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by

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A CALCULATION METHOD FOR THE TURBULENT BOUNDARY LAYER ON AN INFINITE YAWED WING IN COMPRESSIBLE, ADIABATIC FLOW

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SUMMARY

A method is presented for the calculation of the compressible turbulent boundary layer both at the attachment line and over the surface of an infinite, yawed, thermally-insulated wing. The method uses the momentum integral and entrainment equations for three-dimensional compressible flow. Comparison with the few available experimental results is encouraging.

A Fortran computer program, based upon the method, has been written to calculate the boundary layer development on an infinite yawed wing of given section shape, sweep and pressure distribution at a given Reynolds number, Mach number, stagnation temperature and transition position.

* Replaces RAE Technical Report 72193 - ARC 34388

CONTENTS

1 INTRODUCTION

One of the most pressing requirements of boundary-layer theory is that it should be able to predict the development of turbulent boundary layers on swept wings. For wings of large aspect ratio it seems reasonable, as a first approximation, to neglect the effects of variations along the span and to treat the wing as though it were infinite. This does not reduce the number of physical assumptions involved in calculating the development of the boundary layer but it does considerably reduce the numerical complexity of the calculation procedure.

Such a calculation method was devised for incompressible flow by the present author[1] and also, independently, by Cumpsty and Head[2]. This method used the momentum integral equations in streamline coordinates together with an extension to three dimensions of Head's[3] entrainment method for the calculation of shape factor development.

Green[4] has shown how Head's entrainment method may be extended to compressible flow and so here use is made of Green's ideas to extend the three-dimensional method to compressible adiabatic flow. A brief outline of Green's method is given in section 2. Section 3 then gives details of the method for three-dimensional compressible flow. In section 4 the method of solution for the infinite yawed wing is presented and in section 5 the solution at the attachment line of an infinite yawed wing is given. Comparison between the predictions of the method and the few experimental results that are available is given in section 6.

The calculation method is available as a Fortran computer program which has been in use at the RAE since 1968 and a listing together with details of the input data required may be obtained from the author.

The Report concludes with a discussion of the shortcomings of the present method together with some thoughts on further extensions.

2 OUTLINE OF GREEN'S METHOD FOR TWO-DIMENSIONAL FLOW

Green's[4] method is an extension to compressible flow of a method devised by Head[3] for the calculation of two-dimensional incompressible turbulent boundary layers. The object of the later method is to solve the boundary-layer momentum integral equation,

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \frac{\theta}{U_e}\frac{dU_e}{dx}(H + 2 - M_e^2) \quad , \tag{1}$$

together with an auxiliary equation known as the entrainment equation

$$\frac{d(\delta - \delta_1)}{dx} = C_E - \frac{\delta - \delta_1}{U_e}\frac{dU_e}{dx}(1 - M_e^2) \quad , \tag{2}$$

which may be derived by integrating the continuity equation across the boundary layer and denoting those terms which represent the entrainment of the external flow into the boundary layer by C_E. In the above equations x represents the distance along the surface in the streamwise direction and U_e, M_e respectively denote the velocity and Mach number in that direction at the edge of the boundary layer. The terms θ, $\delta - \delta_1$ and H are defined as

$$\theta = \int_0^\delta \frac{\rho U}{\rho_e U_e}\left(1 - \frac{U}{U_e}\right)d\zeta \quad , \qquad \delta_1 = \int_0^\delta 1 - \frac{\rho U}{\rho_e U_e}\, d\zeta \quad , \tag{3}$$

and

$$H = \frac{\delta_1}{\theta} \quad .$$

We also write $H_1 = \frac{\delta - \delta_1}{\theta}$, where ρ denotes the fluid density, δ the edge of the boundary layer and ζ the distance perpendicular to the wall. The term C_f in equation (1) is the local skin friction coefficient, i.e. the surface shear stress at the point divided by the product $\rho U_e^2/2$. The basis of Head's incompressible method is the assumption that the entrainment coefficient, C_E, is a unique function of H_1, and that H_1 is solely a function of H. For compressible flow Green retains the first of these assumptions and uses Head's original curve of $C_E = F(H_1)$ in the form

$$C_E \equiv F(H_1) = 0.0299(H_1 - 3.0)^{-0.6169} \quad . \tag{4}$$

Green then suggests that to characterise the shape of the velocity profile a 'transformed' shape parameter $\bar{H}$, where

$$\bar{H} = \int_0^\delta \frac{\rho}{\rho_e}\left(1 - \frac{U}{U_e}\right)d\zeta/\theta \tag{5}$$

might be regarded as the equivalent in compressible flow to the parameter H at low speeds. For compressible flow H_1 is thus assumed to be solely a function of $\bar{H}$. The form of this function is taken to be

$$\bar{H} = 1 + 1.12\left[H_1 - 2 - \sqrt{(H_1 - 2)^2 - 3}\right]^{0.915} \quad . \tag{6}$$

The assumption of a parabolic temperature distribution with zero wall heat transfer

$$T = T_r + (T_e - T_r)(U/U_e)^2 \quad , \tag{7}$$

where T_r denotes recovery temperature, produces the relation

$$(H + 1) = \frac{T_r}{T_e}(\bar{H} + 1) = \left(1 + \left(\frac{\gamma - 1}{2}\right)rM_e^2\right)(\bar{H} + 1) \tag{8}$$

where r denotes recovery factor.

The final assumptions of Green's method concern the skin-friction coefficient in flows with pressure gradients. This is assumed to be given by

$$\left(\frac{C_f}{C_{fp}} + 0.5\right)\left(\frac{\bar{H}}{\bar{H}_p} - 0.4\right) = 0.9 \tag{9}$$

where C_{fp} is the skin-friction coefficient on a flat plate at the same R_θ and M_e. $\bar{H}_p$, the flat plate transformed shape parameter, is assumed to be given by

$$\frac{1}{\bar{H}_p} = 1 - 6.8\sqrt{\frac{C_{fp}}{2}} \quad . \tag{10}$$

C_{fp} is assumed to be given by Spalding and Chi's[5] correlation in the form

$$F_C C_{fp} = \frac{0.012}{(\log_{10}(F_R R_\theta) - 0.64)} - 0.0093 \tag{11}$$

with

$$F_C = \frac{T_r/T_e - 1}{\left(\tan^{-1}\left[(T_r/T_e - 1)^{\frac{1}{2}}\right]\right)^2} \tag{12}$$

and

$$F_R = (T_r/T_e)^{-0.702} \quad . \tag{13}$$

The above assumptions are sufficient to allow equations (1) and (2) to be solved numerically as a pair of simultaneous, ordinary first-order differential equations with dependent variables θ and $\delta - \delta_1$. With these quantities known at any station the other quantities of normal aerodynamic interest such as C_f and H are easily derived from the above relations.

