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A Comparison of the Surface-Source Solution with an Exact Solution for the Twodimensional Inviscid Flow about a Slotted-Flap Aerofoil

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SUMMARY

An exact analytic test case for the twodimensional inviscid flow about a slotted-flap aerofoil is compared with a numerical solution by a surface-source method. Some of the main causes of error in the surface-source method are identified and a general scheme for producing consistent solutions is proposed.

** Replaces RAE Technical Report 72008 - ARC 33611*

CONTENTS

1 INTRODUCTION

A recent report[1] has demonstrated that the incompressible flow about an aerofoil fitted with a slotted flap is relatively simple. The solution depends upon the inviscid flow and its interaction with the wakes and boundary layers. It was suggested in Ref.1 that confirmation of the accuracy of the method used for calculating these inviscid flows would be advantageous. At present these flows can only be calculated by numerical methods. The most suitable approach appeared to be the surface-source method of A.M.O. Smith[2]. However, to the author's knowledge, its performance had been judged only on its capacity to predict the flow about single aerofoils. An exact test case for the inviscid flow about two adjacent, lifting aerofoils has been produced by the present author (see Appendix B), so that it became possible to make a comparison between this and the results of the surface-source method applied to calculate the flow about a slotted-flap aerofoil. The present Report gives details of this comparison.

The examination of the performance of the numerical method against an exact test case cannot prove any general results about the best way of approximating the aerofoil surfaces by a distribution of straight-line elements. However, by the examination of several different distributions, some general trends are indicated and this leads to the definition of a general form of distribution, which produces consistent solutions and avoids some of the grosser errors.

2 DISTRIBUTION OF THE SOURCE ELEMENTS

In the surface-source method, the aerofoil is approximated by a distribution of straight-line source elements placed on the surface of the aerofoil. In Appendix A, the method is briefly summarised and it is seen that the determination of the strength of the line sources forms the crux of the method, whilst the distribution of the elements is not specified. However, the solution is dependent on the way the elements are distributed and different distributions lead to markedly different solutions. In this section, the effect of several different distributions on the solution is examined.

2.1 Distribution of elements on each aerofoil

A short description of the exact test case is given in Appendix B. The coordinates for the main aerofoil and the flap of the test case are listed in Table 1 and the aerofoils are illustrated in Fig.1. The corresponding pressure distributions for a flow at zero angle of incidence for the main aerofoil obtained by the exact method are shown in Figs.2 and 3. The initial distribution of elements, which have as their end points the coordinates of Table 1, had a total of 122 elements divided equally between the aerofoils. This set of elements was deliberately chosen to be 'irregular' in that:

(a) on both the main aerofoil and the flap, the elements on the lower surface are shorter near the trailing edge and longer near the leading edge than those on the upper surface

(b) on the flap, the elements are generally shorter on the upper surface than on the lower.

The forces obtained from the surface-source method with this 'irregular' distribution of elements are compared with the exact forces in Table 2(a). There is a large error in the overall forces and the pressure distributions do not contain the essential features of the exact solution. In particular, there is a cross-over in the pressure distribution near the trailing edge of the main aerofoil, as illustrated in Fig.4.

In fact, this feature indicates the main cause of error in this solution. The two elements adjacent to the trailing edge are of different length. The element on the lower surface is shorter than the element on the upper surface. The approximate form of the Kutta-Joukowski condition used in the surface-source method expresses the equality of the velocities at the midpoints of these elements. This is a poor approximation to the Kutta-Joukowski condition in this case, so that the estimation of the circulation, a vital part of the calculation, contains an error. It seems, therefore, that elements around the trailing edge must be placed so that the Kutta-Joukowski condition is more closely approximated.

The re-definition of the elements around the trailing edge required that some new profile coordinates had to be interpolated from the original set. The set of points defining the aerofoil was transformed to the (θ,z) plane by the transformation:

$$\theta = \arccos\left\{\frac{2x}{c} - 1\right\}$$

$$z = y$$

where c is the chord of the aerofoil.

The curve was then single-valued, thus the interpolation could be performed by a cubic-spline method[4].

Now for this transformation:

$$\frac{dz}{d\theta} = \frac{dy}{dx}\frac{\partial x}{\partial \theta}$$

$$= -\frac{dy}{dx}\frac{\sin\theta}{2}c .$$

At the trailing edge, $\theta = 0, 2\pi$ thus

$$\frac{dz}{d\theta} = 0 .$$

These equalities provided the two conditions at the endpoints, which are necessary for the complete definition of the cubic spline.

Points, which were placed 'regularly' around the aerofoils, were found by dividing the θ-axis into equal intervals and transforming the corresponding points on the curve to the physical plane. There was a total of 120 elements, divided equally between the aerofoils. As shown in Table 2(a), this produces answers for $C_{L(\Gamma)}$ which are in closer agreement with the exact solution. The lift coefficients based on the integration of the pressure distributions defined by only 60 points should be treated with some caution. Fig.4 also demonstrates that the pressure distribution in the region of the trailing edge of the main aerofoil is more closely approximated*, and it is concluded that the symmetrical placing of elements about the trailing edge causes the Kutta-Joukowski condition to be more closely approximated.

The cubic-spline interpolation was used to define various distributions of elements around the aerofoils. It seems reasonable to assume that the quality of the solution will increase with the number of elements and this is examined more closely in section 2.2. In comparing the different forms of distribution to determine the most acceptable, the total number of elements was increased to 180, and 120 of these elements were placed on the main aerofoil. For these cases

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* However, it should be noted that the numerical method only gives values of the pressure coefficient at the centre of the source elements, and makes the pressures equal at the elements on the upper and lower surfaces closest to the trailing edge. These last elements are several times larger in the regular distribution than in the irregular and therefore the pressure distribution in Fig.4 appears to 'close' at 0.998 c.

the calculation was first performed with a 'regular' distribution of elements. This ensured that the elements were placed symmetrically around the trailing edge, but that relatively few elements occurred around the leading edge. This was a region of high curvature and could well need more elements to define the flow accurately. A second distribution was calculated, where the θ-axis was subdivided such that $\cos \theta/2$ decreased in equal intervals. This 'cosine' distribution placed more elements around the leading edge and was used for both aerofoils. Finally, in an attempt to define the gap between the aerofoils more closely, a 'cosine' distribution was used on the flap, whilst a 'regular' distribution was used on the main aerofoil.

The forces are compared in Table 2(a) and the pressure coefficients around the leading edges of the main aerofoil and flap are plotted against y/c in Figs.5,6 respectively. The 'cosine' distribution does not improve the solution. Even though there are more points around the leading edge, the pressure distributions are not an improvement on the solution with 'regular' spacing. This is attributed to the lack of points around the trailing edge, which leads to an inaccurate estimate of the circulation.

For 'regular' spacing on the main aerofoil and 'cosine' spacing on the flap, the elements are placed more densely around the trailing edge of the main aerofoil and the estimate of the circulation is improved. The flow in the gap between the aerofoils is defined in more detail, but the poor definition of the trailing edge of the flap leads to errors in both flap and main-aerofoil circulations. This form of distribution gives estimates of the lift which are lower than the ones obtained by using the 'regular' spacing.

A distribution was constructed such that there were 120 elements on the main aerofoil and 60 elements on the flap, with each set of elements at 'equal' spacing. The results are included in Table 2(a) and Figs.5 and 6. This does not produce any improvement in the solution.

It is concluded that, in this case, the 'regular' spacing of elements around the aerofoils produced the most accurate answers. It places the elements symmetrically around the trailing edge, thus the approximate Kutta-Joukowski condition more closely represents the real Kutta-Joukowski condition. It also provides enough elements around the remainder of the aerofoil to give a reasonable approximation of the profile and of the flow. In all further calculations in this Report, the elements are distributed 'regularly' around the aerofoils.

2.2 Distribution of elements between the aerofoils

After a satisfactory method of defining each aerofoil had been determined, the question of the division of the elements between the aerofoils was considered. A number of calculations were performed with a total of 180, 120, 90 and 60 elements. The elements were divided between the aerofoils in different proportions and the parameter χ was defined as (number of elements on the flap)/(number of elements on the wing). Fig.7 shows the variation of percentage error in $C_{L(P)}$, the lift coefficient derived from the pressure distribution, against χ for various total numbers of sources. The percentage error in $C_{L(P)}$ is defined as

$$\frac{C_{L(P)} \text{ exact} - C_{L(P)} \text{ surface-source}}{C_{L(P)} \text{exact}} \times 100$$

As the total number of elements increases, the curves become flatter. The figure indicates that with 60 elements the best results could be obtained with $\chi = 1.4$, whilst with increasing number of elements the error becomes less dependent on χ.

On examination of the errors in $C_{L(P)}$ for the individual aerofoils, it is found that there is a definite minimum in the error in the lift of the wing at $\chi \doteq 0.7$; whilst the error in the flap lift decreases more-or-less inversely with χ in the range 0 to 2.0 (see Figs.8, 9). These trends are explained by considering the variation of the circulation about the individual aerofoils with χ, for a solution with a total of 120 elements. In Fig.10a it can be seen that the circulation around the main aerofoil obtained a maximum at $\chi = 1.1$, the value of which is still below the exact value. This is reflected in the pressure distributions around the leading edge. In Fig.11, the pressure distributions for various χ are given and the approximate C_p never attains the suction peak of the exact solution. However, the circulation around the flap increased with χ and passed through the exact value at $\chi = 1.2$ (Fig.10b). This trend is reflected in the pressure distributions around the leading edge. Although there are relatively few points around the suction peak on the flap for small values of χ, it can be reasonably inferred from Fig.12 that the suction peak increases with χ past the value of the exact solution.

Thus, although reasonable values of the total lift coefficient can be attained with values of χ between 1.0-2.0, the points must be divided equally between the bodies (χ = 1.0) to obtain reasonable pressure distributions and lift coefficients for the individual aerofoils. This would be essential if the inviscid solution was going to be used as the basis of a boundary-layer calculation.

In Fig.13, the dependence of the percentage error in $C_{L(P)}$ on the number of elements is compared for two different values of the parameter χ. The error is inversely proportional to the number of elements and decreases to 1.5% for 180 sources with χ = 1.0.

The best solutions are therefore obtained from the surface-source method, if the elements are placed regularly over the aerofoils, divided equally between them, and if the largest possible number of elements are employed.

3 SURFACE-SOURCE METHOD WITH EXACT VALUE OF CIRCULATION

In the surface-source method, the calculation of the circulations by using an approximation to the Kutta-Joukowski condition is one cause of error. Another error lies in the approximation of a continuous source distribution by a set of discrete source elements. In the last section it was shown that the approximate solution approaches the exact solution as the number of elements is increased. The size of this error for an approximation by 180 source elements, was estimated by combining the basic flows, using the exact values of the circulations as the coefficients. The comparisons are given in Table 2(b). The lift, which was calculated by integrating the pressure distribution, is improved on both the main aerofoil and the flap. It is noteworthy that, with the correct circulation, there is still a 0.83% error in total $C_{L(P)}$ as opposed to a 1.83% error, with the approximate Kutta-Joukowski circulation. The approximate Kutta-Joukowski condition underestimates the circulation on the main aerofoil, whilst it overestimates the circulation on the flap. This is reflected in the comparisons, in Figs.14 and 15, of the pressure distributions around the leading edges for the three methods.

