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A Computer Program to Calculate the Pressure Distribution on an Annular Aerofoil

by

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A COMPUTER PROGRAM TO CALCULATE THE PRESSURE DISTRIBUTION ON AN ANNULAR AEROFOIL

by

C. Young

SUMMARY

A computer program has been written in FORTRAN to calculate the pressure distribution on an annular aerofoil at zero angle of incidence at subsonic speed. The theory and the program are described and some comparisons between the predicted pressure distribution and experimental results are presented. Close agreement between theory and experiment is obtained.

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* Replaces RAE Technical Report 71199 – ARC 33665

CONTENTS

1 INTRODUCTION

There has been a renewed interest in methods of calculating the pressure distribution on an annular aerofoil or engine cowl in recent years. This is a result of the need to design improved fan cowls for engines of high by-pass ratio.

The theory advanced here is a logical extension of the earlier work on annular aerofoils. Küchemann and Weber[1] developed a theory for calculating the velocity distribution on thin annular aerofoils which was extended by Bagley *et al*[2] to include thickness and incidence effects, but again using distributions of singularities placed on a cylinder. With the increasing availability of large, high speed digital computers, it has been possible to develop a method using singularities distributed over the body surface, which should in practice, give more accurate results.

The method of surface singularities was placed on a firm foundation by A.M.O. Smith[3] but this method as published, is not capable of calculating the pressure distribution over the whole surface of an annular aerofoil; the afterbody of the aerofoil has to be replaced by a semi-infinite cylinder. This is a serious deficiency because the effect of the afterbody becomes increasingly important as the length to diameter ratio of the aerofoil is reduced, and the circulation developed around the aerofoil plays a large part in determining the overall forces and pressure distribution on the body.

The fan cowl of an engine of high by-pass ratio has to cope with a wide range of operating conditions varying from the take off condition, when the mass flow is high, to the engine failure condition, when the fan is windmilling and the mass flow is low. It is essential, therefore, to be able to calculate the pressure distribution on the aerofoil at any specified mass flow ratio.

The method of surface singularities and the extensions that have been made for the annular aerofoil problem are described in section 2. The computer program is explained in section 3 and some examples of its use are presented in section 4. The present theory is compared with other calculation methods in section 5.

2 THE THEORY OF SURFACE SINGULARITIES APPLIED TO ANNULAR AEROFOILS

2.1 The method of A.M.O. Smith for bodies of revolution

The principles on which the method of surface singularities is based[3] are now well-established and only a brief description of the theory is given below.

The surface on which the pressure distribution is to be calculated is specified by a number of ordinates, and surface elements are formed by joining these ordinates with straight lines. Thus for axisymmetric bodies with N ordinates specified, the surface is approximated by N - 1 conical frustra, Fig.1. A control point at which the boundary conditions are applied is selected on each element; this point is usually taken as the mid-point of the element for convenience.

A surface source density of unit strength is placed on each element and the velocity component normal to the surface induced at every control point by all the other elements is calculated by numerical integration. This leads to a matrix $[V_{n_{ij}}]$ whose elements are the normal velocity components induced at the ith control point by the source density on the jth element. The diagonal entries of the matrix represent the normal velocity induced at the ith control point by the source density on its own surface element. To obtain the actual normal velocities the elements of the matrix must be multiplied by the proper values of the source density q_j, which are as yet unknown. Thus the quantity $\sum_{j=1}^{N-1} V_{n_{ij}} q_j$ is the total normal velocity at the ith control point due to the complete set of N - 1 surface elements.

The boundary condition applied at each control point is that the total normal velocity is zero, i.e. the flow is tangential to the surface of the aerofoil. A set of simultaneous linear equations can be written down which is equivalent to the application of the boundary condition at each control point. The equations are of the form

$$\sum_{j=1}^{N-1} V_{n_{ij}} q_j = V_0 \vec{\sin} \theta_i + F_i \qquad i = 1,2...N-1 \qquad (1)$$

where θ_i is the surface slope of the aerofoil at the ith control point and F_i is any other prescribed normal velocity boundary condition, e.g. suction or blowing. The term $V_0 \vec{\sin} \theta_i$ in the equations is the contribution from the free stream velocity flowing through the surface which must be cancelled. This term must be evaluated in the correct sense.

The set of linear equations can be solved for the unknown source strengths q_j and the tangential velocity component and the pressure coefficient at the control point calculated.

The theory developed by A.M.O. Smith for bodies of revolution at zero angle of incidence goes no further, so it cannot be used for the annular aerofoil problem as no Kutta condition has been applied, and the circulation around the aerofoil is undefined. Furthermore there is no convenient way of changing the mass flow through the aerofoil.

2.2 Controlling the mass flow ratio

The mass flow through the aerofoil can be changed by the addition of a uniform vortex distribution whose strength can be varied to give the required intake flow. This vortex distribution, which is referred to as the 'fan' vortex, extends from the leading edge of the aerofoil to infinity downstream. This distribution could be placed anywhere on the surface or inside the aerofoil, and could vary in strength along the chord. The particular choice made here, of a uniform vortex distribution placed on the camber surface of the aerofoil and on a cylinder downstream of the trailing edge, has proved satisfactory in all cases so far examined.

The 'fan' vortex itself induces a normal velocity component at the control points on the surface of the aerofoil which must be cancelled. The set of equations (1) are modified to

$$\sum_{j=1}^{N-1} V_{n_{ij}} q_j = V_0 \sin\vec{\theta}_i - V^*_{n_i} \gamma_F \qquad i = 1,2...N-1 \qquad (2)$$

where $V^*_{n_i} \gamma_F$ is the normal velocity induced at the ith control point by the 'fan' vortex of strength γ_F.

The strength of the 'fan' vortex required to give a specified mass flow ratio is not known initially. In the computer program, two values of the 'fan' vortex are specified and the corresponding mass flow ratios calculated. From these, the 'fan' strength required to give the required mass flow ratio is deduced. It is shown in section 3 that this does not lead to a lot of extra computing.

2.3 The Kutta condition

The circulation around the aerofoil is undefined until a Kutta condition is applied at the trailing edge. The condition normally applied in surface singularity methods for twodimensional aerofoils is that there should be no difference in the tangential velocity between the first and last control points, i.e. at the points nearest the trailing edge of the aerofoil on the lower and upper surface. This condition has to be modified in the annular aerofoil problem to allow for the velocity jump across the trailing edge due to the trailing vortex cylinder.

Another uniform vortex distribution has to be added to apply the Kutta condition. This distribution is also placed on the camber surface and only extends over the chord length of the aerofoil.

The 'Kutta' vortex as this distribution will be called, also induces a normal velocity at the control points, thus the set of equations (2) becomes

$$\sum_{j=1}^{N-1} V_{n_{ij}} q_j + V_{n_i} \gamma_k = V_0 \vec{\sin}\, \theta_i - V^*_{n_i} \gamma_F \qquad i = 1,2 \ldots N-1 \qquad (3)$$

where $V_{n_i} \gamma_k$ is the normal velocity induced by the 'Kutta' vortex of strength γ_k.

The tangential velocities at the control points nearest to the trailing edge have to be carefully written down because of the sense in which the velocity components are evaluated. The calculation is always made in the direction of increasing i, Fig.1, thus along the inner surface, the calculation is proceeding against the free stream velocity, and this component evaluated in the correct sense is negative. On the outer surface, the calculation is made in the opposite direction, and the component of the free stream velocity is positive.

To evaluate the velocity jump at the trailing edge we require the velocities to be measured in the sense of x increasing. Thus on the outer surface, the tangential velocity at the last control point is

$$V_0 \vec{\cos}\, \theta_{N-1} + \sum_{j=1}^{N-1} V_{t_{N-1,j}} q_j + V_{t_{N-1}} \gamma_k + V^*_{t_{N-1}} \gamma_F$$

and on the inner surface at the first control point the tangential velocity is

$$-\left(V_0 \vec{\cos}\, \theta_1 + \sum_{j=1}^{N-1} V_{t_{1,j}} q_j + V_{t_1}\gamma_k + V^*_{t_1}\gamma_F\right)$$

where $\sum_{j=1}^{N-1} V_{t_{1,j}} q_j$ is the total tangential velocity induced by the source distribution on the complete set of surface elements, and $V_{t_1}\gamma_k$, $V_{t_1}\gamma_F$ are the tangential velocity components induced by the 'Kutta' and 'fan' vortex distributions respectively.

The difference in these tangential velocity components must be equal to the strength of the 'fan' vortex, thus the equation used to satisfy the Kutta condition is

$$- \sum_{j=1}^{N-1} (V_{t_{1,j}} + V_{t_{N-1,j}})q_j - (V_{t_1} + V_{t_{N-1}})\gamma_k = V_0(\vec{\cos}\, \theta_1 + \vec{\cos}\, \theta_{N-1}) + (V^*_{t_1} + V^*_{t_{N-1}})\gamma_F + \gamma_F \quad . \tag{4}$$

The equations (3) and equation (4) form a set of N simultaneous linear equations from which the $N-1$ source strengths q_j and the strength of the 'Kutta' vortex γ_k can be determined.

2.4 Centrebodies and spinners

The effect of a centrebody or spinner can be included in the calculation with only a small alteration. If NC is the number of ordinates specified on the centrebody there will be an additional $NC-1$ surface elements and control points making a total of $N + NC - 2$. The summations in all the equations must therefore be made over all $N + NC - 2$ elements and the range of i in equations (3) is similarly increased. The equation used to satisfy the Kutta condition is unchanged except for the range of the summation.

2.5 Compressibility considerations

The theory described in sections 2.1 to 2.4 is based on incompressible flow but the effect of changing the free stream Mach number can be investigated using the Prandtl-Glanert transformation. The radial ordinates of the body are scaled by a factor of $\beta(=\sqrt{1 - M^2})$ and the incompressible flow calculated on the analogous body. The velocity increments thus calculated are rescaled by a factor of $1/\beta$, on the radial velocity, and $1/\beta^2$ on the axial velocity. The tangential velocity at the ith control point then becomes

$$V_{t_i} = V_0 \vec{\cos} \theta_i + \frac{V_x}{\beta^2} \cos \theta_i + \frac{V_r}{\beta} \sin \theta_i$$

and the pressure coefficient is calculated using the formula

$$C_p = \frac{2}{\gamma M^2} \left\{ \left[1 - \frac{\gamma - 1}{2} M^2 (V_{t_i}^2 - 1) \right]^{3.5} - 1 \right\}$$

3 THE COMPUTER PROGRAM

The computer program has been written in FORTRAN for an ICL 1907 computer. A listing of the program is given in Appendix A and a flow chart in Fig.2.

The program consists of a MASTER segment: A34R; five subroutines: XFAN, CAM, FORM, ELE, INVERT; two library subroutines: F4ELC1, F4ELC2; and four function segments: SIMPSN, DIR, TERP, VR. The MASTER segment is described in section 3.1 and the subroutines and functions in section 3.2. The core store requirements and running time of the program are discussed in section 3.3.

The numbers in brackets in the following text refer to the line numbers in the listing of the program.

3.1 The MASTER segment

The MASTER segment controls the running of the program and all the input and output operations. The physical quantities represented by the main arrays and variables used in the segment are listed in Appendix B.

The initial statements (0110-0170) are the normal FORTRAN statements for declaring the size of arrays and the type of variable used. The program has been written to accept up to 89 control points which is equivalent to 90 body ordinates for an isolated aerofoil, or 91 ordinates for an aerofoil and centrebody. The pressure distribution at up to five mass flow ratios can be produced with a single run of the program. These limits can be changed by altering the dimensions of the arrays throughout the program.

After setting some initial constants used in the segment (0180-0200) the input data is read (0210-0420). For the following text, it is assumed that the input data is punched on 80 column cards and that the reader is familiar with the FORMAT statement. The input data is summarised in Appendix C.

The first data card contains a case number, CASEN, of eight characters, and a case description, stored in the array TEXT, of up to 72 characters. The characters are read using an 'A' field descriptor and may therefore consist of any characters in the FORTRAN set, in particular, the case number need not necessarily be an integer. These quantities take no useful part in the calculation and are only used to identify the output.

The number of ordinates on the aerofoil surface, N, is read followed by N pairs of ordinates X,R. The ordinates must be specified from the trailing edge on the inner surface to the trailing edge on the outer surface of the aerofoil. No special distribution of points is necessary though it is advisable to space the ordinates closely in regions of high curvature and to avoid rapid changes in the spacing between the points. The first and last input points must be at the trailing edge of the aerofoil and one point must be at the leading edge, X = 0. The error in the calculated circulation γ_k decreases as the point at which the Kutta condition is applied is moved nearer to the trailing edge[4] so it is recommended that the second, and last but one input points, are fairly near to the trailing edge.

The number of ordinates on the centrebody, NC, is read, and if NC is non-zero, the centrebody ordinates. These points should be in order of increasing axial ordinate. The last pair of ordinates is followed by the quantity RD, which is the radius of the centrebody at the leading edge of the aerofoil. The program can therefore deal with spinners which protrude from the aerofoil. If the centrebody does not extend to the leading edge, RD should be zero.

The number of mass flow ratios, NF1, at which the pressure distribution is to be calculated is read followed by a card containing up to eight quantities. The first three numbers are respectively, the trailing edge radius of the aerofoil, RO, the chord length of the aerofoil, CHORD, and the free stream Mach number. The remaining quantities are the values of the mass flow ratio, AOAI. All the data referring to the geometry of the aerofoil and the centrebody must be measured in the same coordinate system with the leading edge of the aerofoil at X = 0.

The input peripheral is released (0430) and two arbitrary values of the strength of the 'fan' vortex are specified (0500-0520). The mass flow ratio produced by these values of the 'fan' strength is calculated and linear interpolation is used to derive the 'fan' strength which will give the specified mass flow ratio. A matrix formulation is used so the matrix of velocities corresponding to the left hand side of equations (3) and (4) has only to be evaluated once as these velocities depend on the geometry of the configuration and not on the 'fan' strength. The main matrix is inverted and a solution of the equations can be obtained for any number of 'fan' strengths by a simple matrix multiplication. Most values of the 'fan' strength required to give mass flow ratios of practical interest have been found to lie between the two values chosen, which are 0 and -0.3.

The input data is transformed according to the Prandtl-Glanert compressibility laws (0530-0610) and the ordinates of the control points XP, RP calculated (0620-0670).

The ordinates of the camber surface are not required to a high degree of accuracy and linear interpolation is used. A dummy call to the interpolation function TERP is made (0690) to transform the axial ordinate X(I) to the array TH(I). The elements of this array are simply the axial ordinates of the aerofoil but multiplied by -1 if the point is on the inner surface; it is then possible to distinguish between the inner and outer surfaces of the aerofoil. The camber ordinates are calculated by the subroutine CAM, at every 2% chord over the chord length of the aerofoil, and specified at every 4% chord on the cylinder downstream of the trailing edge. The camber surface is covered with a uniform vortex distribution density so the choice of the axial location of the camber ordinates is fairly arbitrary; in this respect the present method is more flexible than is the case if discrete vortex rings are used.

The velocity components induced at the control points by the two vortex distributions are calculated by the subroutine XFAN (0720-0830). The subroutine calculates the radial and axial velocity components because these are required again later in the program, but then they are scaled by the appropriate compressibility factors. The normal and tangential velocity components are put in the arrays VNG, VTG for the 'Kutta' vortex distribution and in the arrays VNF, VTF for the 'fan' vortex distribution.