3 THE METHOD FOR THREE-DIMENSIONAL COMPRESSIBLE FLOW

For three-dimensional flow it has been found[6] extremely useful to define boundary-layer parameters in terms of streamline coordinates. This is a system in which one family of coordinate curves is the projection of the external streamlines on to the surface of the body, whilst the other family consists of the orthogonal trajectories of the first family. The direction of an external streamline is called the streamwise direction. The cross-flow, or cross-wise component of flow in the boundary layer, is the component at right angles to the streamwise direction. It has been found[14] with streamline coordinates that the streamwise flow has similar properties to a two-dimensional boundary layer. In particular empirical relations derived for two-dimensional flow, e.g. skin-friction formulae shape-parameter relations and velocity-profile families, provide good approximations for the streamwise component of a three-dimensional flow.

The momentum integral and entrainment equations for a three-dimensional compressible flow with an irrotational external flow may be written in streamline coordinates as

$$\frac{\partial\theta_{11}}{\partial s} + \frac{\partial\theta_{12}}{\partial n} = \frac{\tau_{01}}{\rho U_e^2} - \frac{1}{U_e}\frac{\partial U_e}{\partial s}\theta_{11}(H + 2 - M_e^2) - \theta_{11}\frac{1}{r}\frac{\partial r}{\partial s} + \theta_{22}\frac{1}{r}\frac{\partial r}{\partial s} + M_e^2\,\theta_{12}\frac{1}{U_e}\frac{\partial U_e}{\partial n} \tag{14}$$

$$\frac{\partial\theta_{21}}{\partial s} + \frac{\partial\theta_{22}}{\partial n} = \frac{\tau_{02}}{\rho U_e^2} - 2\theta_{21}\left(\frac{1}{U_e}\frac{\partial U_e}{\partial s} + \frac{1}{r}\frac{\partial r}{\partial s}\right) - \theta_{11}(H + 1)\frac{1}{U_e}\frac{\partial U_e}{\partial n} - \frac{1}{U_e}\frac{\partial U_e}{\partial n}\theta_{22} + \theta_{21}M_e^2\frac{1}{U_e}\frac{\partial U_e}{\partial s} + \theta_{22}M_e^2\frac{1}{U_e}\frac{\partial U_e}{\partial n} \tag{15}$$

$$\frac{\partial(\delta - \delta_1)}{\partial s} - \frac{\partial \delta_2}{\partial n} = C_E - (\delta - \delta_1)\left[\frac{1}{r}\frac{\partial r}{\partial s} + (1 - M_e^2)\frac{1}{U_e}\frac{\partial U_e}{\partial s}\right] - \delta_2 M_e^2 \frac{1}{U_e}\frac{\partial U_e}{\partial n} \quad . \qquad (16)$$

Here s represents distance along an external streamline, n distance parallel to the body surface and normal to an external streamline, $\partial r/r\partial s$ is a measure of the convergence or divergence of the external streamline and the various displacement and momentum thicknesses are defined as

$$\delta_1 = \int_0^\delta \left(1 - \frac{\rho U}{\rho_e U_e}\right) d\zeta \qquad \delta_2 = -\int_0^\delta \frac{\rho V}{\rho_e U_e} d\zeta$$

$$\theta_{11} = \int_0^\delta \left(1 - \frac{U}{U_e}\right)\frac{\rho U}{\rho_e U_e} d\zeta \ , \qquad \theta_{12} = \int_0^\delta \left(1 - \frac{U}{U_e}\right)\frac{\rho V}{\rho_e U_e} d\zeta \qquad (17)$$

$$\theta_{21} = -\int_0^\delta \frac{\rho U V}{\rho_e U_e^2} d\zeta \ , \qquad \theta_{22} = -\int_0^\delta \frac{\rho V^2}{\rho_e U_e^2} \quad .$$

The two shape factors H and H_1 are defined by

$$H = \delta_1/\theta_{11} \qquad (18)$$

and

$$H_1 = (\delta - \delta_1)/\theta_{11} \quad . \qquad (19)$$

τ_{01}, τ_{02}, U and V are the components of skin friction and velocity in the s and n directions respectively. ζ denotes the direction normal to the surface, ρ is the density, M the Mach number and subscript 'e' refers to conditions at the outer edge of the boundary layer, $\zeta = \delta$. Following Green we also define $\bar{H}$ as

$$\bar{H} = \frac{\int_0^\delta \left(1 - \frac{U}{U_e}\right)\frac{\rho}{\rho e}}{\theta_{11}} d\zeta \qquad (20)$$

In order to derive relationships between the various crosswise and the streamwise profile integrals, we now assume explicit velocity profile families for the streamwise and crosswise flows. The streamwise velocity profile is assumed to be of the form suggested by Spence for two-dimensional flow

$$\frac{U}{U_e} = \left(\frac{z}{z_\delta}\right)^n \tag{21}$$

where
$$z = \int_0^\delta \frac{\rho}{\rho_e}\, d\zeta \quad \text{and} \quad z_\delta = \int_0^\delta \frac{\rho}{\rho_e}\, d\zeta \quad . \tag{22}$$

For incompressible flow Mager[7] has suggested the following form for the cross-flow velocity profile

$$\frac{V}{U_e} = \left(1 - \frac{\zeta}{\delta}\right)^2 a \frac{U}{U_e} \tag{23}$$

where $a = \tan \beta$ and β is the angle between an external streamline and the direction of the limiting streamline on the body surface. For compressible flow we assume that the Mager profile may be generalised as

$$\frac{V}{U_e} = \left(1 - \frac{z}{z_\delta}\right)^2 a \frac{U}{U_e} \quad . \tag{24}$$

Experimental support for the introduction of the correlating variable z/z_δ in equation (24) consists solely of the observation by Hall and Dickens[8] that such a change of variable made an already poor agreement between measured and predicted velocity profiles no worse.

