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Radio Propagation through Rocket Exhaust Jets:
Part 1,
Electromagnetic Wave Propagation in an Ionised
Medium

By
H. Williams

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RADIO PROPAGATION THROUGH ROCKET EXHAUST JETS:
PART I
ELECTROMAGNETIC WAVE PROPAGATION IN AN IONISED
MEDIUM

by

H. Williams

SUMMARY

The attenuation per unit path length, phase shift per unit path length and reflection coefficient at the boundary are derived theoretically for a medium containing free electrons, in terms of the electron density, electron collision frequency with heavy molecules and the applied radio frequency.

Numerical results are tabulated over wide ranges of the three variables, and these results are also presented graphically.

Useful approximations to the complete theory are derived and the regions in which these may be applied in practice are indicated.

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1 INTRODUCTION

The purpose of this paper is to present as simply as possible the generally accepted theory of electromagnetic wave propagation in an ionized gas, together with tables of computed values for the attenuation, phase shift, and reflection at the boundary of such a medium.

The emphasis is placed on the practical use of this theory and to facilitate this, approximations are derived.

It may appear that the treatment in places is oversimplified and perhaps incomplete but as the final aim is the presentation of equations and data of practical importance anything not directly bearing on this or of use in obtaining an understanding of the processes is omitted.

Throughout it is assumed that a free electron concentration exists and no mention is made of the probable source of these electrons.

2 ELECTROMAGNETIC PROPAGATION IN A PARTIAL CONDUCTOR

There exists for an ionised gas an effective electrical conductivity and dielectric constant due to the presence of free ions. The medium may therefore be considered as a partial conductor. Consequently the general equations for propagation in a partial conductor are derived before proceeding to the specific case of an ionised medium.

We assume that the medium is homogeneous and isotropic and that in any given volume, due to equal amounts of positive and negative charge, the net charge density is zero. Maxwell's equation¹ may thus be simplified to

$$\text{div } \underline{E} = 0$$

$$\text{div } \underline{H} = 0$$

$$\text{curl } \underline{E} = - \frac{\mu}{c} \frac{\partial \underline{H}}{\partial t}$$

$$\text{curl } \underline{H} = \frac{\epsilon}{c} \frac{\partial \underline{E}}{\partial t} + \frac{4\pi\sigma \underline{E}}{c} \quad .$$

From these the following wave equations may be derived:

$$\nabla^2 \underline{E} = \frac{\epsilon\mu}{c^2} \frac{\partial^2 \underline{E}}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2} \frac{\partial \underline{E}}{\partial t}$$

$$\nabla^2 \underline{H} = \frac{\epsilon\mu}{c^2} \frac{\partial^2 \underline{H}}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2} \frac{\partial \underline{H}}{\partial t} \quad .$$

The above equations have many solutions. We shall confine ourselves to the simplest, but probably most important one, the plane wave. In general the behaviour described by this solution may be carried over to cases not so simple.

For a wave having its electric vector confined solely in the plane (y, z), constant over this plane and parallel to the y axis we have for its components:

$$E_x = E_z = 0, \quad \frac{\partial E_y}{\partial z} = \frac{\partial E_y}{\partial y} = 0$$

and thus

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\epsilon\mu}{c^2} \frac{\partial^2 E_y}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2} \frac{\partial E_y}{\partial t} \quad . \quad 2.1$$

This is a scalar wave function and describes a plane wave propagated along the x axis. A similar equation may be derived for Hz. The electric vector will be a function of both distance (x) and time (t) and because of the finite conductivity will be attenuated with increasing values of x. If we assume the commonest case of a wave sinusoidal in both distance and time we may describe it in a general fashion by

$$E_y = E_0 \exp(j\omega t - \gamma x) \quad . \quad 2.2$$

Substitution of equation 2.2 in 2.1 leads to the condition that 2.2 is a solution of 2.1 if

$$\gamma = j \frac{4\pi\mu\sigma\omega}{c^2} - \frac{\epsilon\mu\omega^2}{c^2} \quad . \quad 2.3$$

The constant γ is therefore, in general, complex and may be written as

$$\gamma = \alpha + j\beta \quad . \quad 2.4$$

Confining ourselves to a wave propagated in the positive direction of x we have

$$E_y = E_0 \exp(-\alpha x) \exp(j(\omega t - \beta x)) \quad . \quad 2.5$$

The equation 2.5 describes a sinusoidal wave progressing with a velocity $V = \omega/\beta$, of wavelength $\lambda = 2\pi/\beta$ and angular frequency ω rad/sec. The term in α represents a falling off in intensity with distance, α therefore being known as the attenuation coefficient. The phase constant of the medium is β . The constant γ describes completely the propagation characteristics of the medium and is known as the propagation constant.

It is useful at this point to make a change in our parameters by inserting

$$k = \frac{\alpha c}{\omega}, \text{ and } n = \frac{\beta c}{\omega} \quad . \quad 2.6$$

Substituting 2.6 in 2.4 and using 2.3 we obtain

$$\left(\frac{\omega k}{c} + j \frac{\omega n}{c}\right)^2 = j \frac{4\pi\mu\sigma\omega}{c^2} - \frac{\epsilon\mu\omega^2}{c^2} .$$

Equating real and imaginary parts, we get

$$\left. \begin{aligned} k^2 - n^2 &= -\epsilon\mu \\ kn &= \frac{\mu\sigma}{\nu} \end{aligned} \right\} \quad 2.7$$

where ν is the frequency in cycle/sec = $\omega/2\pi$.

Solving for k and n gives

$$n^2 = \frac{1}{2} \left(\sqrt{\epsilon^2 \mu^2 + \frac{4\mu^2 \sigma^2}{\nu^2}} + \epsilon\mu \right) \quad 2.8$$

$$k^2 = \frac{1}{2} \left(\sqrt{\epsilon^2 \mu^2 + \frac{4\mu^2 \sigma^2}{\nu^2}} - \epsilon\mu \right) . \quad 2.9$$

It follows therefore that

$$\alpha = \frac{\omega}{c} \left[\frac{1}{2} \left(\epsilon\sqrt{\mu^2 + \frac{4\mu^2 \sigma^2}{\epsilon^2 \nu^2}} - \epsilon\mu \right) \right]^{\frac{1}{2}} ; \quad 2.10$$

$$\beta = \frac{\omega}{c} \left[\frac{1}{2} \left(\epsilon\sqrt{\mu^2 + \frac{4\mu^2 \sigma^2}{\epsilon^2 \nu^2}} + \epsilon\mu \right) \right]^{\frac{1}{2}} . \quad 2.11$$

The signs of the roots are to be selected in every case such that α , β , k and n are all real numbers.

It is important that the physical significance of the real numbers n and k be fully understood. Substituting 2.6 into 2.5 leads to

$$E_y = E_o \exp \left[j\omega \left(t - \frac{(n - jk)}{c} x \right) \right] . \quad 2.12$$

For a plane wave in a dielectric ($\sigma = 0$) we have

$$E_y = E_o \exp \left[j\omega \left(t - \frac{nx}{c} \right) \right]$$

and remembering that the velocity of such a wave is given by $\nu = \omega/\beta = c/n$, we see that a partial conductor may be considered to have an effective complex refractive index given by $n - jk$. The real part is commonly termed the refractive index and the imaginary the damping coefficient.

An equivalent way of considering the medium is in terms of a complex propagation constant (equation 2.4). Further, Ampere's rule states that $\text{curl } \underline{H} = \frac{4\pi \underline{i}}{c}$ where \underline{i} is the total current flowing. Comparison with the last of Maxwell's equations shows therefore that the total current flowing is given by

$$\underline{i} = \left(\sigma + j \frac{\omega \epsilon}{4\pi} \right) \underline{E} \quad , \quad 2.13$$

and thus another possible way of describing the medium is in terms of a complex conductivity, the real and imaginary parts being given by

$$\sigma_r = \sigma \quad , \quad \sigma_i = \frac{\omega}{4\pi} (1 - \epsilon) \quad 2.14$$

respectively.

3 ELECTROMAGNETIC PROPAGATION IN AN IONISED GAS

Let us assume a free ion of charge "e" and mass "m" under the influence of an electromagnetic oscillation. The ion will vibrate in sympathy with the applied electric field but will lose such directed momentum that it acquires on collision with a heavy molecule. (Ion-ion collisions are relatively infrequent.) In the equation of motion of the ion we represent this randomisation of the momentum, when averaged over a large number of impacts, by a frictional damping force proportional to the velocity of the ion. The equation of motion becomes

$$m \frac{d^2 y}{dt^2} + g \frac{dy}{dt} = eEy = eE_0 \exp(j\omega t) \quad 3.1$$

or

$$m \frac{du}{dt} + gu = eE_0 \exp(j\omega t) \quad ,$$

where u is the ion velocity at time t. In an ionised gas the current density is Neu , where N is the ion density, and thus

$$m \frac{di}{dt} + gi = Ne^2 E_0 \exp(j\omega t) \quad . \quad 3.2$$

The steady state solution to 3.2 is

$$i = \frac{Ne^2 E_0 \exp(j\omega t)}{m(j\omega + g/m)} \quad ,$$

which becomes

$$i = \left[\frac{Ne^2 g}{m^2 (\omega^2 + g^2/m^2)} - j \frac{Ne^2 m\omega}{m^2 (\omega^2 + g^2/m^2)} \right] E_y \quad . \quad 3.3$$

To obtain the total current flowing we must add the displacement current due to the time-varying field which is

$$i_d = \frac{1}{4\pi} \frac{\partial E_y}{\partial t} = \frac{j\omega}{4\pi} E_y$$

$\left[\frac{1}{4\pi} \right]$ is the vacuum capacitance of the unit condenser .

Finally we obtain for the total current density

$$i = \left[\frac{Ne^2 g}{m^2 (\omega^2 + g^2/m^2)} + j \frac{\omega}{4\pi} \left(1 - \frac{4\pi Ne^2 m}{m^2 (\omega^2 + g^2/m^2)} \right) \right] E_y \quad 3.4$$

Comparison of 3.4 with 2.13 leads to the following values of the effective conductivity and dielectric constant

$$\sigma = \frac{Ne^2 g}{m^2 (\omega^2 + g^2/m^2)}$$

$$\epsilon = 1 - \frac{4\pi Ne^2 m}{m^2 (\omega^2 + g^2/m^2)} \quad .$$

So far we have retained the frictional damping term g but in order to proceed further we need to know its value, which at the present time is the subject of considerable speculation. However, published experimental evidence supports the assumption $g = m\omega_1$ where ω_1 is the collision frequency of ions with heavy molecules. Consequently

$$\sigma = \frac{Ne^2 \omega_1}{m (\omega^2 + \omega_1^2)} \quad 3.5$$

$$\epsilon = 1 - \frac{4\pi Ne^2}{m (\omega^2 + \omega_1^2)} \quad . \quad 3.6$$

Both the dielectric constant and the conductivity are functions of $1/m$ so that the lighter ions will have the major effect. In practice, where electrons are present, the overall effect will be due to the free electron population. In all that follows therefore it will be assumed that N and ω_1 refer to the free electron.

Substituting into 2.8 and 2.9 and assuming $\mu = 1$, as is nearly always the case for a gas, we obtain

$$n^2 = \frac{1}{2} \left(\sqrt{\left[1 - \frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \right]^2 + \left[\frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \frac{\omega_1}{\omega} \right]^2} + \left[1 - \frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \right] \right)$$

3.7

$$k^2 = \frac{1}{2} \left(\sqrt{\left[1 - \frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \right]^2 + \left[\frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \frac{\omega_1}{\omega} \right]^2} - \left[1 - \frac{4\pi N e^2}{m(\omega^2 + \omega_1^2)} \right] \right).$$

The above equations may be used to describe completely the propagation of radio waves within an ionised gas in terms of electron density, electron collision frequency and angular radio frequency.

The physical picture of electrons vibrating in sympathy with the electric field and subsequent collisions randomizing the momentum demands two assumptions. First, that the strength of the electromagnetic field is such that it does not upset the collision frequency of the electrons and secondly, that several oscillations of the electron may occur along one free path. The former assumption is generally valid, but the latter is not and together with the assumption $g = m\omega_1$ involves an erroneous averaging of electron velocities. The energy distribution of the electron must be taken into account. This has been done by Margenau².

For Maxwellian distribution of electron velocities Margenau derives

$$\sigma_r = \frac{4}{3} \frac{e^2 \lambda' N}{(2\pi m k T)^{\frac{1}{2}}} K_2(x_1) \quad 3.8$$

$$\sigma_1 = \frac{4}{3} \frac{e^2 \lambda' N}{(2\pi m k T)^{\frac{1}{2}}} x_1^{\frac{1}{2}} K_{3/2}(x_1) \quad 3.9$$

where λ' is the mean free path of the electrons, and

$$x_1 = \frac{1}{2} \frac{m}{k T} (\omega \lambda')^2$$

$$K_2(x_1) = 1 - x_1 - x_1^2 \exp(x_1) E_1(-x_1)$$

$$K_{3/2}(x_1) = \left(\frac{1}{2} - x_1\right) \pi^{\frac{1}{2}} + \pi x_1^{\frac{3}{2}} \exp(x_1) (1 - \operatorname{erf}(x_1^{\frac{1}{2}})) \quad .$$

Ei and erf represent the exponential integral and the error function respectively.

In Fig.1 are plotted the conductivity and dielectric constant, as determined by the simple theory and Margenau's, against ω for $\omega_1 = 10^8 \text{ sec}^{-1}$, $N = 10^{10} \text{ cm}^{-3}$. It can be seen that there is little difference between the two when considering the electromagnetic properties of rocket exhausts. The simple theory is adequate and easier to apply.

Before leaving the complete theory of Margenau there are two special cases of interest:

(a) Where the ionised medium is at high pressure and the radio frequency is low. Under this condition we have $x_1 \rightarrow 0$ and

$$\lim_{x_1 \rightarrow 0} K_2 = 1, \quad \lim_{x_1 \rightarrow 0} K_{3/2} = \frac{\sqrt{\pi}}{2}.$$

Therefore

$$\sigma_r (\text{Marg}) = \frac{4}{3} \frac{e^2 \lambda' N}{(2\pi m K T)^{1/2}}, \quad \sigma_i (\text{Marg}) = \frac{\omega e^2 \lambda'^2 N}{3 K T}.$$

Substituting the value of λ' from gas kinetic theory

$$\lambda' = \frac{2}{\omega_1} \sqrt{\frac{2 K T}{\pi m}},$$

$$\sigma_r (\text{Marg}) = \frac{8}{3\pi} \frac{N e^2}{m \omega_1}, \quad \sigma_i (\text{Marg}) = \frac{8}{3\pi} \frac{N e^2 \omega}{m \omega_1^2}.$$

The same condition on the simple theory is described by $\omega^2 \ll \omega_1^2$, whence

$$\sigma_r (\text{Simp}) = \frac{N e^2}{m \omega_1}, \quad \sigma_i (\text{Simp}) = \frac{N e^2 \omega}{m \omega_1^2}$$

and therefore

$$\frac{\sigma_r (\text{Marg})}{\sigma_r (\text{Simp})} = \frac{\sigma_i (\text{Marg})}{\sigma_i (\text{Simp})} = \frac{8}{3\pi} = 0.849.$$

(b) At low gas pressures and high radio frequencies, we have $x_1 \rightarrow \infty$ and thus

$$\lim_{x_1 \rightarrow \infty} K_2 = \frac{2}{x_1}, \quad \lim_{x_1 \rightarrow \infty} K_{3/2} = \frac{3\sqrt{\pi}}{4x_1}.$$

It follows that

$$\sigma_r (\text{Marg}) = \frac{4}{3} \frac{Ne^2 \omega_1}{m\omega^2}, \quad \sigma_i (\text{Marg}) = \frac{Ne^2}{m\omega}.$$

On the simple theory this condition is described by $\omega_1^2 \ll \omega^2$ and thus

$$\sigma_r (\text{Simp}) = \frac{Ne^2 \omega_1}{m\omega^2}, \quad \sigma_i (\text{Simp}) = \frac{Ne^2}{m\omega},$$

$$\frac{\sigma_r (\text{Marg})}{\sigma_r (\text{Simp})} = \frac{4}{3} = 1.33,$$

$$\frac{\sigma_i (\text{Marg})}{\sigma_i (\text{Simp})} = 1.0.$$

4. ATTENUATION IN A PARTIAL CONDUCTOR

Let the peak powers at two points x_1, x_2 on the x axis be P_1, P_2 respectively. From 2.5 we may write

$$P_1 = P_0 \exp(-2\alpha x_1)$$

$$P_2 = P_0 \exp(-2\alpha x_2)$$

where P_0 is the power at some arbitrary point $x = 0$.

The attenuation experienced over this interval is by definition

$$\begin{aligned} D &= 10 \log_{10} \frac{P_2}{P_1} = 20 \log_{10} \frac{\exp(-\alpha x_2)}{\exp(-\alpha x_1)} \text{ dB} \\ &= 20 \log_{10} \exp(\alpha x_1 - \alpha x_2) \text{ dB} . \end{aligned}$$

Putting $x_2 - x_1 = 1$ we obtain as the attenuation per unit path length

$$\delta = - \frac{20\alpha}{2.303} = - \frac{20}{2.303} \frac{kw}{c} \text{ dB cm}^{-1}$$

with $c = 2.998 \times 10^{10} \text{ cm sec}^{-1}$

$$\delta = -2.897 \cdot 10^{10} kw \text{ dB cm}^{-1} \quad 4.1$$

In Tables I to XIII values of δ , as computed on the simple theory, are tabulated against the variable ω for the two parameters N and ω_1 . The same information is presented graphically in Fig. 2-8.

5 PHASE SHIFT IN A PARTIAL CONDUCTOR

We have (equation 2.5)

$$E_y = E_0 \exp(-\alpha x) \exp(j(\omega t - \beta x)) \quad .$$

We may write from equation 2.6

$$\beta = \frac{\omega n}{c}$$

where n is the refractive index of the medium given by 3.7 for an ionised gas. The phase constant for free space is $\beta_0 = \frac{\omega}{c}$ and thus the phase shift experienced by the electromagnetic disturbance with respect to the wave in free space is

$$\beta_0 - \beta = \frac{\omega}{c} (1 - n) \text{ rad cm}^{-1} \quad . \quad 5.1$$

When $n < 1$ the phase of the wave in the medium is retarded; when $n > 1$ it is advanced with respect to the wave in free space.

Values of $\beta_0 - \beta$, computed by the use of equations 3.7 and 5.1, are tabulated with the variable ω and parameters N and ω_1 in Tables XIV - XXVI. The same information is presented graphically in Fig. 9-15. For convenience they are plotted on a logarithmic scale and the curves therefore appear discontinuous; this however is not the case, as can be seen in Fig. 24 where $\beta_0 - \beta$ is plotted on a linear scale.

6 REFLECTION AT NORMAL INCIDENCE TO THE BOUNDARY OF A PARTIAL CONDUCTOR

The determination of the voltage reflection coefficient normal to the boundary of an ionised medium demands a knowledge of the relative magnitudes of the magnetic and electric vectors within the medium.

For a plane wave we may write Maxwell's second equation as

$$\frac{\partial E_y}{\partial x} = - \frac{\mu}{c} \frac{\partial H_z}{\partial t} \quad . \quad 6.1$$

We shall assume a solution to this equation to be some function of $(x - vt)$ v being the phase velocity, in the medium. Therefore

$$E_y = f(x - vt) \quad .$$

From the wave equation of section 2 (equation 2.1) we know that H_z will have a similar dependence on x and t . Therefore let

$$H_z = af(x - vt + b) \quad ,$$

where a and b are constants yet to be determined.

The above is a solution of 6.1 only if

$$b = 0$$

$$a = \frac{c}{\mu v} \quad . \quad 6.2$$

Let us for the moment consider a dielectric medium ($\sigma = 0$) so that the wave equation 2.1 becomes

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\epsilon \mu}{c^2} \frac{\partial^2 E_y}{\partial t^2} \quad .$$

In this case the velocity of propagation of the wave in the medium is given by

$$v = \frac{c}{\sqrt{\epsilon \mu}}$$

and thus 6.2 becomes

$$a = \sqrt{\frac{\epsilon}{\mu}} \quad ,$$

and we immediately have

$$\frac{E_y}{H_z} = \sqrt{\frac{\mu}{\epsilon}} \quad . \quad 6.3$$

As shown previously (section 2) a partial conductor may be considered to have a complex refractive index ($n - jk$). Therefore, in order to put 6.3 into a form immediately applicable to a partial conductor, the refractive index =

$$\frac{c}{v} = \sqrt{\epsilon\mu} = n - jk$$

Therefore

$$\frac{H_z}{E_y} = \frac{n - jk}{\mu} \quad 6.4$$

Having derived 6.4 we may now proceed.

Consider an electromagnetic wave incident normally on a vacuum/partial conductor interface of infinite extent. The boundary condition is the common one of tangential E and H continuous across the interface. Remembering that the reflected wave reverses direction of travel and that therefore either E_r or H_r must undergo phase reversal, we have

$$E_i + E_r = E_t$$

$$H_i - H_r = H_t$$

$$E_i = H_i$$

$$E_r = H_r$$

$$H_t = \frac{n - jk}{\mu} E_t$$

(the subscripts "i" refers to the incident wave, "t" to the transmitted wave and "r" to the reflected wave) and thus

$$\frac{E_i - E_r}{E_i + E_r} = \frac{n - jk}{\mu}$$

The voltage reflection coefficient may be defined by

$$\Gamma = \frac{E_r}{E_i}$$

Therefore

$$\Gamma = \frac{\mu - n + jk}{\mu + n - jk} \quad 6.5$$

The voltage reflection coefficient is therefore complex, indicating a phase change on reflection. The magnitude of the coefficient is given by

$$|\Gamma|^2 = R = \frac{(\mu - n)^2 + k^2}{(\mu + n)^2 + k^2} \quad . \quad 6.6$$

As before, $\mu = 1$ and thus finally

$$R = \frac{(1 - n)^2 + k^2}{(1 + n)^2 + k^2} \quad . \quad 6.7$$

The phase change on reflection may be determined from 6.5 as follows:

$$\Gamma = \frac{(\mu - n) + jk}{(\mu + n) - jk} = \sqrt{\frac{(\mu - n)^2 + k^2}{(\mu + n)^2 + k^2}} \frac{\exp(j\theta_1)}{\exp(-j\theta_2)}$$

where

$$\tan \theta_1 = \frac{k}{\mu - n}$$

$$\tan \theta_2 = \frac{k}{\mu + n} \quad .$$

We may therefore write

$$\Gamma = A \exp(j\theta)$$

where A is the modulus and θ the argument of the complex number Γ . We have

$$E_r = \Gamma E_i = AE_i \exp(j\theta) \quad .$$

If, as we have so far assumed, E_i may be described by

$$E_i = E_0 \exp(j\omega t) \quad ,$$

then

$$E_r = AE_0 \exp(j\omega t + \theta) \quad .$$

Thus the phase change is given by

$$\theta = \theta_1 + \theta_2 = \tan^{-1} \frac{2\mu k}{\mu^2 - n^2 - k^2} \quad \text{radian} \quad .$$

The power reflection coefficient R is tabulated in Tables XXVI - XXXV and graphically presented in Fig. 16-21 for a practical range of values in N_1 , ω and ω_1 .

7 APPROXIMATIONS TO THE SIMPLE THEORY FOR A GASEOUS MEDIUM

7.1 High values of ω_1

The general complication of equations 3.7 makes it difficult to determine immediately the effect of changes in the variable N , ω and ω_1 on the final result. This difficulty may be overcome by a consideration of the relative magnitudes of the conduction current and displacement current. Such a course is doubly useful in allowing the derivation of a good approximation and in presenting a clear physical picture of the controlling processes.

