C.P. No. 273
(13,803)
A.R.C. Technical Report

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The Hot-Wire Anemometer for Turbulence Measurements

Part I

by

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1956

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The Hot-wire Anemometer for Turbulence Measurements
Part I.

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O.U.E.L.52

Presented by Prof. A. Thom.

10th February, 1951

SUMMARY

An equivalent circuit is developed for the hot-wire anemometer, as used for turbulence measurements. Improvements in the frequency response, which can be produced by feedback systems, and by the use of radio-frequency heating, are also described.

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1. Introduction

When a hot-wire is used for turbulence measurements, it is usual to heat it by means of a constant direct current. Velocity variations cause the temperature of the wire to change, and therefore its resistance. The constant current flowing through the varying resistance causes a voltage which, for small variations and low frequencies, may be taken as indicating the turbulence.

At/

At higher frequencies, however, the thermal lag of the wire causes distortion, and it is well known that the effect is that of a simple time-constant. (Ref.1). The falling response can be compensated by a condenser-resistance time-constant of equal magnitude, but this has to be changed with any change in steady velocity or direct current. It has been proposed (Refs.2,3), by means of positive feedback, to reduce the temperature variations, and hence the inherent time-constant.

In Section 2 a simple equivalent circuit is described, from which it is easy to see what is the effect of positive feedback, and what is required in the design of such a system. It may, however, be expected that in practice difficulty will be experienced in keeping stable any positive-feedback system which gives a large reduction in the time-constant.

In Section 3 it is suggested that, if radio-frequency current heating is used instead of direct current, the positive feedback is more easily arranged, and the instability difficulty does not arise. An approximate equivalent circuit for this system is found, from which the frequency response may be predicted.

In Section 4, a method of testing the radio-frequency system by means of a direct-current input signal, simulating velocity changes, is described.

In Section 5 it is suggested that, if both direct-current heating and radio-frequency heating are employed, negative feedback may be used to improve the frequency response still further.

2. The Equivalent Circuit for Direct Current Heating.

If the temperature of the wire at any instant is θ, and the air temperature is θ_a, and the corresponding resistances are R, R_a, we have the approximate relation

$$\theta - \theta_a = \alpha (R - R_a) \qquad \ldots(1)$$

where α is a constant depending on the wire.

If the current flowing at any instant is i, heat is being supplied at a rate i^2R, and we may assume that heat is being lost at a rate $\phi(\theta,V)$, where V is the air velocity, and ϕ is an unknown function. It follows that

$$W \frac{d\theta}{dt} = i^2R - \phi(\theta,V) \qquad \ldots(2)$$

where W is the water-equivalent of the wire.

From (1) and (2) we may write

$$\lambda \frac{dR}{dt} = i^2R - f(R,V) \qquad \ldots(3)$$

where $\lambda = \alpha W$, and $f(R,V) = \phi(\theta,V)$.

Now/

Now suppose that the temperature is steady at θ_1 when the velocity is V_1, and the current is i_1, and that the corresponding resistance is R_1. If we let $V = V_1 + \Delta V$, $i = i_1 + \Delta i$, and $R = R_1 + \Delta R$, we have, from (3), neglecting higher powers of ΔV, Δi and ΔR,

$$\lambda \frac{d(\Delta R)}{dt} = 2i_1R_1\Delta i + i_1^2\Delta R - \frac{Jf}{JR_1}\Delta R - \frac{Jf}{JV_1}\Delta V, \qquad \ldots(4)$$

Writing $i_1\Delta R = v_c$, and dividing by $2i_1R_1$, we have

$$\frac{1}{2i_1^2R_1}\left\{\lambda \frac{dv_c}{dt} + \left(\frac{Jf}{JR_1} - i_1^2\right)v_c\right\} = \Delta i - \frac{1}{2i_1R_1}\frac{Jf}{JV_1}\Delta V. \qquad \ldots(5)$$

$$\text{With } C = \frac{\lambda}{2i_1^2R_1}, \quad \frac{1}{R_b} = \frac{1}{2i_1^2R_1}\left(\frac{Jf}{JR_1} - i_1^2\right), \text{ and } i_a = -\frac{1}{2i_1R_1}\frac{Jf}{JV_1}\Delta V \qquad \ldots(6)$$

$$\text{we get } C\frac{dv_c}{dt} + \frac{v_c}{R_b} = \Delta i + i_a. \qquad \ldots(7)$$

In this equation we may regard v_c as a voltage response to the current generators i_a and Δi, as shown in Fig.(1).

