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A Simplified Treatment of Losses for One-Dimensional Mixing Between Hot and Cold Gas Streams at Constant Pressure and Low Velocity

By

B.S. Stratford and J.G. Williams

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A Simplified Treatment of Losses for One-dimensional Mixing Between Hot and Cold Gas Streams at Constant Pressure and Low Velocity

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B. S. Stratford and J. G. Williams

June, 1962

SUMMARY

Standard theory for the one-dimensional mixing between hot and cold gas streams is straightforward in principle, but the results can be rather difficult to interpret, as well as quite lengthy to calculate. The interpretation can be difficult either in obtaining a physical understanding, or for seeing general trends.

Several simplified formulae may be put forward to supplement the existing methods of analysis. Most of the formulae in the present paper are concerned with the loss of total pressure during mixing; these formulae are limited to flow at low Mach number. A further analysis concerns the gain of thrust which results when two streams of air supplying a propelling nozzle in compressible flow are mixed before the nozzle.

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1. Introduction

Several methods exist for calculating the pressure losses during mixing between hot and cold streams, see, for example, References 1 to 3. These methods are accurate within the assumptions of one-dimensional flow and complete mixing. The formulae which they provide, however, do not readily show the general trends of behaviour, while the calculations are fairly long. Moreover, if the pressure losses are a small proportion of the total pressure the result may be given as the small difference of large quantities and difficulty can be experienced in performing the calculation accurately - particularly when allowance is made for the variation of specific heat. An alternative approach is therefore provided in the present paper.

There could also be some advantage in a method of calculation which gave an extremely simple answer even though it were not highly accurate. Such a method could readily show general trends, it might facilitate an intuitive or a physical understanding of the flow, and it might be of assistance in assessing the behaviour of more complex flows not satisfying the idealised one-dimensional conditions of the theory. Consequently the exact results are further simplified with these possibilities in view.

2. The Pressure Loss due to Mixing Between Streams of Constant Specific Heat

When two streams of mass flow rates m_1, m_2 and velocities u_1, u_2 mix completely at constant static pressure to form a stream m_3, u_3, as in Fig.1, the final velocity is given by conservation of momentum to be

$$u_3 = (m_1 u_1 + m_2 u_2)/m_3 \qquad \ldots (1)$$

where

$$m_3 = m_1 + m_2 \qquad \ldots (2)$$

For incompressible or low Mach number streams of the same perfect gas the final temperature is

$$T_3 = (m_1 T_1 + m_2 T_2)/m_3 \qquad \ldots (3)$$

so that the final density is given by

. $1/\rho_3$/

$$1/\rho_3 = (m_1/\rho_1 + m_2/\rho_2)/m_3 \qquad \ldots (4)$$

Thus the final dynamic head, $\frac{1}{2}\rho_3 u_3^2$, is

$$\tfrac{1}{2}\rho_3 u_3^2 = \frac{1}{2}\,\frac{(m_1 u_1 + m_2 u_2)^2}{(m_1 + m_2)\,(m_1/\rho_1 + m_2/\rho_2)} \qquad \ldots (5)$$

Equation (5) will be found to be equivalent to

$$\tfrac{1}{2}\rho_3 u_3^2 = \frac{\frac{1}{2}\rho_1 u_1^2 m_1/\rho_1 + \frac{1}{2}\rho_2 u_2^2 m_2/\rho_2}{m_1/\rho_1 + m_2/\rho_2} - \frac{1}{2}\,\frac{m_1 m_2 (u_2 - u_1)^2}{(m_1 + m_2)\,(m_1/\rho_1 + m_2/\rho_2)} \qquad \ldots (6a)$$

$$\text{i.e.,}\quad P_3 = \left\{\left(\frac{m_1}{\rho_1}P_1 + \frac{m_2}{\rho_2}P_2\right)\Big/\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)\right\} - \frac{\frac{1}{2}\,m_1 m_2 (u_2 - u_1)^2}{(m_1 + m_2)\,(m_1/\rho_1 + m_2/\rho_2)} \qquad \ldots (6b)$$

The first term on the right hand side of Equation (6b) is the volume flow mean total pressure before mixing, and is invariant with respect to the static pressure at which the mixing occurs. The second term, which is entirely dependent on the static pressure, may therefore be interpreted as the loss of total pressure, say $(-\Delta P)_a$, suffix 'a' denoting constant specific heat. Thus, if the volume flow mean dynamic head before mixing is q, and the ratios m_2/m_1 and u_2/u_1 are denoted m and u,

$$q = \left(\frac{m_1}{\rho_1}(\tfrac{1}{2}\rho_1 u_1^2) + \frac{m_2}{\rho_2}(\tfrac{1}{2}\rho_2 u_2^2)\right)\Big/\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)$$

$$= \tfrac{1}{2}(m_1 u_1^2 + m_2 u_2^2)\,/\,(m_1/\rho_1 + m_2/\rho_2) \qquad \ldots (7a)$$

$$\frac{(\Delta P)_a}{q} = -\frac{m_1 m_2 (u_2 - u_1)^2}{(m_1 + m_2)\,(m_1 u_1^2 + m_2 u_2^2)} \qquad \ldots (7b)$$

$$= -\frac{m(u - 1)^2}{(1 + m)\,(1 + mu^2)} \qquad \ldots (7c)$$

Equation (7b)/

Equation (7b) can be interpreted physically as showing that the proportional loss of dynamic head when two streams mix at constant static pressure is the same as the proportional loss of K.E. when two inelastic masses collide. The two phenomena are in fact essentially the same and the equation can be derived from this assumption. The loss in each instance is due to a dissipation of kinetic energy proportional to the square of the difference of the velocities.

Equation (7) is a reasonably convenient formula for the pressure loss in one-dimensional incompressible flow at constant static pressure and constant specific heat, for which conditions it is exact. It will now be simplified for special applications.

2.1 Equal total pressure before mixing

When the initial total pressures of the two streams are equal,

$$\tfrac{1}{2}\rho_1 u_1^{\,2} = \tfrac{1}{2}\rho_2 u_2^{\,2} \qquad \dots (8)$$

and therefore

$$u_1^{\,2}/u_2^{\,2} = \rho_2/\rho_1 = T_1/T_2 \qquad \dots (9)$$

Equation (7b) then becomes

$$\frac{(\Delta P)_a}{q} = -\frac{m_1 m_2 \, (T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}})^2}{(m_1 + m_2)\,(m_1 T_1 + m_2 T_2)} \qquad \dots (10)$$

or, from Equation (3),

$$\frac{(\Delta P)_a}{q} = -\frac{m_1 m_2 \, (T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}})^2}{(m_1 + m_2)^2 \, T_3} \qquad \dots (11)$$

If a quantity δ' is defined by

$$\delta' = (T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}})/T_3^{\frac{1}{2}} \qquad \dots (12)$$

Equation (11) becomes

$$\frac{(\Delta P)_a}{q} = -\frac{m_1 m_2}{(m_1 + m_2)^2}\,\delta'^2 \qquad \dots (13)$$

Equation (13)/

Equation (13) is still exact for the same conditions as Equation (7) with the additional limitation that the initial total pressures are equal.

