C.P. No. 1208



PROCUREMENT EXECUTIVE, MINISTRY OF DEFENCE

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Boundary-Layer Pressure Fluctuations at High Reynolds Numbers on a Free-Flight Test Vehicle

by

D. R. Roberts

Structures Dept., R.A.E., Farnborough

LONDON: HER MAJESTY'S STATIONERY OFFICE

1972





UDC 532.526.4 : 533.6.048.2 : 533.665.5 : 533.6.011.12 : 533.6.011.5

*CP No.1208 March 1971

BOUNDARY-LAYER PRESSURE FLUCTUATIONS AT HIGH REYNOLDS NUMBERS ON A FREE-FLIGHT TEST VEHICLE

bу

D. R. Roberts

SUMMARY

Measurements have been made of the boundary-layer pressure fluctuations on the body of a free-flight aerodynamic test vehicle powered by a solid-fuel rocket motor. The vehicle reached a maximum Mach number of 2.2 with a maximum Reynolds number of about 215 millions.

Pressure spectra have been deduced, and have been found to compare reasonably with a theoretical spectrum for homogeneous isotropic turbulence.

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1.5 to 2.2, while being essentially equal to 50% of this thickness over the range Ma = 2.0 to Ma = 2.2.

At Ma = 2.2 the root mean square boundary-layer pressure was equal to 0.0045 of the free stream dynamic pressure.

^{*} Replaces RAE Technical Report 71033 - ARC 33171





CONTENTS

			Page
1	INT	RODUCTION	5
2	DES	CRIPTION OF TEST EXPERIMENT	5
	2.1	Description of test vehicle	5
	2.2	Aerodynamic environment	6
	2.3	Range facilities	6
3	MET	HOD OF DATA ANALYSIS	7
4	DIS	CUSSION OF RESULTS	12
	4.1	Comparison of experimentally-determined spectra with theoretical spectrum for homogeneous isotropic turbulence	12
	4.2	The scale of the turbulence	13
5	CON	CLUSIONS	14
Table		Density of (pressure squared $ imes$ 10000) spectrum with respect to frequency factor 'k'	16
Table	2	Frequency and corresponding frequency factor 'k'	24
Table	3	Experimental data and derived quantities for individual runs	25
Table	4	Experimental data and derived quantities for collective run	26
Symbol	ls		27
Refere	ence	s	29
Illust	trat	ions Figures	1-12
Dotack	ah I	a abetract cards	_



ILLUSTRATIONS

	Fig.
General arrangement drawing of 'Shark 1' test vehicle	la
Side view photograph of 'Shark 1'	1b
Plot of pressure against time for the first 5 milliseconds of	
runs 32, 33 and 34	2a-c
Plot of velocity against elapsed time	3
Plot of altitude against elapsed time	4
Plot of Mach number against elapsed time	5
Plots of autocorrelation function, normalised autocorrelation	
function, and unsmoothed, hanning-smoothed and hamming-smoothed	_
spectral density estimates	6а-е
Plot of $\hat{\lambda}_{\mathbf{u}}$ against elapsed time	7a
Plot of $\lambda_{\mathbf{u}}^{-}$ against Mach number	7b
Plot of $\$_{\mathrm{T}}$ against elapsed time	8a
Plot of \$_ against Mach number	8ъ
Plot of unsmoothed spectral density against frequency factor, for	
collective run	9
Plot of $(\lambda_{11}/\$_{T})$ against elapsed time	10
Plot of $(\lambda_{11}/\$_{T})$ against Mach number	11
Plot of unsmoothed spectral density against frequency, for run 35	12a
Plot of unsmoothed spectral density against frequency, for run 45	12b
Plot of unsmoothed spectral density against frequency, for collective	
run	12c



1 INTRODUCTION

A series of three free-flight aerodynamic test vehicles, designated the 'Shark'* series, was envisaged for the investigation of boundary-layer characteristics at high Reynolds number.

On Shark 1, measurements were made of skin friction drag by means of surface pitot tubes, and boundary-layer pressure fluctuations using a piezo-electric pressure transducer. On Shark 2, it was planned to make direct measurements of skin friction drag by means of a modified force-balance accelerometer, for comparison with the results obtained from Shark 1. Difficulties with the instrument resulted in the abandonment of the experiment. On Shark 3, measurements were made of boundary-layer pressure fluctuations employing two piezo-electric pressure transducers, one at a rear station similar to that used on Shark 1, and the other at a station positioned to give a Reynolds number half that at the rear station. This Report presents the pressure-fluctuation results from Shark 1.

2 DESCRIPTION OF TEST EXPERIMENT

2.1 Description of test vehicle

2.1.1 General

The vehicle consisted of a 4 calibre tangent circular ogival nose fitted to a cylindrical body 8 calibre long followed by a 2 calibre tail section, to the tail section were fitted three stabilising fins, radially equally distributed. The body and tail tube were constructed of resin bonded paper tube, and the nose was a glass fibre moulding. The vehicle was propelled by a non-separating solid-fuel rocket motor mounted inside the body. The overall length was about 233 inches (5.92 m) and the launch weight was about 1204 pour ds (546 kg). A general arrangement drawing is given in Fig.1a.

2.1.2 Instrumentation

Transducers were installed to measure:

(a) Boundary-layer pressure fluctuations (special piezo-electric lead-zirconate bimorph transducer, ± 0.2 lbf/in² [± 1379 N/m²]).

^{*}These vehicles were designed and constructed by Aerodynamics Department, RAE, and launched by them at the RAE Aberporth firing range.



- (b) Pitot and static pressure differential at each of five surface pitot tubes (variable inductance pressure transducers).
- (c) Acceleration normal to the vehicle body surface near the boundary-layer pressure measuring station (piezo-electric accelerometer).
- (d) Accelerations normal and perpendicular to a given vehicle-body radius (variable inductance accelerometers).

The output signals of these transducers were translated through suitable sequence-switches and modulator units to frequency-modulate the transmission of three 465 MHz oscillator units. The signals from these oscillators were received and translated at the ground receiving station, and recorded on magnetic tape and oscilloscope record films.

One transmitter was used to telemeter the data from (b) and (d), to give measurements of vehicle accelerations and skin-friction. The second transmitter telemetered the output of (c) to give measurements of the environmental vibration local to the boundary-layer pressure-fluctuation transducer. The third transmitter telemetered the output from (a); the information derived from this transducer is considered in this paper.

2.2 Aerodynamic environment

The maximum Mach number achieved was about 2.2 $[q_{max} \simeq 6870 \text{ lbf/ft}^2]$ (330 kN/m²).

The maximum local Reynolds number was about 215 millions. Maximum altitude was about 1456 ft (444 m).

2.3 Range facilities

In addition to the telemetry receiving and recording station, the firing range provided the following:-

- (a) a number of observation posts suitably sited, equipped with highspeed cine cameras and kinetheodolites, which tracked the vehicle during
 flight, and afforded data from which was deduced velocity and altitude
 information.
- (b) Radar cover to provide additional velocity and altitude data, particularly when the vehicle was beyond optical range of the observation posts, or obscured by cloud.
- (c) A central source of timing pulses which were transmitted to all the data recording systems, and also initiated a firing sequence which resulted in the electrical pulse sent to the rocket motor igniter circuit. Measurement of the roll of the vehicle was also made by the telemetry receiving station.



The transmission of data from the boundary-layer pressure fluctuation transducer ceased after about 5 seconds of flight. This record deals with the data received during this time.

3 METHOD OF DATA ANALYSIS

The signal representing the output of the boundary-layer pressure fluctuation transducer was frequency-translated after reception and recorded on $\frac{1}{2}$ inch magnetic tape at 60 inches per second. It was later played back at 7.5 inches per second, and re-recorded at 30 inches per second. Thus one second of vehicle flight time was represented by 240 inches of magnetic tape. This operation was carried out by Instrumentation and Ranges Department at the Royal Aircraft Establishment, Farnborough, using the equipment known as BRAMBLE¹. This analogue record was digitised by means of a high-speed digital recorder made to an RAE design by English-Electric/Leo/Marconi Ltd., the output from which was taken to a five-hole paper tape punch. For ease of handling, the record of the five seconds of flight time was divided into fifty punched paper tapes, each representing approximately 0.1 second of flight time, and designated runs 00 to 49, the designation representing the approximate time of the beginning of that section of data. Thus run 23, for instance, refers to the data received during the section beginning at about 2.3 seconds after the commencement of the flight.

Each of these fifty runs consisted of approximately 6170 discrete readings of pressure amplitude, expressed in units of analogue/digital converter output. For convenience in the computing process, these output units were retained throughout the analysis. The relationship between these units and standard physical units is given below:-

1 pressure unit =
$$0.0031163 \text{ lbf/in}^2$$
 = 0.02149 kN/m^2
1(pressure)² unit = $9.7113 \times 10^{-6} (\text{lbf/in}^2)^2$ = $0.4618 \times 10^{-6} (\text{kN/m}^2)^2$.

The readings were spaced at intervals of 15.5 microseconds of flight time. Figs.2a, 2b and 2c show plots of pressure-readings for the first 5 milliseconds of each of the three runs, 32, 33 and 34.

Using the RAE Mathematics Department ICL 1907 computer, the data for each run was treated separately, the following operations being performed.

(a) The arithmetic mean of all the values in the run was found, and subtracted from each value in turn:-

$$y_t = (recorded value) - (arithmetic mean)$$
.

(b) The root mean square of the pressure fluctuations was evaluated using the expression

$$\sigma_{\rm p} = \left[\frac{1}{(N+1)} \sum_{t=1}^{N+1} y_t^2 \right]^{\frac{1}{2}}$$

where (N + 1) is the number of discrete values contained in the run.

(c) The autocorrelation function is defined as

$$R_{\tau} = \frac{1}{(N+2-\tau)} \sum_{t=1}^{N+2-\tau} y_t y_{(t+\tau-1)}$$

where $\tau = 1, 2, 3 \dots (M + 1)$

where M is the number of correlated intervals used in the spectral density analysis.

The expression was evaluated for $\tau = 1, 2, 3 \dots$ (100).

(d) The unsmoothed spectral density estimates were calculated from 2:-

$$V_{k} = \Delta t \left\{ R_{1} + R_{(M+1)} \cos \left[(k-1)\pi \right] + 2 \sum_{\tau=2}^{M} R_{\tau} \cos \left[\frac{\left[(k-1)(\tau-1)\pi \right]}{M} \right] \right\}$$

where $k = 1, 2, 3, \dots$ (M + 1)

and At is the time interval between each two consecutive discrete values.

Tabulated values of unsmoothed spectral density are presented in Table 1.

(e) The frequency to which this estimate refers is given by:-

$$f_k = \frac{(k-1)}{2\Delta tM} .$$

Values of k with the corresponding values of frequency are presented in Table 2.

The expression in (d) and (e) were evaluated for $k = 1, 2, 3 \dots 100$.

(f) The estimates of hanning-smoothed and hamming-smoothed spectral densities are given by:-

hanning smoothing

$$W_{k} = 0.5 V_{1} + 0.5 V_{2}$$

$$= 0.25 (V_{k-1} + V_{k+1}) + 0.5 V_{k}$$

$$= 0.5 V_{M} + 0.5 V_{M+1}$$

$$k = 1$$

$$1 < k < M$$

$$k = M + 1$$

hamming smoothing

.

These expressions were also evaluated for $k = 1, 2, 3 \dots 100$.

