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SUMMARY

The existence and uniqueness of the solutions obtained from the streamline curvature method of calculating flow through turbomachines[1] are examined for several operating points of Rolls-Royce compressor. It is shown that under certain conditions the truncation errors in the numerical solution can become large and hence give rise to the violation of the uniqueness conditions. The computer programme may then give wrong answers to the physical problem. The conditions for existence and uniqueness may be violated when the meridional velocities are small (e.g., near stall) or when there are regions of choked flow. Flow for an operating point in the stall region is computed by suitable modifications to minimize the truncation errors and hence to obtain a unique solution. This is compared with the results of the actuator disc theory and experiment reported in Ref. 8. Also the effect of variation of losses on the calculation is examined together with the effect of a correction term due to a dissipative body force, which should be included in the momentum equation, when losses are introduced.

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* Replaces A.R.C.32 727

Notation

a	velocity of sound
C	absolute velocity
C_p	specific heat at constant pressure
f	equation (11)
F	equation (12)
F_r	radial component of the dissipative body force
G	equation (11)
h	enthalpy
H	equation (11)
i	incidence
I	relative stagnation enthalpy
$\bar{K}$	blockage coefficient
m	meridional direction
$\dot{m}_t$	specified mass flow rate
M	Mach number
n	number of estimates for the streamline pattern
p	pressure
Q	entropy function
r	radius
r_m	radius of curvature of meridional projection of a streamline
R	gas constant
s	entropy
T	temperature
U_m	blade speed at the mean radius $\left(\frac{r_h + r_t}{2}\right)$
W	velocity relative to the rotor
α	blade angle
β	flow angle $\tan^{-1}\left(\frac{W_\theta}{C_m}\right)$
$\bar{\beta}$	$90° - \beta$
γ	ratio of specific heats
δ	ratio of the body force term to the other terms on the right hand side of equation (1)

δ_r/

δ_r	streamline shift
μ	relaxation factor
ρ	density
ϕ	$\tan^{-1} \frac{C_r}{C_x}$
ω	rotational speed of the rotor
$\bar{\omega}$	total pressure loss coefficient

Subscripts

h	hub
i	upstream reference station
m	meridional direction
R	relative
t	tip
x	axial direction
θ	circumferential direction
0	stagnation condition - also value at radius r_0
1	inlet of the blade row
2	outlet of the blade row

Introduction/

Introduction

During recent years the numerical methods of streamline curvature[1] and matrix throughflow[2], both using digital computers, have been widely used for the calculation of throughflow in turbomachinery. The two methods are based on the same mathematical models but differ in their numerical techniques. The general procedure, common to both methods, is to start with an initial approximation of the streamline positions and then by using equations of motion, energy and continuity calculate more accurate positions. It is found that this process overcorrects the initial errors and that the change in such parameters as streamline position or radius of curvature must be damped by a factor less than 1 to obtain convergence.

In general the above procedures are idealized and cannot always be realized in practice. Choking and negative velocities (flow reversals) may occur in the process of completing the calculation. If these conditions occur for an intermediate streamline pattern and are not present in the final solution, then the basic techniques may be modified to deal with them. However, if the final solution involves choked or reversed flows (reversed flows occur mainly when a compressor is operated in the surge region), then the techniques can not be used without substantial modifications. In this case the only way the system of equations can be satisfied is to allow for negative meridional velocities to occur, a condition not allowed for in either of the methods at present.

Mathematically when choking or zero meridional velocities occur, the conditions for the existence and uniqueness of the solution of the governing system of differential-integral equations is violated. Moreover, since these equations are solved numerically, it is the behaviour of the finite difference equations (replacing the original differential-integral equations in the computer programme) which is of interest. Numerical errors may give rise to violation of the existence and uniqueness conditions and if solutions exist the programme may give wrong answers for the physical problem. For the throughflow calculations this condition may occur at small values of the meridional velocity i.e. near stall when large losses are present.

One object of the present investigation was to study the flow in this region in order to determine the limits of applicability of the throughflow methods. It was therefore essential to investigate the conditions for the existence and uniqueness of the finite difference equations used in the programme. Since flow in this region is highly irreversible, it was necessary to include high losses in the calculation. It has been shown by Horlock[3] that when losses are included in the computation a dissipative body force should be included in the momentum equation. This gives rise to a 'body force term' in the differential equation for the meridional throughflow velocity. This term is usually neglected if the losses are small. Another object of the present work was to study the effect of inclusion of this term on the accuracy of the streamline curvature calculation when losses are high.

Analysis

The flow field is calculated by using the laws of conservation of mass and energy and Newton's second law of motion.

Momentum/

Momentum equation

For axisymmetric flow $\left(\frac{\partial}{\partial\theta} = 0\right)$, the momentum equations and the equation of state may be used to give the radial variation of the meridional velocity[1] as

$$\frac{\partial C_m^2}{\partial r} + 2\sin^2\bar{\beta}\left[-\frac{\sin\phi}{C_m}\frac{\partial C_m}{\partial m} + \frac{\cos\phi}{r_m} + \frac{\operatorname{cosec}^2\bar{\beta}}{2}\left(\frac{1}{Q}\frac{\partial Q}{\partial r}\right) + \frac{1}{2}\frac{\partial}{\partial r}\cot^2\bar{\beta} + \frac{\cot^2\bar{\beta}}{r} + \frac{2\,\omega\cot\bar{\beta}}{C_m}\right]C_m^2$$

$$= 2\sin^2\bar{\beta}\left[\frac{1}{Q}\frac{\partial IQ}{\partial r} + \frac{\omega^2 r^2}{2}\left(\frac{1}{Q}\frac{\partial Q}{\partial r}\right)\right] \qquad \ldots (1)$$

where Q is a function of the entropy

$$Q = e^{-s/C_p} = \left(\frac{P_o}{P_i}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Big/ \left(\frac{T_o}{T_i}\right)$$

and $\bar{\beta} = (90 - \beta)$ with β the flow angle downstream of the blade row, measured along a streamline

$$\beta = \tan^{-1}\left(\frac{W_\theta}{C_m}\right)$$

Behind a stationary blade row the angle β represents the absolute flow angle with the angular velocity $\omega = 0$. For a rotating blade row (ω finite) β is the flow angle relative to the blade.

The quantity I is defined as

$$I = h_o - \omega r C_\theta$$

and

$$\frac{\sin\phi}{C_m}\frac{\partial C_m}{\partial m} = -\frac{\left(1 + M_\theta^2 + \frac{r}{r_m\cos\phi}\right)\frac{\sin^2\phi}{r} + \tan\phi\,\frac{\partial\phi}{\partial r}}{1 - M_m^2} \qquad \ldots (2a)$$

in/

in a duct region,

$$\frac{\sin\phi}{C_m}\frac{\partial C_m}{\partial m} = -\frac{\left(1 + M_\theta^2 + \frac{r}{r_m \cos\phi}\right)\frac{\sin^2\phi}{r} + \tan\phi\frac{\partial\phi}{\partial r} - \frac{\sin\phi}{C_m}\left(\frac{W_\theta}{r}M_m^2\frac{\partial}{\partial m}(r\tan\beta) + 2\omega M_\theta M_m \sin\phi\right)}{1 - M_{rel}^2} \quad \ldots (2b)$$

in a bladed region, where the tangential component of the blade force is finite. Note that by a duct region it is meant that rC_θ is constant along a streamline and by a bladed region it is meant that $W_\theta = C_m \tan\beta$ where β is a specified blade angle, giving tangential component of the blade force.

Equation (1) is derived by introducing changes in entropy into the equations of motion for inviscid adiabatic flow. As pointed out by Horlock[3] the manipulation of the loss problem in this manner is inconsistent with the assumption of inviscid adiabatic flow unless dissipative body forces are introduced in the momentum equations. Horlock obtains an expression for the radial component of the dissipative body force as

$$F_r = -\frac{C_p T \rho}{Q}\frac{\partial Q}{\partial m}\sin^2\bar{\beta}\sin\phi$$

Introducing this component of the dissipative force into the radial component of the equation of motion, Horlock arrives at a modified equation for C_m, with the correction term $\frac{2F_r}{\rho}$ added to the right hand side of equation (1).

One object of the present investigation was to find the effect of this correction term on the accuracy of the calculation when the losses are large.

Continuity equation

The mass flow rate must be equal to the specified value $\dot{m}_t$. Therefore

$$\dot{m}_t = \int_{r_h}^{r_t} 2\pi r\,\rho\,C_m\cos\phi\,dr \quad \ldots (3)$$

This integral equation gives the constant of integration of equation (1).

Additional/

Additional equations

These equations relate the density ρ, the tangential velocity C_θ, and the Mach numbers M_m and M_θ to C_m.

The enthalpy rise across a blade row can be found from the steady flow energy equation and from the equation relating change of angular momentum to the moment of tangential forces (including viscous forces) about the axis of rotation. This leads to

$$\Delta h_o = (r\,\omega\,C_\theta)_2 - (r\,\omega\,C_\theta)_1 \qquad \dots (4)$$

where

$$C_\theta = C_m \cot \bar{\beta} \qquad \dots (5)$$

Note that this equation does not depend on the dissipative body force and for incompressible flow remains the same with or without this force. Assuming a perfect gas,

$$\Delta T_o = \frac{\Delta h_o}{C_p} \qquad \dots (6)$$

Thus starting with known conditions at a given axial station (station 1) and for specified values of C_m at the next axial station (station 2), values of T_o at station 2 can be obtained from equations (4), (5) and (6) (applied along each streamline between station (1) and (2)).

From the definition of stagnation temperature

$$T = T_o - \frac{C^2}{2C_p}$$

so that

$$a = a_o \left[1 - \frac{\gamma - 1}{2} \left(\frac{C_\theta^2 + C_m^2}{a_o^2} \right) \right]^{\frac{1}{2}} \qquad \dots (7)$$

and the Mach numbers M_m and M_θ can be evaluated. In order to obtain the density at station (2), first the value of the entropy function Q_2 is obtained from the specified losses. It is assumed that the following total pressure loss coefficient is available:

$$\bar{\omega} = \frac{P_{01R} - P_{02R}}{P_{01R} - p_1}$$

where 01R and 02R denote respectively the inlet and outlet stagnation conditions relative to the blade row. Ideally the loss data should account for losses due to skin friction on blade surfaces, tip clearance, secondary

flow/

flow and shocks. If the radial shift of the streamlines is small, then $\bar{\omega}$ can be estimated from two-dimensional cascade data.

If the effect of the radial movement of streamlines is ignored, then $T_{01R} = T_{02R}$ and the change of entropy along a streamline is given by

$$s_2 - s_1 = C_p \log_e \left(\frac{T_{02R}}{T_{01R}} \right) - R \log_e \left(\frac{P_{02R}}{P_{01R}} \right)$$

$$= - R \log_e \left[1 - \bar{\omega} \left(\frac{P_{01R} - p_1}{P_{01R}} \right) \right]$$

The change in the entropy function (Q) can be found from the definition

$$s_2 - s_1 = -C_p \log \left(\frac{Q_2}{Q_1} \right)$$

Hence

$$\frac{Q_2}{Q_1} = \left[1 - \bar{\omega} \left(\frac{P_{01R} - p_1}{P_{01R}} \right) \right]^{\left(\frac{\gamma-1}{\gamma} \right)} \qquad \ldots (8)$$

and in terms of absolute and relative total temperatures and absolute Mach number at inlet to the blade row, we have

$$\frac{Q_2}{Q_1} = \left[1 - \bar{\omega} \left\{ 1 - \frac{1}{\left(\left(\frac{T_{01R}}{T_{01}} \right) \left(1 + \frac{\gamma - 1}{2} M_1^2 \right) \right)^{\frac{\gamma}{\gamma-1}}} \right\} \right]^{\frac{\gamma-1}{\gamma}} \qquad \ldots (9)$$

and the density can be found from

$$\frac{\rho_2}{\rho_1} = \left[\left(\frac{Q_2}{Q_1} \right)^{\gamma} \frac{T_2}{T_1} \right]^{\frac{1}{\gamma-1}} \qquad \ldots (10)$$

Numerical solution

The flow through a turbomachine of given geometry can be analysed by using equations (1) and (3) and the auxiliary equations (2) and (4) to (10). Equation (1) must be solved by the method of successive approximations since it is necessary to assume a set of streamlines that satisfy both equations (1) and (3). These two equations form a system of non-linear simultaneous differential-integral equations and can not be solved explicitly. The numerical solution outlined by Novak[1] is used here. Some details of the present solution will now be given since the actual numerical technique used

for/

for integration of equation (1) has to be decided and the method used for calculating slope and curvature of the meridional streamlines can differ. Also the conditions for the existence and uniqueness of the numerical solution have to be investigated.

Equation (1) (including Horlock's correction term) is of the form

$$\frac{\partial C_m}{\partial r} = \frac{1}{2}\left[\frac{G(r,C_m)}{C_m} - H(r,C_m)\, C_m\right] = f(r,C_m) \qquad \cdots (11)$$

where

$$H(r,C_m) = 2\sin^2\bar{\beta}\left[-\frac{\sin\phi}{C_m}\frac{\partial C_m}{\partial m} + \frac{\cos\phi}{r_m} + \frac{\operatorname{cosec}^2\bar{\beta}}{2}\left(\frac{1}{Q}\frac{\partial Q}{\partial r}\right) + \frac{1}{2}\frac{\partial}{\partial r}\cot^2\bar{\beta} + \frac{\cot^2\bar{\beta}}{r} + 2\omega\frac{\cot\bar{\beta}}{C_m}\right]$$

and

$$G(r,C_m) = 2\sin^2\bar{\beta}\left[\frac{1}{Q}\frac{\partial IQ}{\partial r} + \frac{\omega^2 r^2}{2}\frac{1}{Q}\frac{\partial Q}{\partial r} + \frac{C_p T}{Q}\frac{\partial Q}{\partial m}\sin\phi\right]$$

Equation (11) is solved with an initial condition

$$C_m(r_o) = C_{m_o}$$

The conditions for the existence and uniqueness of solutions of equation (11) are well known[4]. Briefly if f is continuous in some region of (r,C_m) plane, containing the point (r_o,C_{m_o}) then equation (11) has at least one solution. If moreover the partial derivative $\frac{\partial f}{\partial C_m}$ exists and is continuous in that region then the equation has precisely one solution.

The behaviour of f depends on the manner in which the numerical solution is set up. The present solution was based on the "off design alternative" i.e., the equation is solved with the following input data.

(i) Enthalpy distribution at inlet to the blade row.
(ii) Losses for each streamline (specified $\bar{\omega}$).
(iii) Fluid leaving angles relative to the blade row for each streamline.

Losses and leaving angles are derived from cascade data and the enthalpy distribution is specified at the first axial station and subsequently found from equation (4).

The/

The complete method of solution is summarized below.

(1) Some initial guess is made of the streamline pattern in the meridional plane such that a certain percentage of total mass flow will pass through each stream tube. This enables the slope and curvature to be computed at each meridional station.

Initially the flow is assumed to be one-dimensional and the annulus height is divided into a suitable number of annuli of equal areas (to pass equal mass flows).

The most critical part of the streamline-curvature calculation is the estimation of the curvature of the streamlines. A study was made of the curve-fitting methods used for this purpose. In Ref. 5 a reliable technique was proposed and compared with other methods. In brief the proposed technique is based on the spline curve-fitting method and is referred to as the "double spline fit". Instead of fitting one spline to the data points and differentiating it once for slope and once more for curvature, a second spline is fitted to the slopes obtained from the first spline and this is differentiated for curvatures. This "double spline fit" was found to give satisfactory results when the ratio of the wave length to the point spacing of the primary harmonics constituting the curve were greater than 5. However when the calculation stations are between blades this ratio is smaller than 5 and even the "double spline fit" may not be sufficiently accurate. Wilkinson[6] shows that finite difference techniques using polynomials to find the curvature give better results for the curvature than the spline fits, provided the ratio of wave length to point spacing is large (> 10) but these methods are also inaccurate for the present computations.

(2) Some initial guess is made of the value of the meridional velocity C_{m_o} at radius r_o, for the axial station at which the calculation is being done.

(3) From this assumption of C_m the local speed of sound is computed (equations (5) and (7)).

(4) The term $\dfrac{\sin\phi}{C_m}\dfrac{\partial C_m}{\partial m}$, needed to find $G(r,C_m)$ is found from either equation (2a) or (2b).

(5) The terms $\dfrac{\cos\phi}{r_m}$, $\dfrac{\operatorname{cosec}^2\bar{\beta}}{2}\left(\dfrac{1}{Q}\dfrac{\partial Q}{\partial r}\right)$, $\frac{1}{2}\dfrac{\partial}{\partial r}\cot^2\bar{\beta}$, $\dfrac{\cot^2\bar{\beta}}{r}$, $\dfrac{2\omega\cot\bar{\beta}}{C_m}$ needed to find $H(r,C_m)$ are obtained from the specified gas leaving angles β, slope and curvature of streamlines, and the estimated C_m.

(6)/

(6) The quantities $\frac{\partial Q}{\partial m}$, $\frac{\partial IQ}{\partial r}$, $\frac{\partial Q}{\partial r}$ are obtained from results of the previous iteration. (Initially these are set equal to zero). All the derivatives in the present programme are obtained by fitting splines to the functions and differentiating the splines.

(7) Steps (1) to (6) enable the quantities $G(r,C_m)$ and $H(r,C_m)$ to be evaluated.

