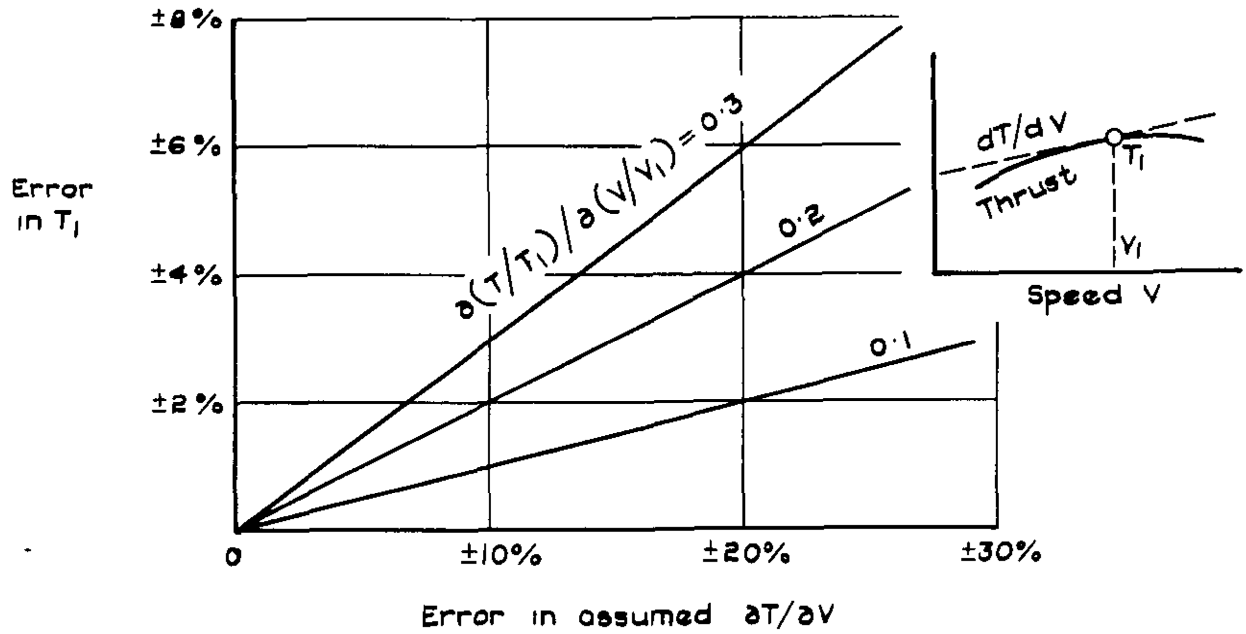


Fig. 4 Effect of error in assumed value of thrust change with speed on thrust evaluation according to equation 13