With the assumption of the velocity profiles, (21) and (24), all the cross-flow thicknesses may be related to the streamwise momentum thickness θ_{11} by relations of the form

$$\left.\begin{aligned} \theta_{21} &= a f_1(\bar{H})\theta_{11} \\ \theta_{12} &= a f_2(\bar{H})\theta_{11} \\ \delta_2 &= a f_3(\bar{H})\theta_{11} \\ \theta_{22} &= a^2 f_4(\bar{H})\theta_{11} \quad . \end{aligned}\right\} \tag{25}$$

The functions f_1 to f_4 are identical with those derived by the present author[1] as functions of H for the incompressible flow and are listed in Appendix A.

We now combine the assumptions of Green and the present author[1] regarding the entrainment coefficient so that $C_E(H_1)$ is assumed to remain unchanged from two-dimensional flow and to be given by equation (4) the relation between $\bar{H}$ and H_1 by equation (6)* and the relation between H and $\bar{H}$ by equation (8). The streamwise skin-friction coefficient $\tau_{01}/\rho U_e^2 \equiv C_f/2$ is assumed to be given by equations (9) to (13). The cross-flow skin-friction coefficient is assumed to be given by

$$\frac{\tau_{02}}{\rho U_e^2} = a \frac{C_f}{2} \quad . \tag{26}$$

With the above assumptions and a given external flow (i.e. U_e and r known as functions of s and n) equations (14), (15) and (16) are reduced to a system of three simultaneous first-order partial differential equations with dependent variables θ_{11}, $\delta - \delta_1$ and a and independent variables s and n. The following sections give details of the process of solution of these equations for certain special cases.

4 SOLUTION FOR THE INFINITE YAWED WING

For an infinite yawed wing it was shown in Ref.1 that we may write $\partial/\partial s = (U_1/U_e)\ (d/dx)$ and $\partial/\partial n = (-\ V_1/U_e)\ (d/dx)$ where x denotes distance along the surface normal to the leading edge and U_1 and V_1 are the velocity components of the external stream in the x direction and parallel to the leading edge respectively. It was also shown that the term $\partial r/r\partial s$ may be written as $\partial r/r\partial s = (V_1^2/U_e^3)dU_1/dx$. With these relations the equations are reduced to a system of three first-order simultaneous ordinary differential equations, which may be solved numerically as an initial-value problem. The equations are

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* It might be thought that there is an inconsistency between equation (6) and equation (21), but there is no physical significance in this. Equation (8) is well matched to the other elements of the two-dimensional method (equations (4), (9), (10), (11)) and is therefore the better semi-empirical representation of the streamwise profile. Equation (21) is simply a convenient device, in combination with equation (24), for deriving the functions f of equation (25) with acceptable accuracy.

$$\left.\begin{aligned}
A_{11}\frac{d\theta_{11}}{dx} + A_{12}\frac{da}{dx} + A_{13}\frac{d\delta - \delta_1}{dx} &= \phi_1 \\
A_{21}\frac{d\theta_{11}}{dx} + A_{22}\frac{da}{dx} + A_{23}\frac{d\delta - \delta_1}{dx} &= \phi_2 \\
A_{31}\frac{d\theta_{11}}{dx} + A_{32}\frac{da}{dx} + A_{33}\frac{d\delta - \delta_1}{dx} &= \phi_3
\end{aligned}\right\} \tag{27}$$

where the coefficients are functions of θ_{11}, a, $\delta - \delta_1$, x and the external flow and are given in full in Appendix B.

These equations cannot be used at the attachment line of an infinite yawed wing since there $A_{11} = A_{13} = A_{21} = A_{22} = A_{23} = A_{31} = A_{33} = 0$. However, by symmetry we may say that at the attachment line $a = d\theta_{11}/dx = d(\delta - \delta_1)/dx = 0$, whilst attachment line values for da/dx, θ_{11} and $\delta - \delta_1$, may be obtained from the theory given in the next section. We thus start the calculation a small distance Δx away from the attachment line with starting values given by $(\theta_{11})_{al}$, $(da/dx)_{al}\Delta x$ and $(\delta - \delta_1)_{al}$ the subscripts denoting attachment line values.

For flows which are laminar at the attachment line and up to some transition position x_T we follow Cooke[9] and evaluate θ_{11} at the transition position from

$$\theta_{11}^2 = 0.45\nu_0\left(\frac{T_e}{T_0}\right)^{-3} U_1^{-2}U_e^{-4} \int_0^{x_T} \left(\frac{T_e}{T_0}\right)^{1.5} U_1 U_e^4 dx \quad . \tag{28}$$

θ_{11} is then assumed to be continuous at transition. For simplicity and in the absence of any experimental evidence to the contrary, the transition value of β is taken to be zero whilst the transition value of H and hence $\delta - \delta_1$ has to be specified for any particular case. Alternatively rather than attempt to compute the flow from the attachment line starting values of θ_{11}, and $\delta - \delta_1$ may be specified at any x station.

5 COMPRESSIBLE TURBULENT ATTACHMENT LINE FLOW ON AN INFINITE YAWED WING

This section is devoted to the solution of equations (14), (15) and (16) under the assumptions of section 2 in the special case of turbulent attachment line flow on an infinite yawed wing. The analysis follows closely that of

Cumpsty and Head[10] for the incompressible case and so only an outline will be given here.

