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# Further Analysis of TSR.2 Flights through Turbulence

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FURTHER ANALYSIS OF TSR 2 FLIGHTS  
THROUGH TURBULENCE  
BY  
T.B. SAUNDERS

SUMMARY

Human pilot describing functions are derived from records of two flights through turbulence by the TSR 2 based on the assumption of the use of pitch rate cues and normal acceleration cues respectively. No indication can be gained of the validity of these assumptions. Small low frequency errors in pitch rate introduced during trace readings were magnified by subsequent integration and produced large errors in the time histories of vertical gust velocity at very low frequencies. This prevented the estimation of turbulence power spectra.



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## 1.0 INTRODUCTION

The aim of the research was to study records of TSR.2 flights through turbulence with a view to defining the action of the pilot and the effect this had on the longitudinal response to turbulence of the pilot-aircraft system. Insufficient time remained for the secondary aims of investigating pilot techniques in approach and landing and the effect of vibration on pilot's response.

In the event, inadequate instrumentation of certain parameters led to difficulties in estimating the gust velocity and the main results consist of estimates of pilot's response.

The theoretical basis of the work is described in Section 2, the available records in Section 3 and the analysis is described and discussed in Sections 4 and 5.

## 2.0 THEORETICAL CONSIDERATIONS

The original motivation for this research came from the discovery of apparently non-Gaussian probability distributions in aircraft response data obtained in flights through turbulence (Reference 1). Although non-Gaussian amplitude distributions have been measured before (e.g. Reference 2) turbulence is often assumed to have a Gaussian distribution and arguments can be advanced in justification of this assumption based on the physical mechanism of turbulence. The departures from the Gaussian distribution usually consist of greater probabilities at large amplitudes, the bulk of the distribution at lower amplitudes being reasonably well fitted by the normal curve. Both the above examples are of this type.

One possible explanation advanced in Reference 1 is the effect of pilot control actions non-linearly related to the aircraft response. The aim of the work reported here was to study the probability distributions of aircraft response variables and the derived gust velocity, and in particular to study the action of the pilot. Because the response of the pilot is of considerable interest in its own right, it will be considered separately below after sections dealing with the estimation of probability distributions and the calculation of turbulence velocity from aircraft response.

### 2.1 Probability Distributions

The amplitude probability density function of a random process  $\{x(t)\}$  can be defined to be:-

$$p(x) = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \quad (\text{Prob. } [x < x(t) < (x + \Delta x)])$$

This is not suitable as a basis for estimating the probability density so we consider the alternative definition, valid for a stationary, ergodic process:

$$p(x) = \lim_{\Delta x \rightarrow 0} \lim_{T \rightarrow \infty} \frac{1}{T} \left( \frac{T_x}{\Delta x} \right) \quad (1)$$

where  $T_x$  is the total amount of time within  $T$  that a single element  $x(t)$  lies in the interval  $x < x(t) < x + \Delta x$

(Note the distinction here between  $x(t)$ , a function of  $t$  and the independent amplitude variable  $x$ ).

Thus an estimate (though not an unbiased one (Reference 3)) of probability density function  $p(x)$  can be obtained from equation (1) by measuring  $T_x$  over a finite time  $T$ . The accuracy of the estimate will depend on the finite length  $T$  of the sample and on the non-zero value  $\Delta x$  of amplitude step chosen. The way in which the accuracy depends on these quantities is in turn determined by the details of the power spectral density of the process.

An equivalent signal bandwidth could be used to specify a minimum value for  $T$  as in Reference 3. However, this leads to difficulties e.g. in the case of a sharp peak at low frequency in the power spectrum. A better method would be to specify several cycles of a "lowest significant frequency"  $f_L$

$$f_L T \gg 1$$

where  $f_L$  would be defined to be:

$$f_L \triangleq \bar{f} = \frac{[\int_0^{\bar{f}} G_x(f) df]^2}{\int_0^{\bar{f}} G_x^2(f) df} \quad (2)$$

$$\text{where } \bar{f} = \frac{1}{\sigma} \int_0^{\infty} f G_x(f) df$$

and  $G_x(f)$  is the power spectral density.

One possible effect of violating this condition can be seen by considering the probability density of a random process  $\{X \sin(a t + \phi_i)\}$  consisting of sine waves with the same amplitude and frequency ( $X$  and  $a$  respectively) and with phase angles  $\phi_i$  chosen in a random manner. This process is stationary and ergodic and with probability density given by:

$$p(x) = \begin{cases} (\pi \sqrt{X^2 - x^2})^{-1} & |x| < X \\ 0 & |x| > X \end{cases} \quad (3)$$



and is illustrated in figure 1A. The Gaussian distribution curve:

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp. \left( \frac{-x^2}{2\sigma^2} \right) \quad (4)$$

is given in Figure 1B for comparison.

Figure 2 shows a short sample of narrow band Gaussian noise. It is part of a time series obtained by passing a long sample of Gaussian white noise through a narrow bandwidth filter. The original series gave an estimate of probability density which was Gaussian at the 6% confidence level. It is clear that the short sample of filtered noise shown in Figure 2 would give a probability density curve much nearer to the curve of 1(A) than to a Gaussian curve. Thus, in this case the effect of analysing too short a sample would be to thicken the tails of the distribution i.e. to predict too high a probability density at large amplitudes.

A similar effect could occur if an attempt were made to analyse a non-stationary process with a varying mean value. The effect could be present to a small extent in measurements of turbulence and could be responsible for non-Gaussian probability distributions being found.

## 2.2. Turbulence Velocity

The fluctuation of the vertical component of wind speed provides the most important input to a pilot - aircraft system in flight through turbulence and must be evaluated with respect to time for a complete analysis of such a system. The turbulence velocity can be obtained from records taken on the aircraft itself given adequate instrumentation:

$$w_G = V (\alpha - \theta) + h \quad (5)$$

Alternatively if angle of pitch and rate of climb are not recorded the derivatives of these two quantities can be used:

$$w_G = V (\alpha - \theta_0 - V \int_0^t q \, dt) + h_0 + 32.2 \int_0^t a_z \, dt \quad (6)$$

Where  $q$  is rate of pitch (rad./sec.)

$1 + a_z$  is normal acceleration (g units)

The initial pitch angle and climb rate  $\theta_0$  and  $h_0$  are constant and can be ignored for most purposes.

### 2.3 Human Pilot Response

The treatment of the human pilot as a servo element in closed loop control of an aircraft has been described in numerous publications of which Reference 4 is given as an example. Although the dynamic response of the human operator has been measured in a wide variety of situations, nearly all these experiments have been carried out in simulators on the ground. Reference 5 contains one example of an experiment that has been made in an aircraft in flight.

One reason for the shortage of measurements in flight is undoubtedly the difficulty encountered in selecting which of several available inputs or cues, the pilot is using. A block diagram is shown in Figure 3A for the longitudinal case considered here. Aircraft attitude and the normal acceleration experienced by the pilot provide two possible cues. Even for this simple case the elaboration of the block diagram in Figures 3B and C shows that the analysis required for the complete system would be very complicated. In these circumstances drastic simplification can be justified as a means of gaining some initial insight to the problem.

One way of making some progress would be to consider each cue in turn as a single loop problem. The analysis can then follow the lines of that in Reference 4, which is reproduced in Appendix 1 for convenience. It must be appreciated, however, that such a policy can give a valid answer for a pilot describing function only if the cue being considered is in fact being used on its own by the pilot. Since the aircraft response variables which provide possible cues are themselves related by linear differential equations, the analysis can give no assessment of their relative importance.

## 3.0 SOURCES AND PROCESSING OF DATA

### 3.1 Sources

The 19th and 20th flights of the TSR.2 each contained a level run at low altitude, the Mach numbers being 0.7 and 0.9 respectively. In the present study, analysis was mainly confined to the handling information recorded at a paper speed of 1 cm/sec. on A.13 trace recorders. Details of the relevant quantities recorded and the recording sensitivities are given in Table 1.

Most of the data recorded on magnetic tape was of relevance only to the engines and systems aspects of the aircraft and was time multiplexed. However, a continuous record of vertical acceleration at the pilot's cockpit was available and was used for rapid surveys of turbulence levels during the flights.

### 3.2 Processing

The analysis of data can be divided into two stages. The first stage consisted of a survey of the whole of the two flights with the aim of defining suitable samples for the more detailed analysis carried out in the second stage.

The times covered by the two low level runs were determined by reference to the pilots identification markers. Height and airspeed throughout the runs were read from the traces by hand and plotted at 10 second intervals. Some idea of the variation of the level of turbulence through the two runs was obtained by analogue power spectral analysis of 50 second samples of the magnetic tape record of vertical acceleration at the pilot's cockpit. The low frequency limit for the analysis is fixed at 10 cycles/second by the Muirhead filters used so a speed increase by a factor of 32 was included to bring this down to a real frequency of 0.315 cycles. A further survey was made using the Muirhead filters on "flat" setting. This gives the mean power extending to frequencies lower than 10 c/s but not down to D.C.

The surveys of flight condition and mean square normal acceleration at the pilot's cockpit were used to choose three samples of apparently stationary conditions (Figures 5 and 6):

Sample A from Flight 19 200 seconds long  
Sample B from Flight 20 250 seconds long  
Sample C from Flight 20 450 seconds long

Sample B contained a significantly higher intensity of turbulence than the other samples. A shorter section of B, 60 seconds in length, was designated B<sub>1</sub> and used in initial analysis.

The detailed analysis of these samples consisted of reading the trace records at  $\frac{1}{10}$  th second intervals using Benson-Lehner OSCAR equipment. The cards produced by this equipment were checked by a Fortran IV program which causes the traces to be replotted to scale on an off-line plotter. They were then used as the input to a calibration program with its output on digital magnetic tape. This was then available for input to further analysis programs.

#### 4.0 ANALYSIS

Because of the large amount of data available, the analysis relied heavily on computer programs, written in Fortran IV language for the IBM 7040 computer at Warton. By this means only the initial gathering of data from the trace records involved manual operations and the large samples necessary for statistical reliability could be more easily dealt with.

Three main analysis programs were used:-

1. To evaluate the vertical component of turbulence,  $w_G$ .
2. To estimate the probability density of sets of data.
- and 3. To measure the power and cross-spectral densities and coherence of the data.

The first two of these were written specifically for the present work; the third was already available. The programs are briefly described in Appendix 2.

Reasons are also given in Appendix 2 for choosing a conventional spectral analysis program instead of one suggested by the Royal Aircraft Establishment involving linear interpolation of the correlation function.

#### 4.1 Vertical Component of Turbulence

The first application of the program to compute the vertical component of turbulence was to the 60 second sample of rather severe turbulence known as B<sub>1</sub> (section 3.2). The result was half a cycle of large amplitude with the more reasonable, higher frequency variation of turbulence velocity superimposed on it, Figure 7. If this had been caused by an error in the program equations or by a true physical phenomenon (such as the wind blowing parallel to slowly varying terrain) a comparable variation should have been visible in the already integrated pressure measurements of height and airspeed. Both these possibilities were ruled out, leaving the choice between instrumentation errors or trace reading errors.

The amplitude of the spurious 120 second period variation was still small enough to allow the results to be fed into the spectral analysis program and this was done. Meanwhile analysis of a further sample (A), of 200 seconds duration, was commenced in the hope that the error might be an isolated example resulting from the way sample B<sub>1</sub> had been read. However, sample A<sub>1</sub> when analysed showed the same low frequency variation to an even greater extent. The program had been modified to print out the two integrated terms in the expression for  $w_G$  and this indicated that the trouble was being caused by the integrated pitch rate term (Figure 8). Small, slowly varying errors in pitch rate were being integrated up over large numbers of data points to give angles of pitch apparently as high as  $10^\circ$  in the middle of the sample. This corresponds to a contribution of 150 ft./sec. to  $w_G$  at the high forward speed ( $V = 853$  ft./ sec.).

Therefore, errors in pitch rate originating at the trace reading stage were investigated by comparing a small sample of readings with very precise measurements of the trace. It is usually assumed that the accuracy in trace reading on the OSCAR equipment is 0.25 mm. at the  $2\sigma$  level. The pitch rate sensitivity given in Table 1 gives a corresponding accuracy of approximately 0.25 degrees per second in pitch rate. This would be acceptable as a random error occurring at fairly high frequency. However, the integration of pitch rate to give angle of pitch has the effect of magnifying any low frequency or constant errors. For this reason the program eliminates constant errors by reducing the rate of pitch time history to zero mean before integration.

In checking the reading errors the measurements of the traces were done more accurately by confining the study to only 30 of the 2000 readings. If these are assumed to be exact the root mean square error of the OSCAR readings is 0.34 mm. which is consistent with the assumed error of 0.25 mm. There is, however, some bias in the errors (mean error = 0.125 mm.) and also a bias between the errors in the first half of the sample (mean error = -0.06 mm.) and those in the second half (mean error = 0.34 mm.). Therefore,

after removing the overall bias the reading errors alone would account for a bias of  $-0.2$  mm. ( $-0.18^\circ$  /sec.) in the first half of the record and a similar bias of opposite sign in the second half. When integrated over half the sample length this would give a pitch angle of  $18^\circ$  in the middle of the sample. Thus, on the basis of this small sample of reading errors it can be said that low frequency reading errors of small magnitude did occur and that, after integration, these would be sufficient to cause the spurious half cycle variation of  $w_G$  encountered.