The two right hand sides of the equations corresponding to the chosen 'fan' strengths are evaluated (0840-0950). The main matrix corresponding to the left hand side of the equations is set up by the subroutine FORM and inverted (0960-0980). The strengths of the singularities are found by multiplying the inverted matrix by the right hand sides (0990-1060). The source strengths are held in the array SOL and the strengths of the 'Kutta' vortex in the array G.

The mass flow ratio is determined (1070-1280) by integrating the axial velocities calculated across the face of the aerofoil. These axial velocities do not need any scaling for compressibility as the calculation is made in the transformed space and the effective Mach number is zero.

The strengths of the 'fan' vortex distribution required to give the specified mass flow ratios are obtained (1310-1320) and the calculation jumps back (0850) to form a new set of right hand sides. The second set of source and vortex strengths are found using the inverted matrix and as a check on the interpolation the true value of the mass flow ratio is calculated. In all the calculations made so far, the value of the mass flow ratio calculated using the interpolated value of the 'fan' strength has agreed with the specified value to an adequate accuracy.

The tangential velocity component at the control points are calculated (1360-1570) by adding the contributions from the source distribution, calculated by the subroutine ELE, and the vortex distributions to the free stream velocity. The appropriate compressibility scaling factors are used throughout.

The computed pressure distributions are then printed out preceded by a tabulation of the input data (1580-1950).

3.2 Subroutines and functions

Five subroutines have been written; two are used to calculate the velocities induced by the source and vortex distributions: ELE, XFAN. The subroutine CAM calculates the ordinates of the camber surface and the subroutine FORM and INVERT set up and invert the main matrix.

The subroutine XFAN (1980-2430) calculates the axial and radial velocity components induced at the control points by the 'fan' and 'Kutta' vortex distributions. A vortex distribution of unit strength is placed on the camber surface and on the cylinder downstream of the trailing edge. There is no closed form for the velocity induced by an element of the camber surface as in the two-dimensional case so an integration has to be made. Each element of the camber surface is divided into a number of vortex rings, the number chosen depending on the relative position of the control point and the element, and an integration using Simpson's rule made. This numerical integration process is also performed on the cylinder from the trailing edge to some convenient point downstream, in this case taken as 3.04 chords. The velocity components induced by the remaining semi-infinite vortex cylinder downstream of 3.04 chord are evaluated at the axial position corresponding to the control point but at a radial ordinate equal to the radius of the cylinder (2370-2410). This allows a closed form for the integral to be used and introduces only a small error. The summation for the 'Kutta' vortex is taken over the first 50 elements of the camber surface corresponding to an integration over the chord length.

The ordinates of the camber surface are calculated by the subroutine CAM (2440-2610). The radial ordinates are calculated using linear interpolation over the chord length (2570-2590) and are set equal to the radius of the trailing edge for axial locations downstream of the aerofoil (2520-2530).

The subroutine FORM (2620-2880) sets up the main matrix of velocities corresponding to the left hand side of equations (3) and (4) of section 2.3. The normal velocity components induced by the source distribution on the surface of the aerofoil are calculated by the subroutine ELE (2890-3460) which is a modified form of the subroutine INX 1 of Ref.5. The velocity components are evaluated in a similar way to those in XFAN, but the subroutine ELE also has to deal with the singular integral when the control point lies on the surface element over which the integral is being made (3290-3420). The subroutine is also used in the MASTER segment to calculate the tangential velocity components. The surface slope TAU in this case is replaced by TAU $- \pi/2$.

The parameter B1 is used to scale the axial and radial velocity components by the correct compressibility factors. When the normal velocities are calculated, B1 is set equal to unity so that no scaling is applied, but in the calculation of the tangential velocities, B1 is set equal to β, and B2 to β^2.

The main matrix is inverted by the subroutine INVERT (3470-3630). The matrix is well-behaved and no sophisticated inversion technique is required. The subroutine listed is the simplest that could be found[6].

Two library subroutines F4ELC1, F4ELC2 are used in the program, to calculate the first and second complete elliptic integrals which are required in the calculation of the velocity components. The first parameter in the subroutine is the argument, k^2, and the second parameter is the value of the integral on return. A simple polynomial approximation to each function is used[14].

The four function segments are self-explanatory and need little comment. The function SIMPSN performs numerical integration using Simpson's rule. The correct sense and value of the surface slope is evaluated by the function DIR which is a modified form of the function PSI of Ref.6. The function TERP performs linear interpolation. The dummy call to this function (0690) is used to set up the array TH(I). The intake velocity ratio corresponding to the mass flow ratio VI is calculated by the function VR. The velocity ratio is found from an iterative solution to the equation[7]

$$VI = VR(0.2M^2(1 - VR^2) + 1.0)^{2.5} .$$

Newton's method for finding the zero of a function is used to give rapid convergence.

3.3 Computing details

It is difficult to give the precise time taken by the program since it varies considerably with the number of input points. On an ICL 1907 computer with a 1.2 μs core cycle time, a calculation with the maximum number of input points needs about 10 minutes of central processor time. The program as listed compiled by XFAT Mk.2E requires 30 k words of core store.

4 COMPARISON BETWEEN THEORY AND EXPERIMENT

The computer program was developed as a complement to some experiments that were made on three annular aerofoils[7]. These aerofoils had a chord to diameter ratio of unity and were tested over a wide range of Mach number and mass flow ratio in the RAE 8ft × 6ft transonic tunnel. The cowls were mounted on a semi-infinite centrebody which was represented in the calculations.

The calculated pressure distribution on cowl 1 at a high mass flow ratio is compared with the measured distribution in Fig.3. The overall agreement between theory and experiment is quite good except on the inner surface downstream of the peak where there was a local flow separation.

The importance of correctly representing the afterbody is demonstrated in Fig.4. The pressure distribution calculated on the forebody of cowl 1 is compared with that calculated on a forebody of the same shape followed by a long cylindrical afterbody. The difference in the pressure distribution is mainly due to the circulation developed around the complete cowl.

Some comparisons between the calculated pressure distribution, made with about 70 control points, and the measured distributions for cowls 2 and 3 are shown in Figs.5 to 8. Again, good agreement is obtained except at the leading edge of the cowl where the theory overestimates the suction level.

The theory has been compared with the experimental results up to a Mach number of 0.70 which is the Mach number at which shock waves started to appear on the cowls. Figs.9 and 10 show the pressure distribution on cowl 3 at a Mach number of 0.70 and at two mass flow ratios. The agreement is reasonable on the inner surface and behind the shock wave on the outer surface of the cowl.

Although the 'fan' vortex is placed on a cylinder downstream of the trailing edge, the stream tubes are curved as shown in Fig.11. The stream tubes were traced by calculating the value of the stream function at several radial positions and at thirty axial stations using the singularity strengths obtained from the program. Specified values of the stream function were found by interpolation. Fig.11 clearly shows the stream tubes expanding ahead of the cowl and contracting downstream of the trailing edge.

The predicted pressure distribution on an annular aerofoil with a chord to diameter ratio of 0.75 is shown in Fig.12. This is the aerofoil B1 designed by the Admiralty Research Laboratory[8] and tested in a low speed wind-tunnel at NPL. The ordinates are not particularly well defined in the reference and the calculation was made with only 50 control points, but the agreement is still good.

5 COMPARISON WITH OTHER THEORIES

Several other methods for calculating the pressure distribution on an annular aerofoil have appeared in recent years and these are compared with experiment and the present method in this section.

The computer program written by Mason[9] at Rolls Royce was one of the first to be developed. The method is similar to that described in section 2.1 except that the surface singularities may be sources or vortices and a variety of boundary conditions can be imposed. Most of the calculations for annular aerofoils have been made using a surface vortex distribution with the boundary condition that the stream function should have a specified value at all the control points. The stream function is related to the inlet velocity ratio by the formula

$$\frac{V_i}{V_0} = \frac{\psi_{TE}}{\frac{1}{2}R_0^2 V_0}$$

so the method can calculate the pressure distribution on the aerofoil for any inlet conditions fairly easily. However, when the mass flow ratio is reduced below the free flow value there should be a trailing vortex system similar to that described in section 2.2 but this is not represented in the Rolls Royce program. The greatest deficiency in the method is that no Kutta condition is applied and generally, there is a singularity in the velocity distribution at the trailing edge.

Some calculations have been made by Rolls Royce on the three annular aerofoils tested at RAE[7]. Fig.13 shows the predicted pressure distribution on cowl 2 at low Mach number. The corresponding pressure distribution calculated with the present program is shown in Fig.5. The infinite velocity at the trailing edge of the Rolls Royce calculation is not apparent in this case, and generally, the agreement is good. A more typical result is shown in Fig.14, for cowl 3, corresponding to Fig.7 for the present method. The calculated pressure distribution breaks down at about 80% chord although the agreement on the forebody and on the inner surface of the aerofoil is quite good.

A considerable amount of theoretical work on annular aerofoils using linearised and non-linearised theory has been done by Geissler[10]. His non-linearised theory uses a surface vortex distribution with the same boundary condition used in the present method, i.e. the normal velocity component is zero at the control points. Another vortex distribution, also placed on the surface of the aerofoil is used to satisfy the Kutta condition. The Kutta condition is applied at the trailing edge and is that the flow should be tangential along a line bisecting the trailing edge angle. There is no convenient way of changing the mass flow ratio and to compare theory and experiment at the same inlet conditions requires a change in the strength of the 'Kutta' vortex distribution. A reduction of about 25% is required to match the results for cowl 2 and about 20% for cowl 3. Once the strength of the vortex distribution has been changed, the Kutta condition is no longer satisfied and the theory predicts an infinite velocity at the trailing edge. However, the agreement between theory and experiment is extremely good over all but the last few per cent of the aerofoil chord.

Another approach to the problem has been adopted by Ryan[11]. This method is based on the work of Martensen[12] and Wilkinson[13] for twodimensional aerofoils and cascades and uses discrete vortex rings instead of a distribution on surface elements. The boundary condition is that the tangential velocity is zero inside the aerofoil. The major disadvantage of the method is that the solution is calculated at specified locations so it is difficult to calculate the pressure distribution in regions of particular interest unless the number of input points is increased significantly. However, this does not necessarily give greater accuracy because errors arise from the use of isolated vortex rings[4].

Ryan uses the same Kutta condition as Wilkinson[13] that the load is zero at the point nearest the trailing edge. This is achieved by setting the vortex strength at the first and last points equal and opposite. This choice of Kutta condition is not the best for cowls or intakes as considerable numerical problems arise if the method is extended to calculate the pressure distribution at different mass flow ratios to simulate the effect of a propeller or screen.

The pressure distribution on the ARL duct B1 calculated by an early version of Ryan's program is shown in Fig.15, and the results from the present theory in Fig.12. The mass flow is incorrect by about 12%, but better agreement is obtained for the B3 duct, Fig.16, particularly on the outer surface.

The program developed at ARA by Langley (unpublished) uses a vortex distribution on the surface of the aerofoil and another vortex distribution on a cylinder downstream of the trailing edge. The boundary condition is that the stream function should have a specified value on the surface as in the Rolls Royce method and the Kutta condition is the same as in the present method. Fig.17 compares the pressure distribution predicted by Langley's theory and the present method on an annular aerofoil with a chord to diameter ratio of unity and a 10% RAE 101 thickness distribution. The agreement between the two methods is quite good.

6 CONCLUSIONS

A theory has been developed and a computer program written to calculate the pressure distribution on an isolated annular aerofoil or an annular aerofoil and centrebody. The method gives results that are in close agreement with experiment over a range of geometries, Mach number, and mass flow ratio.

The present theory has also been compared with several other methods dealing with the same problem. The calculation methods developed by Langley and Geissler use a similar model of the flow and give similar results to the present method though Geissler's method is less flexible since it cannot be used to calculate the pressure distribution at any mass flow ratio. The other methods are deficient or restricted in some respects though good agreement between theory and experiment is obtained in some cases.