Along the attachment line of an infinite yawed wing we may write

$$\frac{\partial}{\partial s} = \frac{\partial}{\partial y} = 0 \; , \quad \frac{\partial}{\partial n} = - \frac{d}{dx} \; ,$$

$$\theta_{12} = \theta_{22} = \theta_{21} = \delta_2 = \tau_{02} = \frac{\partial U_e}{U_e \partial n} = 0$$

With these simplifications (14), (15) and (16) reduce to

$$- \frac{d\theta_{12}}{dx} + \frac{1}{r}\frac{\partial r}{\partial s}\theta_{11} = \frac{\tau_{01}}{\rho U_e^2} = \frac{C_F}{2} \qquad (29)$$

$$\frac{d\theta_{22}}{dx} = 0 \qquad (30)$$

$$\frac{d\delta_2}{dx} + \frac{1}{r}\frac{\partial r}{\partial s}(\delta - \delta_1) = C_E \quad . \qquad (31)$$

A further equation is required and is derived by differentiating equation (15) with respect to x and then applying the simplifying assumptions pertaining to the infinite yawed wing attachment line. Cumpsty and Head[10] showed that it is convenient to introduce the dimensionless variable C^* defined as

$$C^* = \frac{V_1^2}{\nu_e \frac{dU_1}{dx}} = \frac{V_1}{\nu_e \frac{1}{r}\frac{\partial r}{\partial s}} \qquad (32)$$

where ν_e denotes the kinematic viscosity at the edge of the boundary layer. Hence we may write

$$\frac{1}{r}\frac{\partial r}{\partial s}\theta_{11} = \frac{V_1\theta_{11}}{\nu_e C^*} = \frac{R_{\theta_{11}}}{C^*} \; , \qquad (33)$$

where $R_{\theta_{11}}$ the streamwise momentum thickness Reynolds number is defined as $V_1\theta_{11}/\nu_e$ With the assumptions for velocity profiles and skin friction coefficient the same as those detailed in section 3.1 and recalling that $\beta = \tan^{-1} a$ the resulting equations are

$$- f_2\left(\theta_{11} \frac{d\beta}{dx}\right) + \frac{R_{\theta_{11}}}{C^*} = \frac{C_f}{2} \qquad (34)$$

$$f_3\left(\theta_{11} \frac{d\beta}{dx}\right) + H_1 \frac{R_{\theta_{11}}}{C^*} = C_E \qquad (35)$$

$$2f_4\left(\theta_{11} \frac{d\beta}{dx}\right)^2 - \left[3f_1 \frac{R_{\theta_{11}}}{C^*} - \frac{C_f}{2}\right]\left(\theta_{11} \frac{d\beta}{dx}\right) + (H + 1)\left(\frac{R_{\theta_{11}}}{C^*}\right)^2 = 0 \quad . \qquad (36)$$

Equations (34), (35) and (36) appear, when due allowance is made for slight differences in notation, to be identical with those given by Cumpsty and Head. However in the present case the functions f_1 to f_4, C_f and C_E all have an implicit variation with Mach number as they are now functions of $\bar{H}$ rather than H. The technique adopted for the solution of equations (34), (35) and (36) was first to substitute for $\theta_{11}\, d\beta/dx$ from (34) into (35) and (36) and then for a given Mach number and C^* to solve the resulting pair of non-linear simultaneous equations for the unknowns $R_{\theta_{11}}$ and $\bar{H}$ by means of a generalised Newton-Raphson procedure. The results for a range of Mach number and C^* are shown in Fig.1.

6 COMPARISON OF THEORY FOR AN INFINITE YAWED WING WITH EXPERIMENT

The incompressible version[1,2] of the method has been compared with experiments by Cumpsty and Head[15]. They obtained quite tolerable agreement once due allowance was made for the departure in the experiment from true infinite yawed-wing conditions. There is little experimental evidence available for boundary-layer development on swept wings in compressible flow, and even in the example shown the tests were made at a Mach number of 0.55 so that the effects of compressibility are not very large. This example is a 55 degree swept wing tested in the RAE 8ft × 8ft wind tunnel. The model was designed by Lock[11], using sonic theory, to have a rooftop upper surface pressure distribution with isobars following lines of constant percentage chord at a lift coefficient of approximately 0.2. A yawmeter rake was mounted near the trailing edge of the model at the position shown in Fig.2 and this yielded results for the various boundary-layer parameters at that station over a range of Reynolds numbers, Mach numbers and model incidence.

These measurements do not in themselves provide enough information for a direct comparison between experiment and theory since, as is mentioned above, we are dealing with an initial-value problem and in order to provide predictions of boundary-layer parameters at the measuring station we must know values of these parameters at some station upstream. Initial values were estimated by an indirect use of measurements of skin friction by the razor-blade technique which were made at various chordwise stations. At a model incidence of 2^o the pressure distribution was of a roof-top nature as shown in Fig.3 and it was assumed that along the flat roof-top the boundary layer would behave as though it were on a flat plate. With this assumption it was possible to take the measured value of skin friction at 10% chord and substitute this into equations (10) to (13) to obtain starting values of θ_{11} and $\bar{H}$. The values of $\bar{H}$ and θ_{11} were then inserted into equations (7) and (8) to yield initial values of $\delta - \delta_1$ and H. The initial value of $a = \tan \beta$ was taken to be zero. With these initial values the theory of the previous section was then used to produce the results shown in Fig.4.

In view of the uncertainty regarding the initial conditions and the fact that the wing was not the infinite yawed wing assumed by the theory it is felt that the agreement between experiment and theory shown in the figures is good but may well be fortuitous. Cumpsty and Head[15], for example found that their predictions were very sensitive to the effective sweep assumed, and only small changes in this were needed to substantially improve agreement with their experimental results.

Attempts to calculate the flow right from the attachment line were not so successful. Even with transition assumed at the leading edge the values of θ_{11} and β at the measuring station were underestimated by some 10%-25%. This discrepancy is thought to be caused by the attachment-line conditions for this model not being those of an infinite yawed wing.

7 DISCUSSION AND CONCLUSIONS

Although the greement between experiment and theory reported herein is encouraging it is clear that further experimental checks are required. It is hoped that some recent boundary-layer measurements upon a 30^o swept wing in the 8ft × 8ft wind tunnel at RAE Bedford will go some way towards filling this gap.

Extension of the method to the general three-dimensional case, in which derivatives normal to the direction of integration are accounted for by finite differences rather than analytically as was done here, has recently been achieved by Myring[12]. Myring has at the same time generalised the coordinate system used so that it need no longer be orthogonal. (See also Smith[17].)