Even with the correct circulation, the surface-source method does not reproduce the exact solution. However, a closer representation of the Kutta-Joukowski condition could lead to an improvement in total $C_{L(P)}$ of the order of 1%.

4 ERROR IN DEFINITION OF COORDINATES

In most applications of the surface-source method, the coordinates of the aerofoils may not be known exactly. Either an individual point on the profile may be in error or there may be a random error associated with each point. The first form of error can be easily dealt with, as an examination of the geometry of the body or the pressure distributions will reveal the erroneous point. It can then be replaced by using the cubic-spline interpolation.

The second form is more likely to occur in practice, as the points may be read from a drawing. It is far more difficult to correct this error. The situation was simulated by introducing random errors in the fourth and third decimal places of the coordinates of the test-case aerofoil. In section 2 it was shown that the accurate definition of the trailing edge, and so a reasonable approximation to the Kutta-Joukowski condition, was essential. In an attempt to isolate the error associated with the inaccurate specification of the coordinates, the trailing-edge region ($x/c \geqslant 0.95$) was specified exactly and the errors were only introduced into the remaining coordinates. Table 2(c) shows that errors of this magnitude have little effect upon the total forces. However, in Figs.16 and 17, it can be seen that the pressure distributions around the leading edges of the main aerofoil and the flap deviated from the exact solution. As the error increases, the pressure distributions become rather wavy. Thus, with random errors in the coordinates, the surface-source method will produce a reasonable estimate of the lift coefficients, but incorrect pressure distributions.

If there are random errors associated with the coordinates, then some correction can be made by a cubic-spline interpolation in the following manner. A few points are selected around the aerofoil and these are used to define a cubic-spline fit, which will represent a smoothed profile. The remaining points required for an adequate definition of the profile are obtained by interpolation from this smoothed profile. This method has one disadvantage since there is no guarantee that the points selected for the definition of the cubic spline lie on the original profile, thus the smoothed profile does not necessarily coincide with the original profile. However, the errors associated with a calculation around this smoothed profile as opposed to the original are likely to be small compared with the improvement in the quality of the pressure distributions.

5 CONCLUSIONS

The exact solution is closely approximated by the surface-source method, if the following conditions are satisfied. The surface elements must be placed at equal intervals of θ, where $\cos\theta = \frac{2x}{c} - 1$. They must also be divided equally between the aerofoils. Interpolation from a cubic-spline fit provides a suitable means of obtaining this regular distribution. The approximate Kutta-Joukowski condition only produces a good approximation to the circulation, if the elements are placed symmetrically around the trailing edge, which must be defined accurately. Small errors in the coordinates of the profile could also produce erroneous pressure distributions. This is corrected by interpolating a smoothed profile from a cubic-spline fit. These procedures ensure that the surface-source method gives consistent approximations of the pressure distributions and total forces.

Appendix A

THE SURFACE-SOURCE METHOD

In the surface-source method of A.M.O. Smith[2] a continuous distribution of sources is placed over the surface of the aerofoils. Each source satisfies the Laplace equation and the relevant boundary condition at infinity. The linearity of the problem ensures that the distribution of sources will also satisfy these equations. The boundary condition of zero normal velocity on the aerofoil surface leads to a Fredholm integral equation of the second kind for the source strength. The integral equation is approximated by a set of linear equations in the following manner. The surface of the aerofoil is approximated by a series of straight-line elements and over each element the value of the source density is assumed constant. The term source will be used to denote this surface element of constant source density. The solution of these linear equations forms the crux of the method. The scheme employed for their solution does not give any guide, however, to the positioning or the number of sources that will ensure an acceptable solution.

Some basic flows are calculated: the flow due to a uniform stream at zero angle of incidence to the aerofoils; one due to a uniform stream at 90°, and one for each body with a unit circulation around the body. These flows are then combined linearly so that an approximate Kutta-Joukowski condition is satisfied at each trailing edge. This Kutta-Joukowski condition requires the equality of the velocities at the midpoints of the surface elements closest to the trailing edge on the upper and lower surfaces. Thus the Kutta-Joukowski condition gives one equation in the circulations, for each body, and so a method of calculating the circulations. The full solution is then determined by taking a linear combination of the basic flows. The relevant components of the flows at 0° and 90° are combined to give the effect of incidence, whilst the circulatory flow for each body is related to the appropriate circulation.

Two forms of lift coefficient are defined. $C_{L(\Gamma)}$ is twice the circulation around the aerofoil and $C_{L(P)}$ is the force normal to the direction of the uniform stream. $C_{L(P)}$ is obtained by integrating the pressure distribution.

Appendix B

THE EXACT TEST CASE

In Ref.3 the exact test case was constructed in the following manner. The inviscid flow about two lifting circles was calculated by the method of images and then the two circles were mapped conformally onto two aerofoils by applying the Karman-Trefftz transformation twice. If two conformal transformations are used then the final shape cannot be predicted, thus the parameters of the transformation were adjusted until the final shape resembled an aerofoil with a slotted flap, as shown in Fig.1. The pressure distributions around both aerofoils are given in Figs.2 and 3. The coordinates and pressure distributions are listed in Table 1, whilst the lift coefficients based on the circulation and the pressure distributions are given in Table 2(a).

Table 1

EXACT TEST CASE

Main aerofoil

x	y	C_p	x	y	C_p
1.00000	0.00590	1.00000	0.00017	0.00264	-8.34989
0.99931	0.00612	-0.04022	0.00409	0.01242	-8.73166
0.99417	0.00748	0.23687	0.01311	0.02211	-7.14534
0.98434	0.00903	0.50061	0.02707	0.03155	-5.73037
0.96975	0.00941	0.67369	0.04582	0.04056	-4.73940
0.94998	0.00766	0.75477	0.06914	0.04898	-4.05084
0.92461	0.00361	0.77689	0.09681	0.05663	-3.55471
0.89358	-0.00236	0.76914	0.12857	0.06335	-3.18166
0.85728	-0.00965	0.74766	0.16414	0.06902	-2.88974
0.81639	-0.01771	0.72058	0.20321	0.07352	-2.65315
0.77169	-0.02612	0.69205	0.24543	0.07678	-2.45564
0.72396	-0.03451	0.66437	0.29044	0.07875	-2.28674
0.67396	-0.04261	0.63898	0.33785	0.07942	-2.13965
0.62240	-0.05018	0.61699	0.38724	0.07881	-2.00992
0.56993	-0.05699	0.59936	0.43814	0.07700	-1.89477
0.51716	-0.06287	0.58708	0.49010	0.07408	-1.79260
0.46466	-0.06766	0.58115	0.54258	0.07019	-1.70272
0.41297	-0.07124	0.58264	0.59507	0.06550	-1.62525
0.36259	-0.07350	0.59271	0.64697	0.06020	-1.56109
0.31398	-0.07438	0.61252	0.69769	0.05453	-1.51204
0.26759	-0.07386	0.64327	0.74656	0.04870	-1.48106
0.22381	-0.07194	0.68596	0.79290	0.04293	-1.47265
0.18304	-0.06866	0.74118	0.83597	0.03743	-1.49342
0.14563	-0.06409	0.80830	0.87501	0.03232	-1.55225
0.11190	-0.05833	0.88385	0.90929	0.02762	-1.65810
0.08214	-0.05151	0.95724	0.93815	0.02323	-1.80900
0.05663	-0.04378	0.99969	0.96122	0.01893	-1.95727
0.03560	-0.03530	0.93296	0.97850	0.01458	-1.95374
0.01927	-0.02625	0.53705	0.99043	0.01041	-1.60169
0.00783	-0.01681	-0.79779	0.99753	0.00718	-0.92119
0.00143	-0.00714	-4.20249	1.00000	0.00590	1.00000

Table 1 (concluded)

Flap

x	y	C_p	x	y	C_p
1.31389	-0.20363	1.00000	1.01372	-0.01607	-0.77990
1.31360	-0.20335	0.61683	1.02027	-0.01609	-0.96299
1.31121	-0.20083	0.62318	1.02768	-0.01606	-1.20757
1.30635	-0.19598	0.64997	1.03600	-0.01610	-1.48844
1.29886	-0.18893	0.67827	1.04527	-0.01631	-1.78052
1.28864	-0.17996	0.70420	1.05548	-0.01684	-2.06103
1.27564	-0.16939	0.72668	1.06658	-0.01785	-2.31124
1.25995	-0.15765	0.74560	1.07852	-0.01946	-2.51744
1.24177	-0.14518	0.76122	1.09119	-0.02181	-2.67093
1.22146	-0.13243	0.77401	1.10447	-0.02499	-2.76751
1.19948	-0.11982	0.78457	1.11824	-0.02909	-2.80664
1.17640	-0.10766	0.79358	1.13235	-0.03415	-2.79067
1.15285	-0.09619	0.80184	1.14667	-0.04020	-2.72396
1.12944	-0.08553	0.81022	1.16103	-0.04725	-2.61232
1.10676	-0.07572	0.81976	1.17532	-0.05525	-2.46235
1.08535	-0.06674	0.83159	1.18938	-0.06416	-2.28112
1.06565	-0.05854	0.84699	1.20310	-0.07391	-2.07571
1.04799	-0.05105	0.86722	1.21637	-0.08439	-1.85307
1.03263	-0.04423	0.89341	1.22907	-0.09548	-1.61970
1.01972	-0.03807	0.92597	1.24112	-0.10704	-1.38159
1.00930	-0.03258	0.96308	1.25245	-0.11891	-1.14405
1.00134	-0.02781	0.99520	1.26298	-0.13090	-0.91168
0.99572	-0.02381	0.98154	1.27267	-0.14281	-0.68829
0.99226	-0.02065	0.71658	1.28147	-0.15440	-0.47687
0.99073	-0.01835	-1.17476	1.28934	-0.16544	-0.27961
0.99087	-0.01686	-5.75997	1.29624	-0.17566	-0.09789
0.99242	-0.01604	-2.85918	1.30214	-0.18476	0.06777
0.99508	-0.01571	-1.43049	1.30697	-0.19245	0.21788
0.99864	-0.01569	-0.89891	1.31064	-0.19840	0.35456
1.00295	-0.01582	-0.70367	1.31303	-0.20226	0.48483
1.00797	-0.01598	-0.68332	1.31389	-0.20363	1.00000