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Listing of the program

```
      MASTER A34R
C     ANNULAR AEROFOIL PROGRAM
      DIMENSION TEXT(9),X(91),R(91),XP(89),RP(89),F(5),RC(102),X1(102),
     1RF(11),VF(11,5),VNF(89),VTF(89),VNG(89),VTG(89),RHS(90,5),G(5),
     2SOL(90,5),U(89,5),PI(89),VAU(11),XMU(5),AOAI(5),VRF(89),VXF(89),
     3VRG(89),VXG(89)
      LOGICAL DER
      REAL MACH,MACH2
      COMMON BIG(90,90)
      ND=102
      PI24=2.0*ATAN(1.0)
      DER=.FALSE.
C     READ INPUT DATA
      READ(1,100)CASEN,(TEXT(I),I=1,9)
100   FORMAT(10A8)
C     CASEN=CASE NUMBER, TEXT=CASE DESCRIPTION (8,72 CHARACTERS RESP.)
      READ(1,101)N
101   FORMAT(I5)
C     N=NUMBER OF INPUT POINTS
      READ(1,102)(X(I),R(I),I=1,N)
102   FORMAT(8F10.6)
C     X(I),R(I) ARE BODY ORDINATES
      READ(1,101)NC
C     NC=NUMBER OF CENTRE-BODY POINTS, N+NC LESS THAN 91
      NPNC=N+NC
      IF(NC.NE.0)READ(1,102)(X(I),R(I),I=N+1,NPNC),RD
C     READ ORDINATES OF CENTRE-BODY, RD=CENTRE-BODY RADIUS AT X=0
      READ(1,103)NF1
103   FORMAT(I1)
C     NF1=NUMBER OF MASS FLOW RATIOS (MAXIMUM OF 5)
      READ(1,104)R0,CHORD,MACH,(AOAI(I),I=1,NF1)
104   FORMAT(8F10.5)
C     R0=TRAILING-EDGE RADIUS, CHORD=CHORD LENGTH
C     M=MACH NUMBER, AOAI(I)=MASS FLOW RATIOS
      CALL RLEASE(1)
      IF(NC.EQ.0)RD=0.0
      N1=N-1
      NT=1
      NF=2
      NPNC2=NPNC-2
      IF(NC.EQ.0)NPNC2=N-1
```

```
C       SET UP TWO INITIAL FAN STRENGTHS
        F(1)=0.0
        F(2)=-0.3
C       TRANSFORM INPUT DATA
        MACH2=MACH*MACH
        BETA2=1.0-MACH2
        BETA=SQRT(BETA2)
        RC=RO*BETA/CHORD
        RD=RD*BETA/CHORD
        DO 204 I=1,NPNC
        X(I)=X(I)/CHORD
204     R(I)=R(I)*BETA/CHORD
C       CALCULATE CONTROL POINTS
        DO 205 I=1,NPNC2
        L=I
        IF(I.GE.N)L=L+1
        XP(I)=0.5*(X(L)+X(L+1))
205     RP(I)=0.5*(R(L)+R(L+1))
C       CALCULATE CAMBER ORDINATES
        A=TERP(N,X,R,0.3,DER)
        DER=.TRUE.
        CALL CAM(ND,X1,RC,X,R,N,DER,RO)
C       CALCULATE VELOCITIES INDUCED BY FAN AND KUTTA VORTEX DISTRIBUTIONS
        DO 208 I=1,NPNC2
        L=I
        IF(I.GE.N)L=L+1
        TAU=ATAN2(R(L+1)-R(L),X(L+1)-X(L))
        CALL XFAN(XP(I),RP(I),X1,RC,RO,VRF(I),VXF(I),VRG(I),VXG(I),ND)
        SNT=SIN(TAU)
        CST=COS(TAU)
        VNF(I)=VRF(I)*CST-VXF(I)*SNT
        VNG(I)=VRG(I)*CST-VXG(I)*SNT
        VTF(I)=VXF(I)*CST+VRF(I)*SNT
208     VTG(I)=VXG(I)*CST+VRG(I)*SNT
C       SET UP RHS OF EQUATIONS
1000    DO 401 I=1,NPNC2
        L=I
        IF(I.GE.N)L=L+1
        SNT=SIN(DIR(R(L+1)-R(L),X(L+1)-X(L)))
        DO 401 J=1,NF
401     RHS(I,J)=SNT-F(J)*VNF(I)
```

```
      TAU1=DIR(R(2)-R(1),X(2)-X(1))-PI24
      TAU2=DIR(R(N)-R(N1),X(N)-X(N1))-PI24
      A=ABS(SIN(TAU2))-ABS(SIN(TAU1))
      DO 290 J=1,NF
290   RHS(NPNC2+1,J)=F(J)*(VTF(1)+VTF(N1))+A*F(J)
C     FORM MAIN MATRIX AND INVERSE IF NT=1
      IF(NT.EQ.1)CALL FORM(X,R,NPNC,VNG,VTG,PI,XP,RP,NPNC2,N)
      IF(NT.EQ.1)CALL INVERT(NPNC2+1)
C     CALCULATE SOURCE AND KUTTA VORTEX STRENGTHS
      DO 400 I=1,NPNC2+1
      DO 400 J=1,NF
      SOL(I,J)=0.0
      DO 400 K=1,NPNC2+1
400   SOL(I,J)=BIG(I,K)*RHS(K,J)+SOL(I,J)
      DO 209 J=1,NF
209   G(J)=SOL(NPNC2+1,J)
C     CALCULATE MASS FLOW RATIO
1002  DO 89 I=1,11
      DO 89 J=1,NF
89    VF(I,J)=0.0
      A=(RC(1)*RC(1)-RD*RD)/10.0
      RF(1)=0.001+RD
      DO 90 I=2,11
90    RF(I)=SQRT(A*FLOAT(I-1)+RD*RD)
      RF(11)=RF(11)-0.001
      DO 215 I=1,11
      CALL ELE(0.0,RF(I),-PI24,X,R,NPNC,0,PI,1.0,NPNC2,N)
      DO 216 J=1,NF
      DO 216 K=1,NPNC2
216   VF(I,J)=PI(K)*SOL(K,J)+VF(I,J)
      CALL XFAN(0.0,RF(I),X1,RC,RO,VR1,VX1,VR2,VX2,ND)
      DO 214 J=1,NF
214   VF(I,J)=VF(I,J)+VX1*F(J)+VX2*G(J)+1.0
215   CONTINUE
      DO 217 J=1,NF
      DO 213 I=1,11
213   VAU(I)=VF(I,J)
217   XMU(J)=SIMPSN(VAU,1,11,A)/(10.0*A)
      IF(NT.EQ.2)GO TO 901
C     CALCULATE FAN STRENGTHS FOR SPECIFIED MASS FLOW RATIOS
      DO 900 J=1,NF1
900   F(J)=-0.3*(AOAI(J)-XMU(1))/(XMU(2)-XMU(1))
```

```
      NT=2
      NF=NF1
      GO TO 1000
C     CALCULATE TANGENTIAL VELOCITIES
901   DO 510 I=1,NPNC2
      L=I
      IF(I.GE.N)L=L+1
      TAU=ATAN2(R(L+1)-R(L),X(L+1)-X(L))
      SNT=SIN(TAU)
      CST=COS(TAU)
      VTF(I)=VXF(I)*CST/BETA2+VRF(I)*SNT/BETA
510   VTG(I)=VXG(I)*CST/BETA2+VRG(I)*SNT/BETA
      DO 210 I=1,NPNC2
      L=I
      IF(I.GE.N)L=L+1
      TAU=DIR(R(L+1)-R(L),X(L+1)-X(L))-PI/2
      CALL ELE(XP(I),RP(I),TAU,X,R,NPNC,I,PI,BETA,NPNC2,N)
      DO 211 J=1,NF
      U(I,J)=0.0
      DO 211 K=1,NPNC2
211   U(I,J)=U(I,J)+SOL(K,J)*PI(K)
      SNT=SIN(TAU)
      DO 210 J=1,NF
      U(I,J)=U(I,J)+G(J)*VTG(I)+F(J)*VTF(I)*SNT
210   CONTINUE
C     CALCULATE AND PRINT OUTPUT
      DO 218 I=1,NPNC
218   R(I)=R(I)/BETA
      DO 513 I=1,NPNC2
513   RP(I)=RP(I)/BETA
      CALL DATE(A)
      CALL TIME(B)
      WRITE(2,106)A,B,CASEN,(TEXT(I),I=1,9),N,MACH
106   FORMAT(1H1,26X,28HROYAL AIRCRAFT ESTABLISHMENT//16X,46HAERODYNAMIC
     1S DEPARTMENT -- PROPULSION DIVISION///20X,40HCALCULATION OF THE P
     2RESSURE DISTRIBUTION//20X,39HON AN ANNULAR AEROFOIL BY THE METHOD
     3OF//30X,21HSURFACE SINGULARITIES///25X,5HDATE ,A8,4X,5HTIME ,A8//
     4/23X,34HCASE CONTROL DATA FOR PROGRAM A34R //29X,12HCASE NUMBER ,A
     58/24X,17HCASE DESCRIPTION ,9A8/18X,23HNUMBER OF INPUT POINTS ,I3/2
     69X,12HMACH NUMBER ,F8.5//23X,15HMASS FLOW RATIO,6X,12HFAN STRENGTH
     77/)
```

App.A(cont'd)

```
      WRITE(2,107)(XMU(J),F(J),J=1,NF)
107   FORMAT(1H ,23X,F10.5,10X,F10.5)
      WRITE(2,108)
108   FORMAT(1H0,35X,10HINPUT DATA//27X,1HX,29X,1HR/)
      WRITE(2,109)(X(I),R(I),I=1,NPNC)
109   FORMAT(1H ,20X,F10.5,20X,F10.5)
      DO 219 J=1,NF
      XM=VR(XMU(J),MACH2)
      WRITE(2,116)CASEN,(TEXT(I),I=1,9),MACH,F(J),XMU(J),XM
116   FORMAT(1H1,24X,32HCALCULATED PRESSURE DISTRIBUTION//29X,12HCASE NU
     1MBER ,A8//24X,17HCASE DESCRIPTION ,9A8//29X,12HMACH NUMBER ,F8.5//
     228X,13HFAN STRENGTH ,F8.5//25X,16HMASS FLOW RATIO ,F8.5//20X,21HIN
     3LET VELOCITY RATIO ,F8,5///25X,2HXP,8X,2HRP,8X,1HU,8X,2HCP/)
      DO 222 I=1,NPNC2
      IF(MACH.EQ.0.0)GO TO 304
      CP=2.0*((1.0-0.2*MACH2*(U(I,J)*U(I,J)-1.0))**3.5-1.0)/(1.4*MACH2)
      GO TO 305
304   CP=1.0-U(I,J)*U(I,J)
305   CONTINUE
222   WRITE(2,117)XP(I),RP(I),U(I,J),CP
117   FORMAT(1H ,20X,4F10.5)
219   CONTINUE
      STOP
      END
```

```
      SUBROUTINE XFAN(XP,RP,X1,RC,RO,AVR,AVX,GAVR,GAVX,ND)
C     CALCULATES THE AXIAL AND RADIAL VELOCITY COMPONENTS DUE TO
C     FAN AND KUTTA VORTEX DISTRIBUTIONS OF UNIT STRENGTH
      DIMENSION RC(ND),X1(ND),VX(50),VR(50),AAVX(101),AAVR(101)
      REAL K,K2,KK2
      ND1=ND-1
      AVX,AVR,GAVR,GAVX=0.0
      PI2=8.0*ATAN(1.0)
      DO 4 J=1,ND1
      AA=SQRT((X1(J+1)-X1(J))**2+(RC(J+1)-RC(J))**2)
      RS=SQRT((XP-X1(J))**2+(RP-RC(J))**2)+SQRT((XP-X1(J+1))**2+(RP-RC(J
     1+1))**2)
      CC=0.2+16.0*AA/RS
      NRD=CC
      NRD=2*NRD+1
      IF(NRD.LT.3)NRD=3
      DX=(X1(J+1)-X1(J))/FLOAT(NRD-1)
      DR=(RC(J+1)-RC(J))/FLOAT(NRD-1)
      S=SQRT(DR*DR+DX*DX)
      DO 1 IRD=1,NRD
      RR=RC(J)+DR*FLOAT(IRD-1)
      XX=X1(J)+DX*FLOAT(IRD-1)
      A=(XP-XX)*(XP-XX)
      G=(RP-RR)*(RP-RR)
      B=G+4.0*RP*RR+A
      K=4.0*RR*RP/B
      B=SQRT(B)
      CALL F4ELC1(K,C)
      CALL F4ELC2(K,E)
      VX(IRD)=(C-(1.0+2.0*RR*(RP-RR)/(A+G))*E)/(PI2*B)
1     VR(IRD)=(C-(1.0+2.0*RR*RP/(A+G))*E)*(XX-XP)/(PI2*RP*B)
      AAVX(J)=SIMPSN(VX,1,NRD,S)
4     AAVR(J)=SIMPSN(VR,1,NRD,S)
      DO 6 J=1,ND1
      AVX=AVX+AAVX(J)
6     AVR=AVR+AAVR(J)
      DO 7 J=1,50
      GAVX=GAVX+AAVX(J)
7     GAVR=GAVR+AAVR(J)
```

App.A(cont'd)

```
      K2=4.0*RO*RO/((XP-3.04)**2+4.0*RO*RO)
      CALL F4ELC1(K2,KK2)
      CALL F4ELC2(K2,EK2)
      AVX=AVX-(PI2/4.0-SQRT(1.0-K2)*KK2)/PI2
      AVR=AVR+(1.0/PI2)*(SQRT(K2)*(KK2-(2.0*(KK2-EK2)/K2)))
      RETURN
      END


      SUBROUTINE CAM(ND,X1,RC,X,R,N,D1,RO)
C     CALCULATES THE ORDINATES OF THE CAMBER SURFACE
      DIMENSION X1(ND),RC(ND),X(N),R(N)
      LOGICAL D1
      DO 1 I=1,51
1     X1(I)=0.02*FLOAT(I-1)
      DO 2 I=52,ND
2     X1(I)=0.04*FLOAT(I-51)+1.0
      DO 5 I=51,ND
5     RC(I)=RO
      DO 6 I=1,N
      IF(X(I).EQ.0.0)RC(1)=R(I)
6     CONTINUE
      DO 7 I=2,50
      T=X1(I)
7     RC(I)=0.5*(TERP(N,X,R,T,D1)+TERP(N,X,R,-T,D1))
      RETURN
      END
```

App.A(cont'd)

```
      SUBROUTINE FORM(X,R,N,VNG,VTG,PI,XP,RP,N1,N2)
C     SETS UP MAIN MATRIX, I,E, LHS OF EQUATIONS
      DIMENSION X(N),R(N),PI(N1),VNG(N1),VTG(N1),XP(N1),RP(N1)
      COMMON BIG(90,90)
      N3=N2-1
      B=1.0
      PI24=2.0*ATAN(1.0)
      DO 1 I=1,N1
      L=1
      IF(I.GE.N2)L=L+1
      TAU=DIR(R(L+1)-R(L),X(L+1)-X(L))
      CALL ELE(XP(I),RP(I),TAU,X,R,N,I,PI,B,N1,N2)
      DO 1 J=1,N1
1     BIG(I,J)=PI(J)
      DO 2 I=1,N1
2     BIG(I,N1+1)=VNG(I)
      TAU=DIR(R(2)-R(1),X(2)-X(1))-PI24
      CALL ELE(XP(1),RP(1),TAU,X,R,N,1,PI,B,N1,N2)
      DO 3 J=1,N1
3     BIG(N1+1,J)=-PI(J)
      TAU=DIR(R(N2)-R(N3),X(N2)-X(N3))-PI24
      CALL ELE(XP(N3),RP(N3),TAU,X,R,N,N3,PI,B,N1,N2)
      DO 4 J=1,N1
4     BIG(N1+1,J)=BIG(N1+1,J)-PI(J)
      BIG(N1+1,N1+1)=-VTG(1)-VTG(N3)
      RETURN
      END
```

```
      SUBROUTINE ELE(XP,RP,TAU,X,R,N,I,PI,B1,N1,N4)
C     CALCULATES THE VELOCITY COMPONENTS DUE TO THE
C     SURFACE SOURCE DISTRIBUTION
      DIMENSION X(N),R(N),WW(42),PI(N1)
      REAL KK,LGS
      PI2=8.0*ATAN(1.0)
      SNT=SIN(TAU)
      CST=COS(TAU)
      B2=B1*B1
      DO 4 L=1,N1
      J=L
      IF(J.GE.N4)J=J+1
      AA=SQRT((X(J+1)-X(J))**2+(R(J+1)-R(J))**2)
      RS=SQRT((XP-X(J))**2+(RP-R(J))**2)+SQRT((XP-X(J+1))**2+(RP-R(J+1))
     1**2)
      CC=0.2+16.0*AA/RS
      NRD=CC
      NRD=2*NRD+1
      IF(NRD.LT.3)NRD=3
      IF(I.EQ.L)NRD=NRD+1
      DX=(X(J+1)-X(J))/FLOAT(NRD-1)
      DR=(R(J+1)-R(J))/FLOAT(NRD-1)
      S=SQRT(DR*DR+DX*DX)
      DO 1 IRD=1,NRD
      RR=R(J)+DR*FLOAT(IRD-1)
      XX=X(J)+DX*FLOAT(IRD-1)
      XPX2=(XP-XX)*(XP-XX)
      A=RP*RP+RR*RR+XPX2
      B=2.0*RP*RR
      AMB=A-B
      APB=A+B
      VK1=2.0*B/APB
      APB=SQRT(APB)
      CALL F4ELC1(VK1,KK)
      CALL F4ELC2(VK1,EK)
1     WW(IRD)=CST*(RR*(KK-EK)/RP+2.0*RR*(RP-RR)*EK/AMB)/(PI2*APB*B1)-SNT
     1*2.0*RR*(XP-XX)*EK/(PI2*AMB*APB*B2)
      IF(I.EQ.L)GO TO 2
      PI(L)=SIMPSN(WW,1,NRD,S)
      GO TO 3
2     N2=NRD/2
      N3=N2+1
      PI(L)=SIMPSN(WW,1,N2,S)+SIMPSN(WW,N3,NRD,S)
```

