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The Calibration of a 60° Cone to Measure Mach Number, Total Pressure and Flow Angles at Supersonic Speeds

by

D. R. Andrews and W. G. Sawyer

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SUMMARY

The characteristics of a 60° apex angle cone for measuring local Mach number, total pressure, and flow directions have been determined at Mach numbers of 2.47, 3.25, 3.97 and 4.32 at angles of pitch up to 29°.

Replaces R.A.E. Tech. Note No. Aero 2825 - A.R.C. 24,106.

LIST OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Contd)

1 INTRODUCTION

To explore the flow field around a model in a wind tunnel, an instrument is required that will measure the Mach number, total pressure and flow directions simultaneously. One type of instrument suitable for use at supersonic speeds consists of a cone with four equally spaced orifices on its surface together with a pitot pressure orifice at its apex. Reference 1 shows that the sensitivity of such a probe to changes in flow direction increases with increase of the cone apex angle, but that above 60° included angle the increase in sensitivity is only relatively small.

The characteristics of a 60° included angle, conical probe were determined at Mach numbers of 2.47, 3.25 and 4.32 in the 11 × 16 inch R.A.E. Farnborough Tunnel. The results obtained are given in this note. Eleven similar probes were subsequently made, assembled in a rake, and tested briefly at M = 3.97 in the 4 ft × 3 ft High Supersonic Speed Tunnel at R.A.E. Bedford. The mean results from these eleven probes are included in this note for comparison with the earlier results.

2 DETAILS OF PROBE AND PRESSURE MEASURING SYSTEM

A drawing of the probe is shown in Fig.1 and a photograph in Fig.2. The probe is of the plug-in type with a key for location in the support. In spite of its small size, the errors arising in manufacture were very small. Measurements taken on the probe which was tested in the 11 × 16 inch tunnel showed that the cone angle was 59.9° (instead of 60°), and that the angular displacements of the four holes in the cone surface were within $\pm\frac{1}{2}^\circ$ of 90°. For the 11 probes tested in the HSST, the cone angles were within the range 59.6° to 60.0°, and the angular displacements of the surface holes within ±1° of 90°.

All pressures were measured on self-balancing capsule manometers of 0 - 60 ins Hg abs range and ±0.03 ins Hg nominal accuracy. Because of the very small bore of the tubing within the probe, care was taken to keep the pipe lengths to the manometers as short as possible to minimise lag effects.

3 TESTS MADE

Tests were made on a single probe in the R.A.E. Farnborough 11 inch × 16 inch supersonic wind tunnel at Mach numbers of 2.47, 3.25 and 4.32, the corresponding total pressures being 1, 2, and 5 atmospheres (absolute) respectively. To keep the testing time within acceptable limits, the characteristics were explored fully only for roll angles of 0 to 90°. In this range, the pitch angle was varied from -5° to +24° at M = 3.25 and to +28° at M = 2.47 and 4.32, at increments in roll of 10°. Sufficient testing was done at roll angles greater than 90° to show that the characteristics of the probe were symmetrical.

Tests on 11 similar probes, mounted in a rake, were made at M = 3.97 in the 4 ft × 3 ft High Supersonic Speed Tunnel at R.A.E. Bedford. The total pressure was 7 atmospheres absolute. These tests covered an incidence range from 0 to 25° at roll angles of 0, 90°, 180° and 270°.

4 PRECISION OF RESULTS

The estimated uncertainty of the experimental data obtained in both the 11 inch × 16 inch tunnel and the H.S.S.T. is as follows:

θ	$\pm 0.1^\circ$
ϕ	$\pm 0.1^\circ$
$\frac{\Delta p}{q}$	± 0.01
$\frac{p_A}{p_{02}}$	± 0.002
M_1	± 0.002

The accuracy with which the characteristics of local flow could be obtained, following the procedure outlined in Section 6, is estimated to be:

	M_1 = 2.47, p_{01} = 1 atmos abs.		M_1 = 3.25, p_{01} = 2 atmos abs.		M_1 = 4.32, p_{01} = 5 atmos abs.	
	$\alpha = \beta = 0$	$\alpha = \beta = 20^\circ$	$\alpha = \beta = 0$	$\alpha = \beta = 20^\circ$	$\alpha = \beta = 0$	$\alpha = \beta = 20^\circ$
M_1	±0.02	±0.03	±0.04	±0.07	±0.08	±0.15
p_{01}	±2%	±3%	±4%	±6%	±7%	±12%
α, β	$\pm\frac{1}{4}^\circ$	$\pm\frac{1}{2}^\circ$	$\pm\frac{1}{4}^\circ$	$\pm\frac{1}{2}^\circ$	$\pm\frac{1}{4}^\circ$	$\pm\frac{1}{2}^\circ$

As the errors that arise are inversely proportional to the free stream total pressure (p_{01}), the errors tabulated above have been calculated for specific values of p_{01} at each Mach number.

5 RESULTS AND DISCUSSION

Fig.4 shows the variation with Mach number of the ratio of average pressure, p_A, on the cone surface to the pitot pressure, p_{02}. The results shown are for zero pitch angle, θ. The mean value obtained from the 11 probes tested in the H.S.S.T. is seen to agree well with the results from a single probe in the 11 inch × 16 inch tunnel. Cone theory[2] is in good agreement, giving values of p_A/p_{02} only about 2% larger than experiment. It is apparent from Fig.4 that the sensitivity of the probe decreases rapidly with increasing Mach number, so that, for example near $M_1 = 4.0$, a 1% error in measuring p_A results in a 3% error in M_1. This further leads to a 10% error in p_{01} as derived from the normal shock relation and p_{02} (Section 6).

The average of the four surface pressures remains constant up to pitch angles of about 10°, and then increases progressively as the pitch angle is increased beyond this value. This is shown in Figs.5(a), (b) and (c).

In producing these curves, the data from geometrically similar roll angles has been averaged, as the values would be expected to be the same for reasons of symmetry. It is seen that the effect on p_A of variations in ϕ at any given value of θ and M_1 is less than 1%. Also shown in Figs.5(b) and (c) for $\phi = 0, 90^\circ, 180^\circ$ and 270° are the mean results from the 11 probes tested in the H.S.S.T. These results are seen to be consistent with those obtained from the 11 inch × 16 inch tunnel.

Fig.6 shows that the effect of pitch angle on the measured pitot pressure is negligible up to about 13°. Beyond this angle, the pitot pressure decreases with increasing pitch angle. Mach number effects appear to be very small.

The pressure difference, expressed in coefficient form, across opposite pairs of holes in the cone surface is shown in Figs.7(a) to (f) as a function of pitch angle for various roll angles at Mach numbers of 2.47, 3.25 and 4.32. Small alignment errors are present so that the curves are slightly off-centre and are rotated through a small angle. Although no results are shown for roll angles greater than 90°, sufficient measurements were taken for roll angles between 90° and 360° to show that the characteristics were symmetrical. Because of the symmetry of the probe, it will be seen that the sets of curves for $\left(\frac{\Delta p}{q_1}\right)_\alpha$ and $\left(\frac{\Delta p}{q_1}\right)_\beta$ are virtually identical.

The variation of $\frac{\Delta p}{q_1}$ with pitch angle at any given roll angle is approximately linear over the first $\pm 5^\circ$, and analysis of the experimental results shows that, over this range, the value of $\left(\frac{\Delta p}{q_1}\right)\frac{1}{\theta}$ is proportional to $\cos\phi$ or $\sin\phi$ depending on the pair of holes involved. Fig.8 illustrates this point for the $M = 2.47$ results. It also shows that, when account is taken of the small zero setting error in roll, the yawmeter sensitivities $\left(\frac{\Delta p}{q}\right)_\alpha \frac{1}{\theta \cos\phi}$ and $\left(\frac{\Delta p}{q}\right)_\beta \frac{1}{\theta \sin\phi}$ are identical, as would be expected from symmetry. Fig.9 shows the variation of yawmeter sensitivity with Mach number. The H.S.S.T. results are again consistent with those obtained in the 11 inch × 16 inch tunnel. Experiment and cone theory[3,4] are in quite good agreement, and demonstrate that the effect of Mach number on the sensitivity for small flow deflections can be neglected for Mach numbers above about 2.5.

To facilitate the determination of θ and ϕ from measurements of $\left(\frac{\Delta p}{q_1}\right)_\alpha$ and $\left(\frac{\Delta p}{q_1}\right)_\beta$, the results of Fig.7 have been combined into a plot of $\left(\frac{\Delta p}{q_1}\right)_\alpha$ versus $\left(\frac{\Delta p}{q_1}\right)_\beta$ for various θ and ϕ as shown in Fig.10(a), (b), (c). Before plotting the results in this form, the curves of Fig.7 were first corrected for misalignment errors. Because of symmetry, the curves are shown in only one quadrant. Data for the other quadrants can be readily derived providing the correct sign convention is used (Fig.10(a)).

6 METHOD OF USING PROBE TO OBTAIN FLOW CHARACTERISTICS

The method of obtaining Mach number, total pressure, and flow angles from the measured pitot pressure and four static pressures on the cone surface follows that given in Ref.5, but is repeated here for the sake of completeness.

The arithmetic mean of the four surface static pressures is

$$p_A = \frac{1}{4}\left[p_a + p_b + p_c + p_d\right] .$$

The ratio of this static pressure to the pitot pressure is $\frac{p_A}{p_{02}}$ and hence from Fig.4 a first approximation to M_1 is obtained by assuming pitch angle effects on $\frac{p_A}{p_{02}}$ to be negligible. The normal shock relation gives

$$\frac{p_{02}}{p_{01}} = \left[\frac{6\,M_1^2}{M_1^2 + 5}\right]^{7/2} \left[\frac{6}{7\,M_1^2 - 1}\right]^{5/2}$$

so that p_{01}, the total pressure of the ambient flow, is obtained.

The dynamic pressure, q_1, of the ambient flow is then obtained from the relation

$$\frac{q_1}{p_{01}} = \frac{7}{10}\,M_1^2\left[1 + \frac{M_1^2}{5}\right]^{-7/2} .$$

The pressure differences across opposite pairs of holes are

$$\left(\frac{\Delta p}{q_1}\right)_\alpha = \frac{p_c - p_a}{q_1}$$

and

$$\left(\frac{\Delta p}{q_1}\right)_\beta = \frac{p_d - p_b}{q_1}$$

whence from Fig.10 the pitch and roll angles θ and ϕ of the local flow relative to the probe are obtained. These values are then used to correct for flow angle effects on p_{02} and p_A.