Table 2

(a) Comparison of forces for different distributions of elements on each body

	Wing $C_{L(P)}$	Flap $C_{L(P)}$	Total $C_{L(P)}$	Wing $C_{L(\Gamma)}$	Flap $C_{L(\Gamma)}$	Total $C_{L(\Gamma)}$	N
Exact test case	2.9065	0.8302	3.7367	2.7818	0.9568	3.7386	
Surface-source method Irregular distribution	2.7421	0.8299	3.5720	2.6616	0.9786	3.6402	122
Surface-source method Regular distribution	2.8520	0.8035	3.6555	2.7462	0.9538	3.7000	120
Surface-source method Regular distribution	2.8700	0.7984	3.6684	2.7554	0.9574	3.7128	180
Surface-source method Cosine distribution	2.7032	0.8178	3.5210	2.6324	0.9646	3.5970	180
Surface-source method Cosine and regular distribution	2.8234	0.7724	3.5958	2.7096	0.9226	3.6322	180
Surface-source method Equal spacing	2.7908	0.8255	3.6163	2.7000	0.9632	3.6632	180

(b) Surface-source method with exact circulation

Exact test case	2.9065	0.8302	3.7367	2.7818	0.9568	3.7386	
Surface-source method Approximate Γ	2.8700	0.7984	3.6684	2.7554	0.9574	3.7128	180
Surface-source method Exact Γ	2.8989	0.8067	3.7056	2.7817	0.9568	3.7385	180

(c) Effect of errors in the numerical definition of coordinates

Surface-source method	2.8700	0.7984	3.6684	2.7554	0.9574	3.7128	180
Surface-source method Error 1 in 10^4	2.8699	0.7986	3.6685	2.7554	0.9576	3.7130	180
Surface-source method Error 1 in 10^3	2.8679	0.7962	3.6641	2.7546	0.9469	3.7015	180

SYMBOLS

c	chord
$C_{L(\Gamma)}$	lift coefficient derived from circulation
$C_{L(P)}$	lift coefficient derived from pressure distribution
C_P	pressure coefficient
N	total number of elements
x	coordinate along chord of main aerofoil
y	coordinate normal to chord of main aerofoil
z	coordinate in transformation plane
Γ	circulation
χ	number of elements on the flap/number of elements on the main aerofoil
θ	coordinate in transformation plane

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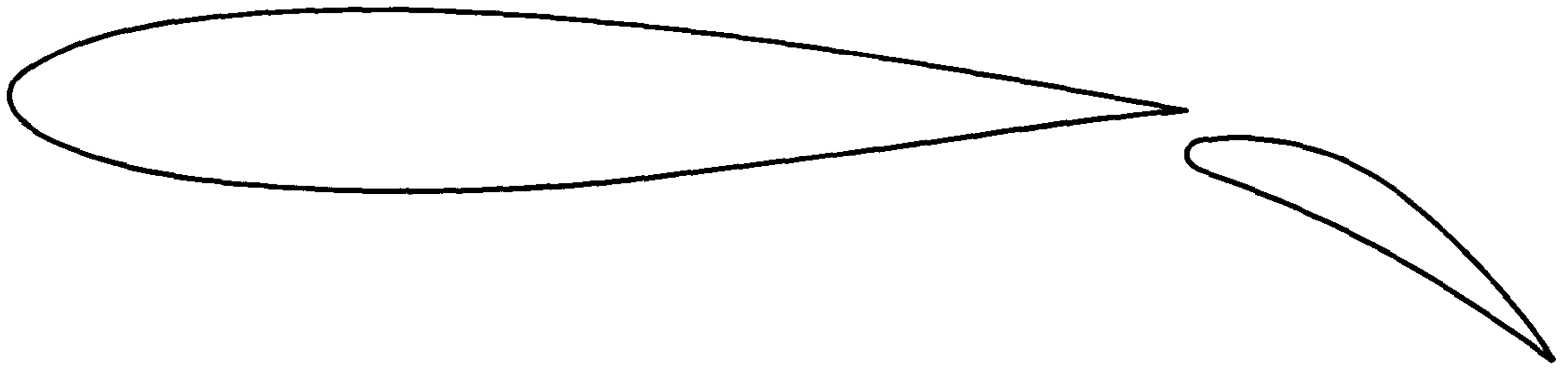