Now for some applications a simpler expression than Equation (13) would be useful even though it were only roughly correct. It is readily shown that when the mass flow is unity Equation (13) becomes

$$-(\Delta P)_a/q = \tfrac{1}{4}\delta^2 + O(\delta^4) \qquad \ldots (14)$$

where

$$\delta = (T_2 - T_1)/2T_3 \qquad \ldots (15a)$$

Fig.2 shows that, although

$$-(\Delta P)_a/q \doteqdot \tfrac{1}{4}\delta^2 \qquad \ldots (15b)$$

might be considered a severe simplification, yet nevertheless it is a reasonable representation of the loss over a range of mass flow ratios, m_2/m_1, from 0·6 to well over 2·5, thus including most practical instances.

2.2 Total pressure nearly equal before mixing

When the total pressures are nearly equal the result corresponding to Equation (15b) becomes

$$-(\Delta P)_a/q \doteqdot (\delta/2 + \phi/5)^2 \qquad \ldots (16)$$

where

$$\phi = (P_2 - P_1)/q \qquad \ldots (17)$$

Comparison with the exact expression of Equation (7) is shown in Fig.3.

2.3 . Sinusoidal temperature distribution

One advantage of Equation (15b) may be seen in the simplicity with which it may be applied to the internal mixing of a single stream having initially a sinusoidal type of temperature variation.

In order to find the loss an element of the warmer flow is paired with an element of the cooler such that the mean temperature of the two elements is equal to the overall mean and their mass ratio the same as the overall mass ratio of hot to cold. The loss is then obtained by integration for all such pairs. Since the loss for each pair is approximately proportional to the square of the value of δ for that pair, the ratio of the loss with a sinusoidal profile, to that with a square profile between the same maximum and minimum temperatures, is just

$$\int_0^{\pi/2} \quad /$$

$$\int_0^{\pi/2} \sin^2 y \, dy \bigg/ \int_0^{\pi/2} dy,$$

i.e., a half. Thus corresponding to Equation (15b), a sinusoidal type of temperature distribution would give

$$-(\Delta P)_a/q \doteqdot \tfrac{1}{8}\delta^2_{m,m} \qquad \ldots (18)$$

where $\delta_{m,m}$ is the value of δ between the maximum and minimum temperatures i.e.,

$$\delta_{m,m} = (T_{max} - T_{min})/2T_3 \qquad \ldots (19)$$

The result from Equation (16) is similarly halved for a sinusoidal distribution, provided the local value of ϕ for a pair of elements is proportional to that of δ.

2.4 The minimum loss of total pressure

Lutz shows that there is a static pressure for which the mixing loss is a minimum[3].

Returning to Equation (6b), i.e.,

$$P_3 = \frac{P_1 m_1/\rho_1 + P_2 m_2/\rho_2}{m_1/\rho_1 + m_2/\rho_2} - \frac{1}{2}\,\frac{m_1 m_2 (u_2 - u_1)^2}{(m_1 + m_2)(m_1/\rho_1 + m_2/\rho_2)}$$

... (6b) bis

the first term on the right hand side is independent of the static pressure at which the mixing occurs, while the second term is essentially negative or zero, being zero if $(u_2 - u_1)$ is zero. Thus the maximum total pressure possible after mixing is equal to the volume flow mean total pressure before mixing, and this total pressure is attained if the stream velocities of u_2 and u_1 are equal - there then being no dissipation of kinetic energy. For the velocities to be equal,

$$(P_1 - p)/\rho_1 = \tfrac{1}{2}u_1^2 = \tfrac{1}{2}u_2^2 = (P_2 - p)/\rho_2 \qquad \ldots (20)$$

which will be found to give

$$p = \frac{P_2 T_2 - P_1 T_1}{T_2 - T_1} \qquad \ldots (21)$$

and/

and hence

$$P_2 - p = T_1 (P_1 - P_2)/(T_2 - T_1) \qquad \ldots (22)$$

Equation (21) gives the value of the static pressure at which there would be zero loss. As would be expected intuitively the velocities can only be equalised if the cooler stream is at the higher total pressure; Equation (22) demonstrates this result as the quantity $(P_2 - p)$ is essentially positive, requiring that $(P_1 - P_2)/(T_2 - T_1)$ is also positive.

When the hot stream is at the higher total pressure Equation (6b) shows that the criterion for minimum loss is that $(u_2 - u_1)$ is a minimum - i.e., that the static pressure is such that

$$(d/dp)(u_2 - u_1) = 0 \qquad \ldots (23)$$

Since

$$\tfrac{1}{2}\rho u^2 = P - p$$

i.e.,

$$\rho u du = - dp \qquad \ldots (24)$$

Equation (23) gives

$$\rho_1 u_1 = \rho_2 u_2 \qquad \ldots (25)$$

or

$$u_1/u_2 = T_1/T_2 \qquad \ldots (26)$$

to be the condition for minimum loss when the hot stream has the greater total pressure.

As would be expected differentiation of $(u_2 - u_1)^2$, in place of $(u_2 - u_1)$ as in Equation (23), gives the criteria for either stream being at the greater total pressure.