App.A(cont'd)

```
      S=S/RP
      SIGMA=DIR(R(J+1)-R(J),X(J+1)-X(J))
      SNS=SIN(SIGMA)
      CSS=COS(SIGMA)
      SNS2=SNS*SNS
      SNS4=SNS2*SNS2
      LGS=ALOG(S/16.0)
      S2=S*S
      PX=-SNS*CSS*S*(1.0+(13.0/6.0+LGS+SNS2)*S2/96.0)/PI2
      PR=-S*(SNS2+LGS-(3.0*(1.0+LGS-SNS2)-2.0*SNS4)*S2/192.0)/PI2
      PI(L)=PI(L)+0.5*(COS(SIGMA-TAU)-PX*SNT/B2+PR*CST/B1)
    3 CONTINUE
    4 CONTINUE
      RETURN
      END


      SUBROUTINE INVERT(N)
C     INVERTS NXN MATRIX IN THE COMMON BLOCK
      COMMON A(90,90)
      DO 1 I=1,N
      TEMP=A(I,I)
      A(I,I)=1.0
      DO 2 J=1,N
    2 A(I,J)=A(I,J)/TEMP
      DO 1 K=1,N
      IF(K-I)3,1,3
    3 TEMP=A(K,I)
      A(K,I)=0.0
      DO 4 J=1,N
    4 A(K,J)=A(K,J)-TEMP*A(I,J)
    1 CONTINUE
      RETURN
      END
```

App.A(cont'd)

```
      FUNCTION SIMPSN(FR,IA,N,H)
C     NUMERICAL INTEGRATION USING SIMPSONS RULE
      DIMENSION FR(N)
      L=(N-IA)/2
      N1=N-1
      IF(N-IA-2*L)21,22,21
22    S=0.0
      DO 23 I=IA,N1,2
23    S=S+H*(FR(I)+4.0*FR(I+1)+FR(I+2))/3.0
      GO TO 24
21    S=H*(5.0*FR(IA)+8.0*FR(IA+1)-FR(IA+2))/12.0
      DO 25 I=IA+1,N1,2
25    S=S+H*(FR(I)+4.0*FR(I+1)+FR(I+2))/3.0
24    SIMPSN=S
      RETURN
      END
```

App.A(cont'd)

```
      FUNCTION DIR(DY,DX)
C     CALCULATES CORRECT SLOPE OF BODY SURFACE
      PI=4.0*ATAN(1.0)
      IF(DY.LE.0.0)GO TO 3
      IF(DX.LE.0.0)GO TO 1
      DIR=ATAN(DY/DX)
      RETURN
1     IF(DX.LT.0.0)GO TO 2
      DIR=PI/2.0
      RETURN
2     DIR=PI-ATAN(ABS(DY/DX))
      RETURN
3     IF(DY.LT.0.0)GO TO 7
      IF(DX.LE.0.0)GO TO 4
      DIR=0.0
      RETURN
4     IF(DX.LT.0.0)GO TO 6
      WRITE(2,5)
5     FORMAT(1H1,10X,27HFUNCTION DIR INDETERMINATE./)
      DIR=0.0
      RETURN
6     DIR=PI
      RETURN
7     IF(DX.LE.0.0)GO TO 8
      DIR=-ATAN(ABS(DY/DX))
      RETURN
8     IF(DX.LT.0.0)GO TO 9
      DIR=-PI/2.0
      RETURN
9     DIR=-PI+ATAN(ABS(DY/DX))
      RETURN
      END
```

```
      FUNCTION TERP(N,X,R,A,D)
C     LINEAR INTERPOLATION FUNCTION
      DIMENSION X(N),R(N),TH(80)
      LOGICAL D
      IF(D)GO TO 2
      N1=N-1
      DO 1 I=1,N1
      TH(I)=X(I)
      IF(X(I).GT.X(I+1))TH(I)=-TH(I)
1     CONTINUE
      TH(N)=X(N)
2     DO 3 I=1,N1
      IF(A.GT.TH(I).AND.A.LE.TH(I+1))GO TO 4
      GO TO 3
4     TERP=R(I)+(R(I+1)-R(I))*(A-TH(I))/(TH(I+1)-TH(I))
      RETURN
3     CONTINUE
      RETURN
      END

      FUNCTION VR(VI,AM)
C     CALCULATES VELOCITY RATIO FROM CORRESPONDING MASS FLOW RATIO
      A=0.2*AM
      NC=1
      VO=VI
2     Y=VO*(A*(1.0-VO*VO)+1.0)**2.5-VI
      Y1=(4.0*A*VO*VO+A+1.0)*(A*(1.0-VO*VO)+1.0)**1.5
      DY=-Y/Y1
      VN=VO+DY
      IF(ABS(DY).LT.0.0000001)GO TO 1
      VO=VN
      NC=NC+1
      IF(NC.GT.100)GO TO 3
      GO TO 2
3     WRITE(2,4)
4     FORMAT(1H0,25HVR FUNCTION NOT CONVERGED)
1     VR=VN
      RETURN
      END
```

Appendix B

LIST OF THE MAIN VARIABLES USED IN THE MASTER SEGMENT

There follows a list of the main variables and arrays used in the MASTER segment of the program with the physical quantity represented by each.

Variable or array	Physical quantity
AOAI	Specified mass flow ratio.
CP	Pressure coefficient.
F	Strength of the 'fan' vortex.
G	Strength of the 'Kutta' vortex.
N	Number of aerofoil ordinates specified.
NC	Number of centrebody ordinates specified.
NPNC	Total number of ordinates specified.
NPNC2	Number of control points.
PI	Velocities induced at a control point by the complete set of surface elements.
SOL	Surface source strengths.
TAU	Surface slope.
U	Tangential velocity.
VF	Axial velocities evaluated across the face of the aerofoil.
VNF, VTF	Normal and tangential velocities induced by the 'fan' vortex.
VNG, VTG	Normal and tangential velocities induced by the 'Kutta' vortex.
VRF, VXF	Radial and axial velocities induced by the 'fan' vortex.
VRG, VXG	Radial and axial velocities induced by the 'Kutta' vortex.
X, R	Aerofoil and centrebody ordinates.
X1, RC	Ordinates of the camber surface.
XP, RP	Ordinates of the control points.

Appendix C

INPUT DATA

The input data and format is summarised below.

Program variable or array	Data format	Physical quantity
CASEN, TEXT	10A8	CASEN=Case number (8 characters) TEXT=Case description (72 characters)
N	I5	Number of aerofoil ordinates.
X, R	8F10.6	Aerofoil ordinates (N pairs).
NC	I5	Number of centrebody ordinates.
if NC $\neq 0$		
X, R (continued), RD	8F10.6	Centrebody ordinates (NC pairs). RD=Radius of the centrebody at the leading edge of the aerofoil.
NF1	I1	Number of mass flow ratios.
Rθ, CHORD, MACH, AOAI	8F10.5	Rθ=Trailing edge radius of the aerofoil. CHORD=Chord length of the aerofoil. MACH=Mach number. AOAI=Mass flow ratios (NF1 values).

SYMBOLS

c	cowl or aerofoil chord length
C_p	pressure coefficient
F_i	a prescribed normal velocity boundary condition at the ith control point
ℓ_F	cowl forebody length
M	Mach number
N	number of ordinates specified on the aerofoil surface
NC	number of ordinates specified on the centrebody surface
q_j	source strength on the jth element
R	radial ordinate
V_i/V_0	inlet velocity ratio
V_0	free stream velocity
V_{n_i}, V_{t_i}	normal and tangential velocity components induced by the 'Kutta' vortex at the ith control point
$V^*_{n_i}, V^*_{t_i}$	normal and tangential velocity components induced by the 'fan' vortex at the ith control point
$V_{n_{ij}}, V_{t_{ij}}$	the normal and tangential velocity components induced at the ith control point by the source density on the jth surface element
V_{t_i}	total tangential velocity at the ith control point
V_x, V_r	axial and radial velocity components
X	axial ordinate
β	$\sqrt{1 - M^2}$
γ_F	strength of the 'fan' vortex distribution
γ_k	strength of the 'Kutta' vortex distribution
θ_i	surface slope at the ith control point
ψ_{TE}	value of the stream function at the trailing edge
μ	mass flow ratio

REFERENCES

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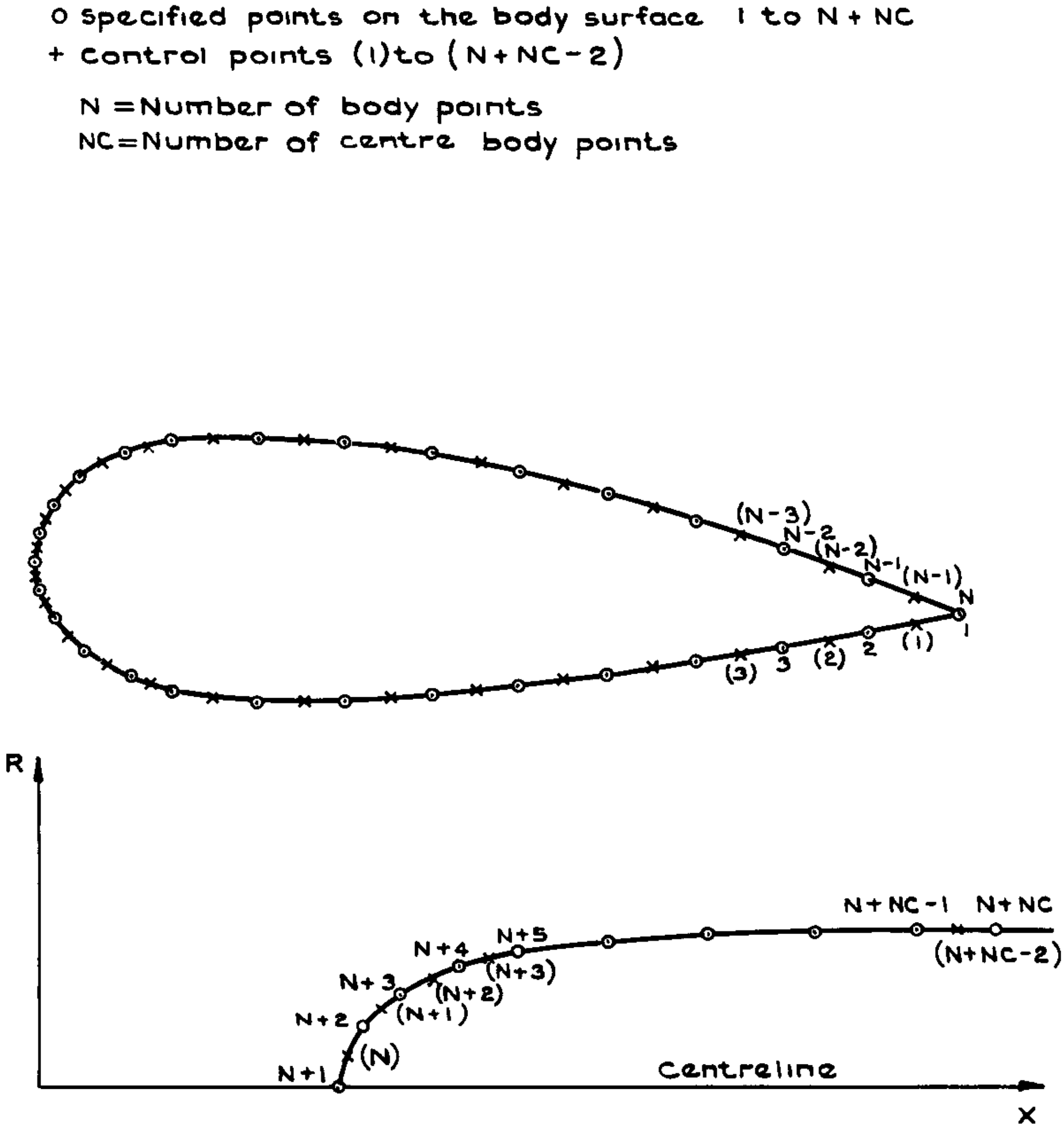