(8) The differential equation (11) is integrated by the Runga-Kutta method and the value of C_m at the next radial station is found. This procedure would fail if the conditions for the existence and uniqueness are violated for any radial step. For a duct region (stations between blade rows) if $C_m \neq 0$ and $M_m \neq 1$ then the functions f and $\frac{\partial f}{\partial C_m}$ are both continuous provided the specified angles $\bar{\beta}$ and loss distribution Q do not give rise to singularities. In a bladed region the conditions become $C_m \neq 0$ and $M_{rel} \neq 1$. These results are obtained by noting that

(i) if $C_m = 0$ then $f(r,C_m)$ is not continuous because $G(r,C_m)$ is divided by C_m. Also the errors in the numerical integration procedure become large if C_m is small.

(ii) if C_m is finite then only the behaviour of $H(r,C_m)$ (equation (11)) needs to be investigated since $G(r,C_m)$ is normally continuous. The only term in $G(r,C_m)$ which can most likely become singular is $\frac{\sin\phi}{C_m}\frac{\partial C_m}{\partial m}$ and the singularity occurs when the local meridional Mach numbers become unity.

Therefore provided numerical errors are small, for finite meridional velocities (C_m), and Mach numbers $\neq 1$, a unique solution exists for C_m at each radial location. However for transonic flows the conditions for existence and uniqueness are violated and the numerical solution outlined here may fail.

(9) If a unique meridional velocity profile (C_m versus r) exists then the density profile can be obtained by using equations (4) to (10).

(10) The mass flow rate across the annulus is found from equation (3) taking into account the effect of annulus wall boundary layers in the form of a blockage factor which may vary through the machine. If the computed mass flow rate does not agree with the specified mass flow rate within a suitable

accuracy/

accuracy range then C_{m_o} is adjusted and the steps (1) to (10) are repeated. If the process by means of which C_{m_o} is adjusted is convergent then a value of C_{m_o} can be found to give a mass flow rate which agrees with the specified value within any accuracy range.

The required value of C_{m_o} should satisfy the following equation

$$F(C_{m_o}) = \dot{m}_t - \dot{m}(C_{m_o}) = 0 \qquad \ldots (12)$$

where $\dot{m}_t$ is the true mass flow rate and

$$\dot{m}(C_{m_o}) = \int_{r_h}^{r_t} 2\pi r \, C_m \, \rho \cos\phi \, dr$$

If $F(C_{m_o})$ is well behaved, i.e., it changes monotonically as shown in Fig. 1, then the method of False Position[4] can be used to find the next approximation to C_{m_o}.

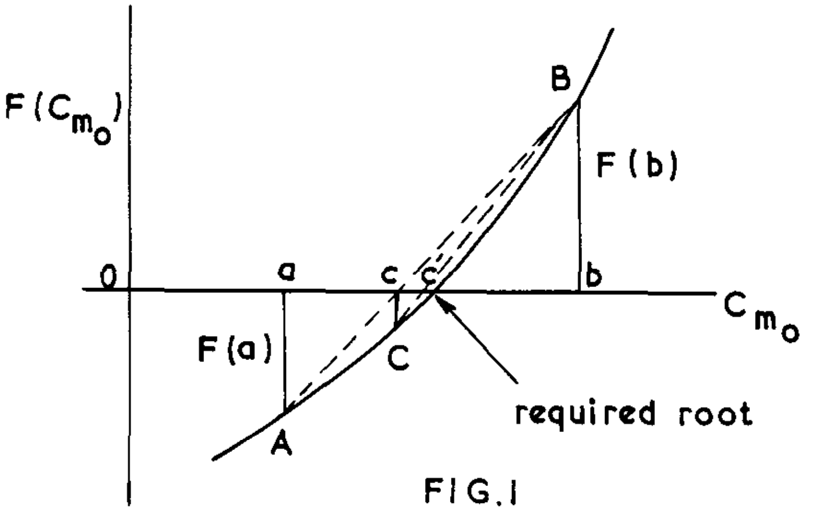

FIG.1

In this method, the next approximation to the root of equation (12) is given by

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

where c is the point where the chord AB intersects the axis. The procedure can be repeated using values at B and C to obtain c' and so on.

Now/

Now provided the conditions for the existence and uniqueness of solution for C_m are satisfied the value of $\frac{\partial C_m}{\partial r}$ at any radial position is unique and the family of meridional velocity profiles $(C_m \sim r)$ obtained (at a given axial station) for different values of C_{m_o} cannot intersect. Considering this fact Marsh[7] has shown that the mass flow rate, obtained from the basic differential-integral equations (1) and (3) increases monotonically with C_{m_o}. This will also be true for the finite difference equations used for the numerical solution provided numerical errors are small so that the behaviour of the two systems is the same. The numerical errors will be small if C_m everywhere along the radius remains sufficiently large, for the Runga-Kutta procedure to remain accurate. A difficulty arises when for a value of C_{m_o}, the meridional velocity at a radius becomes nearly zero. The programme is arranged to discontinue the calculation of the meridional velocity profile with this C_{m_o} and calculate a new profile with C_{m_o} increased by a small increment. This procedure overcomes the difficulty provided the final results do not include reverse flows.

(11) When the overall continuity is satisfied a new streamline pattern is derived as follows:

The local mass flow function $2\pi r \bar{K} \rho C_m \cos\phi$ is plotted against r and the area under the curve is divided into radial increments such that each stream tube will contain the percentage of mass flow chosen in 1. To do this a spline is fitted to the points of $2\pi r \bar{K} \rho C_m \cos\phi$ versus r (Fig. 2a)

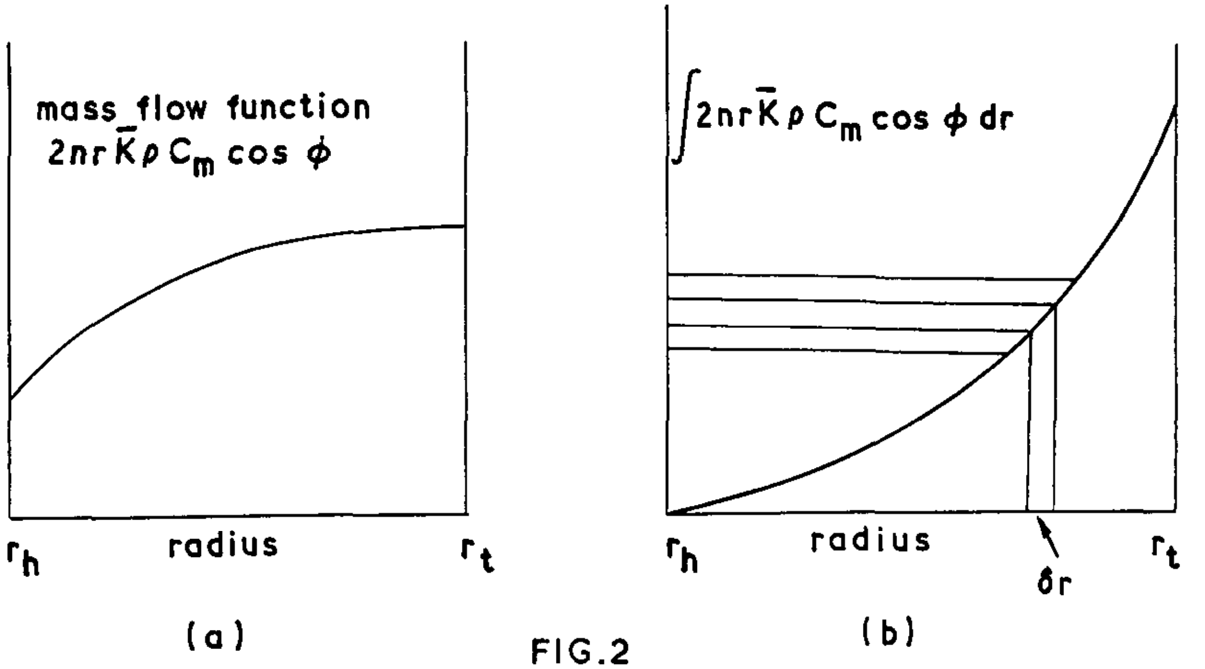

FIG.2

The/

The spline is then integrated and the values of the integral are plotted versus radius as shown in Fig. (2b). The ordinate of the curve at r_h is then divided into the number of stream tubes specified in the first approximation. The new positions of the streamlines are then obtained by a suitable method of inverse interpolation.

Experience has shown that the computation process outlined above overcorrects the initial errors and the change in the streamline positions, and the parameters which depend on the streamline positions (such as $\frac{\partial IQ}{\partial r}$, $\frac{\partial Q}{\partial m}$ etc.), must be damped by a factor less than 1·0 to obtain convergence. Recommended damping factors are based almost entirely on experience and may range from 0·5 to 0·025. Usually for any new case the damping factor is found by trial and error so that a number of calculations with different values must be done. Even when convergence has been obtained there is the possibility that a slightly different damping factor would have given the answer in fewer interactions. For a simple two-dimensional flow Wilkinson[6] derives explicit expressions for the optimum damping factor as a function of grid aspect ratio, Mach number and the differentiation method used for finding slope and curvature of the meridional streamlines. A similar analysis to give the damping factor for the present problem (flow in axial compressors) would be very complex and is therefore not attempted. In the present computer programme the relaxation is applied to

(i) Streamline shift

$$\delta r = r_{n-1} - r_n$$

$$r = r_{n-1} - \mu_1 \delta r$$

where
δr = streamline shift
r_{n-1} = previous streamline position
r_n = new streamline position (unweighted)
r = new streamline position (weighted)
μ_1 = relaxation factor

(ii) quantities $\frac{\partial IQ}{\partial r}$, $\frac{\partial Q}{\partial r}$ and $\frac{\partial Q}{\partial m}$ which depend on the streamline location. For example:

$$\frac{\partial IQ}{\partial r} = (1 - \mu_2)\left(\frac{\partial IQ}{\partial r}\right)_{n-1} + \mu_2\left(\frac{\partial IQ}{\partial r}\right)_n$$

Again suffixes $(n-1)$ and n denote any two consecutive approximations. In the present programme $\mu_1 = 0{\cdot}2$ and $\mu_2 = 0{\cdot}15$.

Results/

Results and Discussion

The objects of the investigation were:-

(a) effect of inclusion of losses on the streamline curvature calculation

(b) limits of applicability of the through-flow calculations (Matrix through-flow - streamline curvature) for axial compressors

(c) effect of Horlock's correction term on the accuracy of streamline curvature calculations

Test data for a Rolls Royce axial flow compressor was available and this machine was chosen for the present investigation. Axial velocities had been measured for this machine and compared with the actuator disc theory (Ref. 8): a comparison between the latter and the streamline curvature was therefore also possible.

Data was provided for several possible compressor builds but the arrangement chosen for the present calculations consisted of three rows of blades; a set of stationary inlet guide vanes, a rotating row and a stationary row. The hub-tip ratio is constant (0·4) through the compressor and the ratio of blade height to blade axial spacing is 2·1. The tip diameter is 14 in. and the tip speed is of the order of 370 ft/sec or 283 ft/sec depending on the gear ratio of the drive gear box.

In the through-flow calculations it is usual to relate the losses and angles to the last (n-1) iteration using a plot of loss and angle against i, the blade row incidence, entering the curves at $i_{n-1} = \beta_{n-1} - \alpha$, where β is the entry gas angle and α the blade angle. A relaxation factor may be applied to the losses as above.

However in the present calculation the main emphasis was on the stability of the flow calculation and on the effect of the dissipative force. It was therefore decided to accept the angle and loss distributions (except loss distributions II) determined from Horlock's actuator disc calculations[8], and not to change these during the successive iterations described here. Essentially therefore we have investigated the flow through three rows of blading which provide prescribed unchanging distributions of loss and outlet angle.

Fig. (3) illustrates the position of the calculating planes, the gas leaving angles and the loss distributions used in the calculations. The program was run for three flow coefficients (Cx_1/U_m = 0·67, 0·585, 0·417) and the results will be discussed in turn.

(i)/

(i) $\frac{Cx_i}{U_m} = 0{\cdot}67$ (Figs. 4, 5)

This flow coefficient is well within the operating range of the compressor (away from the stall region). Figs. (4) and (5) compare axial velocity profiles downstream of rotor and stator, from experiment, actuator disc theory and streamline curvature calculation with and without losses. The agreement between various calculations and experiment is reasonable (note the large scale for $\frac{Cx}{Cx_i}$).

The losses are small for this flow coefficient and little difference is made in the velocity profiles calculated from the streamline-curvature method with and without losses. Two loss distributions have been tried: the distribution with higher losses near the tip of the rotor (Fig. 3) produces a dip in the axial velocity profile (Fig. 4), as might be expected. Since the velocity distribution must satisfy the continuity equation, the axial velocity is increased near the hub (the flow can be considered incompressible, maximum Mach No. < 0.15). The axial velocity profile downstream of the stator is dependent on the axial velocity profile downstream of the rotor and the stator loss distribution. The comparatively higher losses near the root of the stator (Fig. 3) push the flow towards the tip of the stator: thus the dip in the axial velocity profile downstream of the rotor is not apparent downstream of the stator.

For the streamline curvature calculation no difficulties were experienced with convergence, negligible shift being noted in the streamline patterns in less than 20 cycles (each cycle using a new streamline pattern), with or without losses. Also the effect of Horlock's correction term was negligible (see Appendix 1).

(ii) $\frac{Cx_i}{U_m} = 0{\cdot}585$ (Figs. 6, 7)

Again comparatively higher losses near the tip of the rotor (Fig. 3) decrease the axial velocity there: axial velocity near the hub is then increased to satisfy the continuity equation. The rotor losses are small and therefore cause little modification in the axial velocity profile downstream of the rotor. The effect of comparatively high losses near the tip of the stator (Fig. 3) can be seen on Fig. 7 where the axial velocities near the tip are appreciably reduced and mass flow near the hub is increased. Again no difficulties were experienced with the convergence of the streamline curvature calculation and the effect of Horlock's correction term was negligible.

(iii) $\dfrac{Cx_i}{U_m} = 0{\cdot}413$ (Figs. 9-17)

This operating point was within the stall region of the compressor characteristics. It was chosen to study the accuracy of the streamline curvature calculation when the meridional through-flow velocities may become small for a given streamline pattern. The truncation error in the Runga Kutta procedure, used to integrate equation (11), is a function of f, $\dfrac{\partial f}{\partial r}$ and $\dfrac{\partial f}{\partial C_m}$. The actual expression is too complicated to be of much practical use, but for small C_m, these values may become large and hence the error may become large. Referring to Fig. 8, it can be seen that the numerical solution cannot be used for calculating C_m below the value C_{m_E} where the numerical errors become unacceptable.

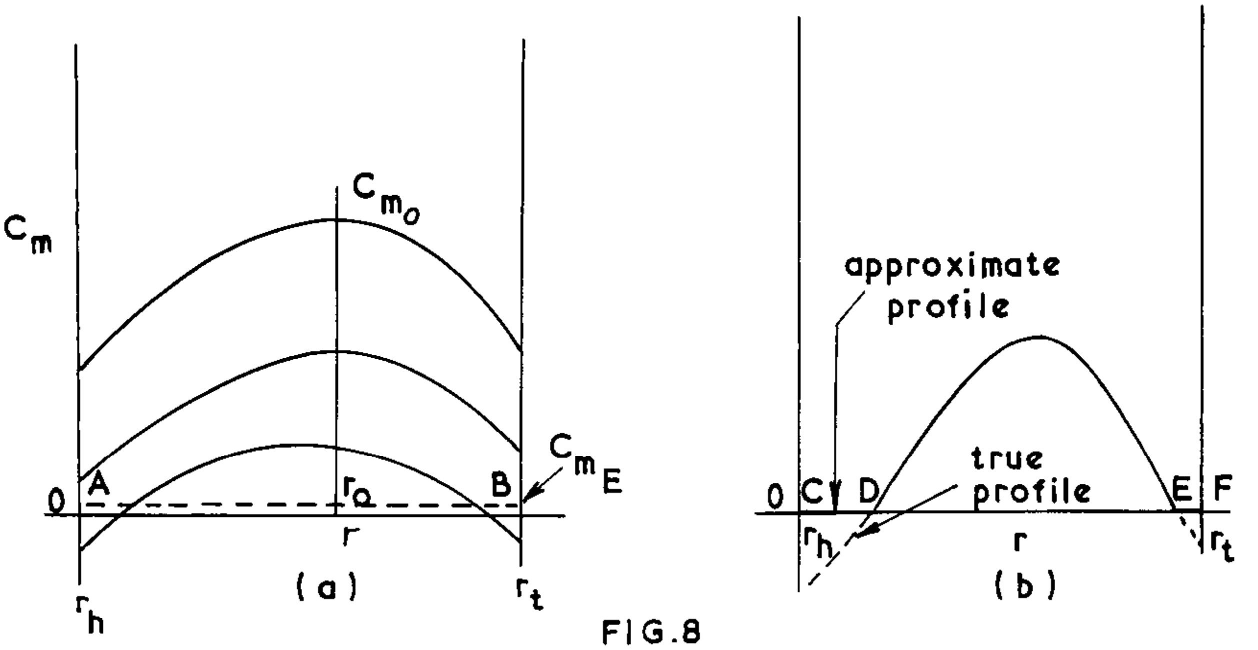

FIG.8

To overcome this difficulty the programme can be modified in either of two ways. If for the given streamline pattern the velocity profile to give the correct mass flow rate is above line AB, then the programme can be arranged to discontinue the computation if C_m falls below AB and increase C_{m_o} until this condition is satisfied. However, if this procedure gives too large a value for mass flow rate then the velocity profile must cross AB. In this case values of C_m below AB are replaced by a small positive value so that the resulting meridional velocity profile, to satisfy the mass flow, would appear as in Fig. (8b). This is of course inaccurate in regions CD and EF where the true velocities may be even negative but the numerical method can not be used to predict the true velocities in this region.