2.5 Comparison between mixing at constant static pressure and mixing in a duct of constant cross-sectional area

For the constant pressure mixing the area ratio is

$$\frac{a_3}{a_1 + a_2} \Big/$$

$$\frac{a_3}{a_1 + a_2} = \frac{(m_1 + m_2)(m_1/\rho_1 + m_2/\rho_2)}{(m_1/\rho_1 u_1 + m_2/\rho_2 u_2)(m_1 u_1 + m_2 u_2)} \quad \ldots (27)$$

If $\rho_1 u_1 = \rho_2 u_2$ this ratio is seen to be unity, so that mixing at constant pressure is then identical with mixing at constant area. If the area ratio is a little greater than unity the mean flow area during mixing is slightly greater than for mixing in a duct of constant area; hence the mean dynamic head during the mixing is slightly less and therefore the loss will be slightly less. This result holds typically for mixing between two streams at equal total pressure, the difference between the losses being typically about 10% of the loss. For example, using the method of Reference 1 the loss when $T_2 = 4T_1$, and $m_2 = m_1$, becomes $(\Delta P)_a/q = 11{\cdot}1\%$ when the flow area is constant, compared with 10% given by Equation (13) of the present paper, for mixing when the static pressure is constant. Since in this example the total pressures and hence the dynamic heads are equal, $\rho_1 u_1^2 = \rho_2 u_2^2$ and hence $\rho_1 u_1/\rho_2 u_2 = (T_2/T_1)^{\frac{1}{2}} = 2$. Thus the difference between the two types of mixing would only be expected to be large when the products $\rho_1 u_1$ and $\rho_2 u_2$ in Equation (27) are very different from each other.

As noticed by Lutz[3], the analysis for mixing at constant static pressure is rather simpler than for that at constant area.

3. The Pressure Loss with Varying Specific Heat

Returning to the initial analysis of Section 2, Equations (1) and (2), for the velocity and mass flow after mixing at constant static pressure, hold independently of the specific heat. Thus the only effect of a varying specific heat on the final dynamic head of Equation (5) would be in the effect on the density. If the actual final temperature is denoted T_{ac} and that for ideal gases T_{id}, the proportional reduction in the final density as a result of the variation in specific heat is

$$(\rho_{id} - \rho_{ac})/\rho_{id} = (T_{id}^{-1} - T_{ac}^{-1})T_{id} = (T_{ac} - T_{id})/T_{ac} \quad \ldots (28)$$

The corresponding reduction in total pressure, say $(-\Delta P)_b$ - suffix 'b' denoting the increment resulting from the variation of specific heat - is

$$-(\Delta P)_b = (\tfrac{1}{2}\rho_3 u_3^2)_{id}\,(T_{ac} - T_{id})/T_{ac} \quad \ldots (29)$$

so that

$$-(\Delta P)_b/\{q - (\Delta P)_a\} = (T_{ac} - T_{id})/T_{ac} \quad \ldots (30)$$

Equation (30) is a reasonably convenient formula for the additional pressure loss which results from the variation of specific heat; it should be exact for one-dimensional mixing of incompressible flows at constant static pressure. The right hand side of Equation (30) is independent of the initial difference in total pressure between the two streams.

Now/

Now if in a gas the increase in specific heat were linear with temperature the right hand side of Equation (30) would be found algebraically to be proportional to δ^2, for small values of δ; δ is defined as in Section 2 by Equation (15a), T_3 being understood as $T_{3,id}$. Consequently the right hand side of Equation (30) has been evaluated numerically for a real gas - air, with zero fuel - and values of a coefficient 'A' sought which would enable Equation (30) to be replaced by

$$- (\Delta P)_b / \{q - (\Delta P)_a\} \doteqdot A\delta^2 \qquad \ldots (31)$$

The calculations were made for a range of values of δ at each of several values of the mean tempature, for four mass flow ratios. For a mass flow ratio of unity the quantity $(T_{ac} - T_{id})/T_{ac}$ was found to be very closely proportional to δ^2, as shown, for example, in Fig.4 for a final temperature T_{id} of 1000°K. For the other mass flow ratios the result did not fit a δ^2 curve so well, but the absolute discrepancy is small. The resulting values for 'A', as given in Fig.5, when used in conjunction with Equation (31), gives $(\Delta P)_b / \{1 - (\Delta P)_a/q\}$ correct to within ½% of q, for temperature ratios up to 4/1. The values of 'A' are seen to be only slightly affected by the mass flow ratio of the two streams and, for temperatures T_{id} above 1000°K, may be represented by

$$A = 0{\cdot}080 - 0{\cdot}030 \{ (T_{id}/1000°K) - 1 \} \ldots T_{id} > 1000°K \qquad \ldots (32)$$

To the accuracy of Equation (14b) Equation (31) may be simplified to

$$- (\Delta P)_b/q = A\delta^2 \qquad \ldots (33)$$

Equations (15b) and (33) may then be combined to give that the loss of total pressure for two streams of air of equal total pressures mixing at constant static pressure is

$$- \Delta P/q = - (\Delta P)_a/q - (\Delta P)_b/q \doteqdot (\tfrac{1}{4} + A)\delta^2 \qquad \ldots (34)$$

where A is given by Fig.5 or Equation (32).

In the range of temperatures likely to be met in a turbojet engine Equation (34) may be replaced by

$$- \Delta P/q \doteqdot 0{\cdot}32\ \delta^2 \qquad \ldots (35)$$

For streams whose initial total pressures are nearly equal Equations (16) and (33) give

$$- \Delta P/q \doteqdot (\delta/2 + \phi/5)^2 + A\delta^2 \qquad \ldots (36)$$

As for Equation (18) the loss for an initial sinusoidal temperature distribution is a half of that for a rectangular distribution between the same maximum and minimum temperatures.

For air containing fuel the values of A are greater than shown in Fig.5. A limited number of calculations at unit mass flow ratio gave increases in A of 50 and almost 100% for fuel/air ratios in the hot stream of 0·02 and 0·04 respectively. The coefficients in Equation (35) increase correspondingly to 0·36 and 0·39.

3.1 Comparison with the exact method of Lewis and Drabble

The method of calculation for constant pressure mixing in the Appendix of Reference 2 has been used to calculate the following example:

$$m_2 = m_1, T_2 = 1800°K, T_1 = 600°K,$$

with zero fuel/air ratio and the Mach number in each stream 0·500. The two total pressures are very slightly different (by 6% of the static pressure) owing to the values of γ being different.

Taking values of γ appropriate to the mean of the static and total temperatures for each stream, and using four significant figures in the calculations, the loss given by the method of Reference 2 becomes about $8\frac{1}{4}$% of the initial dynamic head.

Using the "$0{\cdot}32\ \delta^2$" formula of Equation (35), $\delta = 0{\cdot}500$, and the loss is 8·0%.

Using Equation (13), and ignoring the slight difference of total pressure mentioned previously, the loss for a constant specific heat is 6·7%, while Equation (31) and Fig.5 give a 1·7% increment due to variation of specific heat; thus the total loss using Equations (13) and (31), together with Fig.5, is 8·4%.