Fig.1 Profile of aerofoil used in exact test case

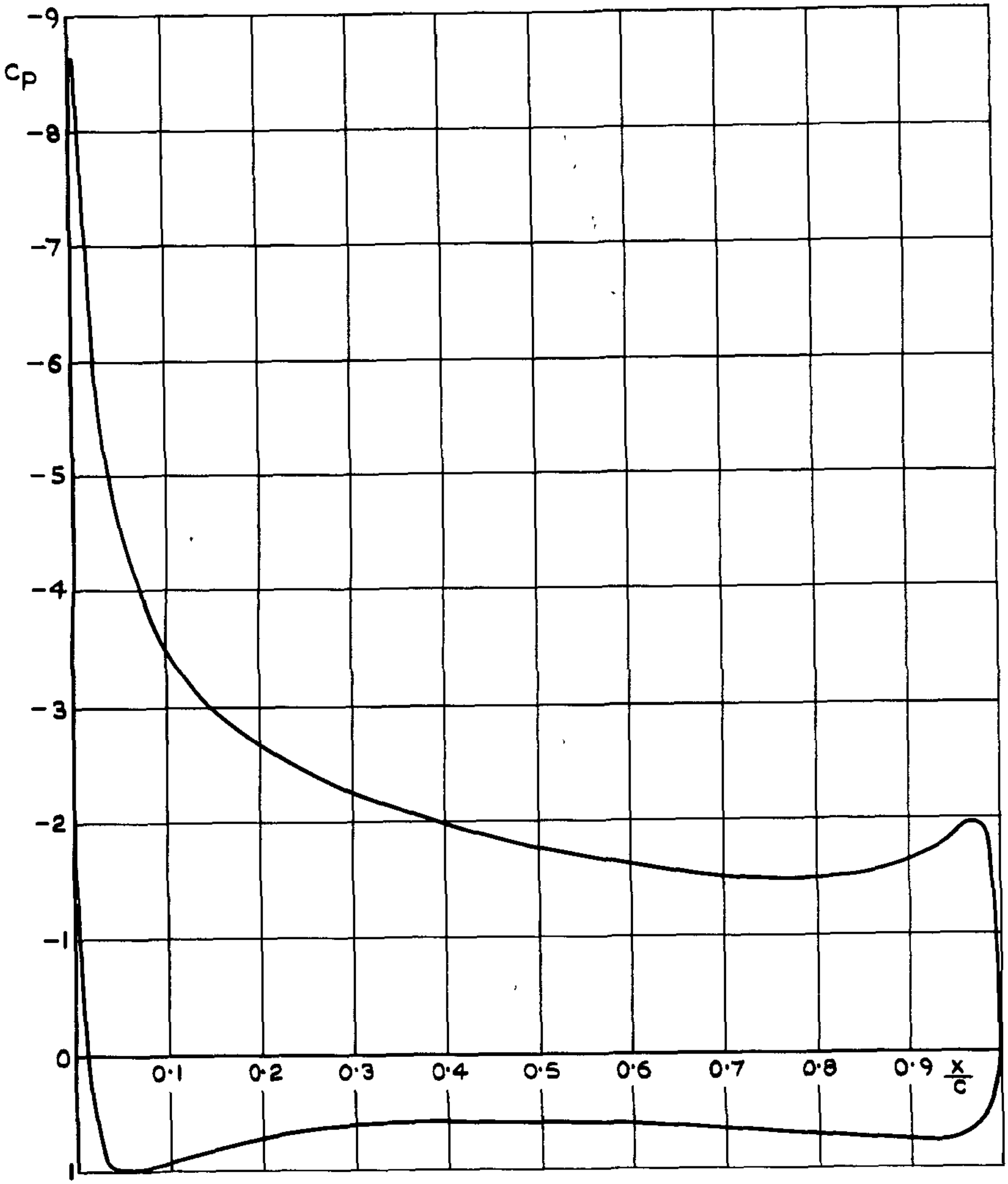


Fig.2 Exact test case. Pressure distribution for main aerofoil

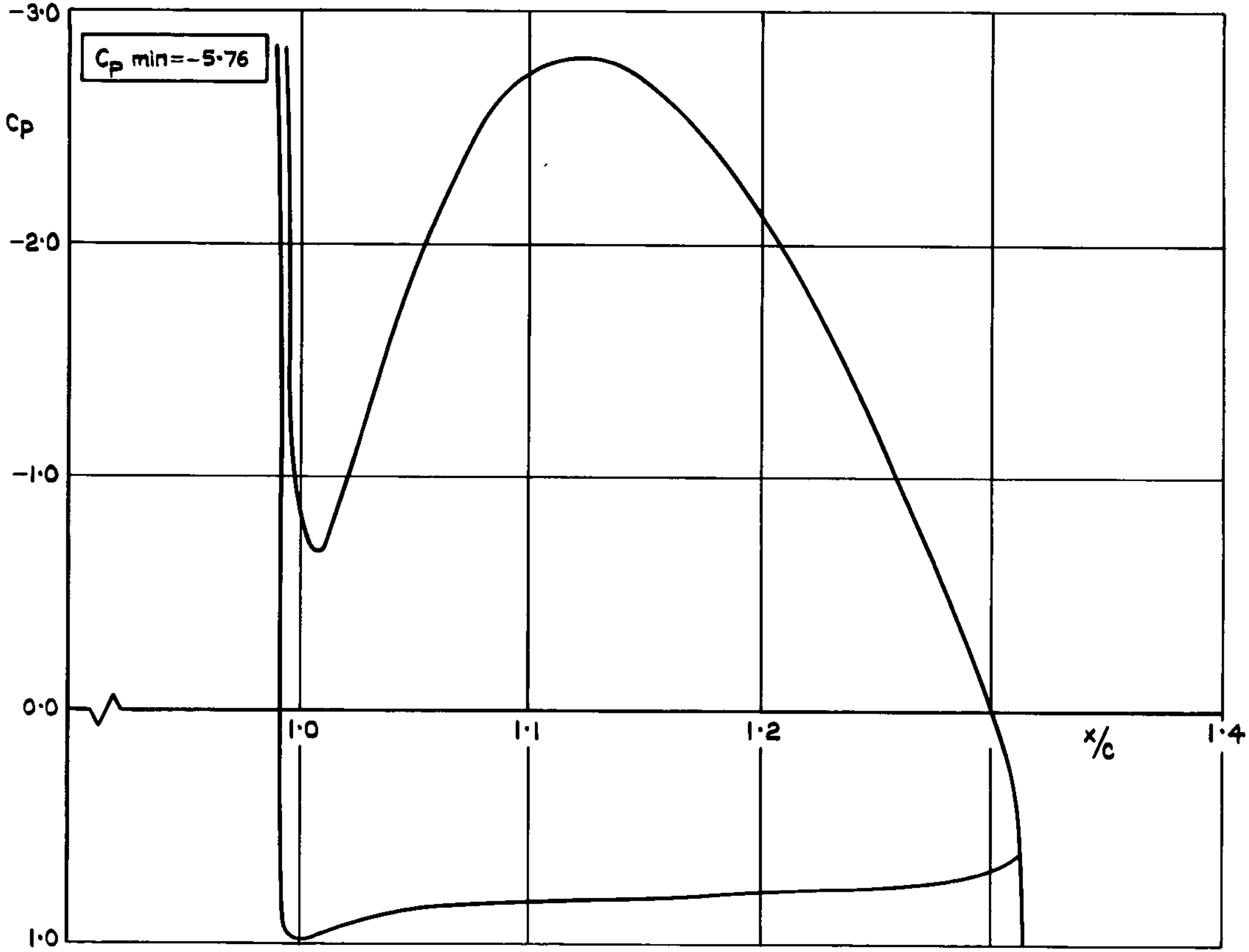


Fig.3 Exact test case. Pressure distribution for flap

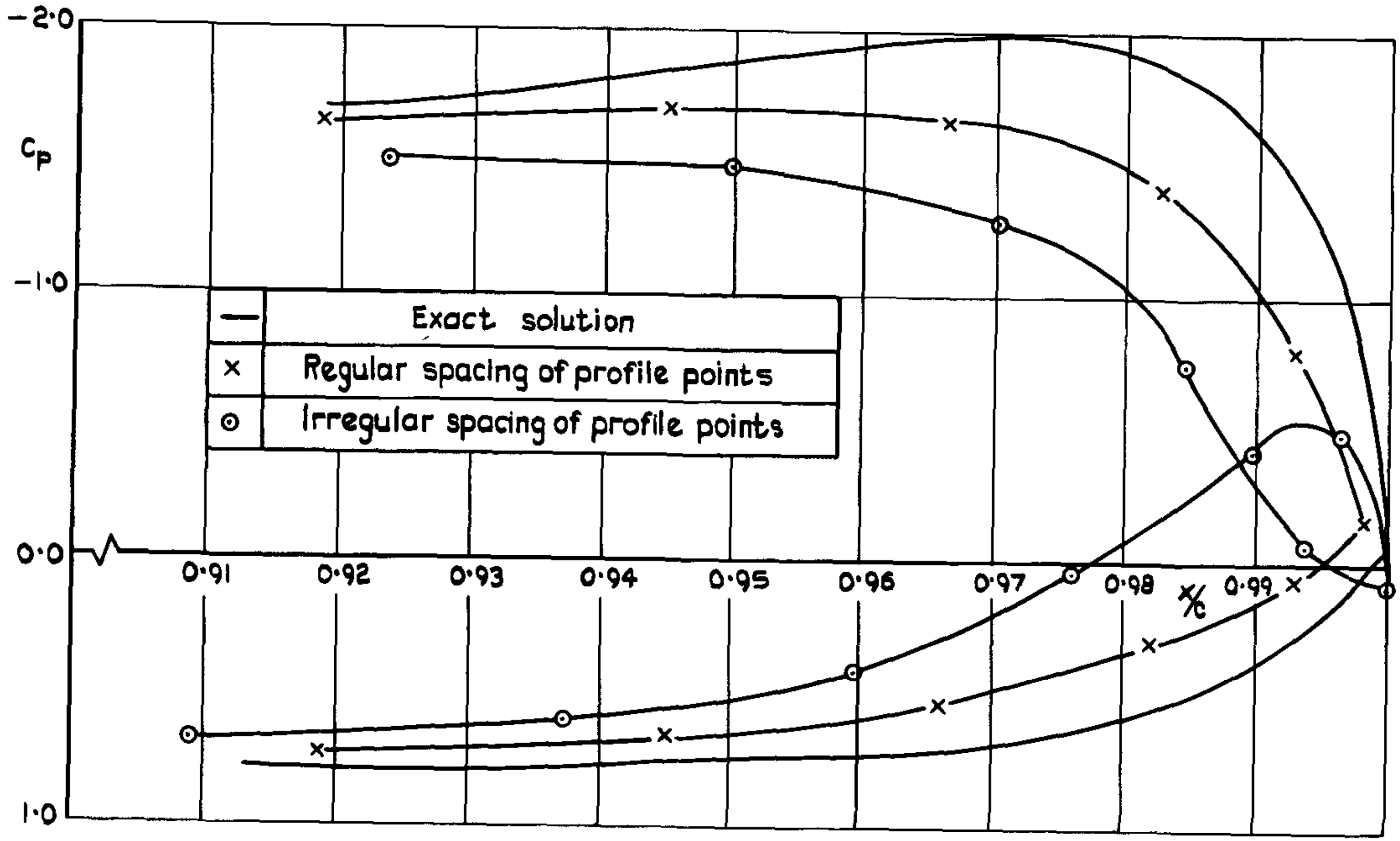


Fig.4 Pressure distribution at trailing edge of main aerofoil

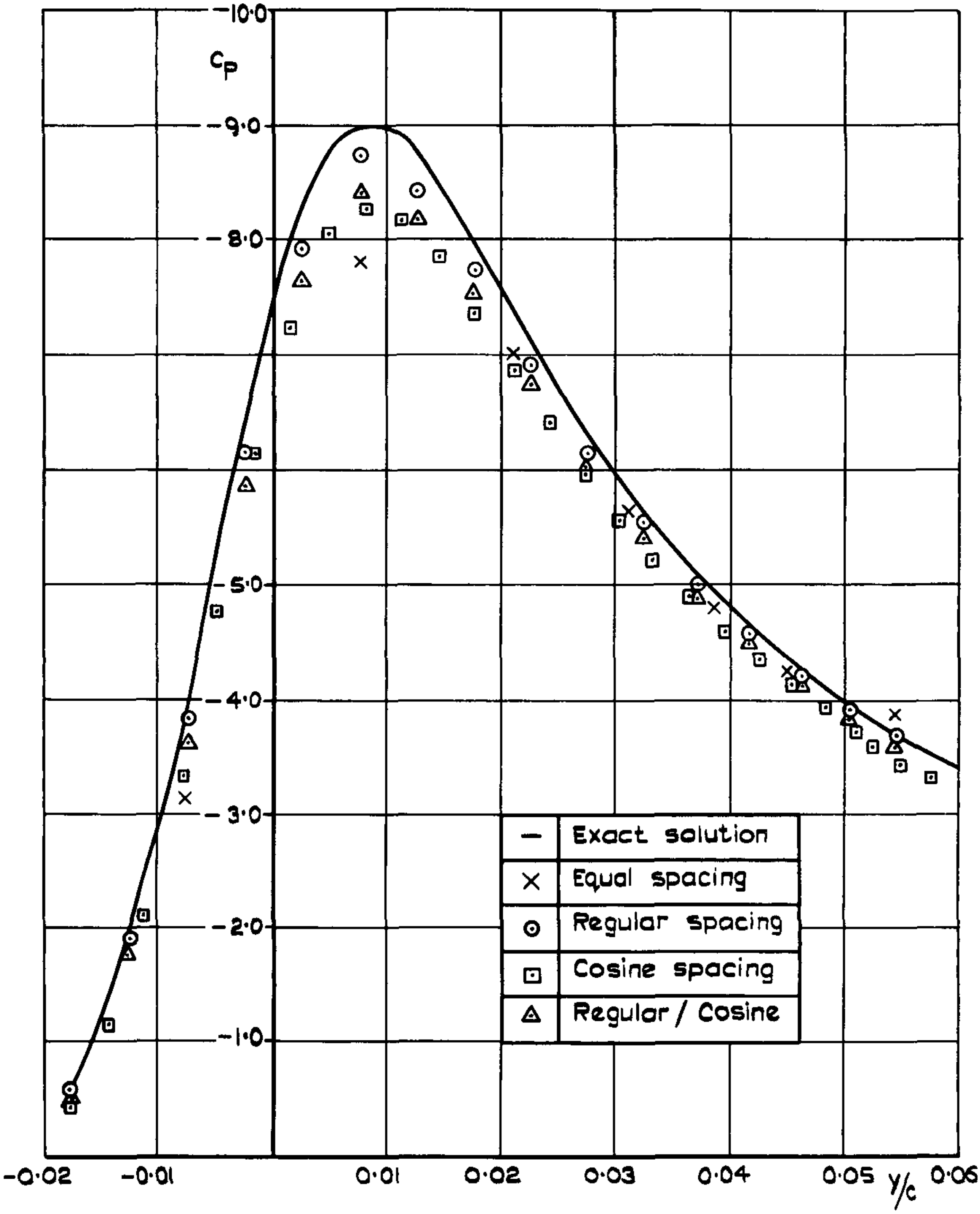


Fig.5 Pressure distribution around leading edge of main aerofoil for various distributions of 180 source elements

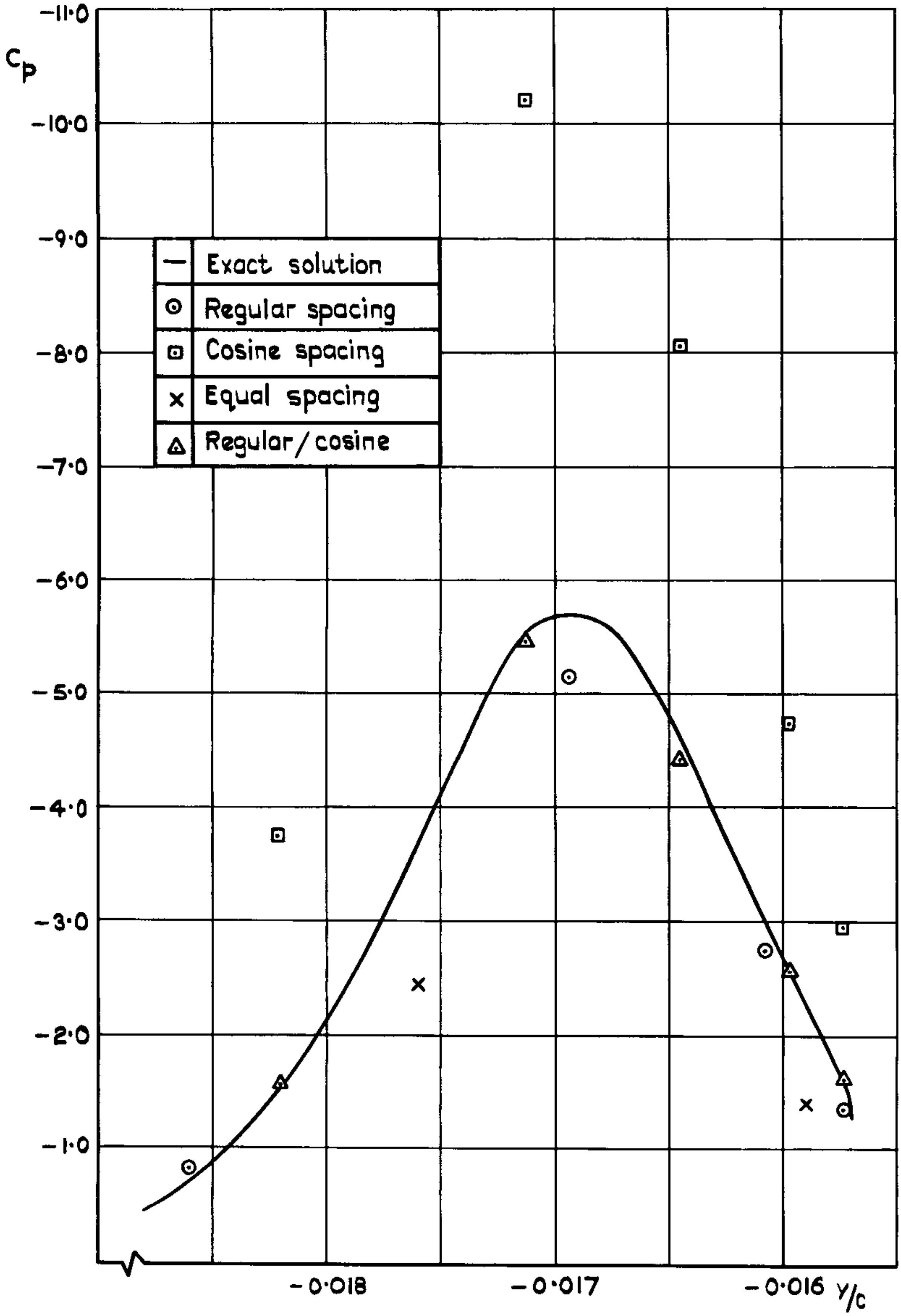


Fig. 6 Pressure distribution around leading edge of the flap for various distributions of 180 source elements