Autocorrelation functions plotted against τ , and smoothed and unsmoothed spectral density estimates plotted against k, are presented for a number of representative runs in Figs.6a to 6e. The maximum value of unsmoothed spectral density and the frequency f_m at which it occurred, were determined, for each run. It should be noted that f_m was taken as the frequency of the peak value of the unsmoothed spectral density, regardless of fluctuations about the mean path through the experimental points. In a few runs, two peaks of similar magnitudes occurred at fairly close frequencies, causing some doubt as to which represented the proper value of f_m . In such a case, reference was made to the plots of smoothed spectral density, which indicated one sensible peak value. The frequency of this value was taken as the value of f_m .

(g) For very high Reynolds numbers (non-dimensional viscosity function A = 0) it was shown³ that the maximum (pressure)² spectral density occurred at

$$X = 2.111$$

where X, the non-dimensional frequency, was defined as

$$X = a\omega = \frac{2\pi fa}{U}$$

where ω was the wave-number, f the measured frequency, U the velocity of the stream relative to the measuring station, and a was a length to make wave-numbers non-dimensional.



The frequency at which the spectral density is a maximum is defined as $\mathbf{f}_{\mathtt{m}}$ so that

$$2.111 = \frac{2\pi f_{m}a}{U}$$

or

$$a = \frac{2.111U}{2\pi f_m} .$$

The scale of the turbulence was defined as

and

$$LU = 0.74677$$
 when $A = 0$.

If the frequency position of the maximum value of the experimental spectrum is made to coincide with that of the theoretical spectrum, the scale of the turbulence is therefore given by -

$$\hat{x}_{u} = \frac{2.111 (0.74677U)}{2\tau f_{m}}$$
$$= \frac{0.25088U}{f_{m}}.$$

This expression was used to calculate the value of ℓ_u for each run.

- (h) The local Reynolds number $R_{_{X}}$ was calculated for each run, assuming that the effective start of the turbulence was at a distance x upstream of the measuring station, equal to the axial distance of the station from the nose of the test vehicle, and using the data of velocity and altitude.
- (1) Taylor 4 has concluded that a reasonable estimate of the turbulence thickness $\$_T$ is equal to about 80% of the total boundary-layer thickness \$ predicted by Spalding 5 , so that

$$R_{\xi_T} = 0.8 R_{\xi}$$

The relationship between R_{\sharp} and R_{χ} has also been defined by Spalding, and using this relationship together with the expression

$$\$_{T} = \left(\frac{0.8 R_{\$}}{R_{\times}}\right) x$$

the values of $\$_{\mathsf{T}}$ were calculated for each run.



- (j) The ratio $\ell_{_{11}}/\xi_{_{\rm T}}$ was then calculated for each run.
- 3.1 For runs 00 to 20, (Ma = 0 to Ma ~ 1.2) the signal/noise ratio of the recorded data was too small for the effective determination of a peak in the plot of spectral density against frequency factor. Under these conditions, a spurious peak imposed by the reading frequency was often apparent, at the lowest frequency used. This would lead, of course, to a false value of $\ell_{\rm u}$ being obtained if this frequency was used in the calculation of the scale of the turbulence in the expression given in 3(g) above. For this reason, the values of the computed spectral density estimates for these runs have not been tabulated, or used in analysis.

The data for runs 21 to 24 inclusive have not been fully analysed because the signal/noise ratios, though higher than those for earlier runs, were still too low for really adequate identification of a significant peak. The data have been included in Table 1 merely for possible use in transonic studies.

The spectral density estimates for runs 21 to 49 are tabulated for a range of k=1 to k=100, in Table 1, and for runs 25 to 49 (Ma ≈ 1.5 to Ma ≈ 2.2) the values of the scale of the turbulence ℓ_u and of the ratio ℓ_u/ℓ_T were calculated. For runs 33 and 49, however, the spectral density plots show very low mean values and low signal/noise ratios, and in these cases the results have not been tabulated. It is suspected that these two runs were affected by transient telemetry faults, and immediately after run 49, the transmission of data effectively ceased.

The values of ℓ_u obtained are plotted against time in Fig.7a and against Mach number in Fig.7b. The positions of runs 33 and 49 are indicated, but no values plotted. Similarly, positions are indicated, but no values plotted, for runs 33 and 49 in the plots of ℓ_u/ℓ_T against time (Fig.10) and against Mach number (Fig.11).

Values of all the environmental experimental data and derived quantities are presented in Table 3, and the vehicle trajectory data in Figs. 3, 4 and 5.

3.2 To reinforce the conclusions derived from the analyses described above, the computing technique was modified to accept the pressure readings for five consecutive runs and use them as the data for one run.

The readings for runs 41 to 45 were used as the data for this collective run, which thus contained about 30800 discrete readings, separated by intervals $\Delta t = 15.55$ microseconds. The mean velocity and mean altitude over the



considered time span were used to derive Reynolds numbers and hence $_T$, and the autocorrelation function, unsmoothed and smoothed spectral density estimates, scale $_u$, ratio $_u$ / $_T$ and root mean square pressure $_p$ were obtained as for the individual runs.

The unsmoothed spectral density estimates are shown in graphical form in Figs.9 and 12c, and the experimental environmental data are given in Table 4.

4 DISCUSSION OF RESULTS

The pressure-transducer used to measure the wall pressure was carefully mounted and inspected to ensure that the measuring diaphragm was flush with the exterior skin of the vehicle. The diaphragm diameter was 0.125 inch (3.175 mm) and the maximum projection above the surrounding surface was 0.003 inch (0.076 mm).

It was arbitrarily assumed that pressure-fluctuations having a wavelength of less than ten times the diameter of the transducer diaphragm would be attenuated to such an extent that they should be regarded as being unreliable, i.e. the highest usable frequency of pressure-fluctuation is given by

$$f = \frac{v}{10d}$$

where v is the vehicle velocity and d is the diaphragm diameter. For the range of vehicle velocities considered (i.e. 1692 ft/s to 2444 ft/s) and the transducer diaphragm diameter of 0.01042 ft, the value of f lies in the range 16.24 kHz to 23.45 kHz, corresponding to values of k = 51 to k = 73. Since all the significant peaks in the plots of spectral density against k occur well below k = 40, it is assumed that errors due to spatial resolution of the transducer may be ignored.

The data obtained from the piezo-electric accelerometer (c) showed that although the vibration local to the piezo-electric pressure transducer was of considerable amplitude in the frequency-range 100 to 300 Hz, the amplitude at higher frequencies was very much smaller, and would have negligible effect on the accuracy of the pressure data.

4.1 Comparison of experimentally determined spectra with a theoretical spectrum for homogeneous isotropic turbulence

In Ref.3, Taylor tabulated a theoretical (pressure) 2 spectrum for very high Reynolds numbers, (A = 0) in a non-dimensional form, and this was used as a basis for comparison with the experimental spectra.



Figs. 12a, b and c show respectively the experimental spectra determined for run 35 (Ma = 2.060), run 45 (Ma = 2.188) and the collective run described in section 3.2 (mean Ma = 2.193). The strong resemblance between these three spectra confirms a reasonably consistent state of turbulence throughout this portion of the available data.

The theoretical spectrum has been superimposed on each of these three figures. The full line indicates the spectrum soplaced as to have its maximum value at the same frequency as that of the maximum experimental value, and to represent the same value of $(\sigma_p)^2$ as the experimental data. The broken line indicates the spectrum placed so as to obtain a good fit with the experimental points over the high frequency range, whilst retaining the same value of $(\sigma_p)^2$.

Although the test vehicle was subject to longitudinal acceleration ranging, over the time span considered, from +22 g to -4 g, and could not therefore be regarded as being in a steady state and thus not directly comparable with the theoretical case, nevertheless a surprisingly good agreement was apparent. It may be noted that the collective run considered in Fig.12c was derived from data obtained during the time span centred on the 'zero g' point of the flight.

4.2 The scale of the turbulence, $\ell_{ m u}$

The values of ℓ_u given in Tables 3 and 4 are based on the values of f_m indicated by the coincident peaks of the experimental points, and the full-line theoretical spectrum. If the values of ℓ_u were to be based on values of f_m given by the peak of the broken-line spectrum, they would be greater by a factor of about 2.6. The values of ℓ_u obtained from the two values of f_m for the three runs illustrated in Figs.12a, b and c are given below:

Run	(Full-line) ^l u(f)	(Broken-line) ^l u(b)	^l u(b) ^l u(f)
35	0.09849 ft (0.03002 m)	0.2616 ft (0.0797 m)	2.66
45	0.08953 ft (0.02729 m)	0.2266 ft (0.0691 m)	2.53
Collective	0.08970 ft (0.02734 m)	0.2266 ft (0.0691 m)	2.53

The true value of $\,^\ell u$ associated with the pressure fluctuations due entirely to turbulence probably lies somewhere between these two extremes.

The values of ℓ_u presented throughout this paper are obtained from calculations based on the frequency of the experimental peak values. However, since the flight of the test vehicle was of such short duration, it is possible that the turbulence was not fully established and that the values of ℓ_u given are somewhat low.

The plot of ℓ_u against elapsed time Fig.7a indicates a mean value of about 0.09 ft (27.4 mm), the individual points showing very little deviation in level after 3.3 seconds. Fig.7b, a plot of ℓ_u against Mach number, again shows very little deviation from the mean above a value of Ma = 2. As the value of ℓ_u is sensibly constant over the considered range of Mach number, the values of ℓ_u/ℓ_T shown in Figs.10 and 11 also exhibit this constancy.

The mean value of ℓ_u for the runs 41 \rightarrow 45 is 0.09245 ft (28.2 mm) and ℓ_u for the collective run is 0.0897 ft (27.3 mm). The mean value of $\ell_u/\$_T$ for the runs 41 \rightarrow 45 is 0.5275, and the value for the collective run is 0.5128. Therefore, the results obtained from the collective run may be assumed to be representative of those from the individual runs.

Taylor 4 cites an example of experimental results derived from an investigation of turbulence at subsonic velocities, which are compared below with the results from the present investigation.

	Ma	R _{\$}	\$ _T	\$	^l u	^ℓ u *T	d d
Taylor	0.176	3.8 × 10 ⁴	0.328 ft (100 mm)	0.410 ft (125 mm)	0.0975 ft (29.7 mm)	0.297	0.0056
Shark	2.2	2 × 10 ⁶	0.175 ft (53.3 mm)	0.219 ft (66.8 mm)	0.0897 ft (27.3 mm)	0.513	0.0045

It is seen that although the Mach number has increased from the first case to the second by a factor of 12.5, and the turbulence Reynolds number by a factor of 53, the scale of the turbulence has remained almost the same, and the ratio $\ell_{_{11}}/\ell_{_{12}}$ has increased only by a factor of 1.73.

5 CONCLUSIONS

Measurements of boundary-layer pressure fluctuations have been made for a short flight-time on a free-flight aerodynamic test vehicle at Mach numbers in the range 1.5 to 2.2, and local Reynolds numbers of 1.51 to 2.15×10^8 .



Autocorrelation functions and (pressure)² spectral densities have been determined and are presented in tabular and graphical form. Representative experimental spectra are compared with a theoretical spectrum for homogeneous isotropic turbulence and are found to be in reasonable agreement.

Values have been obtained of the scale of the turbulence, and the ratio of scale of the turbulence to turbulence boundary-layer thickness, and these are similarly presented. Over the considered range of Ma = 1.5 to Ma = 2.2, the scale of the turbulence was found to fluctuate between 0.47 and 0.76 of the turbulence boundary-layer thickness. At Ma = 2.2 the scale of the turbulence was about 0.51 of the turbulence boundary-layer thickness, and the root mean square pressure was 0.0045 of the free-stream dynamic pressure.