From Fig.6 we obtain

$$\frac{(p_{02})_\theta}{(p_{02})_{\theta=0}}$$

and thus derive a better approximation to the value of $(p_{02})_{\theta=0}$, the pitot pressure at zero pitch angle. Similarly from Fig.5, a better approximation to the value of p_A/p_{02} at zero pitch angle is obtained as

$$\left(\frac{p_A}{p_{02}}\right)_{\theta=0} = \frac{\left(\frac{p_A}{p_{02}}\right)_{\theta}}{\left[\left(\frac{p_A}{p_{02}}\right)_{\theta} \Big/ \left(\frac{p_A}{p_{02}}\right)_{\theta=0}\right]} .$$

The initial procedure is then repeated with these new values of p_{02} and $\frac{p_A}{p_{02}}$ to obtain a second approximation to M_1, p_{01}, θ and ϕ. The process is repeated until the desired accuracy is obtained.

The estimated accuracy with which these quantities can be derived in this way, using the same type of instrumentation and probe as used in the present series of tests is given in Section 4. Note that the errors are inversely proportional to the total pressure, p_{01}. The large errors in M_1 and p_{01} at high Mach number arise from the fact that the curve of $\frac{p_A}{p_{02}}$ becomes very flat as Mach number is increased (Fig.4). Hence a small error in p_A gives a large error in M_1 and hence in p_{01}. At high Mach numbers, the % error in p_{01} may be as much as 5 times the % error in M_1 since the pressure ratio across a normal shock becomes proportional to M_1^5 as $M_1 \to \infty$.

7 CONCLUSIONS

A conical probe having a 60° apex angle has been calibrated so that the pressure at its apex and at four equally spaced points on the conical surface can be used to determine the local flow Mach number, total pressure and flow angles. Pressure measurements were made at Mach numbers of 2.47, 3.25, 3.97 and 4.32 at angles of pitch up to 29°. The data are presented in charts such that the characteristics of the local flow can be readily derived.

It is estimated that, with the same instrumentation and total pressures as used in the present series of tests, such a probe will enable local Mach number to be determined to about ±0.02 at $M = 2.5$ and ±0.08 at $M = 4.3$. Measurements of total pressure should be accurate to about ±2% at $M = 2.5$ and ±7% at $M = 4.3$, and the flow angles to about $\pm\frac{1}{4}^{\circ}$ at all Mach numbers. The errors become larger than these values at high pitch angles, and also vary in inverse proportion to the magnitude of the total pressure.

The poor accuracy in the determination of local Mach number and total pressure at high Mach number arises from the comparative insensitivity of cone surface pressure to Mach number when the Mach number is high.

LIST OF SYMBOLS

$\left(\frac{\Delta p}{q_1}\right)_\alpha$	difference in pressure coefficient between orifices c and a, $\frac{(p_c - p_a)}{q_1}$ (Fig.3)
$\left(\frac{\Delta p}{q_1}\right)_\beta$	difference in pressure coefficient between orifices d and b, $\frac{(p_d - p_b)}{q_1}$ (Fig.3)
p_{02}	pitot pressure measured behind normal shock at cone apex
p_{01}	total pressure ahead of normal shock at cone apex
p_1	static pressure ahead of normal shock at cone apex
q_1	dynamic pressure ahead of normal shock at cone apex
M_1	Mach number ahead of normal shock at cone apex
p	static pressure on cone surface
p_A	arithmetic mean of four cone surface static pressures, $\frac{1}{4}(p_a + p_b + p_c + p_d)$
α	angle of attack (Fig.3)
β	angle of sideslip (Fig.3)
θ	angle of pitch of cone axis (Fig.3)
ϕ	angle of roll (Fig.3)

Suffices

θ	quantity at angle of pitch
$\theta = 0$	quantity at zero angle of pitch
a,b,c,d	position of orifices on cone surface (Fig.3)

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No.	Author	Title, etc.
1	Raney, D.J.	Flow direction measurements in supersonic wind tunnels. A.R.C. C.P. 262. September, 1954.
2	Kopal, Z.	Tables of supersonic flow around cones. MIT Tech. Report No.1. P 18356. 1947.
3	Kopal, Z.	Tables of supersonic flow around yawing cones. MIT Tech. Report No.3. 1947.
4	Kopal, Z.	Tables of supersonic flow around cones at large yaw. MIT Tech. Report No.5. 1949.
5	Centolanzi, F.J.	Characteristics of a 40° cone for measuring Mach number, total pressure, and flow angles at supersonic speeds. NACA TN 3967. May 1957.

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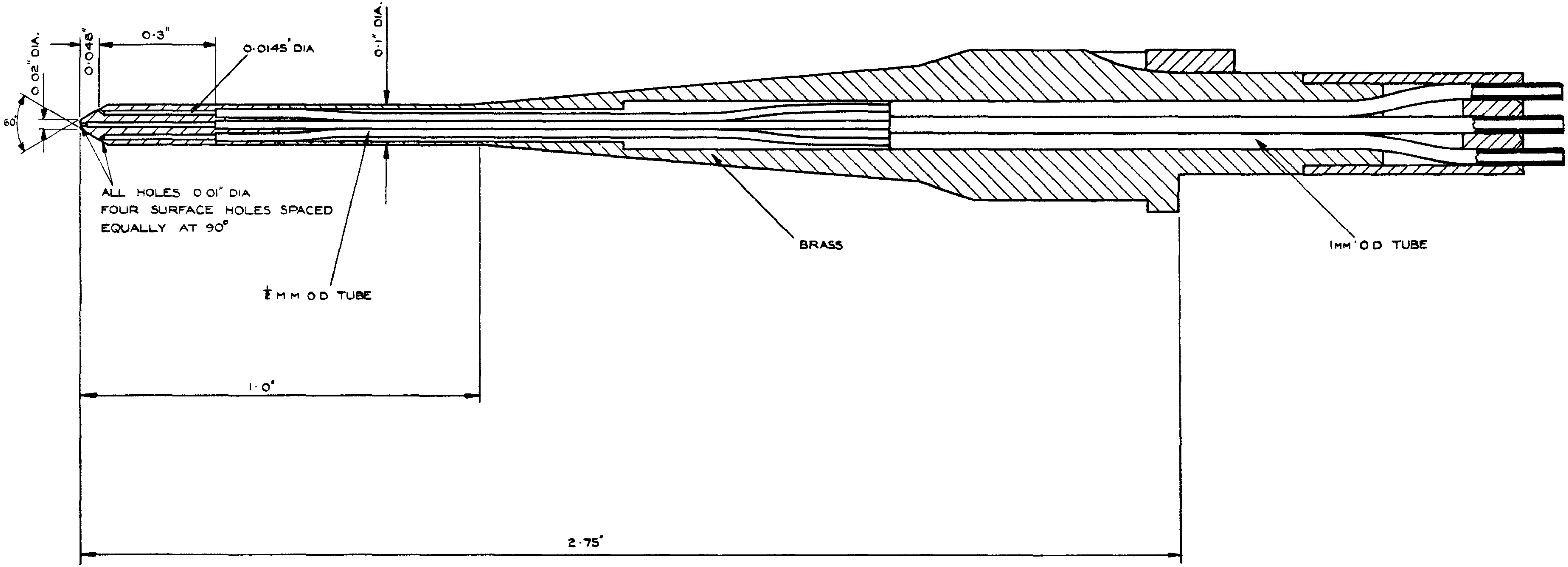


FIG. 1. SECTIONAL VIEW OF FLOW SURVEY PROBE.

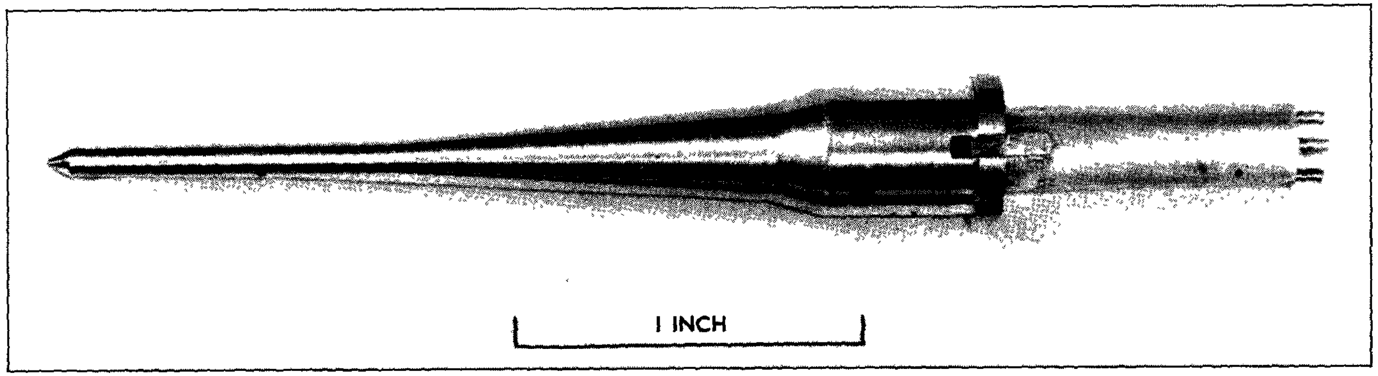
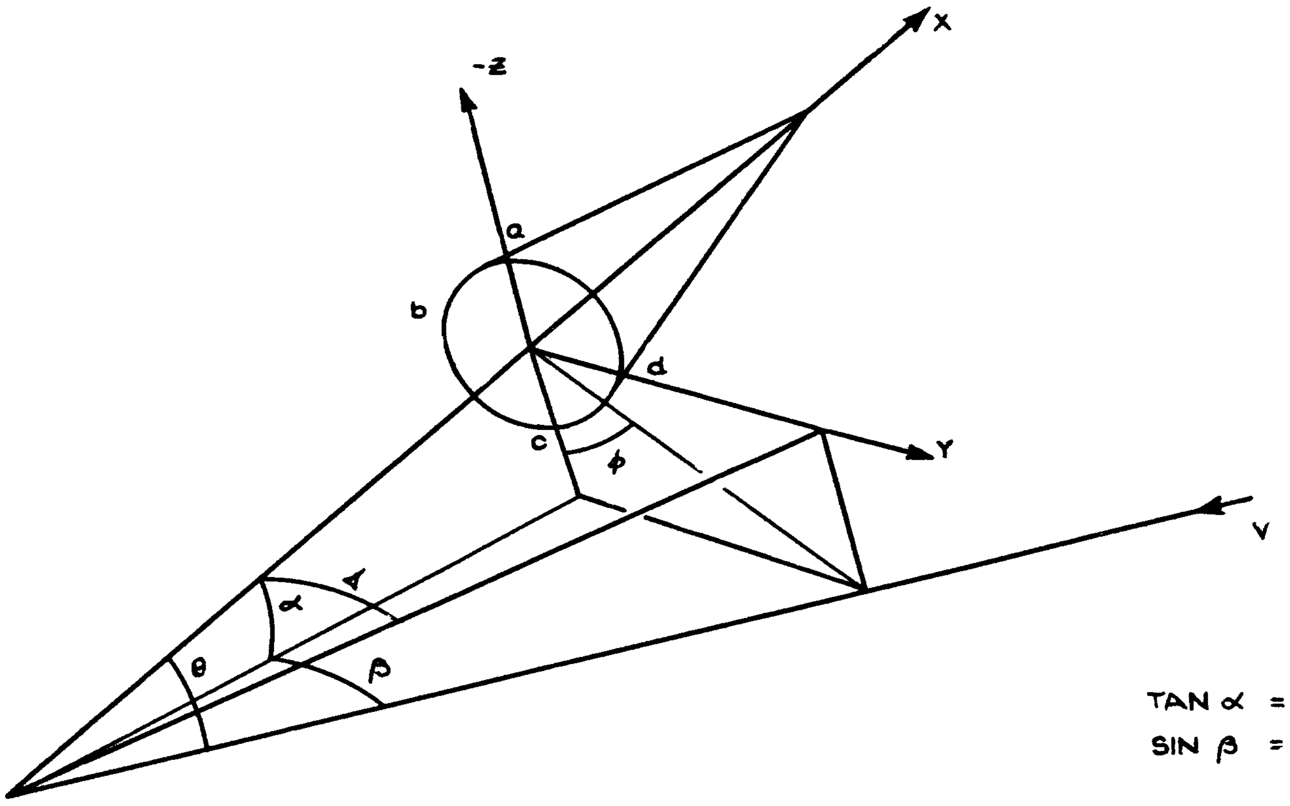