The comparison appears satisfactory and the methods of the present paper would seem applicable at least up to 0·5 Mach number. In the comparison the dynamic head, q, in compressible flow has been taken as the difference of the total and static pressures, rather than as $\frac{1}{2}\rho u^2$. The ratio $(P - p)/\frac{1}{2}\rho u^2$ is 1·064 at $M = 0{\cdot}5$, so that the effect on the calculation is small.

4. The Thrust Gain due to Mixing

The gross thrust M from a jet of mass flow m and uniform nozzle velocity u is

$$M = mu \qquad \ldots (37)$$

If the total temperature of the jet is T and if the fluid is a perfect gas the thrust for inviscid flow may be expressed

$$M = mT^{\frac{1}{2}}f \qquad \ldots (38)$$

where/

where f is a function of the total to static pressure ratio, the flow here being considered compressible. Consequently the gain of thrust obtained when two streams of a perfect gas at the same total pressure, but different temperatures, are mixed at stagnation pressure before expansion through the nozzle (instead of being expanded separately) is

$$(\Delta M)_a = (m_1 + m_2)T_3^{\frac{1}{2}}f - (m_1 T_1^{\frac{1}{2}}f + m_2 T_2^{\frac{1}{2}}f) \qquad \ldots (39)$$

where $(m_1 + m_2)T_3 = m_1 T_1 + m_2 T_2$ ... (40)

Thus

$$\frac{(\Delta M)_a}{M} = 1 - \frac{m_1}{m_1 + m_2}\left(\frac{T_1}{T_3}\right)^{\frac{1}{2}} - \frac{m_2}{m_1 + m_2}\left(\frac{T_2}{T_3}\right)^{\frac{1}{2}} \qquad \ldots (41)$$

M being the thrust from the pre-mixed jet.

From Equation (40) the following relationships may be obtained:-

$$\left.\begin{aligned} T_1/T_3 &= 1 - 2\delta m_2/(m_1 + m_2) \\ T_2/T_3 &= 1 + 2\delta m_1/(m_1 + m_2) \end{aligned}\right\} \qquad \ldots (42)$$

where

$$\delta = (T_2 - T_1)/2T_3 \qquad \ldots (15a) \text{ bis}$$

Equation (41) then becomes

$$\frac{(\Delta M)_a}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}\,\frac{\delta^2}{2} + O(\delta^3) \qquad \ldots (43)$$

the term $O(\delta^3)$ becoming $O(\delta^4)$ when $m_1 = m_2$. For a reasonable range of mass flow ratios Equation (43) may be simplified to

$$(\Delta M)_a/M \doteqdot \tfrac{1}{8}\delta^2 \qquad \ldots (44)$$

Fig. 6 shows the results from Equation (44) in comparison with the exact results from Equation (41). For sinusoidal distributions the effect is approximately halved as in previous sections.

The simplification from Equation (43) to Equation (44) loses the property that $(\Delta M)/M$ becomes zero when m_2/m_1 becomes zero or infinite. Use of the full first term of Equation (43) would have retained this property, but, as most practical applications seem likely to be within the range of applicability of the very short $\frac{1}{8}\delta^2$ formula of Equation (44) (see Fig. 6), the $\frac{1}{8}\delta^2$ formula seems the most appropriate simplification.

For/

For a real gas, for which the specific heat varies with temperature, a similar argument to that in Section 3 can be used to determine the additional gain of thrust. Considering initially the jet at low Mach number, so that the total and static temperatures are nearly equal, the proportional gain in u^2, resulting from the actual mixed temperature exceeding the ideal, is the same as the previous proportional loss in $\frac{1}{2}\rho u^2$. Thus the proportional gain in velocity, and hence thrust, is a half the previous proportional loss of dynamic head, i.e.,

$$(\Delta M)_b/M = \tfrac{1}{2}A\delta^2 \qquad \ldots (45)$$

where A is as in Fig.5. For compressible flow the increment of work output by the gas in accelerating the jet relative to the nozzle during any given increment of pressure change during the expansion is proportional to the temperature of the gas, while the temperature drop is proportional to the work output, and therefore to the initial temperature. Following such general arguments, and noting that the variation of A is fairly small within the practical range of turbojet engines, it would seem that Equation (45) would hold also for compressible flow provided A is taken as a mean over the range of static temperature during the expansion. Thus, to the accuracy of Equation (44), the total gain in a gas with varying specific heat may be written

$$(\Delta M)/M = 0{\cdot}16\ \delta^2 \qquad \ldots (46)$$

For fuel/air ratios of 0·02 and 0·04 respectively in the hot stream the coefficient in Equation (46) would become 0·18 and 0·20 respectively.

The following two examples compare results from the preceding equations with those from the standard method of analysis based on the data of Reference 4. In each example the mass flow ratio of the two streams is unity.

For the first example the initial total temperatures are 1800°K and 600°K when unmixed, giving a total temperature of the mixed stream of 1200°K for an ideal gas and 1223°K for air. The nozzle pressure ratio is 30. The standard method of calculation gives a gain of gross thrust of about $4\frac{1}{4}$% using pure air. Equation (41) gives the gain to be 3·45% for an ideal gas having constant specific heat. Equation (45) with Fig.5, taking a mean value of A to be 0·071 between 1200°K and 500°K - the approximate range of static temperature of the mixed stream - gives the increment due to variation of specific heat to be 0·89%. Thus the total gain becomes 4·34%. On the other hand using the "$0{\cdot}16\ \delta^2$" formula from Equation (46), $\delta = 0{\cdot}500$ and so the gain for air is 4·0%. For a fuel/air ratio of 0·04 in the hot stream the standard calculation gives a gain of about 4·95% while the "$0{\cdot}20\ \delta^2$" method gives a gain of 5·0%.

For the second example the initial temperatures are 1200°K and 600°K, and the nozzle pressure ratio 10. The standard method gives a gain of 1·9%. Equation (41) gives 1·45% for an ideal gas and Equation (45) with Fig.5 gives an increment of 0·37%, i.e., a total of 1·82%. The "$0{\cdot}16\ \delta^2$" formula with $\delta = \frac{1}{3}$ gives 1·78%.

In/

In these examples the simplest formula appears quite adequate for calculating the gain of thrust resulting from pre-mixing.