Fig. 1 Specification of the body geometry

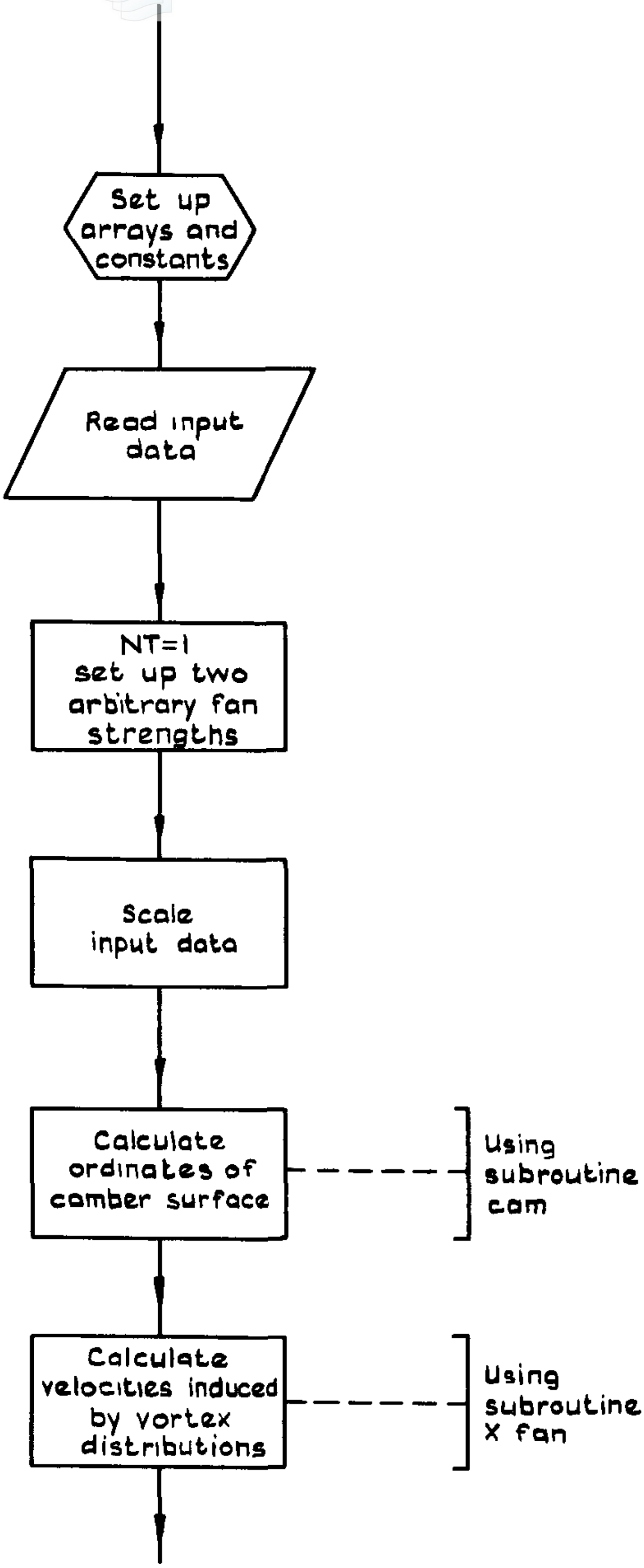


Fig. 2 Computer program flow chart

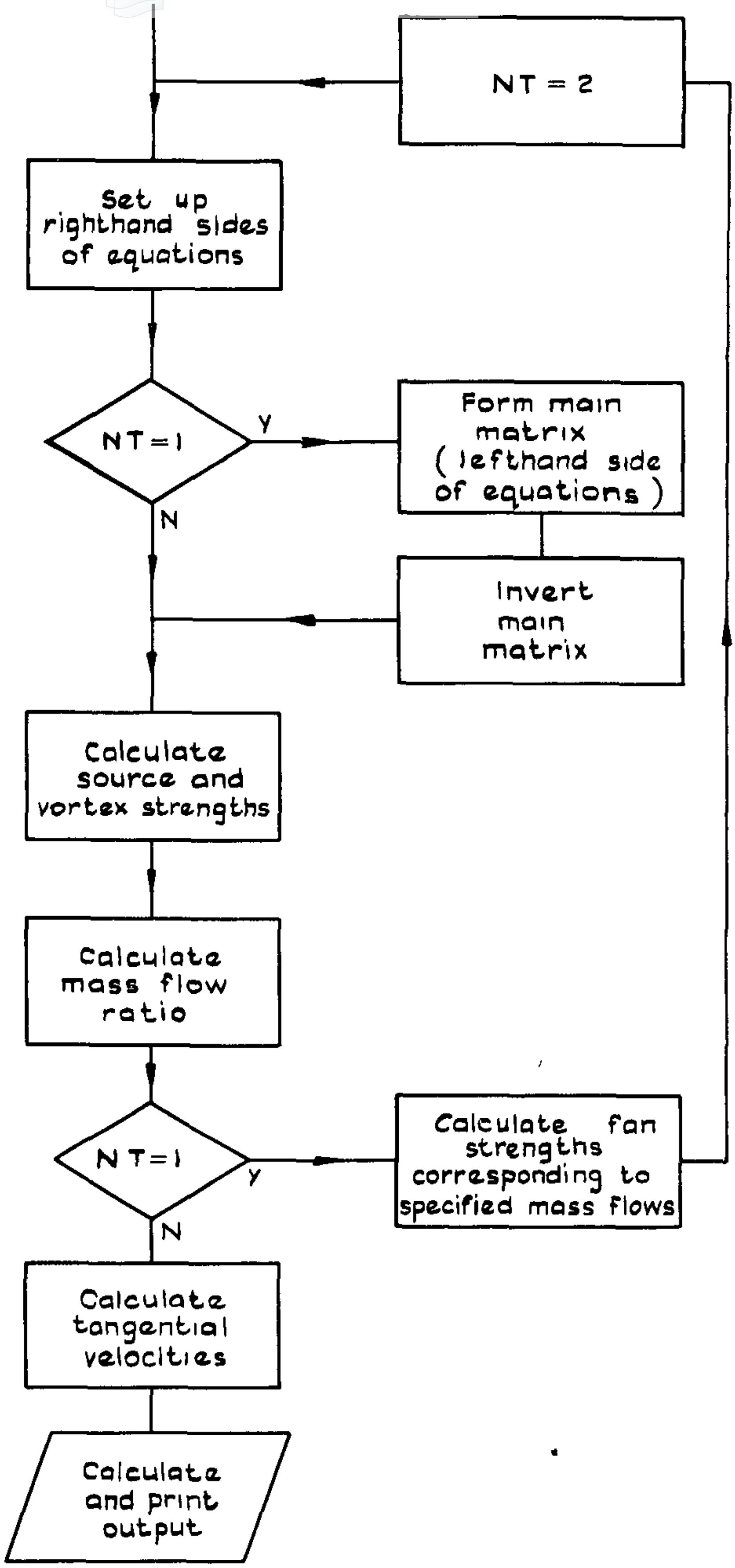


Fig. 2 cont'd Computer program flow chart

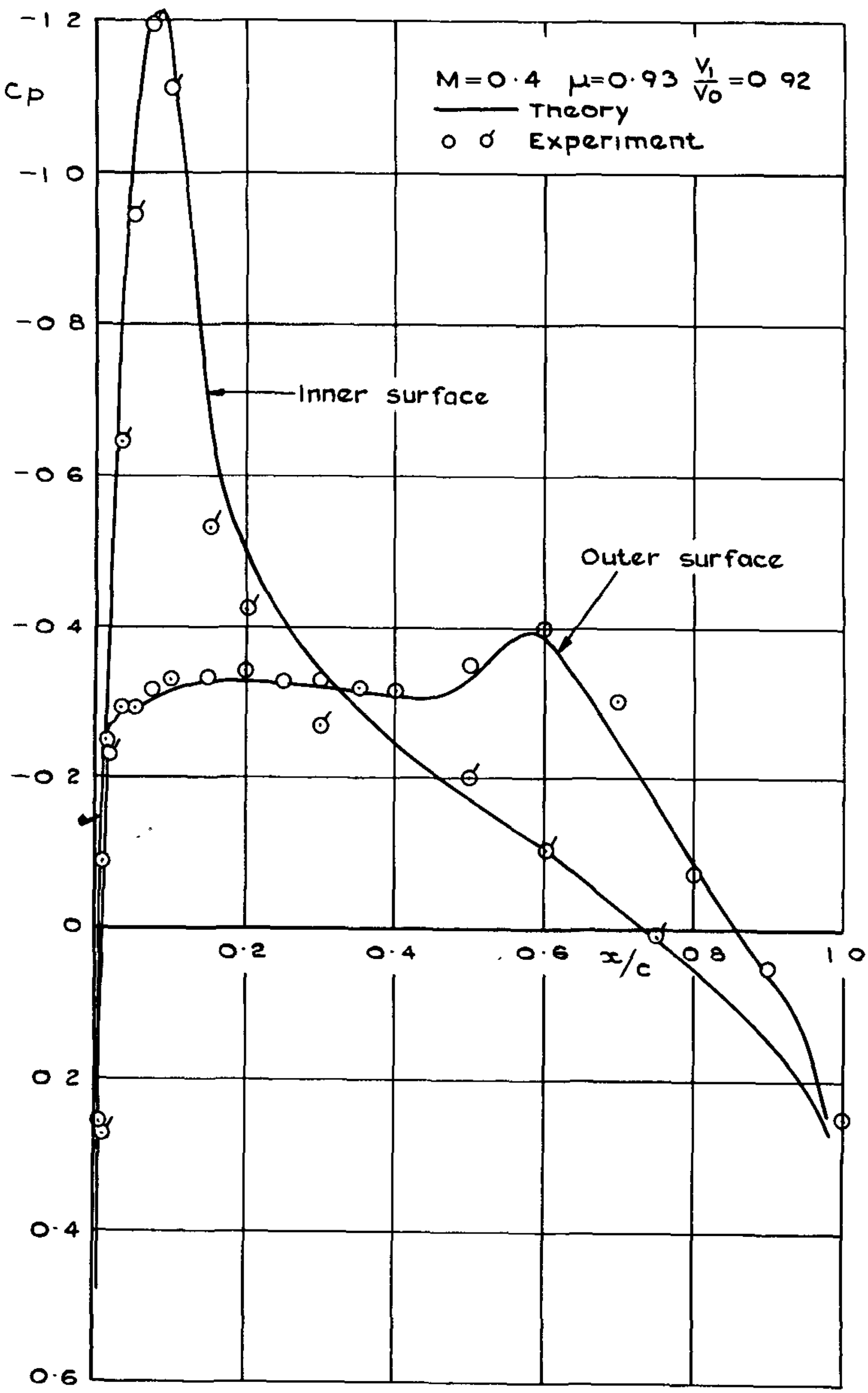


Fig. 3 Comparison between theory and experiment
RAE cowl 1

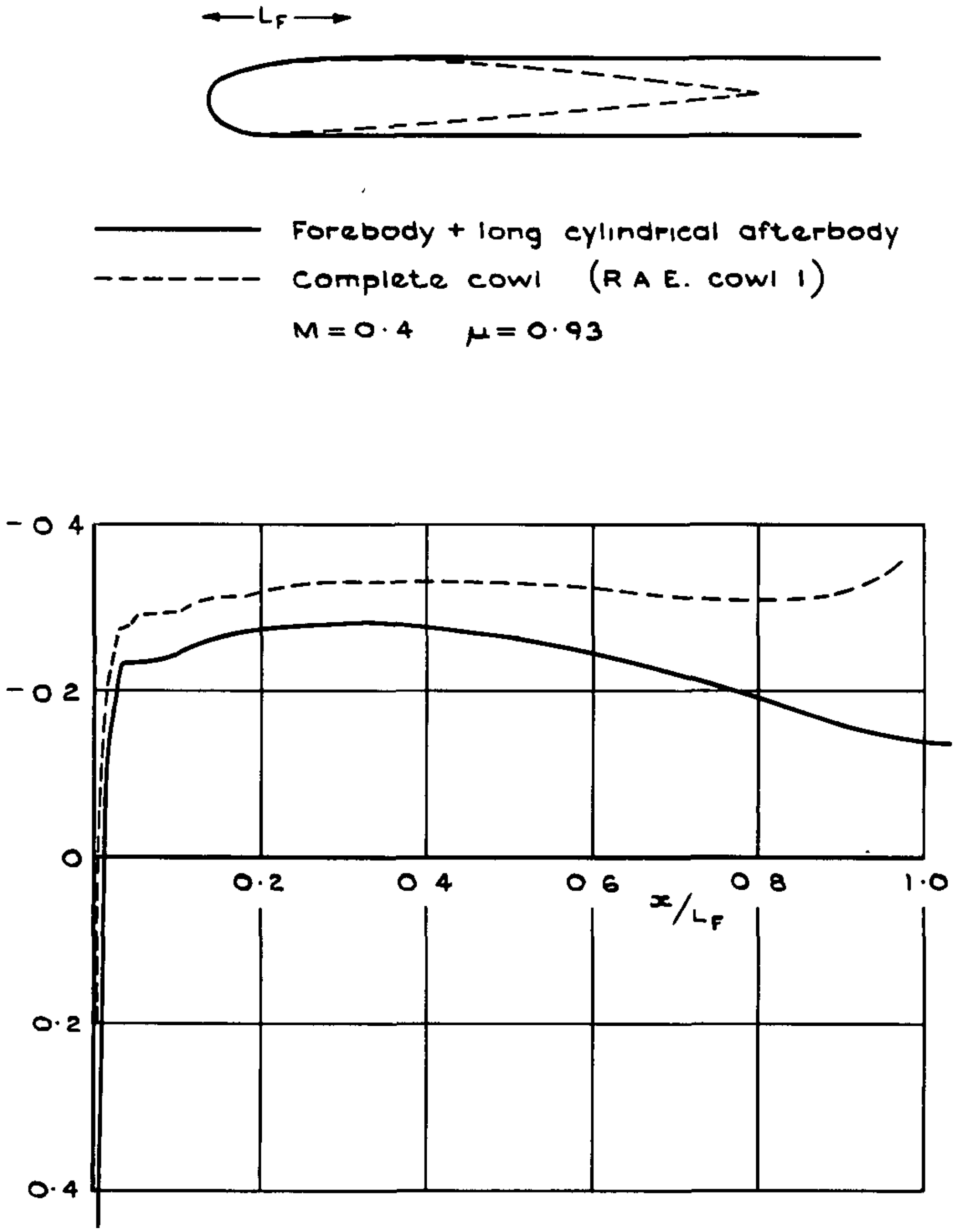


Fig. 4 Comparison between the calculated pressure distribution on the forebody of an intake and a complete cowl