Table.1 Page 1 of 8 pages

11/1-00	IENCY FA		• (100	FRESSU	KE UNII	⇒ - ∪•.	3116 LB/SQ	INCH)
k			R	UN NUMB	ER			
	21	22	23	24	25	26	27	
123456789012345678901234567890123456789012344444444456	902914252275398255515049716246279047773941559073991	3453553344343223223111111111 641959721400216223223111111111 11159888053628966775237 111111111111111111111111111111111111	4343544311286660299646770728779276809440458757486558 4389750621128666023322332212121111 1 1118 11588757486558	621118122323422221221321183100130041763565134084933541 62067812041933883434371883100130041763565134084933541 11111 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	2611783168430864433321222121212111111118995938314 951783364544644433221222121212111111189959388867	23424449570065540932785660454286467890599772683258 234243244555544565443212212222211121212111111 11 11	2476626707725993336800150366971370781807366445479 5720476267077259933368001503669713211111111111111111111111111111111111	



Table. 1. Page 2 of 8 pages

k	JENOT I'A	O TOK		UN NUMB	ER			
	<u> </u>			_				
	21	22	23	24	25	26	27	
555555555666666666677777777778888888888	33422313221221111111111 11111 1 84869270255072897765	41359137230587434909965329381637948568800 1100000000000000000000000000000000	4453552444323222121222212123993701947553877928079912 1022431111111111111111111111111111111111	41732888650318831043076661367088173737371586136570817	755667465655444453355322222343323205463222211212223 7556674656554444533553222223433232054632222211212223	8675584875544333353333243122322223215742212112211121 15742212112211121	9816049324185198166319261638604068271174277393748326 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	



Table.1. Page 3 of 8 pages

	ENCY FA							
k			R	UN NUME	ER			·
	28	29	30	31	32	33	34	
1234567890123456789012345678901234567890123456789012345678901234567890	398 1100 1100 1100 1100 1100 1100 1100 11	103915265647816097965374309455950915070244187967741 1111111111111111111111111111111111	95768658256807625962750336302268894119139572001879587 9212221168682342596275033652334543332331232212211	769740970017317695893028 9037759700173176958930477555754444533323222222 934332221212432232221 877747555754444544333323222222 93433222212124322322221	15819620934979178773739218138032841572860081402063 124242222323232323223453921109968644545454728600814402063	4465433442432323212111111111 1 1 99785575545498787 4465433442432323212111111111 1 1 1 1 1 1 1 1 1 1 1	4312857697103215861431614226711198471908786611503333333333333333333333333333333333	



Table 1. Page 4 of 8 pages

FREQU	ENCY FA	CTOR "k	". (100	PRESSU	RE UNITS	S = 0.3	116 LB/SQ	INCH)
k			R	UN NUMB	ER			
51	28 120	29 123	30 184	31 210	32 196	33 39 38	34 265	
55555555566666666667777777778888888889999999999	111118789687556567635544533442347940351630890488993 111118789687556567635544533442343314333323222312121	1111 1110 1110 1111 1111 1111 1111 111	1474333200734680686707743970226245844733138479120577433	11111111111111111111111111111111111111	222112111111111111 3441255337732601559818670158284125377022238803999216238 1138277022238803999216238	389020113941537847647685941694758891866411159767787	21221221221111112111211111 1111 1111 1	



Table 1. Page 5 of 8 pages

rkegu	UENCY F	ACTOR *	k * • (100	PRESSI	URE UNI	TS = 0.	3116 LB/SQ	INCH)
k			F	RUN NUMI	BER			
	35 664419200556223246482524991674388435324684353333333333333333333333333333333333	3 88887233584438444450281178888465735576348999186555298153448438444450281178888465735575576348999186555298117888846573557634899918655529811788884657355763489991865552981178888465735576348991865552981178888465735576348991				0 8316985110184639851101111111111111111111111111111111111	3116 41 1989 41 1989 41 1989 41 13556 14111 15111 16111 1	INCH)



Table 1. Page 6 of 8 pages

	JENUY FA	0.01		UN NUMB			110 12730	1110117
K			<u></u>	ON NOME	h-1 \			
	35	36	37	38	39	40	41	
555555555666666666677777777778888888888	232323221311212112112111111111111	322222213234222221111111111111111111111	4234073144304783832209998065271719342615628503308924 4234731443047838322112111111111111111 107899924 10899924	3232222222111221111 111111111111169618536264507 32322222211122111 11111111111111169618536264507	23232322121111221111211111111111111111	222122207711892505004189202072986072279410666979389 2221222211221111111111111111111111111	322222322221212222121111111111111111187881227 2222222222	



Table.1. Page 7 of 8 pages

1 IVE &	JENCY FA	ACTOR "I	(*. (100) PRESSI	JRE UNI	rs = 0.	3116 LB/	SQ INCH)
k			F	RUN NUME	BER			
	42	43	44	45	4 6	47	48	49
1234567890123456789012345678901234567890 11111111112222222223333333333344234567890	149860505230022200482214493082929384622092759364499999229 1498605050523002226333332216512293846220927666666666664444499999229	1118744452680177316548807732668593543299887996046291300992 101887445268017731654880732668593543299887996046291300992	19837733741177158932469832164762704980943496353445580 198377337411171123334322111 168532180943496353445580	91111111111111111111111111111111111111	1699815853944587062640844131824700052736130354946294 115771385394458706264084413077666756634445773354455780 116988539944587062640844131776667566344457733544553355455780	2121112297867514058735644251100053433778693443845443370 21211111111111221322132211111 21211111111	0 4719222437104344359319422076201686853540953524981481 1211121112111 12113211111 12113211111 1465745655575565665444081	2 5921835559861258365267477215488912994069933990866375240 111111111111111111111111111111111111



Table.1. Page 8 of 8 pages

FREQU	ENCY FA	CTOR "k			RE UNIT	S = U•)		SQ INCH)
k		·	R	UN NUMB	ER	<u>-</u>		
	42	43	44	45	46	47	48	49
5555555556666666666667777777777888888888	449679026502324184178953758537452502129032049575975 4496779026502324184332222222222222233322222222222222	453442332222222221221111111111111211111111	453134713104532758645334613333307182316900225537921 453134713104532758645334613333307182316900225537921	7621679029284316635533981111111111121897160943996601 13113111111111111111111111111111111	63457451980662284235838478885618153750013623520807158 6345640153322222211112111111111111111111111111	64778183140518791972503513947789936093674511592113 419943183140518791972503513947789936093674511592113 222	556556554342333323222232112322212321232122222222	68657555495463084445567453443544442644464334828 1091656536034630844455674546967518237024237874828 1091656536034630844455674544354444264446433488



Table.2
Frequency and corresponding frequency factor 'k'.

k	FREQUENCY (Hz)	k	FREQUENCY (Hz)	k	FREQUENCY (Hz)	k	FREQUENCY (Hg)
1	0.00	26	8119.77	51	16239.55	76	24359.32
2	324.79	27	8444.57	52	16564.34	77	24684.12
3	649.58	28	8769.36	53	16889.13	78	25008.91
4	974•37	29	9094.15	54	17213.92	79	25333.70
5	1299.16	30	9418.94	55	17538.71	80	25658.49
6	1623.95	31	9743•73	56	17863.50	8 1	25983.28
7	1948.75	32	10068.52	57	18188.30	82	26308.07
8	2273.54	33	10393.31	58	18513.09	83	26632.86
9	2598.33	34	10718.10	59	18837.88	84	26957.65
10	2923.12	35	11042.89	60	19162.67	85	27282.44
11	3247•91	36	11367.68	6 1	19487•46	86	27607.23
12	3572.70	37	11692.48	62	19812.25	87	27932.03
13	3897•49	38	12017.27	63	20137.04	88	28256.82
14	4222•28	39	12342.06	64	20461.83	89	28581.61
15	4547.07	40	12666.85	65	20786.62	90	28906•40
16	4871.86	41	12991.64	66	21111.41	91	29231.19
17	5196.66	42	13316.43	67	21436.21	92	29555•98
18	5521.45	43	13641.22	68	21761.00	93	29880.77
19	5846.24	44	13966.01	69	22085.79	94	30205•56
20	6171.03	45	14290.80	70	22410.58	95	30530.35
21	6495.82	46	14615.59	71	22735•37	96	30855•14
22	6820.61	47	14940.39	72	23060.16	97	31179.94
23	7145 • 40	48	15265•18	73	23384.95	98	31504.73
24	7470.19	49	15589.97	74	23709.74	99	31829.52
25	7794•98	50	15914.76	7 5	24034.53	100	32154.31



49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25		RUN NO•
2397	2409	2418	2426	2434	2440	2444	2444	2439	2429	2413	2392	2365	2333	2295	2241	2202	2149	2091	2030	1965	1899	1830	1762	1692	(ft/s)	VELOCITY
1456	1412	1368	1324	1280	1236	1192	1148	1104	1060	1018	976	944	868	850	800	766	726	682	640	606	572	538	504	470	(ft)	HE I GHT
2.1564	2.1668	2.1746	2.1814	2.1883	2.1933	2.1966	2.1962	2.1914	2.1821	2.1674	2.1482	2.1237	2.0944	2.0601	2.0113	1.9761	1.9282	1.8759	1.8209	1.7624	1.7030	1.6409	1.5797	1.5168		MACH NO.
209.503	210.775	211.787	212.713	213.642	214.396	214.976	215.206	214.995	214.342	213.148	211.509	209.284	206.836	203.556	199.010	195.709	191.185	186.226	180.980	175.331	169.584	163.559	157.613	151.478	$(x10^{-6})$	× ×
2.598	2,612	2.623	2.634	2.644	2.653	2.659	2.662	2,660	2.652	2.639	2.620	2.595	2.568	2.531	2.479	2.442	2.390	2.334	2.274	2.209	2.143	2.074	2.005	1.934	-6 (x10 ·)	,

0.176	0.176 21 0	0.175 20 0	0.175 21 0	0.175 22 0	0.175 21 0	0.175 21 0	0.175 21 0	0.175 22 0	0.175 20 0	0.175 21 0	0.175 20 0	0.176 22 0	0.176 19 0	0.176 19 0	0.176 20 0	0.177 *	0.177 19 0	0.178 13 0	0.178 13 0	0.178 14 0	0.179 13 0	0.180 14 0	0.180 17 0	0.181 14 0	(ft)	3 ×
*	0.31045	0.34427	0.39302	0.37992	0.41882	0.36527	0.32889	0.39923	0.43791	0.39385	0.41451	0.42521	0.79481	0.56015	0.50926	*	0.57465	0.40512	0.40361	0.20259	0.12635	0.09150	0.06800	0.06335		MAX
•	0.09303	0.09830	0.09370	0.08953	0.09424	0.09439	0.09439	0.08971	0.09875	0.09319	0.09725	0.08699	0.10012	0.09849	0.09111	*	0.09222	0.13459	0.13067	0.11676	0.12223	0.10873	0.08506	0.10054	(ft)	JOHUE
*	0.5307	0.5602	0.5342	0.5106	0.5376	0.5386	0.5387	0.5119	0.5634	0.5314	0.5541	0.4952	0.5693	0.5592	0.5163	*	0.5207	0.7578	0.7342	0.6541	0.6825	0.6056	0.4720	0.5559		SCALE/\$
0.2229	0.2475	0.2321	0.2146	0.2111	0.2167	0.2130	0.2261	0.2100	0.2126	0.2094	0.2112	0.2171	0.2269	0.2232	0.2184	0.0835	0.2355	0.2152	0.1842	0.1544	0.1332	0.1148	0.1015	0.0964	(lbf/in)	PRESSURE
49	4 8	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25		NO.