The total current density in the medium may be written as

$$\underline{j} = \left(\sigma + j \frac{\omega \epsilon}{4\pi} \right) \underline{E} \quad .$$

The ratio of the conduction current to the total displacement current is therefore described by the term $\frac{4\pi\sigma}{\epsilon\omega} = \frac{2\sigma}{\epsilon\nu}$. Reference to equations 2.10 and 2.11 shows that it is just this ratio which predominantly controls the values of α and β , and thus the overall properties of the medium. There are two cases of interest. First, where the displacement current predominates, described by $\frac{2\sigma}{\epsilon\nu} \ll 1$, and secondly where the conduction current predominates, described by $\frac{2\sigma}{\epsilon\nu} \gg 1$.

(a) $\frac{2\sigma}{\epsilon\nu} \ll 1$

This arises when the medium is a poor conductor or when it is a good conductor with a high applied radio frequency. We have from equation 2.10, with $\mu = 1$,

$$\alpha = \frac{\omega}{c} \left[\frac{1}{2} \left(\sqrt{\epsilon^2 + \frac{4\sigma^2}{\nu^2}} - \epsilon \right) \right]^{\frac{1}{2}} \quad .$$

Expanding by means of the binomial theorem,

$$\alpha = \frac{\omega}{c} \left[\frac{1}{2} \left(\pm \epsilon \left(1 + \frac{2\sigma^2}{\epsilon^2\nu^2} \right) - \epsilon \right) \right]^{\frac{1}{2}} \quad .$$

There are two solutions;

$$\alpha_1 = \frac{\omega}{c} \left[\frac{1}{2} \left(\frac{2\sigma^2}{\epsilon\nu^2} \right) \right]^{\frac{1}{2}} \quad ,$$

and

$$\alpha_2 = \frac{\omega}{c} \left[\frac{1}{2} \left(-2\epsilon - \frac{2\sigma^2}{\epsilon\nu^2} \right) \right]^{\frac{1}{2}} \quad ,$$

according to which sign of the square root is to be taken. The choice is determined by the fact that α is defined as being real and positive.

Where ϵ is positive

$$\alpha = \frac{\omega}{c} \frac{\sigma}{\nu} \frac{1}{\sqrt{\epsilon}} = \frac{2\pi\sigma}{\omega\sqrt{\epsilon}} \quad . \quad 7.1$$

Similarly, approximating from equation 2.11,

$$\beta = \frac{\omega\sqrt{\epsilon}}{c} \quad . \quad 7.2$$

If we further assume that the dielectric constant differs little from unity, as is generally the case under the conditions being considered, the attenuation now becomes from equations 3.5, 7.1 and 4.1

$$\delta = -1.82 \cdot 10^{-9} \frac{Ne^2 \omega_1}{m(\omega^2 + \omega_1^2)} \text{ dB cm}^{-1} \quad , \quad 7.3$$

and the phase shift from equations 7.2 and 3.6

$$\beta_0 - \beta = \frac{\omega}{c} (1 - \sqrt{\epsilon}) = \frac{\omega}{c} \frac{2\pi Ne^2}{m(\omega^2 + \omega_1^2)} \quad . \quad 7.4$$

The regions in which 7.3 and 7.4 are good approximations are shown shaded and labelled on Fig. 2-8 for attenuation and Fig. 9-15 for phase shift. As may be seen, both attenuation and phase shift are proportional to electron concentration. Where ω_1 is large compared to ω the attenuation is independent of radio frequency but where ω_1 is small compared with ω the phase shift is independent of the electron collision frequency. This behaviour is well illustrated in the figures.

(b) $\frac{2\sigma}{\epsilon\nu} \gg 1$

This condition arises when the medium is a good conductor or when it is a poor conductor with a low applied radio frequency.

We now obtain from equations 2.10 and 2.11

$$\alpha = \beta = \frac{\omega\sqrt{\sigma}}{c\nu} = \frac{1}{c} \sqrt{2\pi\sigma\omega} \quad . \quad 7.5$$

It is to be noted that α and β are now equal and independent of the dielectric constant of the medium. Combining equations 2.6, 3.5, 4.1 and 7.5 the attenuation becomes

$$\delta = -7.26 \cdot 10^{-10} \sqrt{\frac{Ne^2 \omega_1 \omega}{m(\omega^2 + \omega_1^2)}}$$

When applicable at low radio frequencies we generally have $\omega_1^2 \gg \omega^2$ and therefore with little error

$$\delta = -7.26 \cdot 10^{-10} \sqrt{\frac{Ne^2}{m} \frac{\omega}{\omega_1}} = -1.15 \cdot 10^{-5} \sqrt{\frac{N\omega}{\omega_1}} \quad . \quad 7.6$$

From equation 7.5 the phase shift becomes

$$\beta_0 - \beta = \frac{\omega}{c} \left(1 - 3.99 \cdot 10^4 \sqrt{\frac{N\omega_1}{\omega(\omega^2 + \omega_1^2)}} \right) \quad . \quad 7.7$$

The regions in which 7.6 and 7.7 are good approximations are shaded and labelled in Fig. 2-8 for attenuation; Fig. 9-15 for phase shift.

7.2 Low values of ω_1

Inspection of Fig. 7, 8, 14 and 15 for attenuation and phase shift at electron collision frequencies less than 10^8 sec^{-1} shows an interesting phenomenon. Almost at a unique value of ω , the attenuation changes in value by several orders of magnitude. At this value of ω , the phase shift has a sharply defined maximum, almost coincident with the value $\beta_0 - \beta = \omega/c$. This value of ω is known as the critical frequency.

It is to be noted that where the critical frequency is clearly defined the reflection coefficient (R) is effectively unity for $\omega < \omega_c$ (ω_c = critical frequency).

From the figures we see that in the region of the critical frequency we may safely place $\omega_1^2 \ll \omega^2$ and obtain the possibility that the dielectric constant becomes zero. This condition is described (from equation 3.6) by

$$\frac{4\pi Ne^2}{m\omega^2} = 1 \quad .$$

The condition $\epsilon = 0$ defines the critical frequency so that

$$\omega_c^2 = \frac{4\pi Ne^2}{m} \quad . \quad 7.8$$

The position of the critical frequency depends therefore solely on the value of the electron concentration and is independent of ω_1 .

Values of critical frequency are tabulated below:-

$N \text{ cm}^{-3}$	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
$\omega_c \text{ rad sec}^{-1}$	$1.8 \cdot 10^8$	$5.6 \cdot 10^8$	$1.8 \cdot 10^9$	$5.6 \cdot 10^9$	$1.8 \cdot 10^{10}$	$5.6 \cdot 10^{10}$

At the critical frequency, since $\epsilon = 0$, the current is completely conductive and thus the equations derived in (b) apply at this point.

For $\omega > \omega_c$ the dielectric constant differs little from unity but for $\omega < \omega_c$ it becomes negative and decreases rapidly with decreasing ω until $\omega = \omega_1$ whereupon it reaches a constant value with further decrease in ω given by

$$\epsilon = -\frac{4\pi Ne^2}{m\omega_1^2} .$$

In general we may write for $\omega < \omega_c$, $|\epsilon| \gg 1$.

The conductivity (equation 3.5) initially low at high ω increases as the radio frequency is reduced ($\sigma \propto \frac{1}{\omega^2}$) until once again $\omega = \omega_1$ whereupon it also reaches a steady value given by

$$\sigma = \frac{Ne^2}{m\omega_1}$$

as ω is still further reduced.

Thus both ϵ and σ become independent of ω for $\omega < \omega_1$ and we obtain a ratio of the conduction current to the displacement current in this region of

$$\left| \frac{2\sigma}{\epsilon\nu} \right| = 2 \frac{Ne^2}{m\omega_1} \frac{m\omega_1^2}{4\pi Ne^2} \frac{2\pi}{\omega} = \frac{\omega_1}{\omega} .$$

Since $\omega < \omega_1$ the conduction current predominates in this region of very low radio frequencies. The equations to be applied in determining attenuation and phase shift are therefore equations 7.6 and 7.7 and we note that $\alpha = \beta$. In the region $\omega_1 < \omega < \omega_c$ we obtain once again

$$\left| \frac{2\sigma}{\epsilon\nu} \right| = \frac{\omega_1}{\omega}$$

and thus the displacement current predominates.

Remembering that ϵ is now negative we have, after taking the appropriate root and approximating in equations 2.10 and 2.11,

$$\alpha = \frac{\omega}{c} \sqrt{|\epsilon|} \tag{7.9}$$

$$\beta = \frac{2\pi\sigma}{c\sqrt{|\epsilon|}} . \tag{7.10}$$

The attenuation now becomes (combining equations 2.6, 3.5, 4.1 and 7.9)

$$\delta = \frac{-20}{2.303} \frac{\omega}{c} \sqrt{\frac{4\pi Ne^2}{m\omega^2}} = -\frac{20}{2.303} \frac{\omega_c}{c}$$

and is thus dependent only on critical frequency. More specifically, substituting for ω_c from equation 7.8,

$$\delta = -1.63 \cdot 10^{-5} \sqrt{N} \text{ dB cm}^{-1} \tag{7.11}$$

The phase shift is given (from equations 7.10 and 3.6) by

$$\beta_0 - \beta = \frac{\omega}{c} \left(1 - \frac{2\pi\sigma}{\sqrt{\epsilon}} \right)$$

which becomes, where ω is high enough to make ω_1^2 negligible with respect to ω^2 ,

$$\beta_0 - \beta = \frac{\omega}{c} \left(1 - \frac{1}{2c} \frac{\omega_1 \omega_c}{\omega} \right) \text{ rad cm}^{-1} .$$

In general, the second term in the bracket is negligible compared with unity, at the highest radio frequencies in the region $\omega_1 < \omega < \omega_c$. Thus, immediately below the critical frequency the phase shift approximates to

$$\beta_0 - \beta = \frac{\omega}{c} . \tag{7.12}$$

The line described by 7.12 is drawn in on Fig. 11 - 15 which show the phase shift at low electron collision frequencies.

The equation 7.12 indicates that the phase constant in the medium is zero and that therefore the phase velocity (v) is infinite. The group velocity, that is the velocity at which energy is propagated in the medium, will therefore be zero. In other words if 7.12 described the true state of affairs the medium would no longer be able to maintain an oscillation and all the power would be reflected at the boundary. In practice this is very nearly the case. There remains the region $\omega > \omega_c$. Here the dielectric

constant is effectively unity. The ratio $\frac{2\sigma}{\epsilon\nu}$ is small, since the radio frequency is high, and thus once again the displacement current predominates. It follows therefore that equations 7.3 and 7.4 may be used.

With $\omega_1^2 \ll \omega^2$ they become

$$\delta = -1.82 \cdot 10^{-9} \frac{Ne^2 \omega_1}{m\omega^2} \text{ dB cm}^{-1}$$

$$\beta_0 - \beta = \frac{2\pi Ne^2}{cm\omega} \text{ rad cm}^{-1}$$

and therefore both attenuation and phase shift decrease with increasing radio frequency.

7.3 The reflection coefficient

As seen in Fig. 16-21 showing the reflection coefficient at normal incidence the behaviour with N , ω , and ω_1 is generally simple. In fact, for high ω_1 the coefficient may be described by means of a single curve of R plotted against $\frac{N\omega_1}{\omega}$. This is shown in Fig. 22 and tabulated in Table XXXVI. The regions in which it may be used are shaded in Fig. 16-21.

At low ω_1 and radio frequencies above the critical frequency, a plot of R against N/ω^2 is nearly a straight line and describes the behaviour well, the reflection coefficient being independent of the electron collision frequency. This is shown in Fig. 23 and tabulated in Table XXXVII. The regions in which it may be used are again shown shaded and labelled on the curves for R plotted against radio frequency. Between them, these two approximations cover most of the practical range of N , ω and ω_1 . Where a critical frequency is clearly defined, R is effectively unity for $\omega < \omega_c$.

8 DISCUSSION

It is first necessary to know the electron collision frequency to be expected.

The classical kinetic theory of gases, assuming point electrons, leads to

$$\omega_1 = pd^2 \sqrt{\frac{\pi}{2mKT}}$$

where p is the pressure (dyn/cm²) and d the molecular collision diameter, derived from viscosity measurements.

Right now there is no satisfactory quantum mechanical evaluation of the collision frequency and although the classical theory may be expected to be near the truth, it must be in error in treating the electron as a point and applying data obtained from viscosity measurements at room temperature to flame gases.

Belcher and Sugden³, have, however, determined by means of attenuation measurements that the electron collision frequency in a coal-gas air flame is $8.8 \times 10^{10} \text{ sec}^{-1}$ at atmospheric pressure and 2200°K. In the classical equation this leads to an effective collision diameter (d) of 2.10^{-8} cm for flame-gas molecules. This is in good agreement with values derived from viscosity measurements. Taking this value of Sugden's we obtain

$$\begin{aligned} \omega_1 &= \frac{8.8 \cdot 10^{10} \sqrt{2200}}{10^6} \frac{p}{\sqrt{T}} \\ &= 4.1 \cdot 10^6 \frac{p}{\sqrt{T}} \quad . \quad 8.1 \end{aligned}$$

Although the above value for ω_1 is determined for a coal gas/air flame, the products of combustion in this flame and for a rocket are similar. It may be applied with a fair degree of confidence to rocket exhaust gas conditions. We see therefore that ω_1 is dependent on the pressure, the temperature dependence being of second order.

A good average temperature for rocket exhausts is probably of the order of 1800°K. At atmospheric pressure this leads to

$$\omega_1 \cong 10^{11} \text{ sec}^{-1} .$$

Thus in the microwave region, taking X-band in particular ($\omega = 6 \cdot 10^{10} \text{ rad sec}^{-1}$) we obtain for the attenuation and phase shift (equations 7.3 and 7.4 respectively),

$$\delta = - 1.82 \cdot 10^{-9} \frac{N e^2 \omega_1}{m(\omega^2 + \omega_1^2)}$$

or

$$\delta = - 3.35 \cdot 10^{-12} N \text{ dB cm}^{-1} \tag{8.2}$$

$$\beta_0 - \beta = 6.9 \cdot 10^{-13} N \text{ rad cm}^{-1} . \tag{8.3}$$

It is interesting to note that within this region we have

$$\frac{\delta}{\beta_0 - \beta} = - 8.69 \frac{\omega_1}{\omega} \tag{8.4}$$

Equation 8.4 is of practical importance in that the attenuation is often known and the phase shift required.

The particular application of the theory presented here is to the propagation of radio waves through rocket exhaust jets. Although necessary, it is not sufficient for a complete solution of the problem.

An important application of rocket engines is to high altitude flight and as has been shown, the greater the altitude, the lower is the electron collision frequency. There is also a lowering of the free electron density. It is not easy to determine in detail how these variables are influenced by flight conditions, particularly forward velocity and ambient pressure.

A rocket exhaust is not a homogeneous medium; it has regions differing markedly in temperature and pressure. Therefore a radio wave may be refracted and diffracted, complicating the analysis of wave propagation. The true distribution of temperature and pressure within the exhaust jet of a supersonic rocket missile is not known accurately. Assuming, however,

that a true plot of these variables were known, then the propagation problem is still not solved, because of the unknown electron density and collision frequency. We do not have sufficient knowledge available to enable us to determine these quantities with certainty. Perhaps the largest gap in our knowledge relates to the kinetics of the reactions occurring in the gas during its rapid expansion and hence to the electron density existing at any given point and time.

A complete solution of the radio propagation problem in rocket flight to high altitudes must therefore await further knowledge on all these points.

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Title and Units</u>
α	Attenuation coefficient (cm^{-1})
β	Phase constant (cm^{-1})
γ	Propagation constant (cm^{-1})
g	Frictional damping coefficient for the free electron oscillating in sympathy with applied electric field (gm. sec^{-1}) = $m\omega_1$
δ	Attenuation per unit path length (dB cm^{-1})
β_0	Phase constant of free space (cm^{-1}) = ω/c
ϵ	Dielectric constant of the medium
n	Refractive index of the medium
k	Damping coefficient of the medium
μ	Magnetic permeability of the medium
σ_r	Real part of electrical conductivity (statmoh. cm^{-1} = $1.11 \cdot 10^{-12}$ ohm $^{-1}$ cm^{-1})
σ_i	Imaginary part of electrical conductivity (statmoh. cm^{-1})
\underline{E} (vector)	Electrostatic field (e.s.u. cm^{-1} = 299.8 volt. cm^{-1})
\underline{H} (vector)	Magnetic field (e.m.c.g.s.u. = 1 gauss = 1 oersted = 1 gilbert. cm^{-1})
\underline{i} (vector)	Current density (e.s.u. cm^{-2} = 1 statamp. cm^{-2} = $3.336 \cdot 10^{-10}$ amp. cm^{-2})
ω	Angular radio frequency (rad. sec^{-1})
ν	Radio frequency (cycle. sec^{-1})
ω_1	Electron collision frequency with heavy molecules (sec^{-1})
ω_c	Critical radio frequency of the medium (rad. sec^{-1})
N	Electron density (cm^{-3})
Γ	Voltage reflection coefficient
R	Power reflection coefficient

LIST OF SYMBOLS (Contd)

<u>Symbol</u>	<u>Title and Units</u>
P	Electromagnetic power
p	Gas pressure ($\text{dyn.cm}^{-2} = 9.870 \times 10^{-7}$ atmosphere)
T	Absolute gas temperature ($^{\circ}\text{K}$)
e	Electronic charge ($= 4.803 \cdot 10^{-10}$ e.s.u.)
m	Electronic mass ($= 9.107 \cdot 10^{-28}$ gm)
K	Boltzmann's constant ($= 1.380 \cdot 10^{-16}$ erg. deg. $^{-1}$)
c	Velocity of light in vacuo ($= 2.998 \cdot 10^{10}$ cm. sec $^{-1}$)
j	The square root of minus one ($= \sqrt{-1}$)

APPENDIX

Explanation of the Tables

Values of attenuation (dB cm^{-1}), phase shift (rad. cm^{-1}) and power reflection coefficient at normal incidence, are shown tabulated for a range of values of the electron density (N), electron collision frequency (ω_1) and angular radio frequency (ω). Each table corresponds to a particular electron collision frequency. The electron density and angular radio frequency are tabulated as their logarithms to the base ten for convenience. Each vertical column corresponds to a particular value of the electron density, the actual value heading the column. Each of these columns has to its right a sub-column giving the power of ten by which the figure in the main column must be multiplied. In the case of phase shift the value is marked negative or positive according to whether the phase is advanced or retarded with respect to the wave in free space.

For example, given that $N = 10^{10} \text{ cm}^{-3}$, $\omega_1 = 10^{12} \text{ sec}^{-1}$, $\omega = 10^8 \text{ rad sec}^{-1}$ we obtain:-

From table I, $\delta = 4.55 \cdot 10^{-3} \text{ dB cm}^{-1}$

From table XIV, $\beta_0 - \beta = 4.09 \cdot 10^{-5} \text{ rad cm}^{-1}$

From table XXVII, $R = 6 \cdot 10 \cdot 10^{-3}$.

It is unlikely that any great accuracy is required or for that matter is merited in most cases and therefore it should be possible to obtain a reasonable value from the tables directly. Where interpolation is required reference should first be made to the appropriate figure and, if one of the possible approximations is applicable, interpolation becomes a straight-forward process.

TABLE III

Value of attenuation (dB/cm) for electron collision frequency $(\omega_1) = 10^{11} \text{ sec}^{-1}$

$\log_{10} N$	14		13.5		13		12.5		12		11.5		11		10.5		10		9.5		9		8.5		8		7.5		7		6.5	
	$\log_{10} \omega$																															
6	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.06	-2	1.16	-2	6.50	-3	3.65	-3	2.05	-3	1.14	-3	6.18	-4	3.13	-4	1.33	-4	4.55	-5	1.46	-5
6.25	4.87	-1	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.66	-3	4.86	-3	2.72	-3	1.50	-3	7.94	-4	3.73	-4	1.41	-4	4.59	-5	1.46	-5
6.5	6.50	-1	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.15	-2	6.47	-3	3.60	-3	1.96	-3	9.90	-4	4.19	-4	1.44	-4	4.60	-5	1.46	-5
6.75	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.59	-3	4.74	-3	2.51	-3	1.18	-3	4.45	-4	1.45	-4	4.61	-5	1.46	-5
7	1.16	0	6.50	-1	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.14	-2	6.19	-3	3.13	-3	1.33	-3	4.55	-4	1.46	-4	4.61	-5	1.46	-5
7.25	1.54	0	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.66	-2	4.86	-2	2.72	-2	1.50	-2	7.94	-3	3.73	-3	1.41	-3	4.59	-4	1.46	-4	4.61	-5	1.46	-5
7.5	2.06	0	1.16	0	6.50	-1	3.65	-1	2.05	-1	1.15	-1	6.47	-2	3.60	-2	1.96	-2	9.90	-3	4.19	-3	1.44	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
7.75	2.74	0	1.54	0	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.59	-2	4.74	-2	2.51	-2	1.18	-2	4.45	-3	1.45	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
8	3.66	0	2.06	0	1.16	0	6.50	-1	3.65	-1	2.05	-1	1.14	-1	6.19	-2	3.13	-2	1.33	-2	4.56	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
8.25	4.88	0	2.74	0	1.54	0	8.67	-1	4.86	-1	2.72	-1	1.50	-1	7.94	-2	3.74	-2	1.41	-2	4.59	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
8.5	6.51	0	3.66	0	2.06	0	1.16	0	6.48	-1	3.60	-1	1.96	-1	9.92	-2	4.20	-2	1.44	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
8.75	8.69	0	4.89	0	2.75	0	1.54	0	8.61	-1	4.75	-1	2.52	-1	1.18	-1	4.45	-2	1.45	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
9	1.16	+1	6.53	0	3.67	0	2.06	0	1.14	0	6.22	-1	3.15	-1	1.33	-1	4.56	-2	1.46	-2	4.61	-3	1.46	-3	4.60	-4	1.46	-4	4.61	-5	1.46	-5
9.25	1.55	+1	8.74	0	4.90	0	2.74	0	1.51	0	8.01	-1	3.76	-1	1.41	-1	4.60	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
9.5	2.09	+1	1.17	+1	6.57	0	3.65	0	1.99	0	1.00	0	4.24	-1	1.45	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5
9.75	2.81	+1	1.58	+1	8.82	0	4.87	0	2.58	0	1.21	0	4.50	-1	1.45	-1	4.60	-2	1.45	-2	4.60	-3	1.45	-3	4.60	-4	1.45	-4	4.60	-5	1.45	-5
10	3.82	+1	2.14	+1	1.19	+1	6.47	0	3.26	0	1.36	0	4.58	-1	1.45	-1	4.57	-2	1.44	-2	4.57	-3	1.44	-3	4.56	-4	1.44	-4	4.56	-5	1.44	-5
10.25	5.23	+1	2.92	+1	1.61	+1	8.51	0	3.95	0	1.43	0	4.52	-1	1.42	-1	4.48	-2	1.41	-2	4.47	-3	1.41	-3	4.47	-4	1.41	-4	4.47	-5	1.41	-5
10.5	7.20	+1	4.00	+1	2.17	+1	1.09	+1	4.34	0	1.37	0	4.25	-1	1.33	-1	4.20	-2	1.33	-2	4.19	-3	1.33	-3	4.19	-4	1.33	-4	4.19	-5	1.33	-5
10.75	9.78	+1	5.38	+1	2.81	+1	1.23	+1	3.88	0	1.15	0	3.54	-1	1.11	-1	3.51	-2	1.11	-2	3.50	-3	1.11	-3	3.50	-4	1.11	-4	3.50	-5	1.11	-5
11	1.24	+2	6.63	+1	3.10	+1	9.40	0	2.50	0	7.48	-1	2.32	-1	7.31	-2	2.31	-2	7.20	-3	2.31	-3	7.29	-4	2.31	-4	7.29	-5	2.31	-5	7.29	-6