Three/

Three calculations were performed for this flow coefficient, one without losses and two with different rotor loss distributions, to study the effect of variation of losses (when the losses are high).

Loss distribution 1 (Figs. 9-11)

The programme was first run without incorporating the above modifications to minimize truncation errors for small through-flow velocities. This computation was stopped for the 7th streamline pattern because multiple values of C_{m_o} were found to give the specified mass flow rate, downstream of the stator, where some through-flow velocities were small (Figs. 11 and 11a). Therefore, due to growth of numerical errors, the uniqueness conditions for the solution had been violated. Figs. 11a and 11b illustrate that the computed mass flow rate is not monotonically increasing with C_{m_o} and numerical errors have considerably modified the behaviour predicted from consideration of the basic differential-integral equations (1) and (3).

The programme was modified by stopping the computation of C_m by the Runga-Kutta procedure, when C_m was small $\left(\frac{C_m}{Cx_i} < 0{\cdot}05\right)$ and replacing the remaining values of C_m by a small value $\left(\frac{C_m}{Cx_i} = 0{\cdot}01\right)$.

It can be seen from Figs. 11a and 11b that the behaviour is considerably improved but there are still small oscillations in the graphs. These are attributed to errors in the numerical integration of equations (1) and (3). Apart from the slight inaccuracies due to the oscillations, the uniqueness condition is now satisfied and the final results (20th streamline pattern) are compared with the results of the actuator disc theory and experiment reported in Ref. 8. In general the agreement between theory and experiment seems reasonable. Since this operating point is in the stall region of the compressor the losses are not accurately known, and quantitative agreement between theory and experiment is not expected. This is true even when the effect of annulus wall boundary layers are taken into account.

Finally the effect of Horlock's correction term on the accuracy of the calculation was found to be negligible (the difference between results with and without this term was negligible). Conditions for which this term may be neglected are examined in Appendix 1.

Loss distribution II (Figs. 12-14)

Again the programme was first run without the necessary modification to reduce truncation errors in the Runga-Kutta procedure. Multiple values of C_{m_o} were found downstream of stator, for the 6th streamline pattern, giving the specified mass flow rate (Fig. 14a). The uniqueness condition was therefore violated due to the growth of the truncation errors. The modified programme (described above) is seen (Figs. 14a, 14b) to give unique values for C_{m_o}, and the final results obtained from this programme

are/

are given in Figs. 12-14. Loss distribution II gives higher losses near the tip of the rotor as compared with loss distribution I, but gives the same loss distribution downstream of the stator. Comparison of Figs. 10 and 13 shows that the axial velocity near the tip of the rotor is reduced for loss distribution II as compared with the values for loss distribution I. The same trend is obtained downstream of the stator (compare Figs. 11 and 14).

Finally Figs. 14a and 14c demonstrate that Horlock's correction term does not appreciably affect the accuracy of this computation.

<u>No losses</u> (Figs. 15-17)

Results of the tests for uniqueness are given in Figs. 17a to 17d. Again the modified programme gives a unique solution. However the oscillations in the graphs of $\dot{m}$ versus C_{m_o} are not completely eliminated when the programme is modified. These oscillations indicate the existence of appreciable numerical errors in the region where they are large. The curves are well behaved in the region near the specified mass flow rate so that for the present operating point a unique solution can be obtained with reasonable accuracy. The final results (20th streamline pattern) can be compared with the results for flow with losses to study the effect of losses on the calculation. Axial velocity profiles downstream of the guide vanes (Figs. 9, 12, 15) are very similar. For both cases the guide vanes were assumed to introduce zero losses and the changes are due to the change in the slope and curvature of streamlines. The axial velocity profile downstream of the rotor for flow without losses (Fig. 16) is more uniform as compared with flow with losses (Figs. 10, 13) but the differences are not large. However flow downstream of the stator is changed considerably due to losses. The axial velocity downstream of the stator with no losses (Fig. 17) is small near the hub; the velocity for flow with losses (Figs. 11 and 14) is small near the tip. Losses can therefore modify the velocity profiles to a large extent and hence have large effects on the convergence and uniqueness of the solution.

Conclusions/

Conclusions

The conditions for uniqueness of the solutions obtained from the streamline-curvature method of calculating flow through turbomachines may be violated when the meridional through-flow velocity becomes small (e.g. near stall), or when there are regions of choked flow. For a Rolls Royce compressor, flow in the stall region is computed by suitable modifications of the method to minimize the truncation errors and hence to obtain a unique solution. There is agreement between this solution and the results of the actuator disc theory and experiment reported in Ref. 8.

The effect of the losses on the computation is significant when losses are large, but for the axial machine investigated here the effect of the correction term, arising from the inclusion of a dissipative force in the momentum equation, is negligible.

Acknowledgements

The authors are indebted to Professor J. H. Horlock and Dr. H. Marsh for many helpful discussions.

APPENDIX 1

Effect of the dissipative body force on the accuracy of the streamline curvature calculation

The dissipative body force gives rise to the term

$$\frac{2F_r}{\rho} = -\frac{2\,C_p T}{Q}\frac{\partial Q}{\partial m}\sin^2\bar{\beta}\sin\phi$$

on the right hand side of equation (1). If this term is negligible compared with the other terms on the right hand side

$\left(\text{i.e. } 2\sin^2\bar{\beta}\left(\frac{1}{Q}\frac{\partial QI}{\partial r} + \frac{\omega^2 r^2}{2}\left(\frac{1}{Q}\frac{\partial Q}{\partial r}\right)\right)\right)$ then the body force will

not effect the accuracy of the calculation. The ratio of the terms is

$$\delta = \frac{-\frac{C_p T}{Q}\frac{\partial Q}{\partial m}\sin\phi}{\frac{\partial IQ}{\partial r} + \frac{\omega^2 r^2}{2}\left(\frac{\partial Q}{\partial r}\right)}$$

For the present calculations the term $\frac{\partial Q}{\partial m}$ is only finite downstream of the rotor and downstream of the stator; the magnitude of δ can be readily estimated at either station by noting that for the Rolls Royce compressor

$$C_p T \simeq h_o \gg \omega^2 r^2$$

and

$$Q \simeq 1$$

Therefore, downstream of the stator

$$\delta \simeq \frac{\frac{\partial Q}{\partial m}\sin\phi}{\frac{\partial Q}{\partial r} + \frac{1}{h_o}\frac{\partial h_o}{\partial r}}$$

The/

The derivatives can be obtained from Figs. A1, A2 and A3 and values of δ, calculated from these derivatives, are given in Table 1.

Table 1

$\frac{r}{r_t}$	ϕ	$\frac{\partial Q}{\partial r}$	$\frac{1}{h_o}\frac{\partial h_o}{\partial r}$	$\frac{\partial Q}{\partial x} \approx \frac{\partial Q}{\partial m}$	δ
0·5	-2	0	·039	0	0
0·6	-2·9	0	·039	0	0
0·7	-3·2	0	·039	0	0
0·8	-3·2	-.0216	0	-.0164	-·0425
0·9	-2·5	-·043	0	-·076	-·077

It can be seen that δ is negligible. However if there are regions where the denominator can become small and if ϕ can become large (e.g. in centrifugal machines) then the effect of the dissipative body force can become appreciable.

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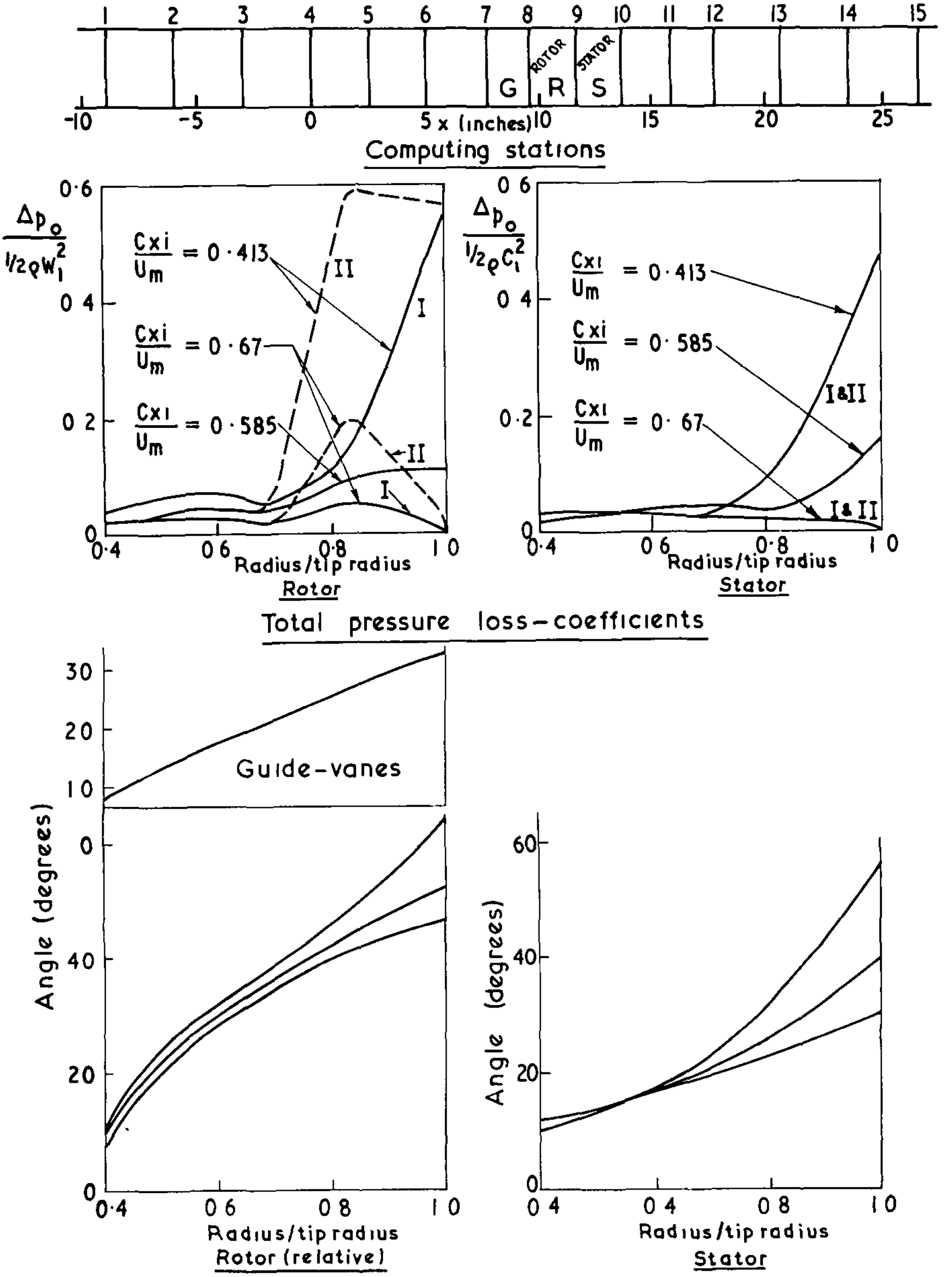