If we now let $Ri = v$, and $R_1i_1 = v_1$, and $v = v_1 + \Delta v$, we have, approximately, $\Delta v = i_1\Delta R + R_1\Delta i = v_c + R_1\Delta i$. The circuit shown in Fig.(1) can thus be modified to give the voltage response Δv, as shown in Fig.(2). This equivalent circuit is interpreted as follows: given any change in the velocity ΔV, we have an equivalent current generator i_a given by (6); given also a current change Δi, the resulting voltage change is Δv.

2.1. The Heat-Loss Function

If it is assumed that the rate of losing heat is given by

$$\phi(\theta, V) = (\theta - \theta_a)\psi(V),$$

we can easily shew that equations (6) become

$$C = \frac{\lambda}{2i_1^2R_1}, \quad \frac{1}{R_b} = \frac{R_a}{2(R_1 - R_a)R_1}, \text{ and } i_a = -\frac{i_1}{2}\frac{\psi'(V_1)}{\psi(V_1)}\Delta V. \qquad \ldots(8)$$

Evidently R_b is typically of the same order as R_1, and is equal to it when $R_1 = 1.5R_a$.

For completeness it should be mentioned here that if it is further assumed, as suggested by King, that $\psi(V) = a + b\sqrt{V}$, the expression for i_a becomes

$$i_a = -\frac{i_1}{4b}\frac{\Delta V}{a\sqrt{V_1} + bV_1}. \qquad \ldots(9)$$

2.2. Constant-current Heating

If the wire is heated by a constant direct current, $\Delta i = 0$, and the equivalent circuit simplifies to that shown in Fig.(3). The time-constant under these conditions, T_c say, is evidently CR_b.

2.3./

2.3. Heating by a General Linear Circuit

If the wire is heated by connection in a direct-current circuit containing linear elements, we may write

$$v = v_0 - R_s i \qquad \ldots(10)$$

and the operating values V_1, i_1, are given by the intersections of this line with the hot-wire characteristic, as shown in Fig.(4). The wire may be regarded as being supplied with current from a constant voltage v_0 through a resistance R_s.

From (10), $\Delta v = - R_s \Delta i \qquad \ldots(11)$

so that the general equivalent circuit of Fig.(2) is here to be modified as shown in Fig.(5). The time-constant is now $\dfrac{CR_b(R_1 + R_s)}{R_b + R_1 + R_s}$.

Evidently the time-constant would be much reduced if R_s could be made negative, and $R_1 + R_s$ small. This can be done by positive feedback, as shown in Fig.(6). Here the constant voltage v_0 and the output of a d-c amplifier of gain m are connected in series to the hot wire H and a resistance r. The voltage across r is connected to the input terminals of the amplifier, so that $v_0 + mir - (r + R_m)i = v$, where R_m is the output resistance of the amplifier. We can write $v = v_0 + [(m - 1)r + R_m]i = v_0 + R_c i$, say. This gives an intersection with the hot-wire characteristics as shown in Fig.(7). Evidently the smaller v_0 is the smaller will be $R_1 - R_c$, and the greater will be the improvement achieved.

The ratio $(R_b + R_1 - R_c)/(R_1 - R_c)$ we may call the improvement factor, being the ratio of the time-constant under constant-current conditions to the new time-constant.

The value of R_c can be made much less dependent on the amplifier gain if additional feedback from v is employed, in some such manner as is shown in Fig.(8). Here the two resistances W are much larger than r or the hot-wire resistance, so that the input to the amplifier is nearly $(ri - v)/2$, and we have

$$v = \frac{2v_0}{2 + m} + \frac{(m - 2)r - 2Rm}{2 + m} . i \, ,$$

so that $R_c = \dfrac{(m - 2)r - 2Rm}{m + 2}$, and the larger m is the more independent of m does this become.

There are various ways in which the method indicated here may be attempted in practice, but the difficulties associated with direct-current amplifiers make it doubtful whether really large improvement factors could be achieved.