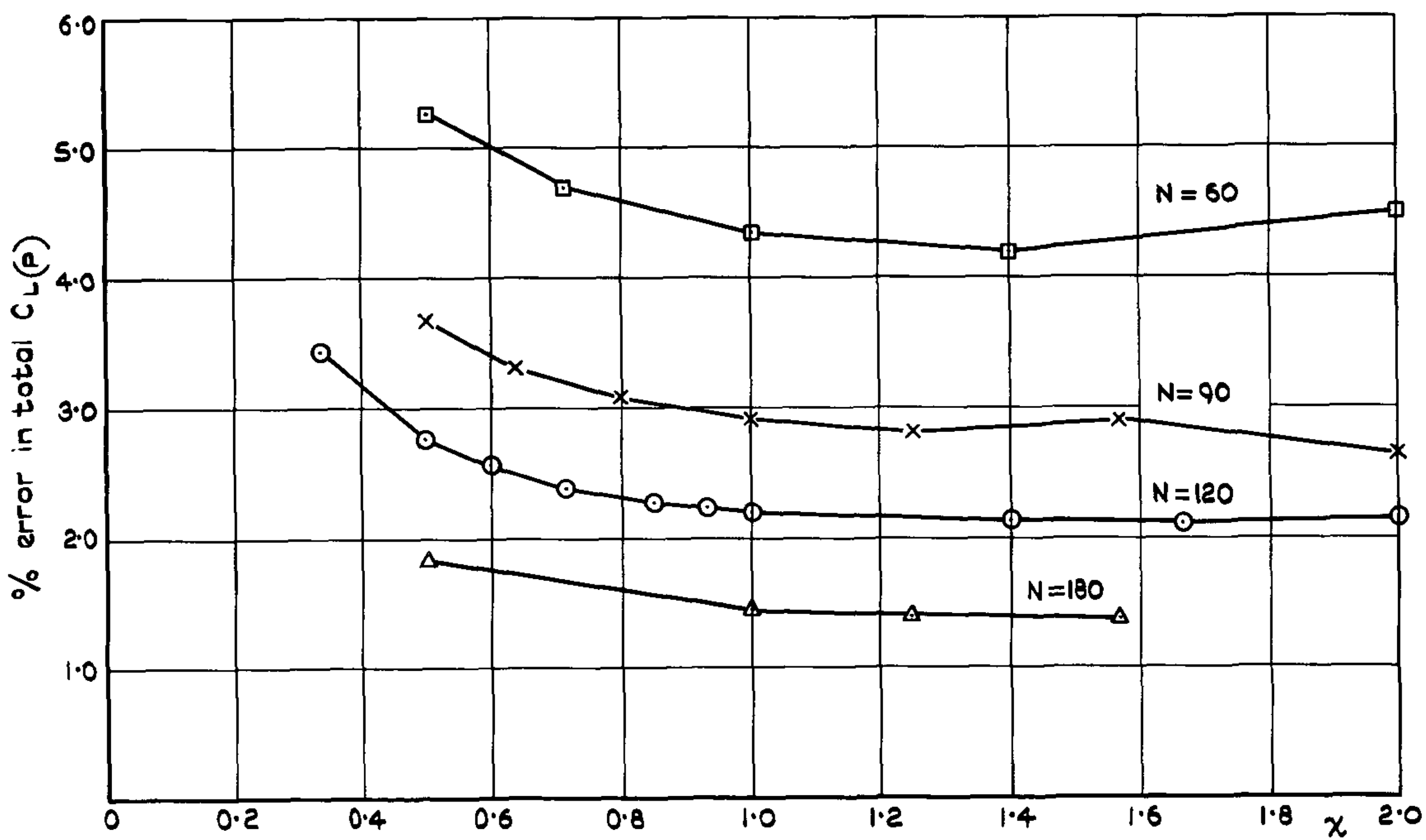


Fig. 7 Percentage error in total $C_{L(P)}$ for various χ and different numbers of elements

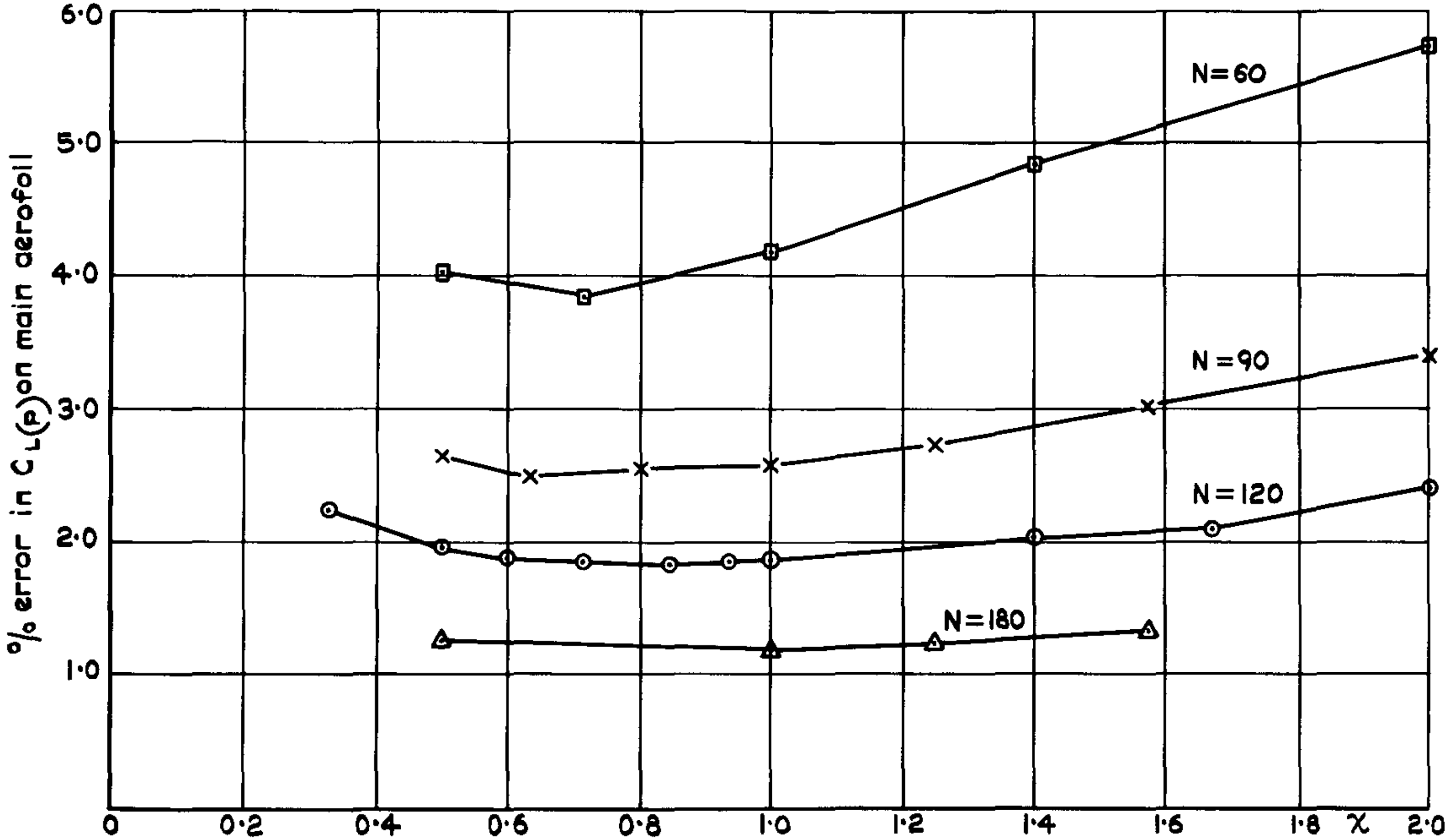


Fig. 8 Percentage error in $C_{L(P)}$ of the main aerofoil for various χ and different numbers of elements