Table.4
Environmental data and derived quantities for collective run.

	IMPERIAL UNITS	UNITS	S.I. UNITS	NITS
MEAN VELOCITY	2440	ft/s	743.7	s/w
MEAN HEIGHT	1192	ft	363•3	E
MEAN MACH NUMBER	2•193			
R (X10) X	214.64			
R (X10 ·)	2.656			
₩	0.175	ft	0.053	ш
π E	22			
S.D. MAX	0.3562			
SCALE	0.0897 ft	ft	0.027	ш
SCALE/\$	0.5128			
R.M.S. PRESSURE	0.2153 lbf/in	.bf/in	1.4844 KN/m	2 kn/m



SYMBOLS

A	non-dimensional viscosity function
LU	l _u /a
Ma	Mach number
M	number of correlated intervals used
N	number of intervals in one run
R _{\$}	boundary-layer Reynolds number
R _{\$T}	turbulence Reynolds number
$\mathtt{R}_{\mathbf{x}}$	local Reynolds number at distance x from the effective start of turbulence
R_{τ}	autocorrelation function for interval τ
U	stream velocity relative to measuring station
$\left. egin{array}{c} v_k \\ sd_k \end{array} \right\}$	unsmoothed spectral density estimate at frequency $f_{\mathbf{k}}$
SD max	maximum value of unsmoothed spectral density estimate for one run
W _k	hanning-smoothed spectral density estimate at frequency $f_{\mathbf{k}}$
X	ωα
z _k	hamming-smoothed spectral density estimate at frequency $f_{\mathbf{k}}$
a	a length to make wave numbers non-dimensional
fk	frequency corresponding to frequency factor k (see Table 2)
$f_{\mathfrak{m}}$	frequency at which the maximum spectral density occurs
k	integer included between 1 and (M + 1); the frequency factor corresponding to frequency $f_{\mathbf{k}}$
k _m	frequency factor corresponding to f_{m}
^l u	the scale of the turbulence
$^{ riangle}$ t	time interval separating each discrete value y from the next
y _t	recorded discrete pressure value at time $$ t, referred to the mean value for that run
\$	total boundary-layer thickness
$\mathbf{\hat{s}_{T}}$	turbulence thickness



SYMBOLS (Contd)

- τ integer included between 1 and (M + 1)
- ω wave number = 2π/wavelength
- o root mean square pressure
- q free stream dynamic pressure

,

-



REFERENCES

No.	Author(s)	Title, etc.
1	E.S. Mallet	'BRAMBLE' - An automatic processing system for
	R.E. Perkins	telemetry data.
	H.W.P. Knapp	RAE Technical Report 65053 (1965)
2	A. Cantin	An autocorrelation and power spectral density analysis
	D. Heckman	computer program for random signals.
	R. Gouge	CARDE Technical Report 565/67 (1967)
3	J. Taylor	The energy and pressure spectra of homogeneous
		isotropic turbulence.
		RAE Technical Report 66346 (ARC 29097) (1966)
4	J. Taylor	Kinetic heating of aircraft structures.
		15 Mitchell Memorial Lecture of the RAeS, February 1968
5	D.B. Spalding	A new analytical expression for the drag of a flat
		plate valid for both the turbulent and laminar regimes.
		Int. J. of heat and mass transfer, Vol.5 (1962)

_



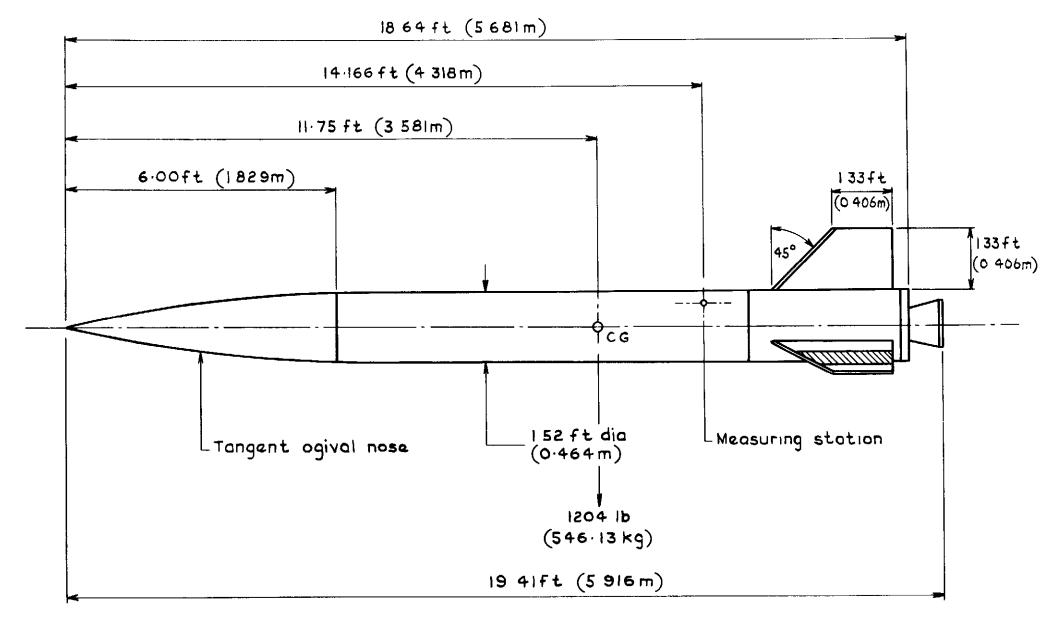


Fig. la General arrangement of 'Shark I'



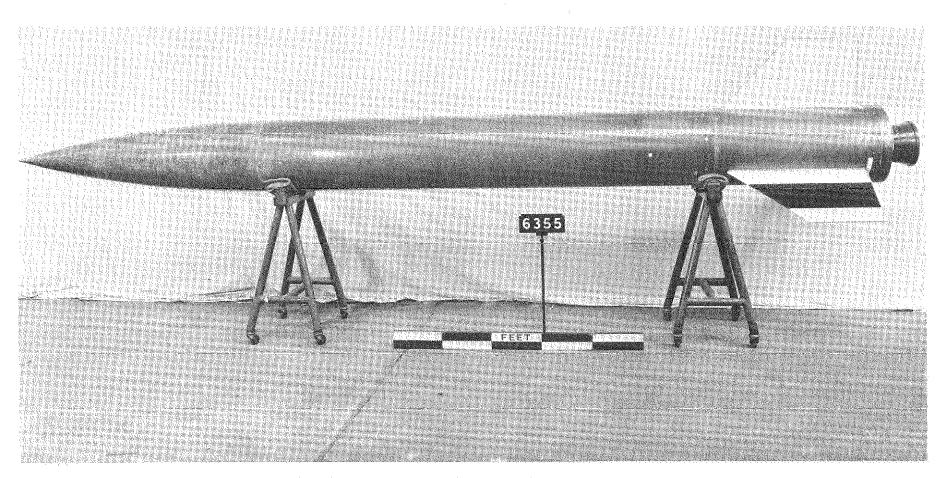
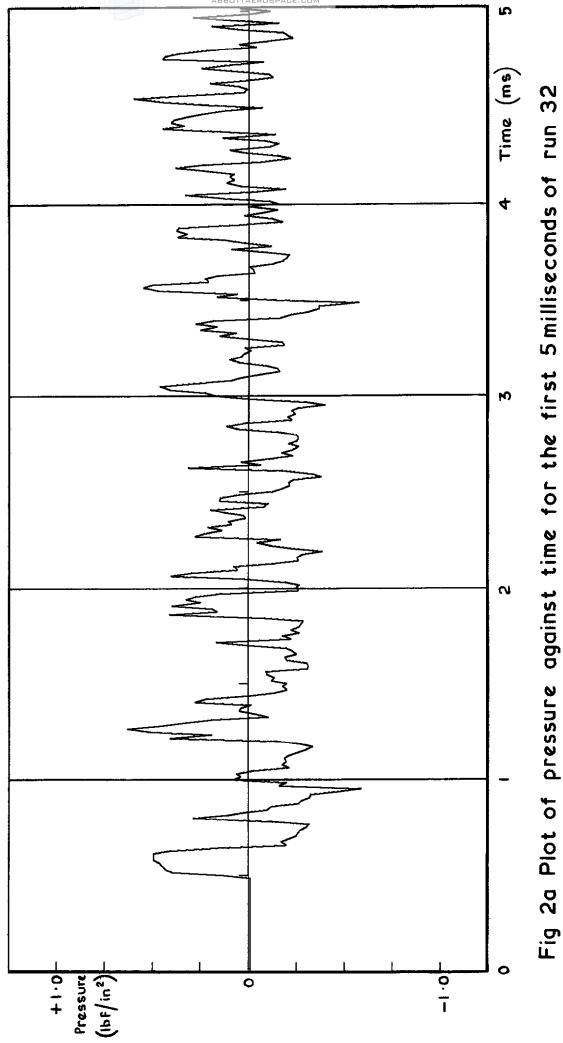


Fig1(b) Side view photograph of 'Shark 1'