Although intuitively one feels that in the long term the future of calculation methods for turbulent boundary layers must lie with the differential methods such as Bradshaw's[13] rather than the integral approach adopted here, at present the advantages of the differential methods are not so clear cut. In two dimensions the most recent development[14] of Head's methods compares favourably in accuracy with Bradshaw's method whilst having a decided advantage in speed of computation. In the general three-dimensional case the generality in the choice of coordinate system offered by Myring's method is a considerable advantage in practical cases whilst the inherent speed advantage of integral methods may well be a major consideration in any design procedure involving an iteration between the boundary layer and the external flow.

In the light of the foregoing it would appear worthwhile to attempt to improve the present method and its extension due to Myring. Two obvious points needing attention are the inclusion of some 'upstream history' effect into the entrainment relation as has been done for the two-dimensional case by Head and Patel[14] and an improvement in the crossflow velocity profile family beyond the simple one used here which does not permit the crossflow to change sign within the boundary layer. On this later point a recent investigation by Klinksiek and Pierce[16] indicates that none of the existing crossflow velocity profile models is really satisfactory and further effort in this direction is required.

Appendix A

THE FUNCTIONS f_1 TO f_4

The functions of $\bar{H}$; f_1, f_2, f_3 and f_4 used in equation (25), and derived from the streamwise and crossflow velocity profile families of equations (21) and (24), are listed below.

$$f_1 = -2/[(\bar{H} - 1)(\bar{H} + 2)]$$

$$f_2 = (14\bar{H} + 30)/[(\bar{H} + 2)(\bar{H} + 3)(\bar{H} + 5)]$$

$$f_3 = -16\bar{H}/[(\bar{H} - 1)(\bar{H} + 3)(\bar{H} + 5)]$$

$$f_4 = -24/[(\bar{H} - 1)(\bar{H} + 2)(\bar{H} + 3)(\bar{H} + 4)]$$

$$f_2 \equiv f_1 - f_3 \quad .$$

Appendix B

THE EQUATIONS USED FOR THE INFINITE YAWED WING PROGRAM

The three first-order simultaneous ordinary differential equations to be solved numerically in this case are

$$A_{11}\frac{d\theta_{11}}{dx} + A_{12}\frac{da}{dx} + A_{13}\frac{d\delta - \delta_1}{dx} = \phi_1$$

$$A_{21}\frac{d\theta_{11}}{dx} + A_{22}\frac{da}{dx} + A_{23}\frac{d\delta - \delta_1}{dx} = \phi_2$$

$$A_{31}\frac{d\theta_{11}}{dx} + A_{32}\frac{da}{dx} + A_{33}\frac{d\delta - \delta_1}{dx} = \phi_3$$

where

$$A_{11} = \frac{U_1}{U_e} - \frac{V_1}{U_e} f_2 a + \frac{V_1}{U_e} a \frac{df_2}{d\bar{H}} \frac{d\bar{H}}{dH_1} H_1$$

$$A_{12} = -\frac{V_1}{U_e} f_2 \theta_{11}$$

$$A_{13} = -\frac{V_1}{U_e} a \frac{df_2}{d\bar{H}} \frac{d\bar{H}}{dH_1}$$

$$A_{21} = \frac{U_1}{U_e} f_1 a - \frac{V_1}{U_e} f_4 a^2 - H_1 \frac{d\bar{H}}{dH_1}\left(\frac{U_1}{U_e} a \frac{df_1}{d\bar{H}} - \frac{V_1}{U_e} a^2 \frac{df_4}{d\bar{H}}\right)$$

$$A_{22} = \frac{U_1}{U_e} f_1 \theta_{11} - 2\frac{V_1}{U_e} f_4 \theta_{11} a$$

$$A_{23} = \frac{U_1}{U_e} a \frac{df_1}{d\bar{H}} \frac{d\bar{H}}{dH_1} - \frac{V_1}{U_e} a^2 \frac{df_4}{d\bar{H}} \frac{d\bar{H}}{dH_1}$$

$$A_{31} = \frac{V_1}{U_e} f_3 a - \frac{V_1}{U_e} a \frac{df_3}{d\bar{H}} \frac{d\bar{H}}{dH_1} H_1$$

$$A_{32} = \frac{V_1}{U_e} f_3 \theta_{11}$$

$$A_{33} = \frac{U_1}{U_e} + \frac{V_1}{U_e} a \frac{df_3}{d\bar{H}} \frac{d\bar{H}}{dH_1}$$

$$\phi_1 = \frac{C_f}{2} - \frac{U_1}{U_e^2}\theta_{11}\left[(H + 2 - M_e^2) + \frac{V_1^2}{U_1^2}(1 - f_4 a_2^2) + M_e^2 f_2 a \frac{V_1}{U_1}\right]\frac{dU_e}{dx}$$

$$\phi_2 = a\frac{C_f}{2} + \frac{U_1}{U_e^2}\theta_{11}\left[af_1\left(M_e^2 - 2\frac{U_e^2}{U_1^2}\right) + \frac{V_1}{U_1}(H + 1 + f_4 a^2(1 - M_e^2))\right]\frac{dU_e}{dx}$$

$$\phi_3 = F(H_1) + \frac{U_1}{U_e^2}\theta_{11}\left[H_1\left(M_e^2 - \frac{U_e^2}{U_1^2}\right) + M_e^2 a f_3\right]\frac{dU_e}{dx} \quad .$$

SYMBOLS

A	coefficient matrix, equation (27)
a	tan β
C*	attachment line parameter defined by equation (32)
$C_f = \tau_{01}/\frac{1}{2}\rho U_e^2$	local skin friction coefficient
C_E	entrainment coefficient defined by equation (4)
F_C	compressibility factor defined by equation (12)
F_R	compressibility factor defined by equation (13)
f_1, f_2, f_3, f_4	functions defined in Appendix A
H, $\bar{H}$, H_1	shape factors defined by equations (18), (19) and (20)
M	local Mach number
R	Reynolds number
r	$-\partial r/r\partial s$ is the geodesic curvature of the lines s = constant
s, n	coordinates along and normal to an external streamline respectively
T	absolute temperature
U, V	velocity components in the directions s and n respectively
U_1, V_1	external flow velocity components in the directions x and y respectively
x, y	coordinates on wing surface normal and parallel to leading edge respectively
z	correlating variable defined by equation (22)
β	angle between external and limiting streamline
γ	ratio of specific heats
δ	boundary layer thickness
ζ	distance normal to wing surface
θ_{11}, θ_{12}, θ_{21}, θ_{22}, δ_1, δ_2	thickness defined by equation (20)
ν	kinematic viscosity of fluid
ρ	density of fluid
τ_{01}, τ_{02}	surface shear stress components in directions s and n respectively