It might be easier to use an a.c. amplifier instead, and, with a method of testing the system, as described in Section 4, there seems no necessity for retaining d.c. amplification.

3. Radio-frequency Heating

Consider the circuit shown in Fig.9, which is that of a tuned regenerative amplifier. It behaves in exactly the manner indicated in Fig.6; a constant voltage v_0 from the oscillator is induced in the tuned circuit, and a voltage is fed back which is proportional to the current flowing through the hot wire R. There is a phase-shift of 90° between the current and the condenser voltage, and a compensating phase-shift between the current in the valve and the feed-back voltage. This circuit thus fulfils the feedback conditions required for improvement of the hot-wire frequency response, as described in Section 2. The greater the degree of feedback the greater the improvement, until there is so much feedback that the circuit becomes self-oscillatory. The input oscillator can then be removed, and we have an arrangement in which resistance, and therefore temperature variations may be expected to have been reduced to a minimum. In practice, in order to produce sufficient r.f. current to heat the wire, it is better to have a tuned-anode arrangement, as shown in Fig.10.

3.1. The Equivalent Circuit with Radio-frequency Heating

If we assume that the r.f. current used is of sufficiently high frequency to give a substantially constant heating effect in the wire, the analysis of Section 2 may be followed through exactly, substituting R.M.S. r.f. current for the direct current, and the same equivalent circuit results. The only modification will be that the magnitude of the r.f. resistance will differ from the d.c. resistance.

It remains now to find a relation between Δv and Δi, as was done in Section 2.3, enabling a circuit similar to Fig.5 to be deduced from the general circuit of Fig.2. This may be done by considering the energy changes which take place in a radio-frequency cycle.

Suppose, as before, the equilibrium values of R and i (which is now R.M.S. r.f. current) to be R_1 and i_1, and $R = R_1 + \Delta R$, $i = i_1 + \Delta i$ at a given instant of time. Consider this instant of time to be taken when the r.f. current has its maximum value, so that the energy stored in the inductance is Li^2. When one half of a radio-frequency cycle has elapsed, say time τ, the energy stored will be approximately $L\left(i^2 + 2i\frac{di}{dt}\tau\right)$, and the increase of energy $2Li\frac{di}{dt}\tau$ will be equal to $v_0 i\tau - (R - R_0)i^2\tau$, approximately.

$$\text{Thus} \qquad 2L\frac{di}{dt} = v_0 - (R - R_c)i \qquad \ldots(12)$$

Under equilibrium conditions this becomes $0 = v_0 - (R_1 - R_c)i_1$,

$$\text{so that} \qquad 2L\frac{d\Delta i}{dt} = -i_1\Delta R - (R_1 - R_0)\Delta i,$$

$$\text{or} \qquad \Delta v = R_c\Delta i - 2L\frac{d\Delta i}{dt} \qquad \ldots(13)$$

This may be compared with the equation (11) of Section 2.3. Evidently the only change required in the equivalent circuit of Fig.5, according to this approximate theory, is to include an inductance 2L as shown in Fig.11.

3.2./

3.2. The Frequency Response with Radio-frequency Heating.

In Fig. 11, either Δv or Δi may be regarded as the response to the velocity variations. In practice, the more convenient to use is Δi, since the r.f. voltage across the tuned circuit, which is proportional to it, is much larger than the voltage across the resistance. The r.f. voltage may be demodulated in any conventional manner.

The results of an analysis of Fig. 11, to find the frequency response, are as follows: if $\frac{1}{LC} - \frac{1}{R_b^2C^2} - \frac{(R_1 - R_c)^2}{4L^2} = \omega_1^2$ say, is positive, the response at the frequency $\frac{\omega 1}{2\pi}$ is the same as at zero frequency. The response at a frequency $\frac{\omega}{2\pi}$ is $10 \log_{10}\left[1 + B^2\omega^2(\omega^2-\omega_1^2)\right]$ decibels below the zero-frequency response, where $B = 2LCR_b/(R_b + R_1 - R_c)$. At the frequency $\frac{\omega 1}{2\pi\sqrt{2}}$, a maximum occurs, where the response is $-10 \log_{10}\left(1 - \frac{B^2}{4}\omega_1^4\right)$ decibels above the zero-frequency response.

