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By
A. ROBINSON, M.Sc., A.F.R.Ae.S.

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A. ROBINSON, M.SC., A.F.R.AE.S.

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Summary.—Lift, drag, and pressure distribution of a triangular flat plate moving at a small incidence at supersonic speeds are given for arbitrary Mach number and aspect ratio. The values obtained for lift and drag are compared with the corresponding values obtained by strip theory. The possibility of further applications of the analysis leading up to the above results is indicated.

1. *General Discussion.*—1.1. *Introduction.*—The pressure distribution on a flat Delta wing (*i.e.*, an isosceles triangular flat plate having its apex pointed against the direction of flow) belongs to one of two different types according to whether the apex semi-angle of the triangle is (i) greater, or (ii) smaller than the given Mach angle. The difference between the two types of flow expresses itself not only in the final result but also in the fact that different methods are best suited for their analytical treatment.

The pressure distribution on a flat Delta wing whose apex semi-angle is larger than the given Mach angle (case (i)) was first calculated by Ward[3]. It was later obtained as a corollary to some work by the present author[2]. The total lift and drag of the aerofoil in that case are also given in R. & M. 2394[2].

The solution of the corresponding problem for a flat Delta wing whose apex semi-angle is smaller than the given Mach angle (case (ii)) has now been obtained by a method which is a counterpart of the treatment of Laplace's equation by systems of orthogonal co-ordinates. Results obtained for this case will be given together with the corresponding results for case (i) which are taken from the above-mentioned papers. These results—without the analysis leading up to them—have already been issued in a preliminary note[1].

1.2. *Notation.*—

ρ	air density
V	free-stream velocity
M	Mach number
μ	Mach angle
S	surface area of Delta wing
b	span

* R.A.E. Report Aero. 2151, received 31st December, 1946.

c	maximum chord
A	aspect ratio
γ	apex semi-angle
x	chordwise co-ordinate (measured from the apex against the direction of flow)
y	spanwise co-ordinate (measured from the centre line)
α	incidence (in radians)
$\overline{\Delta_p}$	pressure difference between top and bottom surfaces of the aerofoil
$l(y)$	spanwise loading
C_L	lift coefficient, based on surface area
C_{Di}	induced drag coefficient based on surface area
λ	$\cot \mu \,.\, \tan \gamma$

1.3. *Results*.—The pressure difference between top and bottom surfaces is given by

In case (i), *i.e.*, when $\gamma > \mu$:

(1, i)
$$\overline{\Delta_p} = \frac{2\rho V^2 \alpha}{\sqrt{(\cot^2 \mu - \cot^2 \gamma)}},$$

when $|x| < |y| \cot \mu$, *i.e.*, outside the Mach cone of the apex,

$$\overline{\Delta_p} = \frac{4\rho V^2 \alpha}{\pi \sqrt{(\cot^2 \mu - \cot^2 \gamma)}} \tan^{-1}\left(\frac{|x|}{\cot \gamma} \sqrt{\left[\frac{\cot^2 \mu - \cot^2 \gamma}{x^2 - y^2 \cot^2 \mu}\right]}\right),$$

when $|x| > |y| \cot \mu$, *i.e.*, inside the Mach cone of the apex,

and by

(1, ii)
$$\overline{\Delta_p} = \frac{2\rho V^2 \alpha \tan^2 \gamma}{E'(\cot \mu \tan \gamma)} \cdot \frac{|x|}{\sqrt{(x^2 \tan^2 \gamma - y^2)}}$$

in case (ii), *i.e.*, when $\gamma < \mu$.

In these formulæ ρ denotes the air density, V the free-stream velocity, α the incidence of the Delta wing in radians, and γ its apex semi-angle; μ is the Mach angle, $\cot \mu = \sqrt{(M^2 - 1)}$, where M is the Mach number, and E' is the elliptic integral defined by

$$E'(u) = \int_0^{\pi/2} \sqrt{\{1 - (1 - u^2) \sin^2 \phi\}}\, d\phi.$$

The chordwise co-ordinate x is measured from the apex against the direction of flow, and the spanwise co-ordinate y is measured from the centre-line of the aerofoil.

(The above formulæ are still valid if the trailing edge of the aerofoil is deformed in any way such that the Mach cones issuing from the trailing edge do not include any portion of the aerofoil.)