TABLE IV

Value of attenuation (dB/cm) for electron collision frequency $(\omega_1) = 10^{10.5} \text{ sec}^{-1}$

$\log_{10} N$	14		13.5		13		12.5		12		11.5		11		10.5		10		9.5		9		8.5		8		7.5		7		6.5	
	$\log_{10} \omega$																															
6	6.50	-1	3.64	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.06	-2	1.16	-2	6.50	-3	3.50	-3	2.05	-3	1.14	-3	6.18	-4	3.13	-4	1.33	-4	4.55	-5
6.25	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.66	-3	4.86	-3	2.72	-3	1.50	-3	7.94	-4	3.73	-4	1.41	-4	4.59	-5
6.5	1.16	0	6.50	-1	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.15	-2	6.47	-3	3.60	-3	1.96	-3	9.90	-4	4.19	-4	1.44	-4	4.60	-5
6.75	1.54	0	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.59	-3	4.74	-3	2.51	-3	1.18	-3	4.45	-4	1.45	-4	4.61	-5
7	2.06	0	1.16	0	6.50	-1	3.66	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.14	-2	6.19	-3	3.13	-3	1.33	-3	4.55	-4	1.46	-4	4.61	-5
7.25	2.74	0	1.54	0	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.66	-2	4.86	-2	2.72	-2	1.50	-2	7.94	-3	3.73	-3	1.41	-3	4.59	-4	1.46	-4	4.61	-5
7.5	3.66	0	2.06	0	1.16	0	6.50	-1	3.66	-1	2.06	-1	1.15	-1	6.47	-2	3.60	-2	1.96	-2	9.91	-3	4.19	-3	1.44	-3	4.60	-4	1.46	-4	4.61	-5
7.75	4.88	0	2.74	0	1.54	0	8.67	-1	4.88	-1	2.74	-1	1.54	-1	8.60	-2	4.74	-2	2.51	-2	1.18	-2	4.45	-3	1.45	-3	4.61	-4	1.46	-4	4.61	-5
8	6.51	0	3.66	0	2.06	0	1.16	0	6.51	-1	3.65	-1	2.05	-1	1.14	-1	6.19	-2	3.14	-2	1.33	-2	4.56	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5
8.25	8.69	0	4.89	0	2.75	0	1.55	0	8.68	-1	4.87	-1	2.72	-1	1.50	-1	7.96	-2	3.74	-2	1.41	-2	4.59	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5
8.5	1.16	+1	6.53	0	3.67	0	2.06	0	1.16	0	6.50	-1	3.62	-1	1.97	-1	9.95	-2	4.21	-2	1.44	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5
8.75	1.55	+1	8.74	0	4.92	0	2.76	0	1.55	0	8.67	-1	4.78	-1	2.53	-1	1.19	-1	4.47	-2	1.45	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5
9	2.09	+1	1.17	+1	6.60	0	3.71	0	2.08	0	1.16	0	6.28	-1	3.18	-1	1.34	-1	4.57	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5
9.25	2.81	+1	1.58	+1	8.89	0	4.99	0	2.88	0	1.54	0	8.15	-1	3.82	-1	1.42	-1	4.60	-2	1.46	-2	4.60	-3	1.45	-3	4.60	-4	1.45	-4	4.60	-5
9.5	3.82	+1	2.15	+1	1.21	+1	6.76	0	3.76	0	2.04	0	1.03	0	4.31	-1	1.45	-1	4.58	-2	1.45	-2	4.57	-3	1.44	-3	4.56	-4	1.44	-4	4.56	-5
9.75	5.24	+1	2.95	+1	1.65	+1	9.24	0	5.09	0	2.69	0	1.25	0	4.52	-1	1.43	-1	4.49	-2	1.42	-2	4.47	-3	1.41	-3	4.47	-4	1.41	-4	4.47	-5
10	7.24	+1	4.06	+1	2.28	+1	1.27	+1	6.87	0	3.43	0	1.37	0	4.34	-1	1.34	-1	4.21	-2	1.33	-2	4.19	-3	1.33	-3	4.19	-4	1.33	-4	4.19	-5
10.25	9.87	+1	5.54	+1	3.09	+1	1.70	+1	8.88	0	3.90	0	1.23	0	3.63	-1	1.12	-1	3.52	-2	1.11	-2	3.50	-3	1.11	-3	3.50	-4	1.11	-4	3.50	-5
10.5	1.27	+2	7.09	+1	3.93	+1	2.10	+1	9.80	0	2.97	0	7.91	-1	2.36	-1	7.35	-2	2.31	-2	7.30	-3	2.31	-3	7.29	-4	2.31	-4	7.29	-5	2.31	-5
10.75	1.47	+2	8.15	+1	4.40	+1	2.12	+1	5.82	0	1.27	0	3.64	-1	1.12	-1	3.52	-2	1.11	-2	3.50	-3	1.11	-3	3.50	-4	1.11	-4	3.50	-5	1.11	-5
11	1.55	+2	8.40	+1	4.10	+1	9.53	0	1.57	0	4.40	-1	1.34	-1	4.21	-2	1.33	-2	4.19	-3	1.33	-3	4.19	-4	1.33	-4	4.19	-5	1.33	-5	4.19	-6

TABLE I

$\log_{10} N$	14		13.5		13		12.5		12		11.5		11		10.5		10		9.5		9		8.5		8		7.5		7		6.5	
	$\log_{10} \omega$																															
6	1.16	-1	6.50	-2	3.65	-2	2.06	-2	1.16	-2	6.50	-3	3.65	-3	2.05	-3	1.14	-3	6.18	-4	3.13	-4	1.33	-4	4.55	-5	1.46	-5	4.61	-6	1.46	
6.25	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.66	-3	4.86	-3	2.72	-3	1.50	-3	7.94	-4	3.73	-4	1.41	-4	4.59	-5	1.46	-5	4.61	-6	1.46	
6.5	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.15	-2	6.47	-3	3.60	-3	1.96	-3	9.90	-4	4.19	-4	1.44	-4	4.60	-5	1.46	-5	4.61	-6	1.46	
6.75	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.59	-3	4.74	-3	2.51	-3	1.18	-3	4.45	-4	1.45	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
7	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.14	-2	6.18	-3	3.13	-3	1.33	-3	4.55	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
7.25	4.87	-1	2.74	-1	1.54	-1	8.66	-2	4.86	-2	2.72	-2	1.50	-2	7.94	-3	3.73	-3	1.41	-3	4.59	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
7.5	6.50	-1	3.65	-1	2.05	-1	1.15	-1	6.47	-2	3.60	-2	1.96	-2	9.90	-3	4.19	-3	1.44	-3	4.60	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
7.75	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.59	-2	4.74	-2	2.51	-2	1.18	-2	4.45	-3	1.45	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
8	1.16	0	6.50	-1	3.65	-1	2.05	-1	1.14	-1	6.19	-2	3.13	-2	1.33	-2	4.55	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
8.25	1.54	0	8.66	-1	4.86	-1	2.72	-1	1.50	-1	7.94	-2	3.73	-2	1.41	-2	4.59	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
8.5	2.05	0	1.15	0	6.47	-1	3.60	-1	1.96	-1	9.90	-2	4.19	-2	1.44	-2	4.60	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
8.75	2.74	0	1.54	0	8.59	-1	4.74	-1	2.51	-1	1.18	-1	4.45	-2	1.45	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
9	3.65	0	2.05	0	1.14	0	6.19	-1	3.13	-1	1.33	-1	4.56	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
9.25	4.86	0	2.72	0	1.50	0	7.94	-1	3.74	-1	1.41	-1	4.59	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
9.5	6.48	0	3.60	0	1.96	0	9.92	-1	4.20	-1	1.44	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
9.75	8.61	0	4.75	0	2.52	0	1.18	0	4.45	-1	1.45	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
10	1.14	+1	6.22	0	3.15	0	1.33	0	4.56	-1	1.46	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
10.25	1.51	+1	8.01	0	3.76	0	1.41	0	4.60	-1	1.46	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
10.5	1.99	+1	1.00	+1	4.24	0	1.45	0	4.61	-1	1.46	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	-6	1.46	
10.75	2.58	+1	1.21	+1	4.50	0	1.45	0	4.60	-1	1.45	-1	4.60	-2	1.45	-2	4.60	-3	1.45	-3	4.60	-4	1.45	-4	4.60	-5	1.45	-5	4.60	-6	1.45	
11	3.26	+1	1.36	+1	4.58	0	1.45	0	4.57	-1	1.44	-1	4.57	-2	1.44	-2	4.56	-3	1.44	-3	4.56	-4	1.44	-4	4.56	-5	1.44	-5	4.56	-6	1.44	

TABLE II

Value of attenuation (dB/cm) for electron collision frequency (ω_p) = $10^{11.5}$ sec⁻¹

$\log_{10} N$	14		13.5		13		12.5		12		11.5		11		10.5		10		9.5		9		8.5		8		7.5		7		6.5	
	$\log_{10} \omega$																															
6	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.06	-2	1.16	-2	6.50	-3	3.65	-3	2.05	-3	1.14	-3	6.18	-4	3.13	-4	1.33	-4	4.55	-5	1.46	-5	4.61	
6.25	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.66	-3	4.86	-3	2.72	-3	1.50	-3	7.94	-4	3.73	-4	1.41	-4	4.59	-5	1.46	-5	4.61	
6.5	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.15	-2	6.47	-3	3.60	-3	1.96	-3	9.90	-4	4.19	-4	1.44	-4	4.60	-5	1.46	-5	4.61	
6.75	4.87	-1	2.74	-1	1.54	-1	8.67	-2	4.87	-2	2.74	-2	1.54	-2	8.59	-3	4.74	-3	2.51	-3	1.18	-3	4.45	-4	1.45	-4	4.61	-5	1.46	-5	4.61	
7	6.50	-1	3.65	-1	2.06	-1	1.16	-1	6.50	-2	3.65	-2	2.05	-2	1.14	-2	6.18	-3	3.13	-3	1.33	-3	4.55	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
7.25	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.66	-2	4.86	-2	2.72	-2	1.50	-2	7.94	-3	3.73	-3	1.41	-3	4.59	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
7.5	1.16	0	6.50	-1	3.65	-1	2.05	-1	1.15	-1	6.47	-2	3.60	-2	1.96	-2	9.90	-3	4.19	-3	1.44	-3	4.60	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
7.75	1.54	0	8.67	-1	4.87	-1	2.74	-1	1.54	-1	8.59	-2	4.74	-2	2.51	-2	1.18	-2	4.45	-3	1.45	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
8	2.06	0	1.16	0	6.50	-1	3.65	-1	2.05	-1	1.14	-1	6.19	-2	3.13	-2	1.33	-2	4.55	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
8.25	2.74	0	1.54	0	8.66	-1	4.86	-1	2.72	-1	1.50	-1	7.94	-2	3.73	-2	1.41	-2	4.59	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
8.5	3.66	0	2.06	0	1.15	0	6.47	-1	3.60	-1	1.96	-1	9.91	-2	4.19	-2	1.44	-2	4.60	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
8.75	4.88	0	2.74	0	1.54	0	8.60	-1	4.74	-1	2.51	-1	1.18	-1	4.45	-2	1.45	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
9	6.51	0	3.65	0	2.05	0	1.14	0	6.19	-1	3.14	-1	1.33	-1	4.56	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
9.25	8.68	0	4.87	0	2.72	0	1.50	0	7.96	-1	3.74	-1	1.41	-1	4.59	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
9.5	1.16	+1	6.50	0	3.62	0	1.97	0	9.95	-1	4.21	-1	1.44	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
9.75	1.55	+1	8.67	0	4.78	0	2.53	0	1.19	0	4.47	-1	1.45	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
10	2.08	+1	1.16	+1	6.28	0	3.18	0	1.34	0	4.57	-1	1.46	-1	4.61	-2	1.46	-2	4.61	-3	1.46	-3	4.61	-4	1.46	-4	4.61	-5	1.46	-5	4.61	
10.25	2.79	+1	1.54	+1	8.15	0	3.82	0	1.42	0	4.60	-1	1.46	-1	4.60	-2	1.45	-2	4.60	-3	1.45	-3	4.60	-4	1.45	-4	4.60	-5	1.45	-5	4.60	
10.5	3.76	+1	2.04	+1	1.03	+1	4.31	0	1.45	0	4.58	-1	1.45	-1	4.57	-2	1.44	-2	4.56	-3	1.44	-3	4.56	-4	1.44	-4	4.56	-5	1.44	-5	4.56	
10.75	5.09	+1	2.69	+1	1.25	+1	4.52	0	1.43	0	4.49	-1	1.42	-1	4.47	-2	1.41	-2	4.47	-3	1.41	-3	4.47	-4	1.41	-4	4.47	-5	1.41	-5	4.47	
11	6.87	+1	3.43	+1	1.37	+1	4.34	0	1.34	0	4.21	-1	1.33	-1	4.19	-2	1.33	-2	4.19	-3	1.33	-3	4.19	-4	1.33	-4	4.19	-5	1.33	-5	4.19	

TABLE XI

Value of attenuation (dB/cm) for electron collision frequency (ω_1) = 10^7 sec⁻¹

log ₁₀ N	11		10.5		10		9.5		9		8.5		8		7.5		7		6.5		6		5.5		5		4.5		4	
	log ₁₀ ω																													
6	1.21	0	6.80	-1	3.82	-1	2.15	-1	1.21	-1	6.80	-2	3.82	-2	2.15	-2	1.21	-2	6.80	-3	3.82	-3	2.14	-3	1.19	-3	6.47	-4	3.27	-4
6.25	1.66	0	9.32	-1	5.24	-1	2.95	-1	1.66	-1	9.32	-2	5.24	-2	2.95	-2	1.66	-2	9.31	-3	5.23	-3	2.92	-3	1.61	-3	8.51	-4	3.95	-4
6.5	2.29	0	1.29	0	7.24	-1	4.07	-1	2.29	-1	1.29	-1	7.24	-2	4.07	-2	2.29	-2	1.29	-2	7.20	-3	4.00	-3	2.17	-3	1.09	-3	4.34	-4
6.75	3.12	0	1.76	0	9.88	-1	5.55	-1	3.12	-1	1.76	-1	9.88	-2	5.55	-2	3.12	-2	1.75	-2	9.78	-3	5.38	-3	2.81	-3	1.23	-3	3.88	-4
7	4.02	0	2.26	0	1.27	0	7.14	-1	4.02	-1	2.26	-1	1.27	-1	7.14	-2	4.01	-2	2.24	-2	1.24	-2	6.63	-3	3.10	-3	9.40	-4	2.50	-4
7.25	4.67	0	2.63	0	1.48	0	8.30	-1	4.67	-1	2.62	-1	1.48	-1	8.29	-2	4.64	-2	2.58	-2	1.39	-2	6.70	-3	1.84	-3	4.01	-4	1.15	-4
7.5	4.99	0	2.80	0	1.58	0	8.87	-1	4.99	-1	2.80	-1	1.57	-1	8.82	-2	4.91	-2	2.66	-2	1.30	-2	3.01	-3	4.96	-4	1.39	-4	4.25	-5
7.75	5.11	0	2.87	0	1.62	0	9.08	-1	5.11	-1	2.87	-1	1.61	-1	8.94	-2	4.84	-2	2.37	-2	4.47	-3	5.37	-4	1.49	-4	4.54	-5	1.42	-5
8	5.15	0	2.90	0	1.63	0	9.15	-1	5.14	-1	2.88	-1	1.60	-1	8.69	-2	4.26	-2	6.35	-3	5.51	-4	1.52	-4	4.64	-5	1.45	-5	4.57	-6
8.25	5.16	0	2.90	0	1.63	0	9.17	-1	5.14	-1	2.86	-1	1.55	-1	7.60	-2	8.90	-3	5.56	-4	1.53	-4	4.67	-5	1.46	-5	4.60	-6	1.45	-6
8.5	5.17	0	2.90	0	1.63	0	9.14	-1	5.08	-1	2.76	-1	1.35	-1	1.26	-2	5.58	-4	1.54	-4	4.68	-5	1.46	-5	4.61	-6	1.46	-6	4.61	-7
8.75	5.17	0	2.90	0	1.63	0	9.04	-1	4.90	-1	2.41	-1	1.82	-2	5.58	-4	1.54	-4	4.68	-5	1.46	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7
9	5.16	0	2.89	0	1.61	0	8.72	-1	4.28	-1	2.76	-2	5.58	-4	1.54	-4	4.68	-5	1.46	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7		
9.25	5.14	0	2.86	0	1.55	0	7.61	-1	4.44	-2	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7				
9.5	5.09	0	2.76	0	1.35	0	7.53	-2	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7						
9.75	4.91	0	2.41	0	1.31	-1	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7								
10	4.28	0	2.32	-1	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7										
10.25	4.12	-1	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7												
10.5	5.58	-4	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7														
10.75	1.54	-4	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.46	-7																
11	4.68	-5	1.47	-5	4.62	-6	1.46	-6	4.61	-7	1.45	-7																		

TABLE XII

Value of attenuation (dB/cm) for electron collision frequency (ω_1) = $10^{6.5}$ sec⁻¹

log ₁₀ N	10		9.5		9		8.5		8		7.5		7		6.5		6		5.5		5		4.5		4				
	log ₁₀ ω																												
6	7.24	-1	4.07	-1	2.29	-1	1.29	-1	7.24	-2	4.07	-2	2.29	-2	1.29	-2	7.24	-3	4.06	-3	2.28	-3	1.27	-3	6.87	-4			
6.25	9.88	-1	5.55	-1	3.12	-1	1.76	-1	9.88	-2	5.55	-2	3.12	-2	1.76	-2	9.87	-3	5.54	-3	3.09	-3	1.70	-3	8.88	-4			
6.5	1.27	0	7.14	-1	4.02	-1	2.26	-1	1.27	-1	7.14	-2	4.01	-2	2.26	-2	1.27	-2	7.09	-3	3.93	-3	2.10	-3	9.80	-4			
6.75	1.48	0	8.30	-1	4.67	-1	2.63	-1	1.48	-1	8.30	-2	4.67	-2	2.62	-2	1.47	-2	8.15	-3	4.40	-3	2.12	-3	5.82	-4			
7	1.58	0	8.87	-1	4.99	-1	2.80	-1	1.58	-1	8.87	-2	4.98	-2	2.79	-2	1.55	-2	8.40	-3	4.10	-3	9.53	-4	1.57	-4			
7.25	1.62	0	9.08	-1	5.11	-1	2.87	-1	1.61	-1	9.07	-2	5.08	-2	2.83	-2	1.53	-2	7.50	-3	1.41	-3	1.70	-4	4.70	-5			
7.5	1.63	0	9.16	-1	5.15	-1	2.89	-1	1.63	-1	9.11	-2	5.07	-2	2.75	-2	1.35	-2	2.01	-3	1.74	-4	4.81	-5	1.47	-5			
7.75	1.63	0	9.18	-1	5.16	-1	2.90	-1	1.62	-1	9.03	-2	4.90	-2	2.40	-2	2.82	-3	1.76	-4	4.85	-5	1.48	-5	4.62	-6			
8	1.63	0	9.18	-1	5.16	-1	2.89	-1	1.61	-1	8.72	-2	4.28	-2	3.97	-3	1.76	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	
8.25	1.63	0	9.18	-1	5.14	-1	2.86	-1	1.55	-1	7.61	-2	5.76	-3	1.76	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	
8.5	1.63	0	9.15	-1	5.09	-1	2.76	-1	1.35	-1	8.73	-3	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	1.46	-7	
8.75	1.63	0	9.05	-1	4.91	-1	2.41	-1	1.40	-2	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.46	-8	
9	1.61	0	8.72	-1	4.28	-1	2.38	-2	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.46	-8	1.46	-8	
9.25	1.55	0	7.61	-1	4.16	-2	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.46	-8	4.61	-9	1.46	-9	
9.5	1.35	0	7.35	-2	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.46	-8	4.61	-9	1.46	-9	1.42	-9	
9.75	1.30	-1	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.46	-8	4.65	-9	1.31	-9					
10	1.77	-4	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.45	-8	4.37	-9	1.65	-9							
10.25	4.86	-5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.44	-8	3.79	-9	2.40	-9									
10.5	1.48	-5	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.60	-8	1.42	-8	4.27	-9													
10.75	4.63	-6	1.46	-6	4.61	-7	1.46	-7	4.61	-8	1.20	-8																	
11	1.46	-6	4.61	-7	1.45	-7	4.47	-8																					



TABLE XIII

Value of attenuation (dB/cm) for electron collision frequency (ω_1) = 10^6 sec^{-1}

log ₁₀ N	10		9.5		9		8.5		8		7.5		7		6.5		6		5.5		5		4.5		4	
	log ₁₀ ω																									
6	1.27	0	7.14	-1	4.02	-1	2.26	-1	1.27	-1	7.14	-2	4.02	-2	2.26	-2	1.27	-2	7.14	-3	4.01	-3	2.24	-3	1.24	-3
6.25	1.48	0	8.30	-1	4.67	-1	2.63	-1	1.48	-1	8.30	-2	4.67	-2	2.62	-2	1.48	-2	8.29	-3	4.64	-3	2.58	-3	1.39	-3
6.5	1.58	0	8.87	-1	4.99	-1	2.80	-1	1.58	-1	8.87	-2	4.99	-2	2.80	-2	1.57	-2	8.82	-3	4.91	-3	2.66	-3	1.30	-3
6.75	1.62	0	9.08	-1	5.11	-1	2.87	-1	1.62	-1	9.08	-2	5.11	-2	2.87	-2	1.61	-2	8.94	-3	4.84	-3	2.37	-3	4.47	-4
7	1.63	0	9.16	-1	5.15	-1	2.90	-1	1.63	-1	9.15	-2	5.14	-2	2.88	-2	1.60	-2	8.69	-3	4.26	-3	6.35	-4	5.51	-5
7.25	1.63	0	9.18	-1	5.16	-1	2.90	-1	1.63	-1	9.17	-2	5.14	-2	2.86	-2	1.55	-2	7.60	-3	8.90	-4	5.56	-5	1.53	-5
7.5	1.63	0	9.19	-1	5.17	-1	2.90	-1	1.63	-1	9.14	-2	5.08	-2	2.76	-2	1.35	-2	1.26	-3	5.58	-5	1.54	-5	4.68	-6
7.75	1.63	0	9.19	-1	5.17	-1	2.90	-1	1.63	-1	9.04	-2	4.90	-2	2.41	-2	1.82	-3	5.58	-5	1.54	-5	4.68	-6	1.47	-6
8	1.63	0	9.19	-1	5.16	-1	2.89	-1	1.61	-1	8.72	-2	4.28	-2	2.76	-3	5.58	-5	1.54	-5	4.68	-6	1.47	-6	4.62	-7
8.25	1.63	0	9.18	-1	5.14	-1	2.86	-1	1.55	-1	7.61	-2	4.44	-3	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7
8.5	1.63	0	9.15	-1	5.09	-1	2.76	-1	1.31	-2	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7				
8.75	1.63	0	9.05	-1	4.91	-1	2.41	-1	1.31	-2	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7				
9	1.61	0	8.72	-1	4.28	-1	2.32	-2	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7						
9.25	1.55	0	7.61	-1	4.12	-2	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7								
9.5	1.35	0	7.33	-2	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7										
9.75	1.30	-1	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7												
10	5.58	-5	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7														
10.25	1.54	-5	4.69	-6	1.47	-6	4.62	-7	1.46	-7																
10.5	4.69	-6	1.47	-6	4.62	-7	1.46	-7																		
10.75	1.47	-6	4.62	-7	1.46	-7																				
11	4.61	-7																								