FIG.2. FLOW SURVEY PROBE



$$\left.\begin{aligned}\tan\alpha &= \tan\theta\ \cos\phi\\ \sin\beta &= \sin\theta\ \sin\phi\end{aligned}\right\}$$

$$\tan\sphericalangle = \tan\theta\ \sin\phi$$

FIG.3 ORIFICE AND ANGLE NOTATION.

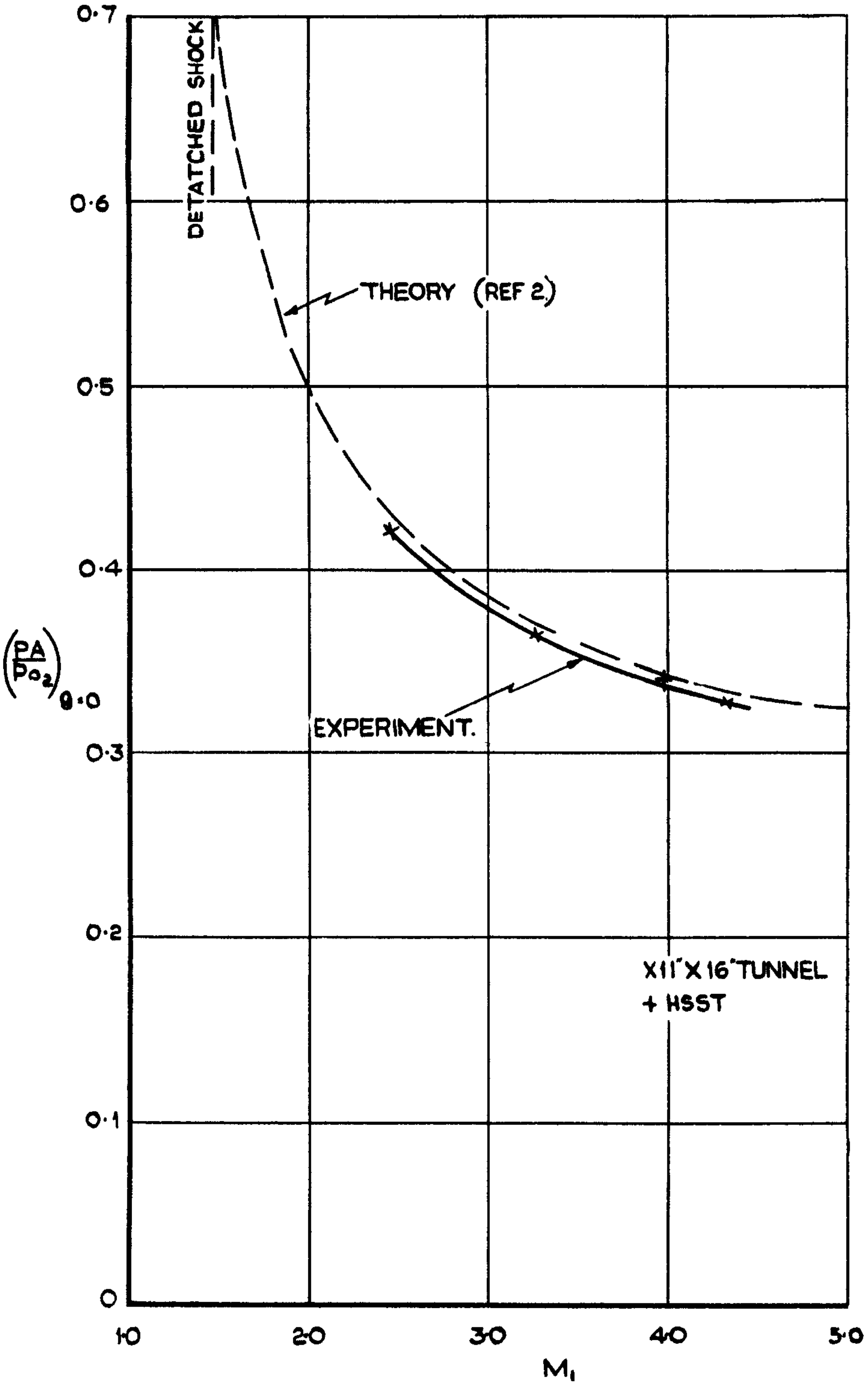
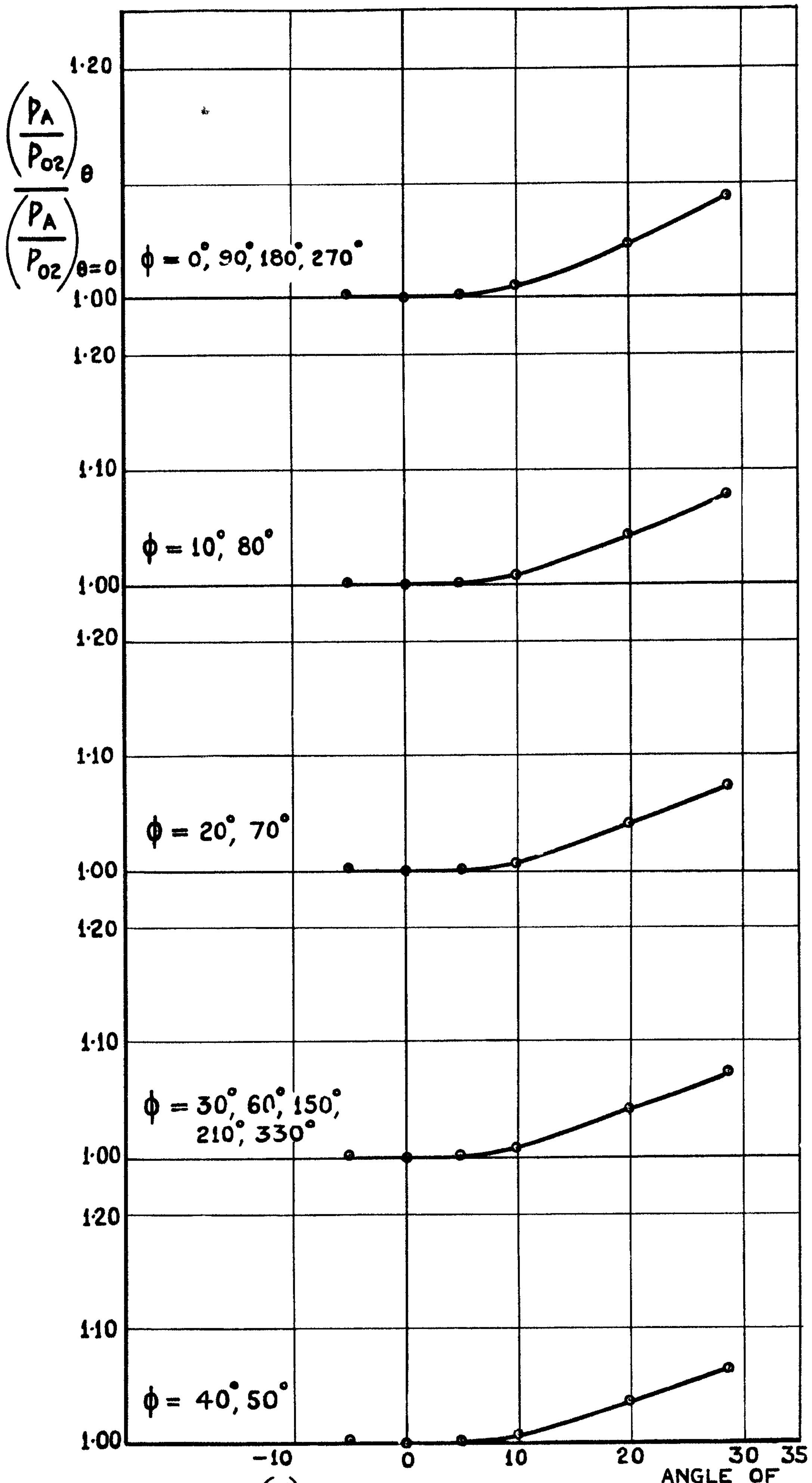
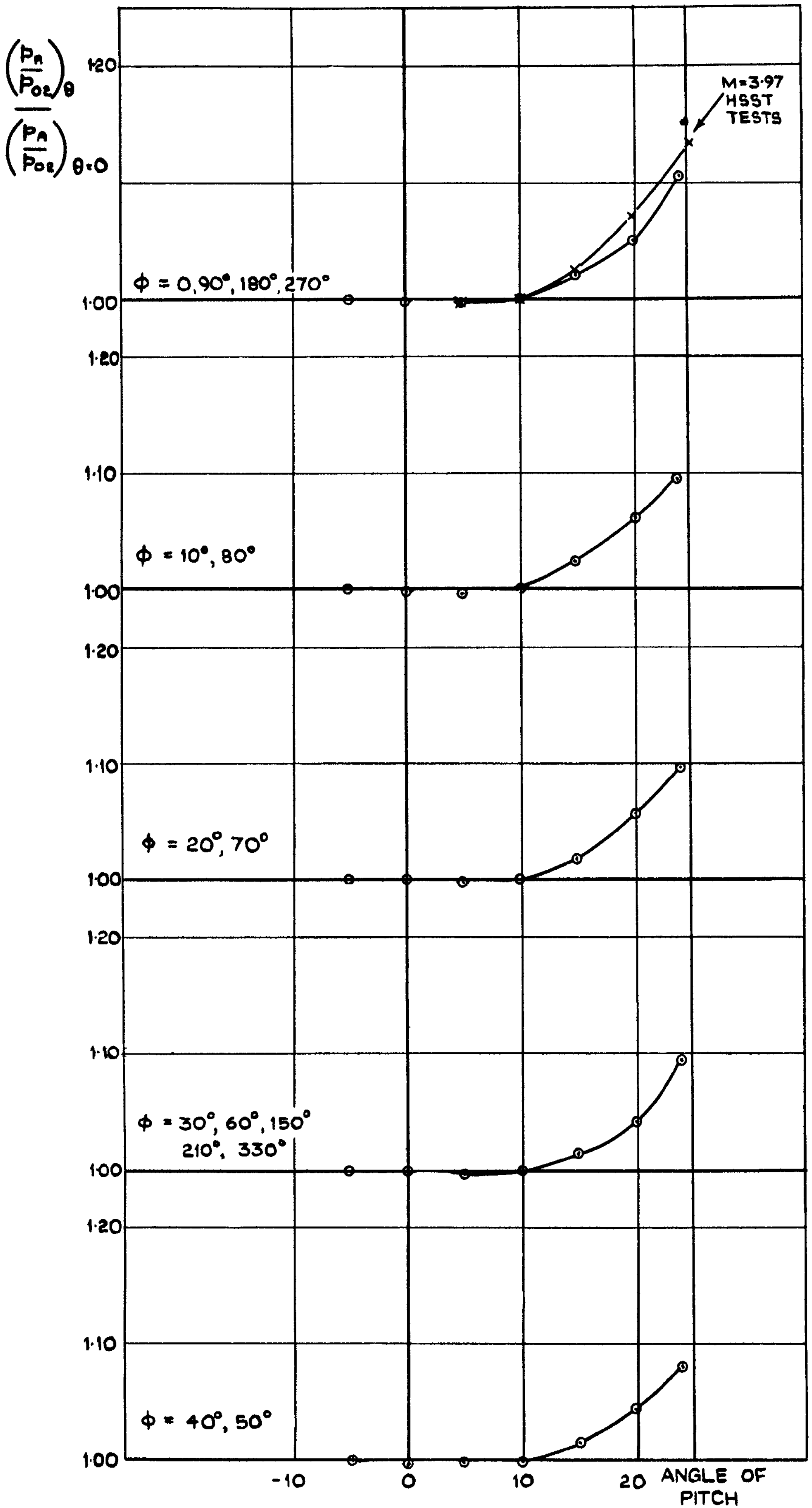