Let c be the maximum chord of the triangular aerofoil and b its span so that the surface area is given by $S = \frac{1}{2}bc$, and the aspect ratio by $A = b^2/S = 2b/c = 4 \tan \gamma$. Then the spanwise lift distribution $l(y)$ is given by

In case (i):

(2, i)
$$l(y) = \frac{2\rho V^2 \alpha (c - |y| \cot \gamma)}{\sqrt{(\cot^2 \mu - \cot^2 \gamma)}} \quad \text{when } c < |y| \cot \mu,$$

$$l(y) = \frac{2\rho V^2 \alpha}{\sqrt{\cot^2 \mu - \cot^2 \gamma}} \left[(c + y \cot \gamma) \tan^{-1} \sqrt{\left\{\frac{\cot \mu - \cot \gamma}{\cot \mu + \cot \gamma} \cdot \frac{c - y \cot \gamma}{c + y \cot \gamma}\right\}} + (c - y \cot \gamma) \tan^{-1} \sqrt{\frac{\cot \mu - \cot \gamma}{\cot \mu + \cot \gamma} \cdot \frac{c + y \cot \gamma}{c - y \cot \gamma}}\right],$$

when $c > |y| \cot \mu$

and by

$$(2,\ \text{ii})\quad l(y) = \frac{2\rho V^2 \alpha}{E'(\cot\gamma\,.\,\tan\gamma)} \sqrt{(c^2 \tan^2\gamma - y^2)} \quad \text{in case (ii)}\ .$$

The lift coefficient, based as usual on surface area, is given by

$$(3,\ \text{i})\qquad C_L = 4\alpha \tan\mu \qquad \text{in case (i), and by}$$

$$C_L = \frac{2\pi\alpha \tan\gamma}{E'(\cot\mu\,.\,\tan\gamma)} \quad \text{in case (ii)}\ .$$

The ratio of this coefficient and of the lift coefficient predicted by two-dimensional (" strip ") theory is shown in Fig. 1. It depends only on the parameter $\lambda = \cot\mu\,.\,\tan\gamma$. Fig. 2 gives C_L for various apex angles (or aspect ratios) plotted against Mach number. As mentioned in R. & M. 2394[2], C_L is equal to its value by strip-theory if $\gamma > \mu$ (case (i)).

A simple dimensional argument shows that the centre of pressure coincides with the centroid of the wing ($x_0 = -2c/3$, $y_0 = 0$).

It will be seen from formula (1, i) that in case (i), the pressure remains finite at the leading edge, and we may therefore assume that the resultant aerodynamic force is normal to the plate. This implies that the drag associated with the lift equals the product of lift and incidence (in radians). To avoid some of the confusion which has arisen in this connection we shall agree to call the whole of this drag " induced drag ". The corresponding coefficient C_{Di} will again be based on surface area. In case (ii) formula (1, ii) shows that, at least according to linear theory, there will be infinite suction at the leading edge, as in subsonic flow, and of the same order of infinity. This indicates the presence of a longitudinal " suction force " which tends to reduce the induced drag. As a result, the induced drag no longer equals the product of lift and incidence.

As formulæ (2, i) and (2, ii) show, the spanwise lift distribution is of elliptic shape as long as $\gamma < \mu$, *i.e.*, in case (ii) but not in case (i). The value of C_{Di} for a given elliptic lift distribution under low-speed conditions is known to be $C_L{}^2/\pi A$, so that the value of $C_{Di}/(C_L{}^2/\pi A)$ measures the deviation of the high-speed régime from the low-speed régime, at least for $\gamma < \mu$. This ratio which again depends only on the parameter $\lambda = \cot\mu \tan\gamma$ is plotted against λ in Fig. 3, and for various apex angles or aspect ratios against Mach number in Fig. 4.

Analytically,

$$(4,\ \text{i})\quad \frac{C_{Di}}{C_L{}^2/\pi A} = \pi \cot\mu \tan\gamma \quad \text{in case (i), and}$$

$$(4,\ \text{ii})\quad \frac{C_{Di}}{C_L{}^2/\pi A} = 2E'(\cot\mu \tan\gamma) - \tan\gamma \sqrt{(\cot^2\gamma - \cot^2\mu)} \quad \text{in case (ii).}$$

For a given spanwise lift distribution, the trailing vortex field in regions far behind the aerofoil is the same in supersonic as in subsonic flow (compare Ref. 7). Accordingly we may subdivide the induced drag into vortex drag, which is associated with the trailing vortex field and is the same for supersonic as for subsonic flow, and induced wave drag, which is peculiar to supersonic flow. The corresponding " vortex drag coefficient " for a Delta wing equals $C_L{}^2/\pi A$ for $\gamma < \mu$. For $\gamma > \mu$ this coefficient increases, for a given lift coefficient, as the spanwise lift distribution curve deviates from the elliptic shape. Inspection of Fig. 3, then shows that even when $\gamma < \mu$ (case (ii)) there still is an induced wave drag in addition to the vortex drag. Thus, while this case shows some affinity with subsonic conditions, the flow is still not truly subsonic. However, as shown by formulæ (3, ii) and (4, ii), as the aspect ratio tends to 0, $C_L \simeq (\pi A/2)\,\alpha$, while $C_{Di} \simeq C_L{}^2/\pi A$,

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both of which are the values given by low-speed theory. This is in agreement with an argument due to R. T. Jones[8] which tends to show that for pointed wings of infinitely small aspect ratio lift and drag are given by the above formulæ in supersonic as in subsonic flow, and serves as a check on the results obtained here.