A variation of similar type occurred in the integrated normal acceleration term (Figure 8) but because of the greater sensitivity of the instrumentation the magnitude of this was much smaller.

In order to obtain a usable output from the program sample A was re-analysed in two parts, each of 100 seconds duration.

#### 4.2. Probability Distributions

Analogue magnetic tape records of normal acceleration at the pilot's cockpit were analysed to find the probability density of samples approximately 50 seconds in length. This was mainly of value as an exercise in automatic data reduction and it also served to emphasise some of the sources of error in the estimation of probability density.

The system used for analysis is shown in block diagram form in Figure 9. The signal was filtered using a 16 c.p.s. linear fourth order filter set up on an analogue computer. The filtered and unfiltered signals were then sampled by a mechanical sampling switch and punched on paper tape using a 7-bit binary code. The tape had then to be rewritten on cards before use as input to the IBM computer.

In spite of careful scaling, the 7-bit code, originally intended for recording TSR.2 engineering data, is inadequate for the present purpose. This was shown by the oscillatory behaviour of some of the probability density plots (e.g. Figure 10). Any attempt to use nonstationary data was also immediately apparent in the results. An extreme example for a sample with time varying mean is shown in Figure 11. Both of the above effects could have contributed to the non-Gaussian level counts originally obtained from trace records (Reference 1). The results obtained from magnetic tape records showed the futility of trying to analyse the derived values of gust velocity with their large amplitude half-cycle variation, in this way.

### 4.3 Pilot's Response

In order to separate the linear response of the pilot from the random "remnant" noise generated by him it is necessary to know the turbulence input to the pilot/aircraft system. Although the derived variations of gust velocity contained large, low frequency errors, these were used for the analysis of pilot's response in the hope of obtaining valid results at high frequencies.

One indication of the validity of the derived pilot describing functions is the value of the coherence (Appendix 1 equation (A8)). The square of this quantity is plotted against frequency in Figures 12, 13, 14 for sample B1 and the two halves of sample A. From these the coherence is seen to be nearly zero except over a finite frequency interval.

In Figures 15, and 16 the pilot's describing functions assuming pitch rate and normal acceleration as cues are plotted over the intervals of relatively high coherence. All the plotted results were obtained using a maximum of 100 lags. Other maximum numbers of lags were used in the analysis of each sample and the 100 lag results were chosen for plotting regardless of record length because these gave the best complete set of results.

## 5.0 DISCUSSION

### 5.1 Vertical Component of Turbulence

The difficulties encountered in deriving the time history of gust velocity (section 4.1) prevented the reliable estimation of the power spectral density of the turbulence. However, some useful conclusions regarding instrumentation requirements can be drawn from this part of the analysis.

The errors in pitch rate were not particularly large but, occurring at low frequencies, they were magnified by the subsequent integration. Thus, where data is to be integrated, or differentiated, the required accuracy is dependent on frequency. To ensure 1% accuracy in pitch angle after integration of pitch rate over 200 seconds, low frequency errors in pitch rate would have to be less than  $1/200\% = .005\%$ . Higher frequency errors in excess of this could be tolerated since they would be weighted less heavily by the integration process. The smallness of this low frequency requirement indicates the inadvisability of integrating long runs of measured data. In the present case integration was necessary because of the absence of direct measurements of pitch angle.

The fact that the low frequency error occurred in trace reading is significant. The reason for the error is not obvious but it is small enough to be caused by minor day to day variations in technique. Resetting of the trace on the equipment is not the obvious reason since this would be done about 4 times during the reading and the setting up of the trace checked three times as often.



Where the necessary standard of measurement and recording cannot be achieved errors outside the valid frequency range can be attenuated by the use of suitable numerical filters. Shortage of time prevented the investigation of this technique in the present case. It is discussed in Reference 2.

## 5.2 Probability Density

The accuracy of amplitude probability estimates is strongly dependent on parameters such as sample length relative to signal frequency content, resolution and stationarity. The last of these can cause serious deformation in the shape of the probability density curve and is the most likely explanation of the non-Gaussian level counting results obtained in Reference 1.

## 5.3 Human Pilot Response

The spurious low frequency variation in the estimates of turbulence input to the pilot - aircraft system gives rise to unreliable results for the analysis of human pilot response at low frequencies. This is indicated by the low values of coherence at low frequency in Figures 13 and 14. The coherence is also low at high frequencies where the motions of the aircraft are too rapid for the pilot to follow. The plots of pilots' describing functions in Figures 15 and 16 have been confined to the intermediate frequencies where the coherence rises to values of the order of 0.7. This is not quite as high as the coherence (0.92) reached at low frequencies on the simulator experiments reported in Reference 4. On the other hand it is high enough to give some confidence in the measurements of pilots' describing function at these frequencies and compares favourably with values of coherence measured both in flight and simulator in Reference 5.

The pilots' amplitude ratio and phase lag are compared with those obtained in Reference 1 for pitch rate control in Figure 15. The amplitude ratio points are consistent in shape with the line from Reference 1, although the level cannot be compared. The phase points exhibit more phase lag than was encountered in the simulator tests. This is consistent with the comparative tests in flight and simulator of Reference 5. There is no known basis for comparison with the normal acceleration cue results in Figure 16. However, the shapes of the amplitude and phase plots are similar to those of the pitch rate results. Therefore, no conclusion can be reached as to which one the pilot was actually using.

## 6.0 CONCLUSIONS

Human pilot describing functions have been deduced from TSR.2 records of flights through turbulence on the assumptions of single loop compensatory tracking of pitch rate and normal acceleration respectively. No indication is given as to which, if either, of these assumptions is valid. The pitch rate describing function results are consistent with previous measurements in flight and simulator.

Small errors introduced in the reading of pitch rate and normal acceleration traces for subsequent integration prevented the estimation of low frequency components of vertical gust velocity.

No mechanism has been found which would lead to the expectation of the non-Gaussian level counting results obtained in Reference 1. However, estimates of probability have been shown to be subject to large errors due to analysis of too short a sample relative to the frequency content, or to the occurrence of a varying local mean value. The results of Reference 1 could have been subject to such errors.

After long consideration a plan for estimating spectral densities from continuous interpolation of the correlation functions was discarded on the grounds that interpolation at this stage would not be justified and would in general give incorrect answers. The possibility that the errors introduced might compensate aliasing errors made in sampling for certain specific cases cannot be ruled out.

Further work of this nature on the TSR.2 records is not justified. It is suggested that a gust research aircraft which would be specially instrumented to overcome the difficulties in evaluation of gust velocity encountered here would provide an ideal vehicle for future studies of human pilot response in flight.





NOMENCLATURE

$x(t)$	Function of $t$ defined for all $t$ unless stated otherwise.
$\{x(t)\}$	Random process consisting of infinite number of "realisations" of $x(t)$
$p(x)$	Amplitude probability density function defined in 2.1.
$G_x(f), G_{xx}(f)$	Power spectral density of $x(t)$
$w_G$	Vertical gust velocity (true) (ft./sec.)
$V$	True air speed (ft./sec.)
$\alpha$	Angle of incidence (radians)
$\theta$	Angle of pitch (radians)
$h$	True pressure altitude (ft.)
$q$	Rate of pitch (rad./sec.)
$a_z$	Incremental normal acceleration (g units)
$Y_c(j\omega) = \frac{q}{\eta}(j\omega)$	Transfer function of "controlled element" i.e. of aircraft + control system (tailplane input)
$Y_G(j\omega) = \frac{q}{W_G}(j\omega)$	Transfer function of controlled element (gust input)
$Y_p(j\omega) = \frac{\eta}{q}(j\omega)$	Pilot's describing function
$G_{xy}(f)$	Cross spectral density of $y(t)$ with respect to $x(t)$ .
$v(t)$	Random portion of pilot's output.
$\rho$	Coherence (defined in Appendix 1, A.8)
$K_\alpha$	Incidence calibration factor.
$\alpha_i$	Indicated incidence.
$x_\alpha$	x-coordinate of incidence vane relative to c of g

$n_i$	Indicated normal acceleration (g units)
$x_n, z_n,$	x,z co-ordinates of normal accelerometer
$\eta_p, \eta_s$	Port and starboard t aileron deflections
$\eta \frac{\eta_p + \eta_s}{2}$	Effective tailplane deflection
$A(l), B(l)..F(l)$	Correlation functions defined in Appendix 1.
$\phi$	Phase angle

TABLE 1

Details of Trace Records Used

Quantity	Recorder Number	Approximate Sensitivity
Altitude	A13/1	1800 ft/cm
Airspeed	A13/1	25 knots/cm
Incidence	A13/5	6.7°/cm
Rate of Pitch	A13/5	9°/sec/cm
Normal Acceleration	A13/6	0.27g/cm
Port Taileron Angle	A13/7	1.6°/cm
Stbd. Taileron Angle	A13/7	1.6°/cm
Fuel Contents (Fwd. Tanks)	A13/14	340 galls/cm
Fuel Contents (Aft Tanks)	A13/14	340 galls/cm

APPENDIX 1

Analysis of Single Loop Pilot-Aircraft System (Ref. 4)

A block diagram of the pilot - aircraft system for a single loop compensatory pitch tracking situation is shown in Figure 4. Pitch rate  $q$  is used to provide a concrete example and could be replaced by any other relevant cue in the ensuing analysis. The following equations can be derived from the block diagram:

$$\eta = v + Y_p q = v + Y_p (Y_G w_G + Y_c \eta)$$

$$\therefore \eta (1 - Y_p Y_c) = v + Y_p Y_G w_G \tag{A1}$$

$$\text{or } \eta = \frac{1}{1 - Y_p Y_c} v + \frac{Y_p Y_G}{1 - Y_p Y_c} w_G$$

$$\text{Also } q = Y_G w_G + Y_c \eta = Y_G w_G + Y_c (v + Y_p q)$$

$$\therefore q = \frac{Y_G}{1 - Y_p Y_c} w_G + \frac{Y_c}{1 - Y_p Y_c} v \tag{A2}$$

Note that all the quantities in the above equations are defined in the frequency domain and are understood to be functions of the imaginary frequency variable ( $j\omega$ ).

Now if we multiply (A1) and (A2) by the complex conjugate of  $w_G(j\omega)$  the following relations between cross and power spectral densities are implied:

$$G_\eta w_G = \frac{1}{1 - Y_p Y_c} G_v w_G + \frac{Y_p Y_G}{1 - Y_p Y_c} G w_G w_G \tag{A3}$$

$$G_q w_G = \frac{Y_G}{1 - Y_p Y_c} G w_G w_G + \frac{Y_c}{1 - Y_p Y_c} G_v w_G \tag{A4}$$

APPENDIX 1 (Continued/....)

Since the pilot's remnant is, by definition, incoherent with the turbulence input  $w_G$ :

$$G_{\nu w_G} = 0$$

and (A3) and (A4) can be written:

$$G_{\eta w_G} = \frac{Y_p Y_G}{1 - \frac{Y_p Y_c}{p c}} G_{w w_G} \quad (A5)$$

$$G_{q w_G} = \frac{Y_G}{1 - \frac{Y_p Y_c}{p c}} G_{w w_G} \quad (A6)$$

Dividing each side of (A5) by the same side of (A6):

$$Y_p = \frac{G_{\eta w_G}}{G_{q w_G}} \quad (A7)$$

Thus an estimate of the pilot's describing function  $Y_p(j\omega)$  can be obtained by cross-spectral analysis of three signals<sup>p</sup>:

The turbulence input  $w_G(t)$

The pilot's output  $\eta(t)$

The aircraft response  $q(t)$

The following relation can also be derived from (A1) in the case where  $w_G$  and  $\nu$  are incoherent:

$$G_{\eta\eta} = \left| \frac{Y_p Y_G}{1 - \frac{Y_p Y_c}{p c}} \right|^2 G_{w w_G} + \left| \frac{1}{1 - \frac{Y_p Y_c}{p c}} \right|^2 G_{\nu\nu}$$

Thus we can define the proportion of the pilot's output power  $G_{\eta\eta}$  that is linearly related to the turbulence input power  $G_{w w_G}$ :

$$\rho^2 = \left| \frac{Y_p Y_G}{1 - \frac{Y_p Y_c}{p c}} \right|^2 \frac{G_{w w_G}}{G_{\eta\eta}}$$

where  $\rho$  is called the coherence.