TABLE XIV

Value of phase shift (rad/cm) for electron collision frequency (ω_1) = 10^{12} sec^{-1}

log ₁₀ N	14		13.5		13		12.5		12		11.5		11		10.5		10		9.5		9		8.5		8		7.5		7		6.5	
	log ₁₀ ω																															
6	-1.33	-2	-7.45	-3	-4.17	-3	-2.33	-3	-1.30	-3	-7.15	-4	-3.88	-4	-2.04	-4	-1.02	-4	-4.53	-5	-1.58	-5	-3.33	-6	-4.10	-7	-4.21	-8	-4.22	-9	-4.22	-10
6.25	-1.77	-2	-9.92	-3	-5.55	-3	-3.10	-3	-1.72	-3	-9.39	-4	-5.03	-4	-2.59	-4	-1.23	-4	-4.96	-5	-1.39	-5	-2.17	-6	-2.35	-7	-2.37	-8	-2.37	-9	-2.37	-10
6.5	-2.36	-2	-1.32	-2	-7.38	-3	-4.10	-3	-2.26	-3	-1.23	-3	-6.46	-4	-3.22	-4	-1.43	-4	-4.98	-5	-1.05	-5	-1.30	-6	-1.33	-7	-1.33	-8	-1.33	-9	-1.33	-10
6.75	-3.14	-2	-1.76	-2	-9.79	-3	-5.43	-3	-2.97	-3	-1.59	-3	-8.19	-4	-3.89	-4	-1.57	-4	-4.41	-5	-6.86	-6	-7.43	-7	-7.50	-8	-7.50	-9	-7.48	-10	-7.42	-11
7	-4.17	-2	-2.33	-2	-1.30	-2	-7.15	-3	-3.88	-3	-2.04	-3	-1.02	-3	-4.53	-4	-1.58	-4	-3.33	-5	-4.10	-6	-4.21	-7	-4.22	-8	-4.21	-9	-4.17	-10	-4.06	-11
7.25	-5.55	-2	-3.10	-2	-1.72	-2	-9.39	-3	-5.03	-3	-2.59	-3	-1.23	-3	-4.96	-4	-1.39	-4	-2.17	-5	-2.35	-6	-2.37	-7	-2.37	-8	-2.34	-9	-2.28	-10	-2.08	-11
7.5	-7.38	-2	-4.10	-2	-2.26	-2	-1.23	-2	-6.46	-3	-3.22	-3	-1.43	-3	-4.98	-4	-1.05	-4	-1.29	-5	-1.33	-6	-1.33	-7	-1.32	-8	-1.28	-9	-1.17	-10	-8.05	-12
7.75	-9.79	-2	-5.42	-2	-2.97	-2	-1.59	-2	-8.19	-3	-3.89	-3	-1.57	-3	-4.41	-4	-6.86	-5	-7.43	-6	-7.47	-7	-7.41	-8	-7.21	-9	-6.57	-10	-4.52	-11	+1.93	-12
8	-1.30	-1	-7.15	-2	-3.88	-2	-2.04	-2	-1.02	-2	-4.53	-3	-1.58	-3	-3.32	-4	-4.09	-5	-4.19	-6	-4.17	-7	-4.06	-8	-3.69	-9	-2.54	-10	+1.08	-11	+1.26	-11
8.25	-1.72	-1	-9.39	-2	-5.03	-2	-2.59	-2	-1.23	-2	-4.96	-3	-1.39	-3	-2.17	-4	-2.34	-5	-2.34	-6	-2.28	-7	-2.08	-8	-1.43	-9	+6.10	-11	+7.06	-11	+2.75	-11
8.5	-2.26	-1	-1.23	-1	-6.46	-2	-3.22	-2	-1.43	-2	-4.98	-3	-1.05	-3	-1.29	-4	-1.31	-5	-1.28	-6	-1.17	-7	-8.05	-9	+3.43	-10	+3.97	-10	+1.54	-10	+5.17	-11
8.75	-2.97	-1	-1.59	-1	-8.19	-2	-3.89	-2	-1.57	-2	-4.40	-3	-6.84	-4	-7.34	-5	-7.20	-6	-6.57	-7	-4.52	-8	+1.93	-9	+2.23	-9	+8.69	-10	+2.91	-10	+9.36	-11
9	-3.88	-1	-2.04	-1	-1.02	-1	-4.52	-2	-1.57	-2	-3.31	-3	-4.04	-4	-4.04	-5	-3.69	-6	-2.54	-7	+1.08	-8	+1.26	-8	+4.89	-9	+1.64	-9	+5.27	-10	+1.67	-10
9.25	-5.03	-1	-2.59	-1	-1.23	-1	-4.95	-2	-1.39	-2	-2.14	-3	-2.26	-4	-2.07	-5	-1.43	-6	+6.10	-8	+7.06	-8	+2.75	-8	+9.20	-9	+2.96	-9	+9.41	-10	+2.98	-10
9.5	-6.45	-1	-3.21	-1	-1.43	-1	-4.96	-2	-1.04	-2	-1.24	-3	-1.16	-4	-8.04	-6	+3.43	-7	+3.97	-7	+1.54	-7	+5.17	-8	+1.57	-8	+5.29	-9	+1.68	-9	+5.31	-10
9.75	-8.16	-1	-3.88	-1	-1.56	-1	-4.35	-2	-6.59	-3	-6.50	-4	-4.52	-5	+1.93	-6	+2.23	-6	+8.69	-7	+2.91	-7	+9.36	-8	+2.92	-8	+9.43	-9	+2.98	-9	+9.44	-10
10	-1.01	0	-4.49	-1	-1.55	-1	-3.20	-2	-3.58	-3	-2.54	-4	+1.08	-5	+1.26	-5	+4.89	-6	+1.64	-6	+5.27	-7	+1.67	-7	+5.30	-8	+1.68	-8	+5.31	-9	+1.68	-9
10.25	-1.22	0	-4.87	-1	-1.34	-1	-1.90	-2	-1.42	-3	+6.10	-5	+7.06	-5	+2.75	-5	+9.20	-6	+2.96	-6	+9.41	-7	+2.98	-7	+9.43	-8	+2.98	-8	+9.44	-9	+2.98	-9
10.5	-1.39	0	-4.74	-1	-9.20	-2	-7.80	-3	+3.43	-4	+3.97	-4	+1.54	-4	+5.17	-5	+1.66	-5	+5.29	-6	+1.68	-6	+5.30	-7	+1.68	-7	+5.30	-8	+1.68	-8	+5.30	-9
10.75	-1.47	0	-3.82	-1	-4.12	-2	+1.93	-3	+2.23	-3	+8.66	-4	+2.90	-4	+9.33	-5	+2.97	-5	+9.40	-6	+2.97	-6	+9.41	-7	+2.98	-7	+9.41	-8	+2.98	-8	+9.41	-9
11	-1.33	0	-1.97	-1	+1.09	-2	+1.25	-2	+4.84	-3	+1.62	-3	+5.21	-4	+1.66	-4	+5.25	-5	+1.66	-5	+5.25	-6	+1.66	-6	+5.26	-7	+1.66	-7	+5.26	-8	+1.66	-8

TABLE XXV

Value of phase shift (rad/cm) for electron collision frequency $(\omega_1) = 10^{6\frac{1}{2}} \text{ sec}^{-1}$

$\log_{10} N$	10		9.5		9		8.5		8		7.5		7		6.5		6		5.5		5		4.5		4		
$\log_{10} \omega$																											
6	-6.10	-2	-3.43	-2	-1.93	-2	-1.08	-2	-6.07	-3	-3.40	-3	-1.90	-3	-1.05	-3	-5.78	-4	-3.11	-4	-1.61	-4	-7.70	-5	-3.10	-5	
6.25	-6.65	-2	-3.73	-2	-2.10	-2	-1.18	-2	-6.59	-3	-3.68	-3	-2.04	-3	-1.12	-3	-6.07	-4	-3.16	-4	-1.53	-4	-6.29	-5	-1.46	-5	
6.5	-6.05	-2	-3.39	-2	-1.90	-2	-1.07	-2	-5.95	-3	-3.30	-3	-1.81	-3	-9.72	-4	-5.01	-4	-2.37	-4	-9.04	-5	-1.05	-5	+2.70	-5	
6.75	-4.43	-2	-2.48	-2	-1.39	-2	-7.73	-3	-4.26	-3	-2.32	-3	-1.22	-3	-6.05	-4	-2.60	-4	-6.73	-5	+3.81	-5	+8.95	-5	+7.46	-5	
7	-2.77	-2	-1.54	-2	-8.53	-3	-4.65	-3	-2.47	-3	-1.24	-3	-5.54	-4	-1.67	-4	+4.85	-5	+1.67	-4	+2.26	-4	+1.87	-4	+5.18	-5	
7.25	-1.58	-2	-8.63	-3	-4.60	-3	-2.33	-3	-1.05	-3	-3.31	-4	+7.16	-5	+2.97	-4	+4.20	-4	+4.81	-4	+4.06	-4	+9.95	-5	+2.96	-5	
7.5	-8.30	-3	-4.20	-3	-1.90	-3	-6.09	-4	+1.18	-4	+5.26	-4	+7.54	-4	+8.80	-4	+9.42	-4	+8.15	-4	+1.82	-4	+5.39	-5	+1.68	-5	
7.75	-3.40	-3	-1.09	-3	+2.05	-4	+9.35	-4	+1.35	-3	+1.57	-3	+1.70	-3	+1.76	-3	+1.57	-3	+3.26	-4	+9.66	-5	+3.00	-5	+9.43	-6	
8	+3.62	-4	+1.66	-3	+2.39	-3	+2.80	-3	+3.03	-3	+3.16	-3	+3.22	-3	+2.95	-3	+5.81	-4	+1.72	-4	+5.35	-5	+1.68	-5	+5.31	-6	
8.25	+4.26	-3	+4.99	-3	+5.40	-3	+5.63	-3	+5.76	-3	+5.82	-3	+5.46	-3	+1.03	-3	+3.06	-4	+9.51	-5	+2.99	-5	+9.44	-6	+2.98	-6	
8.5	+9.61	-3	+1.00	-2	+1.02	-2	+1.04	-2	+1.04	-2	+9.99	-3	+1.84	-3	+5.45	-4	+1.69	-4	+5.32	-5	+1.68	-5	+5.31	-6	+1.68	-6	
8.75	+1.82	-2	+1.85	-2	+1.86	-2	+1.86	-2	+1.81	-2	+3.27	-3	+9.69	-4	+3.01	-4	+9.46	-5	+2.99	-5	+9.44	-6	+2.98	-6	+9.44	-7	
9	+3.31	-2	+3.32	-2	+3.32	-2	+3.27	-2	+5.81	-3	+1.72	-3	+5.35	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	
9.25	+5.91	-2	+5.92	-2	+5.87	-2	+1.03	-2	+3.06	-3	+9.52	-4	+2.99	-4	+9.45	-5	+2.99	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	
9.5	+1.05	-1	+1.05	-1	+1.84	-2	+5.45	-3	+1.69	-3	+5.32	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	
9.75	+1.87	-1	+3.27	-2	+9.69	-3	+3.01	-3	+9.46	-4	+2.99	-4	+9.44	-5	+2.99	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8	
10	+5.81	-2	+1.72	-2	+5.35	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8	
10.25	+3.06	-2	+9.52	-3	+2.99	-3	+9.45	-4	+2.99	-4	+9.44	-5	+2.98	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8	+2.98	-8	
10.5	+1.69	-2	+5.32	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8	+1.68	-8	
10.75	+9.46	-3	+2.99	-3	+9.44	-4	+2.99	-4	+9.44	-5	+2.98	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8	+2.98	-8	+9.44	-9	
11	+5.31	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8	+1.68	-8	+5.31	-9	

TABLE XXVI

Value of phase shift (rad/cm) for electron collision frequency $(\omega_1) = 10^6 \text{ sec}^{-1}$

$\log_{10} N$	10		9.5		9		8.5		8		7.5		7		6.5		6		5.5		5		4.5		4	
$\log_{10} \omega$																										
6	-6.05	-2	-3.40	-2	-1.91	-2	-1.07	-2	-6.02	-3	-3.37	-3	-1.88	-3	-1.04	-3	-5.72	-4	-3.07	-4	-1.59	-4	-7.51	-5	-2.86	-5
6.25	-4.44	-2	-2.50	-2	-1.40	-2	-7.86	-3	-4.39	-3	-2.44	-3	-1.35	-3	-7.32	-4	-3.86	-4	-1.91	-4	-8.22	-5	-2.13	-5	+1.21	-5
6.5	-2.79	-2	-1.57	-2	-8.76	-3	-4.88	-3	-2.70	-3	-1.47	-3	-7.81	-4	-3.93	-4	-1.75	-4	-5.30	-5	+1.54	-5	+5.28	-5	+7.13	-5
6.75	-1.62	-2	-9.04	-3	-5.00	-3	-2.73	-3	-1.45	-3	-7.35	-4	-3.32	-4	-1.05	-4	+2.27	-5	+9.38	-5	+1.33	-4	+1.52	-4	+1.28	-4
7	-9.02	-3	-4.92	-3	-2.62	-3	-1.33	-3	-6.02	-4	-1.93	-4	+3.74	-5	+1.66	-4	+2.39	-4	+2.78	-4	+2.98	-4	+2.58	-4	+5.74	-5
7.25	-4.69	-3	-2.38	-3	-1.08	-3	-3.46	-4	+6.49	-5	+2.96	-4	+4.25	-4	+4.98	-4	+5.38	-4	+5.57	-4	+4.96	-4	+1.03	-4	+3.05	-5
7.5	-1.92	-3	-6.17	-4	+1.14	-4	+5.26	-4	+7.57	-4	+8.87	-4	+9.59	-4	+9.99	-4	+1.02	-3	+9.33	-4	+1.84	-4	+5.44	-5	+1.69	-5
7.75	+2.03	-4	+9.35	-4	+1.35	-3	+1.58	-3	+1.71	-3	+1.78	-3	+1.82	-3	+1.84	-3	+1.73	-3	+3.27	-4	+9.69	-5	+3.01	-5	+9.46	-6
8	+2.39	-3	+2.81	-3	+3.04	-3	+3.17	-3	+3.24	-3	+3.28	-3	+3.30	-3	+3.16	-3	+5.81	-4	+1.72	-4	+5.35	-5	+1.68	-5	+5.31	-6
8.25	+5.40	-3	+5.63	-3	+5.76	-3	+5.84	-3	+5.88	-3	+5.90	-3	+5.74	-3	+1.03	-3	+3.06	-4	+9.51	-5	+2.99	-5	+9.45	-6	+2.99	-6
8.5	+1.03	-2	+1.04	-2	+1.05	-2	+1.05	-2	+1.05	-2	+1.03	-2	+1.84	-3	+5.45	-4	+1.69	-4	+5.32	-5	+1.68	-5	+5.31	-6	+1.68	-6
8.75	+1.86	-2	+1.87	-2	+1.87	-2	+1.87	-2	+1.85	-2	+3.27	-3	+9.69	-4	+3.01	-4	+9.46	-5	+2.99	-5	+9.44	-6	+2.99	-6	+9.44	-7
9	+3.33	-2	+3.33	-2	+3.33	-2	+3.31	-2	+5.81	-3	+1.72	-3	+5.35	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7
9.25	+5.93	-2	+5.93	-2	+5.91	-2	+1.03	-2	+3.06	-3	+9.52	-4	+2.99	-4	+9.45	-5	+2.99	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7
9.5	+1.05	-1	+1.05	-1	+1.84	-2	+5.45	-3	+1.69	-3	+5.32	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7
9.75	+1.87	-1	+3.27	-2	+9.69	-3	+3.01	-3	+9.46	-4	+2.99	-4	+9.44	-5	+2.99	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8
10	+5.81	-2	+1.72	-2	+5.35	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8
10.25	+3.06	-2	+9.52	-3	+2.99	-3	+9.45	-4	+2.99	-4	+9.44	-5	+2.98	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8	+2.98	-8
10.5	+1.69	-2	+5.32	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8	+1.68	-8
10.75	+9.46	-3	+2.99	-3	+9.44	-4	+2.99	-4	+9.44	-5	+2.98	-5	+9.44	-6	+2.98	-6	+9.44	-7	+2.98	-7	+9.44	-8	+2.98	-8	+9.44	-9
11	+5.31	-3	+1.68	-3	+5.31	-4	+1.68	-4	+5.31	-5	+1.68	-5	+5.31	-6	+1.68	-6	+5.31	-7	+1.68	-7	+5.31	-8	+1.68	-8	+5.31	-9

TABLE XXXV

Reflection coefficient at normal incidence for electron collision frequency $(\omega_1) = 10^8 \text{ sec}^{-1}$

$\log_{10} N$	14	13.5	13	12.5	12	11.5	11	10.5	10	9.5	9	8.5	8	7.5	7	6.5																
$\log_{10} \omega$																																
6	1.00	0	1.00	0	1.00	0	1.00	0	9.99	-1	9.98	-1	9.97	-1	9.95	-1	9.91	-1	9.84	-1	9.72	-1	9.51	-1	9.15	-1	8.54	-1	7.55	-1		
6.25	1.00	0	1.00	0	1.00	0	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.93	-1	9.88	-1	9.79	-1	9.63	-1	9.36	-1	8.89	-1	8.11	-1	6.88	-1
6.5	1.00	0	1.00	0	1.00	0	1.00	0	9.99	-1	9.98	-1	9.97	-1	9.95	-1	9.91	-1	9.85	-1	9.73	-1	9.52	-1	9.16	-1	8.55	-1	7.57	-1	6.09	-1
6.75	1.00	0	1.00	0	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.89	-1	9.80	-1	9.64	-1	9.37	-1	8.91	-1	8.14	-1	6.93	-1	5.19	-1
7	1.00	0	1.00	0	1.00	0	9.99	-1	9.98	-1	9.97	-1	9.95	-1	9.92	-1	9.85	-1	9.74	-1	9.53	-1	9.19	-1	8.60	-1	7.64	-1	6.19	-1	4.21	-1
7.25	1.00	0	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.97	-1	9.94	-1	9.89	-1	9.81	-1	9.66	-1	9.41	-1	8.97	-1	8.24	-1	7.08	-1	5.38	-1	3.23	-1
7.5	1.00	0	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.92	-1	9.87	-1	9.76	-1	9.58	-1	9.27	-1	8.73	-1	7.85	-1	6.49	-1	4.56	-1	2.29	-1
7.75	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.97	-1	9.95	-1	9.91	-1	9.84	-1	9.72	-1	9.50	-1	9.13	-1	8.50	-1	7.49	-1	5.93	-1	3.78	-1	1.39	-1
8	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.97	-1	9.94	-1	9.90	-1	9.82	-1	9.68	-1	9.44	-1	9.03	-1	8.33	-1	7.20	-1	5.44	-1	2.89	-1	4.77	-2
8.25	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.97	-1	9.94	-1	9.89	-1	9.81	-1	9.66	-1	9.41	-1	8.97	-1	8.22	-1	6.97	-1	4.79	-1	1.06	-1	6.21	-3
8.5	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.97	-1	9.94	-1	9.89	-1	9.80	-1	9.66	-1	9.39	-1	8.94	-1	8.12	-1	6.55	-1	1.94	-1	7.94	-3	6.32	-4
8.75	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.89	-1	9.80	-1	9.65	-1	9.38	-1	8.89	-1	7.87	-1	3.01	-1	8.71	-3	6.79	-4	6.33	-5
9	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.89	-1	9.80	-1	9.65	-1	9.36	-1	8.74	-1	4.15	-1	8.98	-3	6.95	-4	6.47	-5	6.33	-6
9.25	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.89	-1	9.80	-1	9.63	-1	9.27	-1	5.28	-1	9.07	-5	7.00	-4	6.52	-5	6.37	-6	6.33	-7
9.5	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.89	-1	9.79	-1	9.58	-1	6.33	-1	9.10	-3	7.02	-4	6.53	-5	6.39	-6	6.34	-7	6.33	-8
9.75	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.94	-1	9.88	-1	9.76	-1	7.28	-1	9.11	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.33	-8	6.33	-9
10	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.93	-1	9.87	-1	8.11	-1	9.12	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.34	-8	6.33	-9	6.33	-10
10.25	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.92	-1	8.78	-1	9.12	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.34	-8	6.33	-9	6.33	-10	6.33	-11
10.5	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.96	-1	9.26	-1	9.12	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.34	-8	6.33	-9	6.33	-10	6.33	-11	6.33	-12
10.75	1.00	0	9.99	-1	9.99	-1	9.98	-1	9.57	-1	9.12	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.34	-8	6.33	-9	6.33	-10	6.33	-11	6.33	-12	6.33	-13
11	1.00	0	9.99	-1	9.99	-1	9.75	-1	9.12	-3	7.03	-4	6.54	-5	6.39	-6	6.35	-7	6.34	-8	6.33	-9	6.33	-10	6.33	-11	6.33	-12	6.33	-13	6.33	-14

TABLE XXXVI

Approximation for R at high electron collision frequencies

$N \cdot \omega_1 / \omega$	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}
R	$6.33 \cdot 10^{-5}$	$6.12 \cdot 10^{-3}$	$1.91 \cdot 10^{-1}$	$6.04 \cdot 10^{-1}$	$8.53 \cdot 10^{-1}$	$9.51 \cdot 10^{-1}$	$9.84 \cdot 10^{-1}$

$N \cdot \omega_1 / \omega$	$10^{13.25}$	$10^{14.25}$	$10^{15.25}$	$10^{16.25}$	$10^{17.25}$	$10^{18.25}$
R	$2.00 \cdot 10^{-4}$	$1.80 \cdot 10^{-2}$	$2.95 \cdot 10^{-1}$	$6.86 \cdot 10^{-1}$	$8.88 \cdot 10^{-1}$	$9.63 \cdot 10^{-1}$

$N \cdot \omega_1 / \omega$	$10^{13.50}$	$10^{14.50}$	$10^{15.50}$	$10^{16.50}$	$10^{17.50}$	$10^{18.50}$
R	$6.31 \cdot 10^{-4}$	$4.75 \cdot 10^{-2}$	$4.04 \cdot 10^{-1}$	$7.54 \cdot 10^{-1}$	$9.15 \cdot 10^{-1}$	$9.72 \cdot 10^{-1}$

$N \cdot \omega_1 / \omega$	$10^{13.75}$	$10^{14.75}$	$10^{15.75}$	$10^{16.75}$	$10^{17.75}$	$10^{18.75}$
R	$1.98 \cdot 10^{-3}$	$1.05 \cdot 10^{-1}$	$5.09 \cdot 10^{-1}$	$8.09 \cdot 10^{-1}$	$9.35 \cdot 10^{-1}$	$9.79 \cdot 10^{-1}$

TABLE XXXVII

Approximation for R at low electron collision frequencies

N/ω^2	10^{-10}	$10^{-10.5}$	10^{-11}	$10^{-11.5}$	10^{-12}	$10^{-12.5}$	10^{-13}
R	$9.12 \cdot 10^{-3}$	$7.93 \cdot 10^{-4}$	$6.54 \cdot 10^{-5}$	$6.39 \cdot 10^{-6}$	$6.35 \cdot 10^{-7}$	$6.34 \cdot 10^{-8}$	$6.33 \cdot 10^{-9}$