2. *Analysis.*—2.1. *Pseudo-orthogonal Co-ordinates.*—Let x_1, x_2, x_3 and y_1, y_2, y_3 be two sets of variables interconnected by the relations

$$\left.\begin{aligned} x_j &= f_j(y_1, y_2, y_3)\,, \quad j = 1, 2, 3\,, \\ y_j &= g_j(x_1, x_2, x_3)\,, \quad j = 1, 2, 3\,. \end{aligned}\right\} \qquad (1)$$

The transformation is supposed to be non-singular in a given region,

$$\left|\frac{\partial f_j}{\partial y_k}\right| \neq 0, \quad \left|\frac{\partial g_j}{\partial x_k}\right| \neq 0\,.$$

We have

$$dx_j = \sum_{k=1}^{3} \frac{\partial f_j}{\partial y_k}\, dy_k\,, \quad j = 1, 2, 3\,. \qquad (2)$$

Hence

$$dx_1^2 - dx_2^2 - dx_3^2 = h_1^2\, dy_1^2 - h_2^2\, dy_2^2 - h_3^2\, dy_3^2 + 2h_{12}\, dy_1\, dy_2 + 2h_{13}\, dy_1\, dy_3 + 2h_{23}\, dy_2\, dy_3\,, \qquad (3)$$

where

$$\left.\begin{aligned} h_1^2 &= \left(\frac{\partial f_1}{\partial y_1}\right)^2 - \left(\frac{\partial f_2}{\partial y_1}\right)^2 - \left(\frac{\partial f_3}{\partial y_1}\right)^2\,, \\ h_2^2 &= -\left(\frac{\partial f_1}{\partial y_2}\right)^2 + \left(\frac{\partial f_2}{\partial y_2}\right)^2 + \left(\frac{\partial f_3}{\partial y_2}\right)^2\,, \\ h_3^2 &= -\left(\frac{\partial f_1}{\partial y_3}\right)^2 + \left(\frac{\partial f_2}{\partial y_3}\right)^2 + \left(\frac{\partial f_3}{\partial y_3}\right)^2\,, \end{aligned}\right\} \qquad (4)$$

and

$$h_{jk} = \left(\frac{\partial f_1}{\partial y_j}\right)\left(\frac{\partial f_1}{\partial y_k}\right) - \left(\frac{\partial f_2}{\partial y_j}\right)\left(\frac{\partial f_2}{\partial y_k}\right) - \left(\frac{\partial f_3}{\partial y_j}\right)\left(\frac{\partial f_3}{\partial y_k}\right) \quad j, k = 1, 2, 3, \quad j \neq k\,.$$

Now assume that the functions h_{jk} vanish identically. In that case

$$dx_1^2 - dx_2^2 - dx_3^2 = h_1^2\, dy_1^2 - h_2^2\, dy_2^2 - h_3^2\, dy_3^2\,. \qquad (5)$$

If x_1, x_2, x_3, are rectangular cartesian co-ordinates in three-dimensional space, and another set of co-ordinates y_1, y_2, y_3 is given, such that the functions h_{jk} vanish identically, then y_1, y_2, y_3 will be said to be pseudo-orthogonal co-ordinates in the given space. As a simple example of a system of curvilinear pseudo-orthogonal co-ordinates, we may mention the pseudo-orthogonal counterpart of the familiar spherical co-ordinates. It is given by

$$x_1 = y_1 \cosh y_2, \quad x_2 = y_1 \sinh y_2 \cos y_3, \quad x_3 = y_1 \sinh y_2 \sin y_3.$$

We shall require an expression for the differential parameter $\frac{\partial^2\phi}{\partial x_1^2} - \frac{\partial^2\phi}{\partial x_2^2} - \frac{\partial^2\phi}{\partial x_3^2}$ in terms of general pseudo-orthogonal co-ordinates, where ϕ is an arbitrary scalar function. It is shown in Appendix I that

$$\frac{\partial^2\phi}{\partial x_1^2} - \frac{\partial^2\phi}{\partial x_2^2} - \frac{\partial^2\phi}{\partial x_3^2} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial y_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial\phi}{\partial y_1}\right) - \frac{\partial}{\partial y_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial\phi}{\partial y_2}\right) - \frac{\partial}{\partial y_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial\phi}{y_3}\right)\right]. \qquad (6)$$