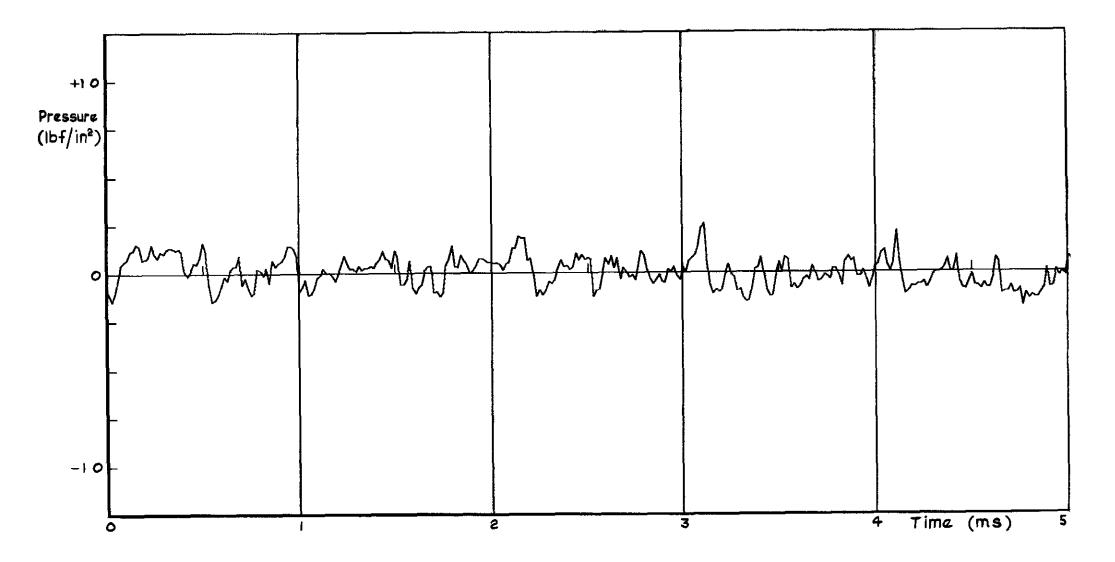


Fig.2b Plot of pressure against time for the first 5 milliseconds of run 33

- N

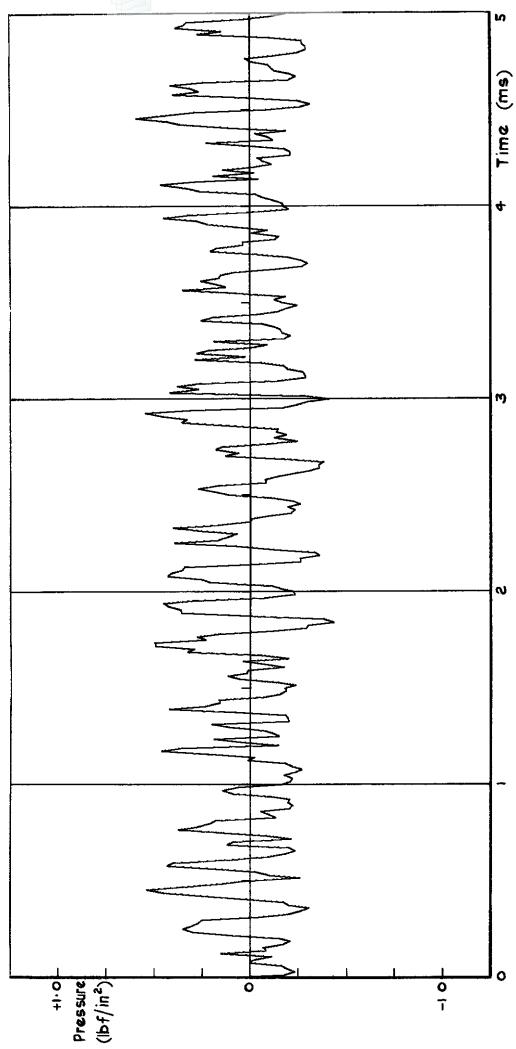


Fig. 2c Plot of pressure against time for the first 5 milliseconds of run 34

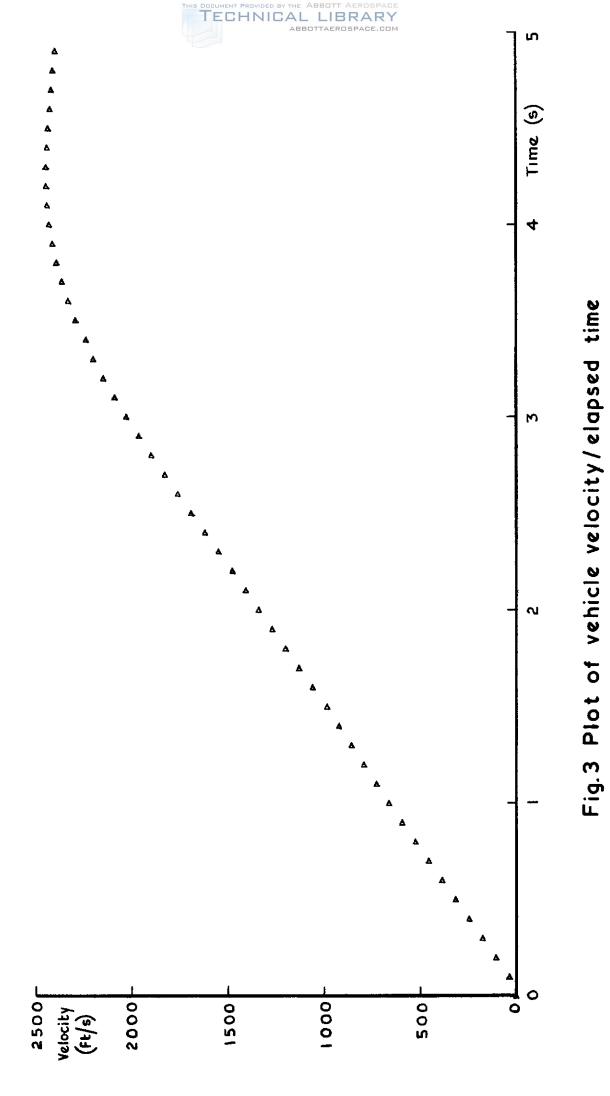


Fig. 4 Vehicle altitude/elapsed time

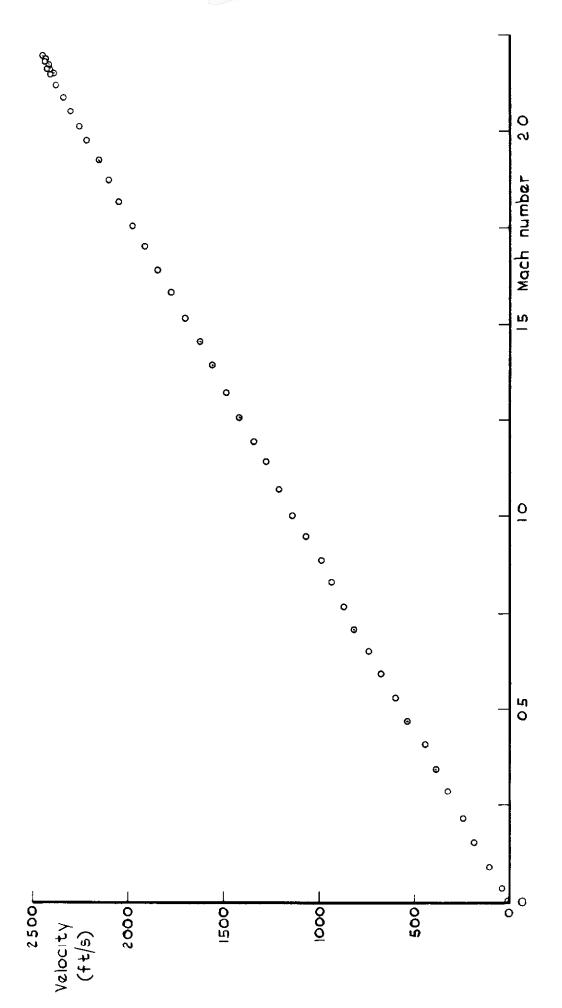
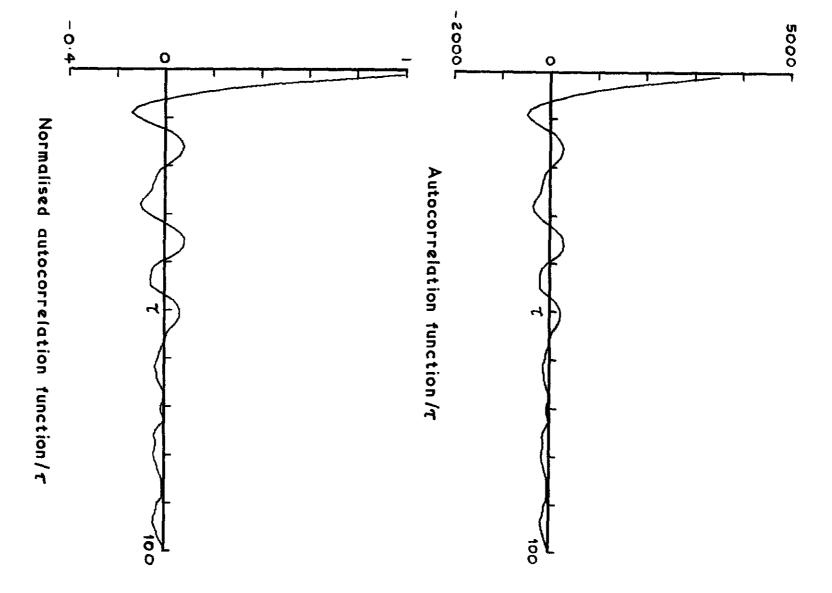


Fig 5 Plot of vehicle velocity/Mach number



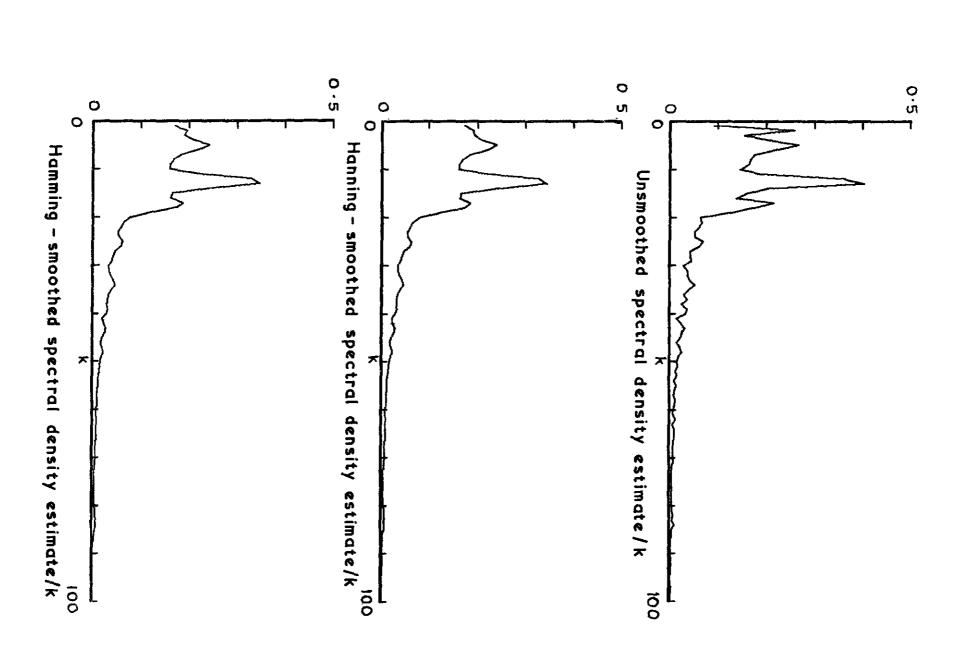
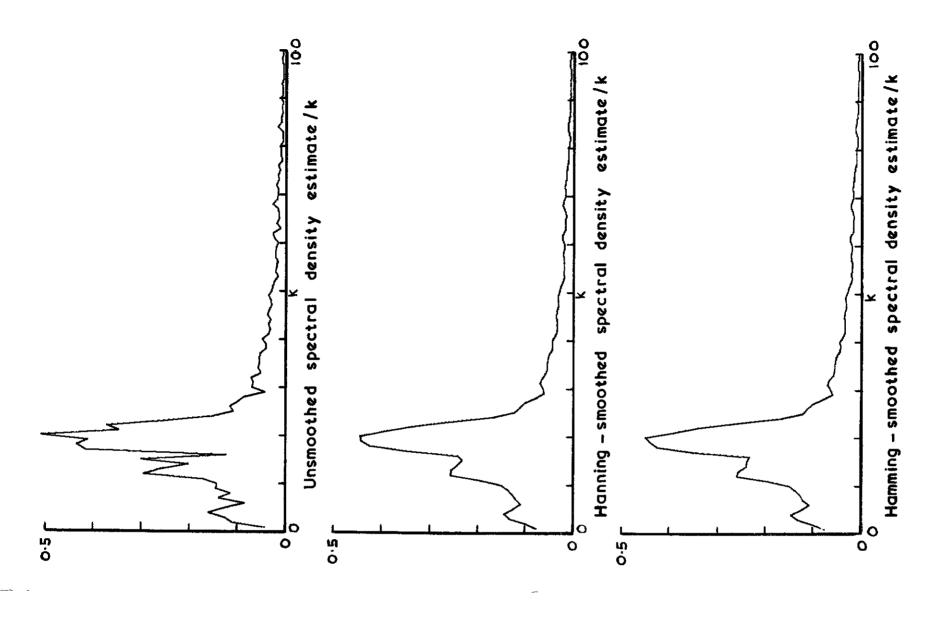