Substituting in (A5):

$$\rho = \frac{|G_{\eta w_G}|}{\sqrt{G_{\eta\eta} G_{w w_G}}} \quad (A8)$$

APPENDIX 2

Computer Programs for Analysis

A.1 Turbulence Velocity

A Fortran IV program was written to evaluate the vertical component of turbulence  $w_G$  from readings of: indicated incidence  $\alpha_i$

pitch rate  $q$

indicated normal acceleration  $n_i$

Equation (5) was used where:

$$\alpha = \left( \frac{1}{K_\alpha} \right) \alpha_i + \left( \frac{x_\alpha}{V} \right) q \quad (A9)$$

$$\theta = \int_0^t (q - \bar{q}) dt \quad (A10)$$

$$\eta = \int_0^t 32.2 (a_z - \bar{a}_z) dt \quad (A11)$$

$$\text{where } a_z = n_i - \frac{x_n}{g} \frac{\pi}{180} \frac{dq}{dt} - \frac{\pi}{180} \frac{z_n}{g} q^2 - 1.0 \quad (A12)$$

$K_\alpha$  is the incidence calibration factor

$x_\alpha$  is the x-co-ordinate of the incidence transducer.

$x_n, z_n$  are the co-ordinates of the normal accelerometer.

- indicates a mean value over all data points.

The correction of  $a_z$  for aircraft pitch angle to give vertical acceleration was assumed to be negligible. The output of the program consists of:-

$$\alpha, a_z, \eta \left( = \frac{\eta_p + \eta_s}{2} \right), q \text{ and } w_G$$

The input and output are at equispaced intervals. Mean values taken over the whole sample are subtracted from the values of the integrands (to eliminate the effects of constant errors in the integrals.) In spite of this some difficulty was encountered in the longer samples because of small errors of very low frequency introduced during trace reading. After integration these produced large variations at about 1/2 cycle over the whole sample length.

APPENDIX 2 (Continued/....)

**A.2 Probability Density**

A further program was written in Fortran IV to estimate the frequency distribution of a series of numbers. The numbers could be the elements of an equispaced time series in which case an estimate of amplitude probability density based on equation (1) would result.

Up to 5000 numbers can be accommodated. The program first computes the mean  $\mu$  and standard deviation  $\sigma$ . The numbers are then sorted into 31 classes each  $0.1 \sigma$  in width and the number in each class stored. Subroutines can be written to fit any standard distribution to the measured frequencies and apply a test of significance. At present only one such subroutine has been written. This fits the Gaussian distribution having the measured mean and standard deviation and applies the chi square test. The results appear in the form of superimposed plots of the measured and fitted distributions annotated with the values of:-

- Number of values
- Mean
- Standard Deviation
- Value of chi square
- and Number of degrees of freedom.

A typical plot is shown in Figure 10.

**A.3 Spectral Analysis**

**A.3.1 Description of Program**

For spectral analysis an existing program (Reference 6) was used. This accepts two simultaneous time series, each having up to 5,500 elements and computes the cross-spectra and two power spectra via the cross correlation and auto-correlation functions. Phase and coherence are calculated from the cross-spectra and power spectra.

In a previous application the original program had been modified and the equations solved in the present form are:

$$A(l) = \frac{1}{n-1} \sum_{i=l+1}^n x_{i-1} x_i - \frac{1}{(n-1)^2} \sum_{i=l+1}^n x_{i-1} \sum_{i=l+1}^n x_i \quad (A.13)$$

$$B(l) = \frac{1}{n-1} \sum_{i=l+1}^n y_{i-1} y_i - \frac{1}{(n-1)^2} \sum_{i=l+1}^n y_{i-1} \sum_{i=l+1}^n y_i \quad (A.14)$$



APPENDIX 2 (Continued/....)

$$C(l) = \frac{1}{n-1} \sum_{i=1}^n x_{i-1} y_i - \frac{1}{(n-1)^2} \sum_{i=1}^n x_{i-1} \sum_{i=1}^n y_i \quad (A.15)$$

$$D(l) = \frac{1}{n-1} \sum_{i=1}^n y_{i-1} x_i - \frac{1}{(n-1)^2} \sum_{i=1}^n y_{i-1} \sum_{i=1}^n x_i \quad (A.16)$$

A(l), B(l), C(l), D(l) are the auto and cross correlations of the time series  $x_i$  and  $y_i$  for  $l = 0 \dots m$ ; n is the number of elements in each time series.

$$E(l) = \frac{D(l) + C(l)}{2} \quad (A.17)$$

is the even part of the cross-correlation and

$$F(l) = \frac{D(l) - C(l)}{2} \quad (A.18)$$

is the odd part:

$$G_{xx}(k) = \frac{\delta_k}{m} \left[ \sum_{l=1}^{m-1} (1 + \cos \frac{\pi l}{m}) \cos \frac{k l \pi}{m} A(l) + A(0) \right] \quad (A.19)$$

$$G_{yy}(k) = \frac{\delta_k}{m} \left[ \sum_{l=1}^{m-1} (1 + \cos \frac{\pi l}{m}) \cos \frac{k l \pi}{m} B(l) + B(0) \right] \quad (A.20)$$

$$RG_{xy}(k) = \frac{\delta_k}{m} \left[ \sum_{l=1}^{m-1} (1 + \cos \frac{\pi l}{m}) \cos \frac{k l \pi}{m} E(l) + E(0) \right] \quad (A.21)$$

$$IG_{xy}(k) = \frac{\delta_k}{m} \left[ \sum_{l=1}^{m-1} (1 + \cos \frac{\pi l}{m}) \cos \frac{k l \pi}{m} F(l) + F(0) \right] \quad (A.22)$$

where  $\delta_k = \frac{1}{2}$  for  $k = 0$  or  $m$ ;  $\delta_k = 1$  otherwise.

$G_{xx}$  and  $G_{yy}$  are the power spectra of  $x_i$  and  $y_i$  and  $RG_{xy}$  and  $IG_{xy}$  are the real and imaginary parts of the cross-spectra. The output also lists the cross spectrum phase, amplitude and decibel amplitude.

Coherence  $\rho(k)$  is given by:

$$\rho^2 = \frac{(RG_{xy})^2 + (IG_{xy})^2}{G_{xx} G_{yy}} \quad (A.23)$$

APPENDIX 2 (Continued/....)

and phase  $\phi(k)$  by:

$$\phi = \left( \text{Arctan} \frac{IG_{xy}}{RG_{xy}} \right) \times \frac{180}{\pi} \text{ degrees} \quad (\text{A. 24})$$

The main change made to the program was the use of the divisors  $(n-1)$  and  $(n-1)^2$  in equations A13 to A16 instead of  $(n-1)$  and  $(n-1)^2$  as in the original program. This was mainly of concern where the small amount of data available for the previous application necessitated the use of small values of the ratio  $\frac{n}{m}$ . In the present application  $m \ll n$  usually but it was not necessary to revert to the original program.

A.3.2 Interpolation

The program described above operates on sampled data and if the original analogue signal contained any significant power at frequencies greater than half the sampling frequency this would be "aliased" and would appear at lower frequencies. Thus the use of sampled data implies the assumption that the original signals had no significant power at frequencies greater than half the sampling frequency.

In informal discussions with the Royal Aircraft Establishment, Bedford, it was learned that measurements of atmospheric turbulence were being analysed there by a modified program which obtained estimates of spectra by Fourier transformation of a continuous function consisting of a linear interpolation between the sample values of the correlation function. This program had been compared with a conventional program by the use of an analytically defined correlation function for which an "exact" power spectrum could be determined analytically. It was found that the linear interpolation method always gave better results at frequencies approaching half the sampling frequency. When applied to measurements of atmospheric turbulence the linear interpolation method gave more plausible results for the high frequency asymptotes of the turbulence power spectra.

A Deuce program using the interpolation method was made available and at the time of changing to an IBM computer it was suggested that such a program should be written for the new computer. An intuitive explanation of the results obtained at the Royal Aircraft Establishment was put forward. It was based on the frequency doubling effect of multiplication of cosine functions in the process of Fourier transforming the correlation functions. However, Mr. A. Stott of the Electronics Department at Warton pointed out that the use of interpolation of previously sampled signals would be inconsistent with the sampling theorems for random data. Therefore the program described in A.3.1 was used.

APPENDIX 2 (Continued/....)

The argument against interpolation depends on the following two theorems:

A. The Sampling Theorem in the Time Domain:

This states that: If the Fourier transform  $X(f)$  of some random time history record  $x(t)$  is zero at all frequencies outside a finite frequency interval from  $-B$  to  $B$  cycles per second it is determined for all  $f$  by the sample values  $x\left(\frac{n}{2B}\right)$  of  $x(t)$  for all  $n$ . Hence  $x(t)$  is completely determined for all  $t$  by these values and it can be shown that:

$$x(t) = \sum_{-\infty}^{\infty} x\left(\frac{n}{2B}\right) \frac{\sin\pi(2Bt-n)}{\pi(2Bt-n)} \quad (\text{A.25})$$

B. Borel's Convolution Theorem

This states that: If  $X_1(f)$  is the Fourier transform of  $x_1(t)$  and  $X_2(f)$  is the Fourier transform of  $x_2(t)$  then the product

$X_1(f)X_2(f)$  is the Fourier transform of the convolution

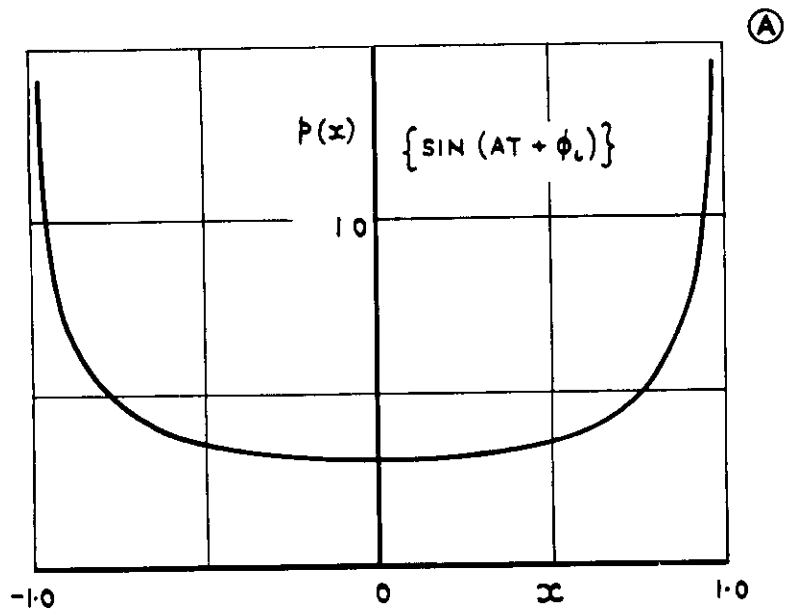
$$\int_0^{\infty} x_1(t) x_2(t - \tau) d\tau$$

The power spectral density is defined to be the square of the signal spectrum and it follows from the above theorems that this is equal to the convolution of the discrete samples  $x\left(\frac{n}{2B}\right)$  with the  $x\left(-\frac{n}{2B}\right)$ . In the case of these discrete values the convolution is merely the sum of products of pairs.

Any interpolation here would be superfluous and unless it were of the particular form given by (A.25) would be equivalent to imposing a hypothetical high frequency variation of the original continuous signal. This would be inconsistent with the original choice of sampling interval. If there is any shortcoming in the estimation of the power spectral density due to high frequency components it can be resolved only by a reduction in the sampling interval.