FIG.4. VARIATION OF SURFACE PRESSURE TO PITOT PRESSURE WITH FREE STREAM MACH NUMBER



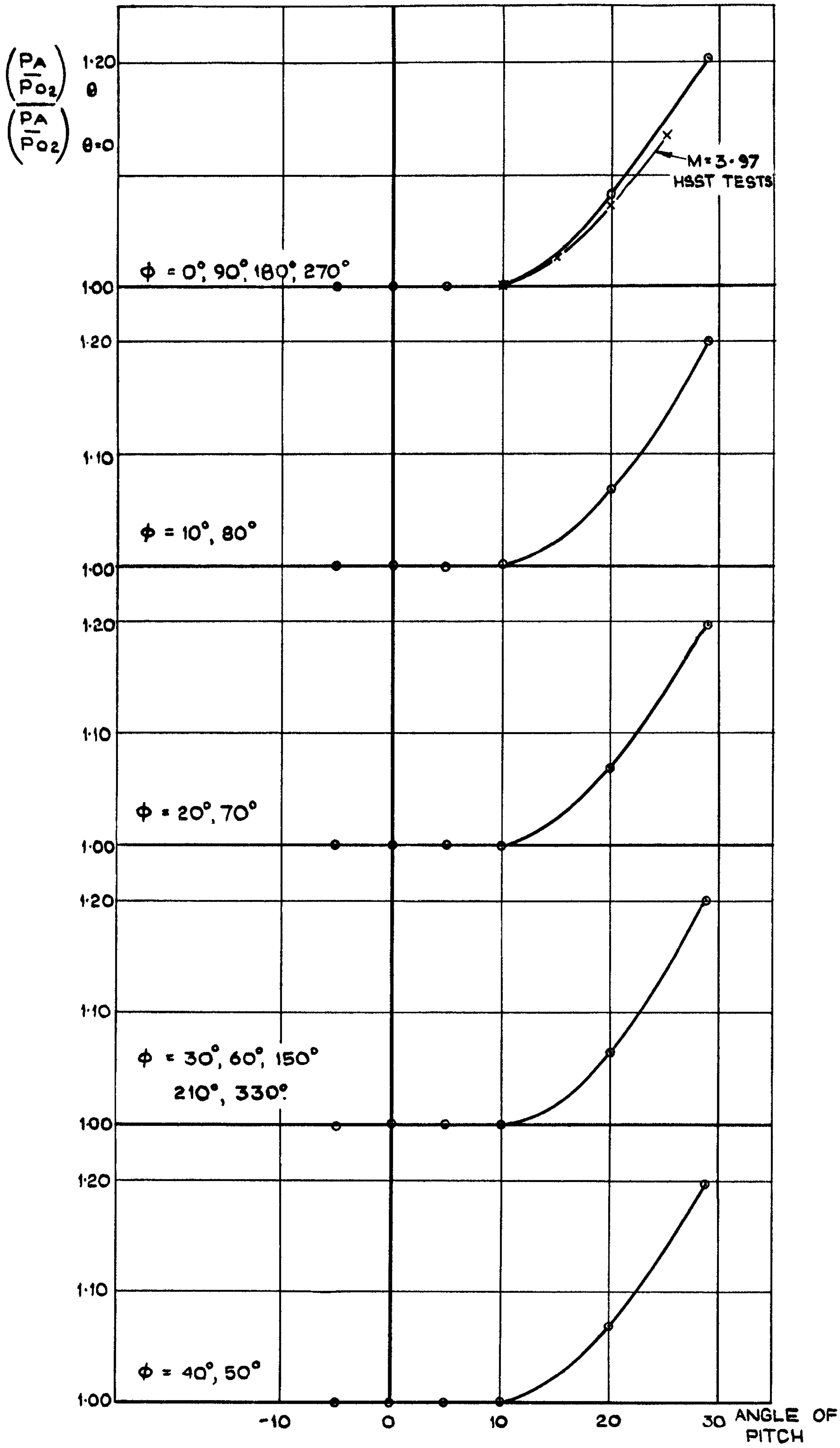
(a) M = 2·47

FIG.5. EFFECT OF ANGLE OF PITCH ON THE RATIO OF AVERAGE STATIC PRESSURE TO PITOT PRESSURE.