Detailed discussion of these results is deferred to a later paper, in which it is hoped to include experimental data. However, some curves are given in Fig. 12 to indicate the sort of results to be expected in a practical case. The time-constant T_c has been taken as one millisecond, and R_b as 10 ohms, giving $C = 100\mu F$.

Curve 1 shows the response under constant-current conditions. Curve 2 shows the response which would be obtained in a d.c. feedback system with $R_1 - R_c = 0.2$ ohms, though it is most unlikely that such a small value could be stably obtained. Curve 3 shows the response in a radio-frequency system, also with $R_1 - R_c = 0.2$ ohms, and $L = 5\mu H$. Curve 4 shows what happens in this r.f. system when $R_1 - R_c = 0$, i.e., the circuit is self-oscillatory.

Actually, in the self-oscillatory system, the effective value of $R_1 - R_c$ is not necessarily zero, and a small deviation from zero has a considerable effect. If the effective value is in fact negative, an audio-frequency oscillation may develop. However, any excessive peaking of the response curve shows up clearly when the system is monitored by a rectangular-wave testing signal, as described in Section 4, and can thus be avoided.

4. Testing

Suppose that a wire is heated both with a direct current i_d and a radio frequency current of R.M.S. value i_r. Let the d.c. resistance be R_d and the r.f. resistance R_r, and let the other variables be distinguished by similar subscripts. Then the basic equivalent circuit, corresponding to Fig. 1, becomes Fig. 13, where, corresponding to equations (6), we have

$$C_x = \frac{\lambda_x}{2i_{x1}^2R_x},\ \frac{1}{R_{xb}} = \frac{1}{2i_{x1}^2R_{x1}}\left[\frac{\partial f_x}{\partial R_{x1}} - i_{x1}^2 - \frac{R_{y1}}{R_{x1}}\, i_{y1}^2\right], \text{ and } i_{xa} = -\frac{1}{2i_{x1}^2R_{x1}}\,\frac{\partial f_x}{\partial V_1}\,\Delta V \quad \ldots(14)$$

If/

If the direct current variations are taken to be the response, we set $x = d$, and $y = r$, while if the r.f. current variations are the response we set $x = r$, and $y = d$.

We may conclude from Fig.13 that when we are using a radio-frequency system, variations in direct current heating the wire will simulate velocity variations. Similarly, with a d.c. system, modulation of a radio-frequency heating current will simulate a variable velocity. This provides a means of testing either system. The testing signal may be sinusoidal, in which case the response at various frequencies is found, or it may be a rectangular wave, when the response is displayed on a cathode-ray oscilloscope.

The foregoing theory assumes variations in the direct current small compared with the mean direct current, and similarly for the R.M.S. radio-frequency current. However, when testing a radio-frequency system it is unnecessary to have a steady direct current. Suppose the direct current, always small compared with the R.M.S. r.f. current is i_d at any instant, and the average value of i_d^2 is $\overline{i_d^2}$. Then the equivalent circuit is exactly as in Fig.1, with parameters corresponding to the r.f. system, with an additional current generator in parallel with i_a giving a current $(i_d^2 - \overline{i_d^2}) \dfrac{R_{d1}}{2 i_{r1} R_{r1}}$. If the testing signal is sinusoidal, say $i_d = I_d \sin \omega t$, $i_d^2 - \overline{i_d^2} = -\dfrac{I_d^2}{2} \cos 2\omega t$, so that this input simulates velocity variations at twice the frequency. When using a wave-analyser for measuring the frequency response, there is a considerable advantage in the fact that the output signal is at twice the frequency of the testing signal, as voltage pick-up difficulties, which arise particularly at high audio-frequencies when the system is attenuating, are thereby much reduced.

A small rectangular input signal, having a non-zero average value, will simulate sudden velocity changes, and, with the response displayed on a cathode-ray oscilloscope, this enables rapid monitoring and adjustment for the best results.

Any d.c. system may be similarly tested by a modulated r.f. input, or by the injection of r.f. pulses.

5. <u>Negative Feedback</u>.