Fig. 6a Plots of derived quantities for run 30



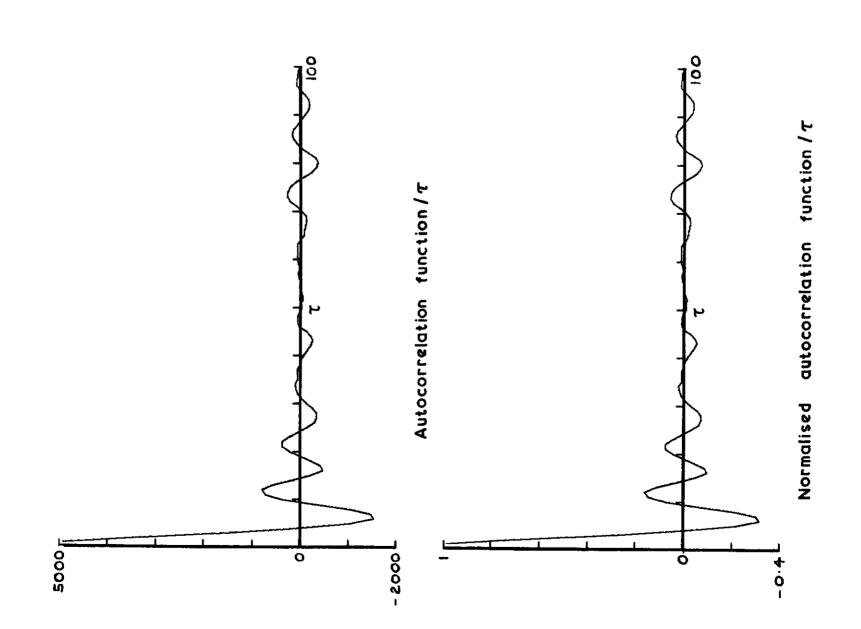
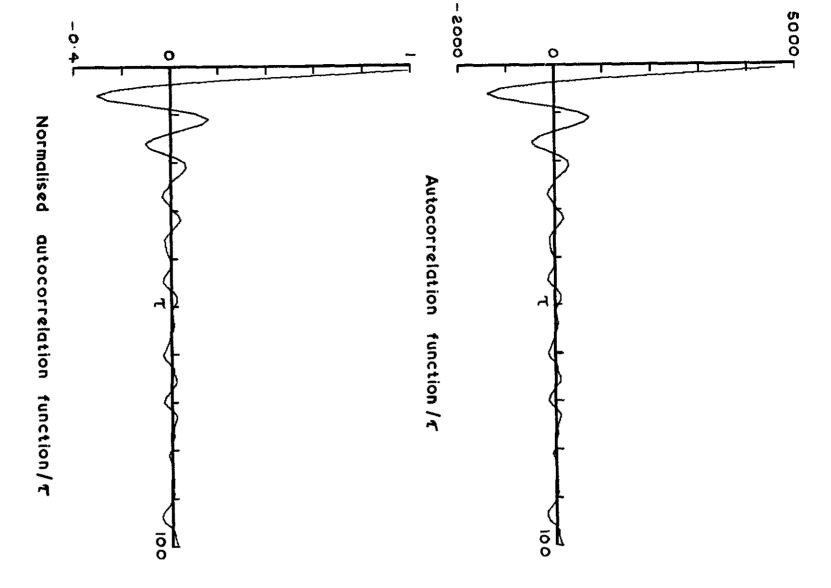


Fig.6b Plots of derived quantities for run 34



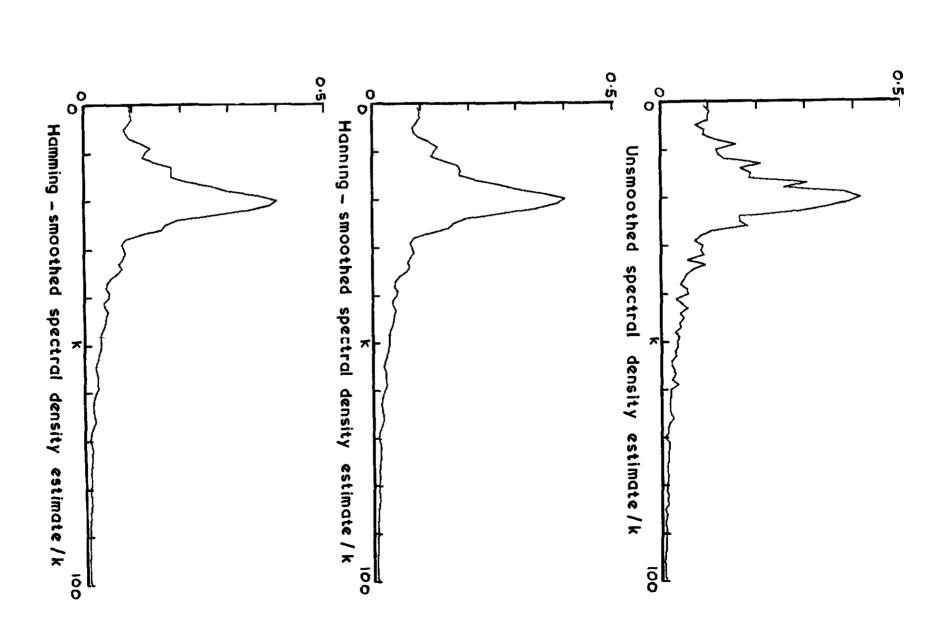
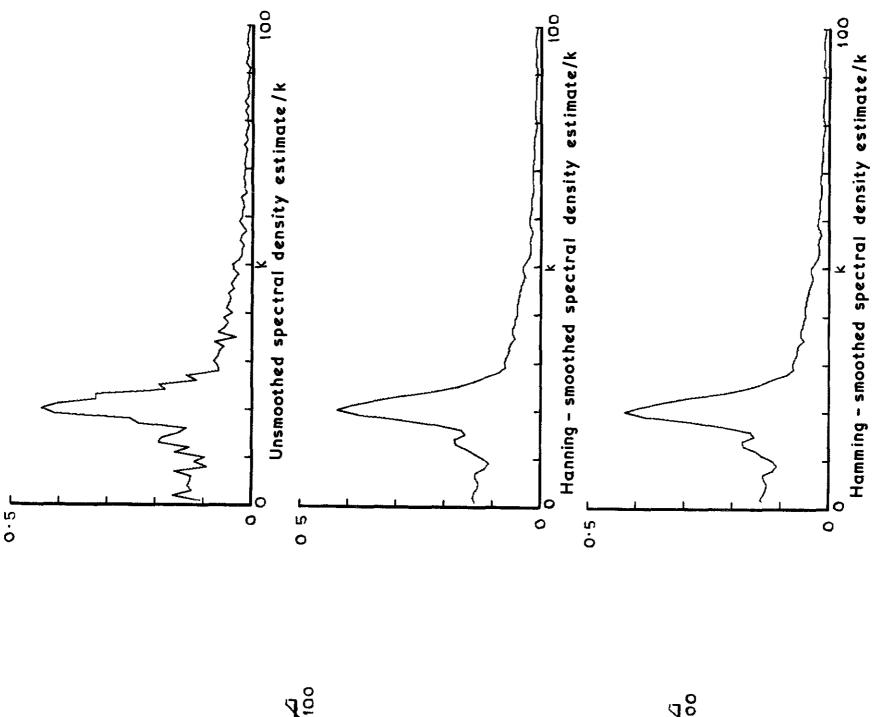


Fig.6c Plots of derived quantities for run 38



Autocorrelation function/au

- 2000

0

5000r

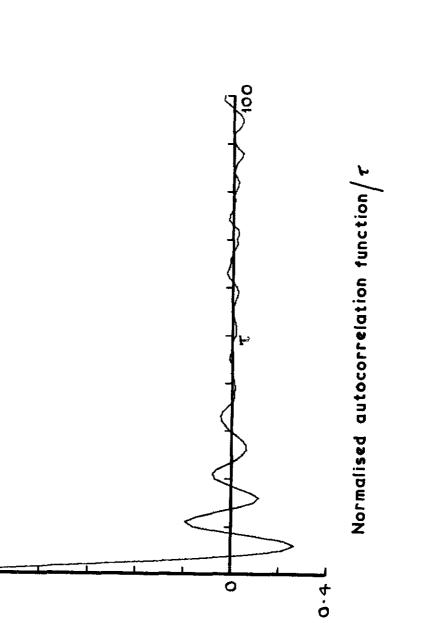
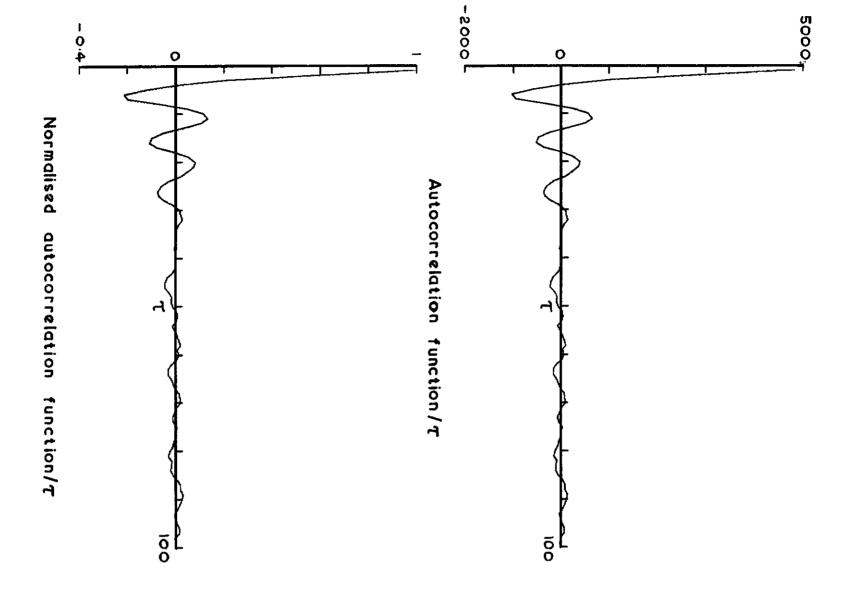