2.2. *Hyperboloido-conal Co-ordinates.*—The solution of problems connected with triangular aerofoils moving at supersonic speeds can be effected by the introduction of a special system of pseudo-orthogonal co-ordinates. Writing x', y', z' and r, μ, ν for x_1, x_2, x_3, and y_1, y_2, y_3, respectively, the connection between the rectangular cartesian co-ordinates x', y', z', and the special system to be introduced, r, μ, ν, will be given by

$$x' = r\frac{\mu\nu}{hk}\ , \quad y' = r\,\frac{\sqrt{(\mu^2 - h^2)}\,\sqrt{(\nu^2 - h^2)}}{h\sqrt{(k^2 - h^2)}}\ , \quad z' = r\,\frac{\sqrt{(\mu^2 - k^2)}\,\sqrt{(k^2 - \nu^2)}}{k\sqrt{(k^2 - h^2)}}\ , \qquad \ldots \qquad (7)$$

where k and h are positive constants, $k > h$. The intervals of variation of r, μ, ν will be taken as

$$0 \leqslant r < \infty\ ,\ k \leqslant \mu < \infty\ ,\ h \leqslant \nu \leqslant k\ .$$

Eliminating μ and ν from (7) we obtain a family of surfaces with r as parameter,

$$x'^2 - y'^2 - z'^2 = r^2. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

Similarly, eliminating r and ν, and r and μ, respectively, we obtain two more families of surfaces,

$$\frac{x'^2}{\mu^2} - \frac{y'^2}{\mu^2 - h^2} - \frac{z'^2}{\mu^2 - k^2} = 0\ , \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

and

$$\frac{x'^2}{\nu^2} - \frac{y'^2}{\nu^2 - h^2} + \frac{z'^2}{k^2 - \nu^2} = 0\ . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

(8) represents a family of hyperboloids of two sheets while (9) and (10) are families of cones. This justifies the name " hyperboloido-conal co-ordinates " for the system under consideration. They are the pseudo-orthogonal counterpart of the system of orthogonal co-ordinates known as " sphero-conal co-ordinates "[4].

Equation (7) shows that for the specified interval of variation, the co-ordinates r, μ, ν can only represent points inside the positive half of the cone $x'^2 - y'^2 - z'^2 = 0$ (*i.e.*, $x' > 0$, $x'^2 - y'^2 - z'^2 > 0$). By solving (7) for r, μ, ν it is found that to every point satisfying $x' > 0$, $x'^2 - y'^2 - z'^2 > 0$ there corresponds exactly one triplet r, μ, ν inside the domain of variation of these variables. On the other hand, to each triplet r, μ, ν there correspond four points x', y', z', according to the determination of the square roots in (7). The ambiguity can be avoided by writing μ and ν as elliptic functions of new variables, but this procedure will not be required in the present report.

For $\mu \to \infty$, the cones of the family (9) tend to approximate the cone $x'^2 - y'^2 - z'^2 = 0$, while for $\mu \to k$ they tend to become equal to the (two-sided) angular region in the x', y' plane given by $\dfrac{x'^2}{k^2} - \dfrac{y'^2}{k^2 - h^2} > 0$. On the other hand, the cones of (10) approximate the complementary angular region in the x', y'-plane $\left(\dfrac{x'^2}{k} - \dfrac{y'^2}{k^2 - h^2} < 0\right)$ as $\nu \to k$, and the y-axis ($x' = 0$, $z' = 0$) as $\nu \to h$. Thus, the intersections of the μ-cones with the plane $x' = 1$ are ellipses, varying between the circle $y'^2 + z'^2 = 1$ and the slit $z' = 0, y'^2 < 1 - \dfrac{h^2}{k^2}$. The intersections of the ν-cones with the same plane are hyperbolae (Fig. 7).

We shall now calculate the quantities h_1, h_2, h_3, h_{12}, h_{13}, h_{23} defined in section 2.1 above. We have

$$\frac{\partial x'}{\partial r} = \frac{\mu\nu}{hk}\ , \qquad \frac{\partial x'}{\partial \mu} = r\,\frac{\nu}{hk}\ , \qquad \frac{\partial x'}{\partial \nu} = r\,\frac{\mu}{hk}\ , \qquad \ldots \quad \ldots \qquad (11)$$

$$\frac{\partial y'}{\partial r} = \frac{\sqrt{(\mu^2 - h^2)}\sqrt{(\nu^2 - h^2)}}{h\sqrt{(k^2 - h^2)}}\ , \quad \frac{\partial y'}{\partial \mu} = r\,\frac{\mu}{h\sqrt{(k^2 - h^2)}}\cdot\frac{\sqrt{(\nu^2 - h^2)}}{\sqrt{(\mu^2 - h^2)}}\ , \quad \frac{\partial y'}{\partial \nu} = r\,\frac{\nu}{h\sqrt{(k^2 - h^2)}}\cdot\frac{\sqrt{(\mu^2 - h^2)}}{\sqrt{(\nu^2 - h^2)}}\ ,$$