(b) M = 3·25

FIG. 5 (CONTD.) EFFECT OF ANGLE OF PITCH ON THE RATIO OF AVERAGE STATIC PRESSURE TO PITOT PRESSURE



(c) M = 4·32

FIG.5(CONCLD)EFFECT OF ANGLE OF PITCH ON THE RATIO OF AVERAGE STATIC PRESSURE TO PITOT PRESSURE

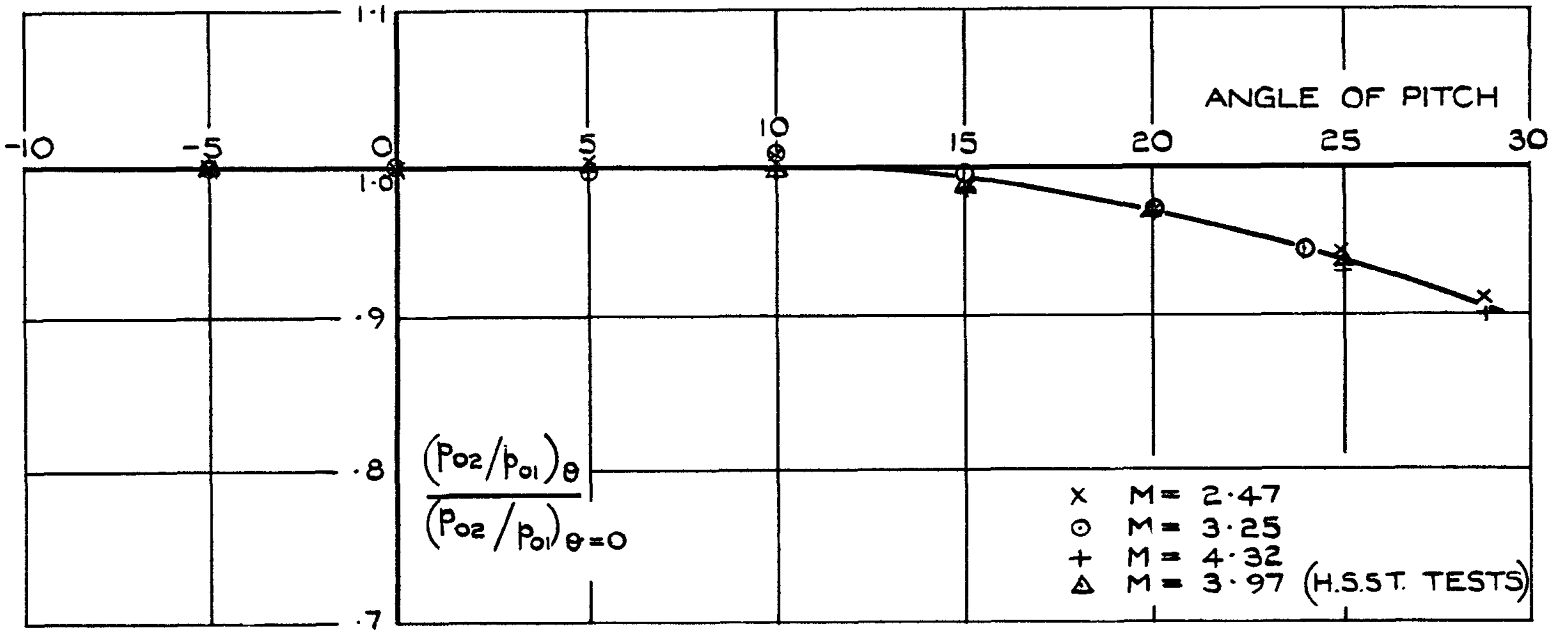
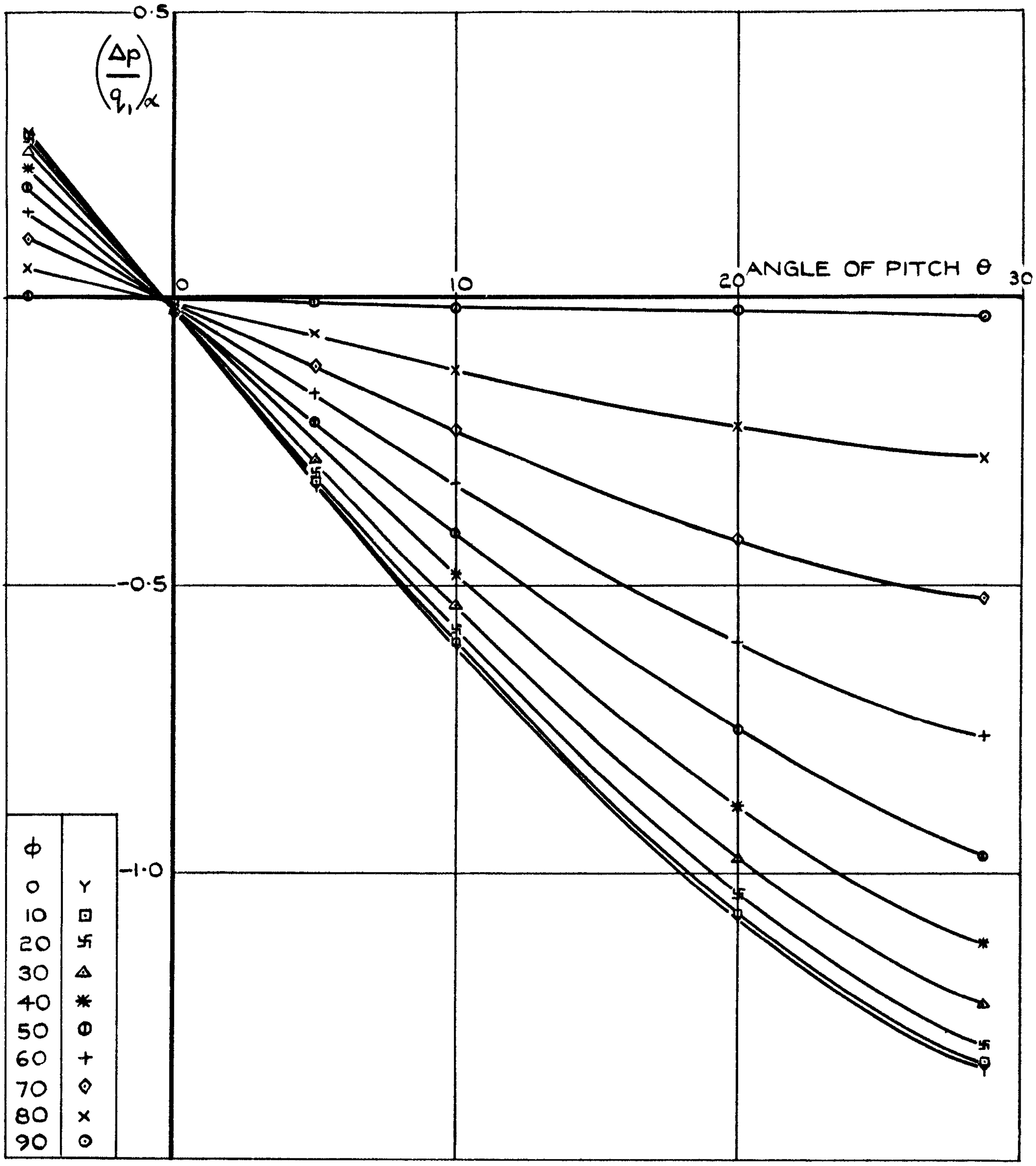
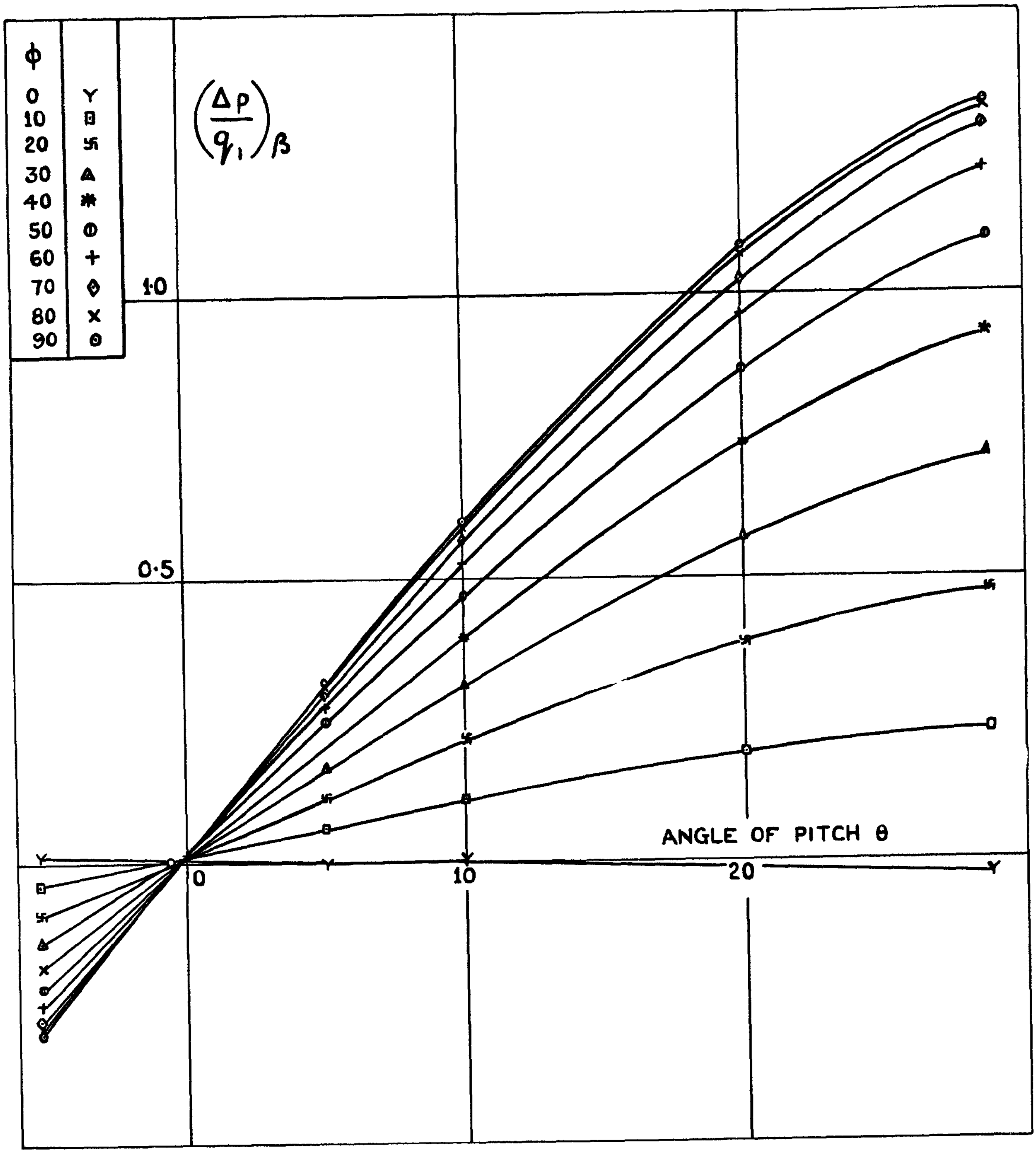


FIG. 6. PITOT PRESSURE VARIATION WITH PITCH ANGLE.