Subscripts

e	denotes conditions at the edge of the boundary layer
p	denotes flat plate conditions
r	denotes recovery conditions
0	denotes stagnation conditions

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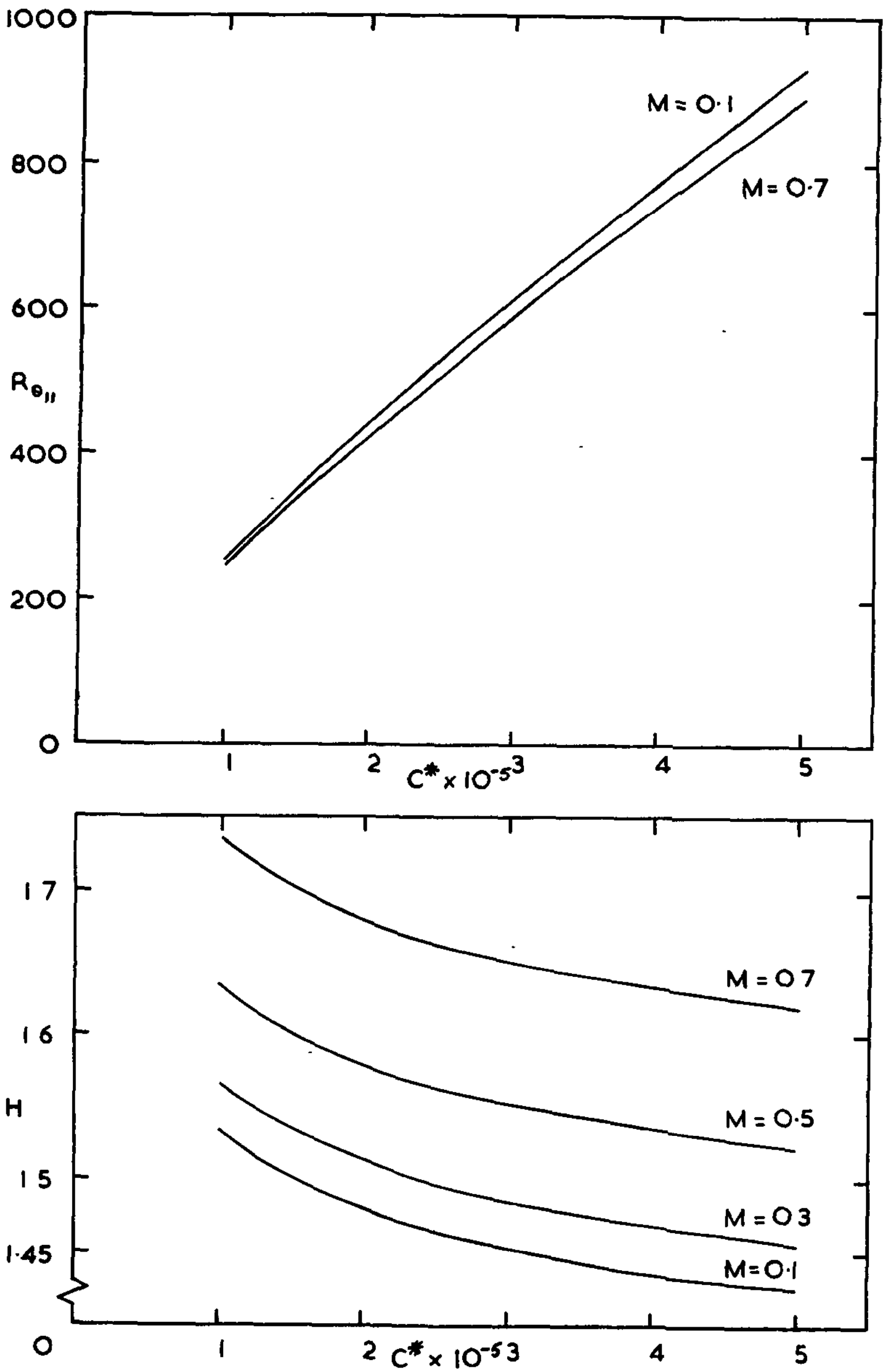


Fig.1 Variation of attachment line values for momentum thickness Reynolds number and form parameter with C^* at various Mach numbers along the attachment line