$$\frac{\partial z'}{\partial r} = \frac{\sqrt{(\mu^2 - k^2)}\sqrt{(k^2 - \nu^2)}}{k\sqrt{(k^2 - h^2)}}\ , \quad \frac{\partial z'}{\partial \mu} = r\,\frac{\mu}{k\sqrt{(k^2 - h^2)}}\cdot\frac{\sqrt{(k^2 - \nu^2)}}{\sqrt{(\mu^2 - k^2)}}\ , \quad \frac{\partial z'}{\partial \nu} = r\,\frac{\nu}{k\sqrt{(k^2 - h^2)}}\cdot\frac{\sqrt{(\mu^2 - k^2)}}{\sqrt{(k^2 - \nu^2)}}\ .$$

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Hence

$$\left.\begin{aligned}
h_1^2 &= \left(\frac{\partial x'}{\partial r}\right)^2 - \left(\frac{\partial y'}{\partial r}\right)^2 - \left(\frac{\partial z'}{\partial r}\right)^2 = 1\,,\\
h_2^2 &= -\left(\frac{\partial x'}{\partial \mu}\right)^2 + \left(\frac{\partial y'}{\partial \mu}\right)^2 + \left(\frac{\partial z'}{\partial \mu}\right)^2 = \frac{r^2(\mu^2-\nu^2)}{(\mu^2-h^2)(\mu^2-k^2)}\,,\\
h_3^2 &= -\left(\frac{\partial x'}{\partial \nu}\right)^2 + \left(\frac{\partial y'}{\partial \nu}\right)^2 + \left(\frac{\partial z'}{\partial \nu}\right)^2 = \frac{r^2(\mu^2-\nu^2)}{(\nu^2-h^2)(k^2-\nu^2)}\,,\\
h_{12} &= \left(\frac{\partial x'}{\partial r}\right)\left(\frac{\partial x'}{\partial \mu}\right) - \left(\frac{\partial y'}{\partial r}\right)\left(\frac{\partial y'}{\partial \mu}\right) - \left(\frac{\partial z'}{\partial r}\right)\left(\frac{\partial z'}{\partial \mu}\right) = 0\,,\\
h_{13} &= \left(\frac{\partial x'}{\partial r}\right)\left(\frac{\partial x'}{\partial \nu}\right) - \left(\frac{\partial y'}{\partial r}\right)\left(\frac{\partial y'}{\partial \nu}\right) - \left(\frac{\partial z'}{\partial r}\right)\left(\frac{\partial z'}{\partial \nu}\right) = 0\,,\\
h_{23} &= \left(\frac{\partial x'}{\partial \mu}\right)\left(\frac{\partial x'}{\partial \nu}\right) - \left(\frac{\partial y'}{\partial \mu}\right)\left(\frac{\partial y'}{\partial \nu}\right) - \left(\frac{\partial z'}{\partial \mu}\right)\left(\frac{\partial z'}{\partial \nu}\right) = 0\,.
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (12)$$

The last three equations show that the system of co-ordinates r, μ, ν is in fact pseudo-orthogonal, as asserted.

Let Φ be an arbitrary scalar function. Then, by (6),

$$\frac{\partial^2\Phi}{\partial x'^2} - \frac{\partial^2\Phi}{\partial y'^2} - \frac{\partial^2\Phi}{\partial z'^2} = \frac{\sqrt{\{(\mu^2-h^2)(\nu^2-h^2)(\mu^2-k^2)(k^2-\nu^2)\}}}{r^2(\mu^2-\nu^2)}\,.$$
$$\times\left[\frac{\partial}{\partial r}\left(\frac{r^2(\mu^2-\nu^2)}{\sqrt{\{(\mu^2-h^2)(\nu^2-h^2)(\mu^2-k^2)(k^2-\nu^2)\}}}\frac{\partial\Phi}{\partial r}\right)\right.$$
$$\left. - \frac{\partial}{\partial\mu}\left(\sqrt{\left\{\frac{(\mu^2-h^2)(\mu^2-k^2)}{(\nu^2-h^2)(k^2-\nu^2)}\right\}}\frac{\partial\Phi}{\partial\mu}\right) - \frac{\partial}{\partial\nu}\left(\sqrt{\left\{\frac{(\nu^2-h^2)(k^2-\nu^2)}{(\mu^2-h^2)(\mu^2-k^2)}\right\}}\frac{\partial\Phi}{\partial\nu}\right)\right].$$

