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Diffraction of Oblique Shock Wave

By

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Diffraction of Oblique Shock Wave

- By -
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Communicated by Dr. M. J. Lighthill

January, 1962

SUMMARY

Some problems of diffraction of oblique shock wave have been considered in this paper. Firstly it has been reviewed that the region between the incident and reflected shock wave remains undisturbed after the shock configuration has crossed the corner. Secondly it has been shown that when the shock configuration has crossed the corner, Mach reflection takes place provided the relative uniform flow behind the reflected shock wave is supersonic.

Introduction

Lighthill[2] investigated mathematically the diffraction of a plane normal shock past a small bend in the year 1949. He has calculated the pressure rise across the shock wave and its shape after it has crossed the corner. In the present paper some aspects of diffraction of oblique shock wave past a small bend have been considered. Myself and Ballabh have earlier[5] established that the region between the incident and reflected shock wave remains undisturbed for all possible shock strengths after the shock configuration has crossed the corner. Whatever is the region of disturbance it is behind the reflected shock wave only. For the consideration of the region behind the reflected shock one has to consider three cases $\alpha_o \gtreqless \alpha_s$ (Ref. 1)**, α_o is the angle of incidence and α_s is that angle of incidence for which the relative flow behind the reflected shock wave before diffraction takes place is sonic. In this paper it has been shown that when $\alpha_o < \alpha_s$ (i.e., when the relative uniform flow is supersonic), Mach reflection takes place after the shock configuration has crossed the corner.

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**Bleakney and Taub have denoted α_o by α.

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2. Formulation of the Problem

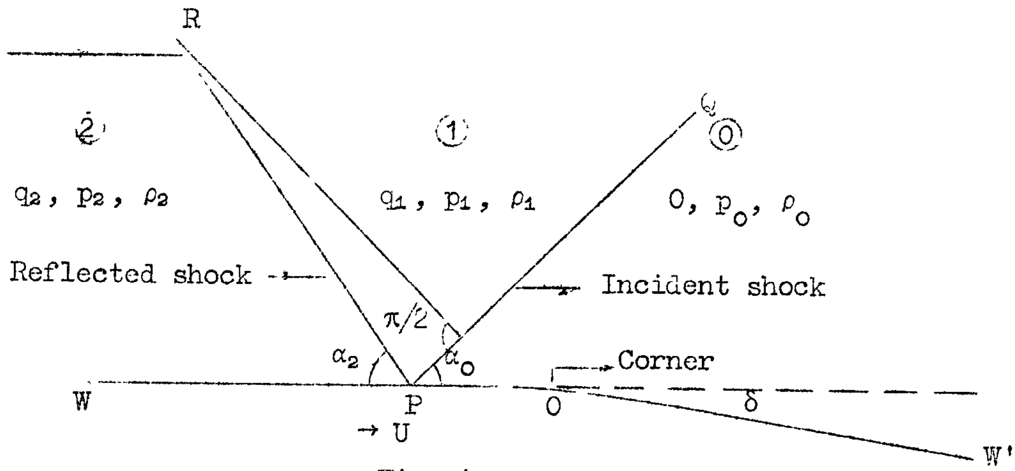

Fig. 1

In the figure WO and OW' are two walls forming an angle δ, where δ is small. PQ is the incident shock, PR is the reflected shock, α_o is the angle of incidence and α_2 is the angle of reflection. The values of velocity, pressure, density and sound velocity, in the region ahead of the shock wave is denoted by subscript zero, that in the region between the incident and reflected shock is denoted by subscript 1 and that behind the reflected shock is denoted by the subscript 2. The region (0) is at rest and therefore q_1 in the region (1) is perpendicular to the incident shock wave. The velocity q_2 in region (2) is parallel to the wall. U in the figure denotes the velocity of the point of intersection P of the incident and reflected shock wave. Now by applying the principle of conservation of mass, momentum and energy, the values of velocity, pressure and density behind the incident and reflected shocks before the configuration has crossed the corner O can be written down in terms of values ahead of the incident shock wave, angle of incidence and angle of reflection (γ has been assumed to be 1.4). These are

Across the incident shock

$$\left.\begin{aligned} q_1 &= \frac{5}{6}\bar{U}\left(1 - \frac{a_o^2}{\bar{U}^2}\right), \quad p_1 = \frac{5}{6}\rho_o\left(\bar{U}^2 - \frac{a_o^2}{7}\right) \\ \rho_1 &= \frac{6\rho_o}{1 + 5\dfrac{a_o^2}{\bar{U}^2}} \end{aligned}\right\} \quad \ldots(1)$$

where $\bar{U} = U \sin\alpha_o$.