PROBABILITY DENSITIES FOR (A) SINE WAVE AND  
(B) GAUSSIAN RANDOM PROCESSES



GAUSSIAN DISTRIBUTION  
SAME VARIANCE

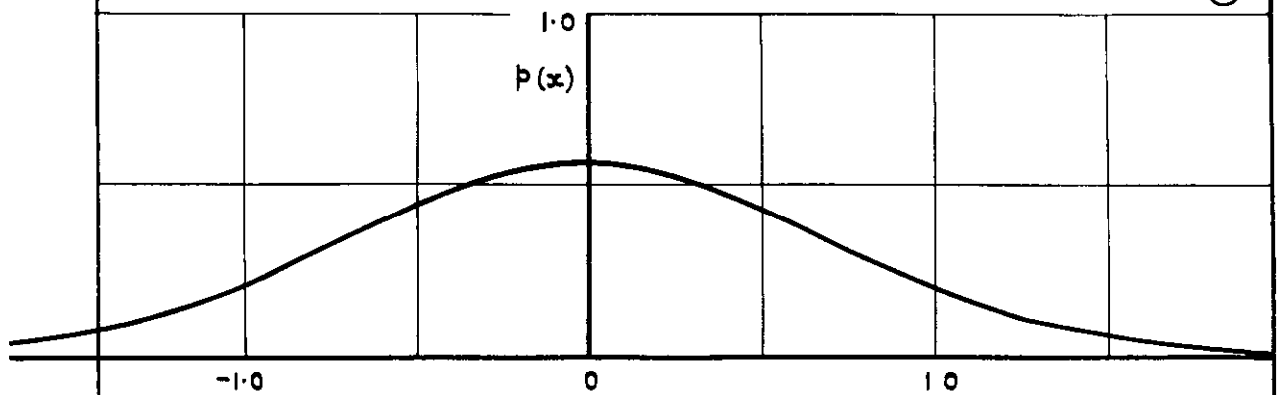
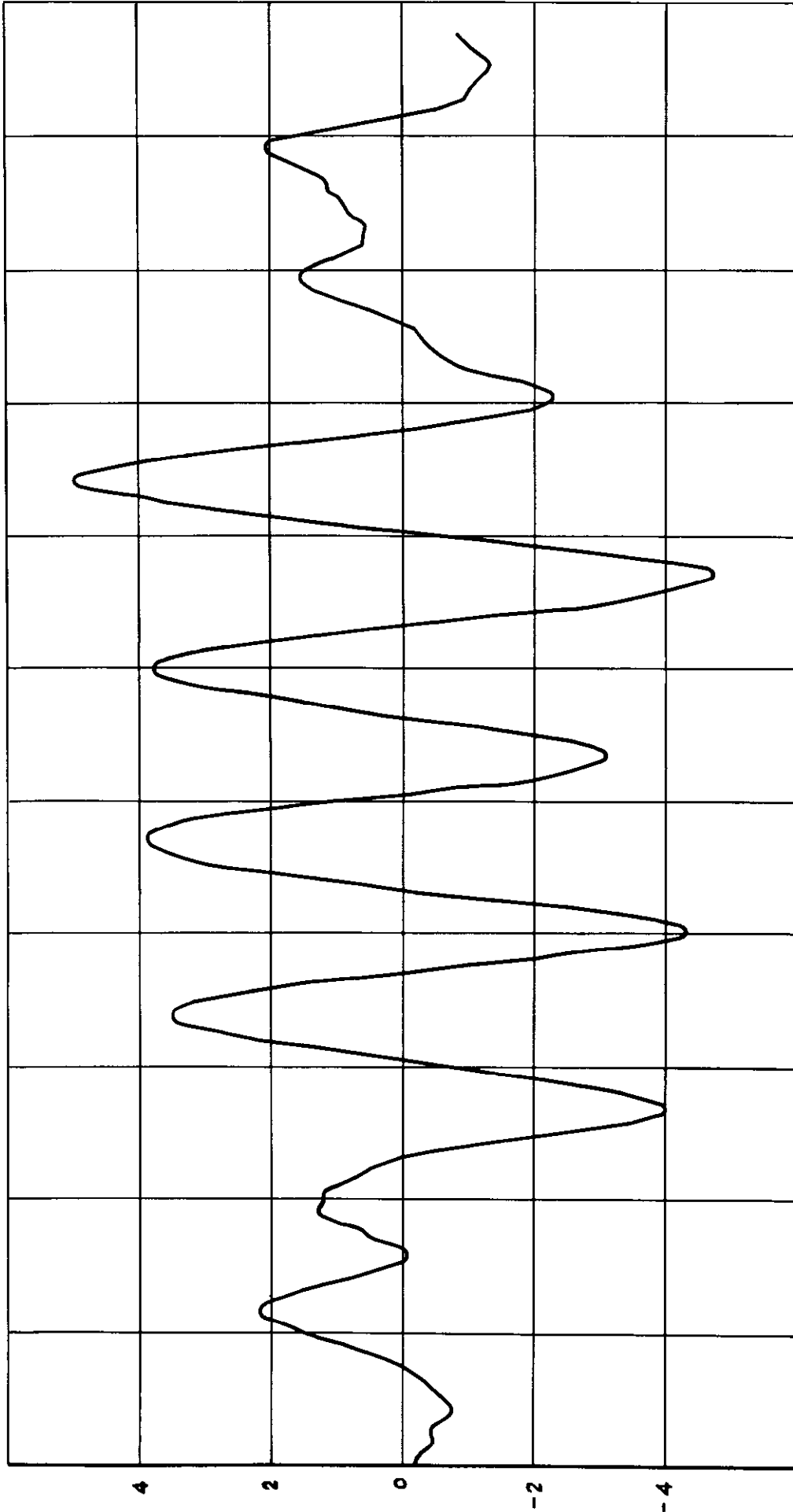


FIG 2.

SHORT SAMPLE OF NARROW BAND GAUSSIAN RANDOM NOISE



EQUIVALENT BLOCK DIAGRAMS  
OF LONGITUDINAL CLOSED LOOP CONTROL.

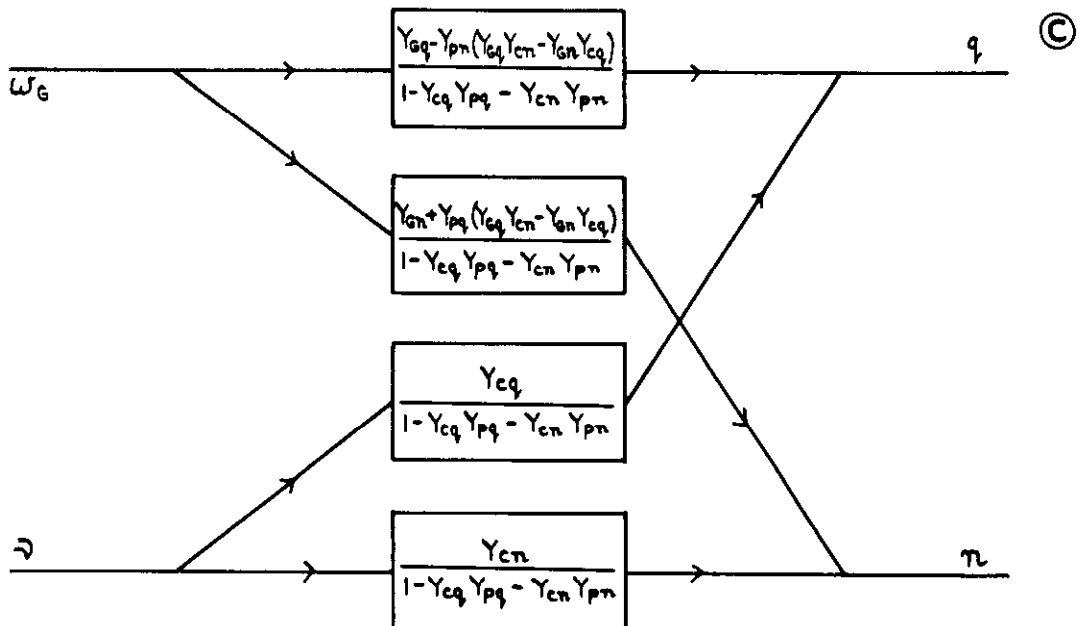
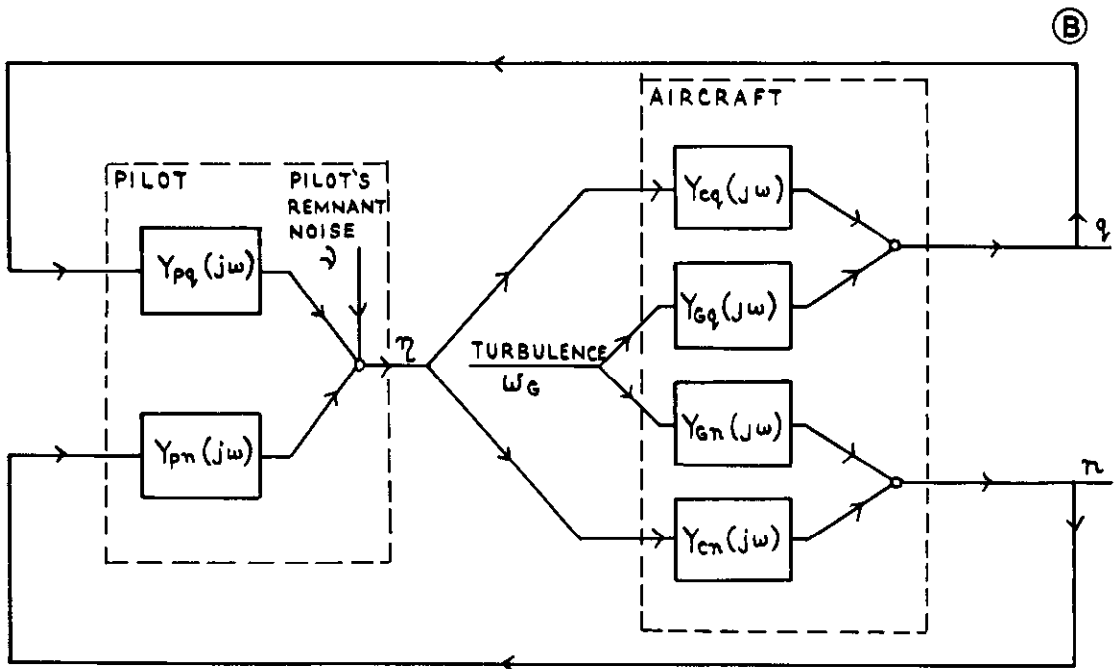
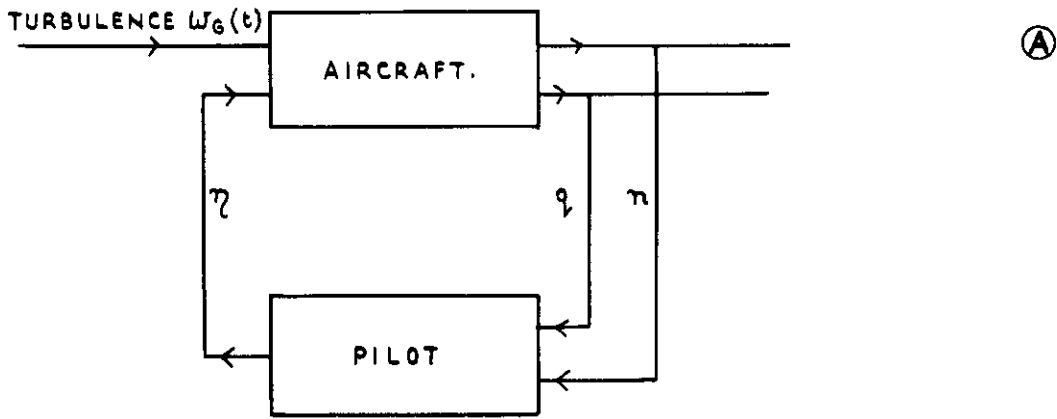


FIG 4.

SINGLE LOOP CONTROL OF PITCH RATE.

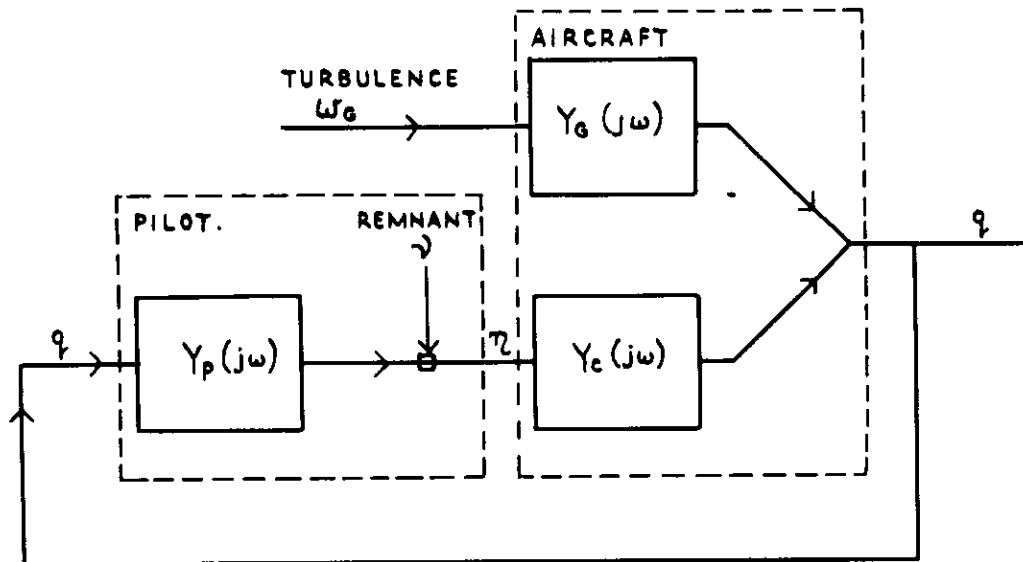
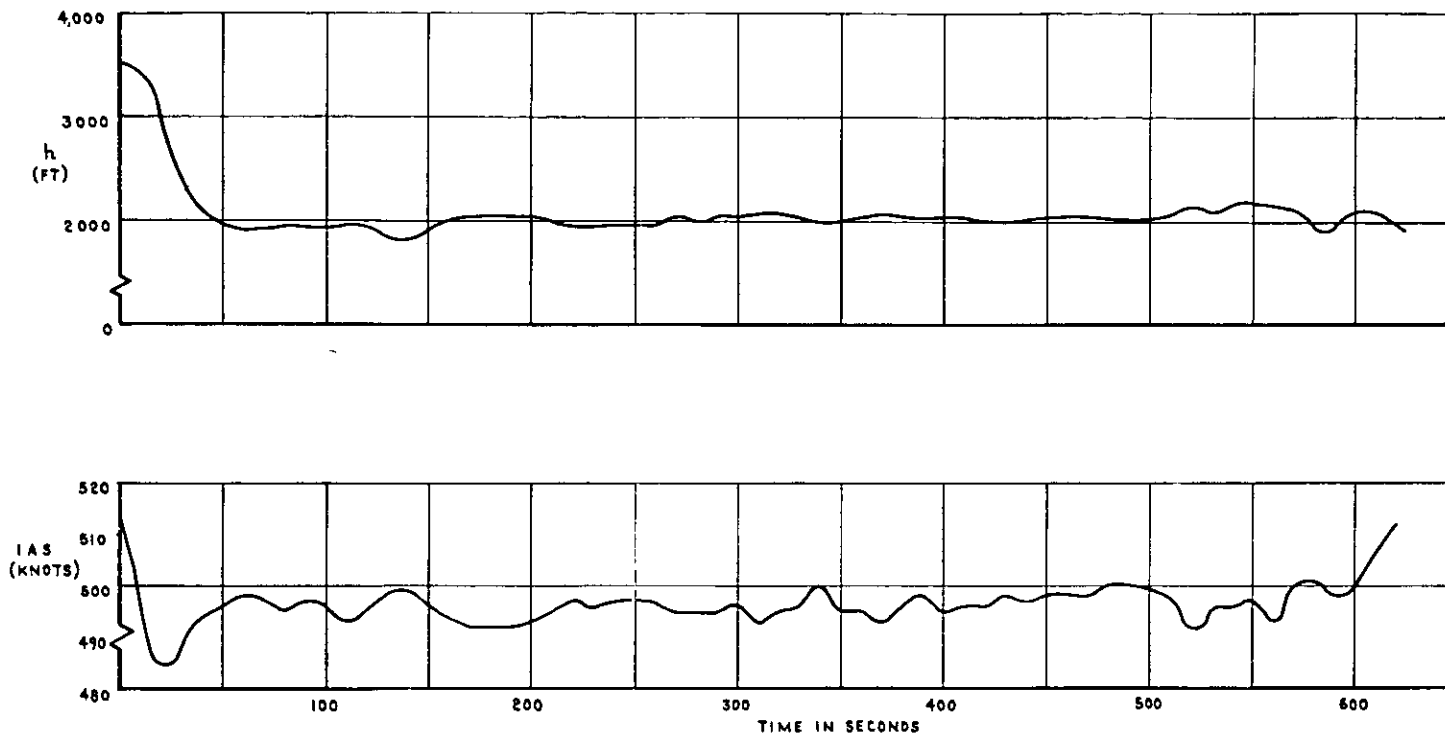