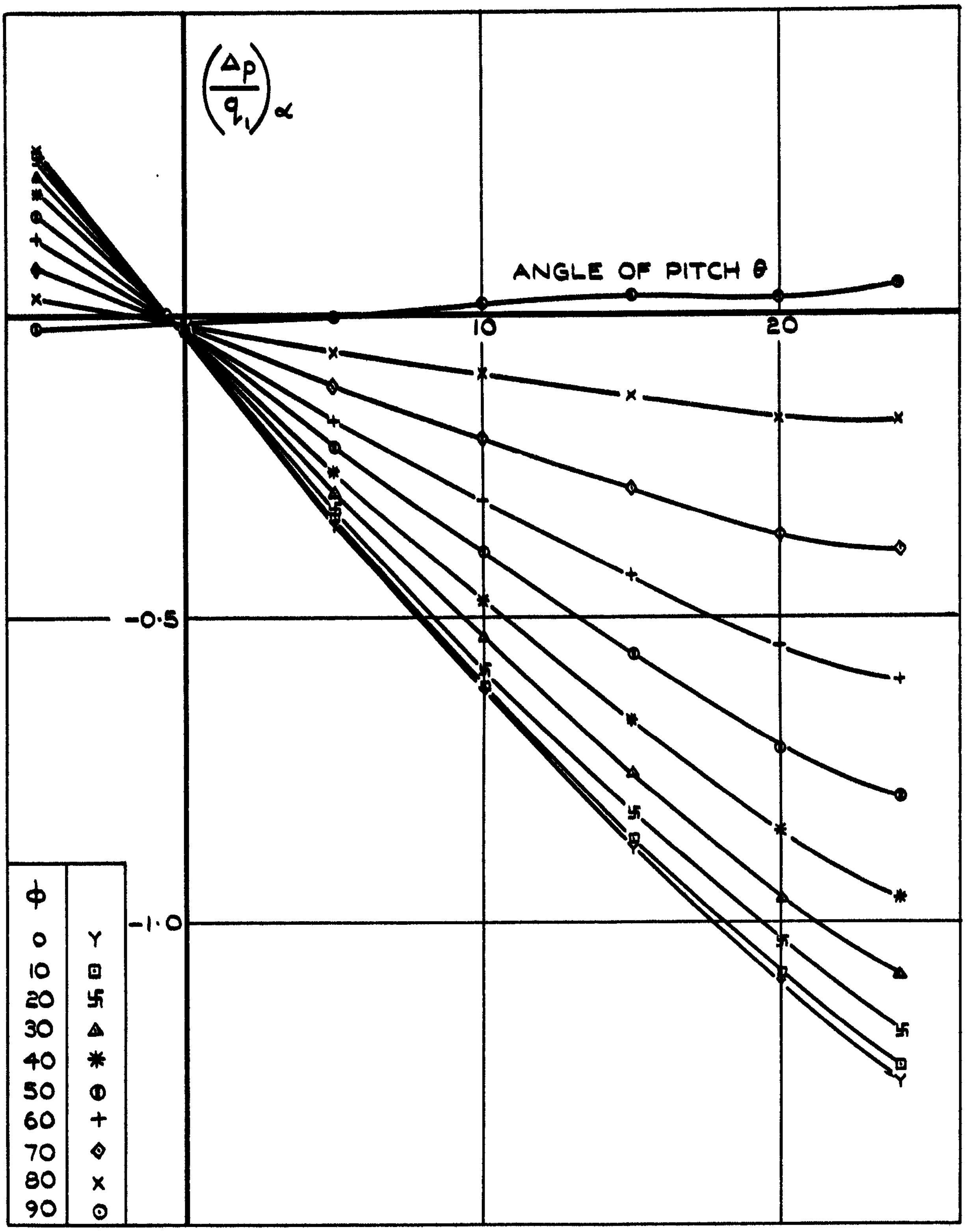
(a) M = 2·47. ORIFICES c AND a

FIG. 7 VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.



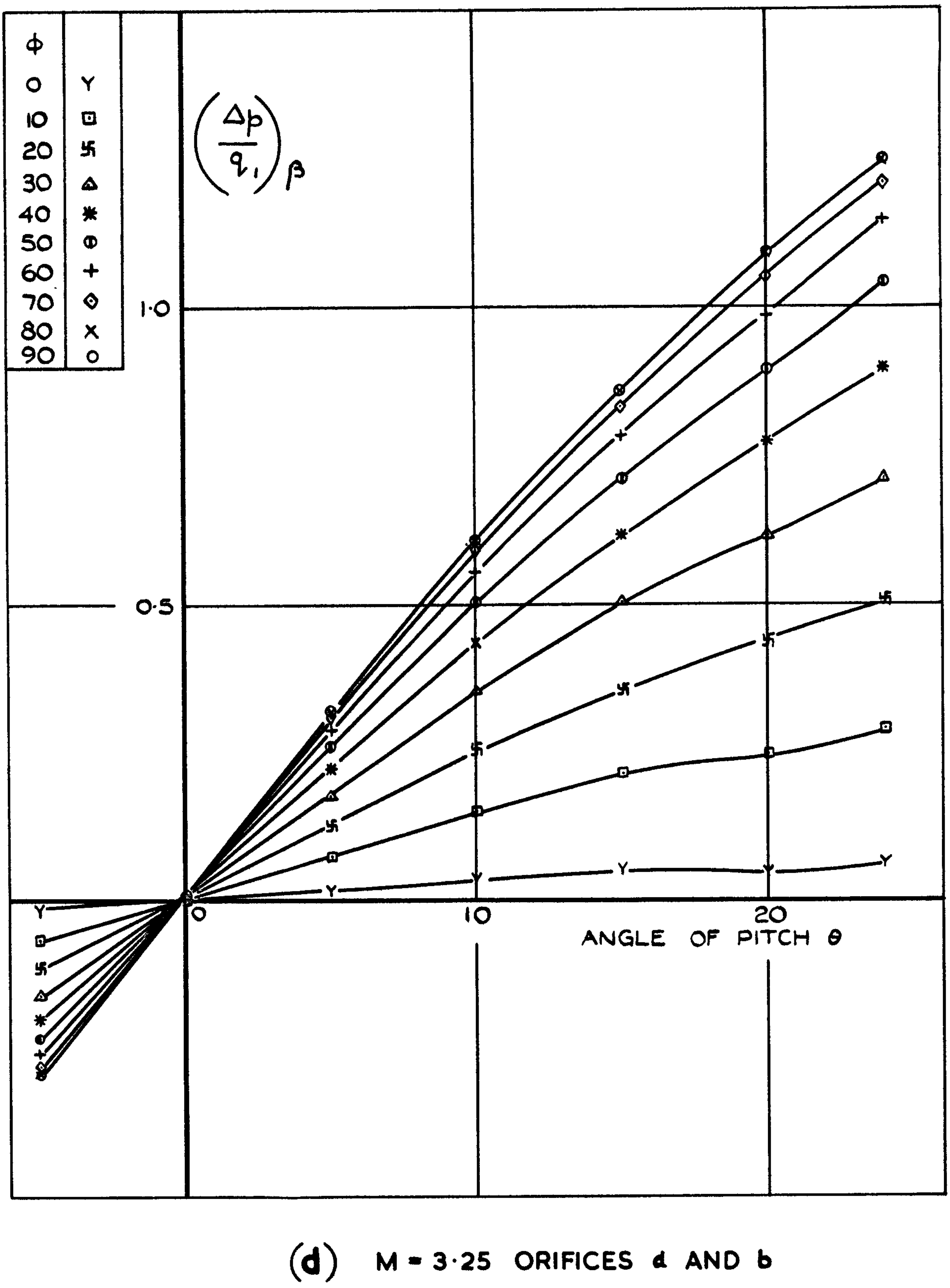
(b) M = 2·47 ORIFICES *d* AND *b*.

FIG. 7. (contd) VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.



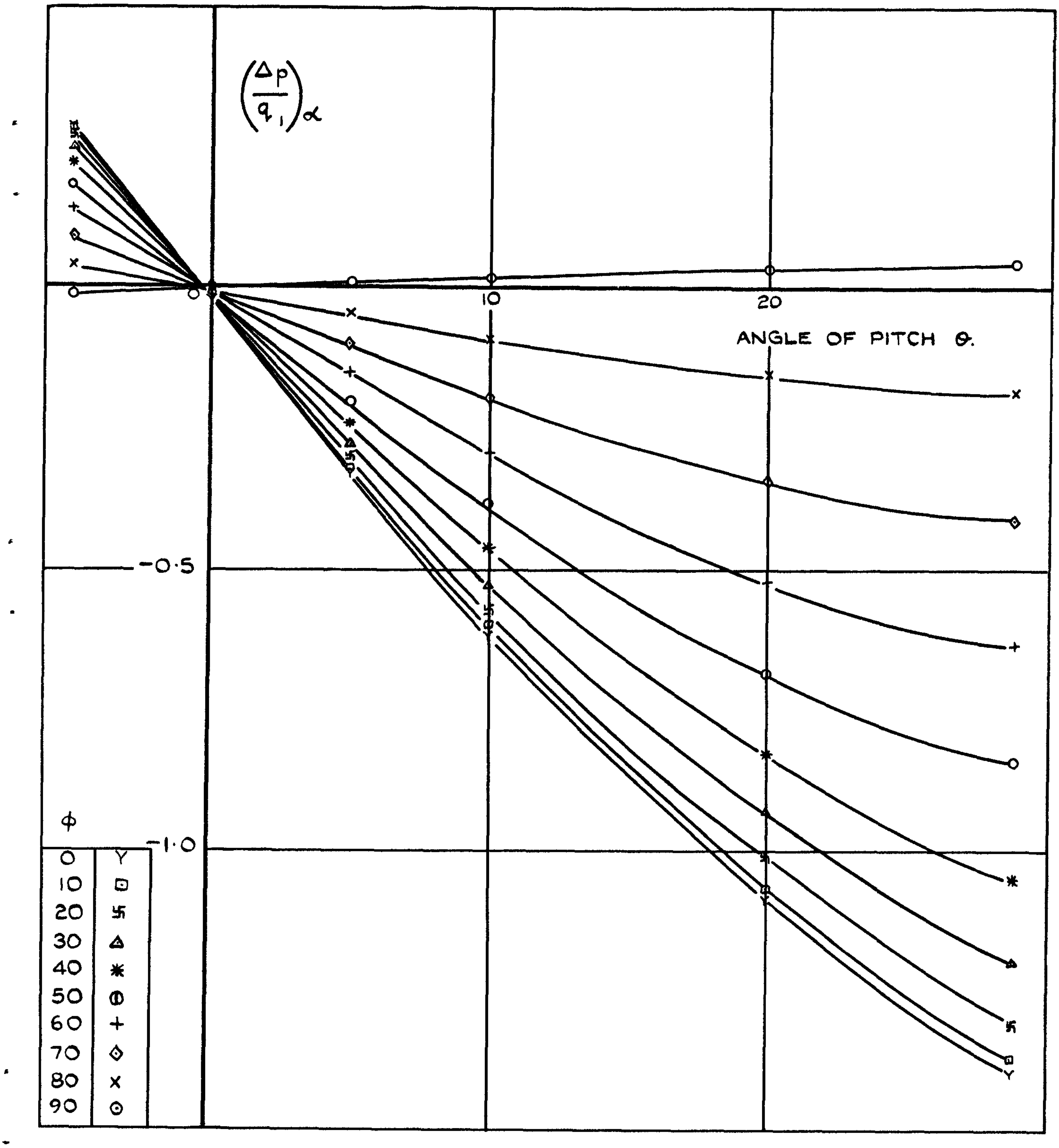
(c) M = 3·25 ORIFICES c AND a

FIG. 7. (contd). VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.