Across the reflected shock

$$\left.\begin{aligned} \overline{q_2} &= \overline{q_1} + \frac{5}{6}(U^*-\overline{q_1})\left\{1 - \frac{a_1^2}{(U^*-\overline{q_1})^2}\right\} \\ p_2 &= \frac{5}{6}\rho_1\left\{(U^*-\overline{q_1})^2 - \frac{a_1^2}{7}\right\} \\ \rho_2 &= \frac{6\rho_1}{\left\{1 + \dfrac{5a_1^2}{(U^*-\overline{q_1})^2}\right\}} \end{aligned}\right\} \quad \ldots(2)$$

where/

where $\overline{q_1} = -q_1 \cos(\alpha_o+\alpha_2)$, $\overline{q_2} = q_2 \sin\alpha_2$

and $U^* = U \sin\alpha_2$.

3. Region between the Incident and Reflected Shock after Diffraction

The region between the incident and reflected shock remains undisturbed for all possible incident shock strengths after the shock configuration has crossed the corner[5]. A brief review of this result is as follows:-

Let at any point the velocity, pressure, density and entropy be $\vec{q_1'}$, p_1', ρ_1' and S_1'. Choose (X,Y) axes with the corner as origin and the original wall produced as X-axis. Following Lighthill's theory of linearization (which is given in detail in Section 4) and by the help of the transformation

$$\left.\begin{aligned} \frac{X-u_1 t}{a_1 t} = x, \quad \frac{Y-v_1 t}{a_1 t} &= y, \quad \vec{q_1'} = \{u_1(1+u),\ v_1(1+v)\} \\ \frac{p_1'-p_1}{a_1\ \rho_1} &= p \end{aligned}\right\} \quad \ldots(3)$$

the equation of continuity and equation of motion give a single second order partial differential equation in p. This equation is

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1 \right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} \right). \quad \ldots(4)$$

The characteristics of this differential equation are all tangents to the unit circle $x^2 + y^2 = 1$. It therefore follows that if there is a region of disturbance the point P where the two shocks intersect will be inside the unit circle $x^2 + y^2 = 1$. Using this as a pointer and using equations (1) it can be shown that the region between the two shocks will or will not be disturbed according as

$$\alpha_o > \text{ or } < \alpha_o^*$$

where $$\sin^2 \alpha_o^* = \frac{(\xi+6)^2}{7(6+6\xi-5\xi^2)}, \quad \xi \text{ being equal to } \frac{p_o}{p_1}. \quad \ldots(5)$$

It has been shown that α_o^* is greater than the extreme value of the angle of incidence consistent with the regular reflection of oblique shocks from rigid wall and therefore the region between the two shocks remains undisturbed.

In order that the unit circle may not interfere with the region between the two shocks beyond the point of intersection P it has also been established that the unit circle is not intersected by the reflected shock wave.

The equation of reflected shock wave is

$$\left(y + \frac{v_1}{a_1} \right) = \tan(\pi-\alpha_2)\left[x - \left(\frac{U-u_1}{a_1} \right) \right].$$

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The condition for the reflected shock not to be intersected by the unit circle will be

$$\left[\frac{U-u_1}{a_1}\right] \tan\alpha_2 - \frac{v_1}{a_1} > \sqrt{1+\tan^2\alpha_2}.$$

On putting $u_1 = q_1 \sin\alpha_0$, $v_1 = -q_1\cos\alpha_0$, the above condition reduces to

$$\frac{U\sin\alpha_2 - \overline{q_1}}{a_1} > 1$$

which is always true as the uniform relative flow in the region between the two shocks normal to reflected shock is always supersonic. This completes the proof.