FIG 5

**XR 219 FLIGHT 19 HIGH SPEED RUN**



**X32 SPEED UP MUIRHEAD ANALYSER ON FLAT SETTING**

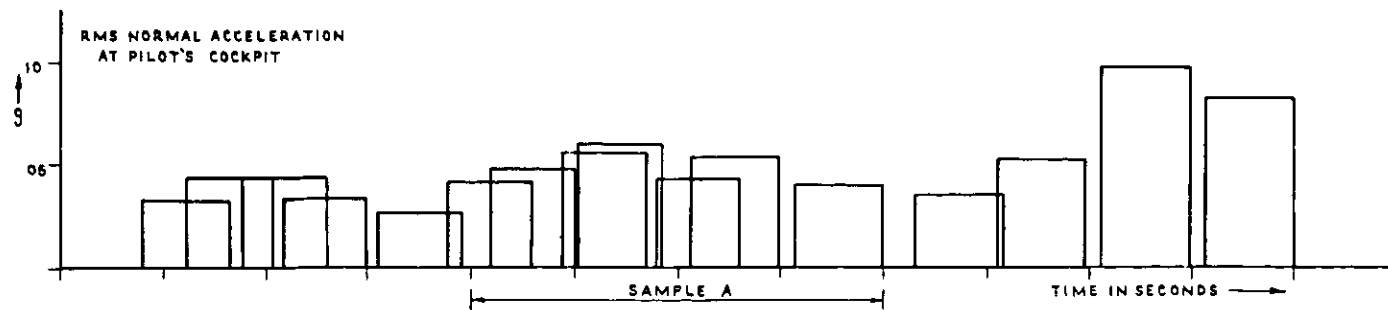
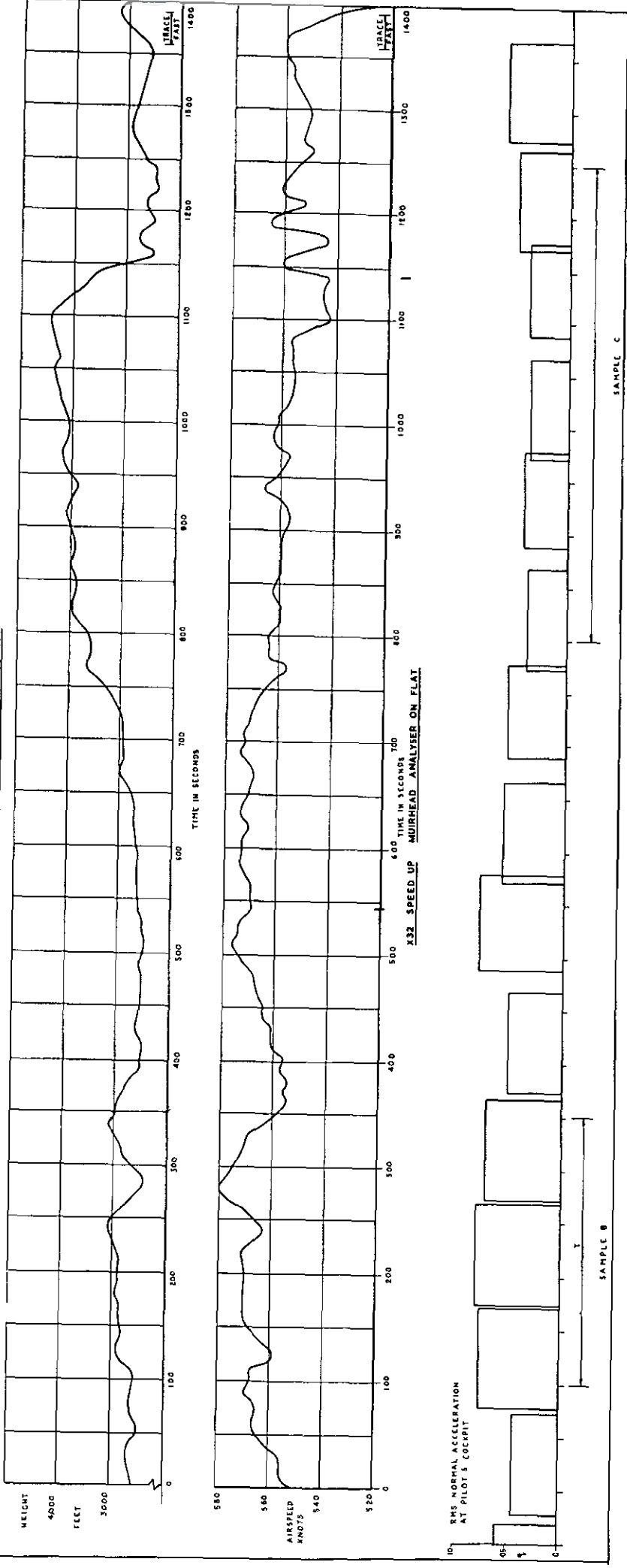