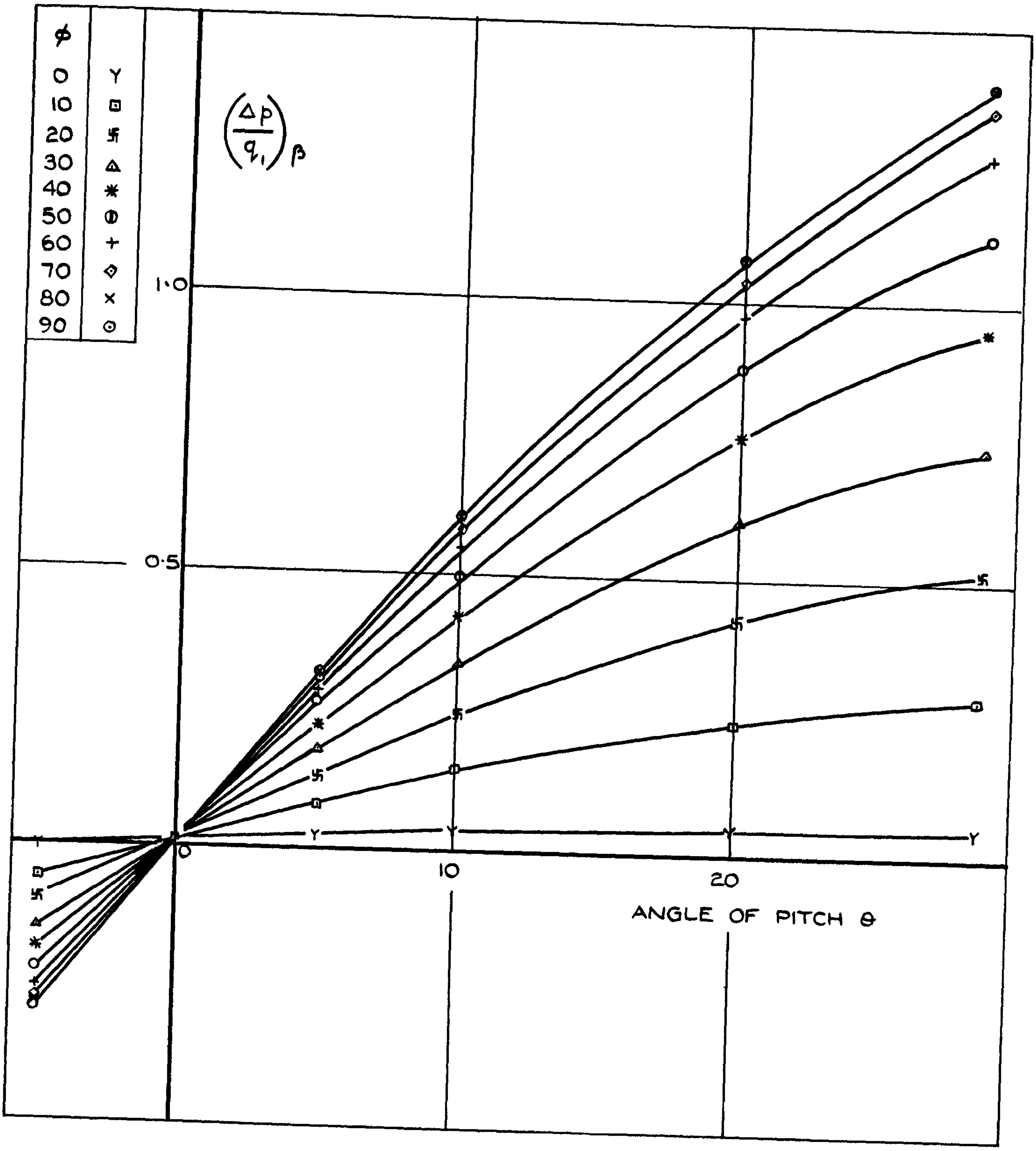
(d) M = 3·25 ORIFICES d AND b

FIG. 7. (contd). VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.



(e) M 4·32, ORIFICES c AND a

FIG. 7. (contd). VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.



(f) M = 4·32 ORIFICES d AND b

FIG. 7. (concld). VARIATION OF STATIC PRESSURE DIFFERENCE WITH PITCH ANGLE.

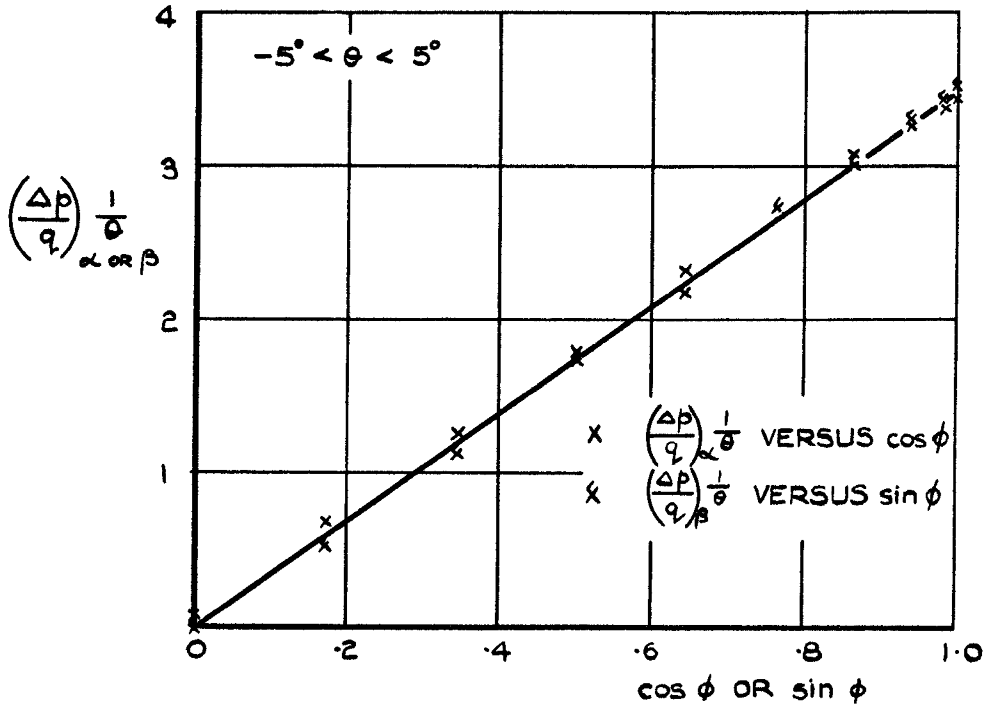


FIG. 8. VARIATION OF $\left(\frac{\Delta p}{q}\right)\frac{1}{\theta}$ WITH ROLL ANGLE ϕ AT M 2·47.