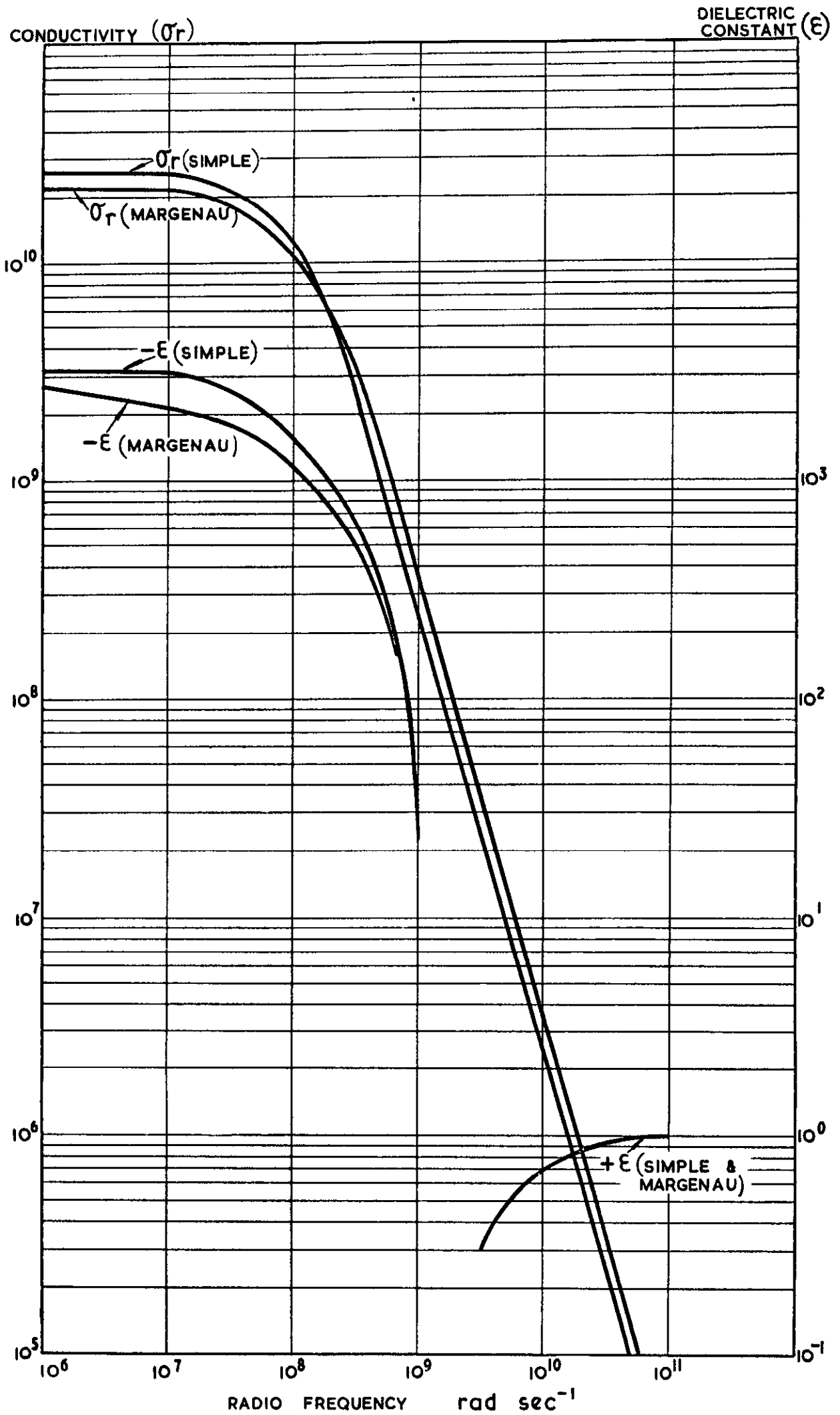


FIG. 1 COMPARISON OF SIMPLE THEORY WITH THAT OF MARGENAU $N=10^{10} \text{ cm}^{-3}$ $\omega_p=10^8 \text{ sec}^{-1}$

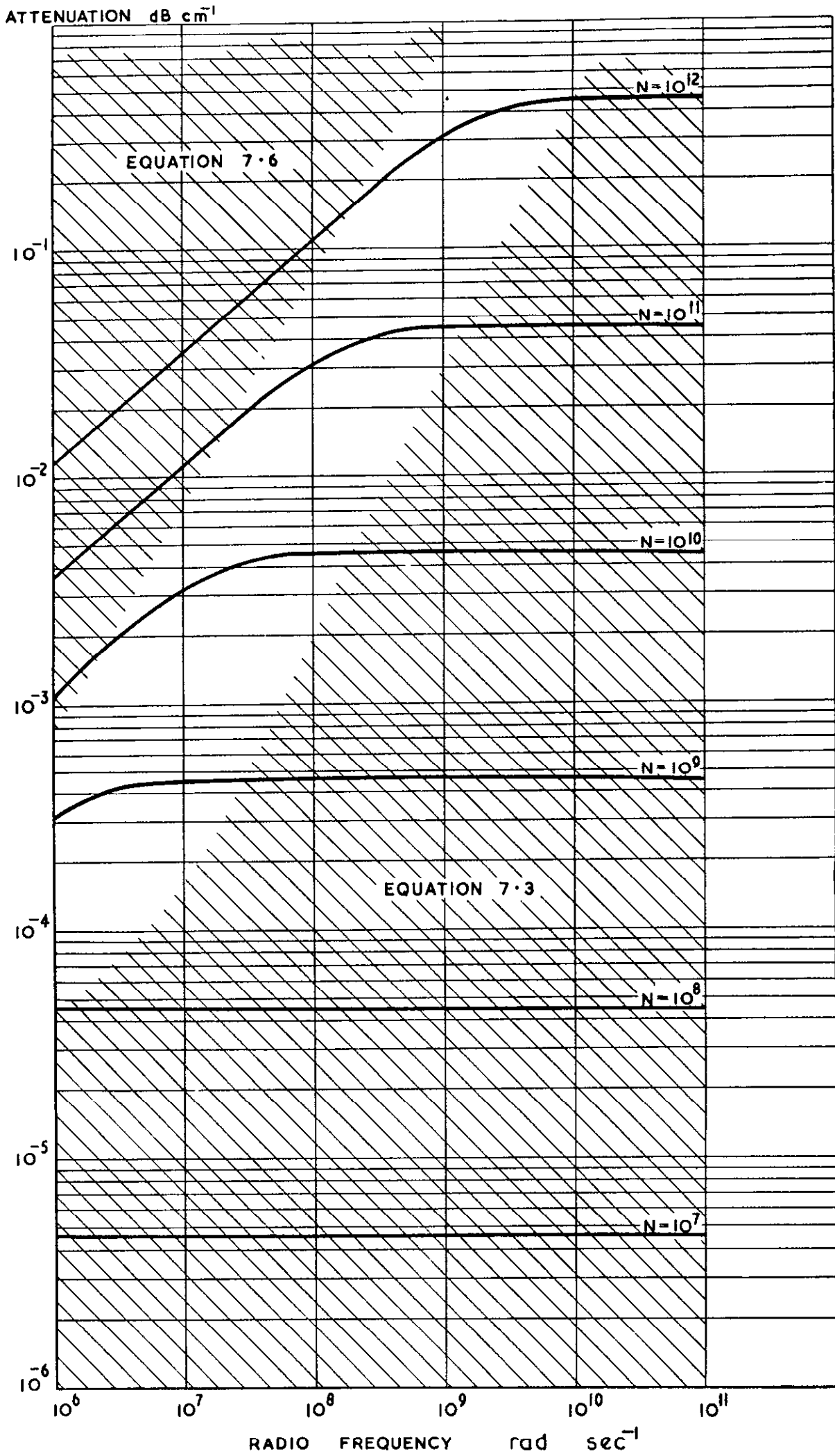


FIG. 2 VARIATION OF ATTENUATION WITH RADIO FREQUENCY $\omega_1 = 10^{12} \text{ sec}^{-1}$

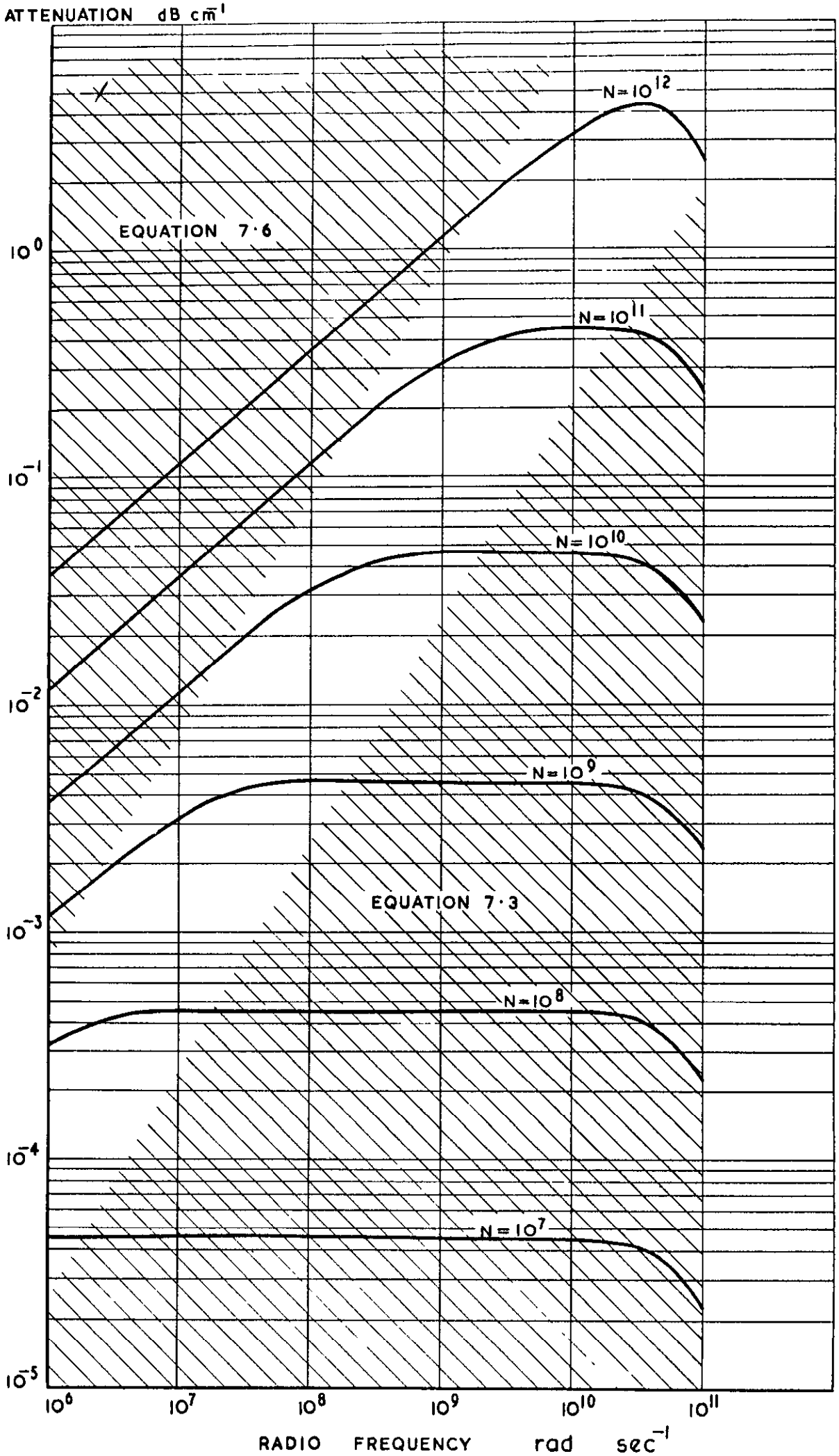


FIG. 3 VARIATION OF ATTENUATION WITH RADIO FREQUENCY $\omega_1 = 10^{11} \text{ sec}^{-1}$

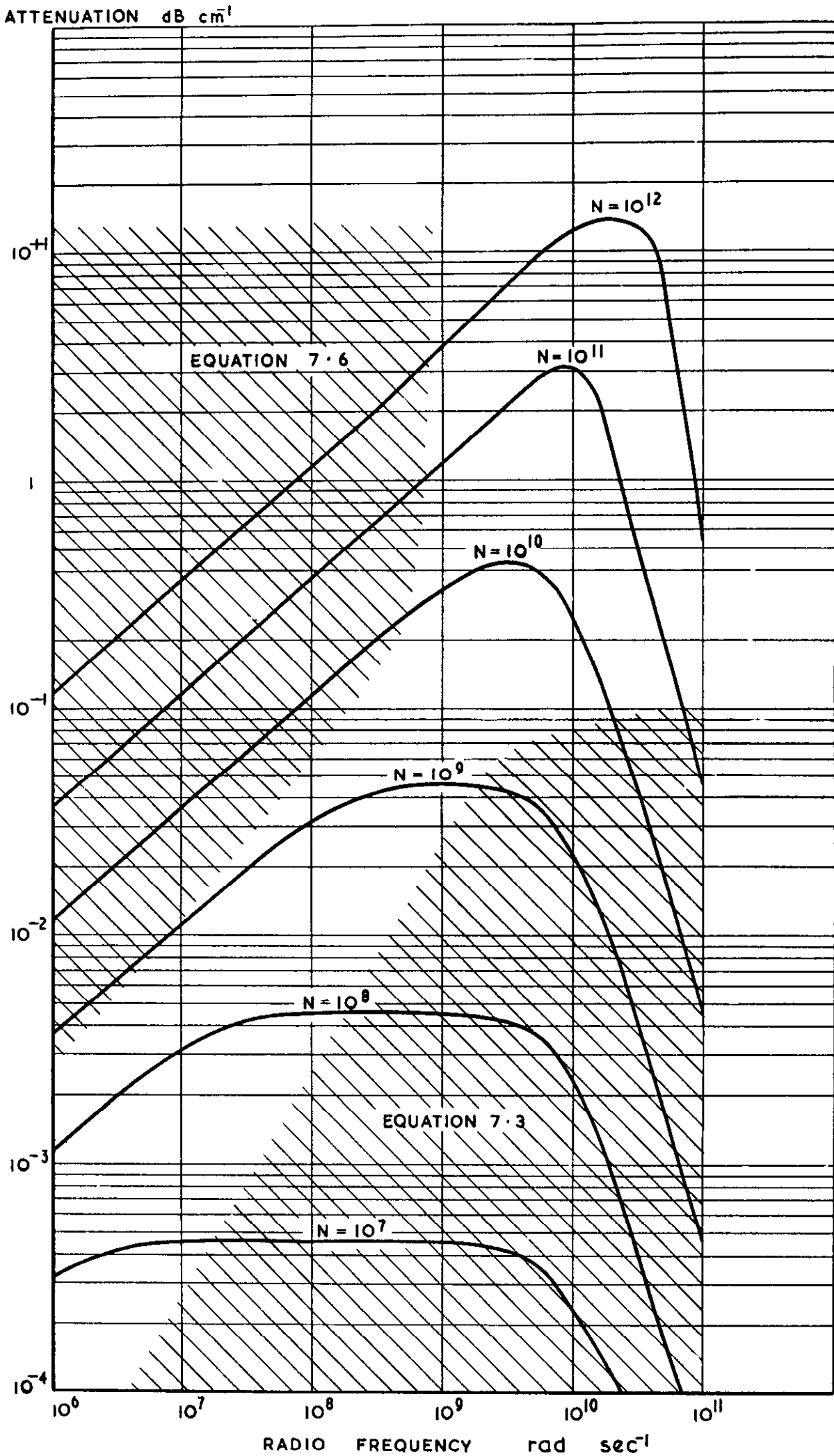


FIG. 4 VARIATION OF ATTENUATION WITH RADIO FREQUENCY $\omega_1 = 10^{10} \text{sec}^{-1}$

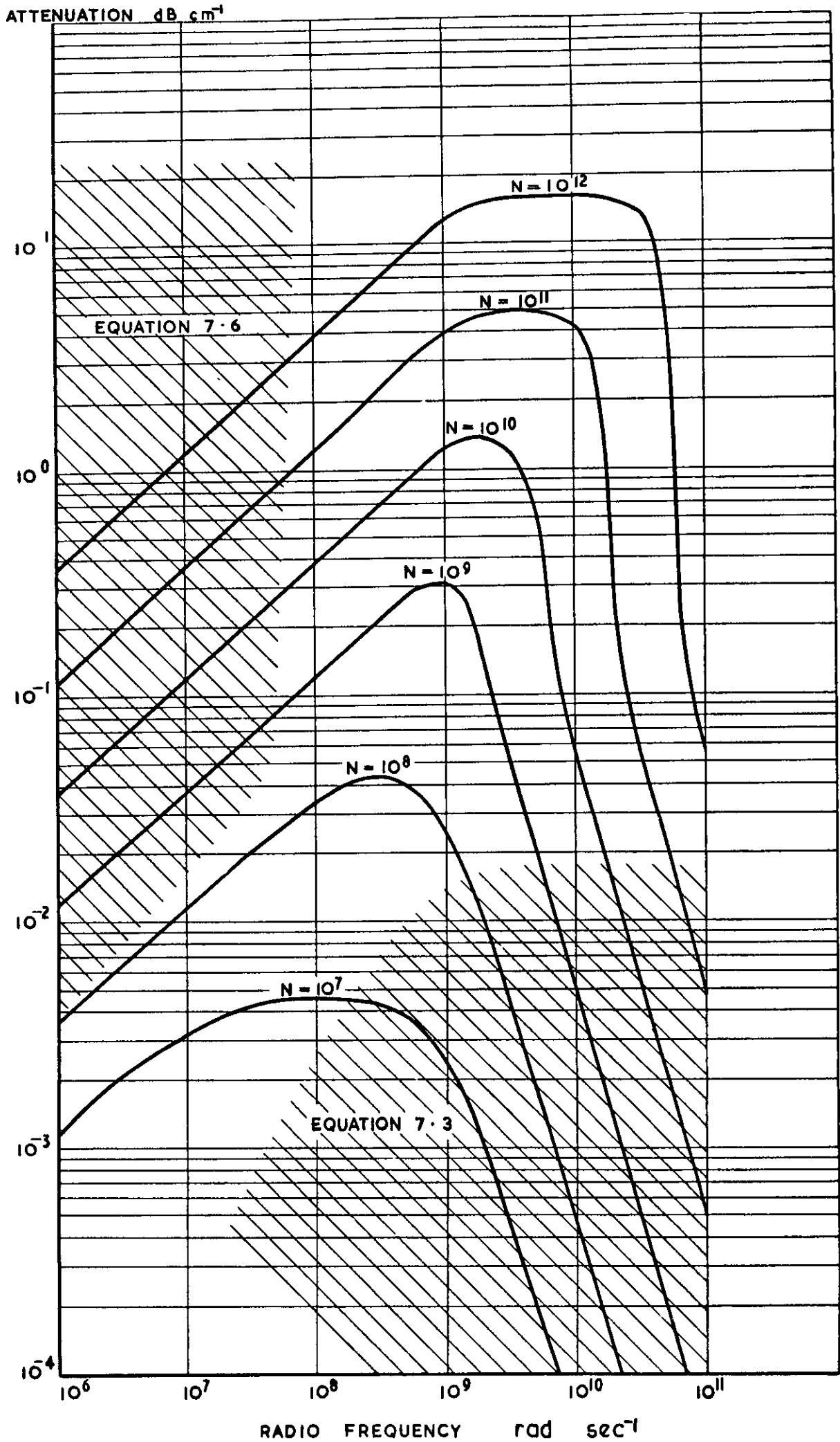


FIG. 5 VARIATION OF ATTENUATION WITH
RADIO FREQUENCY $\omega_1 = 10^9 \text{ sec}^{-1}$

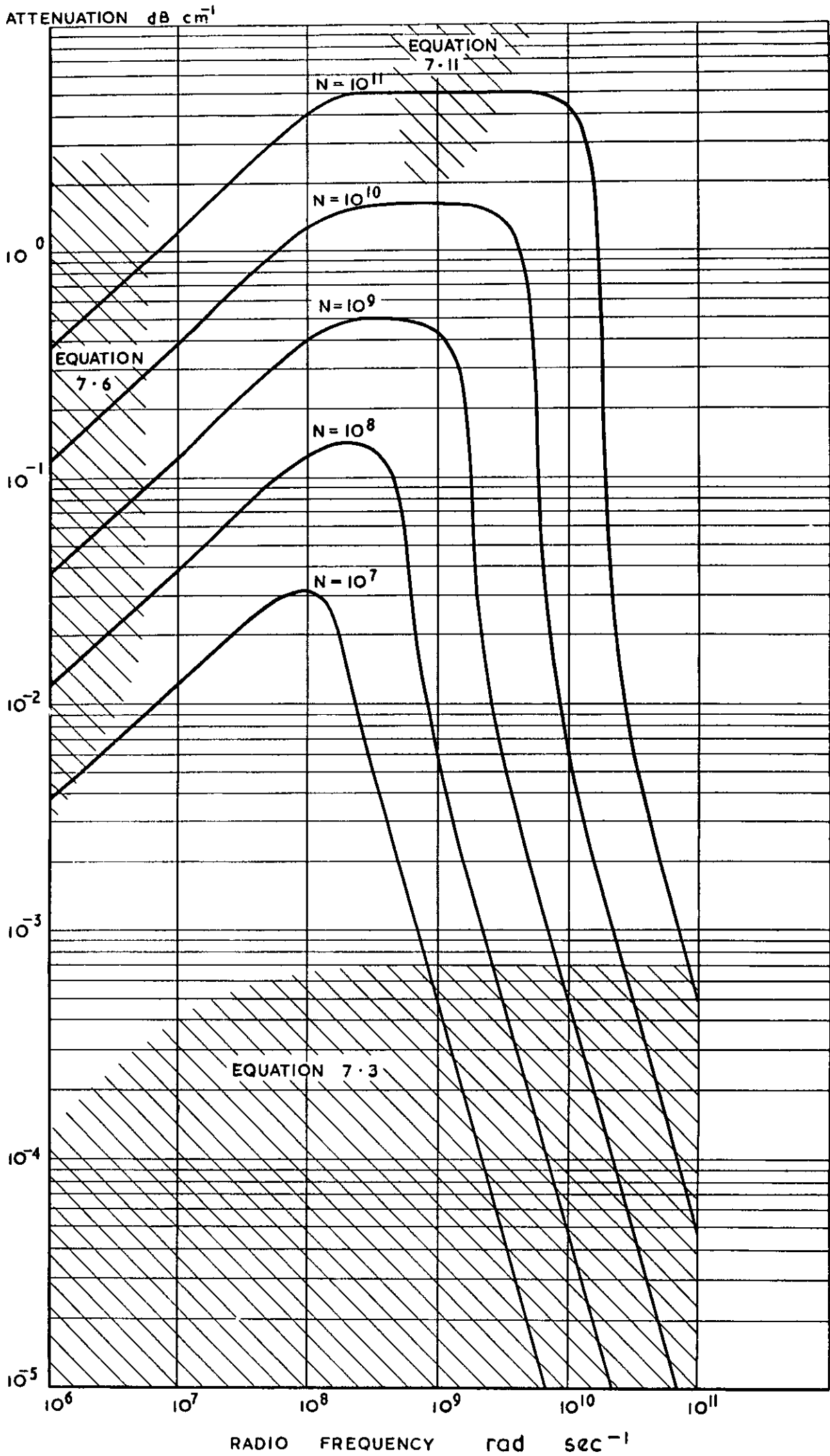


FIG. 6 VARIATIONS OF ATTENUATION WITH RADIO FREQUENCY $\omega_1 = 10^8 \text{ sec}^{-1}$