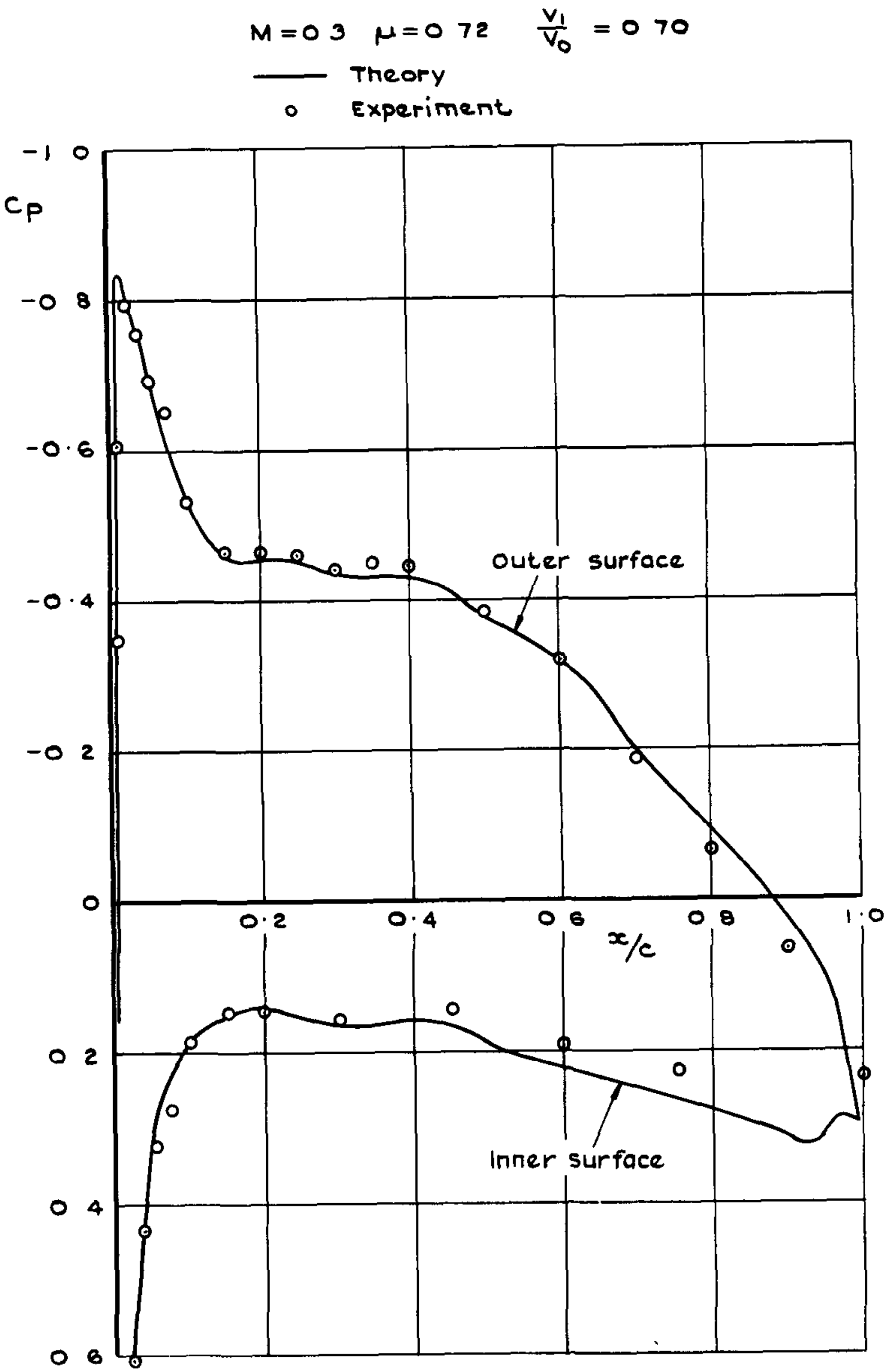


Fig. 5 Comparison between theory and experiment : R.A.E cowl 2

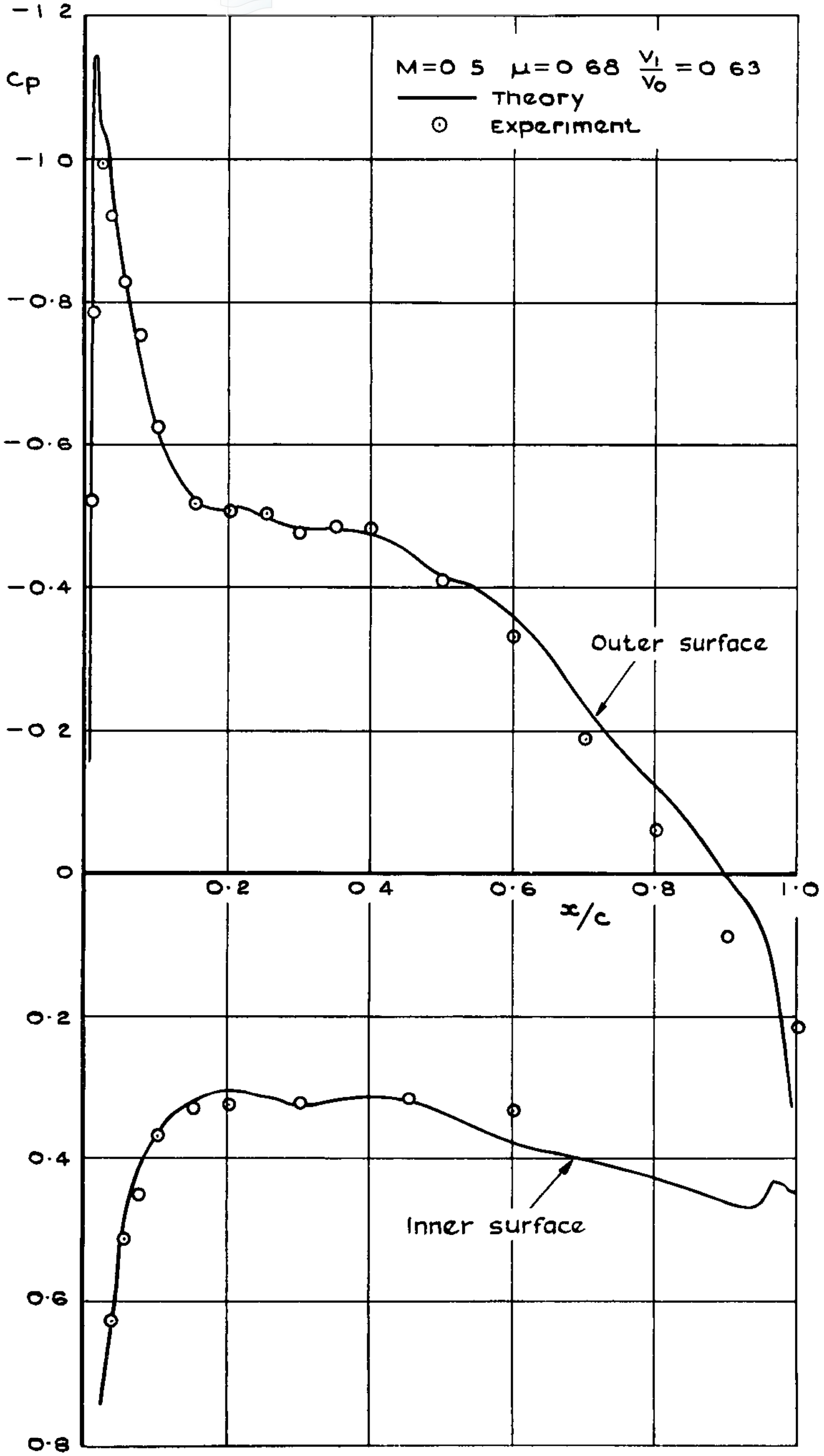
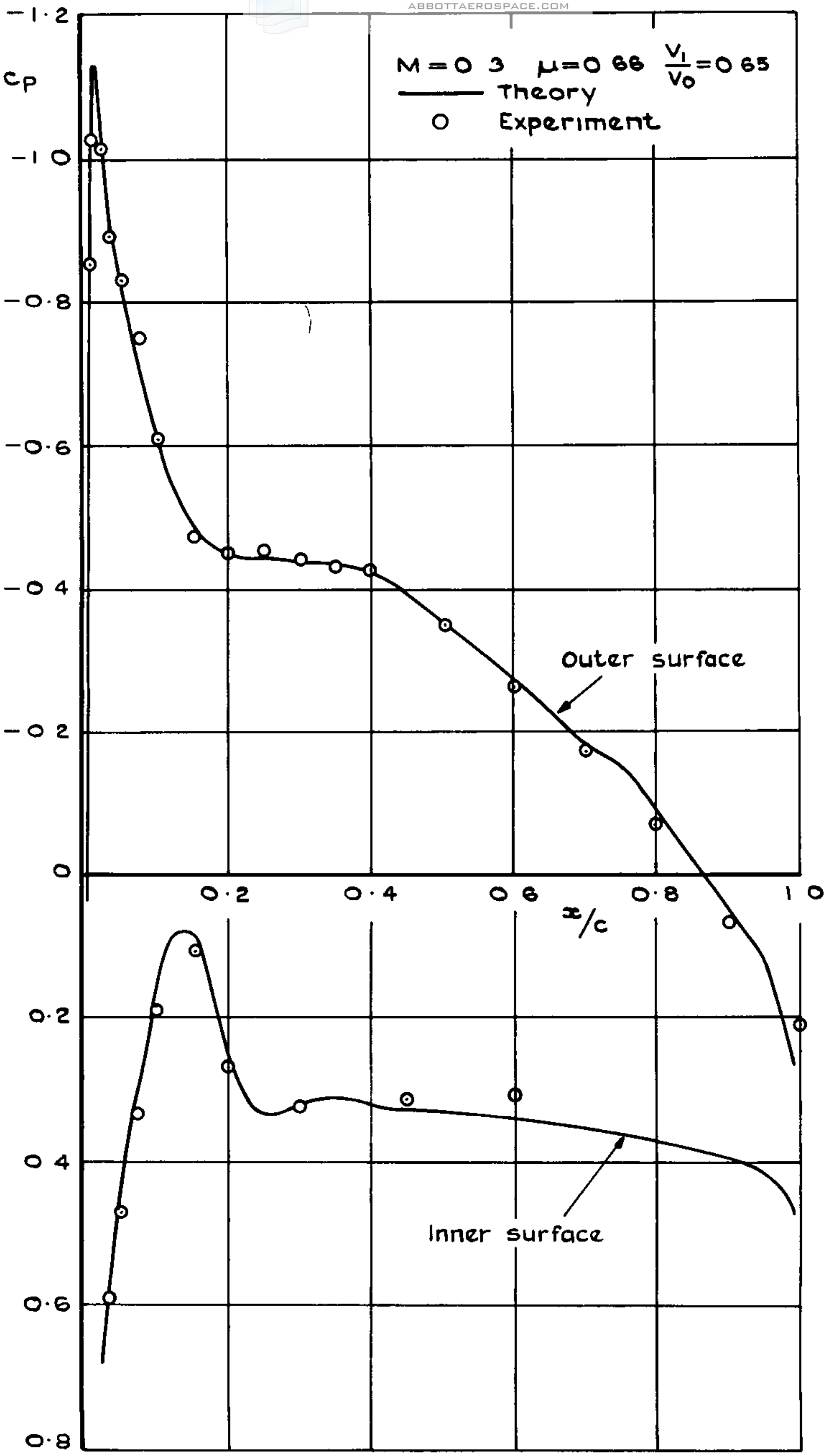