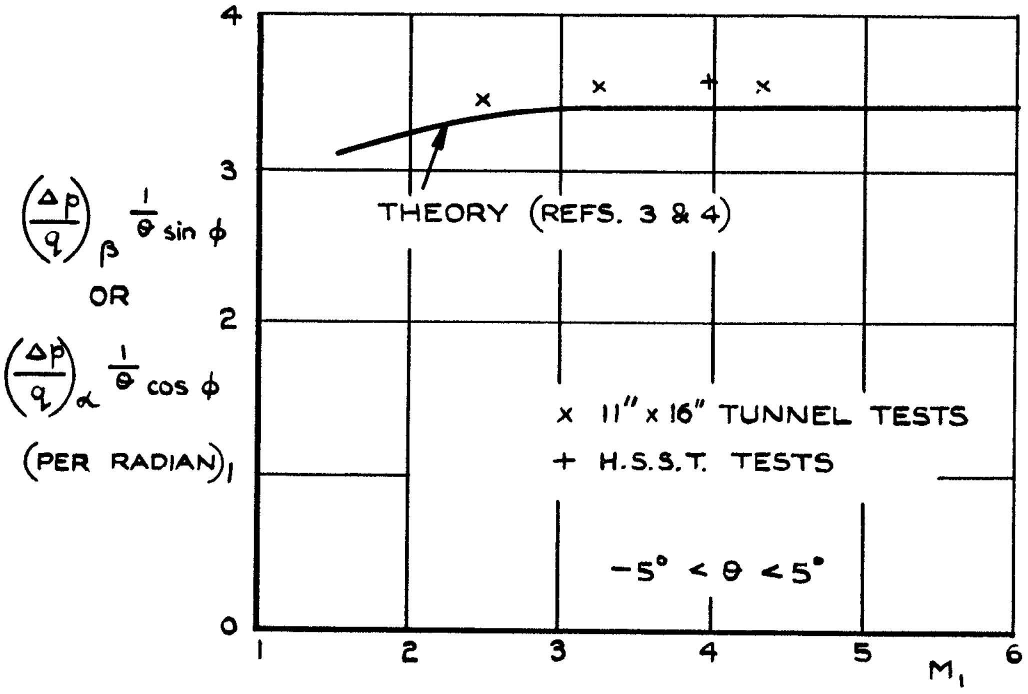
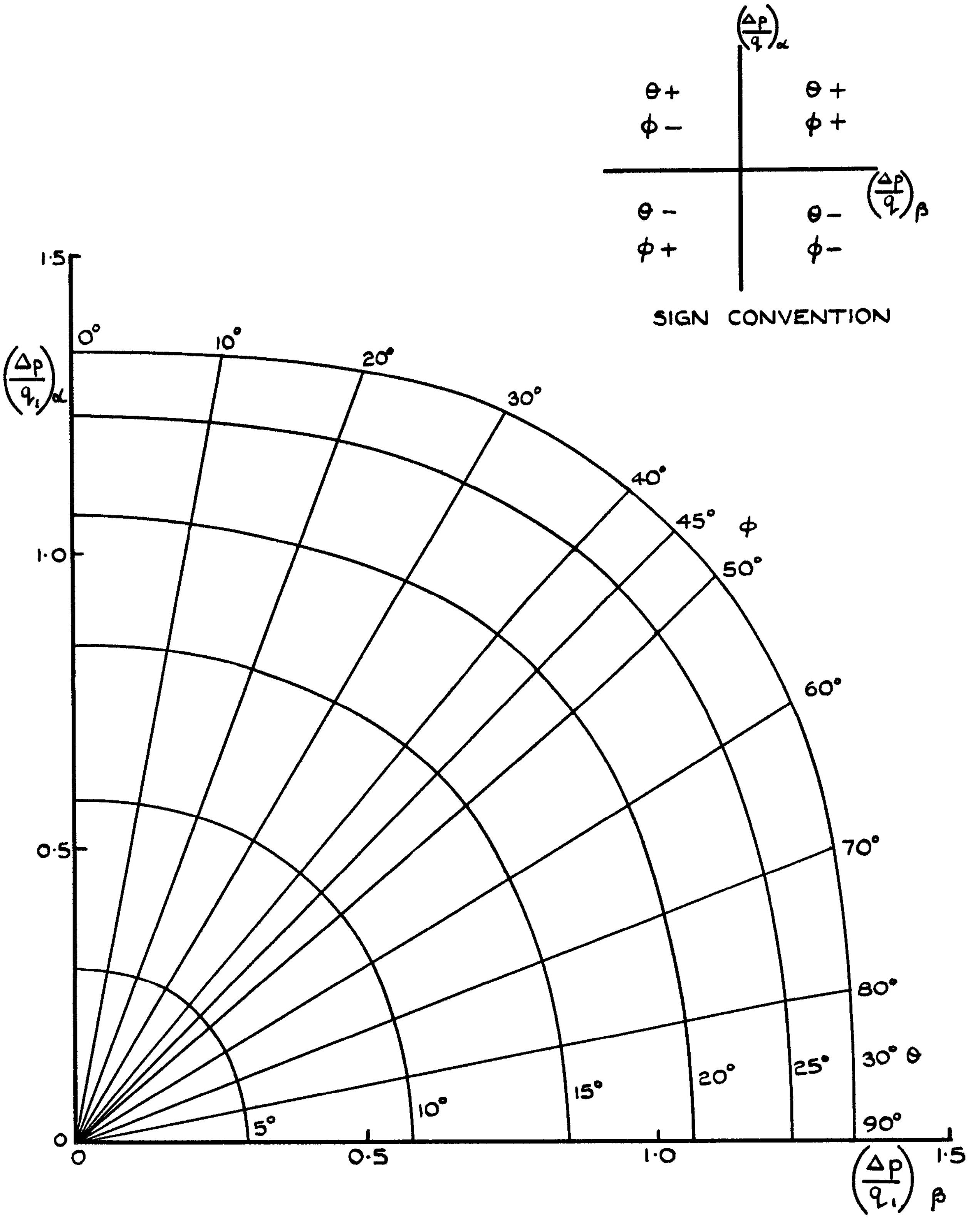
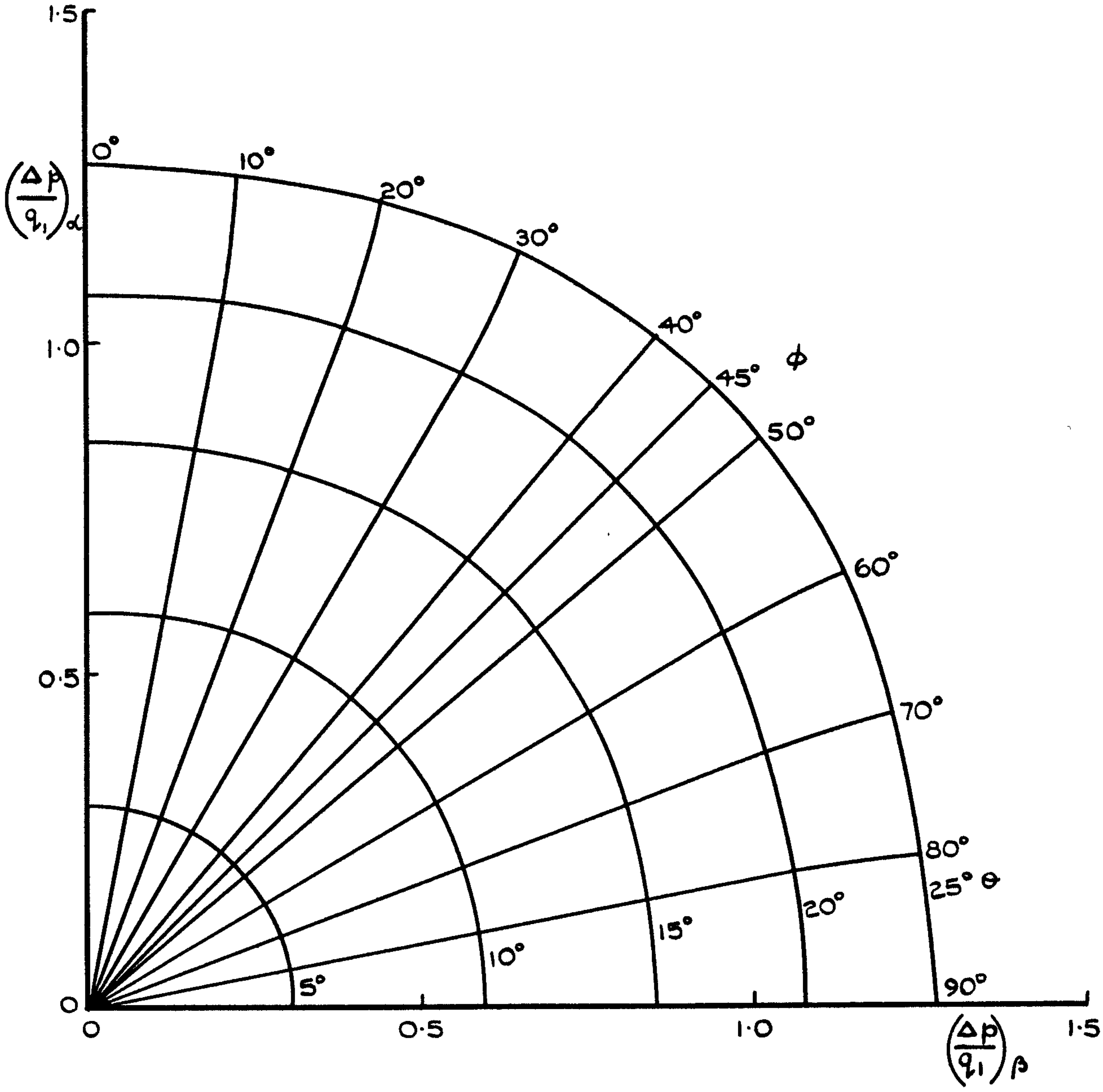


FIG. 9. VARIATION OF YAWMETER SENSITIVITY WITH MACH NUMBER.



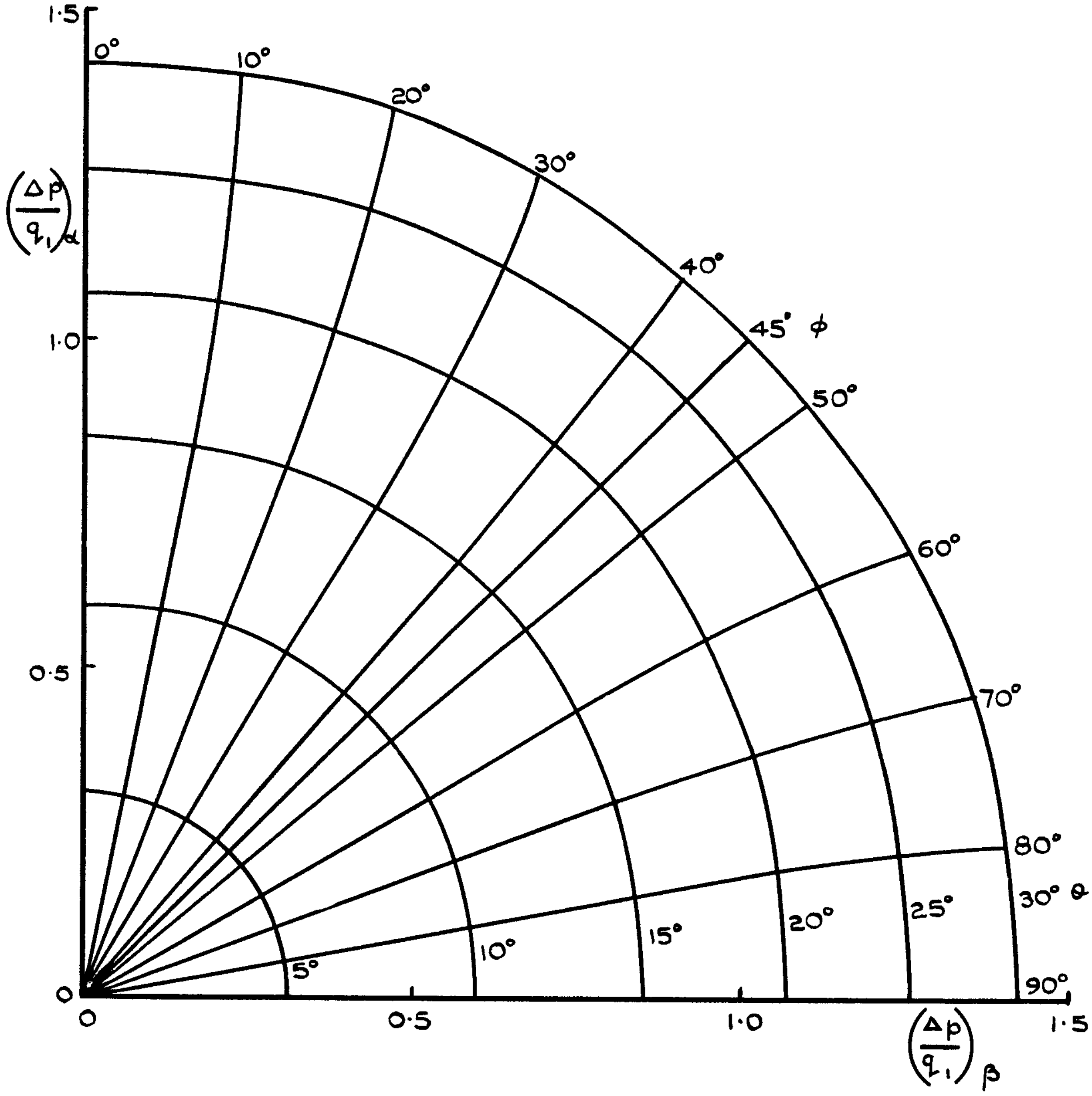
(a) M = 2·47

FIG. 10. CHART FOR DETERMINATION OF PITCH AND ROLL ANGLES.



(b) M = 3·25

FIG. 10. (contd) CHART FOR DETERMINATION OF PITCH AND ROLL ANGLES.



(C) M = 4·32

FIG. 10 (concld). CHART FOR DETERMINATION OF PITCH AND ROLL ANGLES.

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