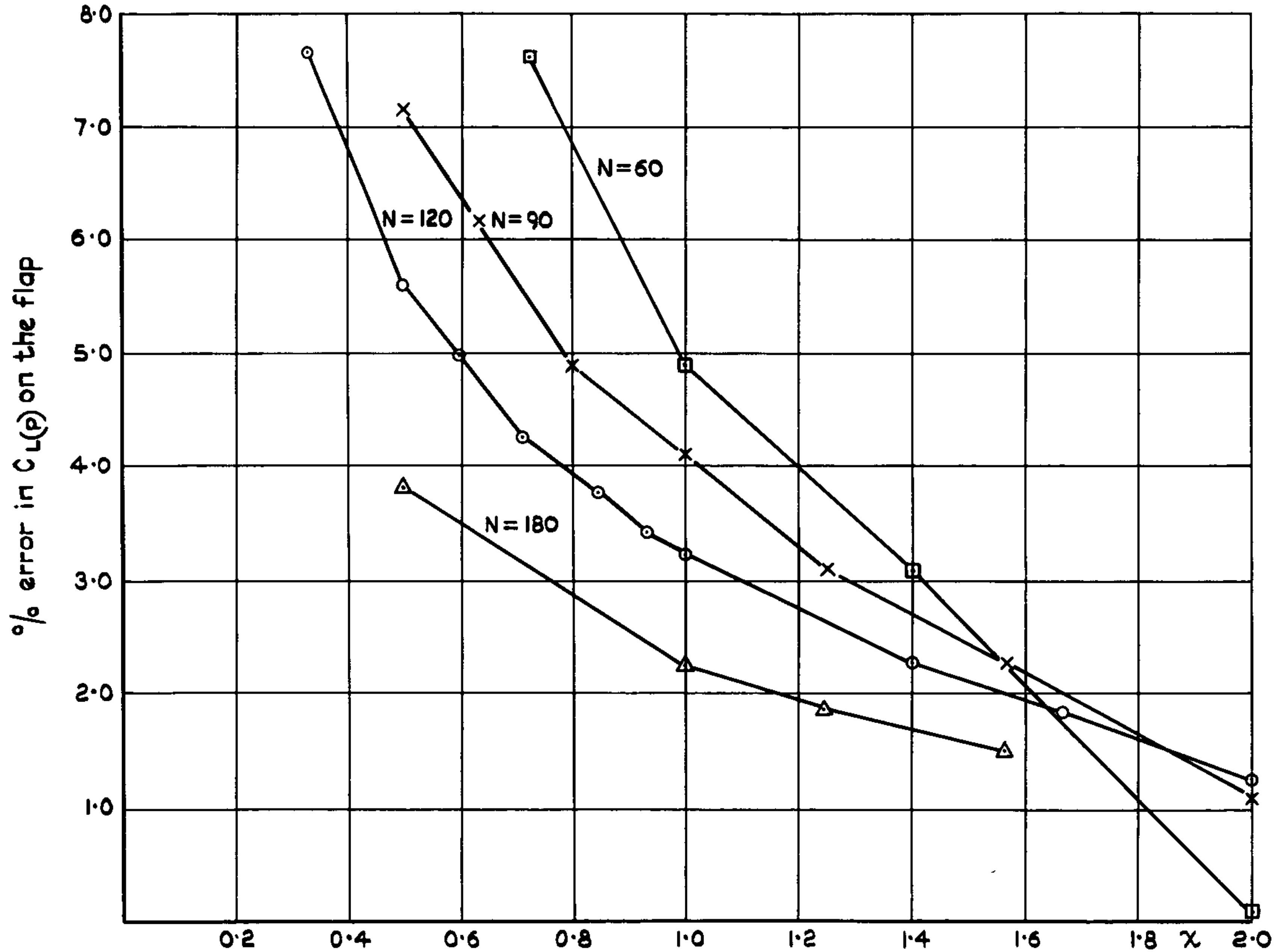


Fig. 9 Percentage error in $C_{L(P)}$ of the flap for various χ and different numbers of elements

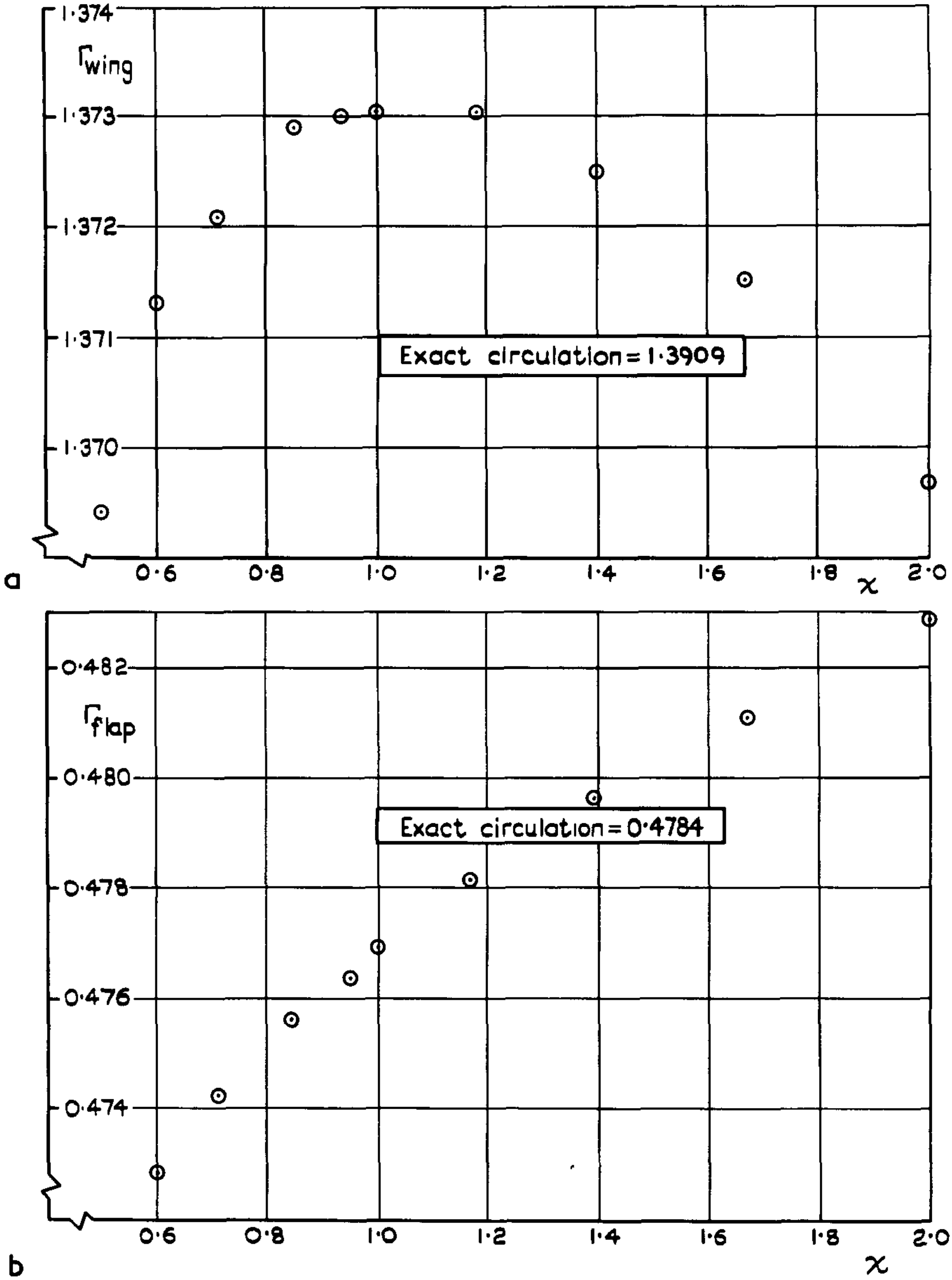


Fig. 10a & b Variation of circulation with χ

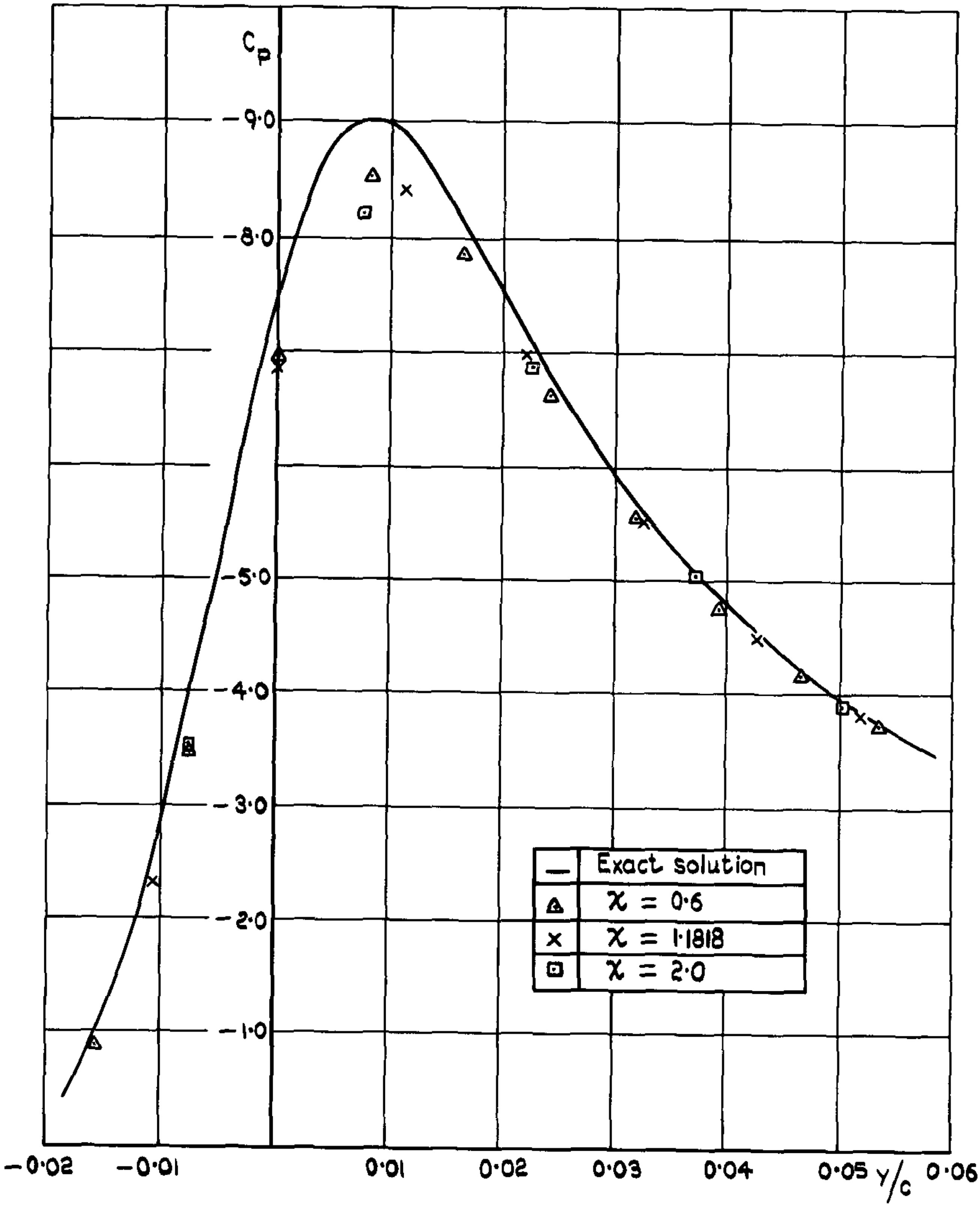
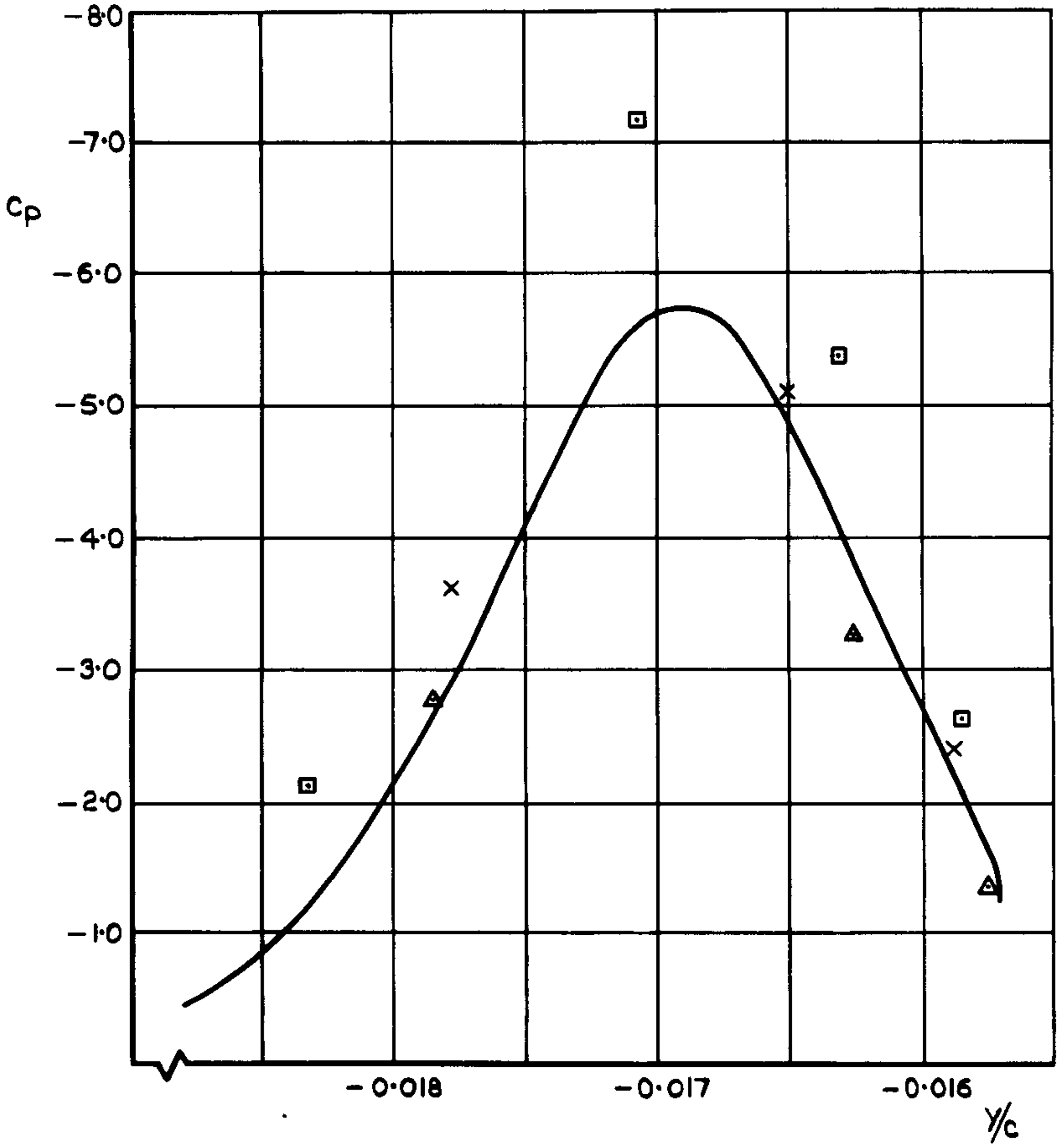