FIG. 3 Downstream gas angles

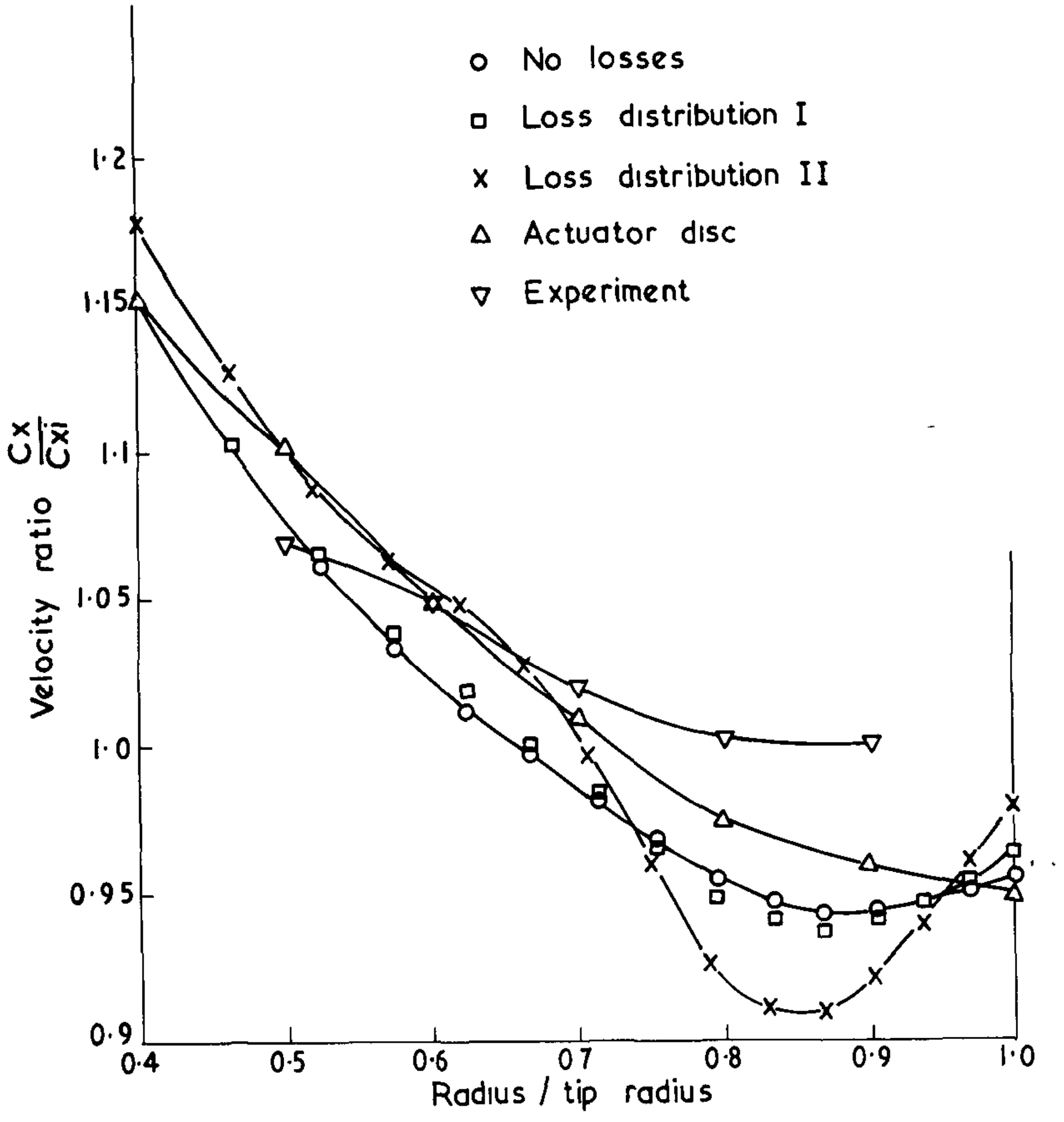


FIG. 4 Axial velocity profiles downstream of the rotor

$$\frac{C_{xi}}{U_m} = 0{\cdot}67$$

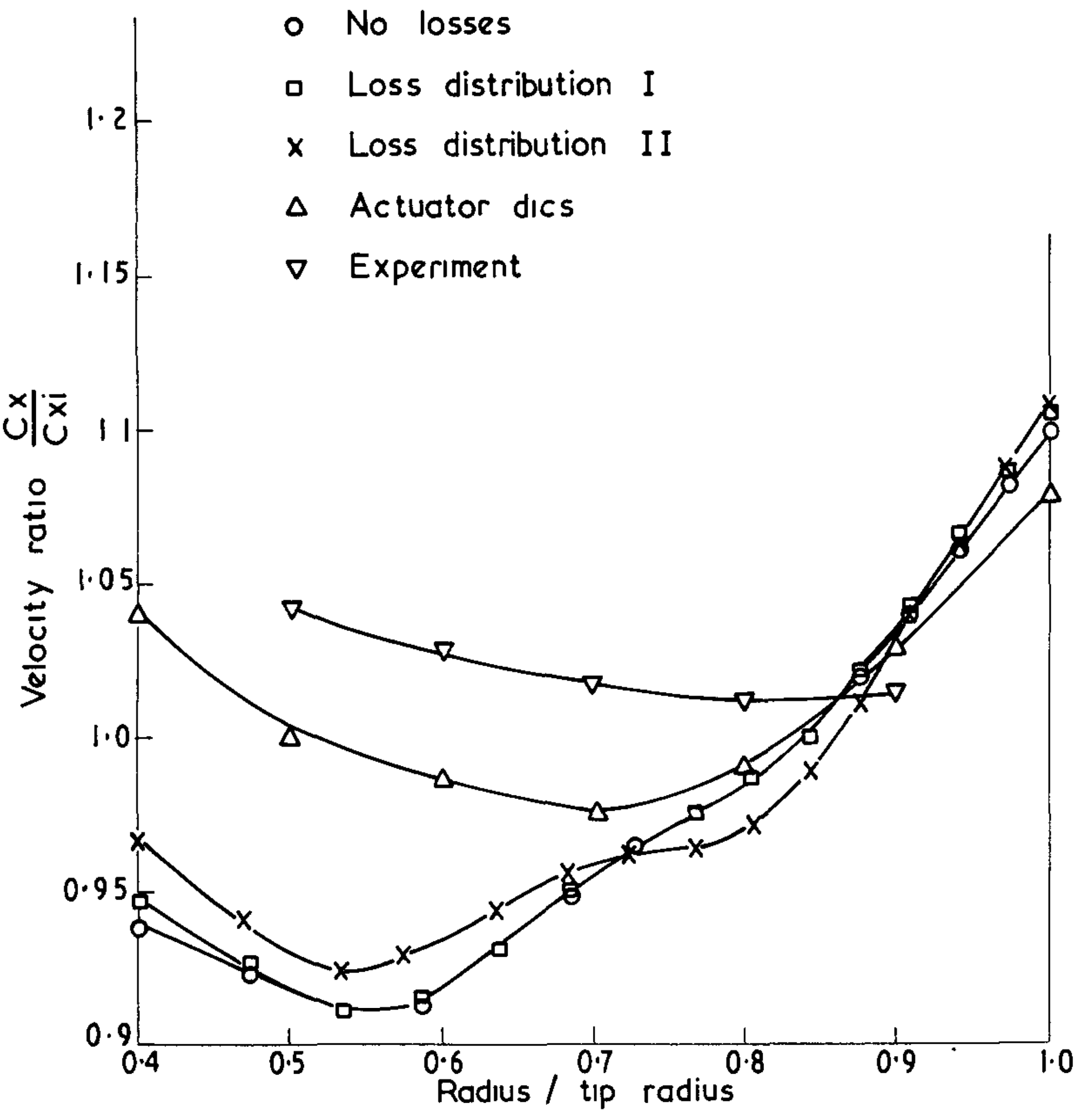


FIG. 5 Axial velocity profiles downstream of the stator

$$\frac{Cxi}{Um} = 0{\cdot}67$$

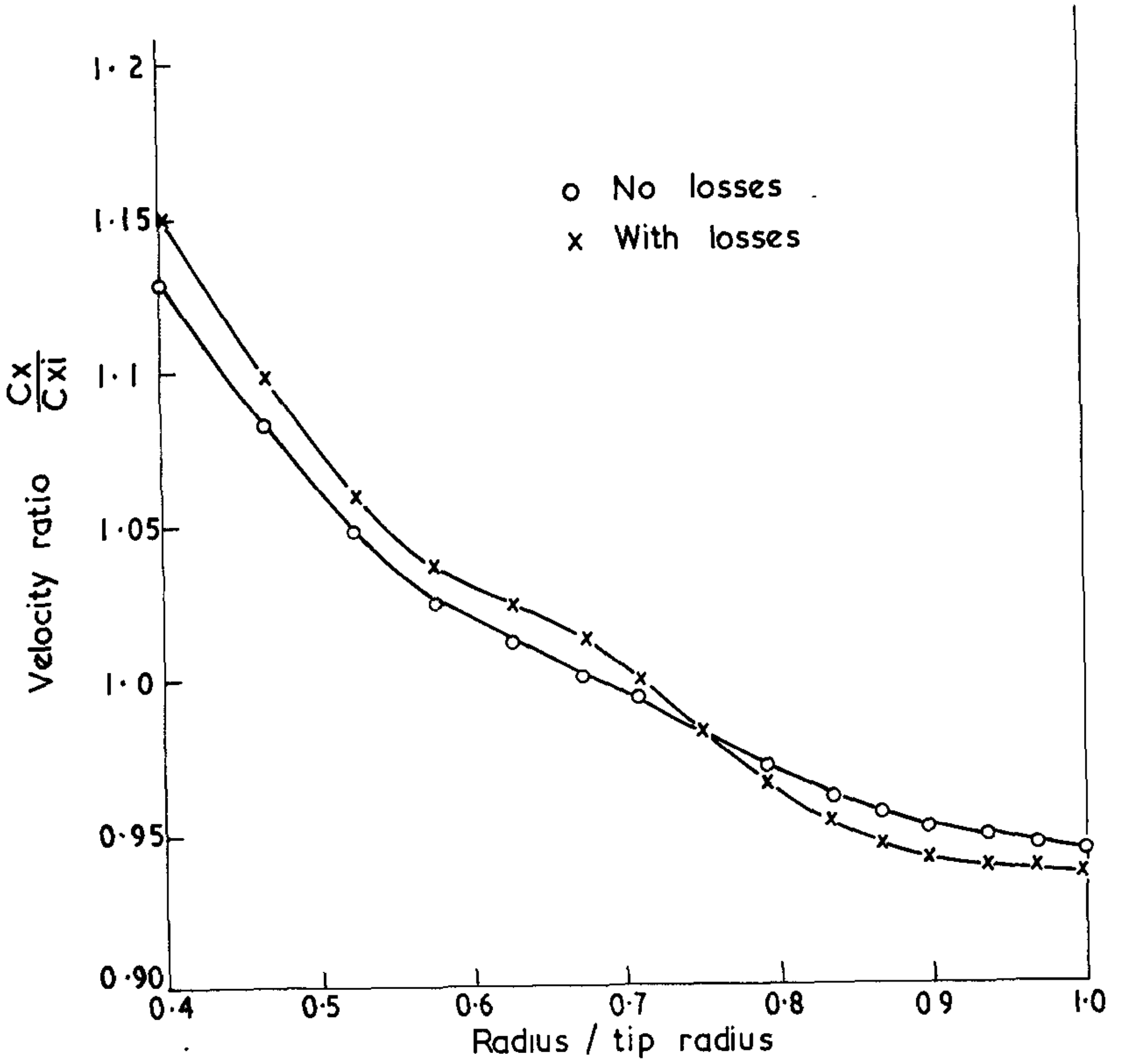


FIG.6 Axial velocity profiles downstream of rotor

$$\frac{C_{xi}}{U_m} = 0{\cdot}585$$

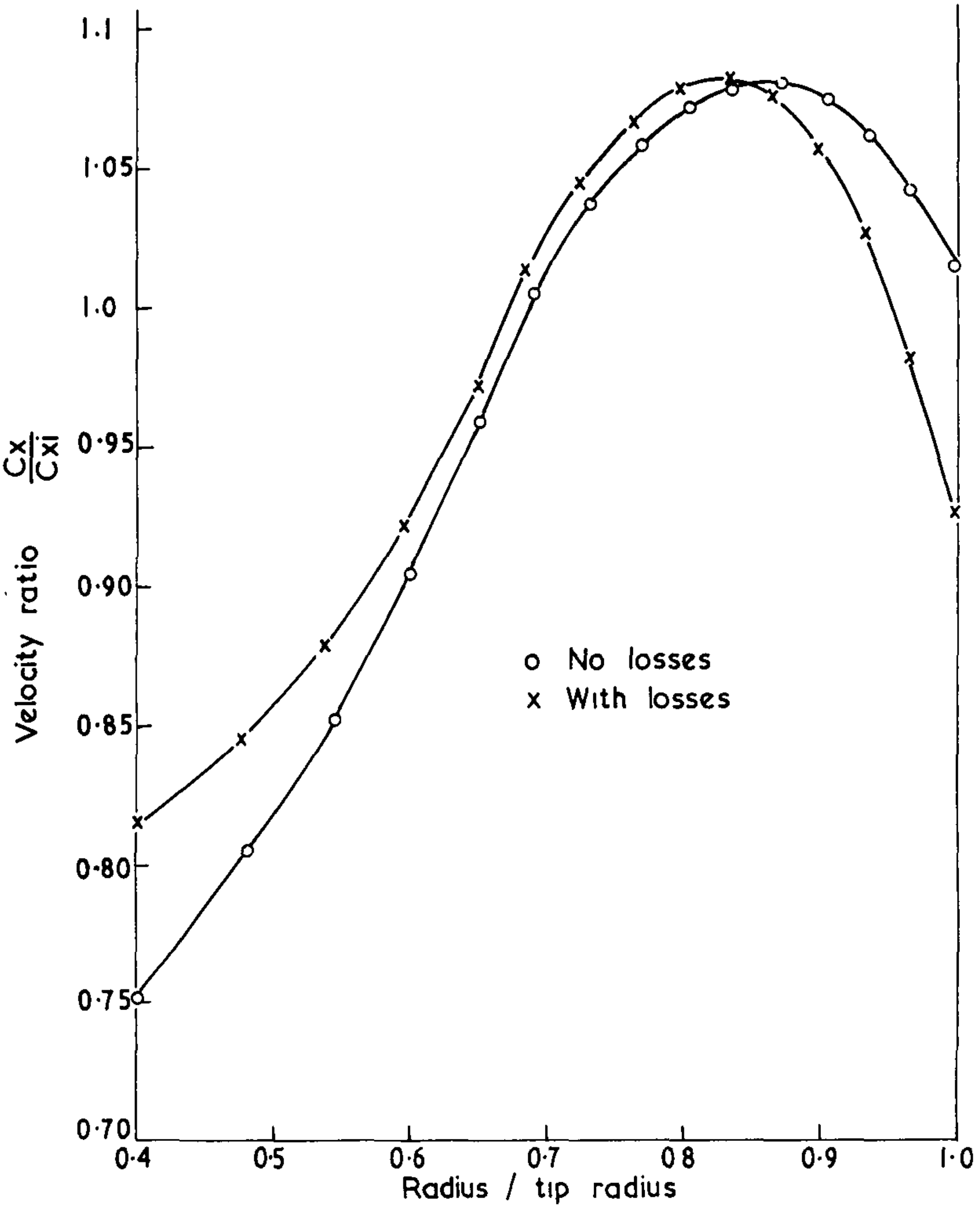


FIG 7 Axial velocity profiles downstream of stator

$$\frac{Cx_1}{U_m} = 0{\cdot}585$$

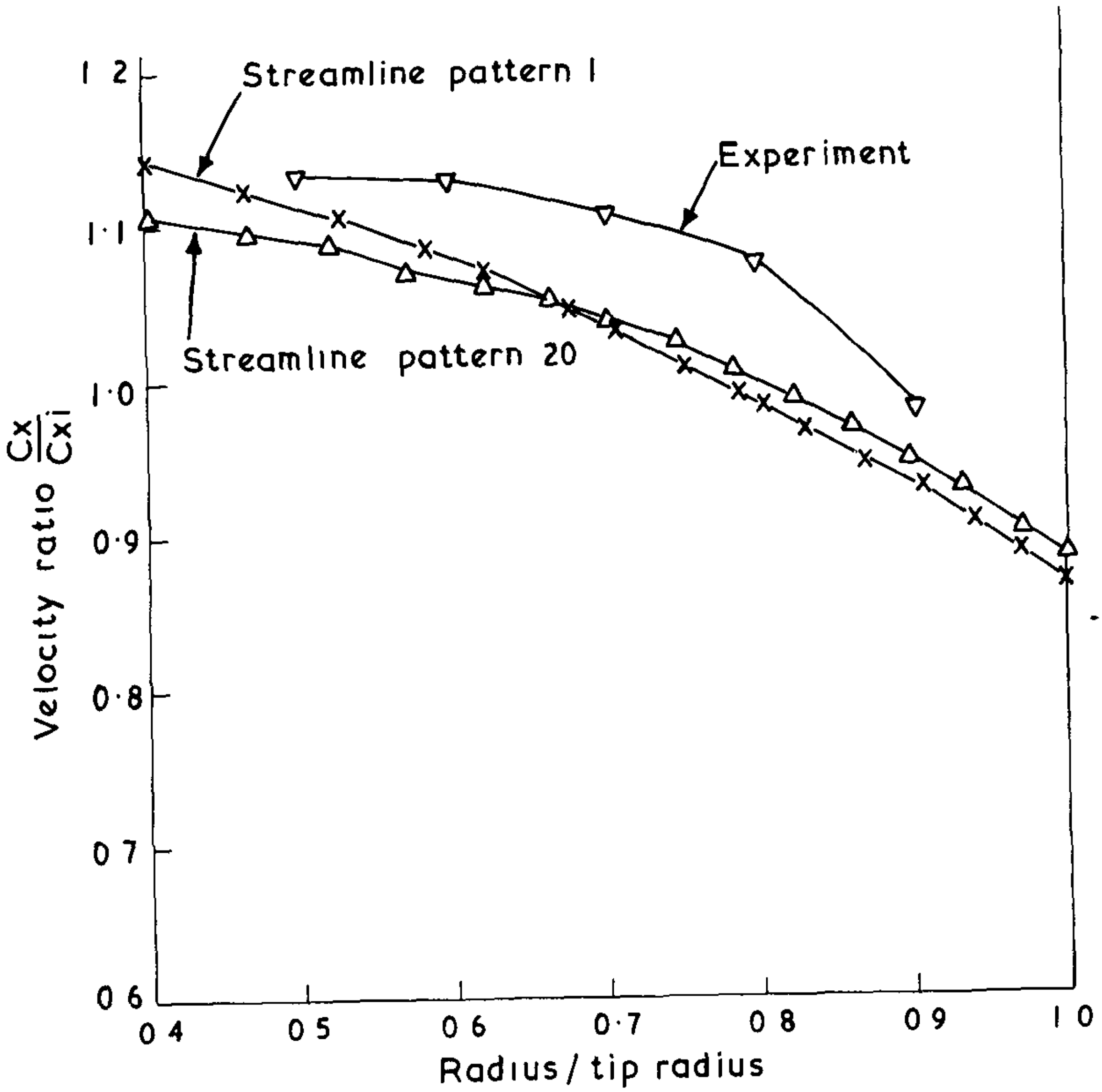


FIG 9 Axial velocity profiles downstream of inlet guide-vanes

Loss distribution I, $\frac{C_{xi}}{U_m} = 0.413$