5. Conclusions

The algebraic analysis for the pressure loss in one-dimensional mixing between two streams of incompressible flow can be treated very simply when the mixing occurs at constant static pressure, while the formulae obtained for the loss hold also as a good approximation for mixing at constant area. By suitable algebraic manipulation the results may be expressed in a manner which is very readily interpreted physically as, for example, for Equation (7). The exact answer when the total pressures of the streams are equal may be simplified even further if an error of the order of 15% of the loss is acceptable - e.g., when the dynamic head is low. The loss of total pressure is then given by

$$-(\Delta P)_a/q \doteqdot \tfrac{1}{4}\delta^2 \qquad \ldots \text{(15b) bis}$$

for an ideal gas in which the specific heat is constant, or

$$-\Delta P/q \doteqdot 0{\cdot}32\,\delta^2 \qquad \text{(35) bis}$$

for pure air within the temperature range of gas turbines. In these formulae q is the initial dynamic head and δ represents the proportional difference of temperature defined as

$$\delta = (T_2 - T_1)/2T_3 \qquad \ldots \text{(15a) bis}$$

For fuel/air ratios of 0·02 and 0·04 in the hot stream the coefficient 0·32 in Equation (35) is replaced by 0·36 and 0·39 respectively. These formulae apply for hot to cold mass flow ratios in excess of 0·6, i.e., for "by-pass" ratios less than about 1·7/1.

For a propelling nozzle having two air supplies at different temperatures but the same total pressure the proportional increase of gross thrust, $\Delta M/M$, which results from mixing the air at stagnation before expansion through the nozzle, may be expressed

$$(\Delta M)_a/M \doteqdot \tfrac{1}{8}\delta^2 \qquad \ldots \text{(44) bis}$$

for compressible flow in an ideal gas having a constant specific heat, and

$$\Delta M/M \doteqdot 0{\cdot}16\,\delta^2 \qquad \ldots \text{(46) bis}$$

for pure air within the temperature range of gas turbines. For fuel/air ratios of 0·02 and 0·04 in the hot stream the coefficient 0·16 in Equation (46) is increased to 0·18 and 0·20 respectively.

The/

The results for temperature distributions which are initially rectangular are halved in a flow having initially a sinusoidal type of temperature distribution between the same maximum and minimum values.

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No.	Author(s)	Title, etc.
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Notation

m_2	mass flow of the hot stream
m_1	mass flow of the cold stream
m_3	mass flow of the mixed stream
m	m_2/m_1
u_2	velocity of the hot stream before mixing
u_1	velocity of the cold stream before mixing
u_3	velocity of the mixed stream
u	u_2/u_1
ρ	density
T	total temperature
P	total pressure
$-\Delta P$	loss of total pressure
p	static pressure
q	volume flow mean dynamic head before mixing (Equation (7a))
a	cross-sectional area of the flow
δ'	$(T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}})/T_3^{\frac{1}{2}}$
δ	$(T_2 - T_1)/2T_{3,id}$
$\delta_{m,m}$	$(T_{max} - T_{min})/2T_3$
ϕ	$(P_2 - P_1)/q$
γ	ratio of the specific heats Cp/Cv
y	distance measured transversely
A	coefficient of δ^2 for Equation (31)
M	gross thrust

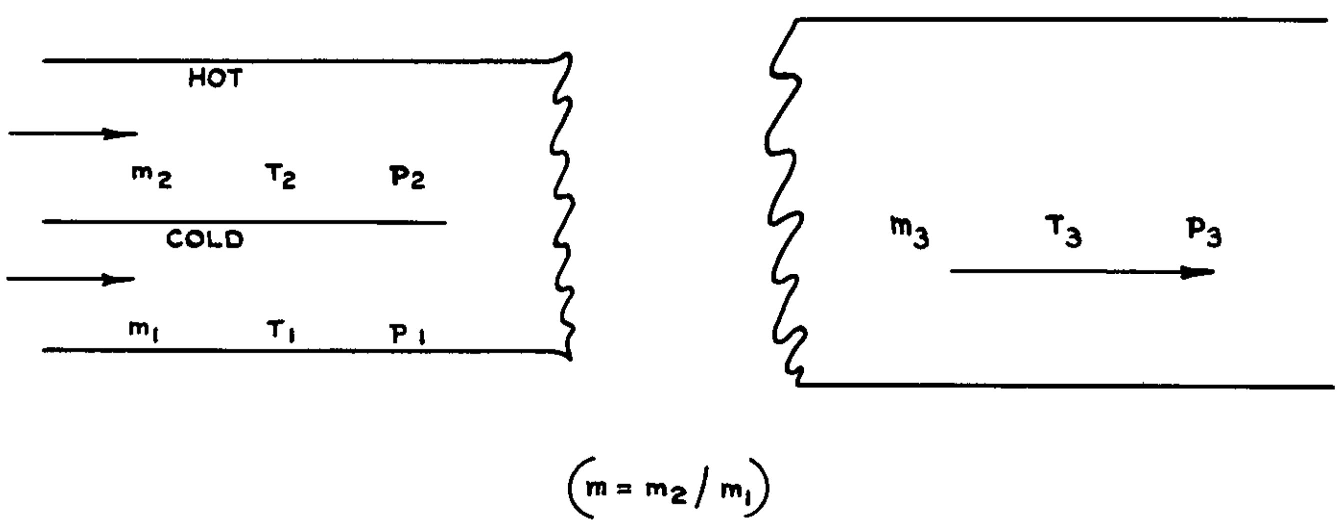
Suffices/

Suffices

$_2$	hot stream
$_1$	cold stream
$_3$	mixed stream
a	result for constant specific heat
b	additional effect resulting from the variation of specific heat
id	after mixing, for an ideal gas with constant specific heat
ac	after mixing, for actual gas (air)