Fig.11 Pressure distribution around the leading edge of the main aerofoil for various χ



—	Exact solution
Δ	$x = 0{\cdot}6$
X	$x = 1{\cdot}1818$
⊡	$x = 2{\cdot}0$

Fig.12 Pressure distribution around the leading edge of the flap for various distributions

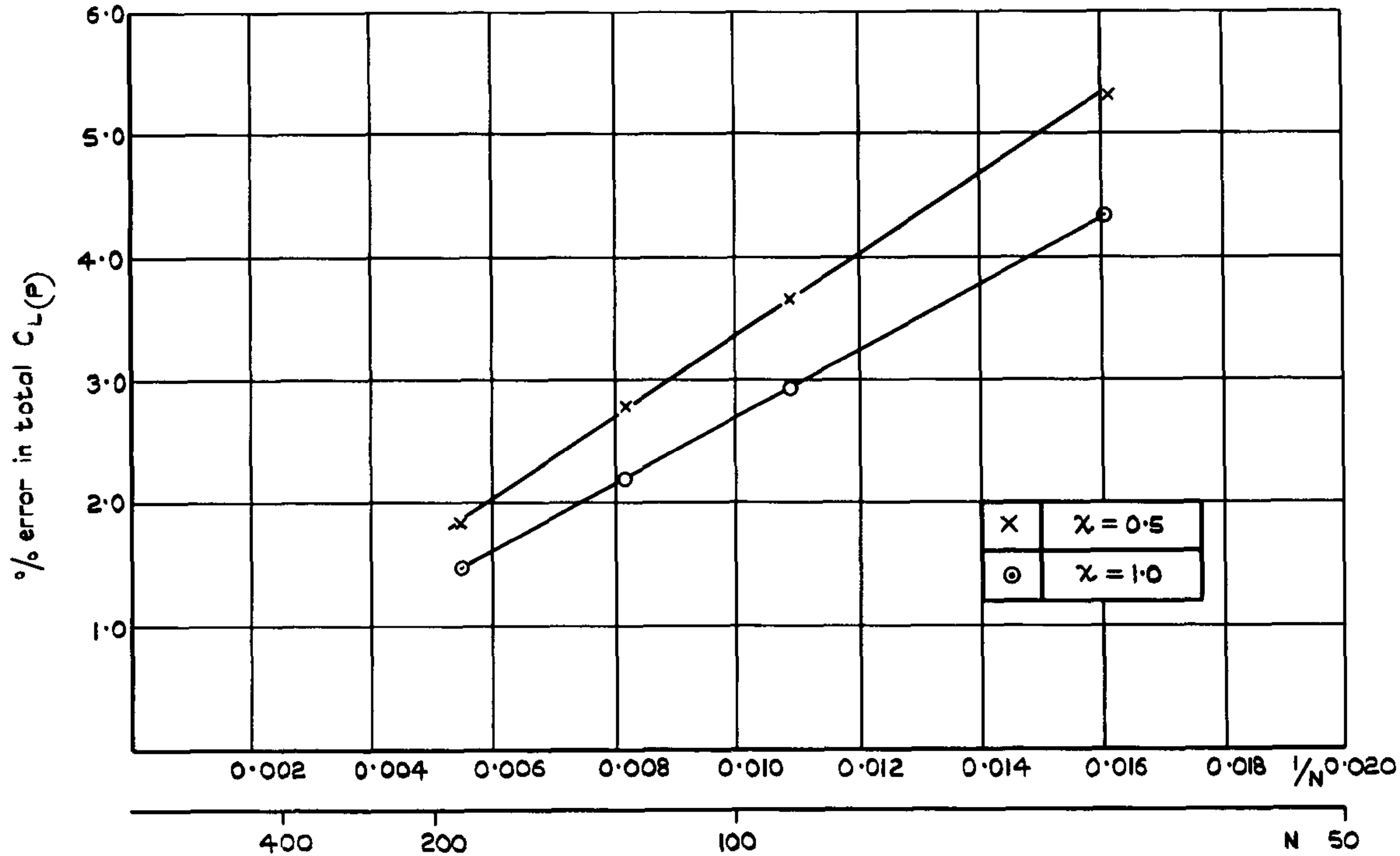


Fig.13 Percentage error in total $C_{L(P)}$ for different numbers of elements

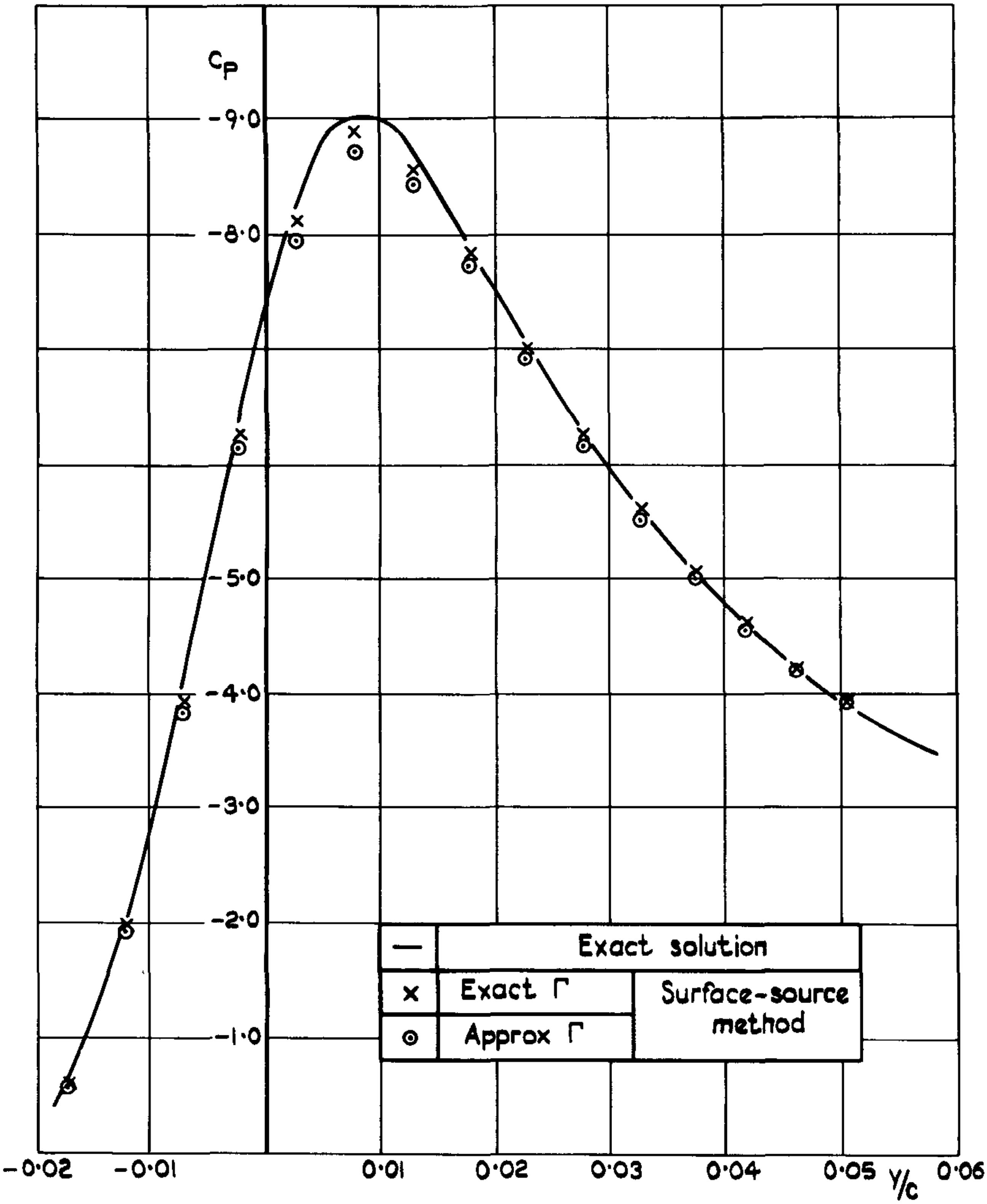
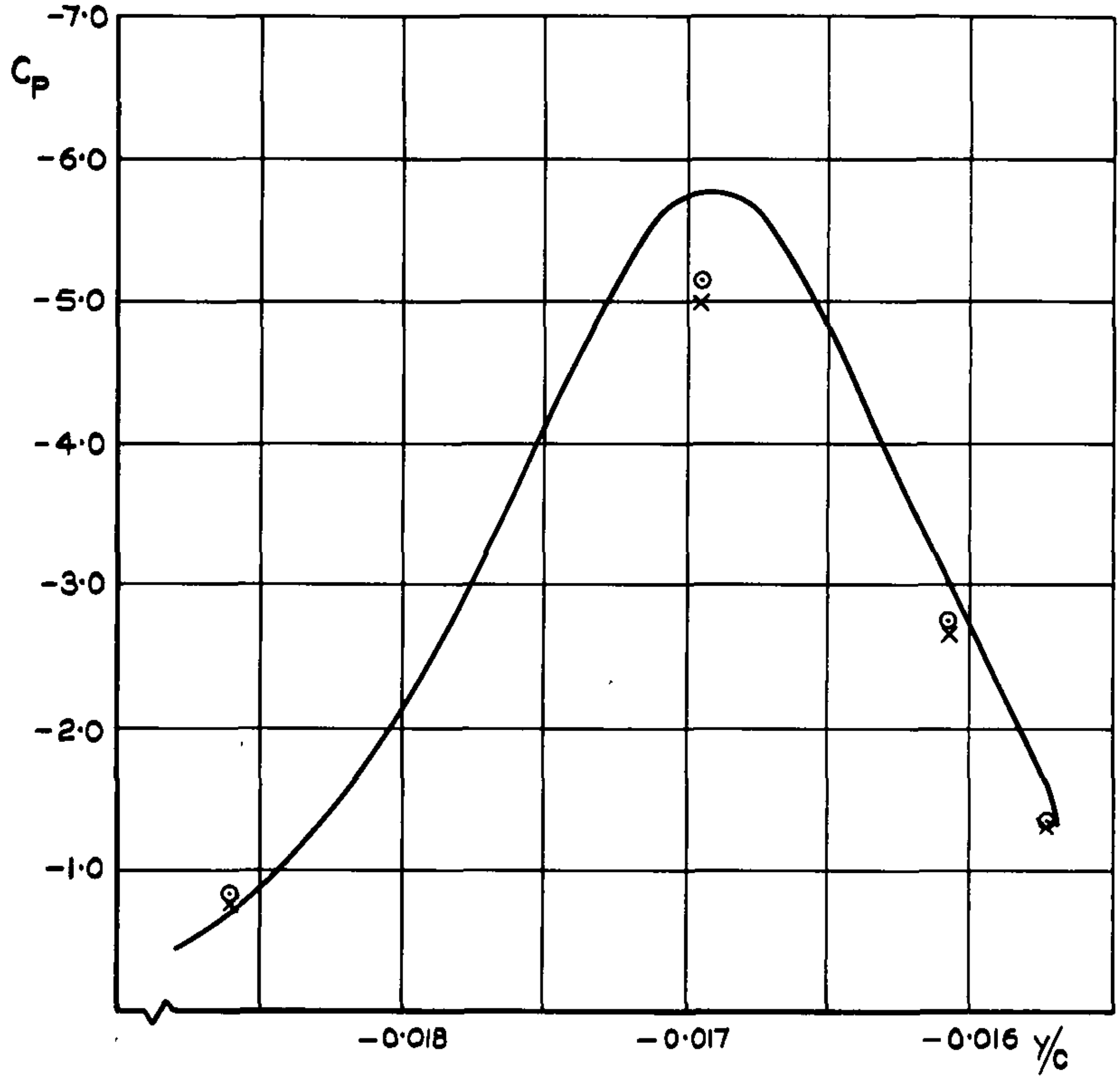