4. Region behind the Reflected Shock after Diffraction

Let the velocity, pressure, density and entropy at any point be $\vec{q}_2'$, p_2', ρ_2' and S_2'. Choose (X,Y) axes with origin at the corner and X axis along the original wall produced. The equation of conservation of mass and momentum can be written as

$$\frac{D\rho_2'}{Dt} + \rho_2' \operatorname{div} \vec{q}_2' = 0 \qquad \ldots(6)$$

$$\frac{D\vec{q}_2'}{Dt} + \frac{1}{\rho_2'} \nabla p_2' = 0 \qquad \ldots(7)$$

and if there is no heat transfer between fluid elements by friction, conduction, or radiation, the entropy will satisfy

$$\frac{DS_2'}{Dt} = 0.$$

On the assumption that $\vec{q}_2'$, p_2', ρ_2' differ only by small quantities from the values $(q_2, 0)$, p_2, ρ_2 which they had before diffraction, the equations (6) and (7) can be approximated as

$$\frac{\partial\rho_2'}{\partial t} + q_2\frac{\partial\rho_2'}{\partial X} + \rho_2 \operatorname{div} \vec{q}_2' = 0 \qquad \ldots(8)$$

$$\frac{\partial\vec{q}_2'}{\partial t} + q_2\frac{\partial\vec{q}_2'}{\partial X} + \frac{1}{\rho_2}\nabla p_2' = 0. \qquad \ldots(9)$$

In the equation (8) $\frac{\partial\rho_2'}{\partial t} + q_2\frac{\partial\rho_2'}{\partial X}$ can be replaced by

$$\left(\frac{\partial\rho_2}{\partial p_2}\right)_{S_2}\left(\frac{\partial p_2'}{\partial t} + q_2\frac{\partial p_2'}{\partial X}\right) = \frac{1}{a_2^2}\left(\frac{\partial p_2'}{\partial t} + q_2\frac{\partial p_2'}{\partial X}\right)$$

S_2 being the entropy in region (2) before diffraction.

If/

If we now introduce the transformation

$$\left.\begin{aligned} \frac{X-q_2 t}{a_2 t} = x, \quad \frac{Y}{a_2 t} = y, \quad \frac{\vec{q}_2'}{q_2} = \{(1+u),\ v\} \\ \frac{p_2'-p_2}{a_2\ q_2\ \rho_2} = p \end{aligned}\right\} \quad \ldots(10)$$

and use the fact that (u, v, p) depend only on x and y, the equations (8) and (9) give the following equations

$$x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad \ldots(11)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} \quad \ldots(12)$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \frac{\partial p}{\partial y}. \quad \ldots(13)$$

In the new axes the origin is at a point on the original wall produced. The straight part of the shock wave lies along a fixed line $x = k - y\cot\alpha_2$ where $k = \frac{U-q_2}{a_2}$. The corner is at the point $(-M_2, 0)$ where $M_2 = \frac{q_2}{a_2}$.

From the equations (11), (12) and (13), by eliminating u and v we get

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right).$$

The equation is hyperbolic for $x^2 + y^2 > 1$ and elliptic for $x^2 + y^2 < 1$, its characteristics are all tangents to the unit circle. So it is justified to assume that the smallest region of disturbance behind the reflected shock will be bounded by the arc of the unit circle, the wall and the reflected shock.

In Lighthill's case the relative outflow from the normal shock is always subsonic but in the case of oblique reflection of shock from rigid wall, the relative outflow from the reflected shock can be supersonic, sonic and subsonic, i.e., $k \gtrless 1$ where $k = \frac{U-q_2}{a_2}$. When $k > 1$ we have $\alpha_o < \alpha_s$ (Ref. 1) and when $k \leqslant 1$ we have $\alpha_o \geqslant \alpha_s$ (Ref. 1). Now from the result of the reflection of oblique shock wave from a rigid wall it is known that the relative outflow from the reflected shock wave is sonic or subsonic for $\alpha_o \geqslant \alpha_s$ which is just slightly smaller than α_e over the entire range of incident shock strength (Fig. 2) (Ref. 4). It has been shown in this paper that when $\alpha_o < \alpha_s$, Mach reflection takes place after the shock configuration has crossed the corner. This means that the problem of an oblique shock

configuration/

configuration passing over a small bend has been completely solved for all angles of incidence which are not in the proximity of the extreme angle over the entire range of incidence shock strength (Fig. 2). Now we proceed to establish the above result.