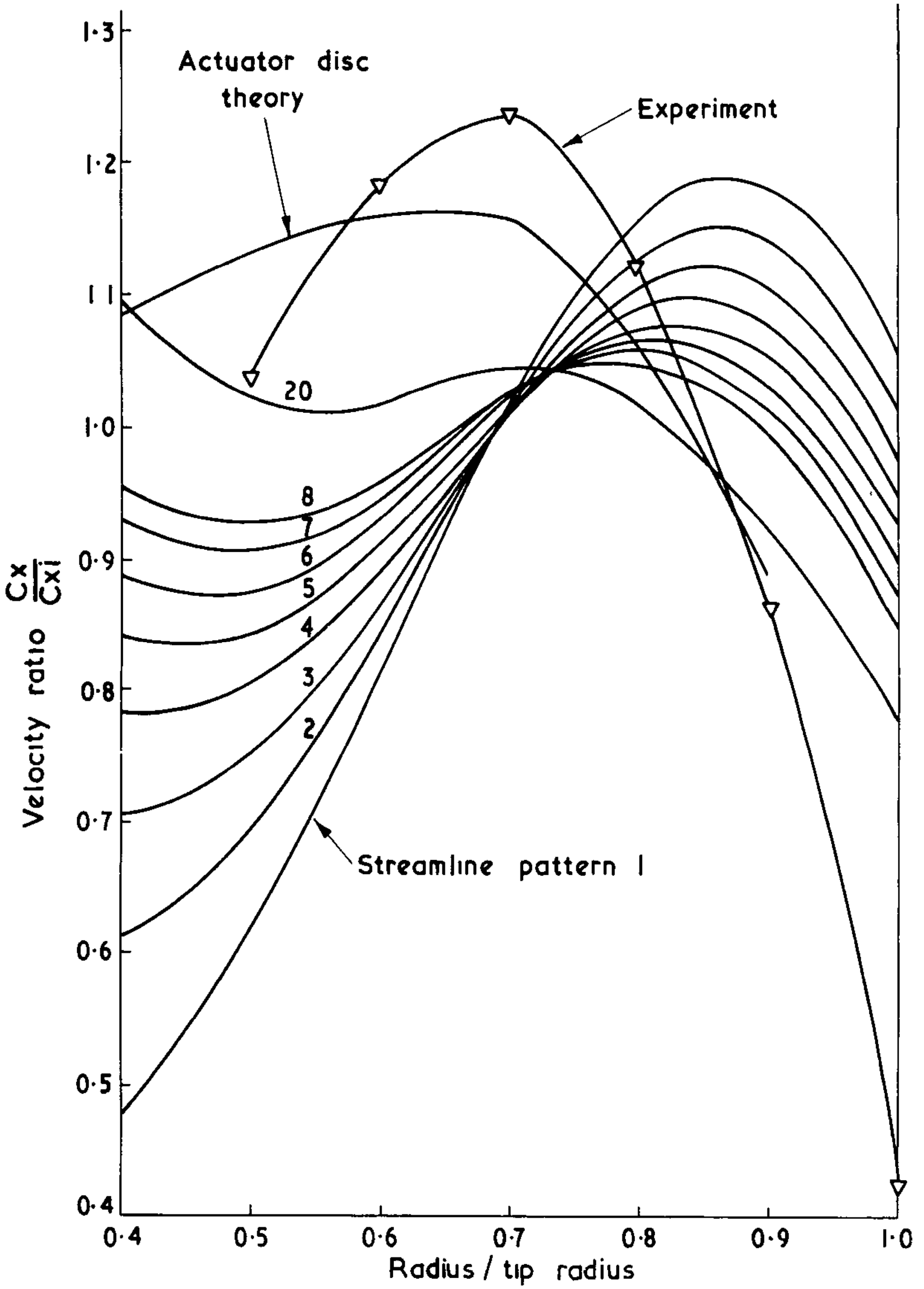


FIG.10 Axial velocity profiles downstream of the rotor

Loss distribution I, $\frac{C_{xi}}{U_m} = 0{\cdot}413$

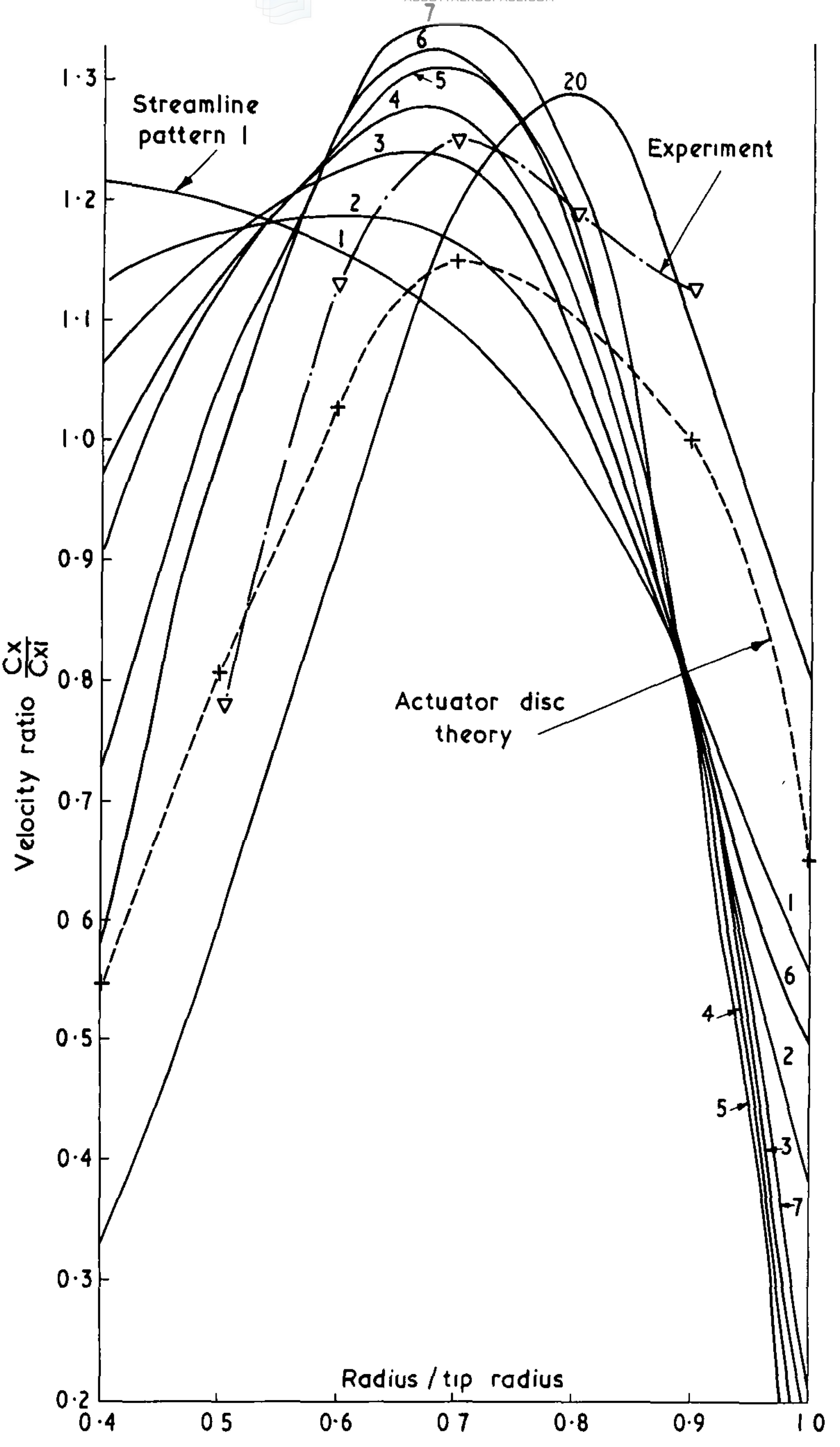


FIG.11 Axial velocity profiles downstream of stator

Loss distribution I, $\frac{C_{xi}}{U_m}$ = 0 413

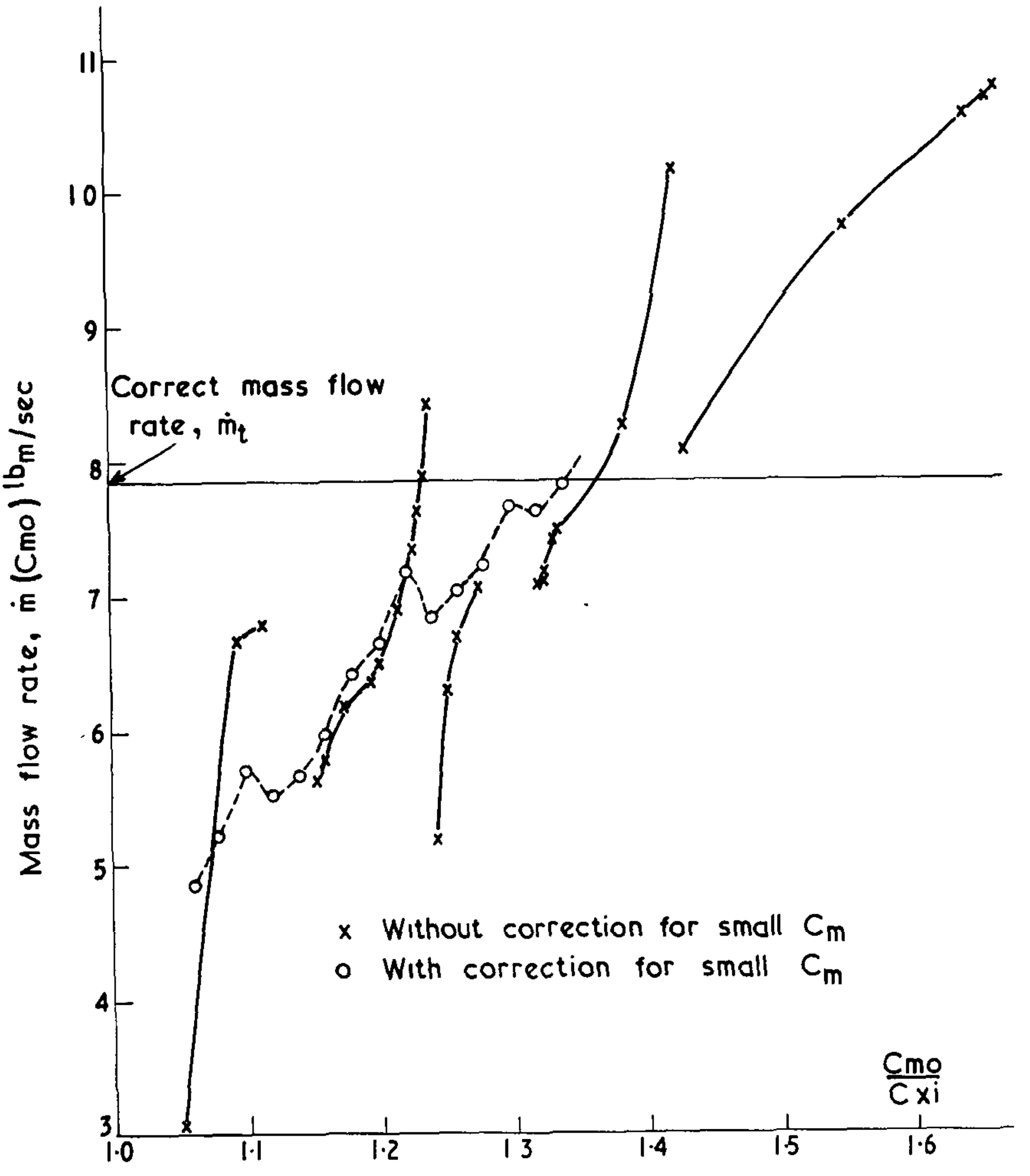


FIG 11a Values of Cmo giving the correct mass flow rate downstream of the stator

7th streamline pattern, loss distribution I, $\frac{C_{xi}}{U_m} = 0.413$

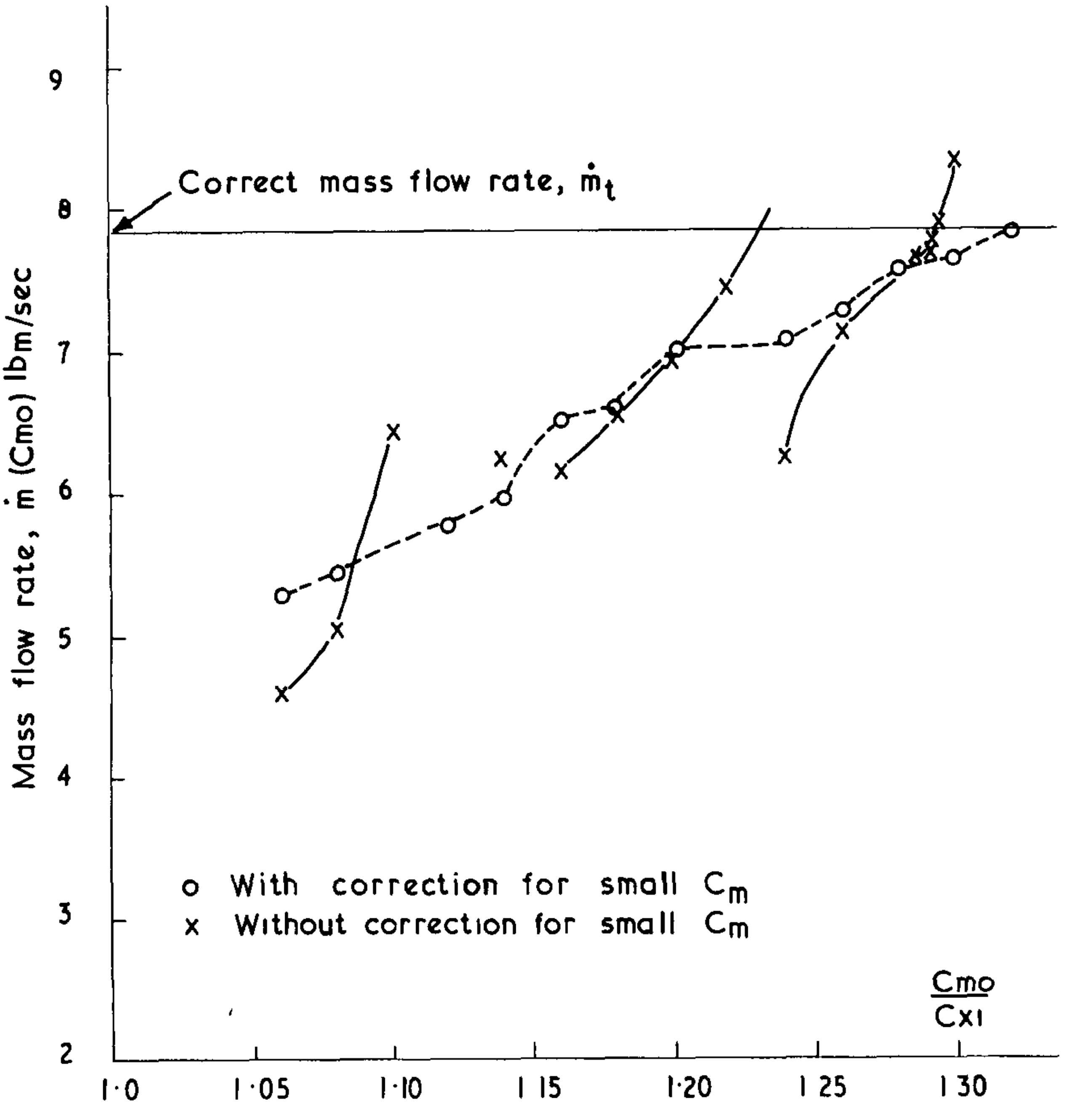


FIG 11b Values of Cmo giving the correct mass flow rate downstream of the stator

6th streamline pattern, loss distribution I, $\frac{C_{xi}}{U_m} = 0.413$

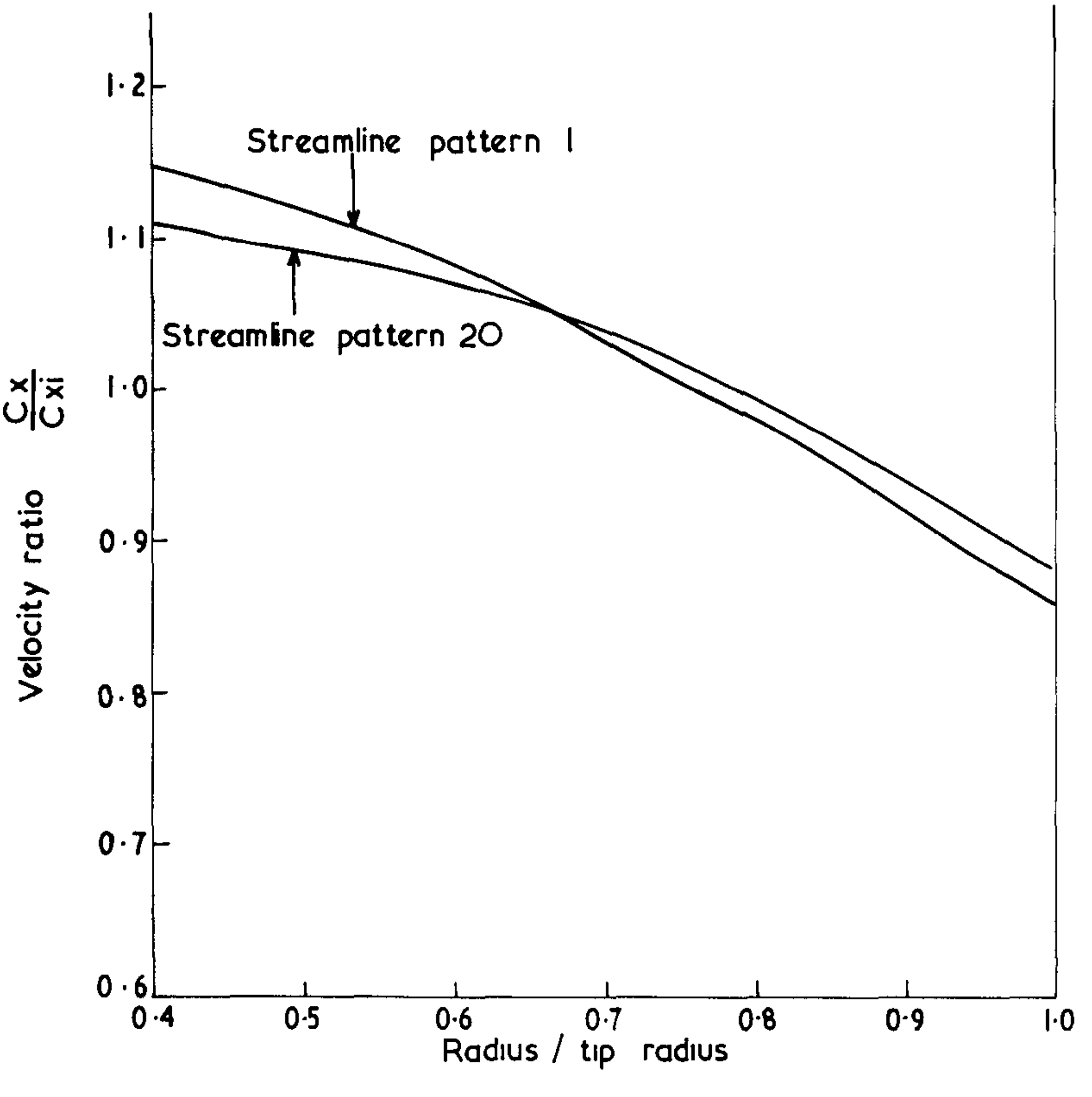