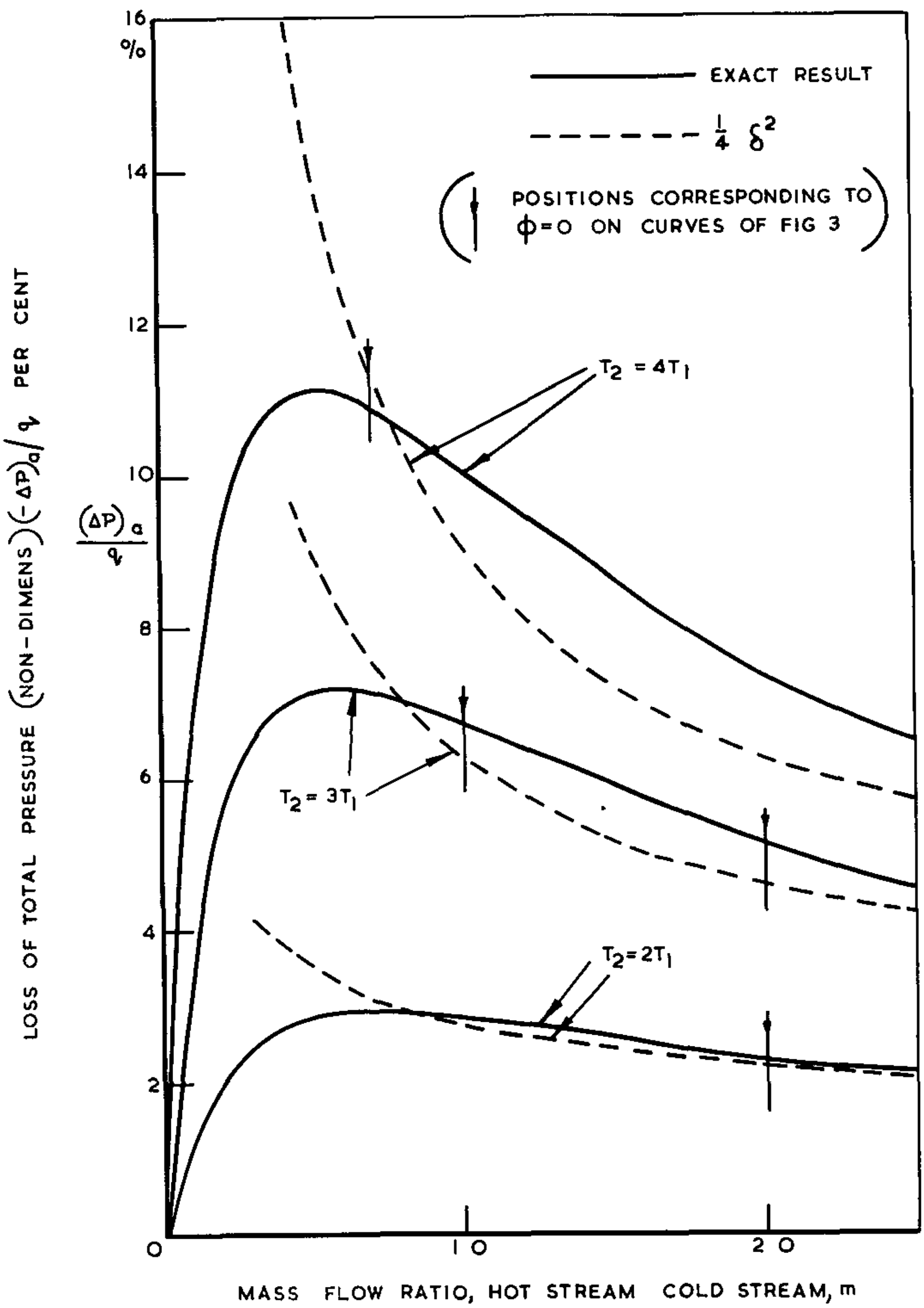
AT

FIG.1.



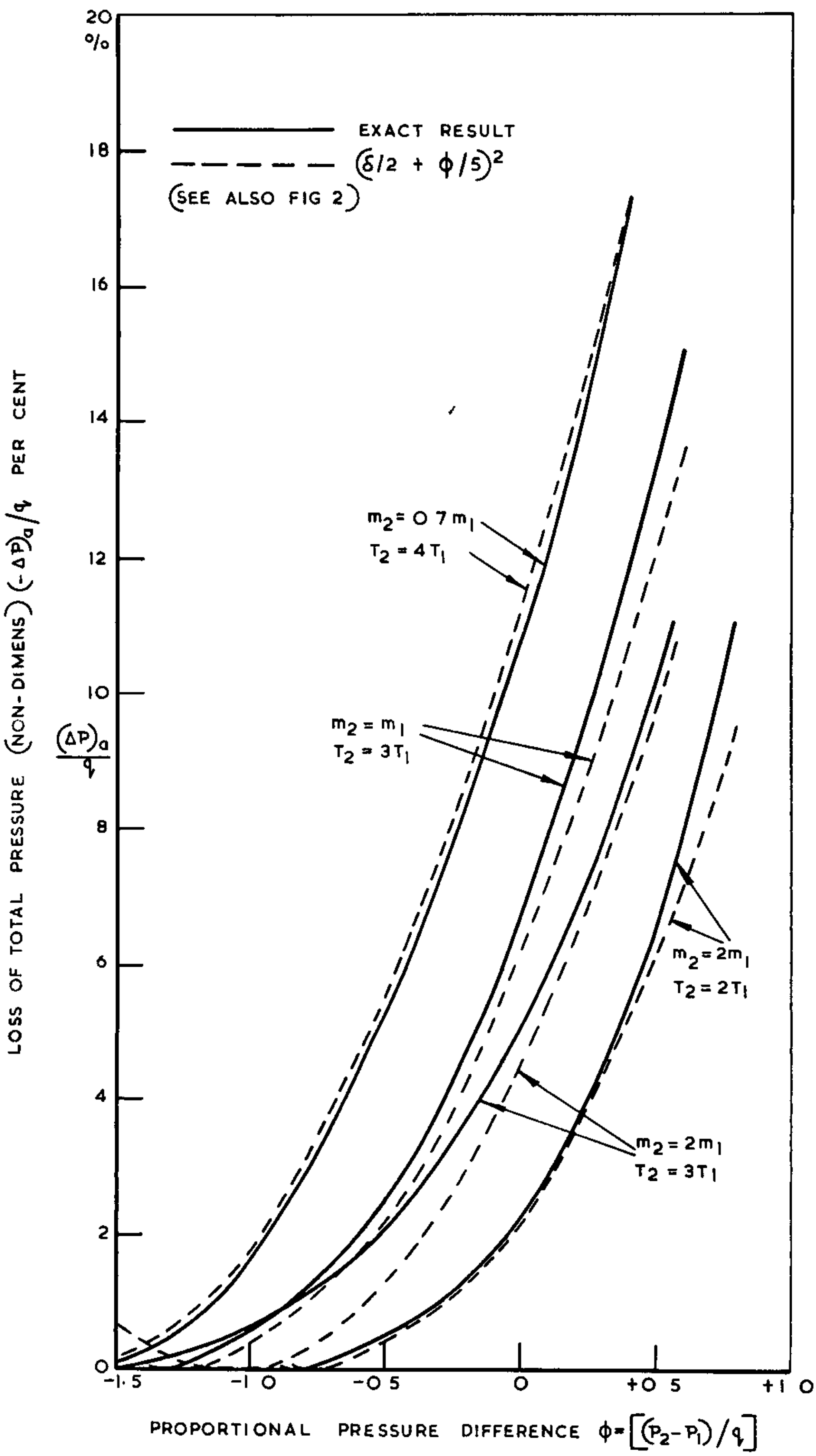
THE MIXING OF TWO STREAMS AT CONSTANT STATIC PRESSURE.