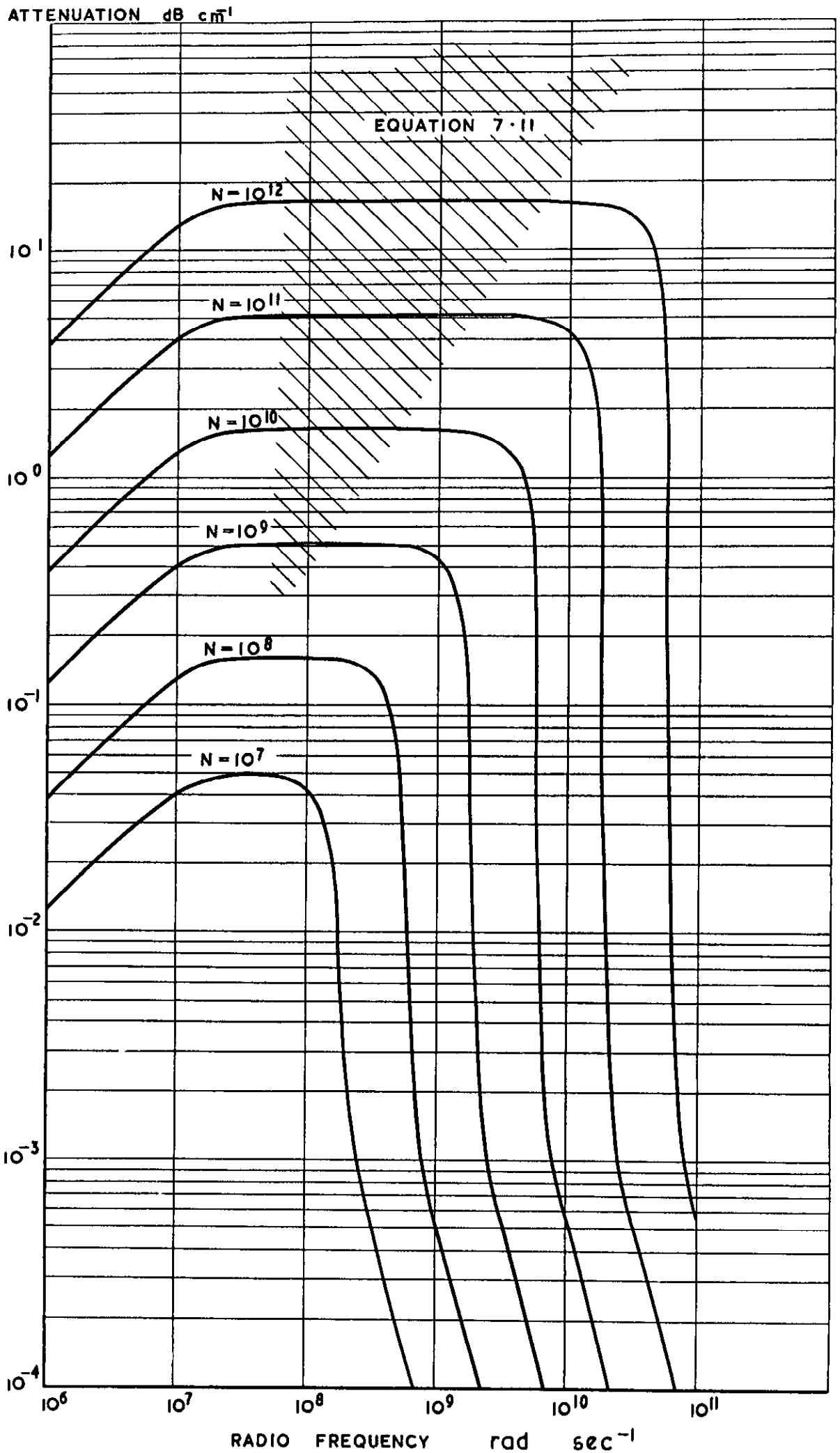


FIG. 7 VARIATIONS OF ATTENUATION WITH RADIO FREQUENCY $\omega_1 = 10^7 \text{sec}^{-1}$

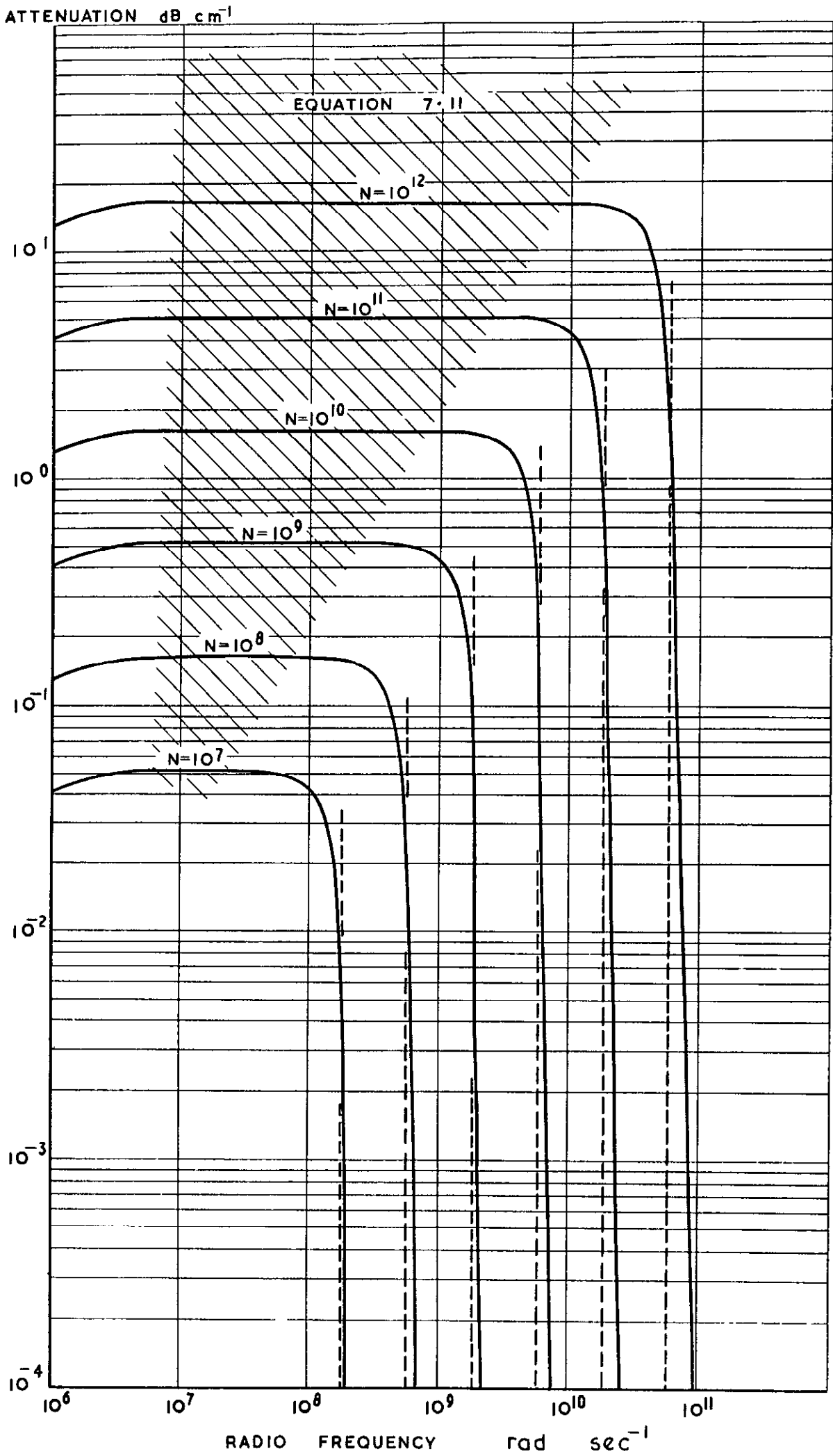


FIG. 8 VARIATION OF ATTENUATION WITH
RADIO FREQUENCY $\omega_1 = 10^6 \text{ sec}^{-1}$

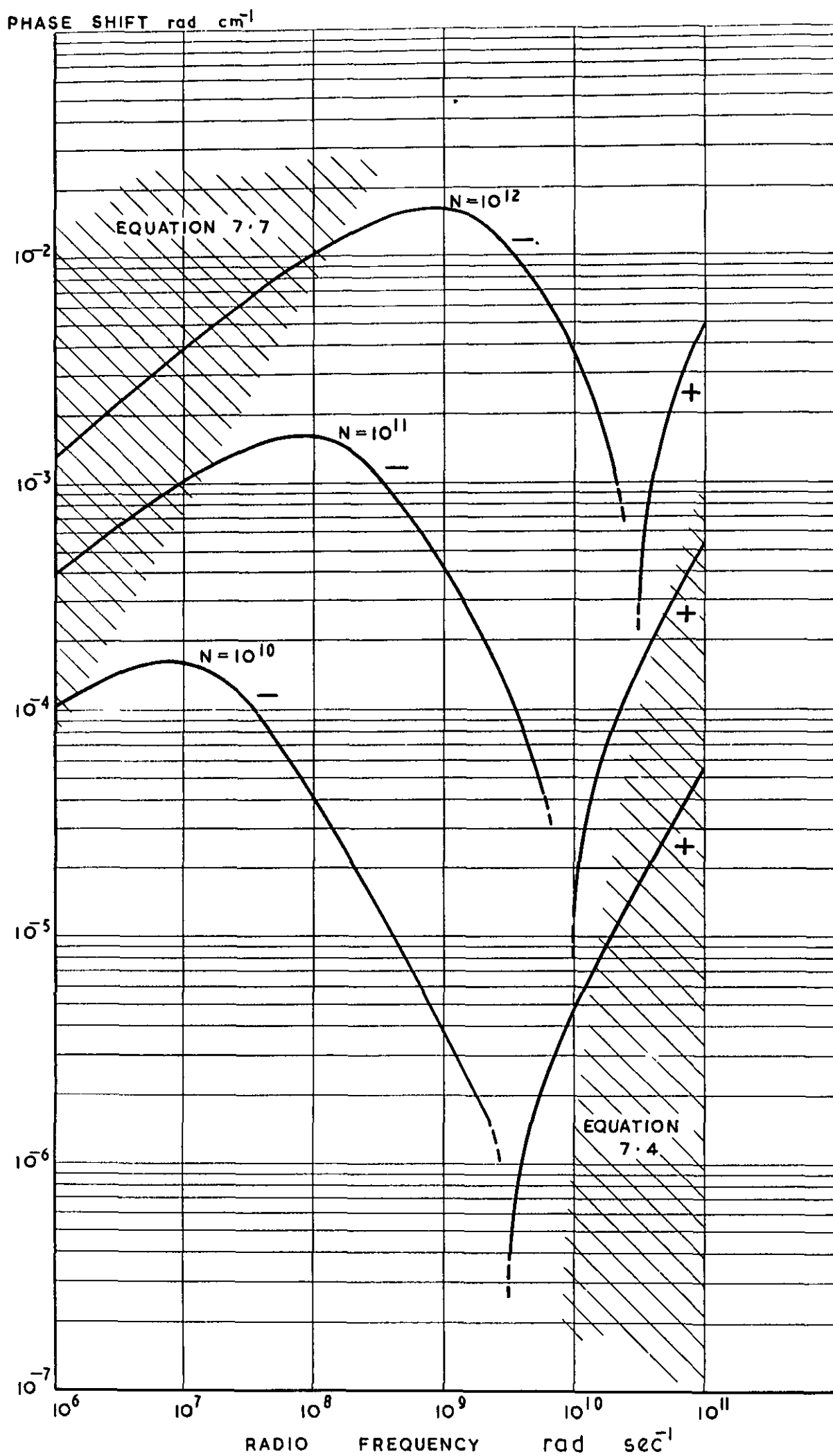


FIG 9 VARIATION OF PHASE SHIFT WITH RADIO FREQUENCY $\omega_1 = 10^{12} \text{sec}^{-1}$

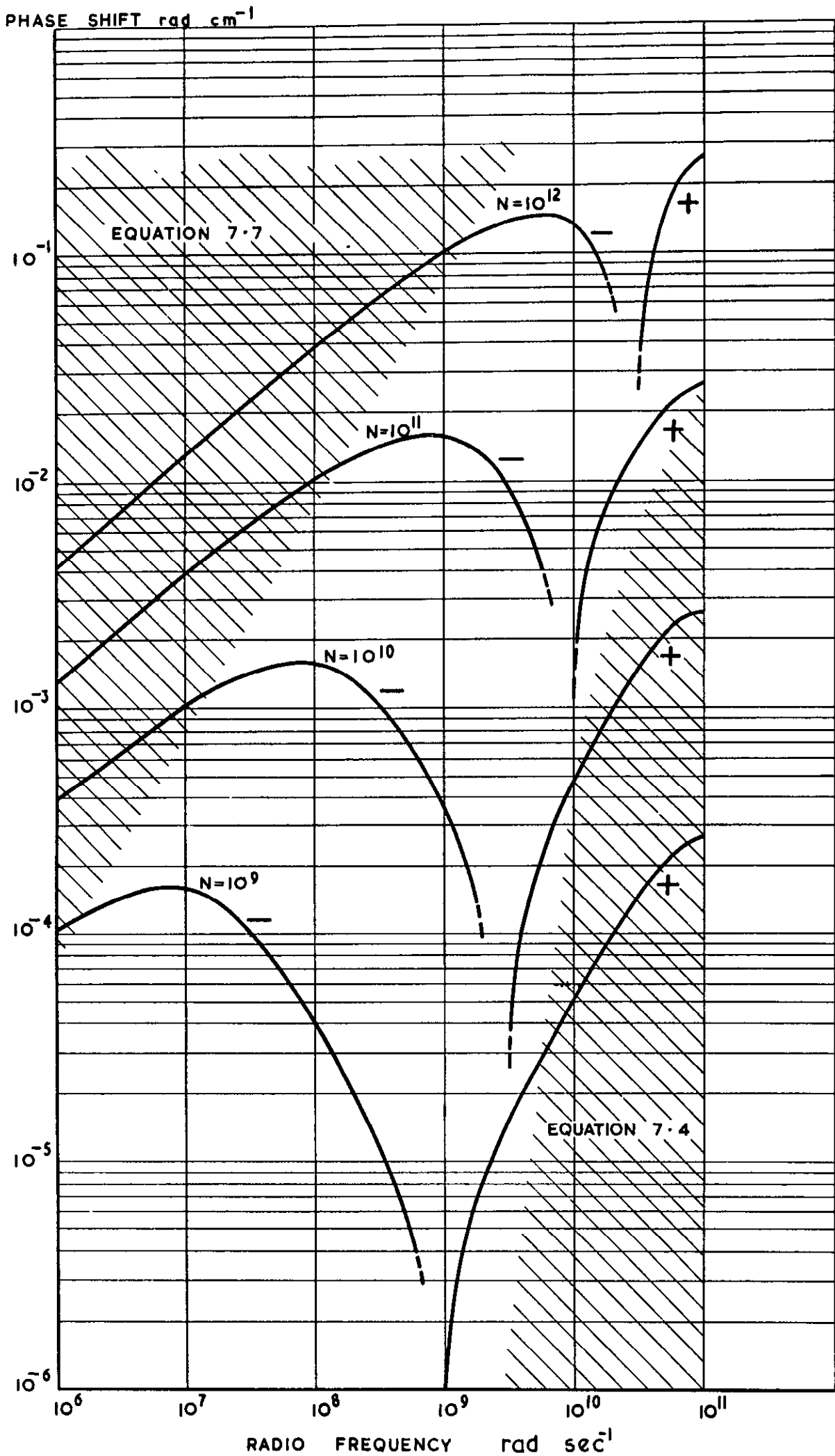


FIG. 10 VARIATION OF PHASE SHIFT WITH
 RADIO FREQUENCY $\omega_1 = 10^{11} \text{sec}^{-1}$

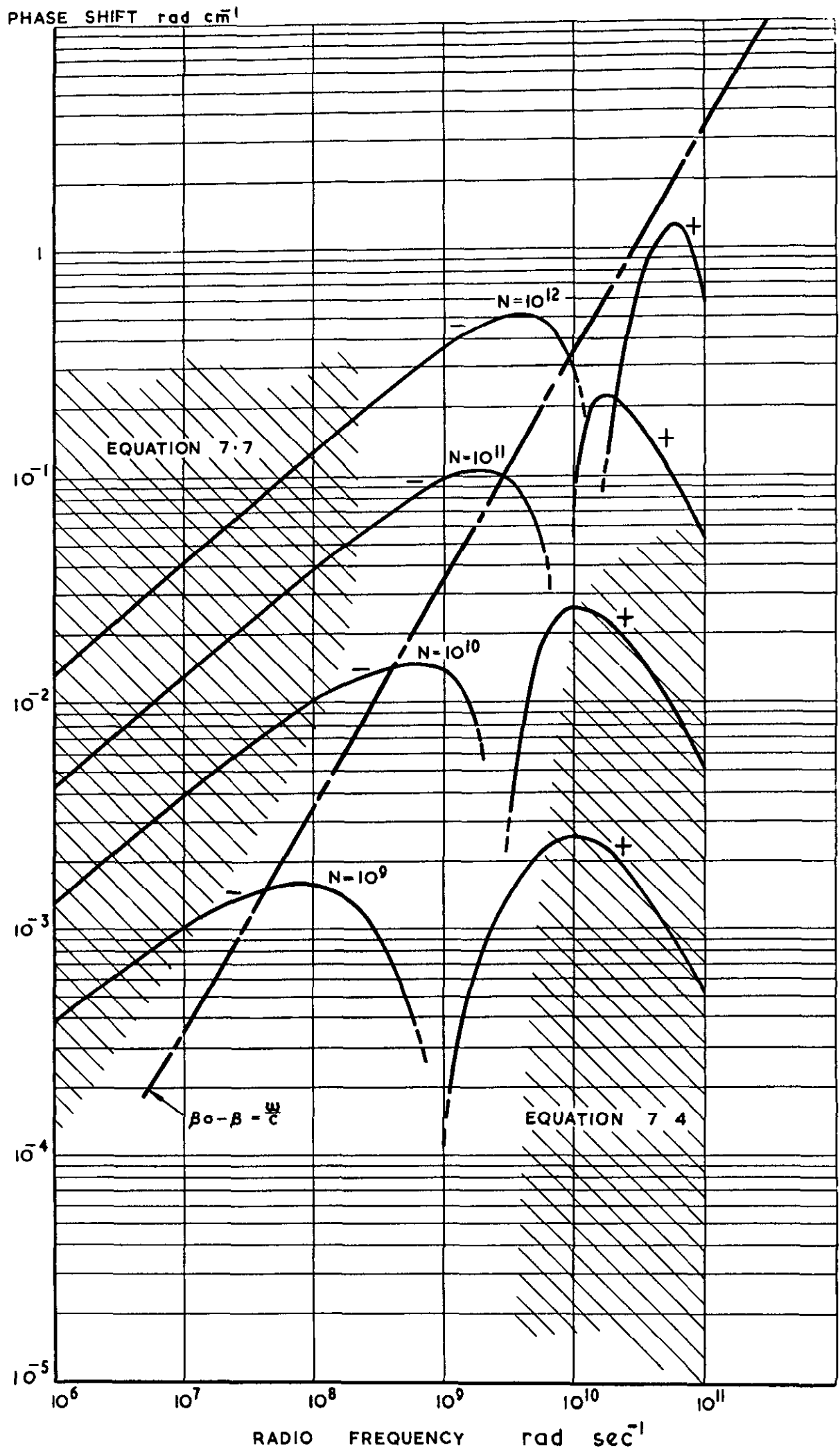


FIG. 11 VARIATION OF PHASE SHIFT WITH RADIO FREQUENCY $\omega_1 = 10^{10} \text{ sec}^{-1}$

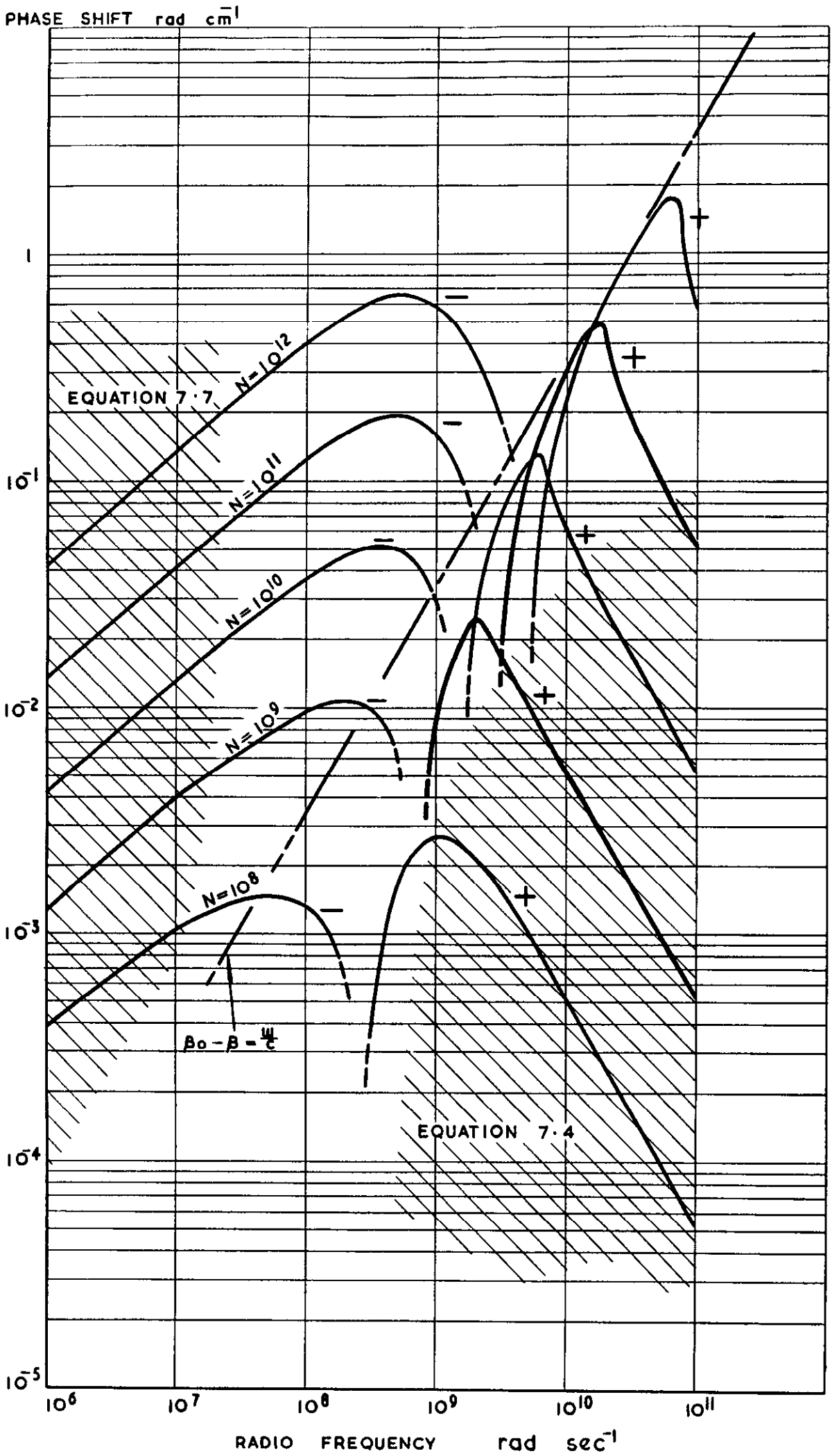


FIG. 12 VARIATION OF PHASE SHIFT WITH RADIO FREQUENCY $\omega_1 = 10^9 \text{ sec}^{-1}$