FIG.12 Axial velocity profiles downstream of the guide vanes

$\frac{Cxi}{Um} = 0{\cdot}413$, loss distribution II

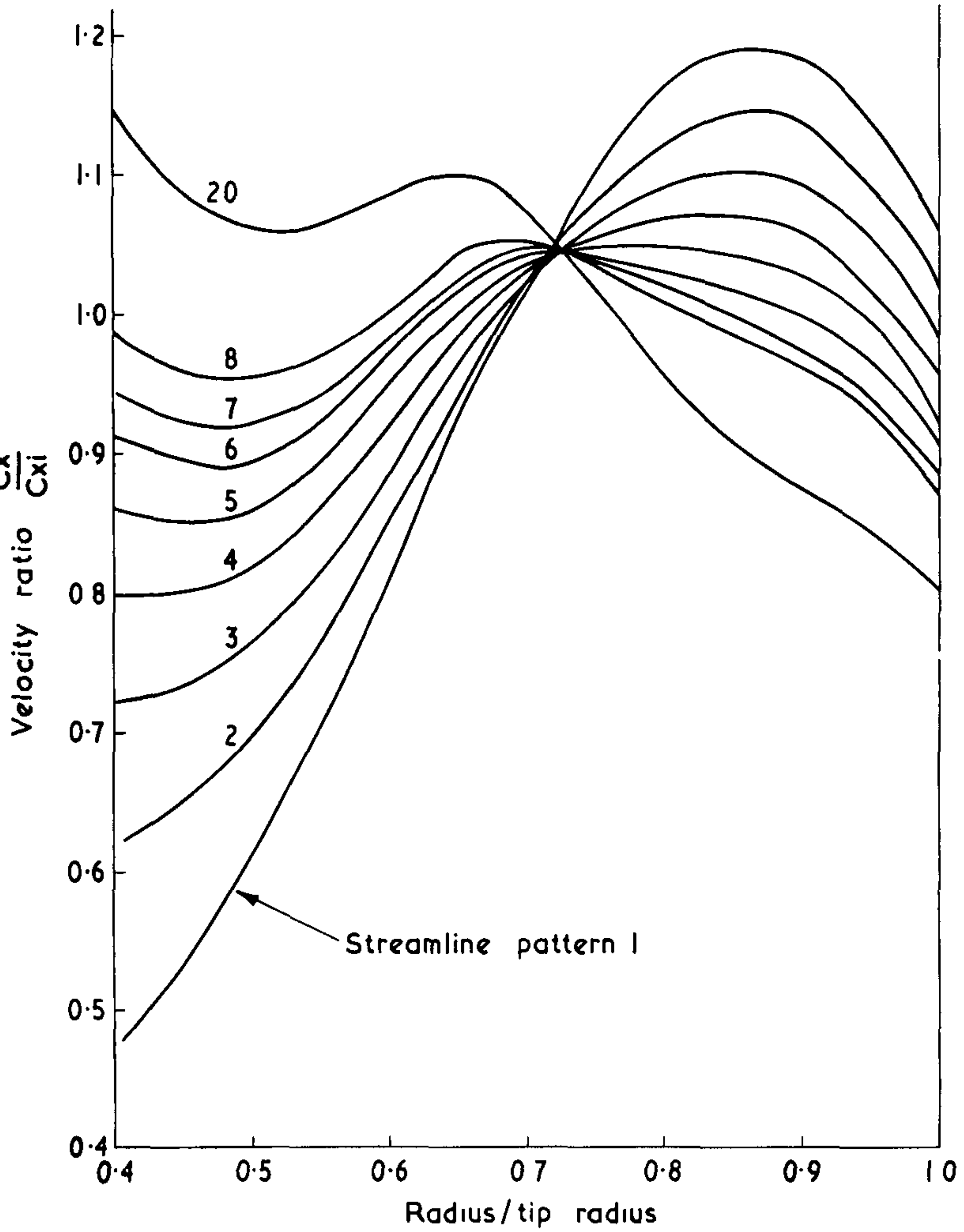


FIG 13 Axial velocity profiles downstream of the rotor

Loss distribution II, $\frac{Cxi}{Um} = 0{\cdot}413$

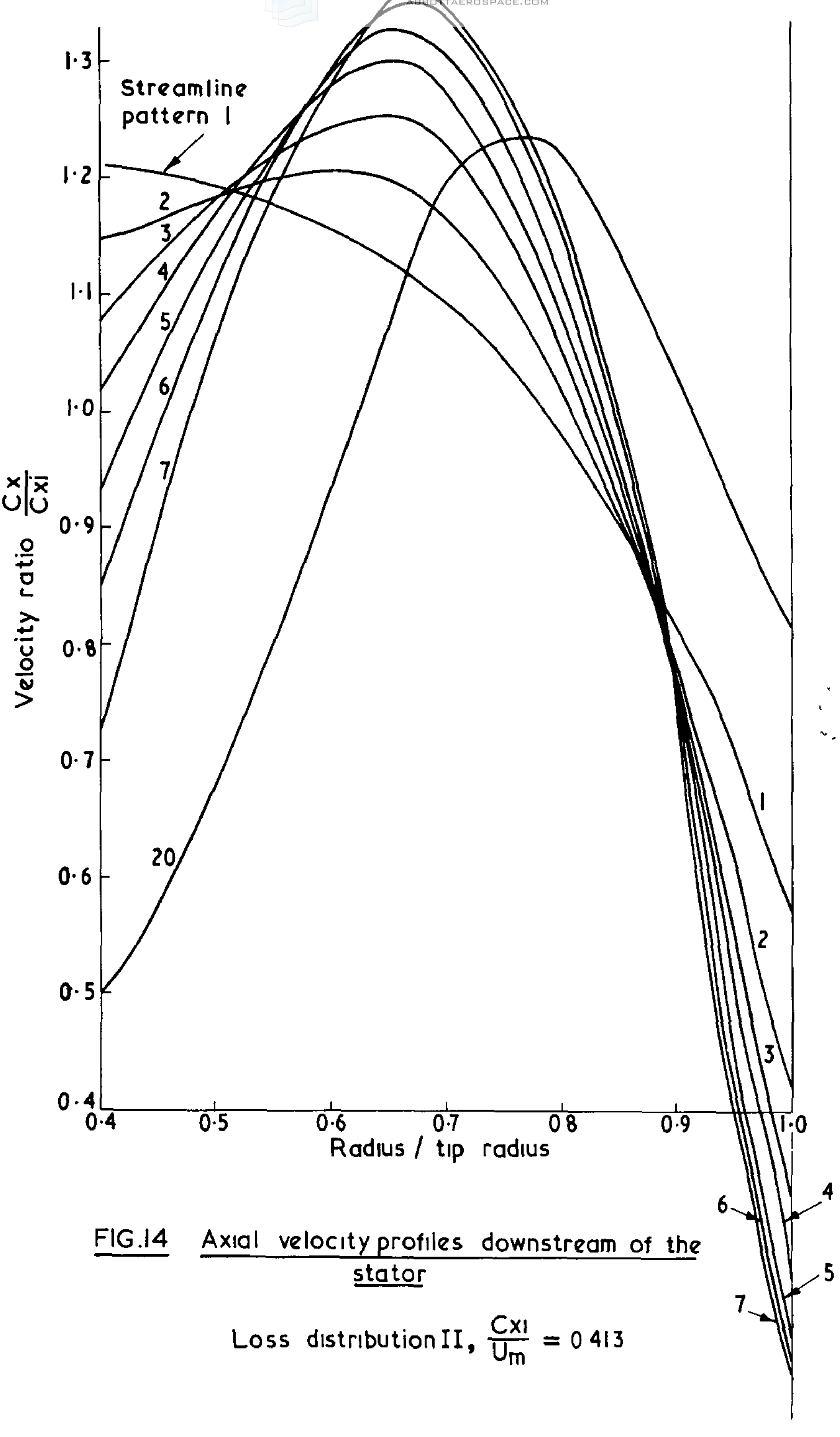


FIG.14 Axial velocity profiles downstream of the stator

Loss distribution II, $\frac{C_{xi}}{U_m} = 0\,413$

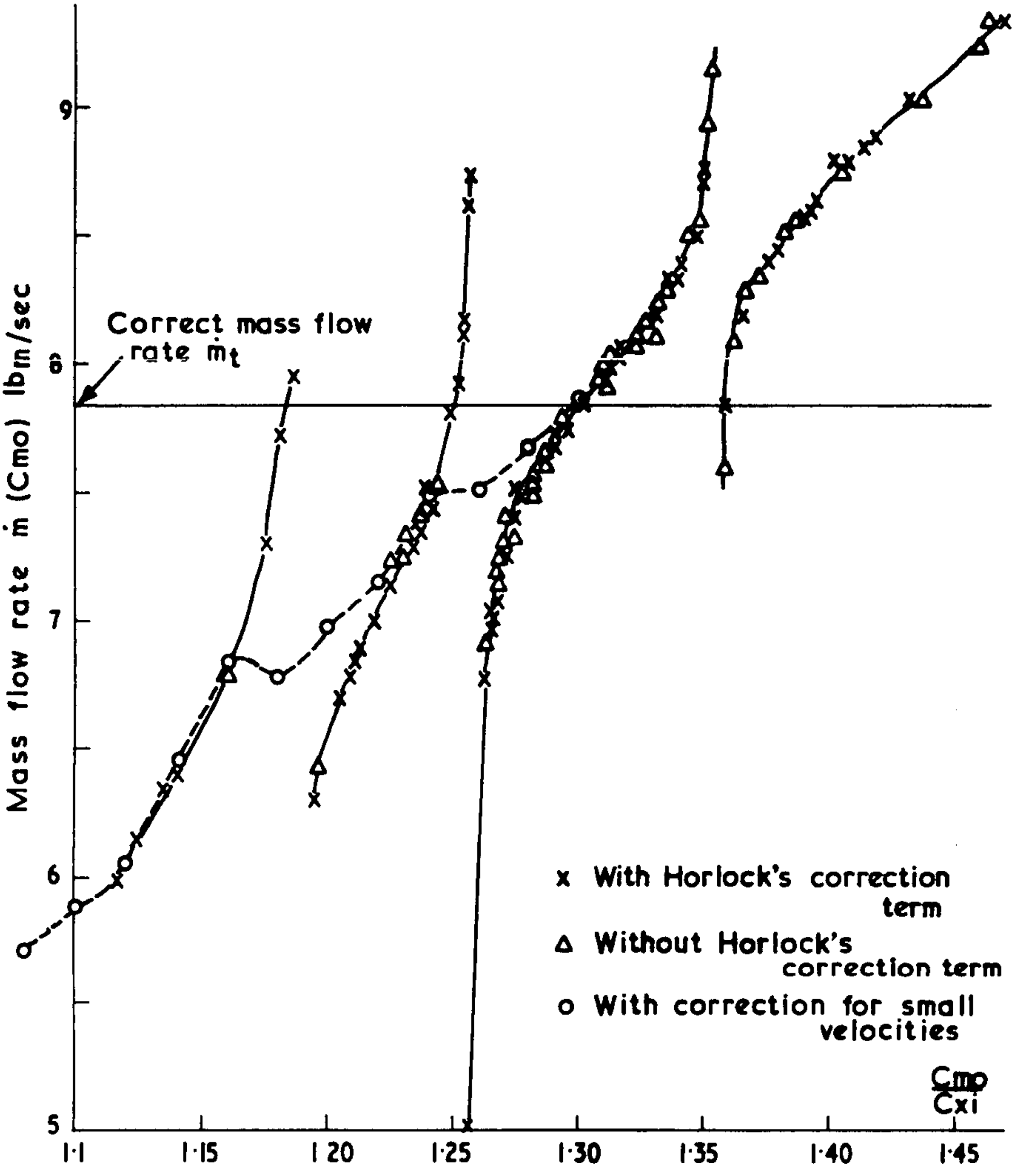


FIG.14a Values of Cmo giving the correct mass flow rate downstream of the stator

6th streamline pattern, loss distribution II, $\frac{C_{xi}}{U_m} = 0{\cdot}413$

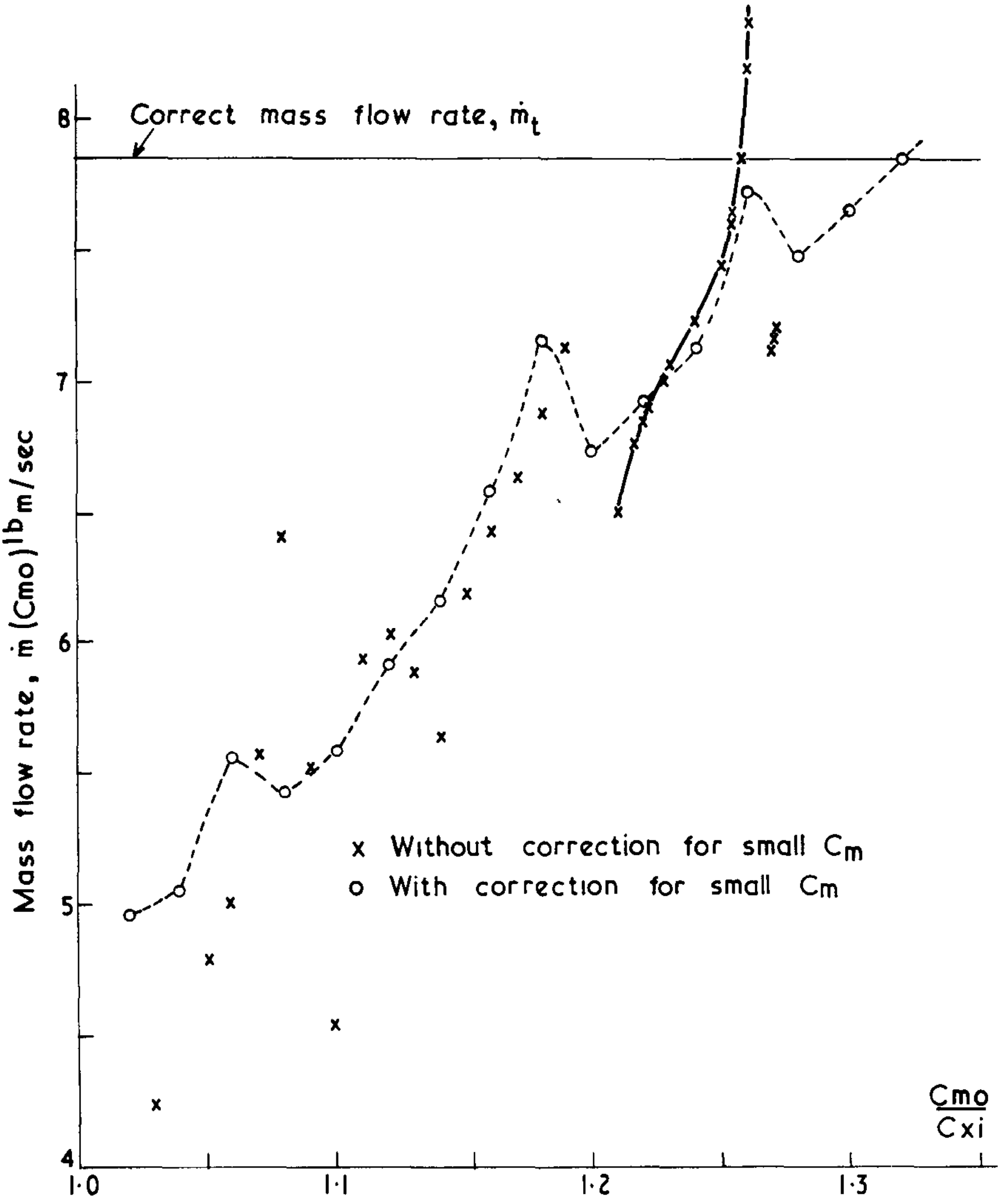


FIG. 14b Values of Cmo giving the correct mass flow rate downstream of the stator