It is evident from Fig.13 that when two types of heating are used the possibility arises of overall negative feedback, since, whichever of the heating agencies is regarded as giving the response, variations in the other correspond to some equivalent velocity variations.

For example, if the direct current is kept constant, as in Section 2.2 we could use the voltage variations to modulate a radio-frequency current which also heats the wire. The equivalent circuit would then be as in Fig.14, where M is a modulator (compare Fig.3). This can easily be shown to be equivalent to Fig.15, so that the time-constant has been reduced by feedback from CR_b to $CR_b/(1 + R_b g)$. It is noteworthy that here the magnitude of the response is reduced by the overall negative feedback, whereas with the positive feedback considered earlier the response may be increased.

It/

It should be made quite clear that the type of feedback considered in this Section is quite different from that of Sections 2 and 3. There the feedback connections only concerned the voltage and current variations, and no signal was produced which corresponded with a velocity variation. The feedback was thus internal to the system, and was unconnected with the input signal. It was not in fact feedback in the normal sense, so that we had the apparently unlikely result that positive feedback increased both the magnitude of the response and the bandwidth. With two types of heating, however, we may indeed have feedback in the usual sense, since we can then produce a feedback signal to add to the input signal.

If a radio-frequency system is used, it should be possible to improve the frequency response by a negative feedback, using direct-current heating for this purpose, and there is no reason why a testing signal should not be superimposed on this direct current in addition. Owing to the more complicated equivalent circuit with r.f. heating, however, the application of negative feedback is not so straightforward as with d.c. heating, and detailed consideration of this matter is deferred to a later paper, in which it is hoped some experimental results may also be given.

6. Conclusion.

It is hoped that the equivalent circuit developed in this paper clarifies the operation of the hot-wire anemometer for turbulence measurements, and that the use of radio-frequency heating will provide a means of obtaining an improved frequency response.

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FIG. 1

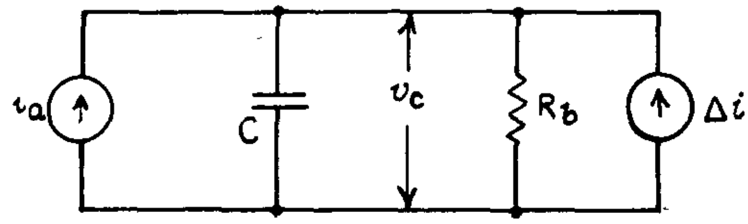

FIG. 2

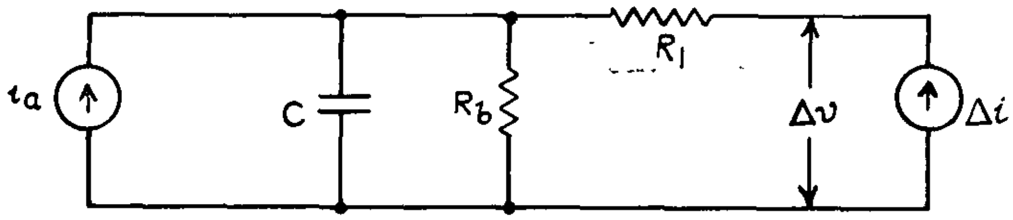

FIG. 3

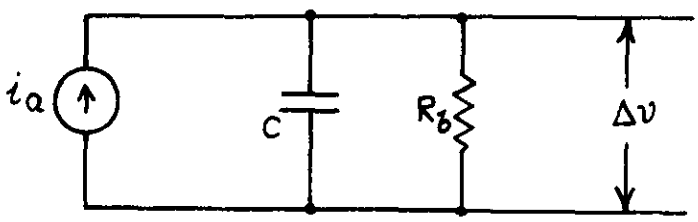

FIG 4.