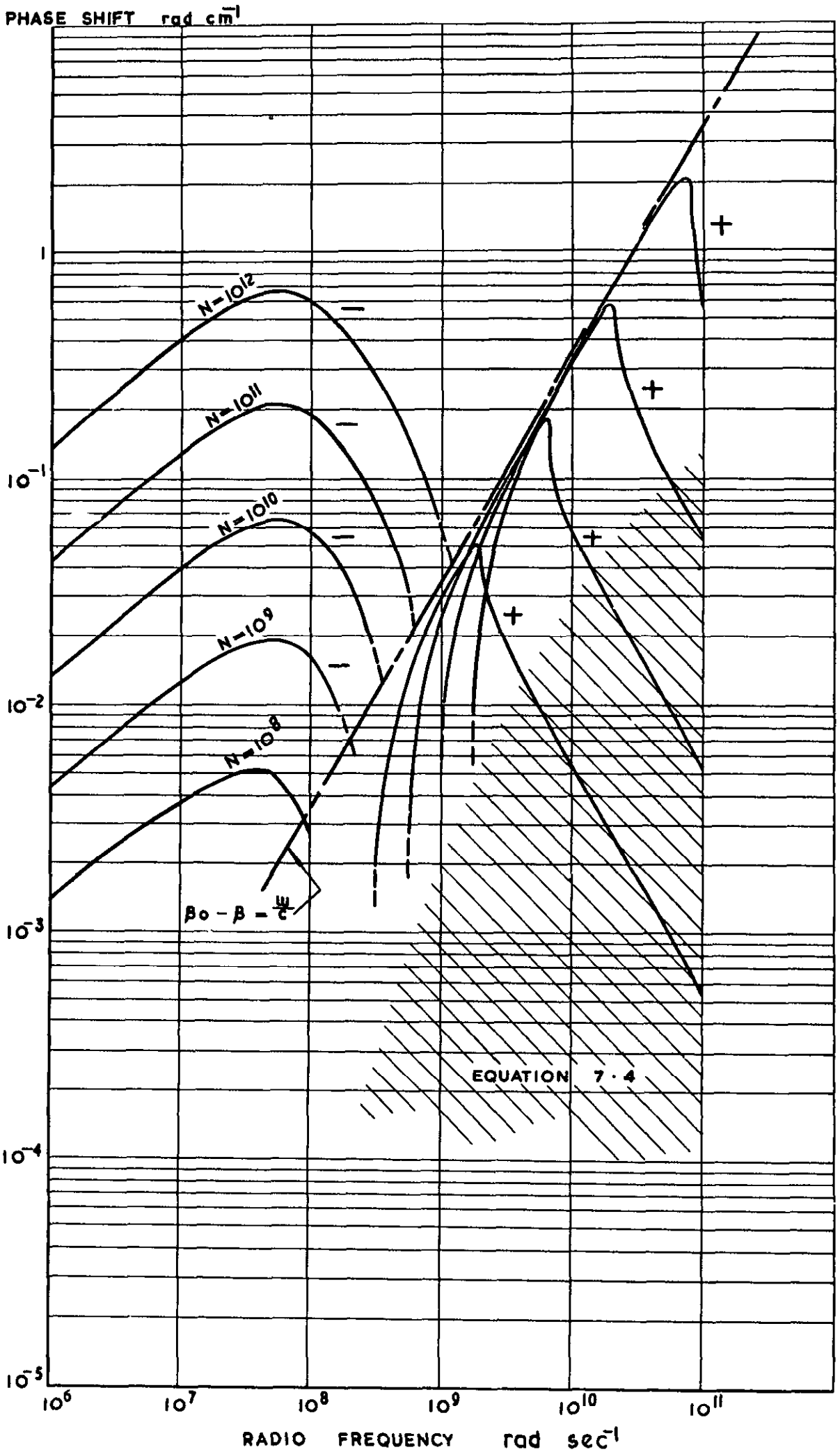


FIG 13 VARIATION OF PHASE SHIFT WITH
RADIO FREQUENCY $\omega_1 = 10^8 \text{sec}^{-1}$

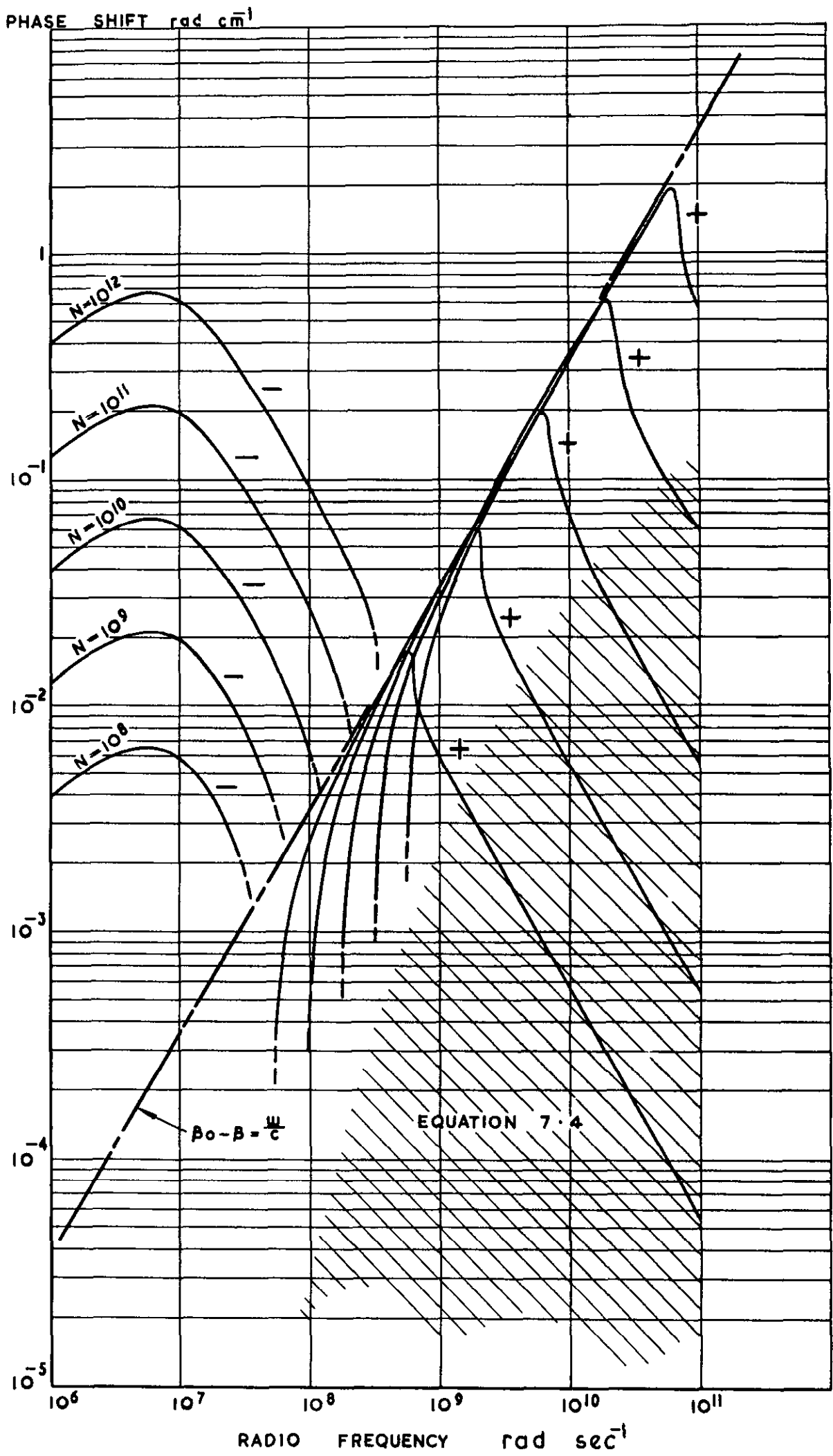


FIG. 14 VARIATION OF PHASE SHIFT WITH
 RADIO FREQUENCY $\omega_1 = 10^7 \text{ sec}^{-1}$

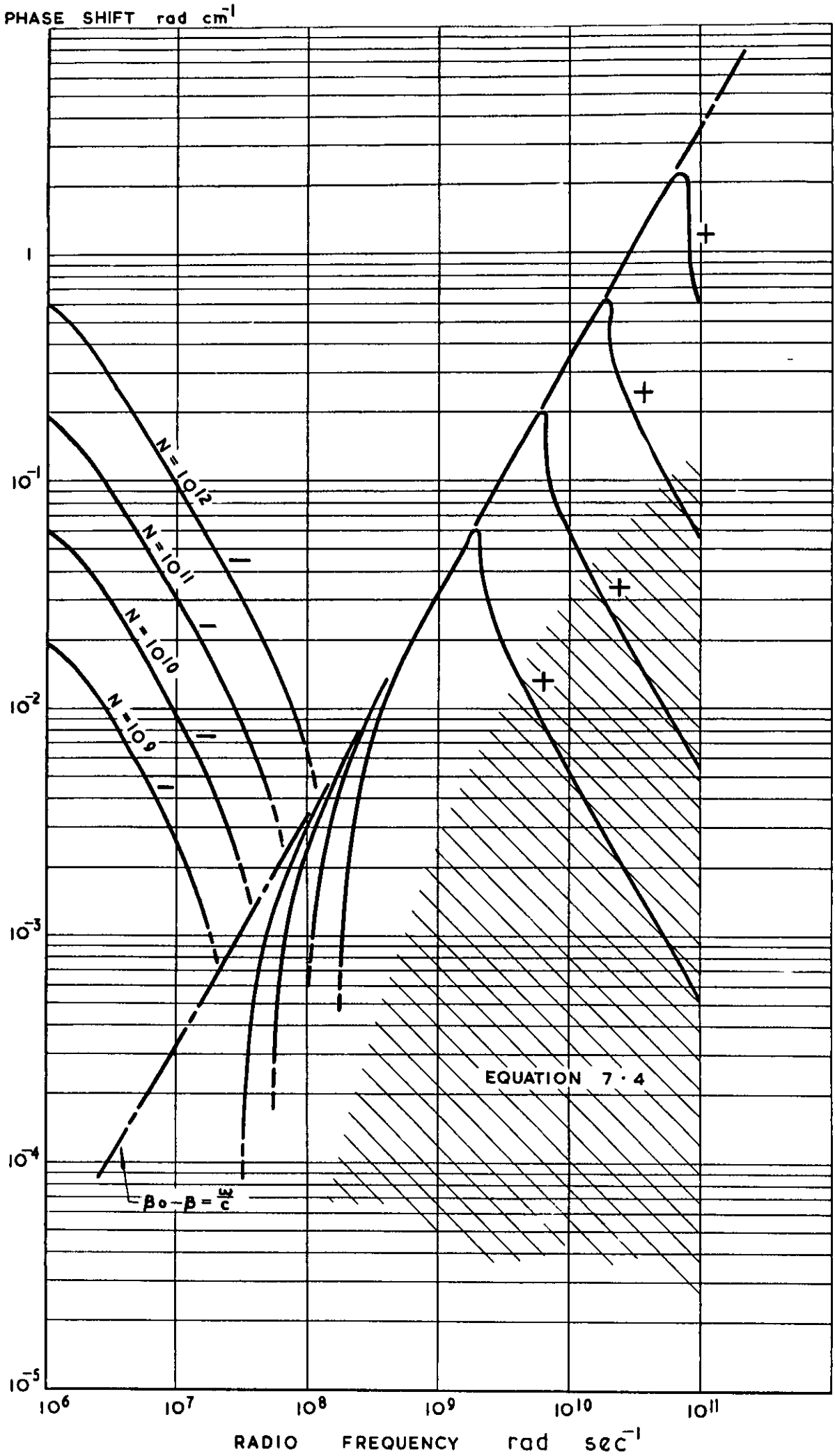


FIG. 15 VARIATION OF PHASE SHIFT WITH
 RADIO FREQUENCY $\omega_1 = 10^6 \text{ sec}^{-1}$

POWER REFLECTION
COEFFICIENT

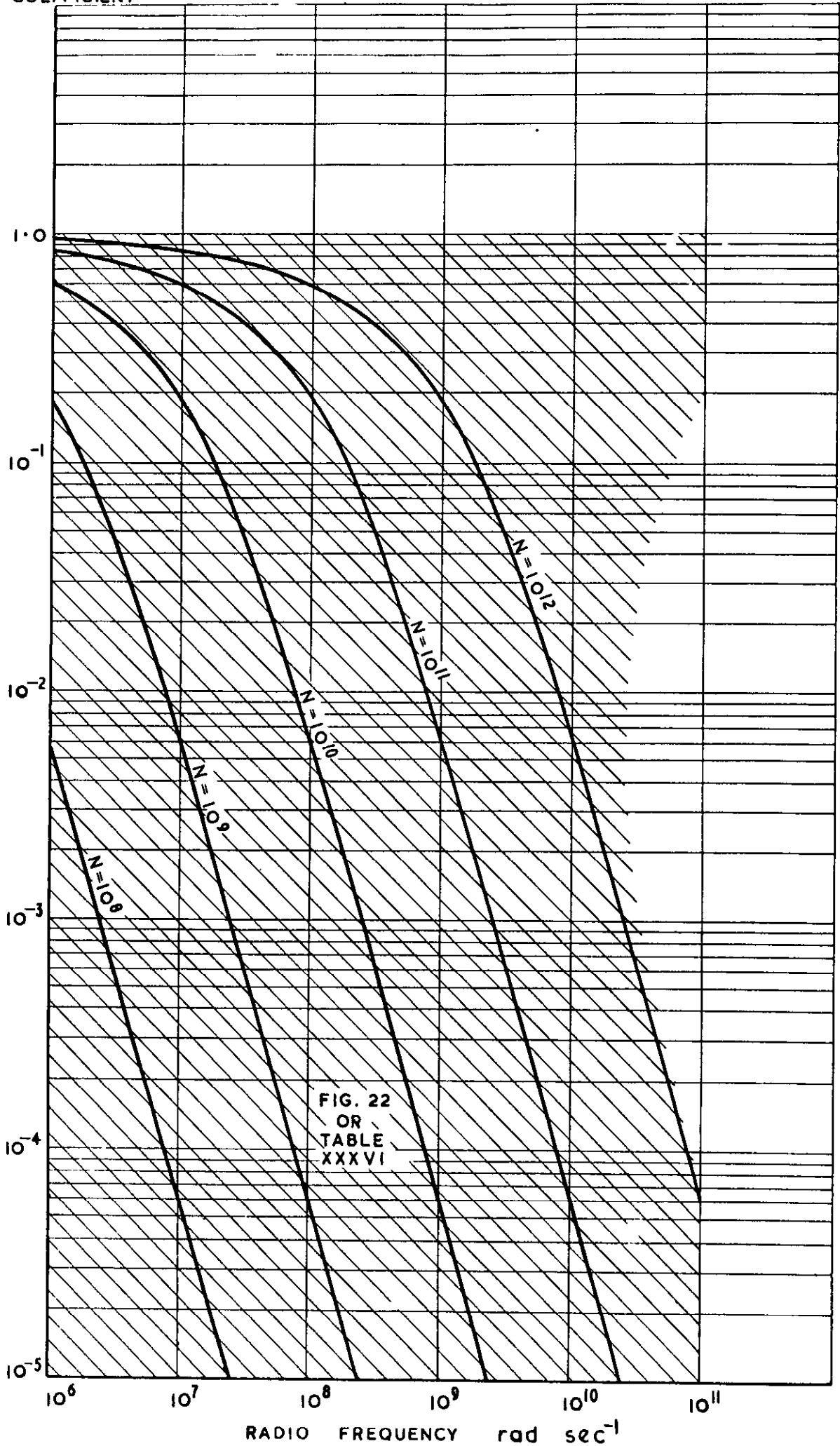


FIG.16 VARIATION OF POWER REFLECTION COEFFICIENT
WITH RADIO FREQUENCY $\omega_1 = 10^{12} \text{sec}^{-1}$

POWER REFLECTION
COEFFICIENT

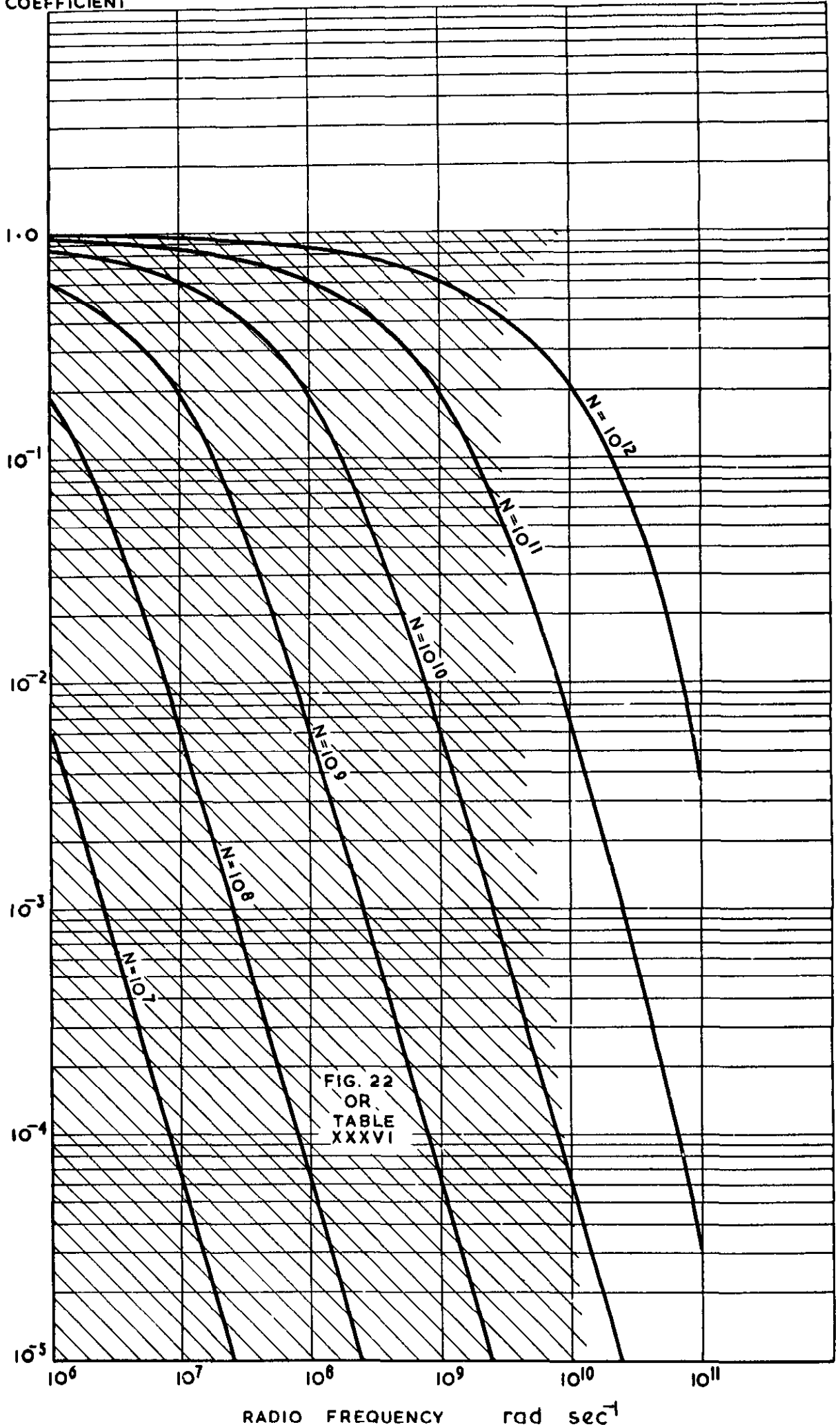


FIG.17 VARIATION OF POWER REFLECTION COEFFICIENT
WITH RADIO FREQUENCY $\omega_1 = 10^{11} \text{sec}^{-1}$

POWER REFLECTION
COEFFICIENT

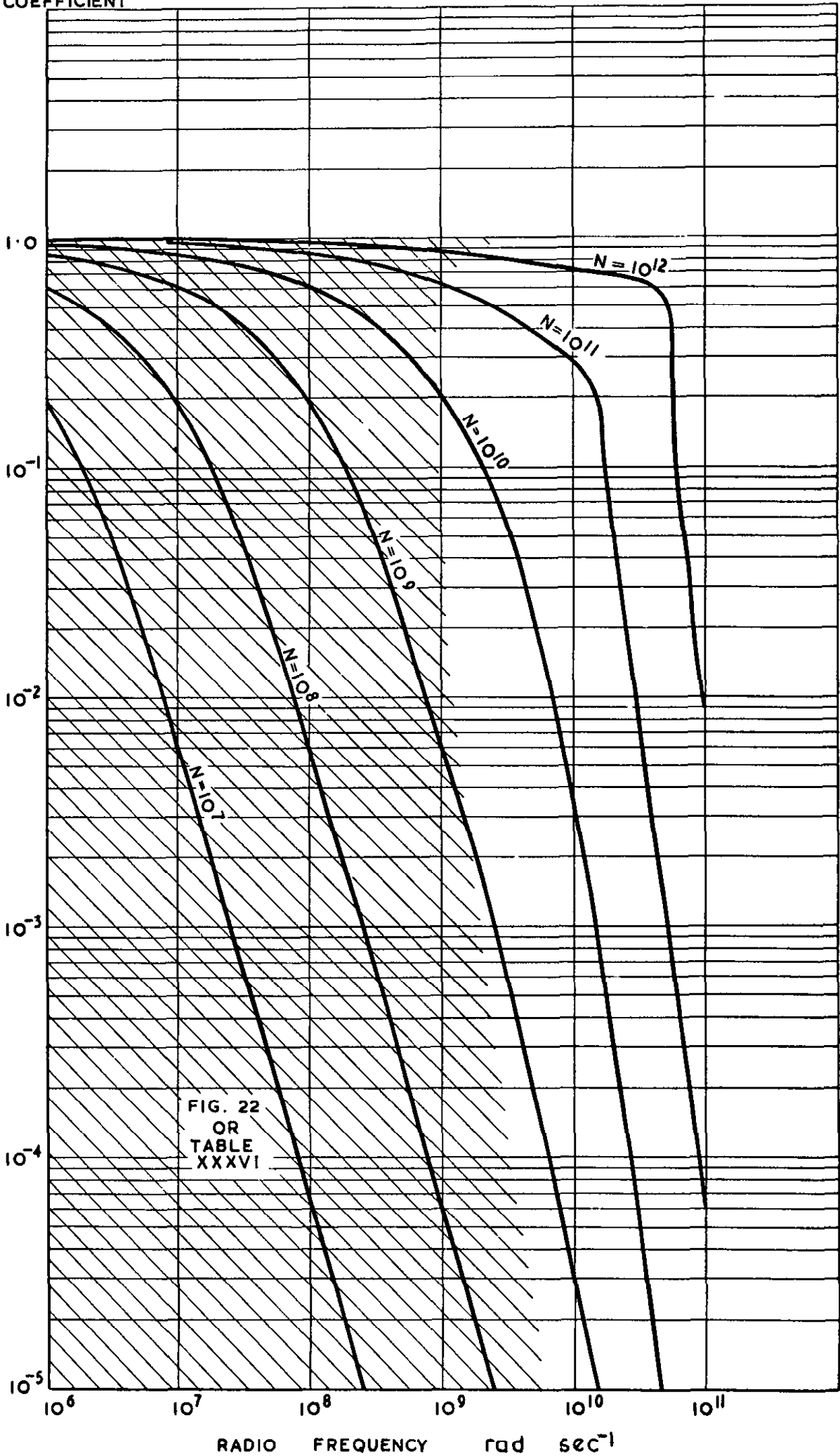
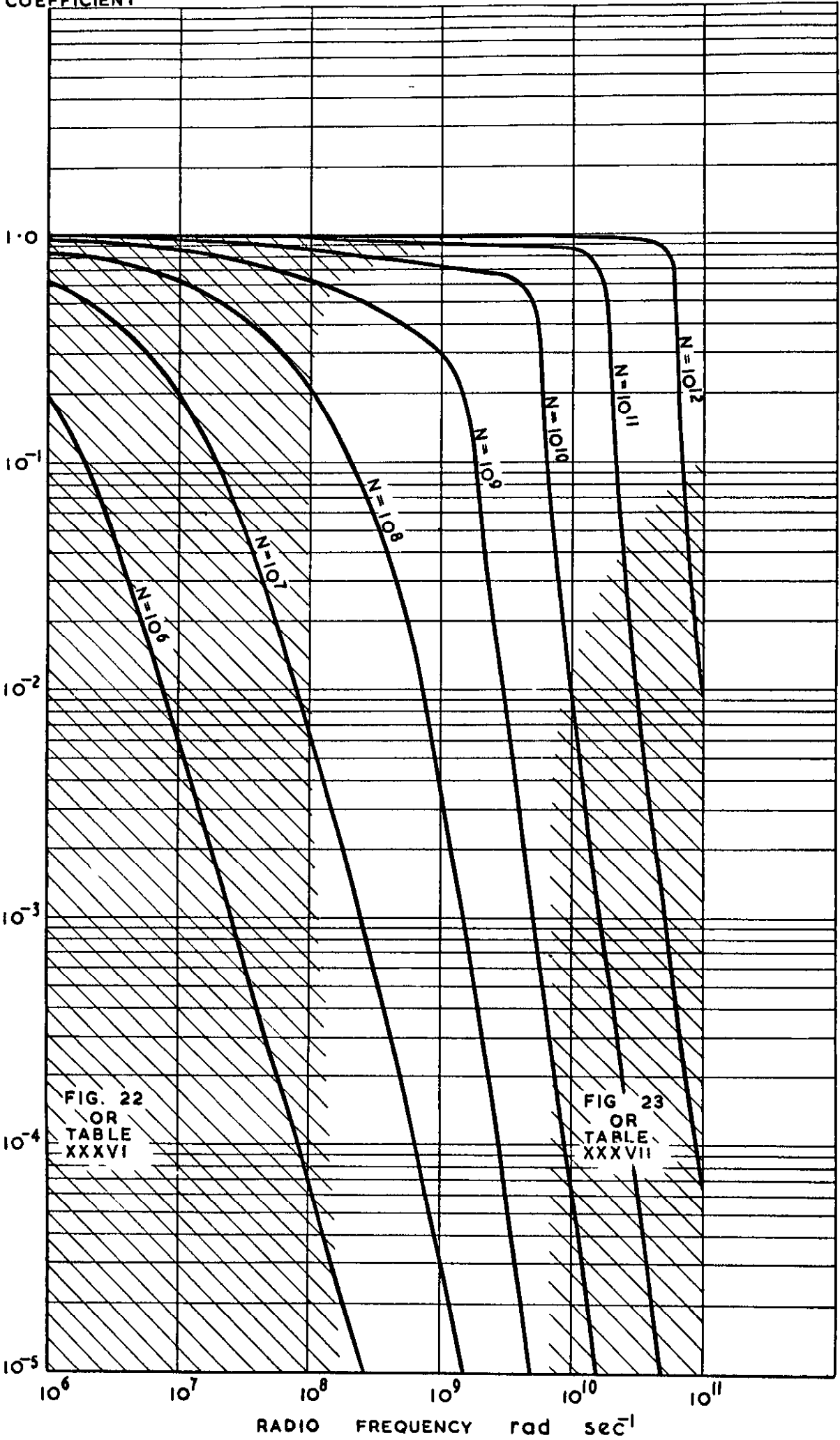