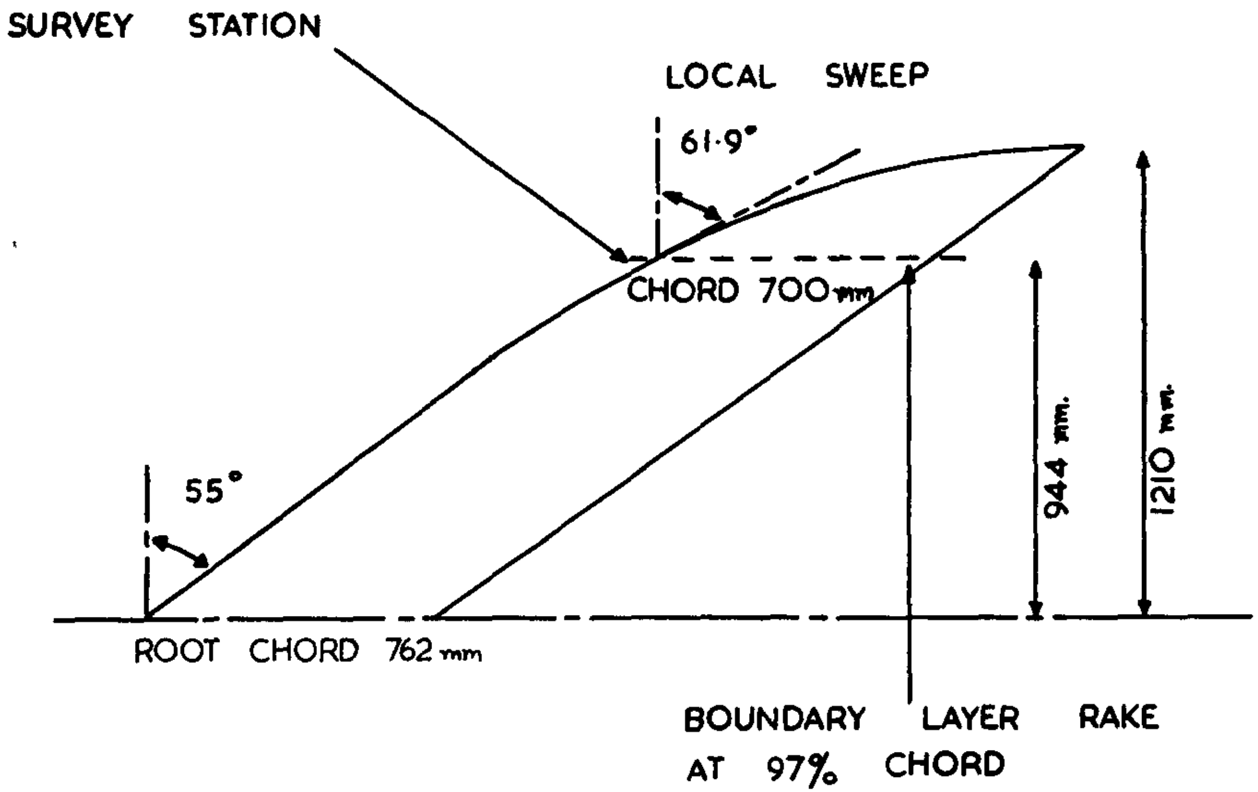


Fig.2 55° swept wing model used in boundary layer survey

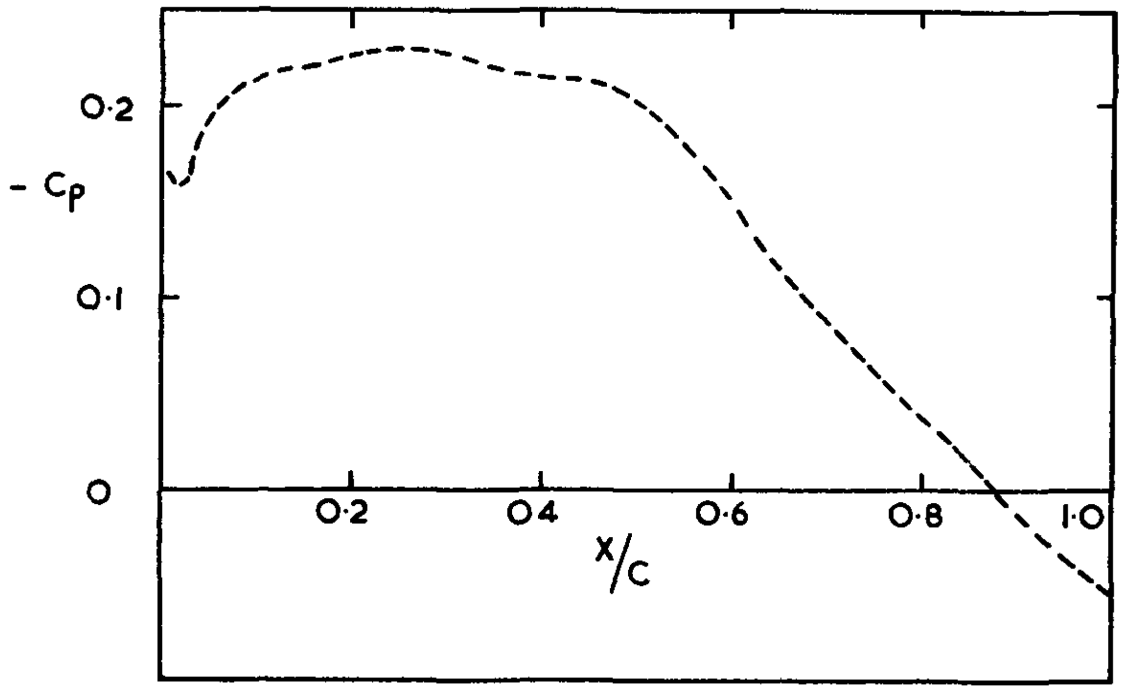


Fig.3 Measured pressure distribution on 55° swept wing model at $M = 0.55$, $\alpha = 2^o$, $R_c = 27 \times 10^6$