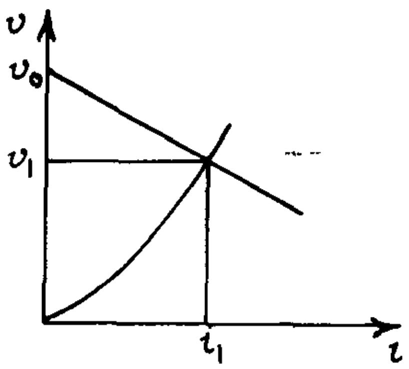

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FIG. 5

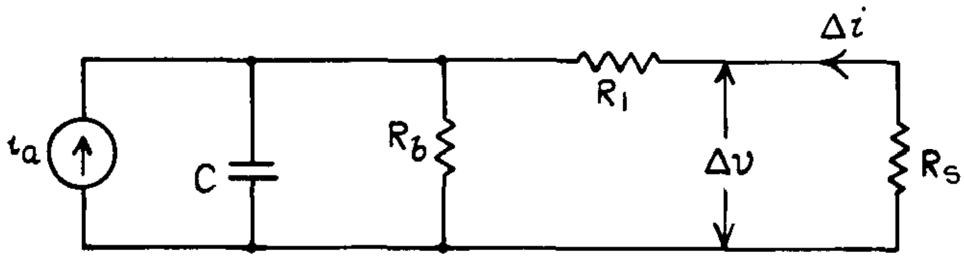

FIG 6

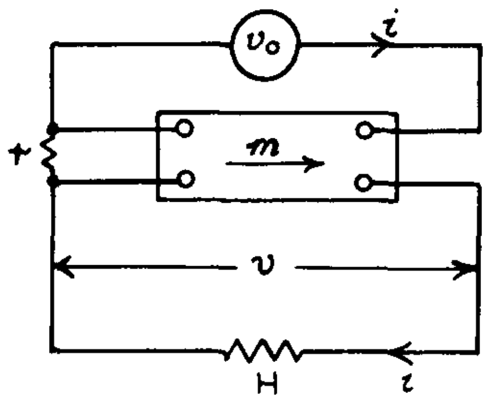

FIG 7

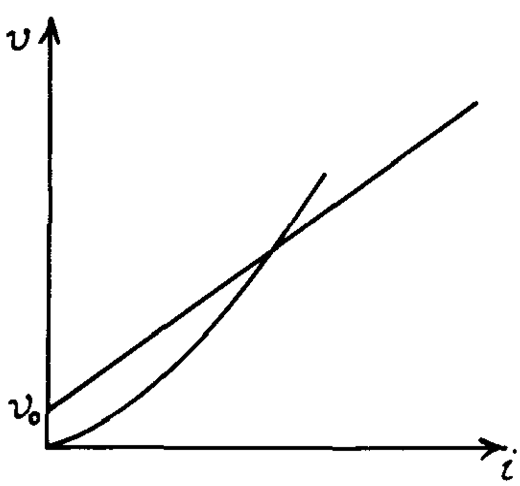

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FIG. 8