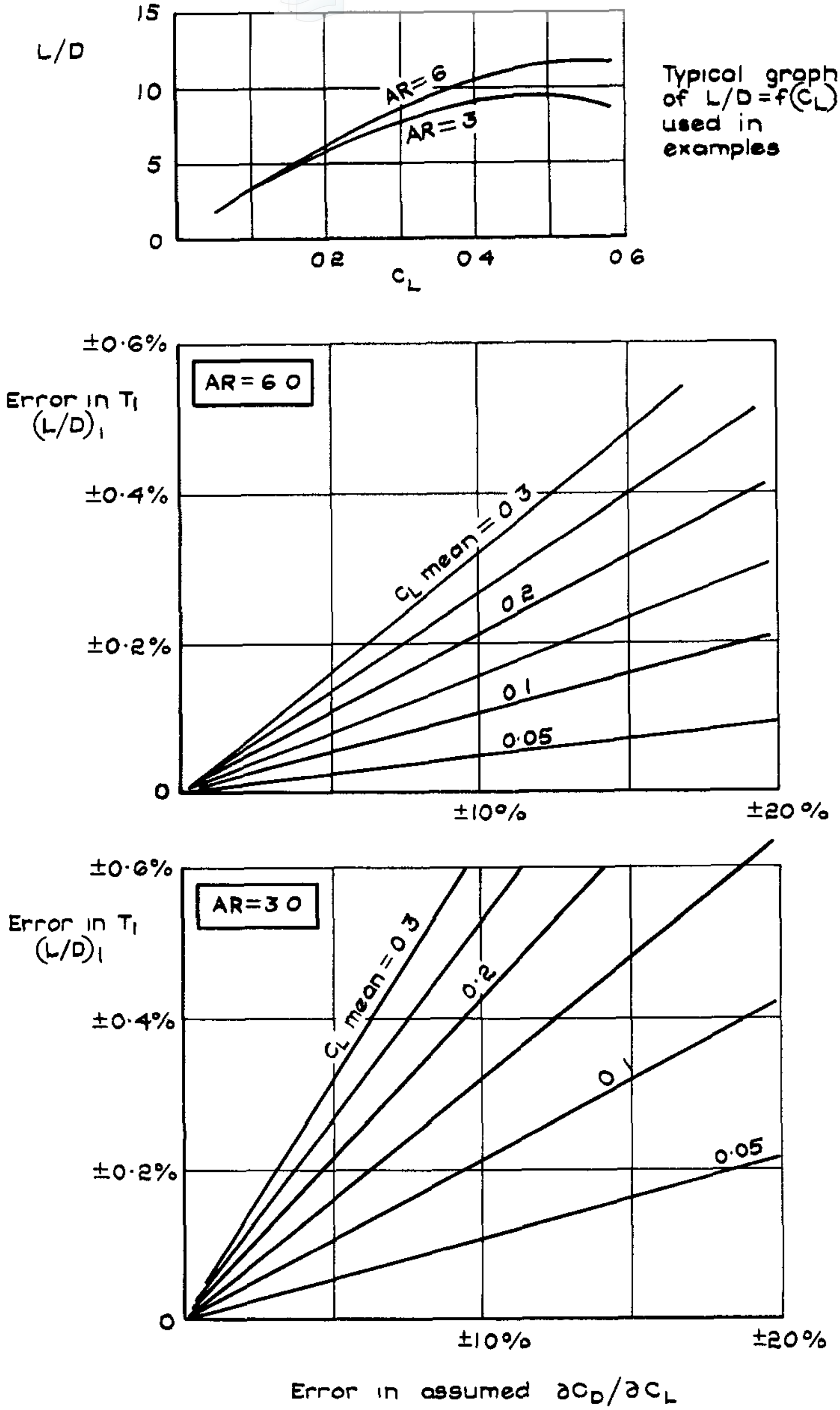


Fig. 5 Sensitivity of thrust evaluation from equation 14 to error in assumed value for induced drag variation with $C_L$

*Printed in England for Her Majesty's Stationery Office by the Royal Aircraft Establishment, Farnborough Dd 502109 K 4*

## DETACHABLE ABSTRACT CARD

A.R.C. C.P. No 1169
December 1969

621 438 018 7
629 13 078.2
629 135 17

Pinsker, W J. G

THEORETICAL ASSESSMENT OF A METHOD FOR THE FLIGHT MEASUREMENT OF NET ENGINE THRUST USING TOWED DRAG DEVICES

It is shown that in theory at least engine net-thrust can be determined from flight tests utilising towed drag devices e g parachutes The thrust evaluation is based on the measurement of the pull exerted by the device on the aircraft and the speed change it produces in level flight A knowledge of the variation of thrust and drag with speed is required, however, and the accuracy of the technique is assessed to be at best 3–5 per cent There are, moreover, flight conditions and configurations where the methods are of little practical value and these are indicated

A potential accuracy of nearer 1 per cent is obtainable, if the technique is used to measure the increment of thrust obtained from change of throttle at a fixed speed. This could be useful as a check on thrust measurements by other methods

(Over)

A.R.C. C.P. No 1169
December 1969

621 438 018 7
629 13 078 2
629 135 17

Pinsker, W J G

THEORETICAL ASSESSMENT OF A METHOD FOR THE FLIGHT MEASUREMENT OF NET ENGINE THRUST USING TOWED DRAG DEVICES

It is shown that in theory at least engine net-thrust can be determined from flight tests utilising towed drag devices e g parachutes The thrust evaluation is based on the measurement of the pull exerted by the device on the aircraft and the speed change it produces in level flight A knowledge of the variation of thrust and drag with speed is required, however, and the accuracy of the technique is assessed to be at best 3–5 per cent There are, moreover, flight conditions and configurations where the methods are of little practical value and these are indicated

A potential accuracy of nearer 1 per cent is obtainable, if the technique is used to measure the increment of thrust obtained from change of throttle at a fixed speed. This could be useful as a check on thrust measurements by other methods

(Over)

A.R.C. C.P. No 1169
December 1969

621 438 018 7
629 13 078.2
629 135 17

Pinsker, W J G

THEORETICAL ASSESSMENT OF A METHOD FOR THE FLIGHT MEASUREMENT OF NET ENGINE THRUST USING TOWED DRAG DEVICES

It is shown that in theory at least engine net-thrust can be determined from flight tests utilising towed drag devices e g parachutes The thrust evaluation is based on the measurement of the pull exerted by the device on the aircraft and the speed change it produces in level flight A knowledge of the variation of thrust and drag with speed is required, however, and the accuracy of the technique is assessed to be at best 3–5 per cent There are, moreover, flight conditions and configurations where the methods are of little practical value and these are indicated

A potential accuracy of nearer 1 per cent is obtainable, if the technique is used to measure the increment of thrust obtained from change of throttle at a fixed speed This could be useful as a check on thrust measurements by other methods.

(Over)

Apart from this particular application, the investigated method does not appear to offer a clear advantage in accuracy over existing procedures, but it might be used where simplicity is more important than high accuracy or where other methods are impracticable for some specific reason

Apart from this particular application, the investigated method does not appear to offer a clear advantage in accuracy over existing procedures, but it might be used where simplicity is more important than high accuracy or where other methods are impracticable for some specific reason

Apart from this particular application, the investigated method does not appear to offer a clear advantage in accuracy over existing procedures, but it might be used where simplicity is more important than high accuracy or where other methods are impracticable for some specific reason

C.P. No. 1169



Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
49 High Holborn, London WC1 V 6HB
13a Castle Street, Edinburgh EH2 3AR
109 St Mary Street, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
50 Fairfax Street, Bristol BS1 3DE
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY
or through booksellers

C.P. No. 1169

SBN 11 470437 6