The point of intersection of the two shocks referred to the transformed axes is (k, 0). Now when $k > 1$ the point lies outside the unit circle. Now if such be the case then we would get a region enclosed by the reflected shock, wall and unit circle just behind the point of intersection P of the two shocks which will be completely undisturbed, but to conceive such a situation is physically not possible, as this should be the region of maximum disturbance. So it appears that the point P leaves the wall and the reflection is taking place in the medium itself. Now this statement can be put on mathematical foundation by making the transformation to polar co-ordinates. Let the transformation be $x = r\cos\theta$, $y = r\sin\theta$ where we take

$\rho = \dfrac{[1-(1-r^2)^{\frac{1}{2}}]}{r}$. With this transformation the circle $r = 1$ becomes the circle $\rho = 1$ and the reflected shock transforms to a circle whose equation is

$$\frac{2\rho\sin(\theta+\alpha_2)}{1+\rho^2} = k\sin\alpha_2.$$

Now if we find out the intersection with the wall we get

$$\frac{2\rho}{1+\rho^2} = k.$$

This gives $$\rho = \frac{1 \pm \sqrt{1-k^2}}{k}.$$

The intersection is imaginary if $k > 1 (\alpha_o < \alpha_s)$ and therefore the point P leaves the wall and Mach reflection takes place. This proves the statement. When $k = 1$, the circle touches the wall and when $k < 1$, the circle intersects the wall at two real points.

5. In Bleakney and Taub's[1] notation $U-q_2$ is z", and a_2 has been denoted by c". They refer that before diffraction takes place, the uniform assumptions in regions (0), (1) and (2) is correct when $\dfrac{U-q_2}{a_2} > 1$. They say that when $\dfrac{U-q_2}{a_2} < 1$ the phenomenon becomes transient and there is no real justification in considering the uniform flows in regions (0), (1) and (2). This is so because when $\dfrac{U-q_2}{a_2} < 1$ the reflected shock wave becomes curved. In the diffraction problems when $\dfrac{U-q_2}{a_2} < 1$ we take the curved shock to be a plane one and hence the flows to be uniform in the regions mentioned. This is an approximation but without which the problems of diffraction of shocks will be extremely complicated[3]. Therefore taking this approximation, diffraction of oblique shock wave in the case when $\dfrac{U-q_2}{a_2} < 1$ can be investigated. The case $\dfrac{U-q_2}{a_2} = 1$ $(\alpha_s = 1)$ is limiting case between

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the two cases $\frac{U-q_2}{a_2} < 1$ and $\frac{U-q_2}{a_2} > 1$. The two cases $\frac{U-q_2}{a_2} \leqslant 1$ $(\alpha_o \geqslant \alpha_s)$ are being considered in a subsequent paper. As stated earlier this case arises only when we are approaching the extreme angle curve. For the case $\left(\frac{U-q_2}{a_2} \leqslant 1\right)$ Dr. M. J. Lighthill's suggestion is being applied for finding out the pressure distribution along the wall behind the reflected shock.

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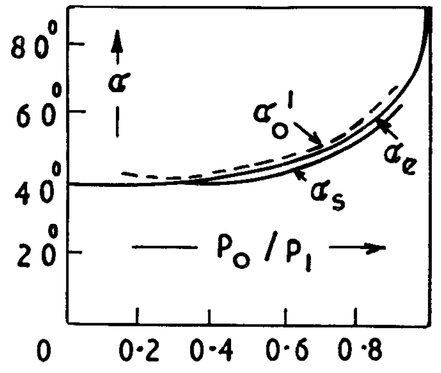
FIG. 2



Critical angles of incidence in shock reflection: theoretical limit for regular reflection a_e, experimentally observed onset of Mach reflection a_o'; and condition for sonic outflow α_s; versus inverse pressure ratio across incident shock front (p_o/p_1)

A.R.C. C.P. No. 612. January, 1962

Srivastava, R. S. Defence Science Laboratory, Delhi.

DIFFRACTION OF OBLIQUE SHOCK WAVE

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