FIG. 2.



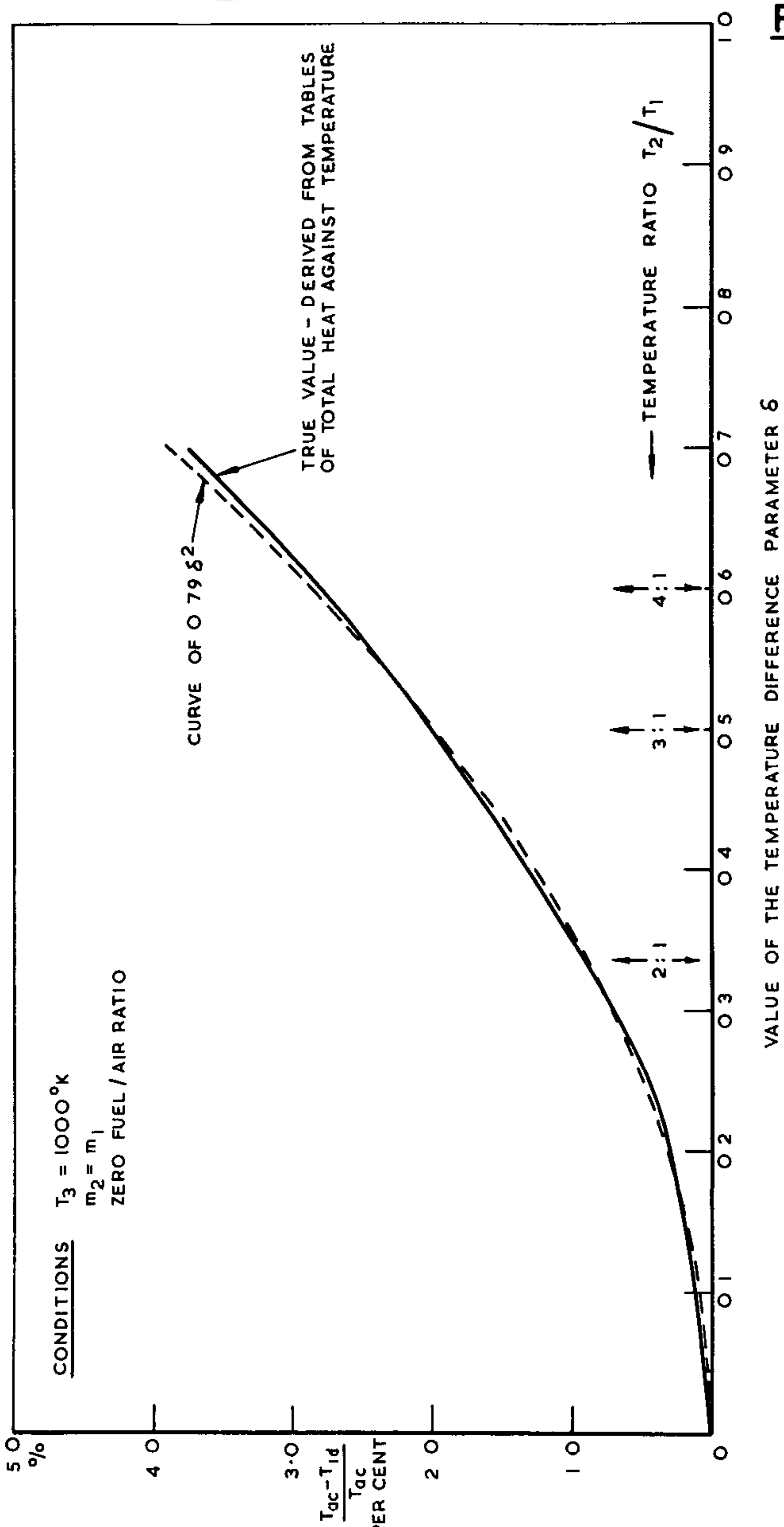
RANGE OF APPLICATION OF THE $\frac{1}{4}\delta^2$ FORMULA

FIG. 3.



RANGE OF APPLICATION OF THE $(\delta/2 + \phi/5)^2$ FORMULA

FIG. 4.



EXAMPLE OF THE EMPIRICAL FITTING TO A δ^2 CURVE OF THE ADDITIONAL LOSS DUE TO VARIATION OF SPECIFIC HEAT.