Fig .6 Comparison between theory and experiment:
R.A.E cowl 2



Fig·7 Comparison between theory and experiment: R.A.E. cowl 3

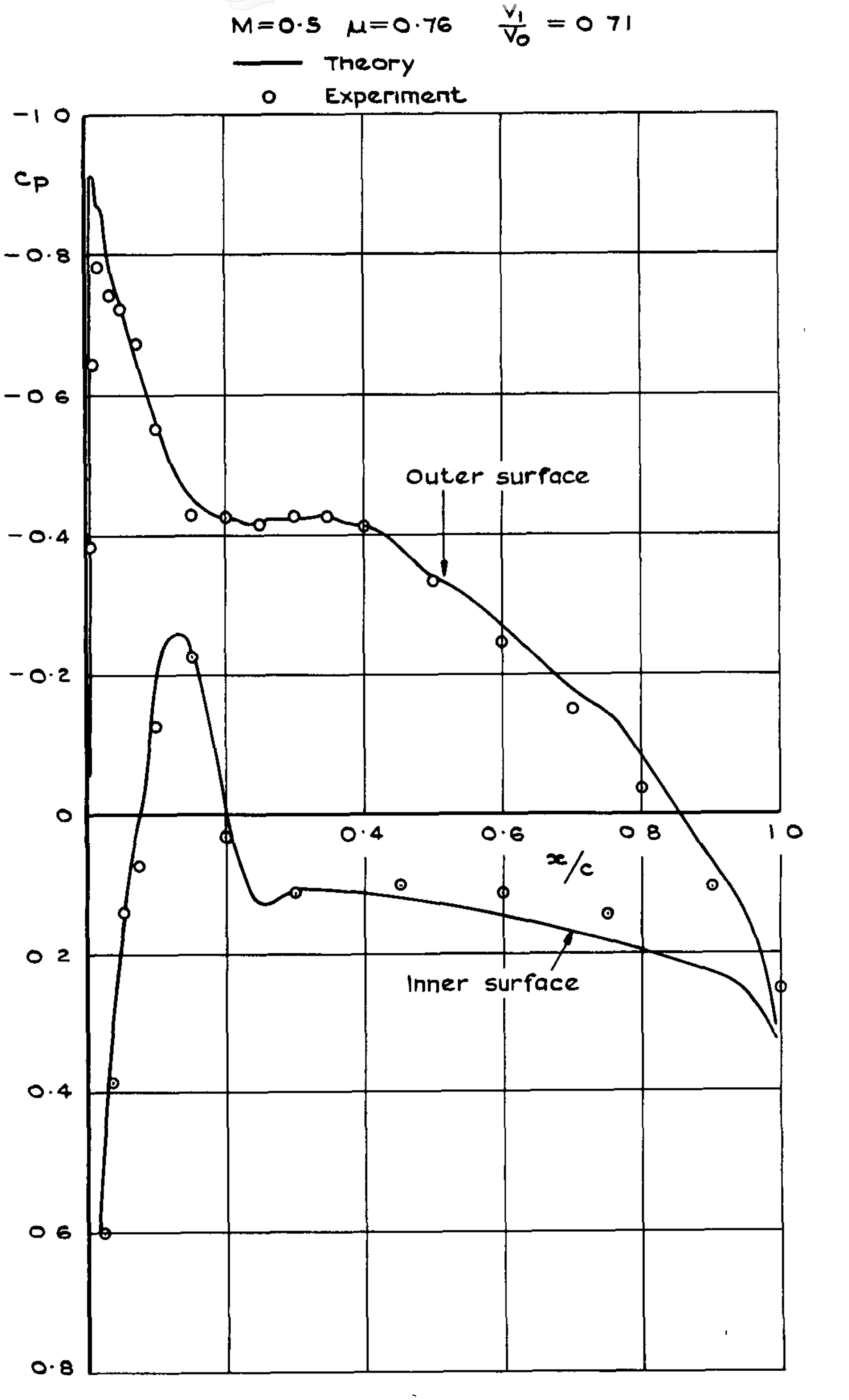


Fig. 8 Comparison between theory and experiment: R.A.E. cowl 3

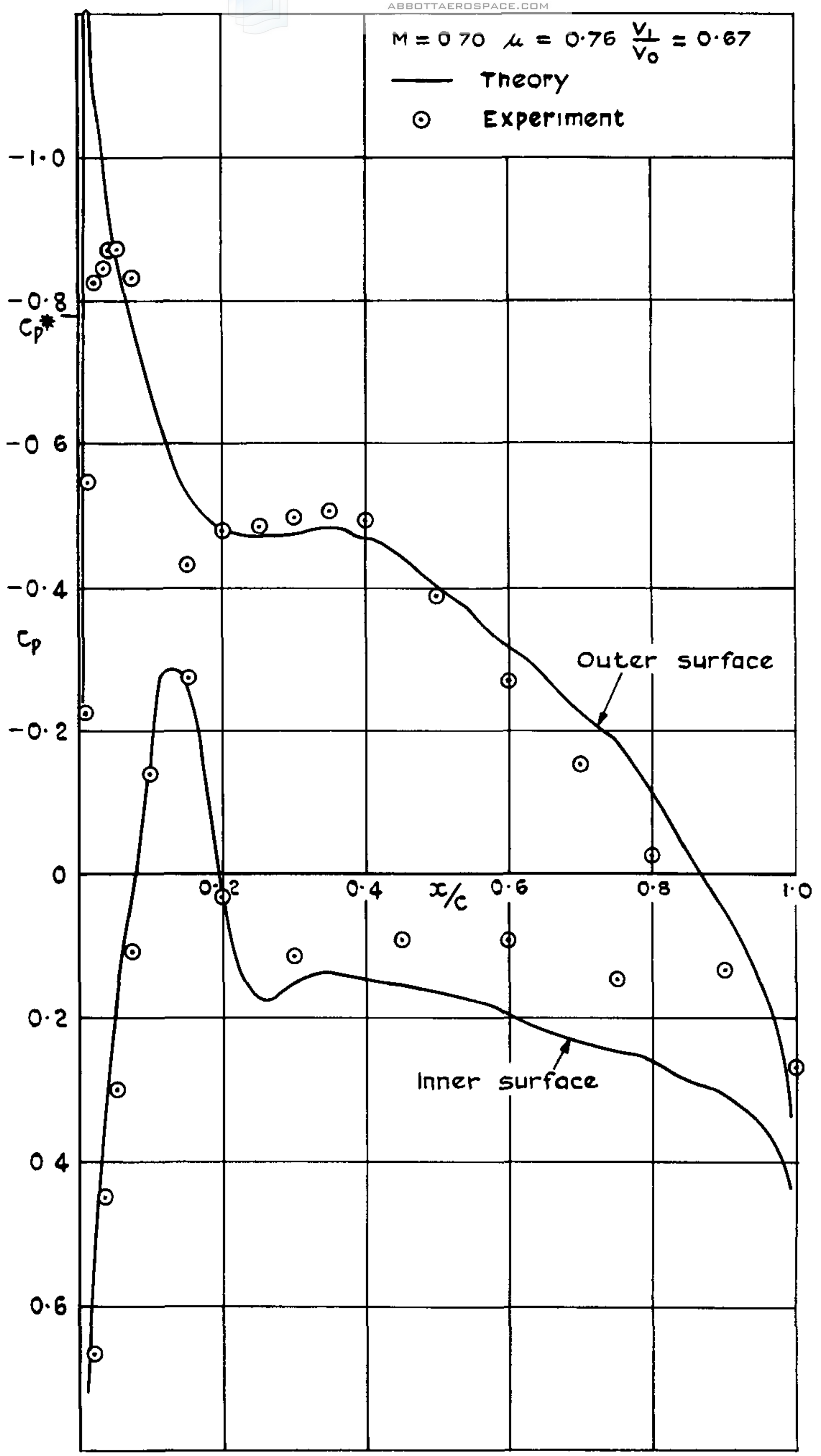


Fig. 9 Comparison between theory and experiment: RAE cowl 3

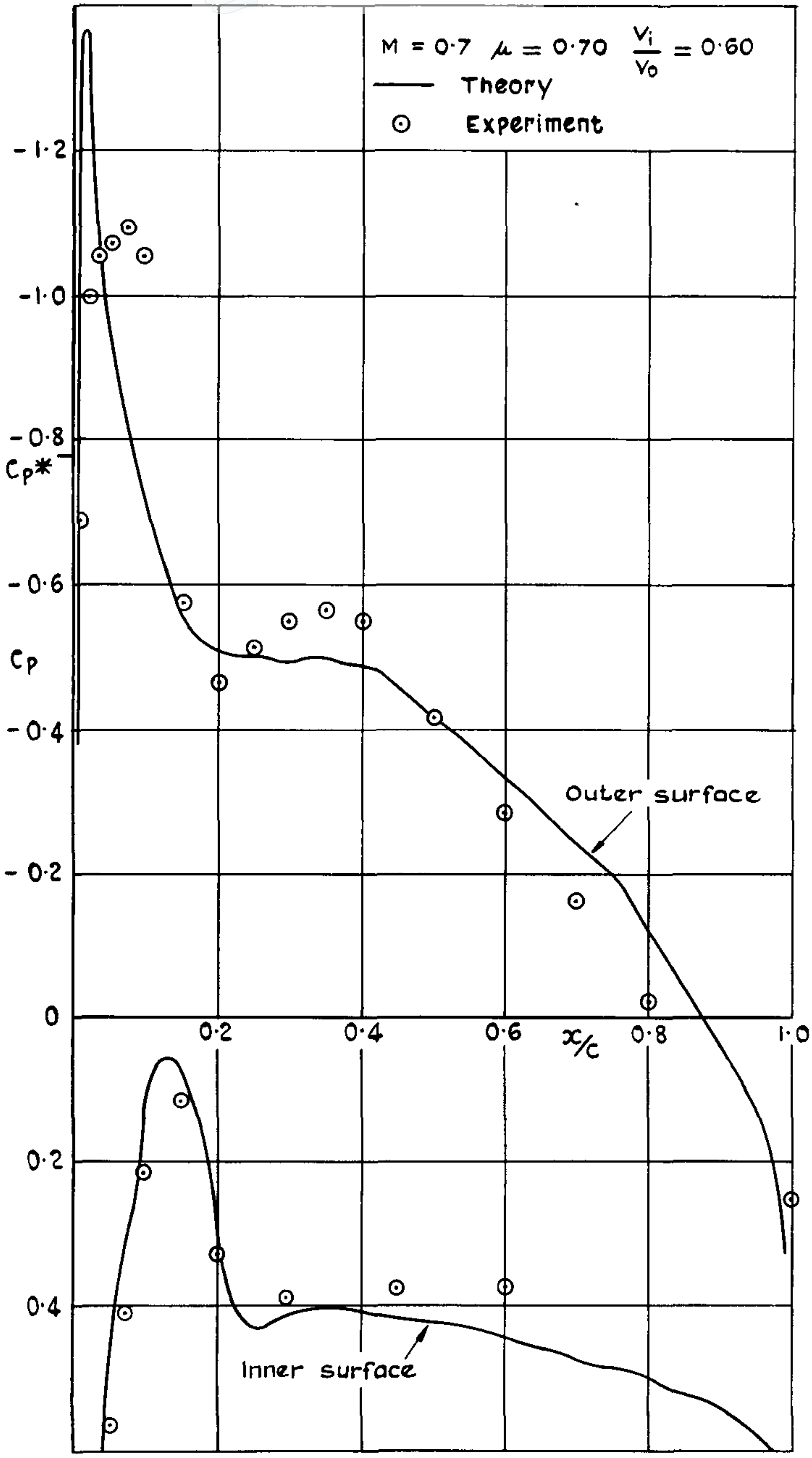


Fig. 10 Comparison between theory and experiment
R.A.E. cowl 3

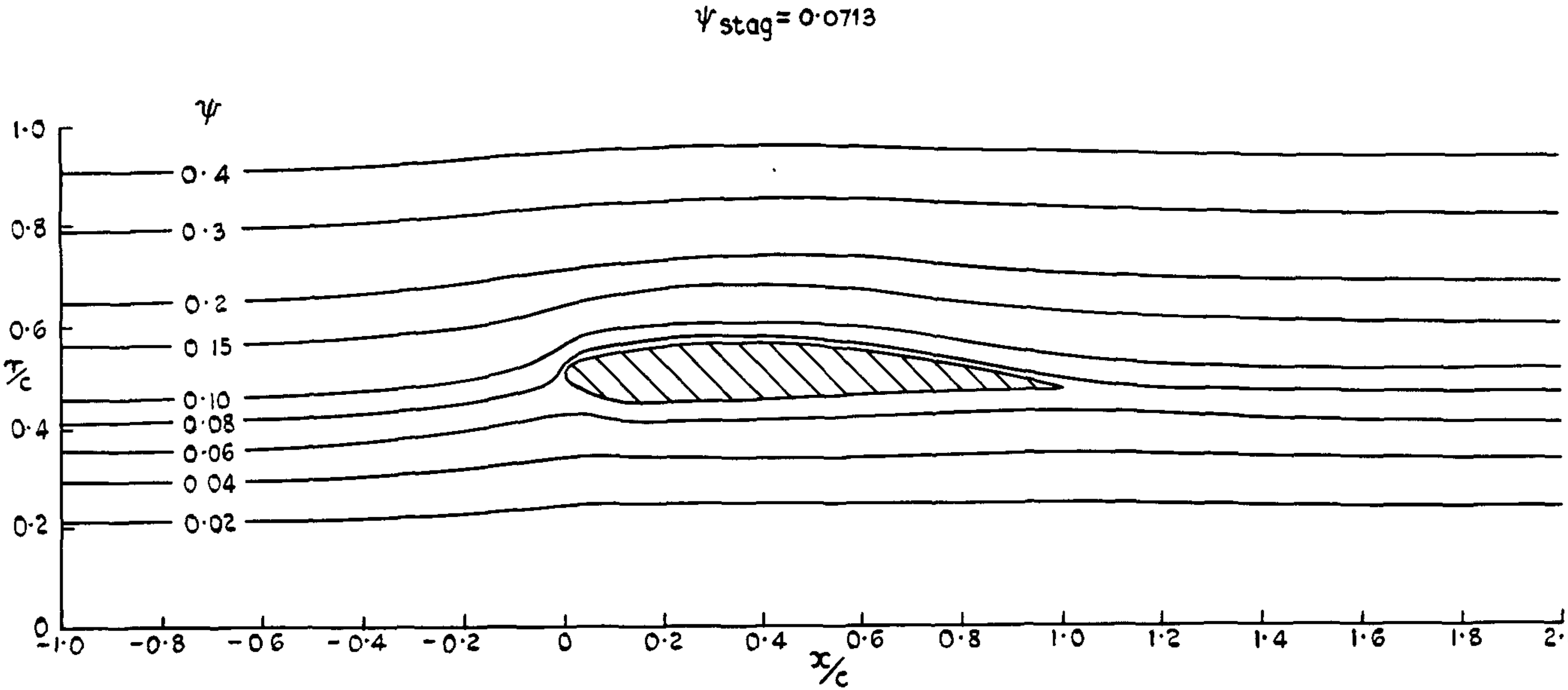


Fig.11 Calculated streamline pattern: cowl 3: $M = 0{\cdot}3$. $\mu = 0{\cdot}57$