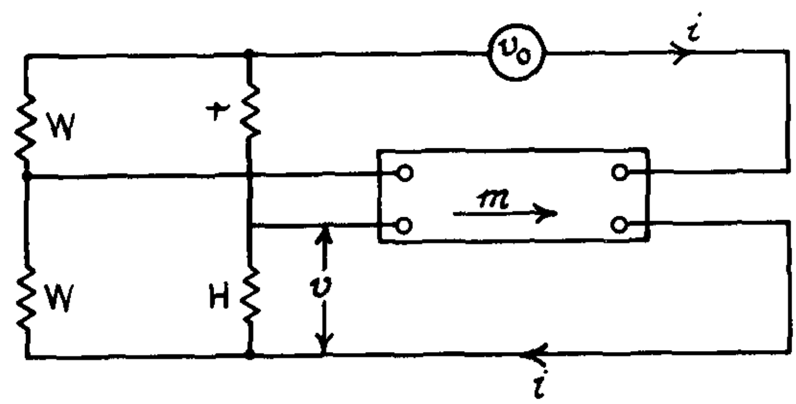

FIG 9

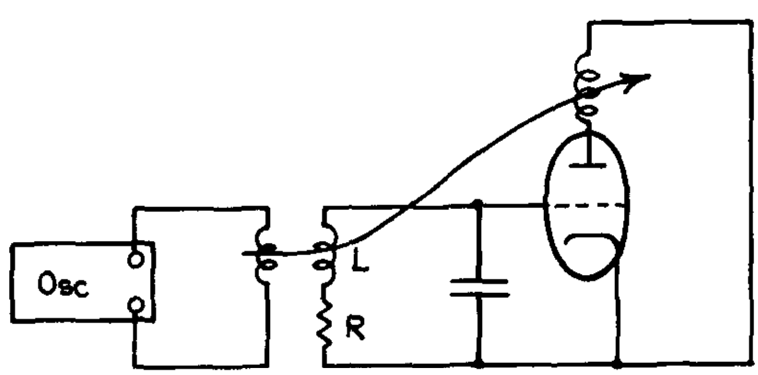

FIG 10

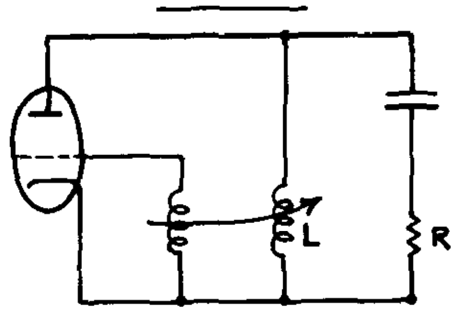

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FIG 11

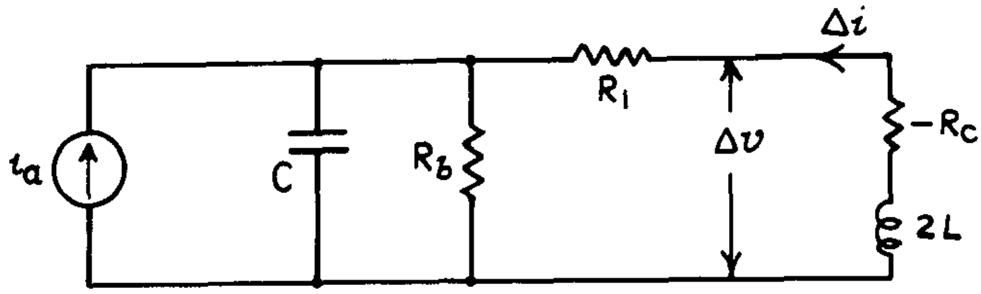

FIG. 12