Fig. 14 Pressure distribution around leading edge of main aerofoil



—	Exact solution	
×	Exact Γ	Surface-source method
⊙	Approx Γ	

Fig.15 Pressure distribution around leading edge of the flap

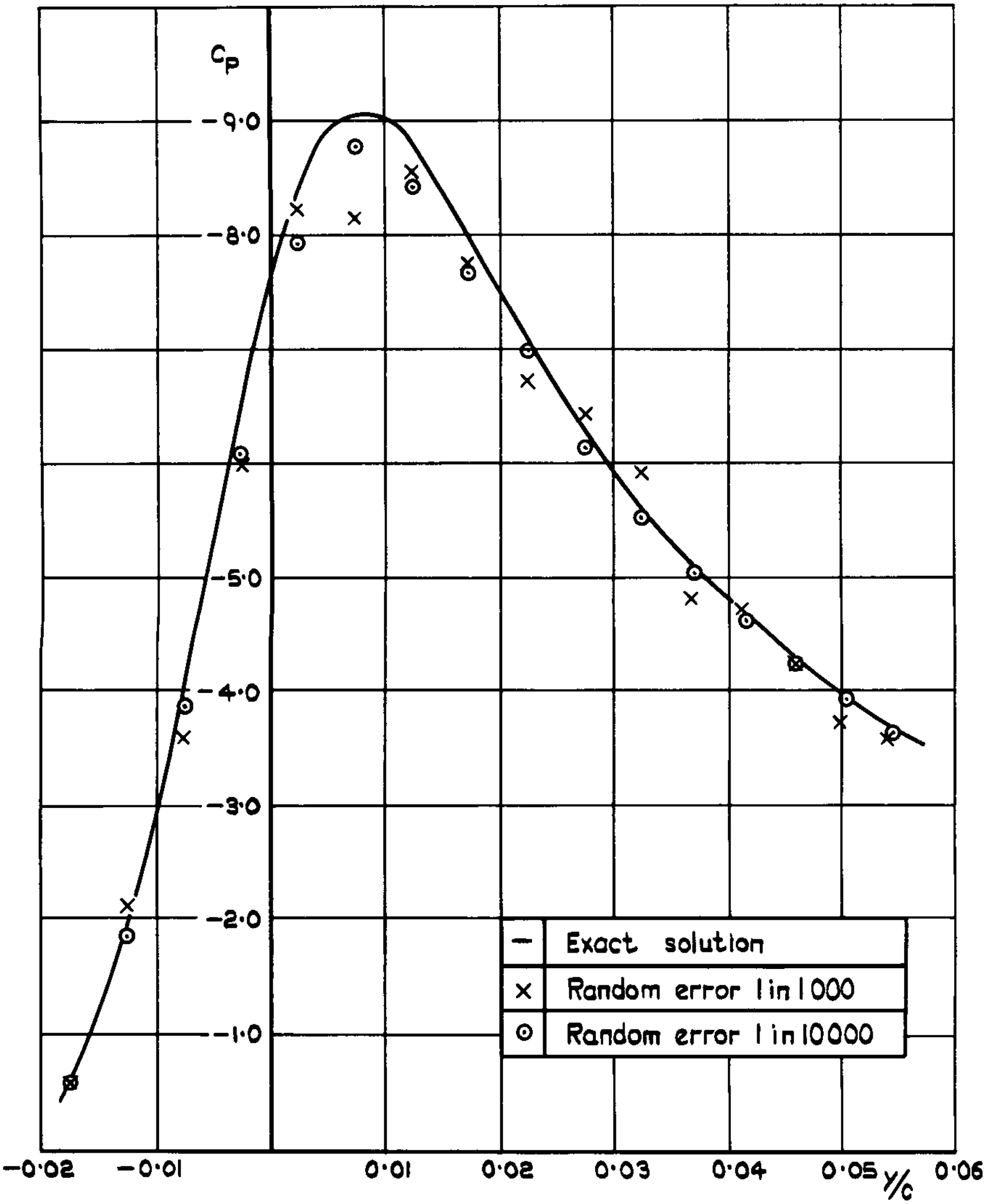


Fig. 16 Pressure distribution around leading edge of main aerofoil with random error in coordinates

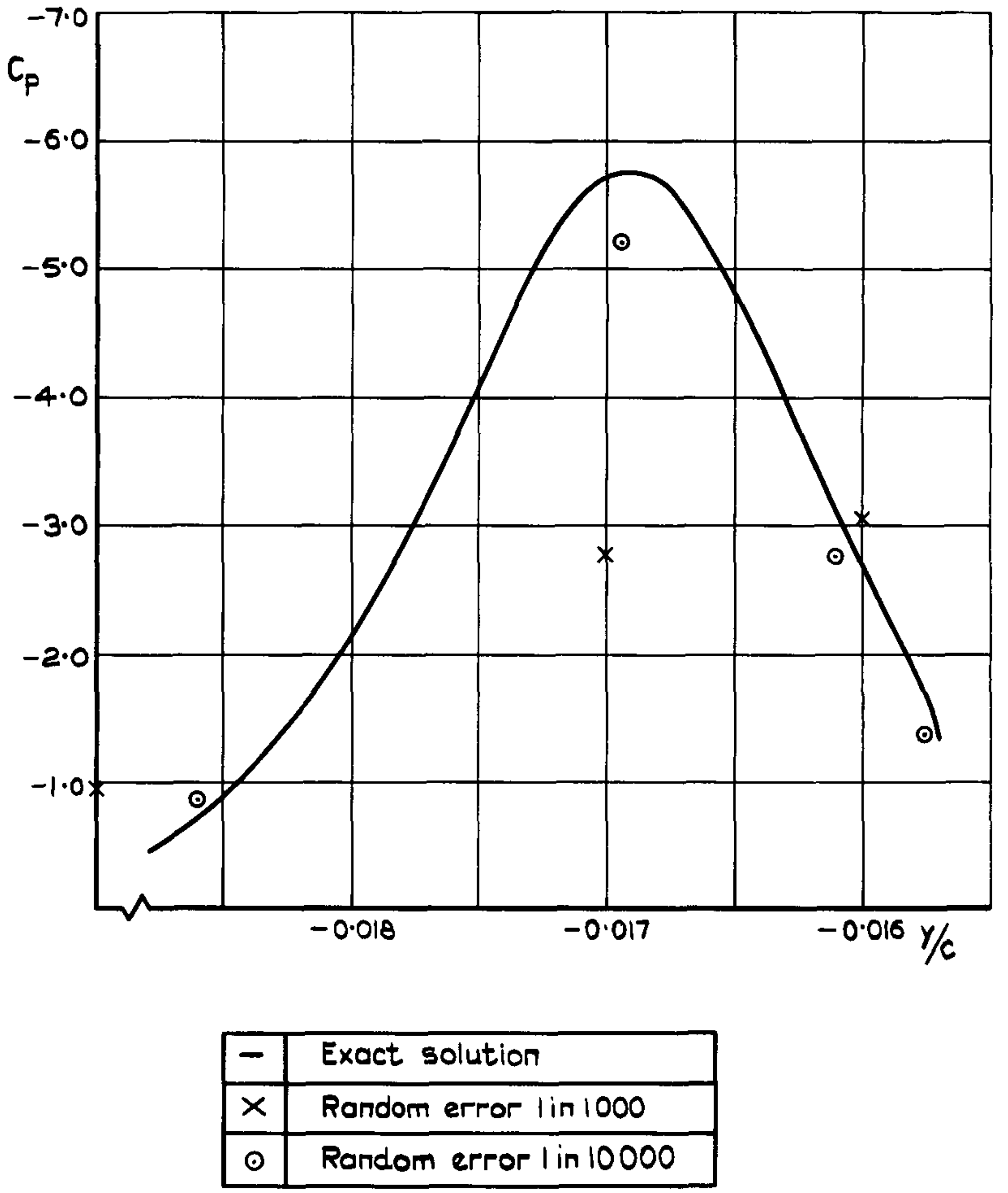


Fig. 17 Pressure distribution around leading edge of flap with random error in coordinates

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