FIG.18 VARIATION OF POWER REFLECTION COEFFICIENT
WITH RADIO FREQUENCY $\omega_1 = 10^{10} \text{ sec}^{-1}$

**POWER REFLECTION
 COEFFICIENT**



**FIG.19 VARIATION OF POWER REFLECTION COEFFICIENT
 WITH RADIO FREQUENCY $\omega_1 = 10^9 \text{sec}^{-1}$**

POWER REFLECTION
COEFFICIENT

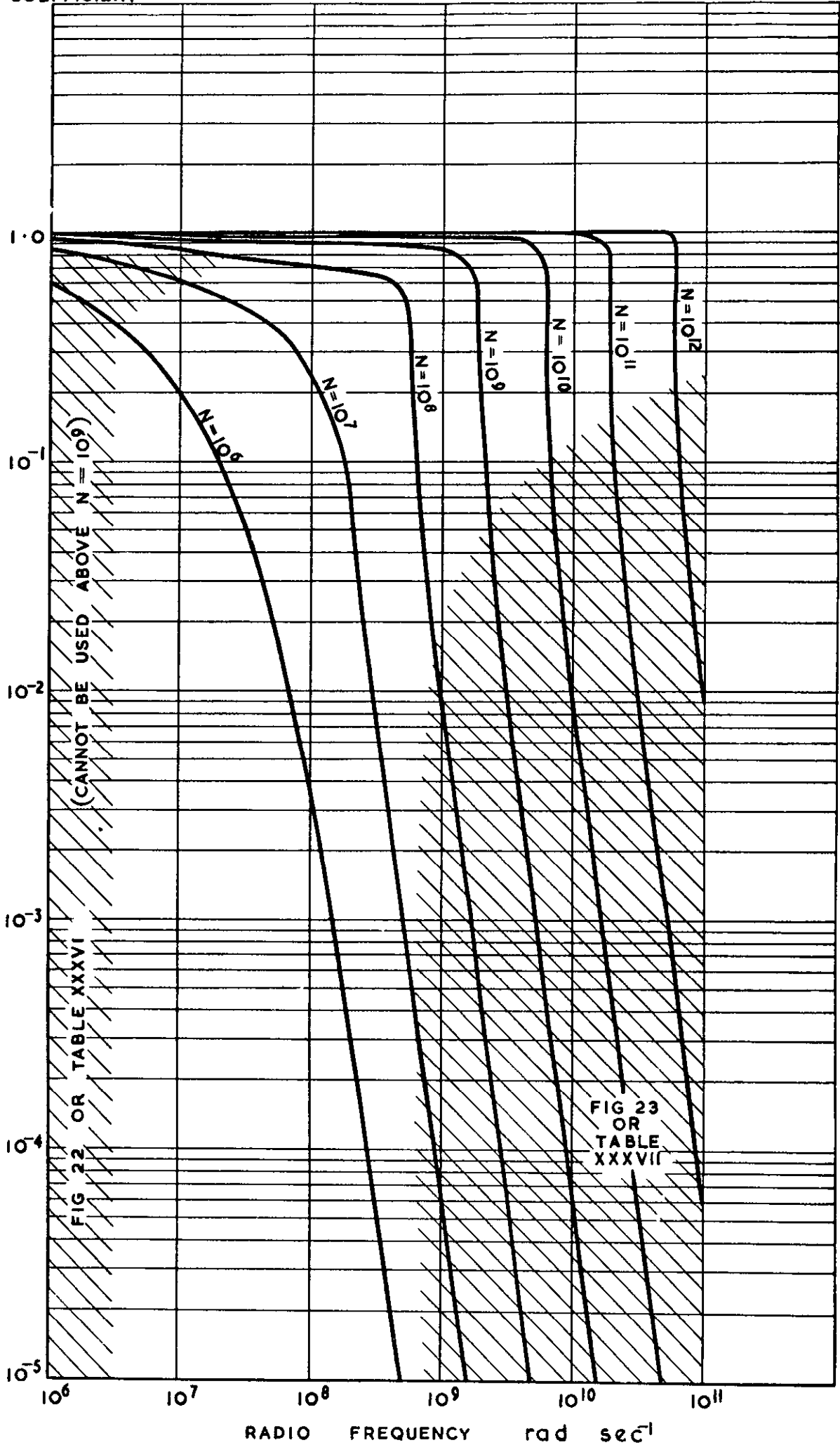


FIG.20 VARIATION OF POWER REFLECTION COEFFICIENT
WITH RADIO FREQUENCY $\omega_1 = 10^8 \text{ sec}^{-1}$

POWER REFLECTION
COEFFICIENT

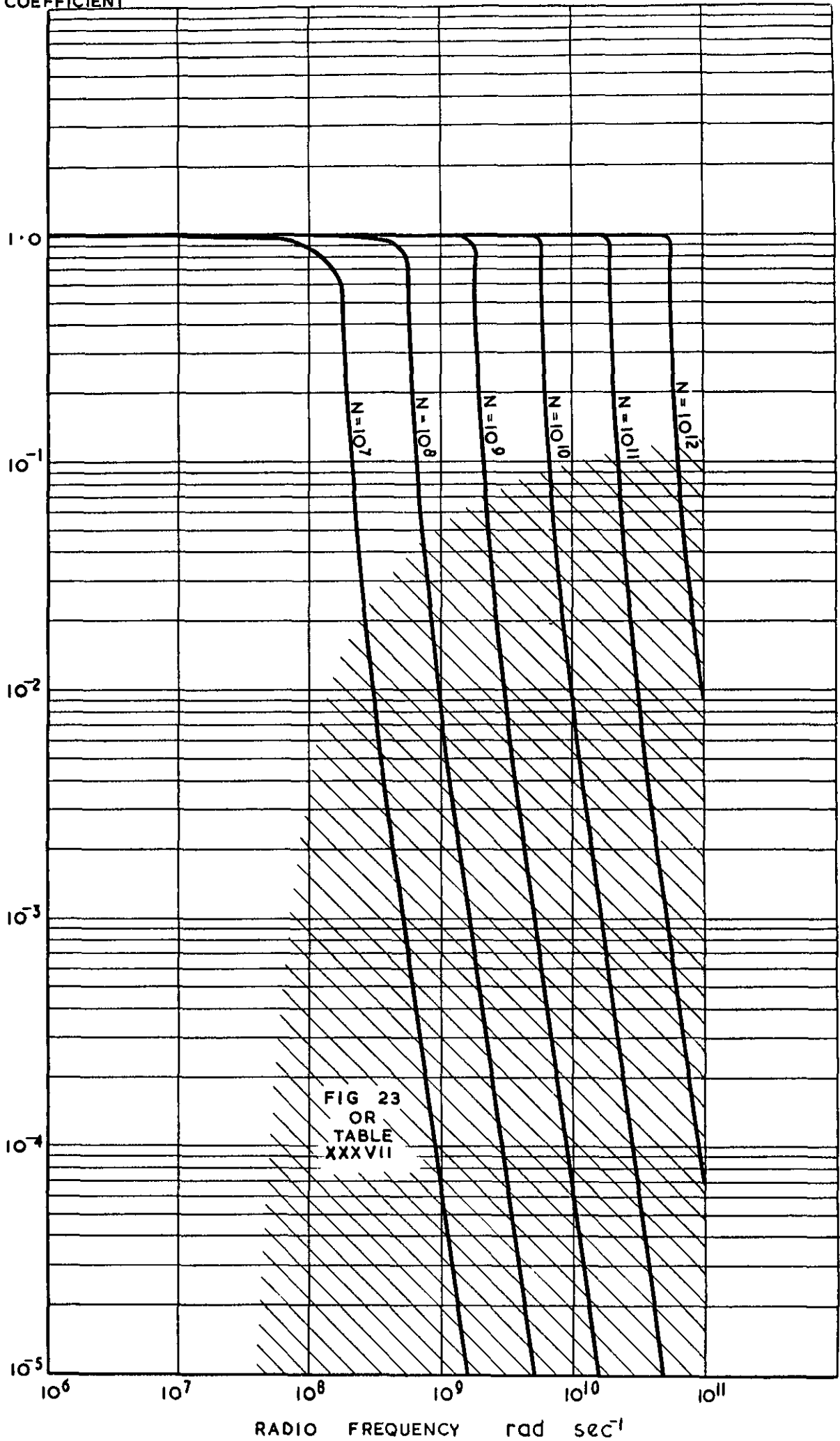


FIG. 21 VARIATION OF POWER REFLECTION COEFFICIENT
WITH RADIO FREQUENCY $\omega_1 = 10^7 \text{sec}^{-1}$

POWER REFLECTION
COEFFICIENT

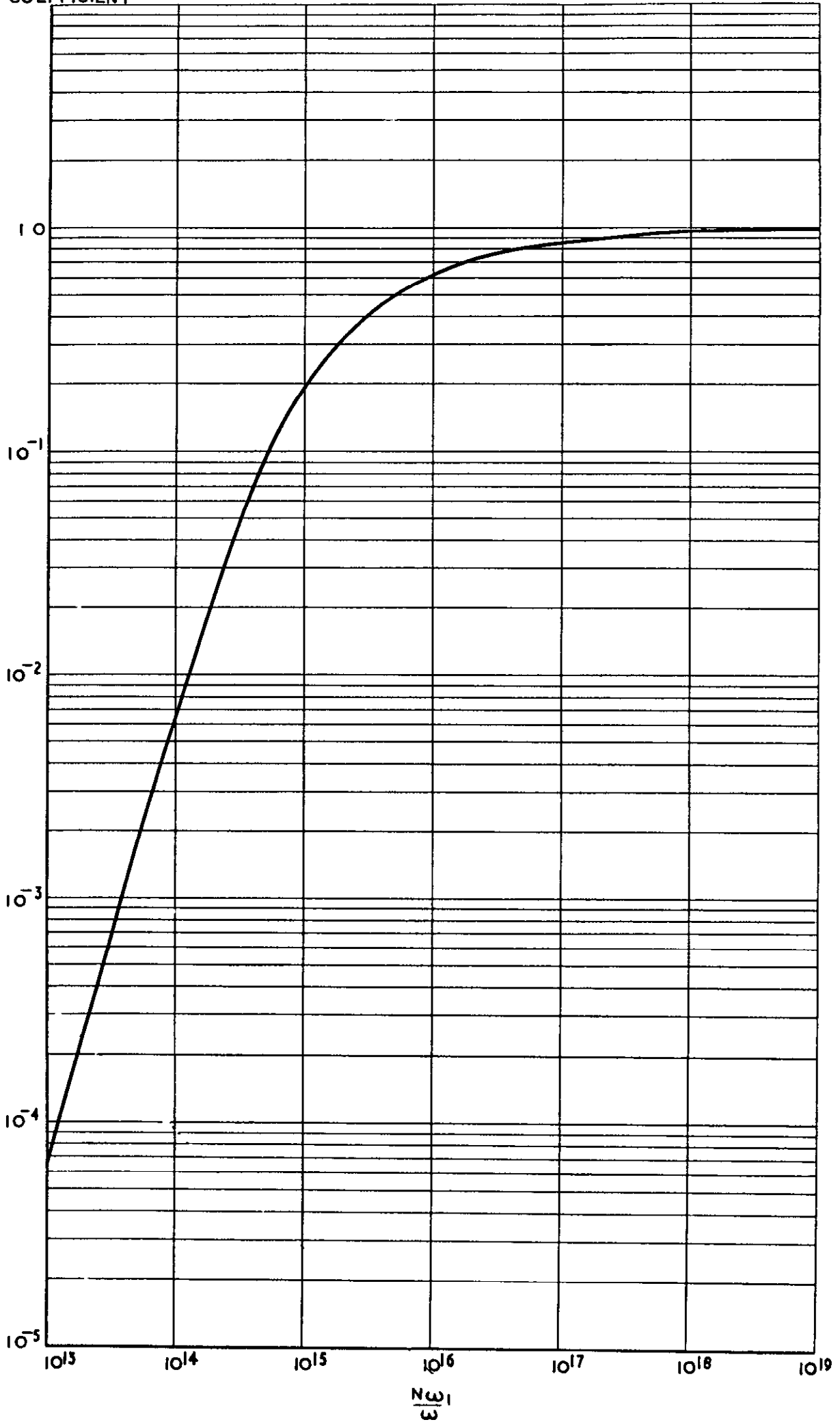


FIG. 22 APPROXIMATION FOR R AT HIGH ELECTRON COLLISION FREQUENCIES

POWER REFLECTION
COEFFICIENT

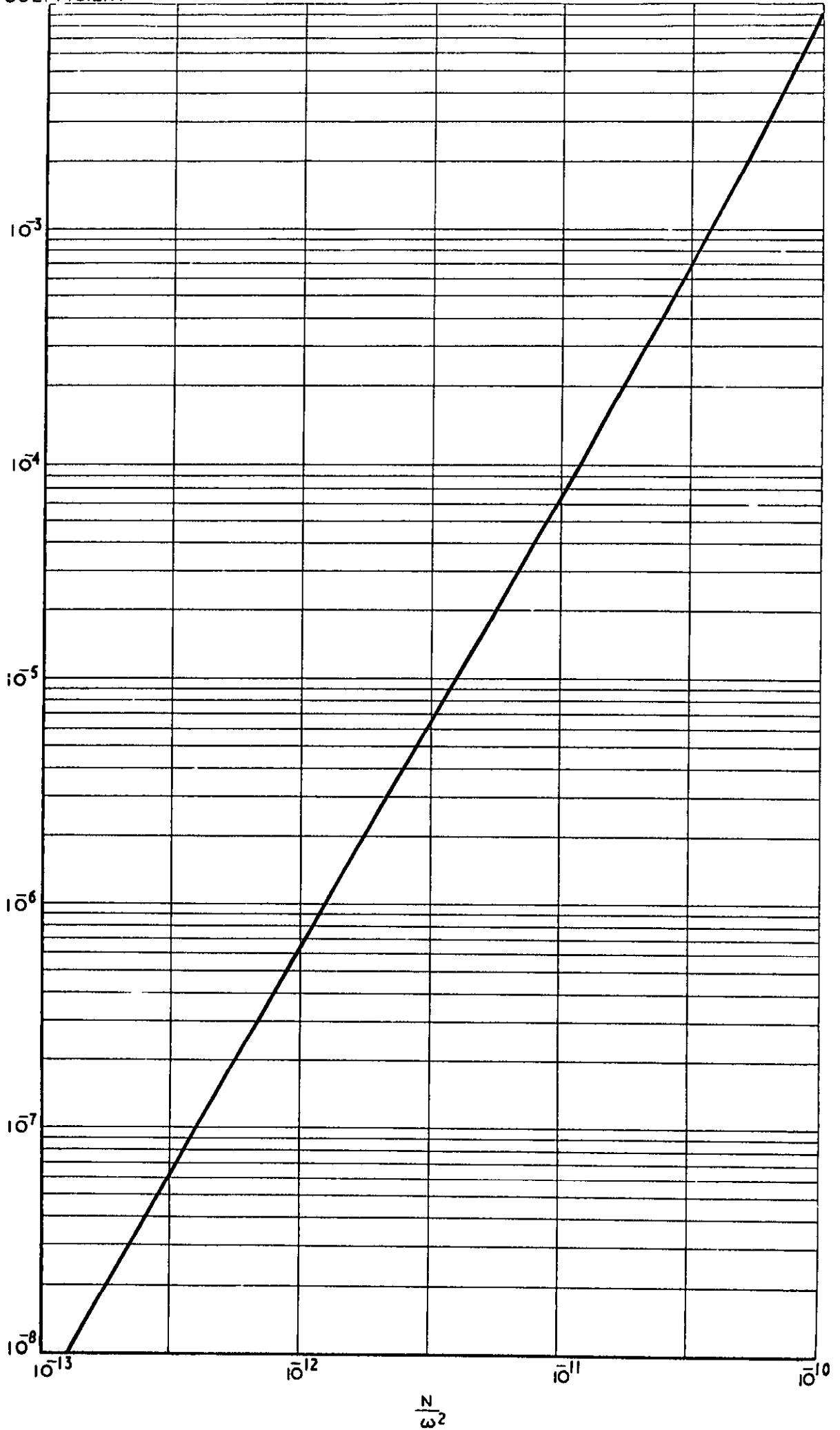


FIG. 23 APPROXIMATION FOR R AT LOW ELECTRON COLLISION FREQUENCIES

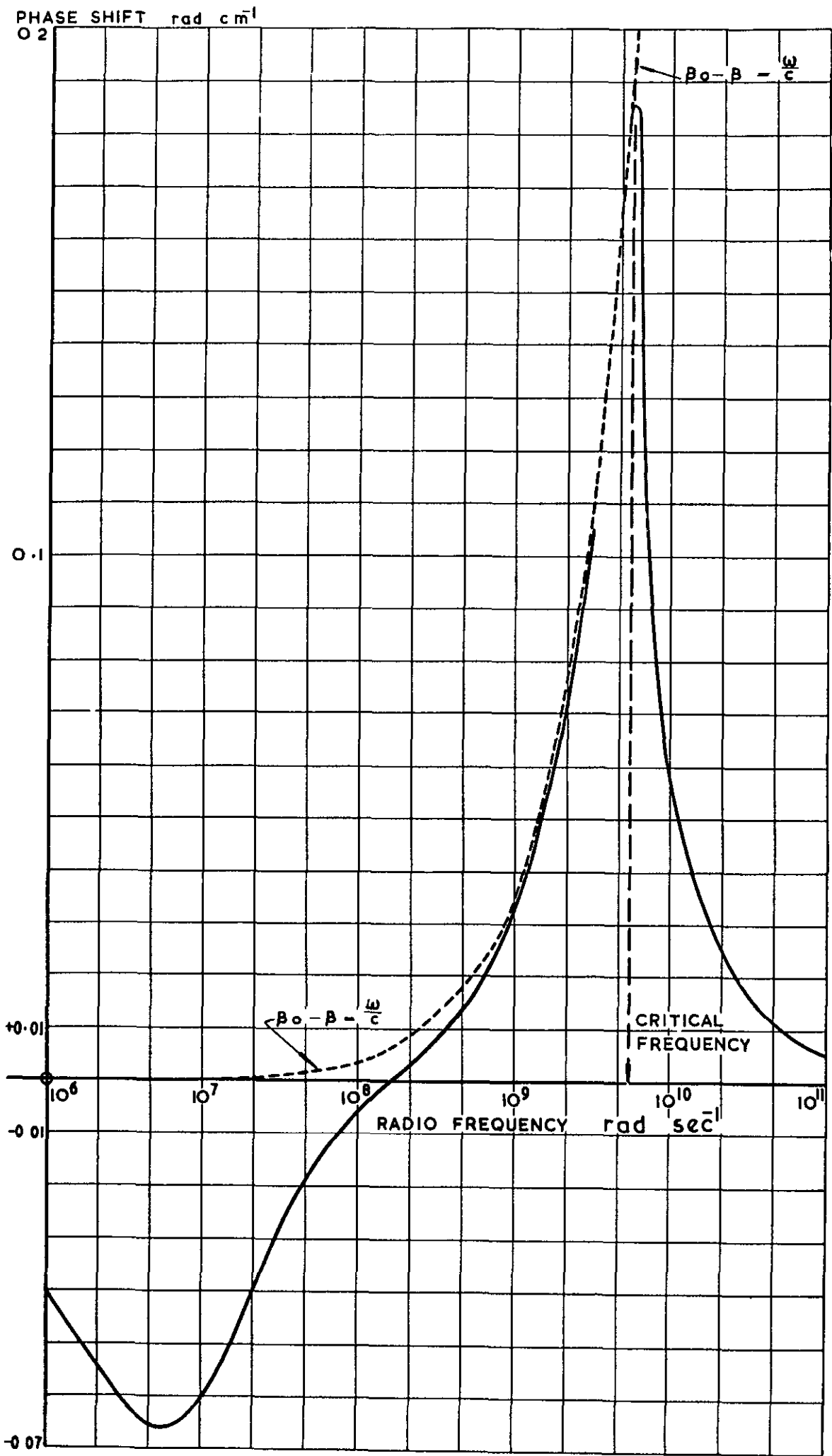


FIG. 24 VARIATION OF PHASE SHIFT WITH
 RADIO FREQUENCY $\omega_1 = 10^7 \text{ sec}^{-1}$. $N = 10^{10} \text{ cm}^{-3}$

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The attenuation per unit path length, phase shift per unit path length and reflection coefficient at the boundary are derived theoretically for a medium containing free electrons, in terms of the electron density, electron collision frequency with heavy molecules and the applied radio frequency.

Numerical results are tabulated over wide ranges of the three variables, and these results are also presented graphically.

Useful approximations to the complete theory are derived and the regions in which these may be applied in practice are indicated

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