Fig. 6d Plots of derived quantities for run 40



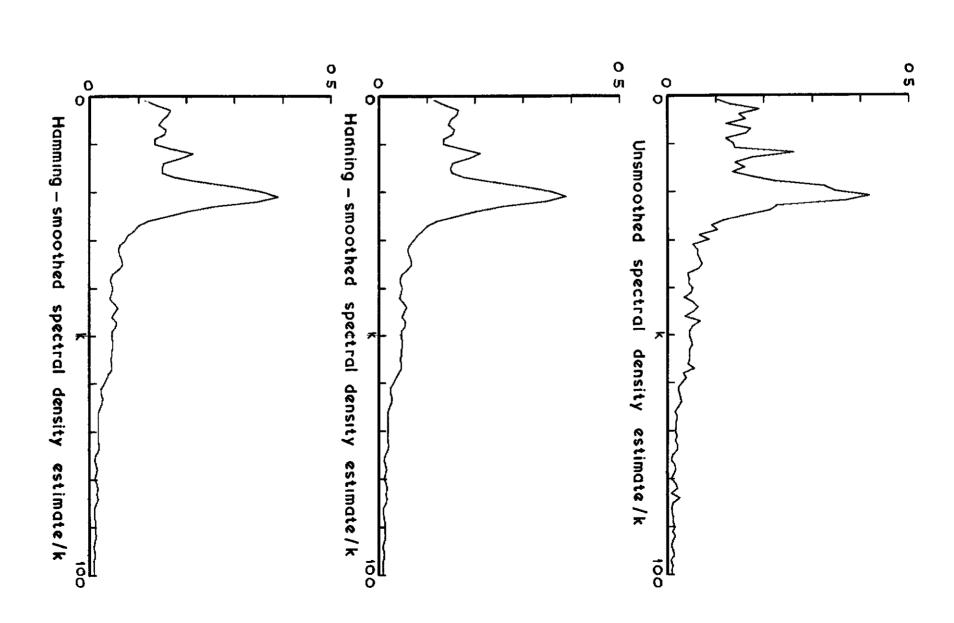


Fig. 6e Plots of derived quantities for run 44

Fig. 7a Scale/elapsed time

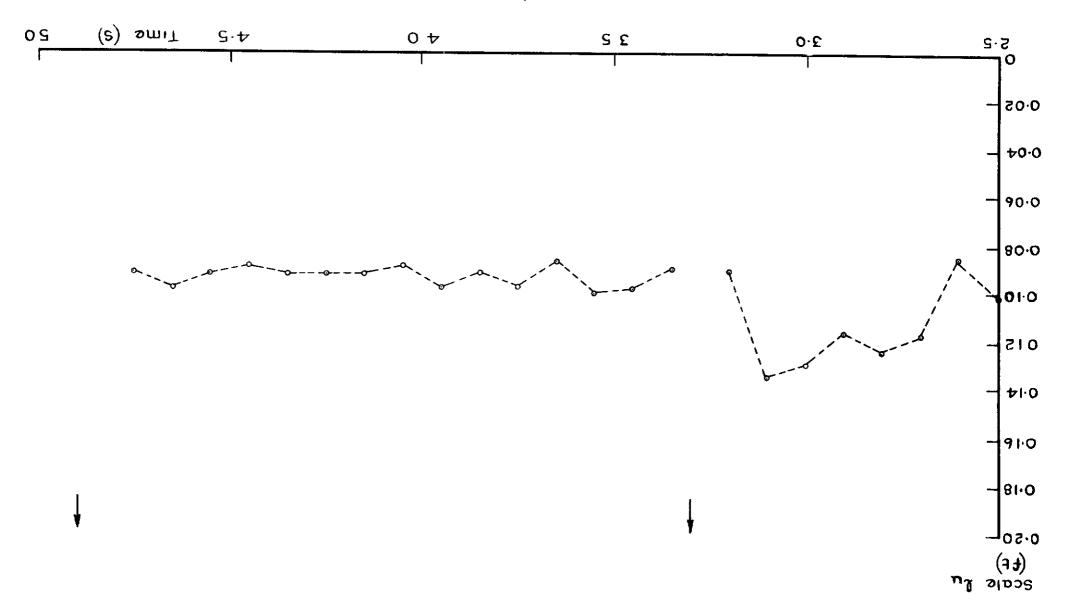
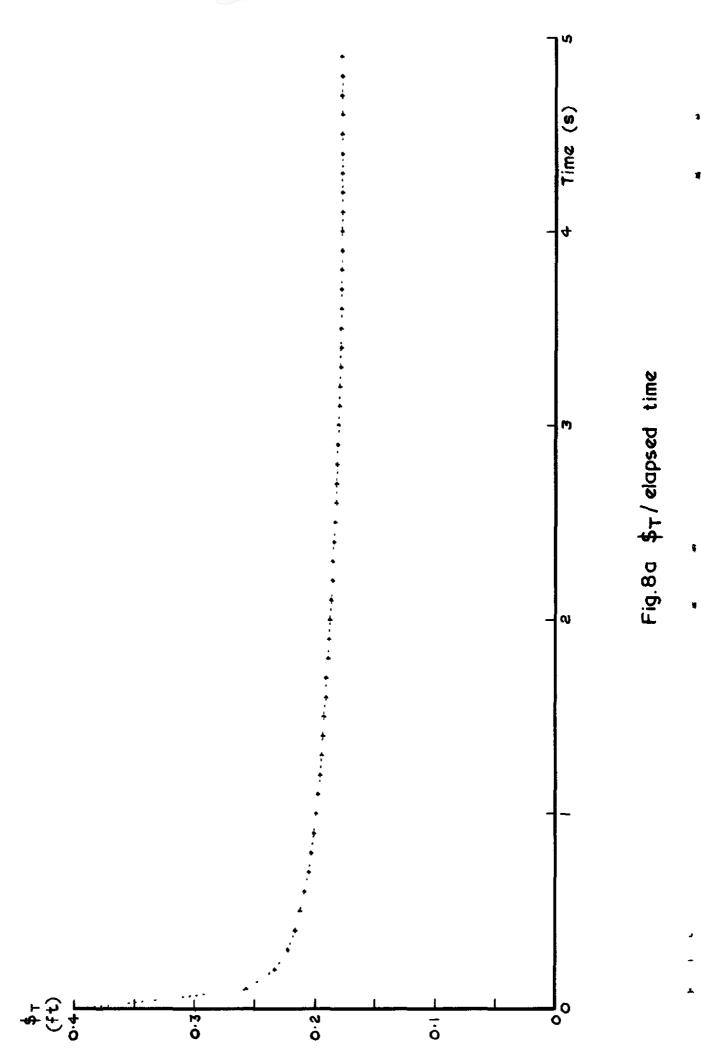


Fig 7b Scale/Mach number



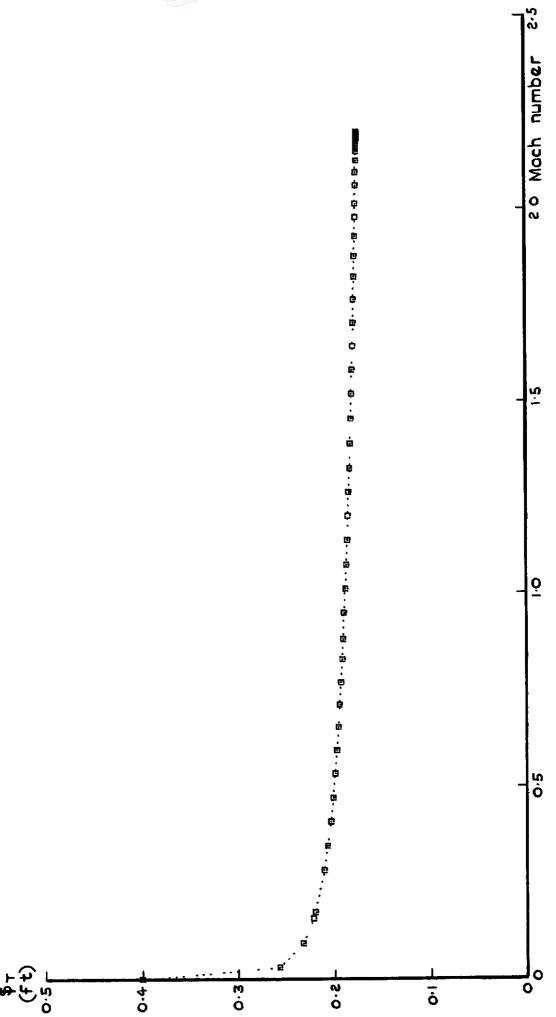


Fig.8b \$7/Mach number



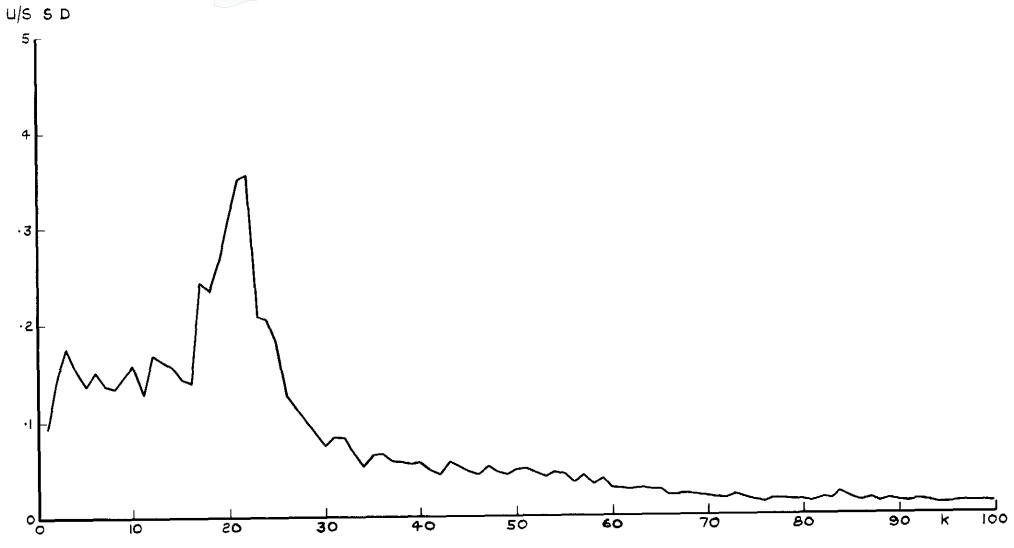
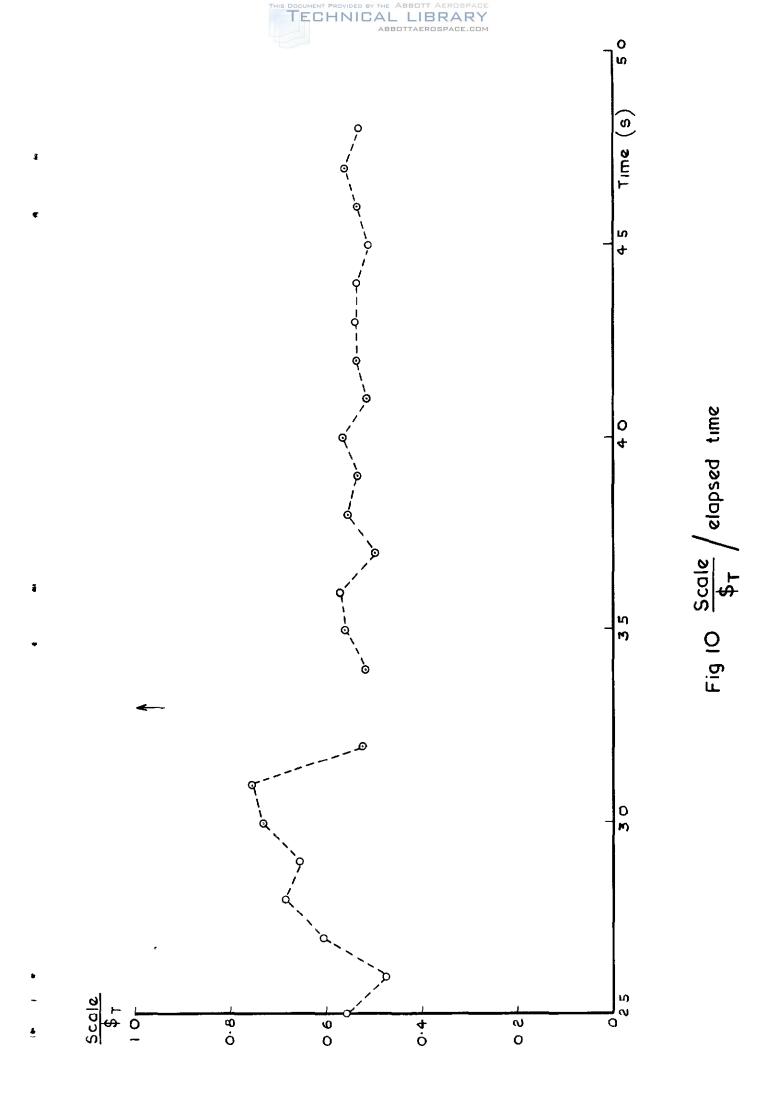
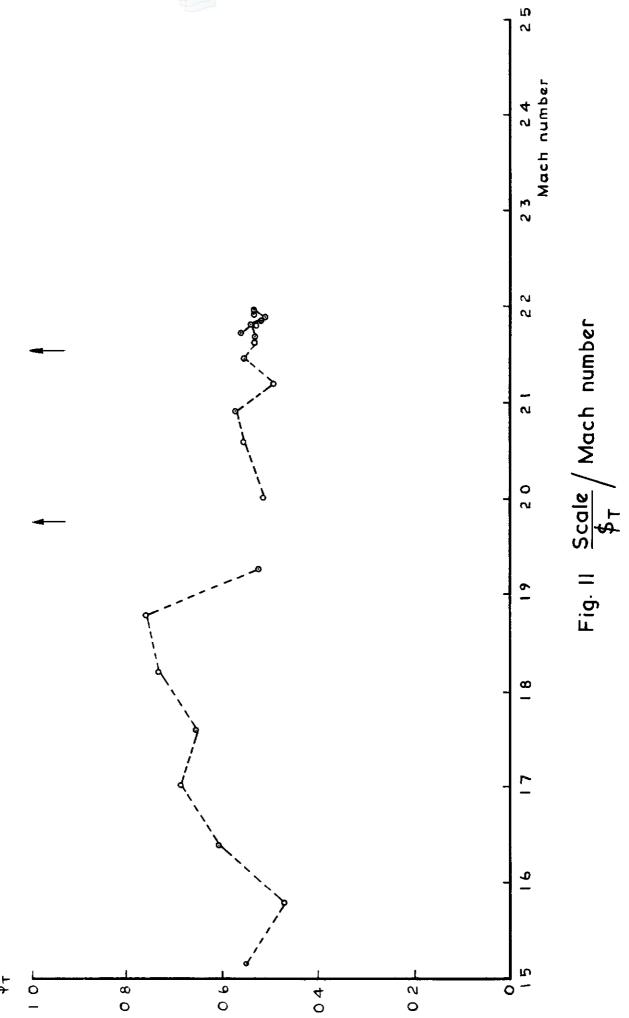