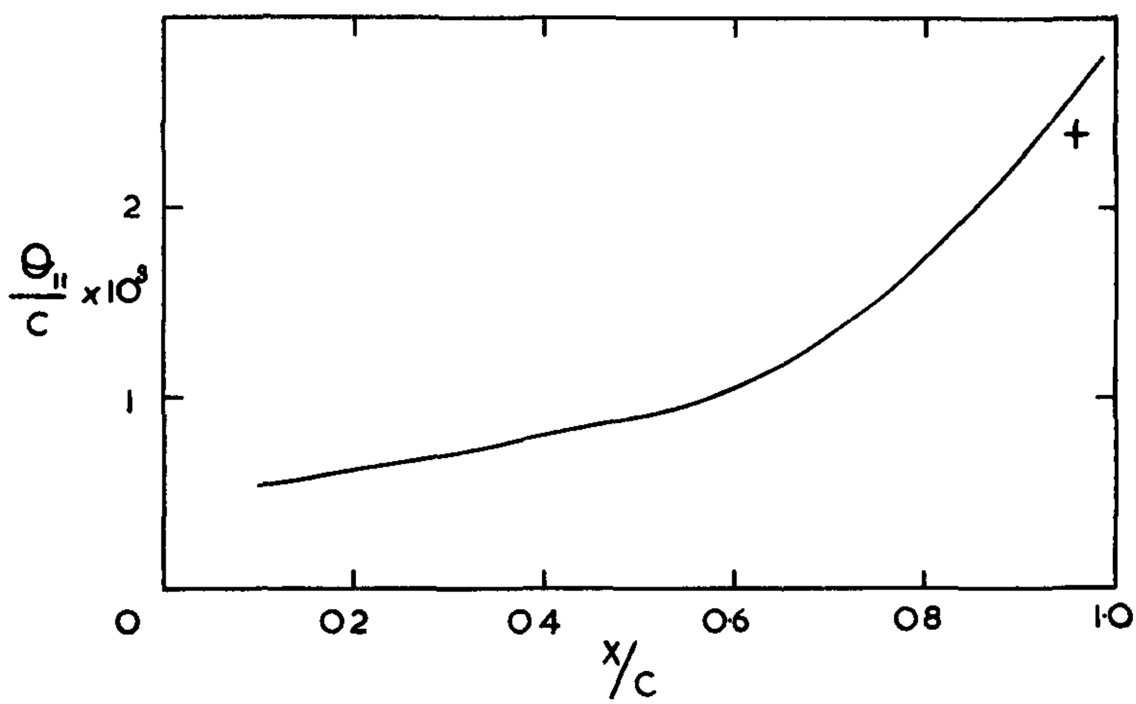
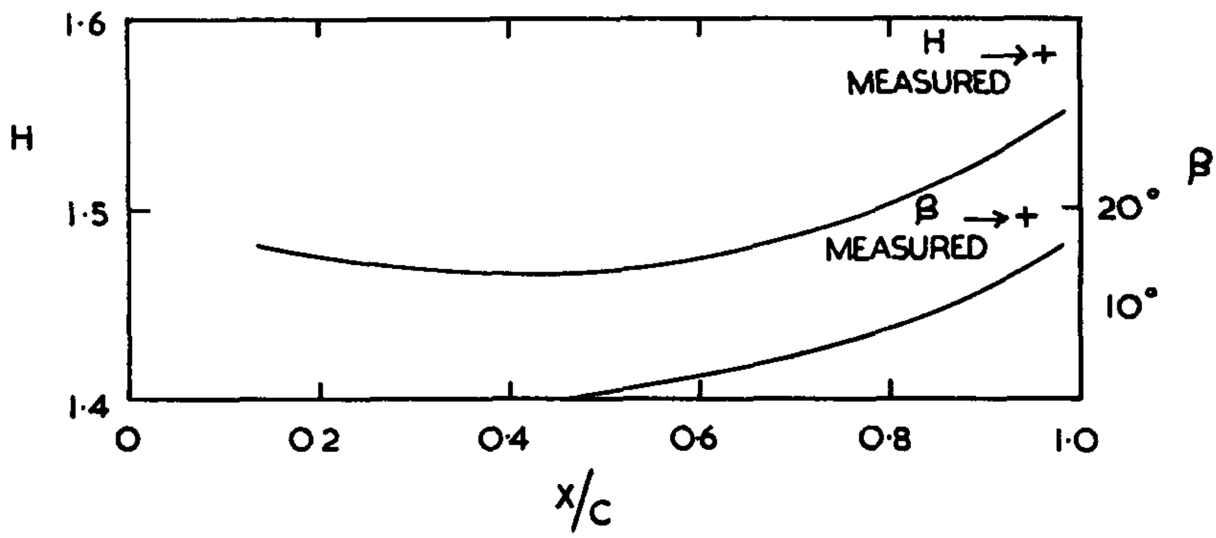


Fig.4a Comparison between measured and predicted values of boundary layer parameters on 55° swept wing model at $M = 0.55$, $\alpha = 2^o$, $R_c = 27 \times 10^6$



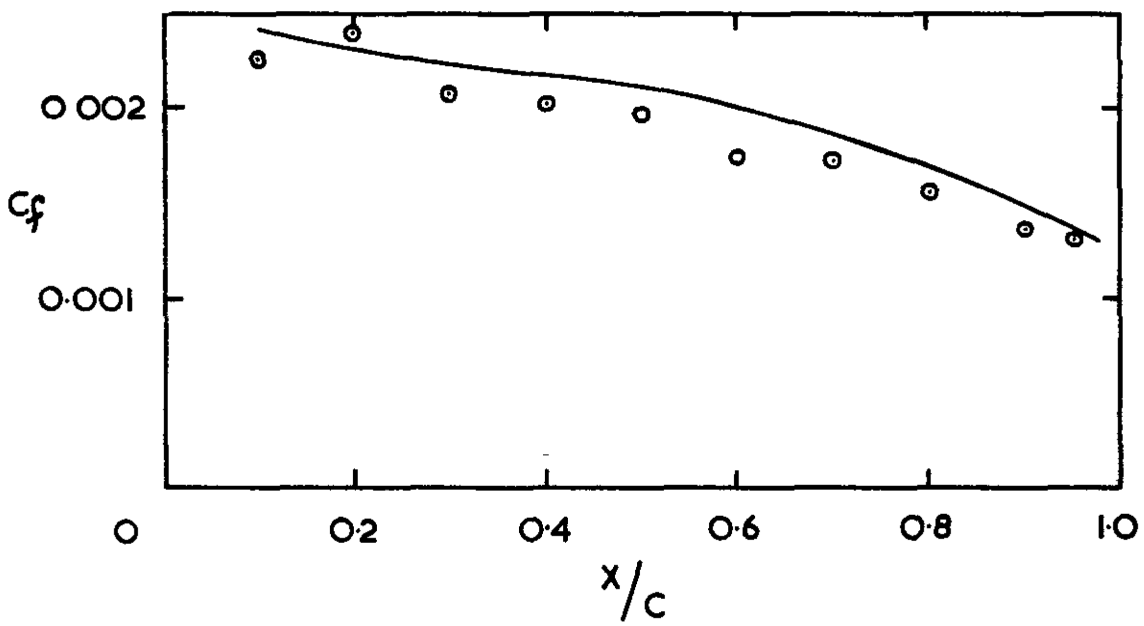


Fig.4b Comparison between measured and predicted values of boundary layer parameters on 55° swept wing model at $M = 0.55$, $\alpha = 2^o$, $R_c = 27 \times 10^6$

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