Or

$$\frac{\partial^2\Phi}{\partial x'^2} - \frac{\partial^2\Phi}{\partial y'^2} - \frac{\partial^2\Phi}{\partial z'^2} = \frac{1}{r^2(\mu^2-\nu^2)}\left[(\mu^2-\nu^2)\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)\right.$$
$$- \sqrt{(\mu^2-h^2)(\mu^2-k^2)}\frac{\partial}{\partial\mu}\left(\sqrt{(\mu^2-h^2)(\mu^2-k^2)}\frac{\partial\Phi}{\partial\mu}\right)$$
$$\left. - \sqrt{(\nu^2-h^2)(k^2-\nu^2)}\frac{\partial}{\partial\nu}\left(\sqrt{(\nu^2-h^2)(k^2-\nu^2)}\frac{\partial\Phi}{\partial\nu}\right)\right]. \quad \ldots \quad \ldots \quad (13)$$

A scalar function Φ which satisfies the equation

$$\frac{\partial^2\Phi}{\partial x'^2} - \frac{\partial^2\Phi}{\partial y'^2} - \frac{\partial^2\Phi}{\partial z'^2} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

will be called a hyperbolic potential function (or, alternatively, a pseudo-harmonic). Equation (14) is equivalent to

$$(\mu^2-\nu^2)\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) - \sqrt{(\mu^2-h^2)(\mu^2-k^2)}\frac{\partial}{\partial\mu}\left(\sqrt{(\mu^2-h^2)(\mu^2-k^2)}\frac{\partial\Phi}{\partial\mu}\right)$$
$$- \sqrt{(\nu^2-h^2)(k^2-\nu^2)}\frac{\partial}{\partial\nu}\left(\sqrt{(\nu^2-h^2)(k^2-\nu^2)}\frac{\partial\Phi}{\partial\nu}\right) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

in hyperboloido-conal co-ordinates.

2.3. *The Triangular Aerofoil.*—Consider a triangular aerofoil of span b and maximum chord length c in a uniform supersonic airstream (Fig. 5). The linearised equation of stationary supersonic flow is (*e.g.*, Ref. 7)

$$n^2 \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial z^2} = 0 \; , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

where $n^2 = M^2 - 1$, $M = V/a$, M being the Mach number, V the free-stream velocity, and a the velocity of sound; x is the longitudinal co-ordinate, measured from the apex of the aerofoil against the direction of flow, y the lateral co-ordinate, positive to starboard and negative to port, and z the vertical co-ordinate, positive downward. Φ is the induced velocity potential so that the three velocity components are given by

$$-V + \frac{\partial \Phi}{\partial x} \; , \; \frac{\partial \Phi}{\partial y} \; , \text{ and } \frac{\partial \Phi}{\partial z} \quad \text{respectively.}$$

In accordance with the conventions of linearised theory, the incidence of the streamlines at the aerofoil is estimated at the vertical projection of the aerofoil, into the x-y plane, thus

$$s = \frac{1}{V} \left(\frac{\partial \Phi}{\partial z} \right)_{z=0} \; , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

where s is the slope of the aerofoil at the point in question, on the upper or lower surface, as the case may be.

Φ must be continuous everywhere except possibly across the wake of the aerofoil. In the present analysis we assume that the aerofoil is completely inside the Mach cone issuing from the apex, so that Φ must be a constant, and may be assumed to vanish, outside the cone. In particular, this yields the condition

$$\Phi = 0 \quad \text{for} \quad x^2 - n^2(y^2 + z^2) = 0 \; . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

The assumption that the aerofoil is inside the Mach cone issuing from the apex means, in symbols,

$$n = \cot \mu < \frac{2c}{b} = \cot \gamma \; , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

where γ is the apex semi-angle of the aerofoil, and μ is the Mach angle.

The longitudinal components of induced velocity is $\partial \Phi / \partial x$, hence, by the linearised Bernouilli equation,

$$p = p_\infty - \rho V \frac{\partial \Phi}{\partial x} \; , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where p is the pressure at the point in question, p_∞ the free-stream pressure, and ρ the air density. The excess pressure $\overline{\Delta p}$ is therefore given by

$$\overline{\Delta p} = -\rho V \frac{\partial \Phi}{\partial x} \; . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

2.4. *Transformation into Hyperboloido-conal Co-ordinates.*—Put $x = -nx'$, $y = y'$, $z = z'$. Expressing equation (10) in terms of x', y', z', we then obtain equation (14). The span of the triangle remains unaltered in the transformation, while the chord is magnified in the ratio of 1 : n. The Mach cone $x^2 - n^2(y^2 + z^2) = 0$ is transformed into $x'^2 - y'^2 - z'^2 = 0$.

Next, transform into hyperboloido-conal co-ordinates, as by (7), with $k = \cot \gamma$, $h = \sqrt{(\cot^2 \gamma - \cot^2 \mu)}$. For these constants, the leading edges of the aerofoil determine the angular region in the x'-y' plane to which the cones of the family (9) approximate as $\mu \rightarrow k$. The triangle itself becomes part of that region.