Fig 9 Collective run-unsmoothed spectral density/k

y i e





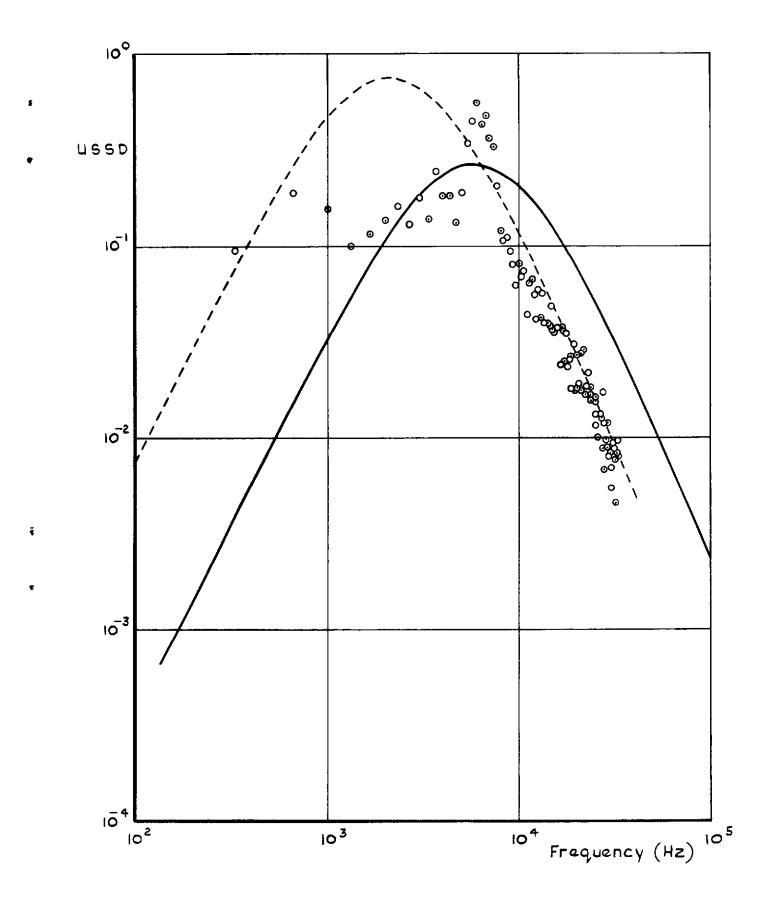


Fig. 12a Plot of unsmoothed spectral density (USSD) against frequency, for run 35

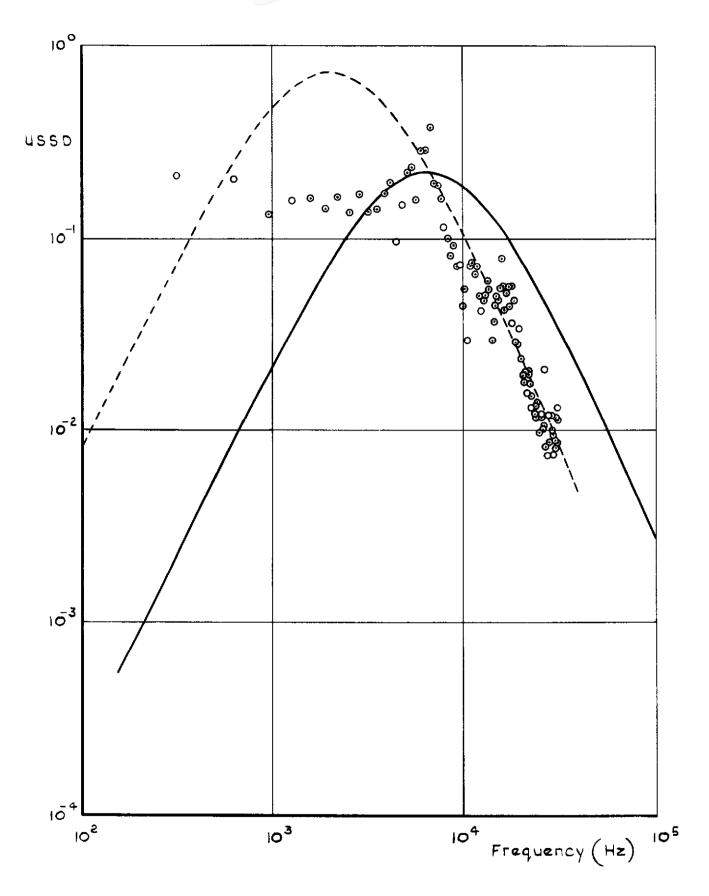


Fig 12b Plot of unsmoothed spectral density (USSD) against frequency, for run 45

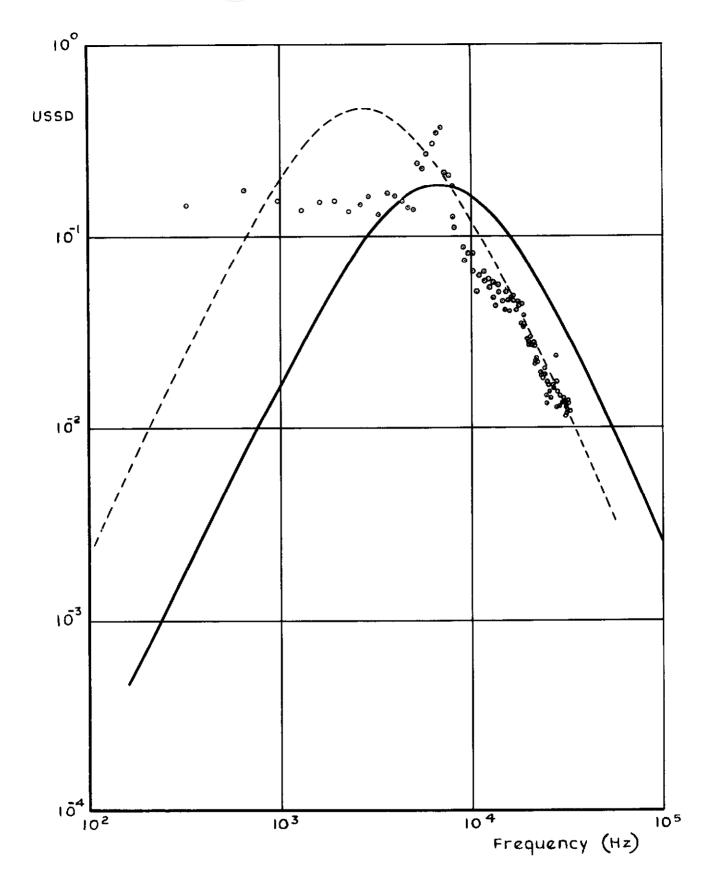


Fig.12c Plot of unsmoothed spectral density (USSD) against frequency, for collective run

•

*

† -

.

Ť



DETACHABLE ABSTRACT CARD

ARC CP No 1208
March 1971

Roberts D R

BOUNDARY-LAYER PRESSURE FLUCTUATIONS
AT HIGH REYNOLDS NUMBERS ON A FREEFLIGHT TEST VEHICLE

532.526,4
533 6 048 2
533 6.011 12
533 6.011 12
533 6.011 5

Measurements have been made of the boundary-layer pressure fluctuations on the body of a free-flight aerodynamic test vehicle powered by a solid-fuel rocket motor. The vehicle reached a maximum Mach number of 2.2 with a maximum Reynolds number of about 215 millions.

Pressure spectra have been deduced, and have been found to compare reasonably with a theoretical spectrum for homogeneous isotropic turbulence

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1.5 to 2 2, while being essentially equal to 50% of this thickness over the range Ma = 2.0 to Ma = 2 2

At Ma = 2 2 the root mean square boundary-layer pressure was equal to 0 0045 of the free stream dynamic pressure

free stream dynamic pressure

At Ma = 2.2 the root mean square boundary-layer pressure was equal to 0.0045 of the

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1 5 to 2 2, while being essentially equal to 50% of this thickness over the range $\,$ Ma = 2 0 to Ma = 2 2

theoretical spectrum for homogeneous ractropic turbulence

гиощіш с

Reastments have been made of the boundary-layer pressure fluctuations on the body of a free-flight acrodynamic test vehicle a free-flight amaximum Reynolds mumber of 2 2 with a maximum Reynolds number of about

FLICHT TEST VEHICLE
BOUNDARY-LAYER PRESSURE FLUCTUATIONS
AT HIGH REYNOLDS NUMBERS ON A FREE-

233 6 011 12 533 6 048 2 533 6 048 2 532 526 4

Roberts, D R

March 1971 ARC CP No 1208 ARC CP No 1208
March 1971

S32.526,4
533.6 048 2

Roberts, D R

BOUNDARY-LAYER PRESSURE FLUCTUATIONS
AT HIGH REYNOLDS NUMBERS ON A FREE-

FLIGHT TEST VEHICLE

Measurements have been made of the boundary-layer pressure fluctuations on the body of a free-flight aerodynamic test vehicle powered by a solid-fuel rocket motor. The vehicle reached a maximum Mach number of 2.2 with a maximum Reynolds number of about 215 millions.

Pressure spectra have been deduced, and have been found to compare reasonably with a theoretical spectrum for homogeneous isotropic turbulence

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1.5 to 22, while being essentially equal to 50% of this thickness over the range $Ma \approx 2.0$ to Ma = 2.2

At Ma = 2 2 the root mean square boundary-layer pressure was equal to 0 0045 of the free stream dynamic pressure







C.P. No. 1208

© Crown copyright 1972

Published by HER MAJESTY'S STATIONERY OFFICE

To be purchased from
49 High Holborn, London WC1 V 6HB
13a Castle Street, Edinburgh EH2 3AR
109 St Mary Street, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
50 Fairfax Street, Bristol BS1 3DE
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY
or through booksellers

C.P. No. 1208

SBN 11 470476 7