FIG. 6

IR219 FLIGHT 20 HIGH SPEED RUN



SAMPLE B

SAMPLE C

X32 SPEED UP  
MURHEAD ANALYSER ON FLAT

10  
05  
0  
RMS NORMAL ACCELERATION  
AT PILOT'S COCKPIT

HEIGHT

4000

FEET

3000

0

TIME IN SECONDS

580

560

AIRFIELD

KNOTS

540

520

0

TIME IN SECONDS

X32 SPEED UP  
MURHEAD ANALYSER ON FLAT

10

05

0

RMS NORMAL ACCELERATION  
AT PILOT'S COCKPIT

0

0

0

0

0

0

0

0

0

0

0

0

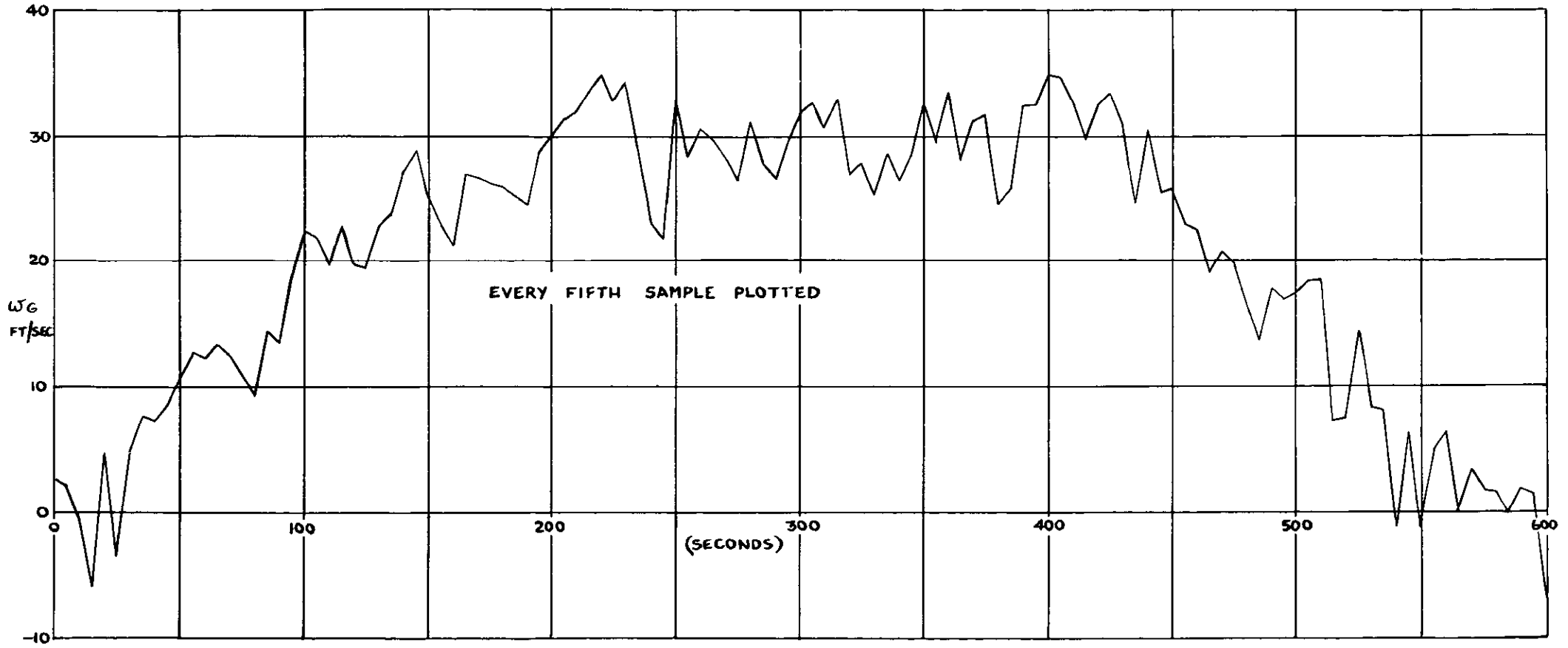
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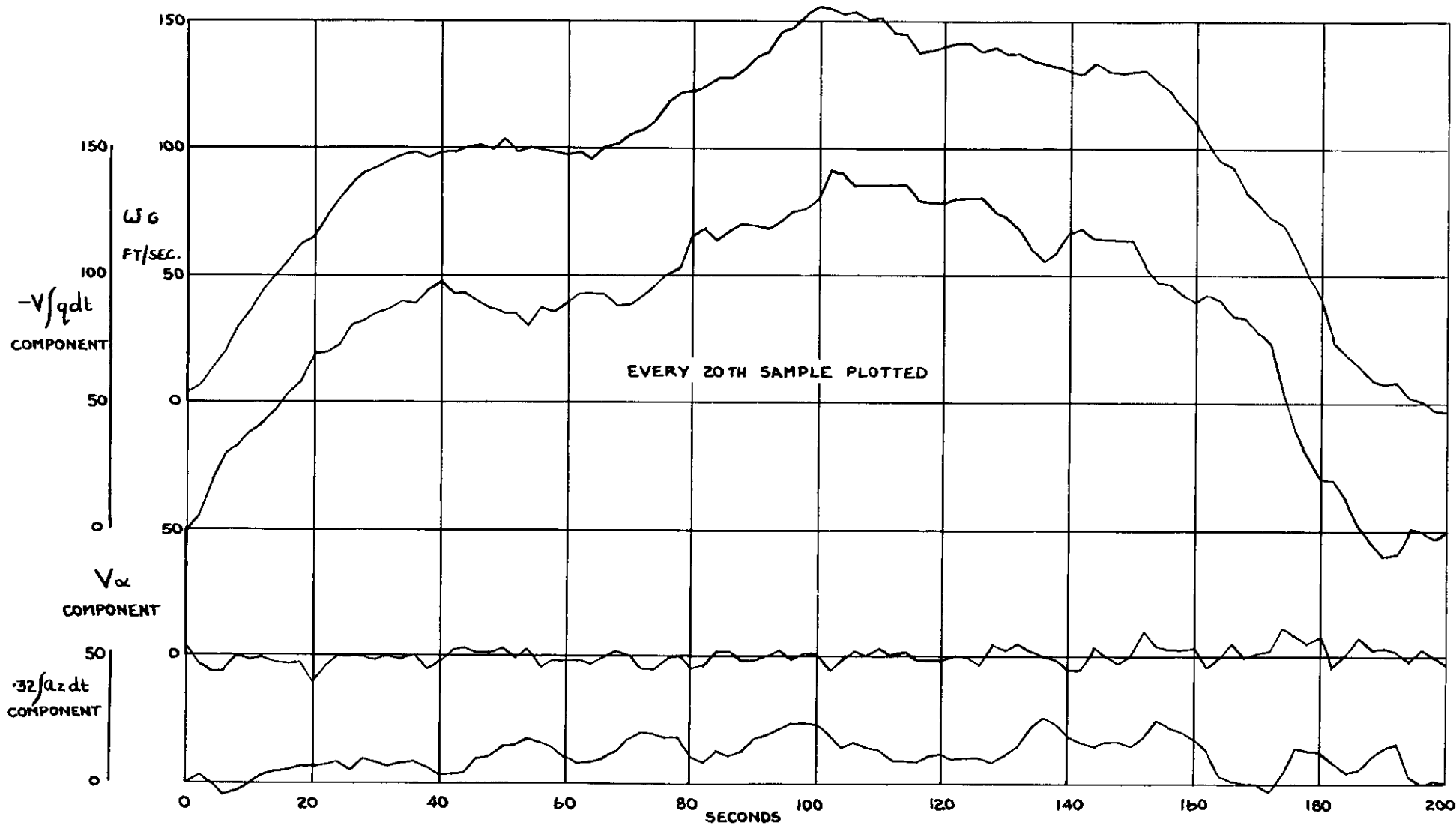
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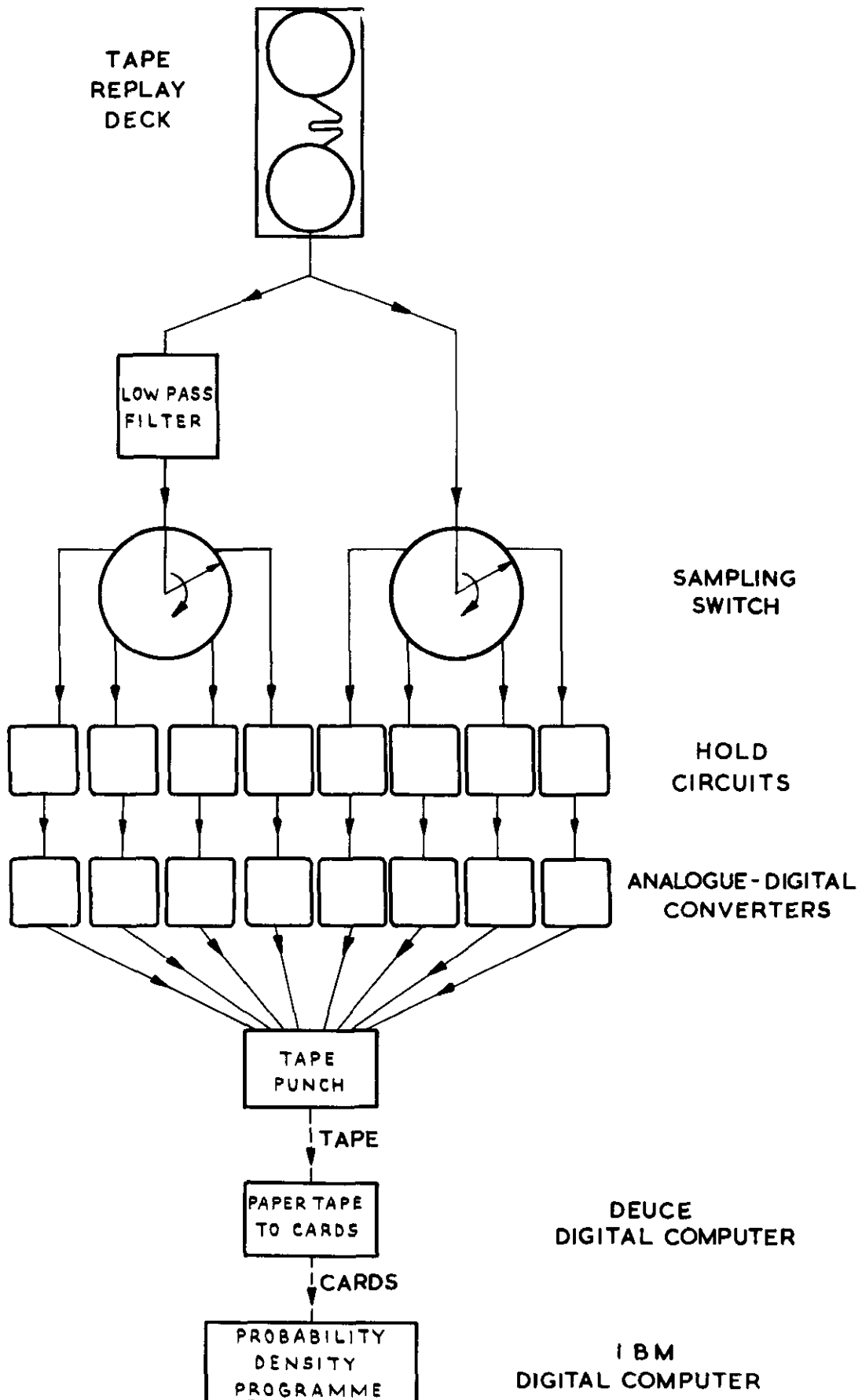


CALCULATED GUST VELOCITY TIME HISTORY FOR SAMPLE B1



COMPOSITION OF VERTICAL GUST VELOCITY TIME HISTORY FOR SAMPLE A

MECHANISED ANALYSIS OF MAGNETIC TAPE RECORDS



FREQUENCY DISTRIBUTION OF FILTERED MAGNETIC TAPE RECORD  
OF NORMAL ACCELERATION AT PILOT'S COCKPIT

(EFFECT OF POOR RESOLUTION DUE TO 7 BIT DIGITISATION)  
PROBABILITY =  $\frac{1}{N} \times (\text{FREQUENCY})$   
N = 4536

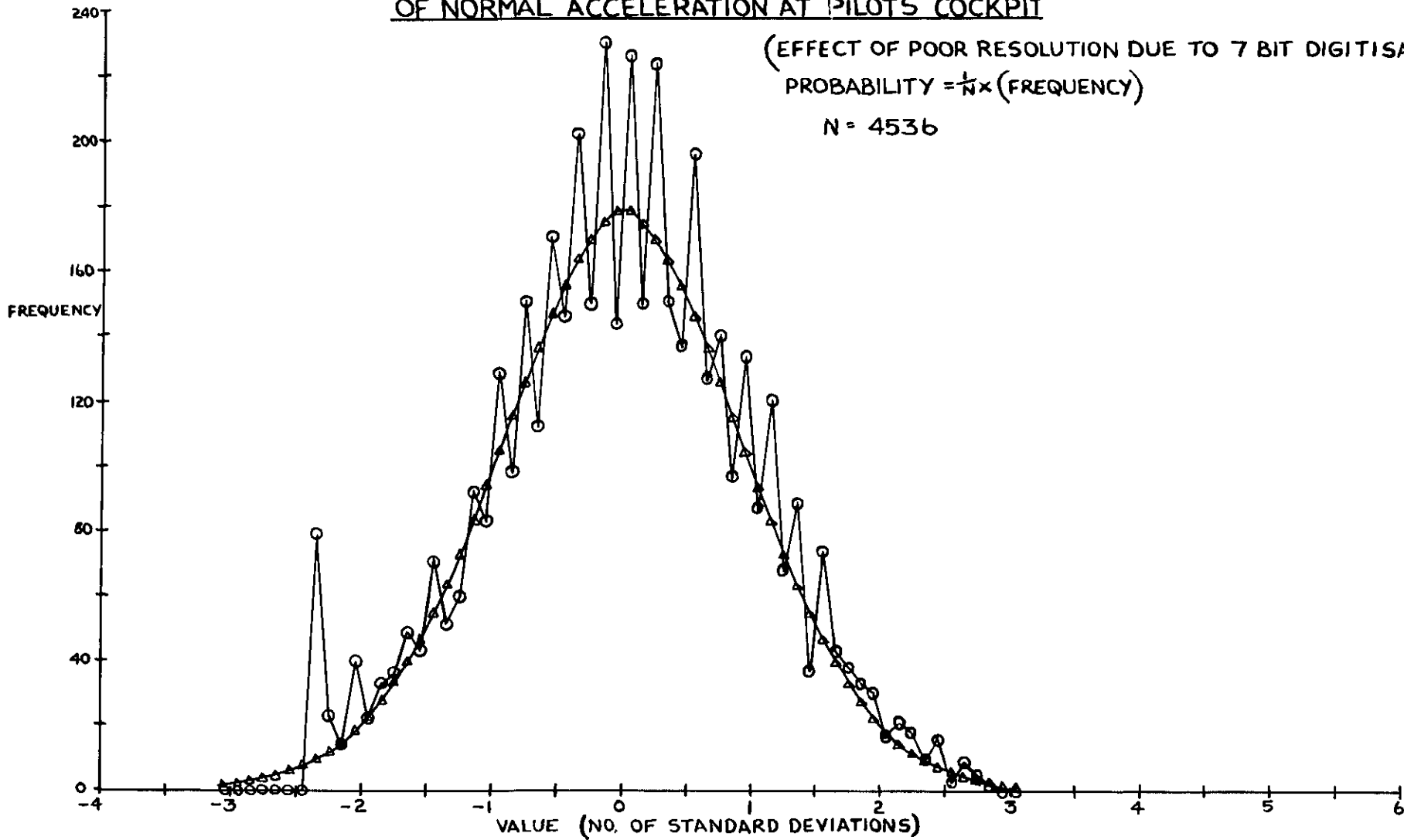


FIG.10

FREQUENCY DISTRIBUTION OF FILTERED MAGNETIC TAPE RECORD OF NORMAL ACCELERATION  
AT PILOT'S COCKPIT (EFFECT OF VARIATION OF LOCAL MEAN VALUE)

PROBABILITY =  $\frac{1}{N} \times$  FREQUENCY      N = 4536

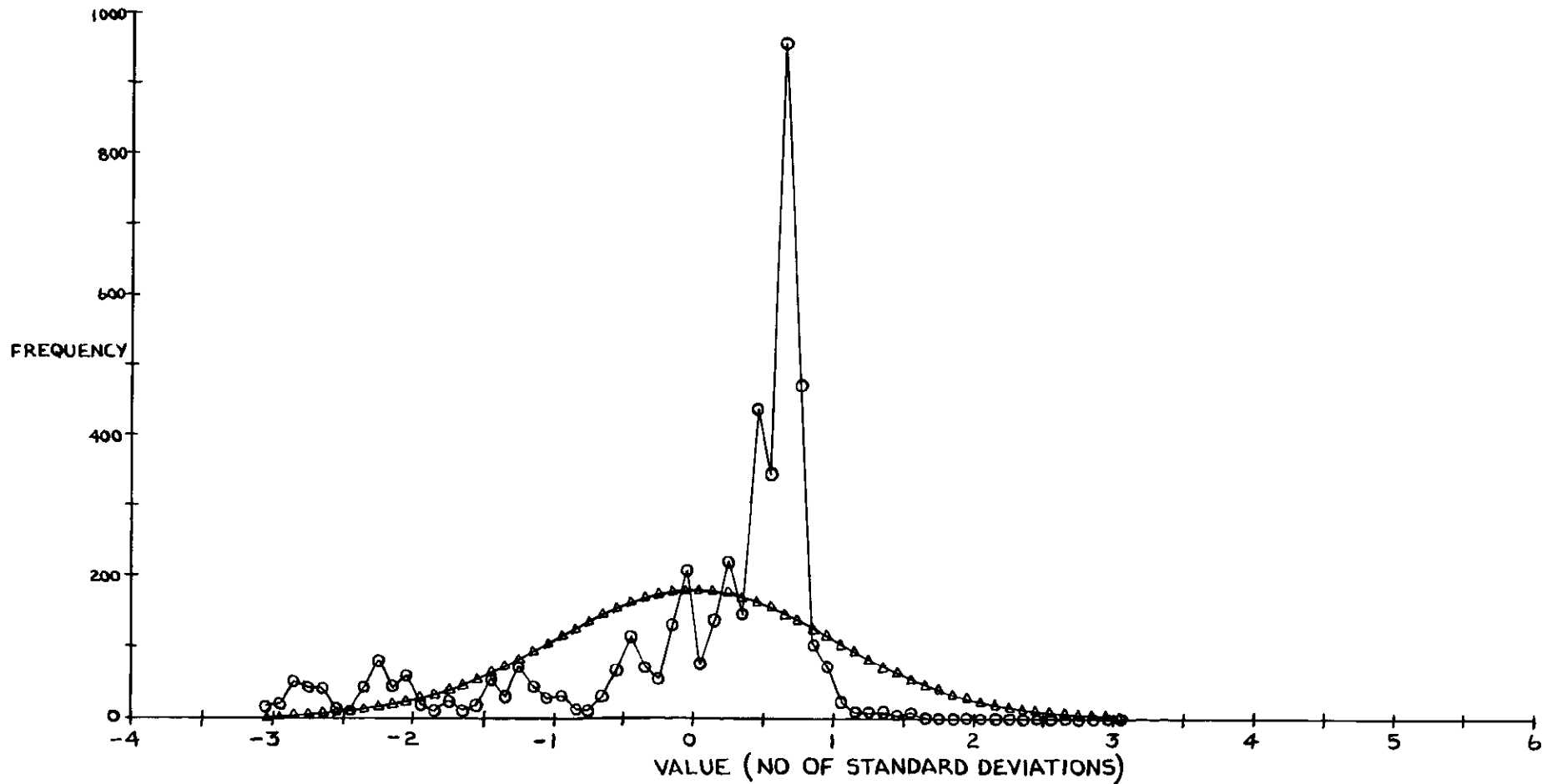


FIG 11.