In order to express the derivatives $\partial f/\partial x$ and $\partial f/\partial z$ of an arbitrary function f in terms of hyperboloido-conal co-ordinates, we first have to calculate the derivatives of r, μ, ν with respect x' and z' in terms of r, μ, and ν. The calculation of these quantities is simplified by the fact that we are dealing with pseudo-orthogonal co-ordinates (*see* Appendix II). Using equations (11) and (12) we find

$$\left.\begin{aligned}
\frac{\partial r}{\partial x'} &= \frac{\mu\nu}{hk}, \qquad \frac{\partial \mu}{\partial x'} = -\frac{\nu(\mu^2-h^2)(\mu^2-k^2)}{hkr(\mu^2-\nu^2)}, \\
\frac{\partial \nu}{\partial x'} &= -\frac{\mu(\nu^2-h^2)(k^2-\nu^2)}{hkr(\mu^2-\nu^2)}, \\
\frac{\partial r}{\partial z'} &= -\frac{\sqrt{(\mu^2-k^2)}\sqrt{(k^2-\nu^2)}}{k\sqrt{(k^2-h^2)}}, \qquad \frac{\partial \mu}{\partial z'} = \frac{\mu(\mu^2-h^2)\sqrt{(\mu^2-k^2)}\sqrt{(k^2-\nu^2)}}{k\sqrt{(k^2-h^2)}\,r(\mu^2-\nu^2)}, \\
\frac{\partial \nu}{\partial z'} &= -\frac{\nu(\nu^2-h^2)\sqrt{(\mu^2-k^2)}\sqrt{(k^2-\nu^2)}}{k\sqrt{(k^2-h^2)}\,r(\mu^2-\nu^2)}.
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (22)$$

Hence

$$\frac{\partial f}{\partial x} = \frac{-1}{nhk}\left[\mu\nu\frac{\partial f}{\partial r} - \frac{\nu(\mu^2-h^2)(\mu^2-k^2)}{r(\mu^2-\nu^2)}\cdot\frac{\partial f}{\partial \mu} - \frac{\mu(\nu^2-h^2)(k^2-\nu^2)}{r(\mu^2-\nu^2)}\cdot\frac{\partial f}{\partial \nu}\right], \quad \ldots \quad \ldots \quad (23)$$

and

$$\frac{\partial f}{\partial z} = \frac{\sqrt{(\mu^2-k^2)}\sqrt{(k^2-\nu^2)}}{k\sqrt{(k^2-h^2)}}\left[-\frac{\partial f}{\partial r} + \frac{\mu(\mu^2-h^2)}{r(\mu^2-\nu^2)}\cdot\frac{\partial f}{\partial \mu} - \frac{\nu(\nu^2-h^2)}{r(\mu^2-\nu^2)}\cdot\frac{\partial f}{\partial \nu}\right]. \quad \ldots \quad \ldots \quad (24)$$

If the induced velocity potential is given as a function of r, μ, and ν, then the excess pressure at any point will be found from (21) and (23). Also, the corresponding shape of the aerofoil will be found from (17) and (24). It is to be noted that the differentiation, as by (24), has to precede the passage to the limit $\mu \to k$. The induced velocity potential, apart from being continuous except possibly across the wake of the aerofoil also has to satisfy equation (15), which is the equivalent of equations (14) and (16) in hyperboloido-conal co-ordinates. Particular solutions of (15) can be obtained by separation of the co-ordinates. This leads to Lamé functions of all kinds and degrees (Appendix IV). To each such solution there corresponds a possible aerofoil whose shape can be calculated, as detailed above. It should be observed that as long as we confine our attention to the region ahead of the trailing edge, conditions behind the trailing edge do not affect the results, so that we may modify the boundary conditions there at our convenience.

In the present paper we shall only consider the special function which corresponds to the flow round a triangular flat plate at incidence.

2.5. *The Flat Delta Wing at Incidence.*—Let α be the incidence of the aerofoil; α is supposed to be small, so that $\tan\alpha \simeq \alpha$. Equation (17) then becomes

$$\alpha = \frac{1}{V}\left(\frac{\partial \Phi}{\partial z}\right)_{z=0}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