7^{th} streamline pattern, loss distribution II, $\frac{Cxi}{U_m} = 0.413$

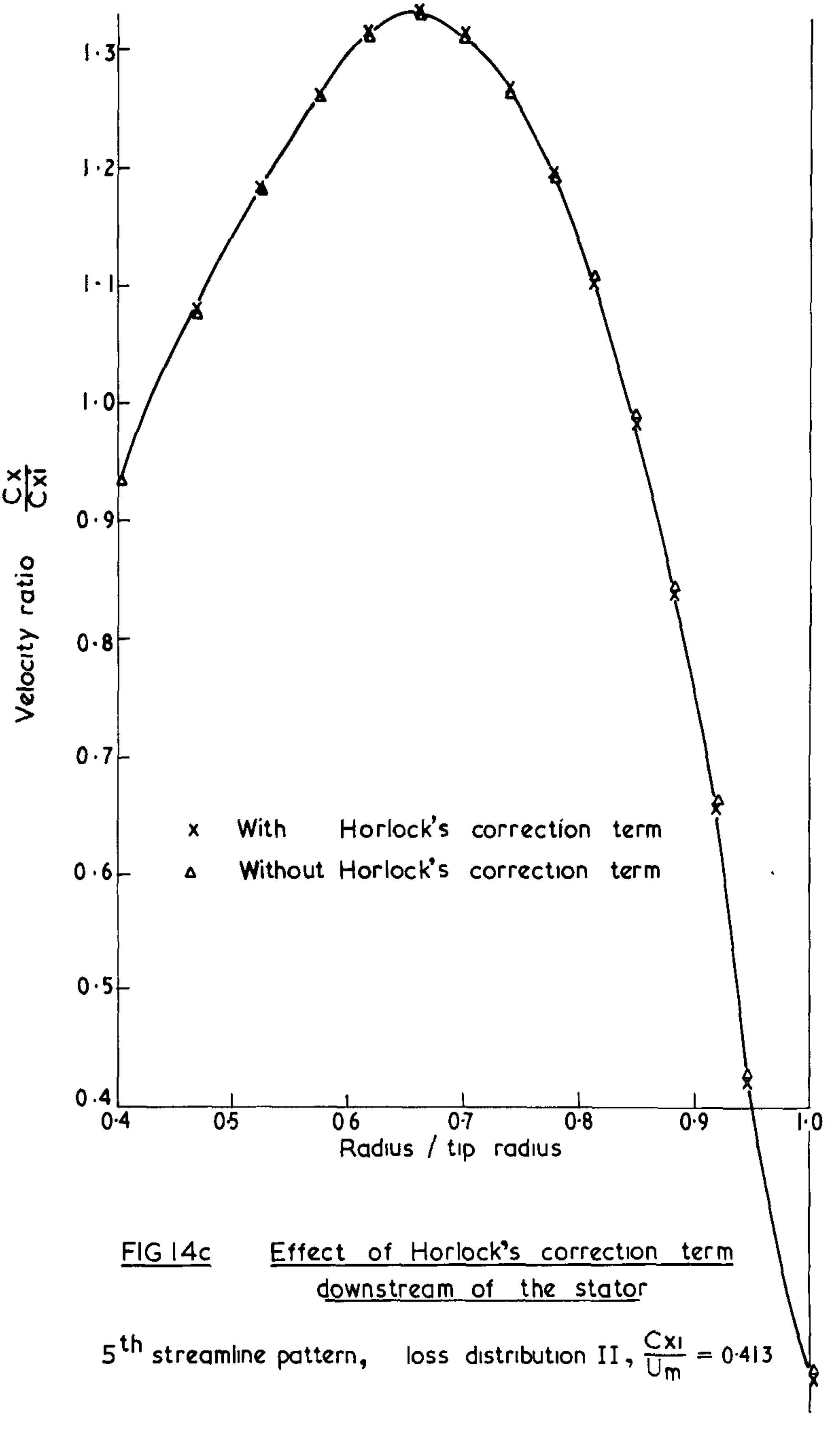


FIG 14c Effect of Horlock's correction term downstream of the stator

5th streamline pattern, loss distribution II, $\frac{C_{x1}}{U_m} = 0{\cdot}413$

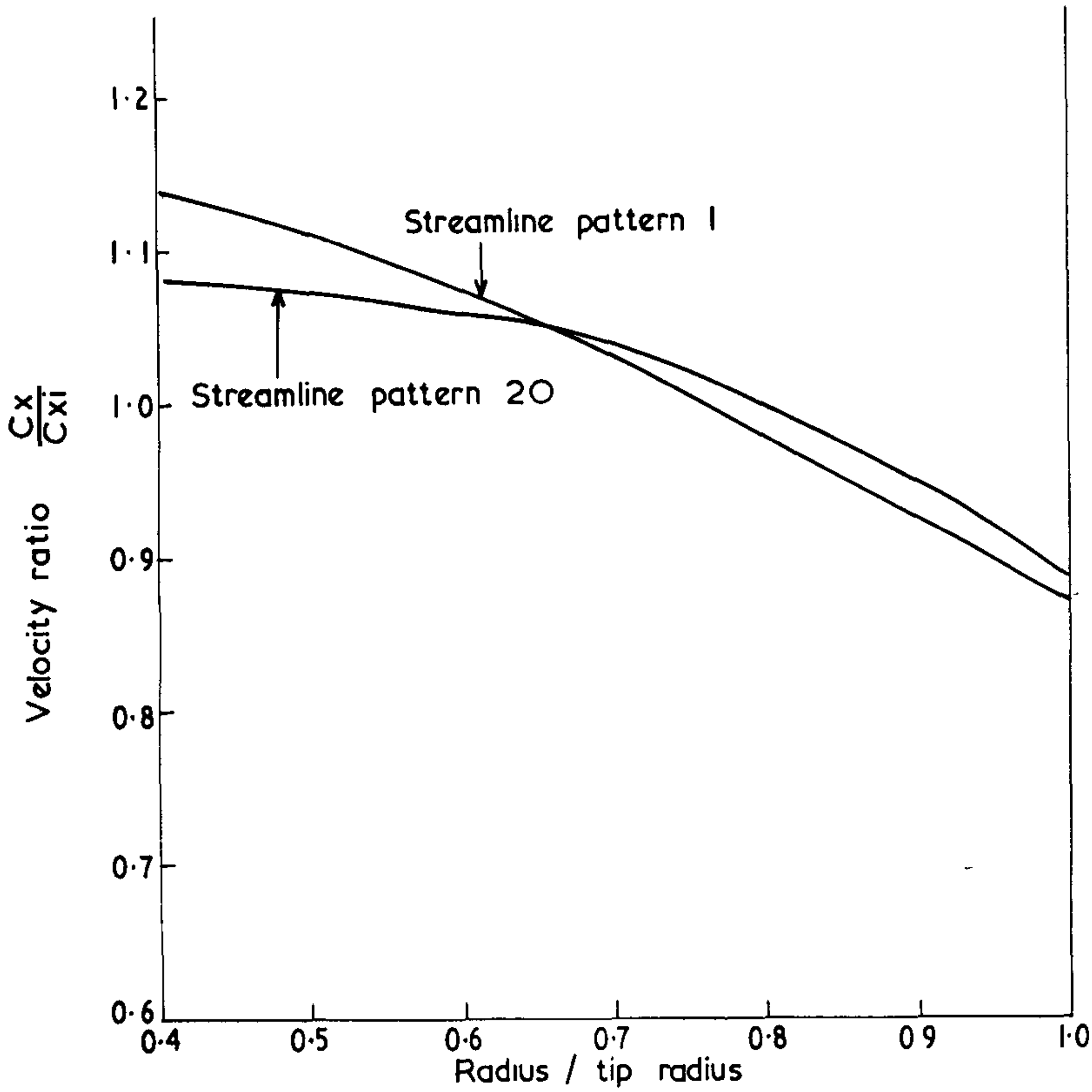


FIG.15 Axial velocity profiles downstream of the inlet guide vanes

$\frac{C_{xi}}{U_m} = 0.413$ No losses

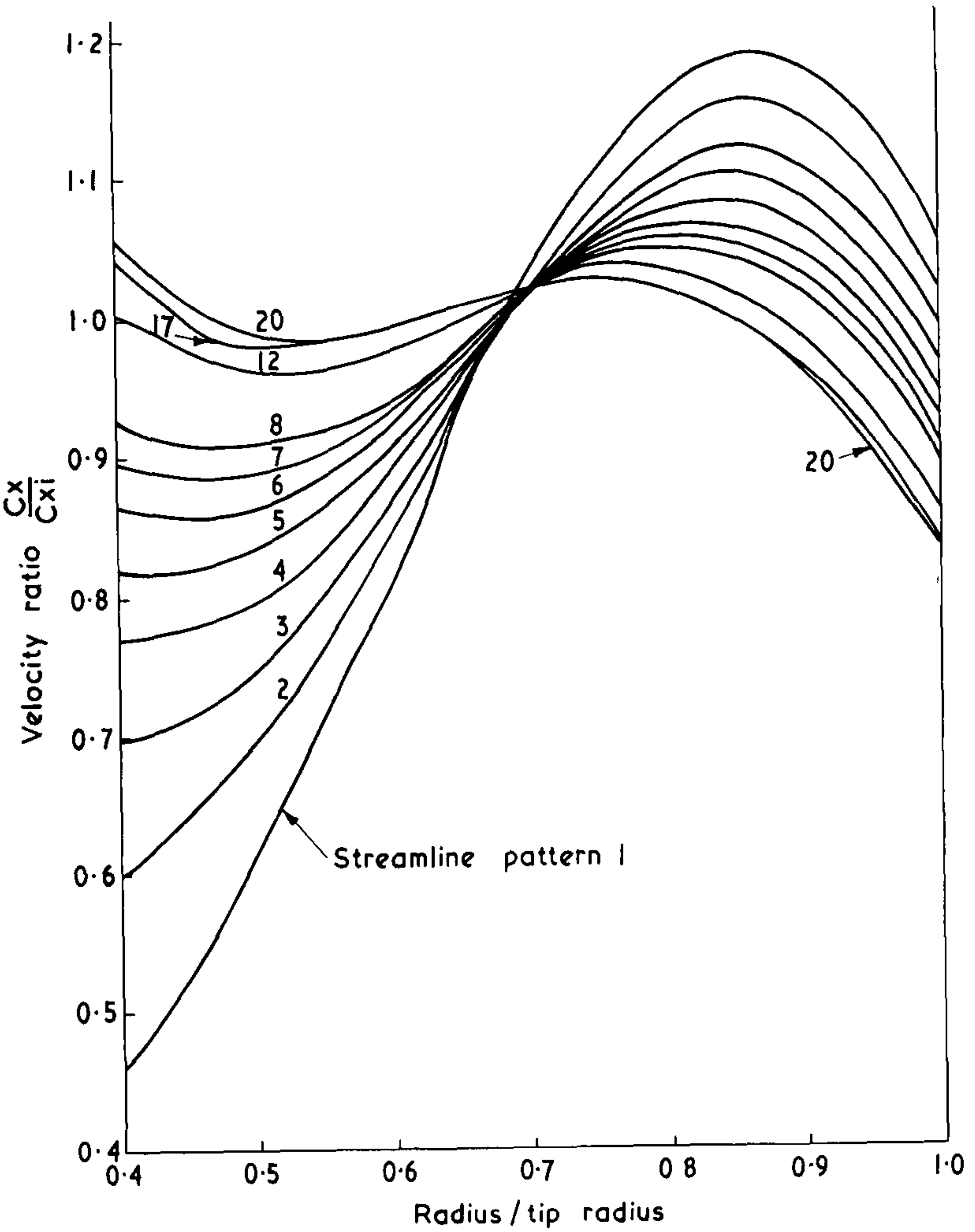


FIG 16 Axial velocity profiles downstream of the rotor

No losses $\frac{C_{xi}}{U_m} = 0\ 413$

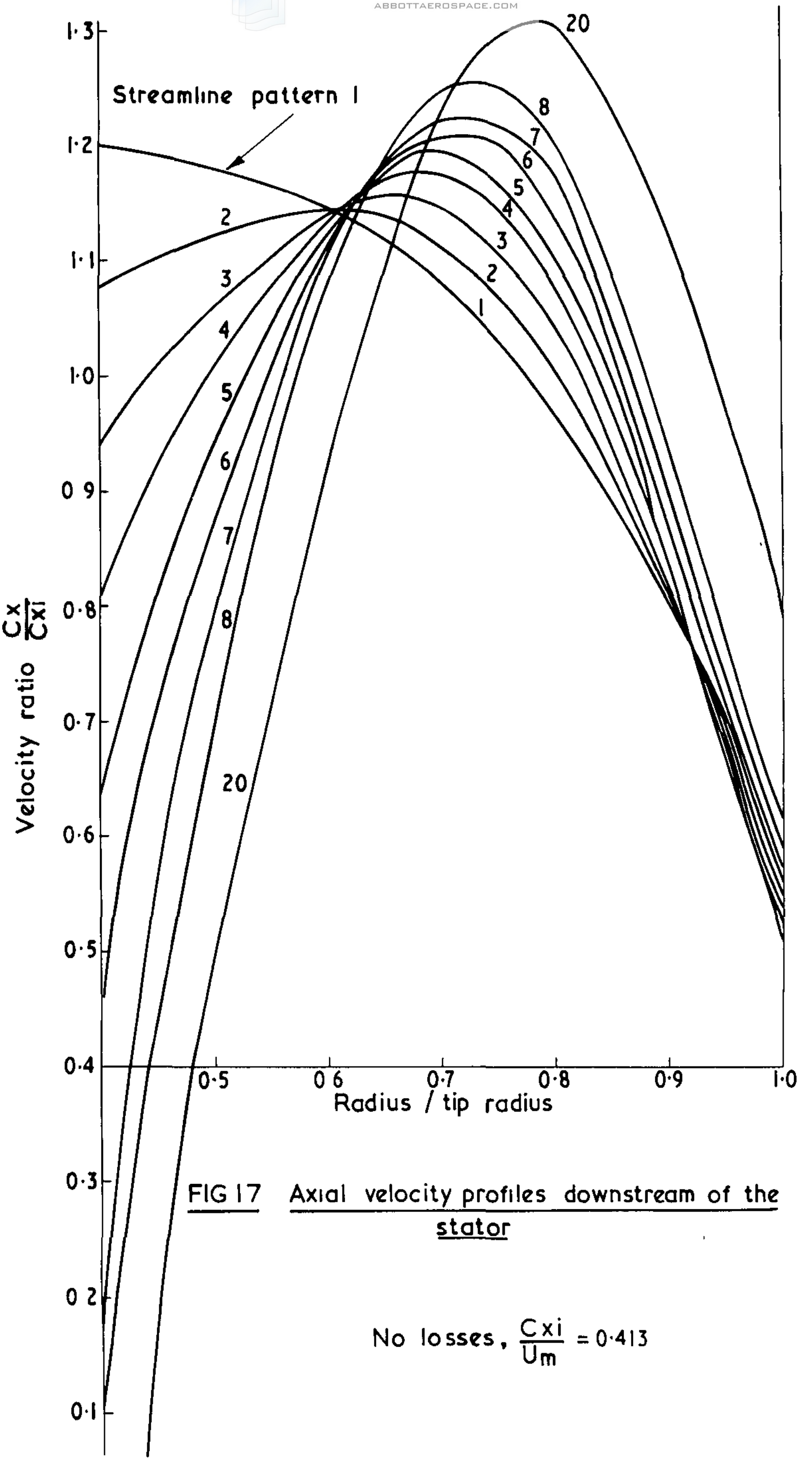
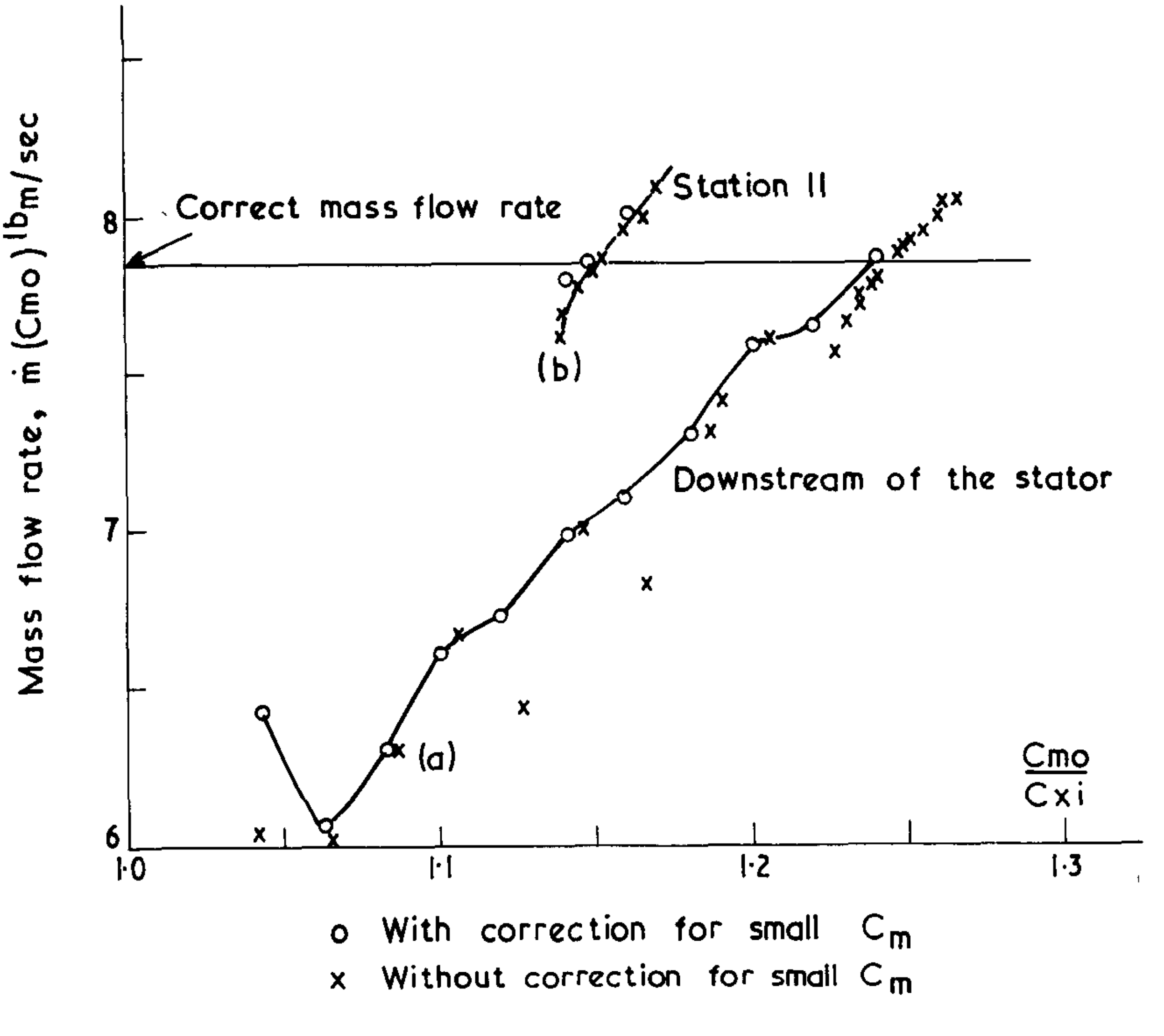


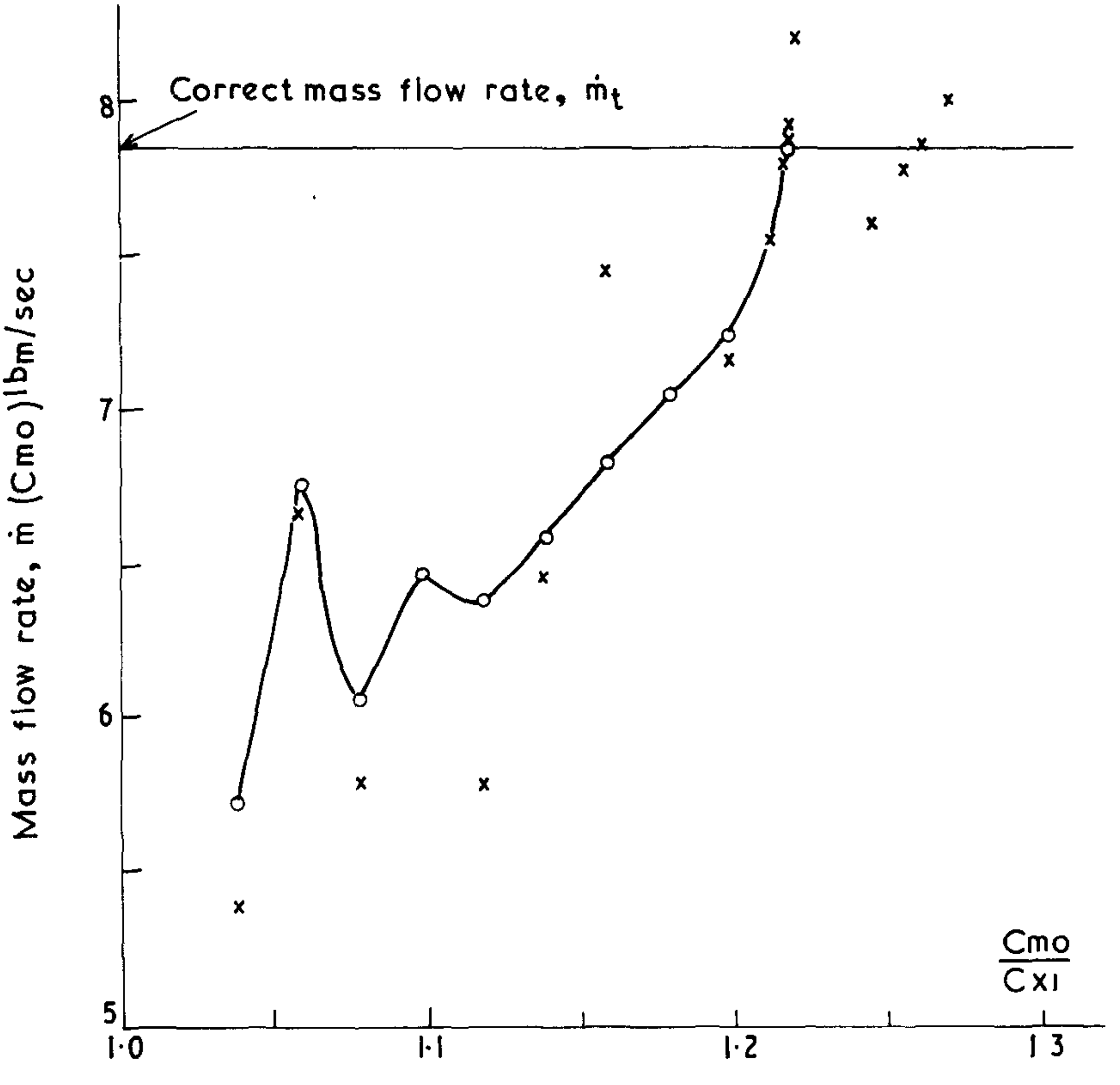
FIG 17 Axial velocity profiles downstream of the stator