FIG 12

(COHERENCE)<sup>2</sup> V FREQUENCY FOR SAMPLE BI  
602 DATA POINTS 100 LAGS

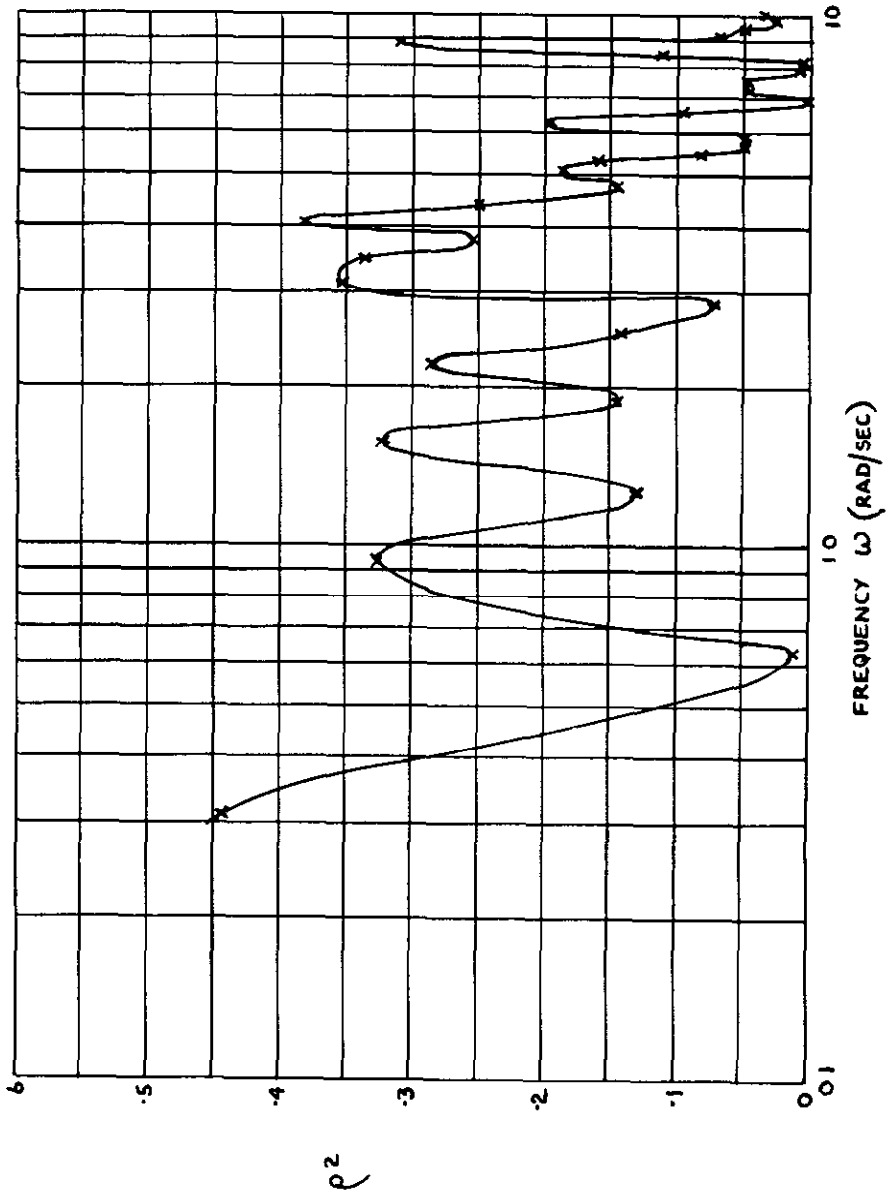




FIG. 13

SAMPLE A1   100 LAGS   COHERENCE <sup>2</sup>   VS   FREQUENCY  
602 DATA POINTS

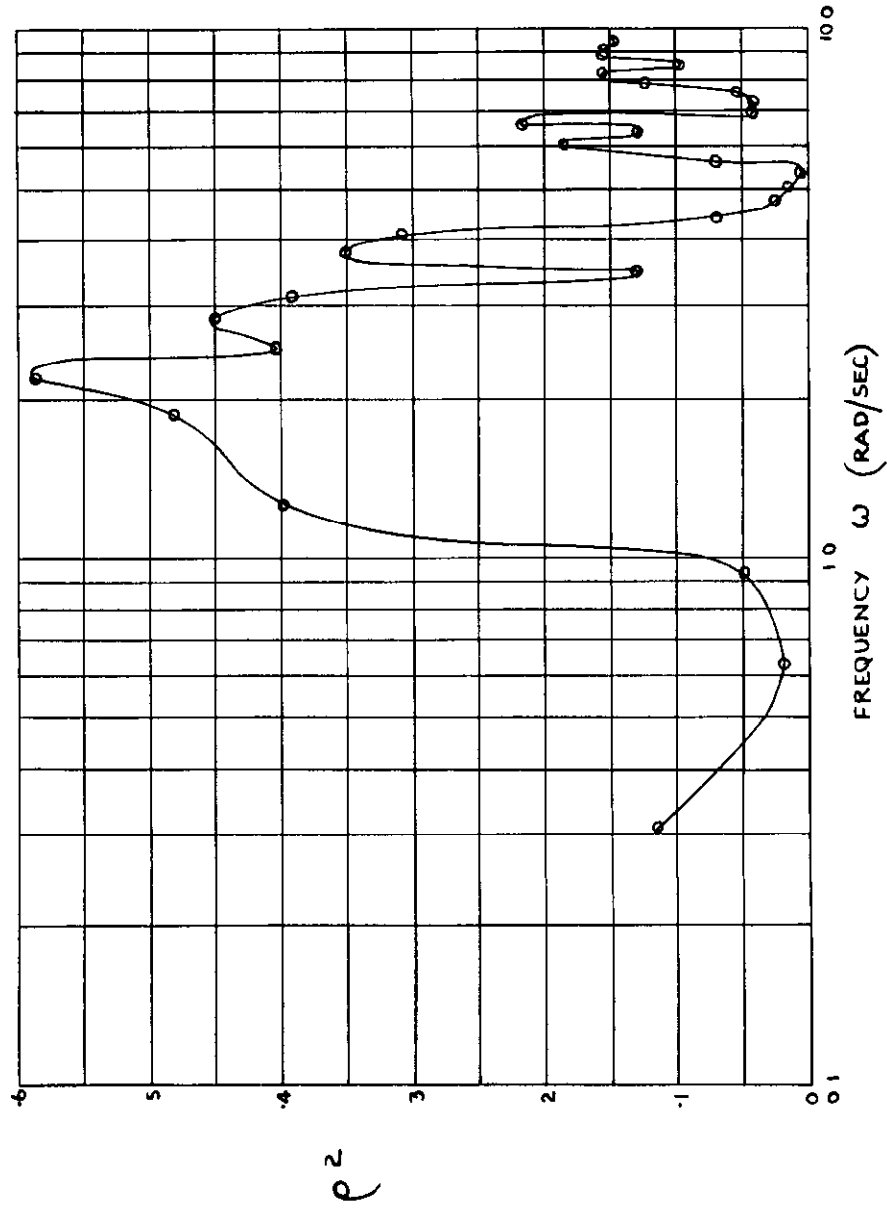
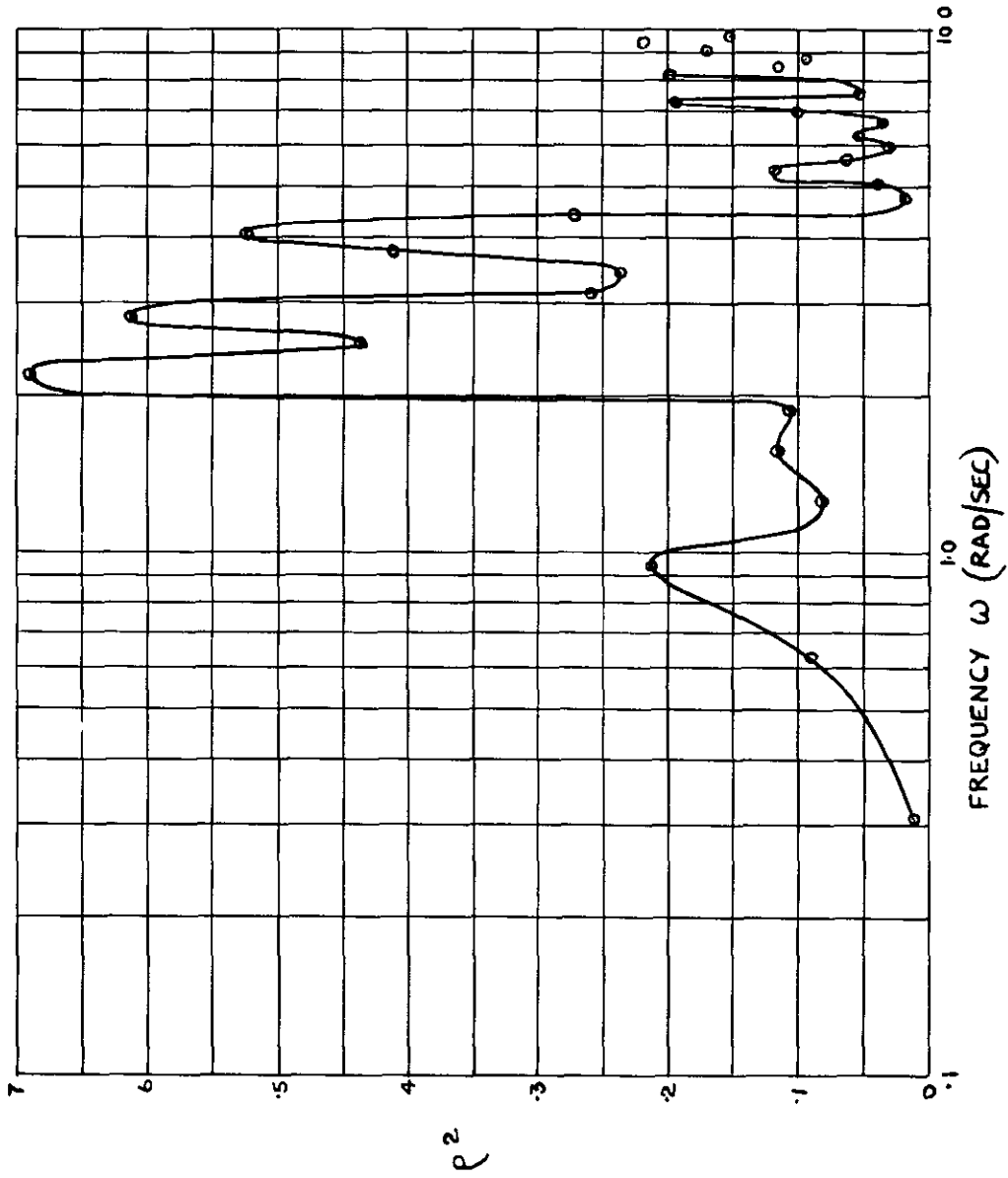