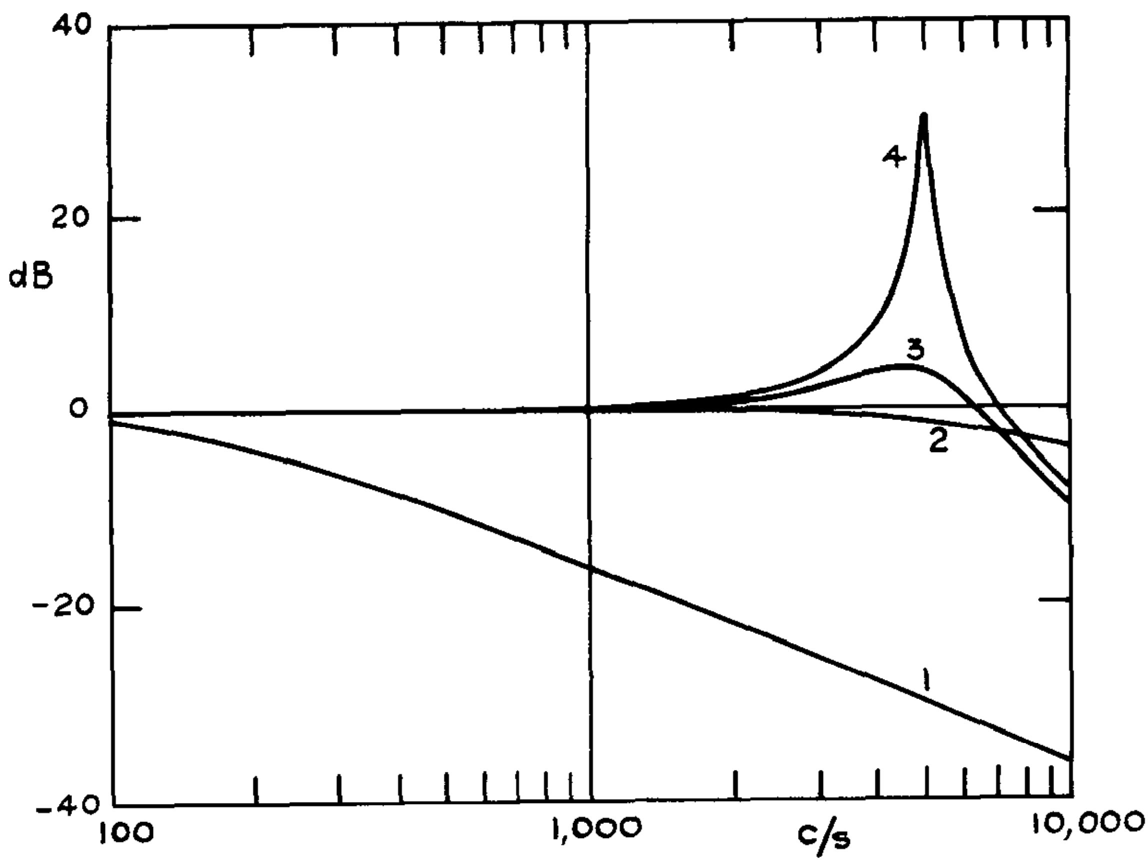

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FIG 13

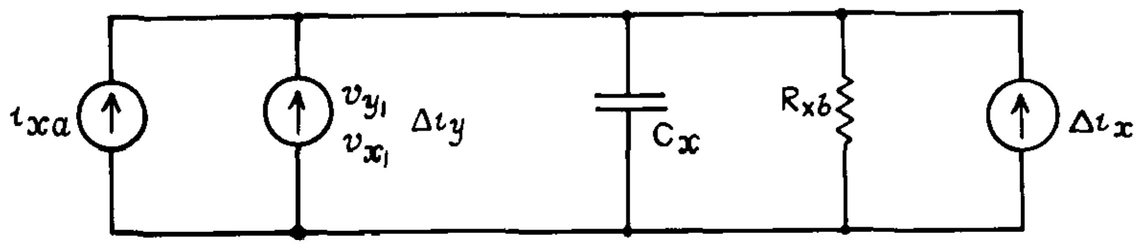


FIG 14

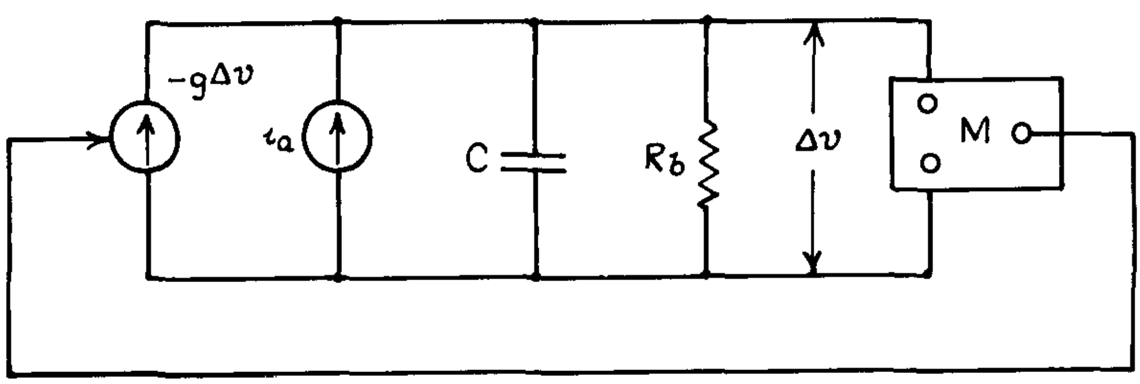
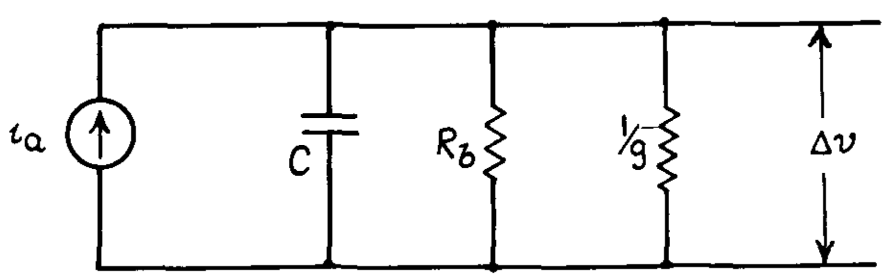


FIG 15



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