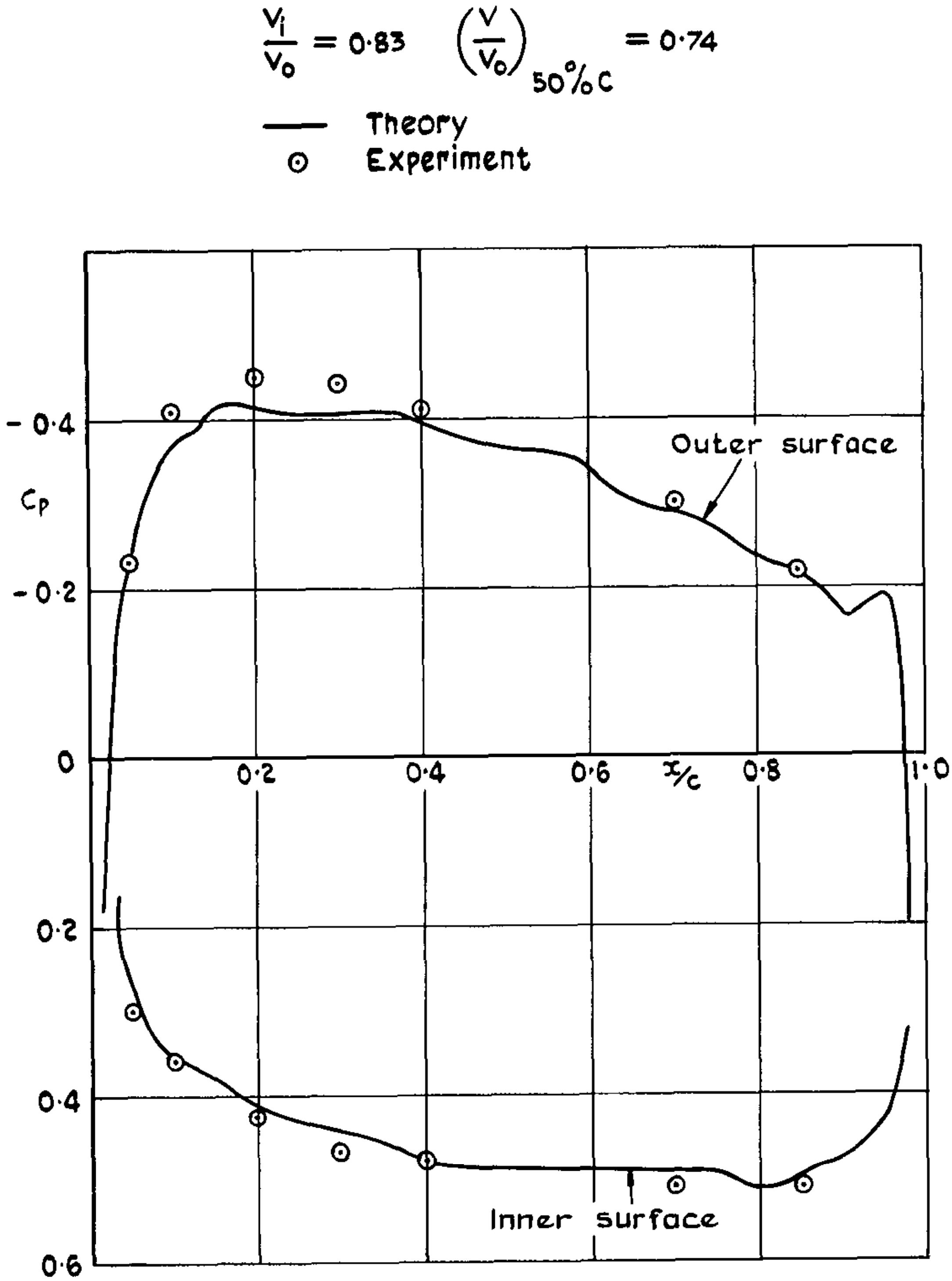


Fig.12 Comparison between theory and experiment: A.R.L. duct B1

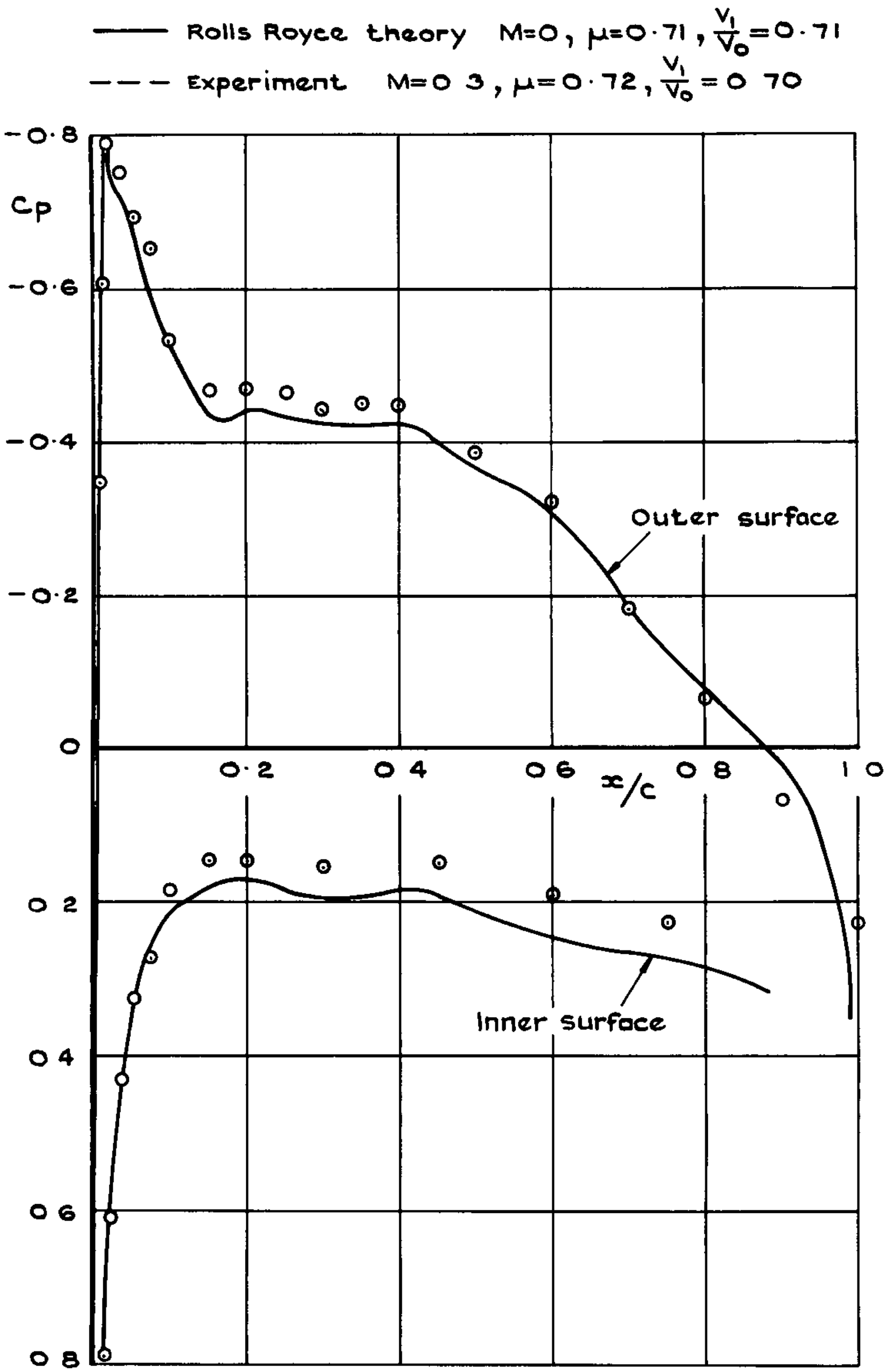


Fig. 13 Comparison between Rolls Royce theory and experiment; cowl 2

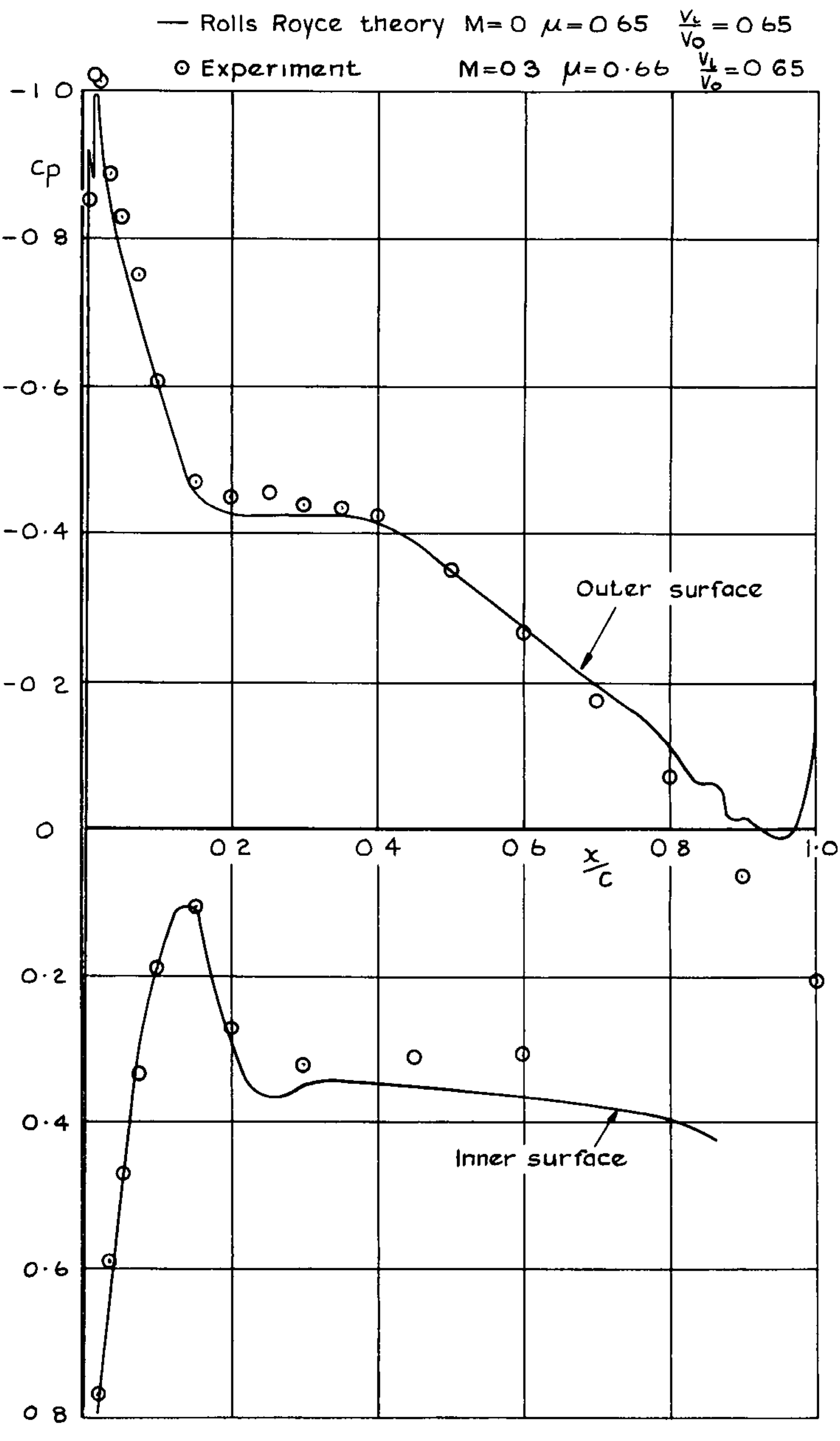


Fig. 14 Comparison with Rolls Royce theory and experiment · cowl 3

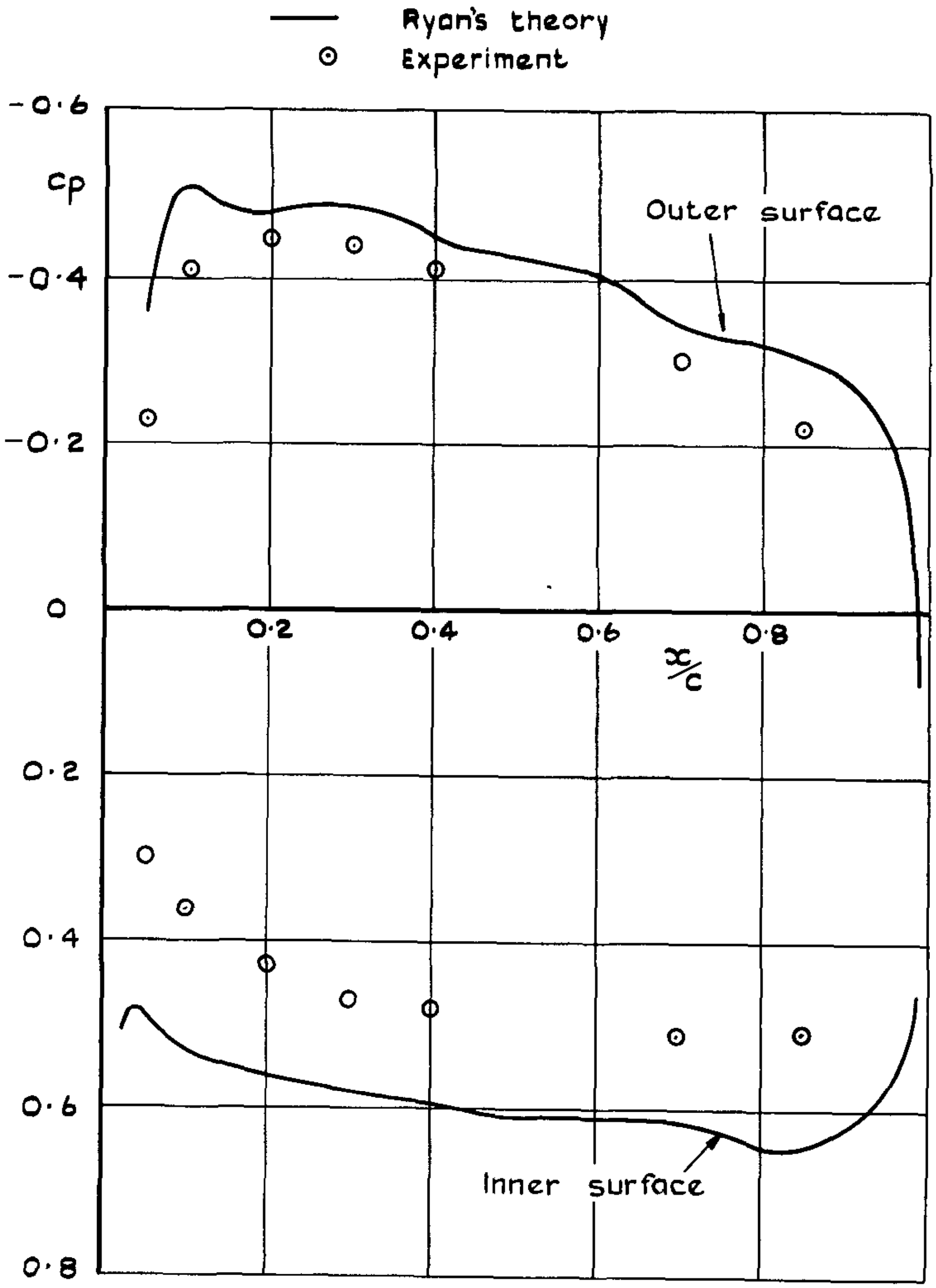


Fig. 15 Comparison between Ryan's theory and experiment:
ARL duct B1

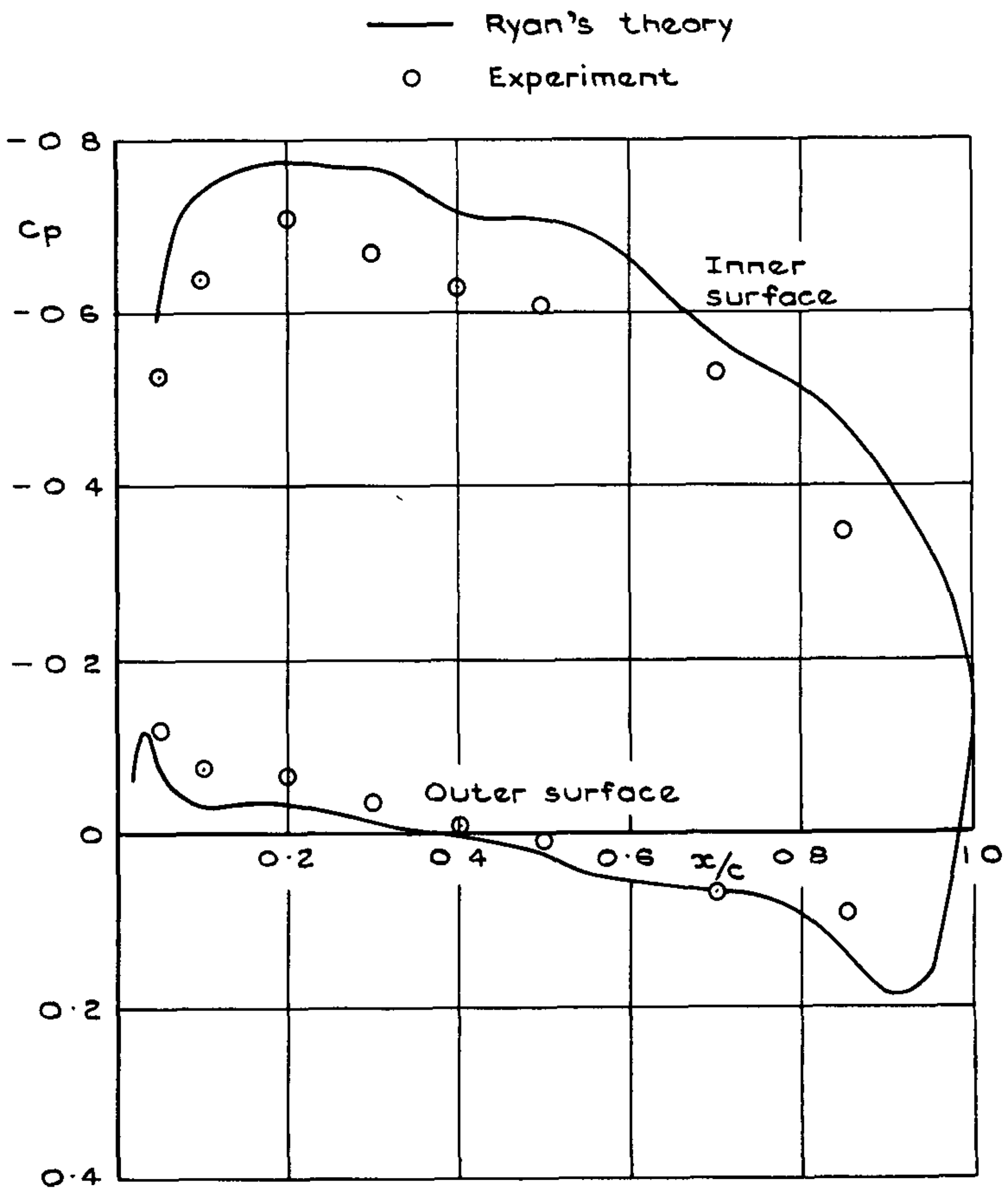


Fig.16 Comparison between Ryan's theory and experiment
A.R.L duct B3

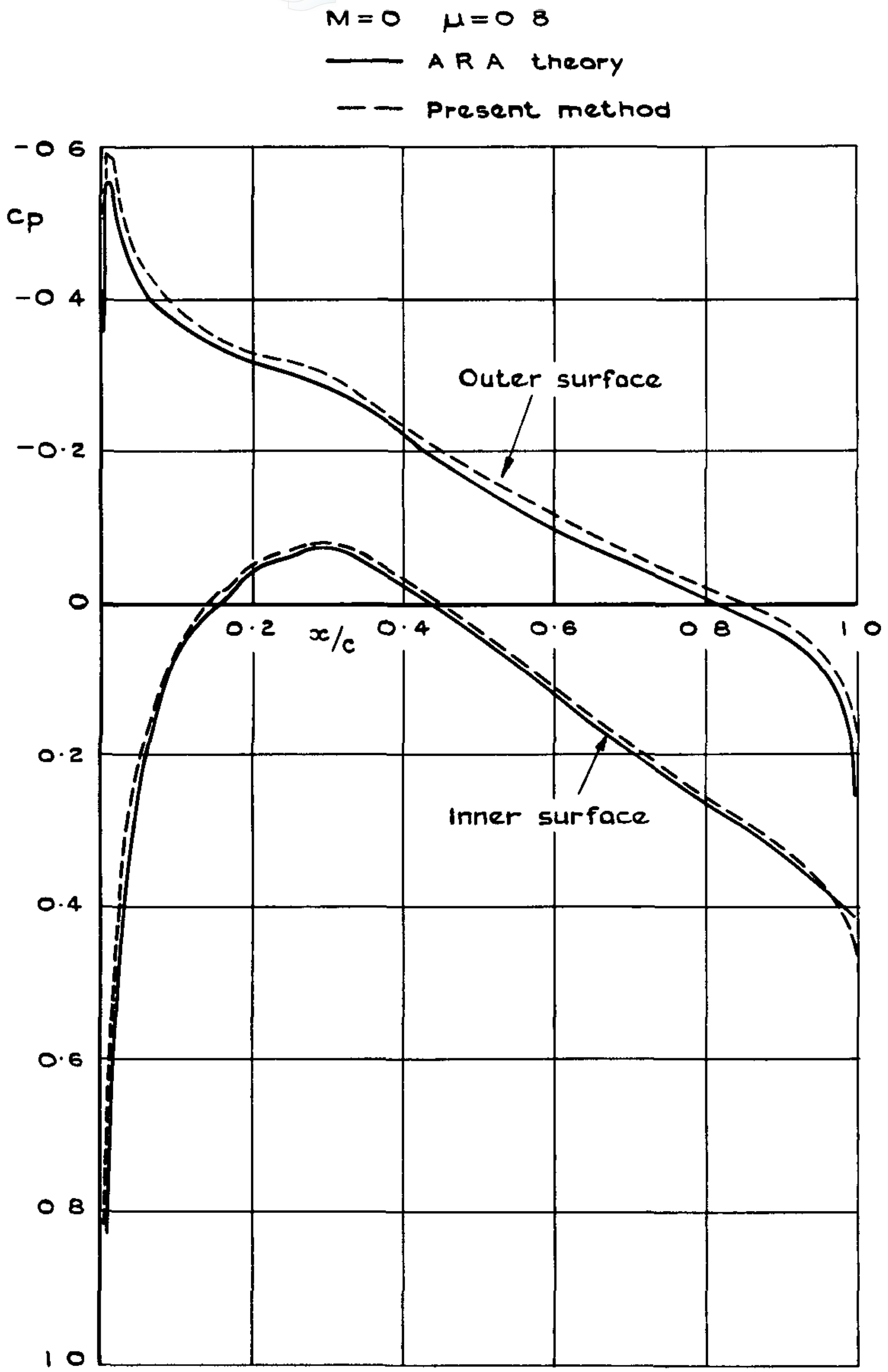


Fig. 17 Comparison between A.R.A. theory and R.A.E. theory: 10% R.A.E. 101 section, chord / diameter ratio = 1·0

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