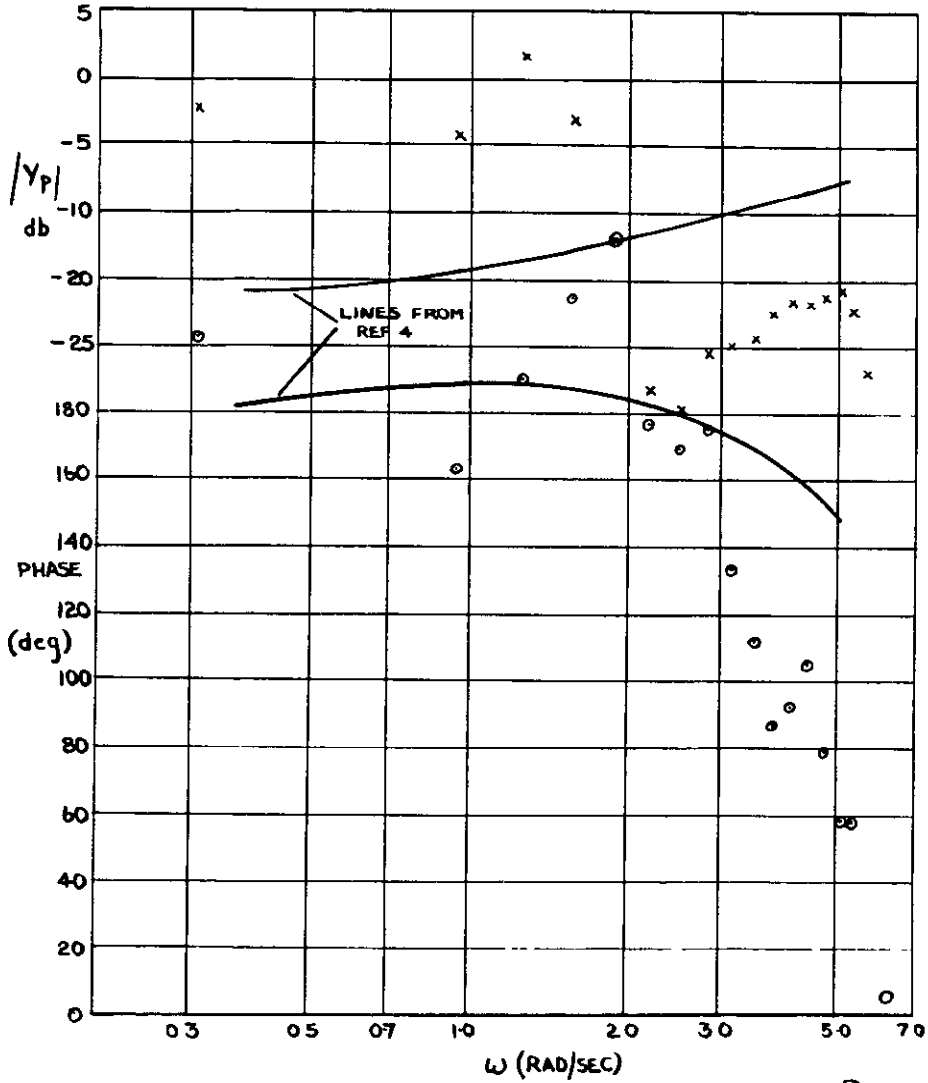
FIG 14

SAMPLE A2 100 LAGS COHERENCE<sup>2</sup> VS FREQUENCY

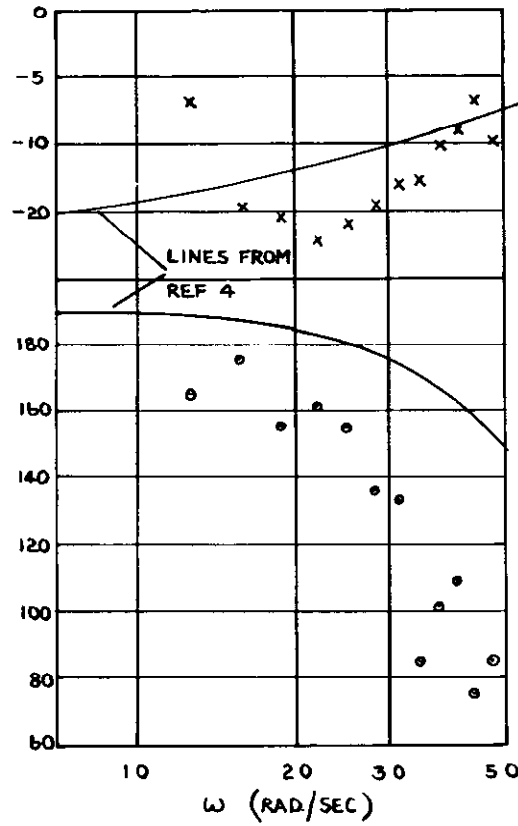
602 DATA POINTS



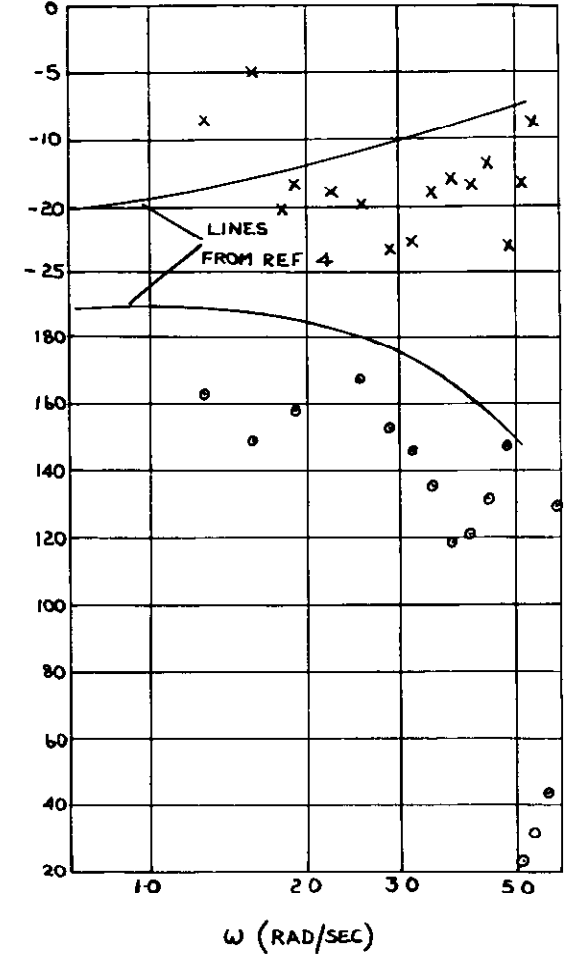
SAMPLE B1



SAMPLE A1



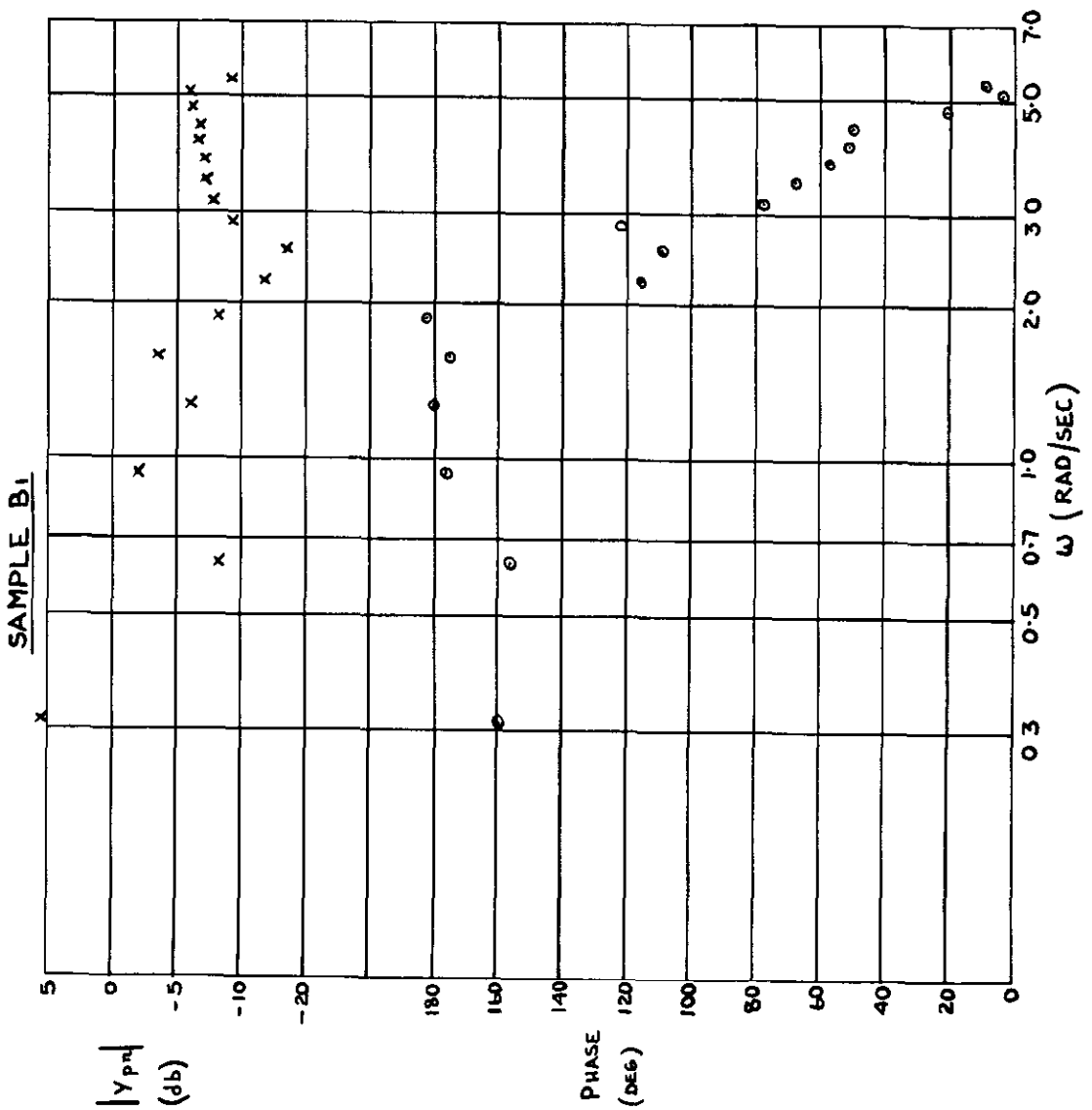
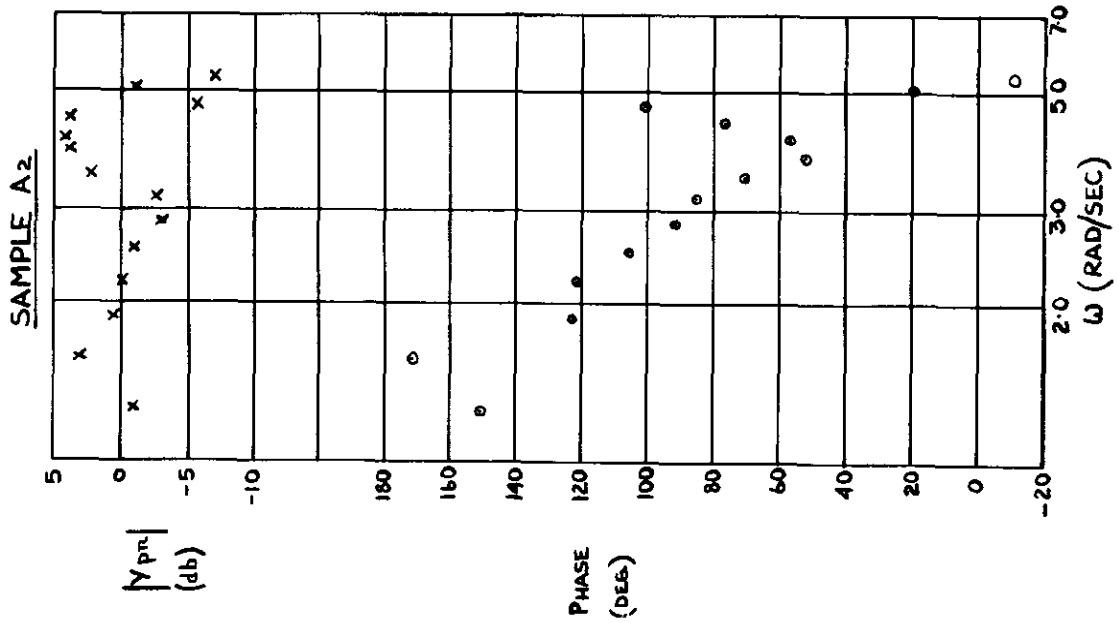
SAMPLE A2



PILOT'S RESPONSE TO PITCH RATE. ( $Y_{pq}$ )

FIG 16

PILOT'S RESPONSE TO NORMAL ACCELERATION. ( $Y_{pn}$ )



A.R.C. C.P.No.997

May, 1967

FURTHER ANALYSIS OF TSR 2 FLIGHTS THROUGH  
TURBULENCE (Report of work by British Aircraft  
Corp. under Min. of Technology contract)  
Saunders, T.B.

629.73 TSR 2  
533.6.013.43  
621-514  
KD/2L/28/CB 42(a)

Human pilot describing functions are derived from records of two flights through turbulence by the TSR 2 based on the assumption of the use of pitch rate cues and normal acceleration cues respectively. No indication can be gained of the validity of these assumptions. Small, low frequency errors in pitch rate introduced during trace readings were magnified by subsequent integration and produced large errors in the time histories of vertical gust velocity at very low frequencies. This prevented the estimation of turbulence power spectra.

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