VALUES OF THE COEFFICIENT A

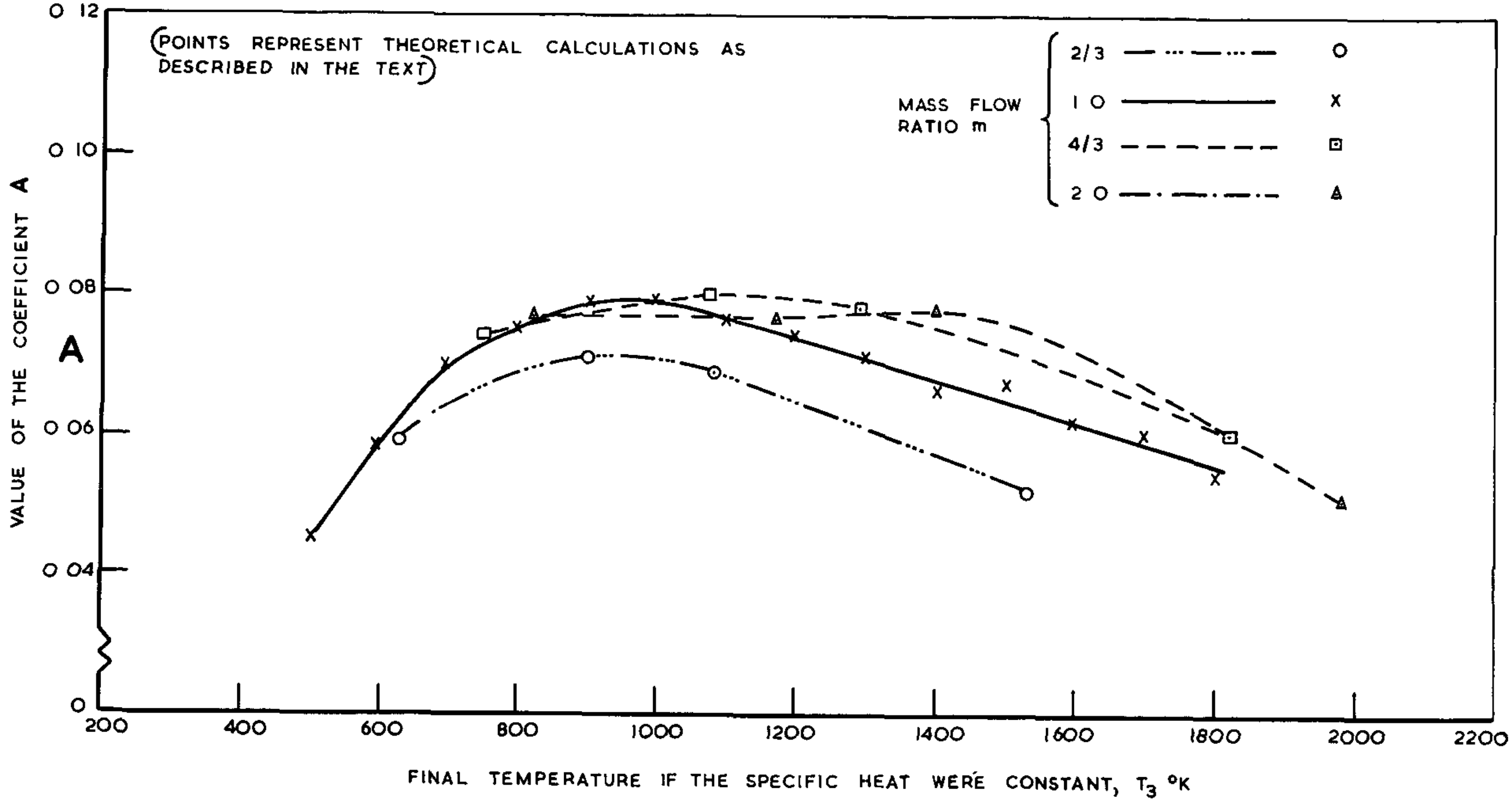
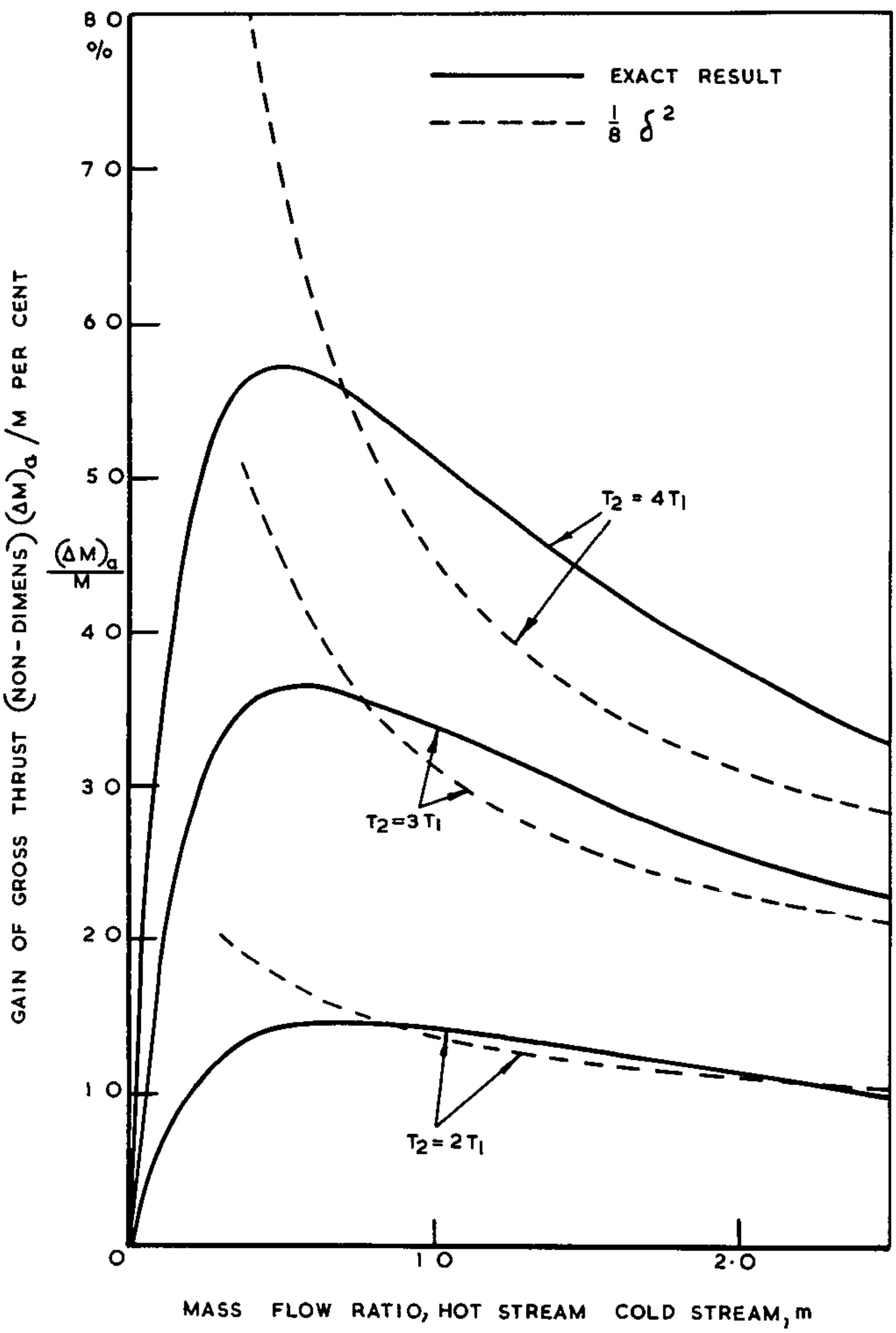


FIG.5.

FIG. 6.



RANGE OF APPLICATION OF THE $\frac{1}{8}\delta^2$ FORMULA FOR THE GAIN OF GROSS THRUST

DS 74687/1/Wt 59 K 4 9/63 XL & CL

A.R.C. C.P.No.686
June, 1962
Stratford, B.S. and Williams, J.G.

A SIMPLIFIED TREATMENT OF LOSSESS FOR ONE-DIMENSIONAL MIXING BETWEEN HOT AND COLD GAS STREAMS AT CONSTANT PRESSURE AND LOW VELOCITY

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Several simplified formulae may be put forward to supplement the existing methods of analysis. Most of

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