No losses, $\frac{Cxi}{Um} = 0{\cdot}413$



FIGS 17a & 17b Values of Cmo giving the correct mass flow downstream of the stator and at station 11

7th streamline pattern, no losses, $\frac{C_{xi}}{U_m} = 0{\cdot}413$



o With correction for small C_m
x Without correction for small C_m

FIG. 17c Values of Cmo giving the correct mass flow rate downstream of the stator

8^{th} streamline pattern, no losses, $\frac{Cx1}{U_m} = 0{\cdot}413$

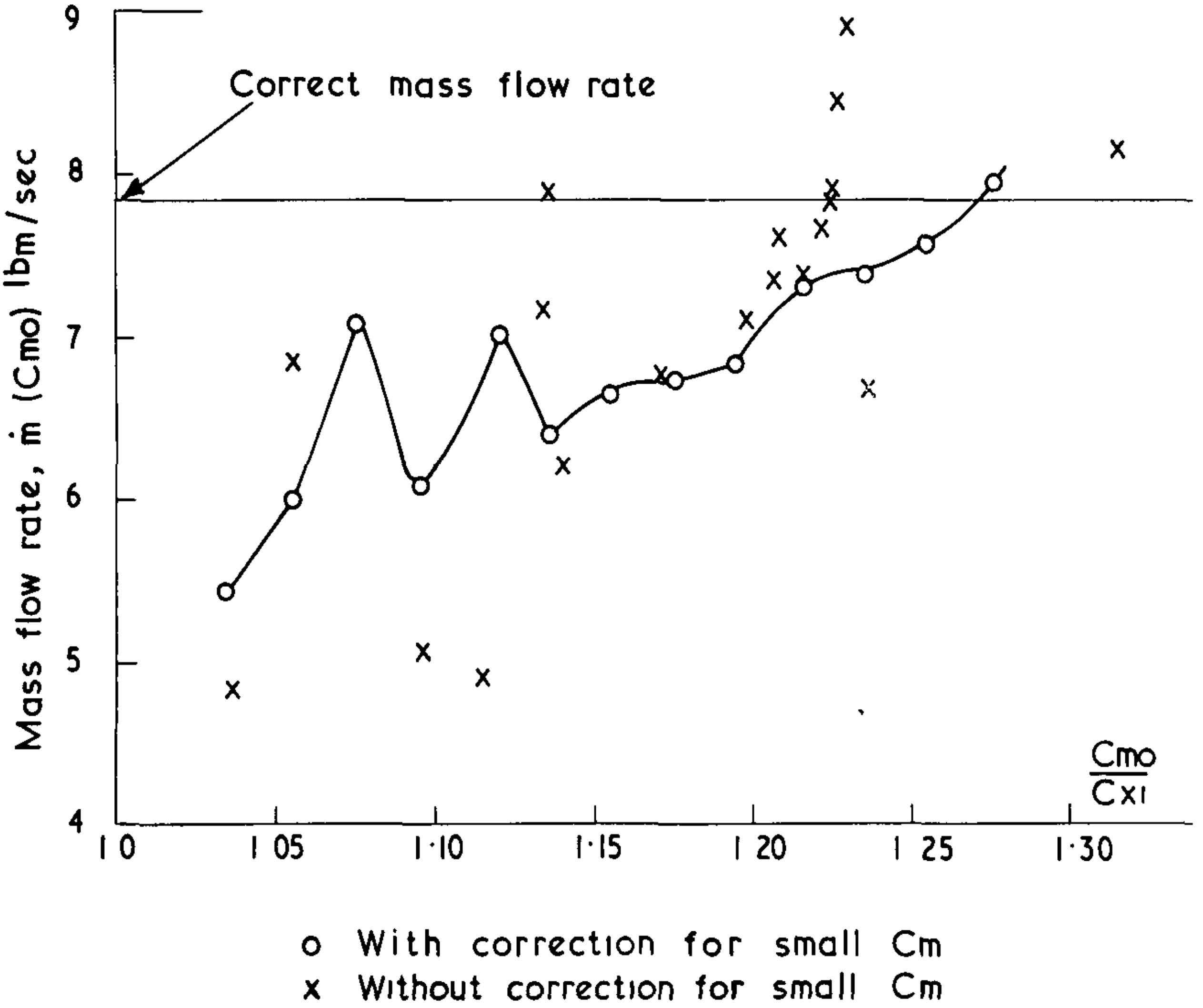


FIG. 17d Values of Cmo giving the correct mass flow rate downstream of the stator

9th streamline pattern, no losses, $\frac{C_{xi}}{U_m} = 0\ 413$

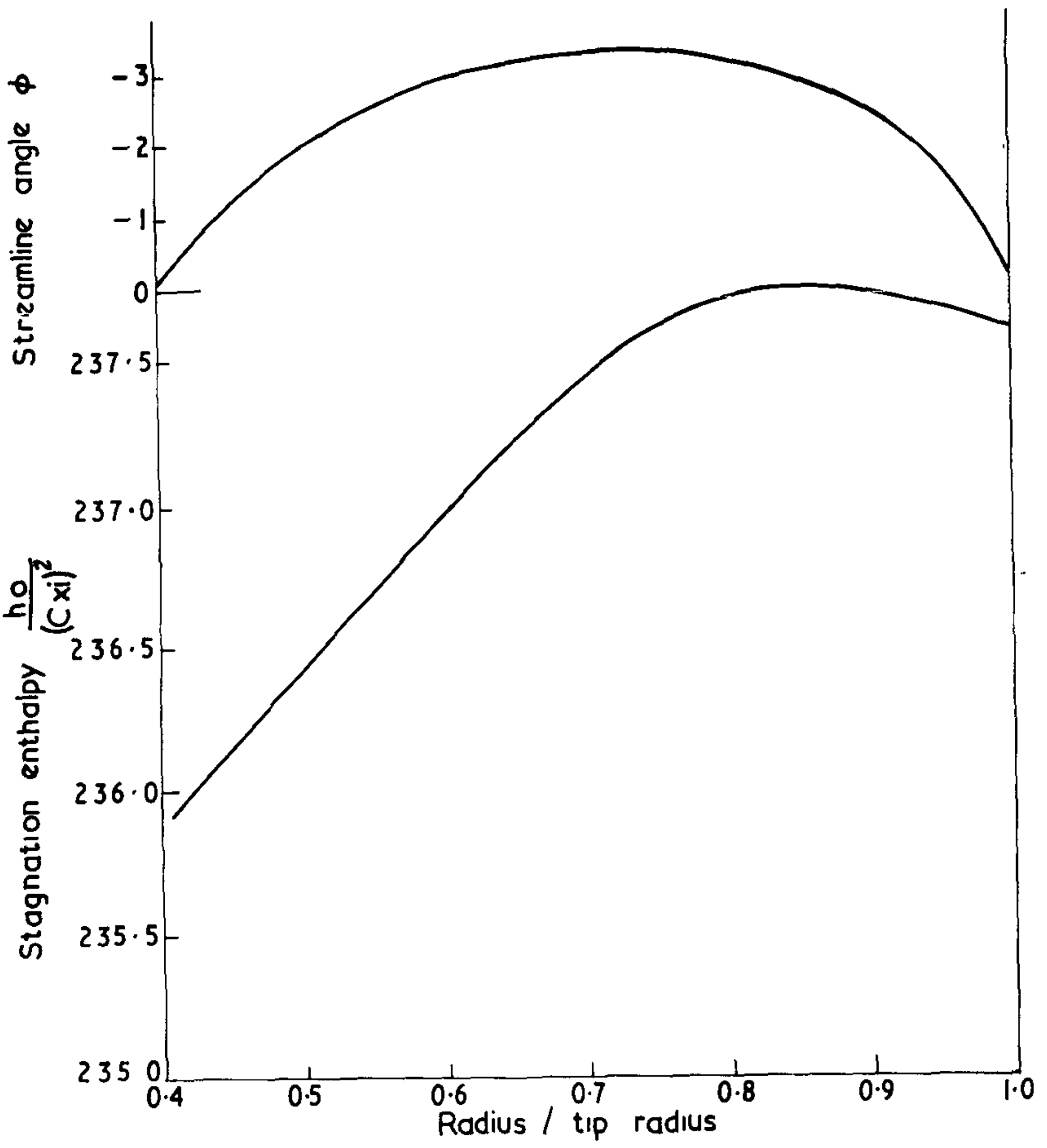


FIG A1 Distribution of stagnation enthalpy downstream of the stator

5th streamline pattern, loss distribution I, $\frac{C_{x1}}{U_m} = 0{\cdot}413$

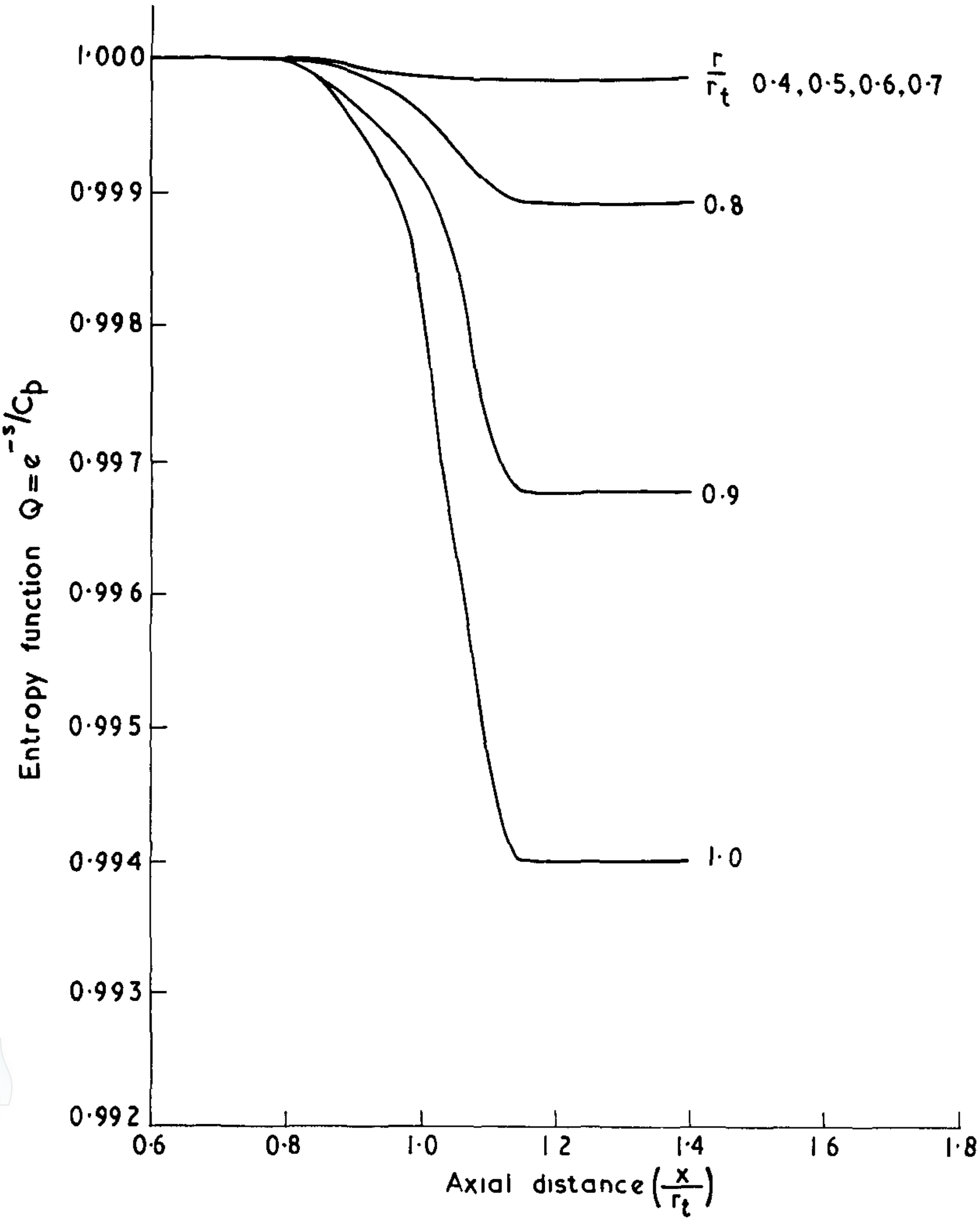


FIG A2 Distribution of the entropy function

5th streamline pattern, loss distribution I, $\frac{C_{x1}}{U_m} = 0{\cdot}413$

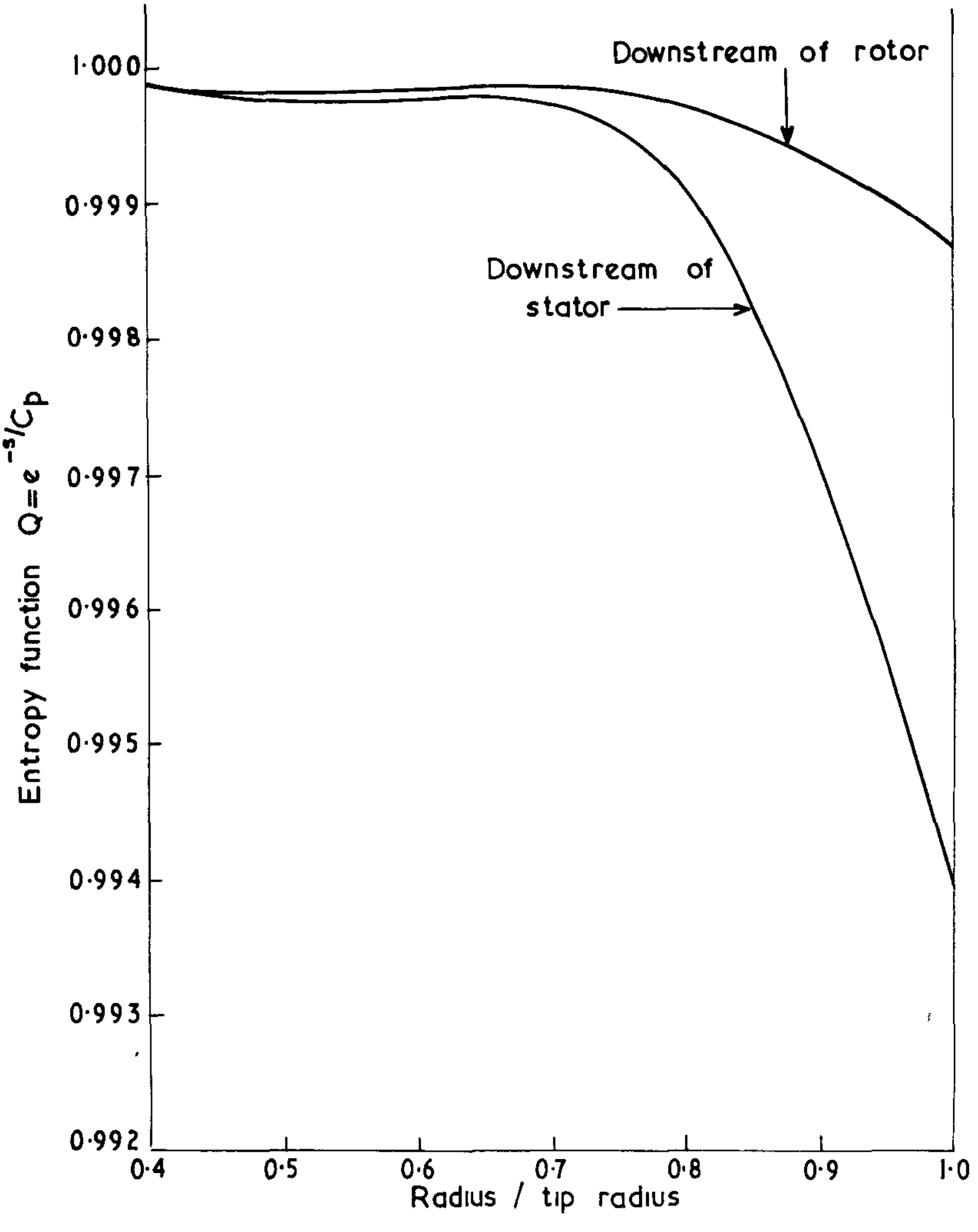


FIG.A3 Distribution of the entropy function

5^{th} streamline pattern, loss distribution I, $\frac{C_{xi}}{U_m} = 0{\cdot}413$

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