According to what has been said in section 2.4 we may assume (25) to hold not only at the aerofoil, but also aft of its trailing edge, between the two straight lines through the leading edges. As there is now no definite length involved in the specification of the boundary conditions, it follows from geometrical considerations that the induced velocity potential Φ must be of the form $r\Phi$, where Φ is a function of μ and ν only. In particular we shall try to find a solution of the form

$$\Phi = r\,\frac{\sqrt{(\mu^2-k^2)}\sqrt{(k^2-\nu^2)}}{k\sqrt{(k^2-h^2)}} = z'g(\mu) = zg(\mu), \quad \ldots \quad \ldots \quad (26)$$

where g is a function of μ yet to be determined. The reason for choosing Φ in this way is that at the aerofoil $\mu = k$, while $z = 0$, so that $\partial\Phi/\partial z$ is constant at the aerofoil, as required. Substituting $\Phi = z'g(\mu)$ in (15) we obtain an ordinary differential equation for $g(\mu)$. In performing the substitution it is useful to remember that $z' = r\,\dfrac{\sqrt{(\mu^2 - k^2)}\,\sqrt{(k^2 - \nu^2)}}{k\sqrt{(k^2 - h^2)}}$ is a solution of (14) and therefore of (15).

We obtain

$$z' \,.\, \sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\,\frac{d}{d\mu}\left(\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\,\frac{d}{d\mu}\right) + 2(\mu^2 - h^2)(\mu^2 - k^2)\,\frac{\partial z'}{\partial \mu}\,.\,\frac{dg}{\partial \mu} = 0\,,$$

or

$$(\mu^2 - k^2)\,\frac{d}{d\mu}\left(\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\,\frac{dg}{d\mu}\right) + 2\mu\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\,\frac{dg}{d\mu} = 0\,. \quad .. \quad (27)$$

Hence

$$(\mu^2 - k^2)\,\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\,\frac{dg}{d\mu} = \text{const.} = C\,,$$

and

$$g(\mu) = C\int_\mu^{\mu_0} \frac{dt}{(t^2 - k^2)\,\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}}\,.$$

In order to ensure that $\Phi \to 0$ as $\mu \to \infty$ for any given r and ν, as required by continuity, we have to take $\mu_0 = \infty$. Hence

$$g(\mu) = C\int_\mu^{\infty} \frac{dt}{(t^2 - k^2)\,\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}}\,, \quad .. \quad .. \quad .. \quad (28)$$

and

$$\Phi = \frac{Cr\,\sqrt{(k^2 - \nu^2)}}{k\,\sqrt{(k^2 - h^2)}}\,.\,\sqrt{(\mu^2 - k^2)}\int_\mu^{\infty} \frac{dt}{(t^2 - k^2)\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}}\,. \quad .. \quad (29)$$

At the aerofoil (*i.e.*, for $\mu = k$) we have $x' = r\nu/h$, $y' = r\sqrt{(\nu^2 - h^2)}/h$, $x'^2 - y'^2 = r^2$, and so $r\sqrt{(k^2 - \nu^2)} = \sqrt{\{x'^2(k^2 - h^2) - y'^2k^2\}}$. Also, by partial integration

$$\lim_{\mu \to k}\left\{\sqrt{(\mu^2 - k^2)}\int_\mu^{\infty} \frac{dt}{(t^2 - k^2)\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}}\right\}$$

$$= \lim_{\mu \to k}\,\sqrt{(\mu^2 - k^2)}\left\{\left[-\,\frac{1}{\sqrt{(t^2 - k^2)}\,.\,t\sqrt{(t^2 - h^2)}}\right]_\mu^{\infty} + \int_\mu^{\infty} \frac{d}{dt}\left(\frac{1}{t\sqrt{(t^2 - h^2)}}\right)\frac{dt}{\sqrt{(t^2 - k^2)}}\right\}$$

$$= \lim_{\mu \to k}\,\frac{\sqrt{(\mu^2 - k^2)}}{\sqrt{(\mu^2 - k^2)}\,.\,\mu\sqrt{(\mu^2 - h^2)}} = \frac{1}{k\sqrt{(k^2 - h^2)}}\,.$$

Hence, at the aerofoil (but not elsewhere in the x-y plane)

$$\Phi = C\,\frac{\sqrt{\left\{\dfrac{k^2 - h^2}{n^2}\,x^2 - k^2y^2\right\}}}{k^2(k^2 - h^2)}\,. \quad .. \quad .. \quad .. \quad (30)$$

The value of the constant C can be determined by means of equation (25). We have, using (24)

$$\frac{\partial\Phi}{\partial z} = \frac{\partial}{\partial z}(zg(\mu)) = g(\mu) + z\frac{\partial g}{\partial z}$$

$$= g(\mu) + \frac{\sqrt{(\mu^2 - k^2)}\sqrt{(k^2 - \nu^2)}}{k\sqrt{(k^2 - h^2)}} \cdot \frac{\mu(\mu^2 - h^2)}{r(\mu^2 - \nu^2)} \cdot \frac{r\sqrt{(\mu^2 - k^2)}\sqrt{(k^2 - \nu^2)}}{k\sqrt{(k^2 - h^2)}} \cdot \frac{dg}{d\mu},$$

$$= C\left[\int_\mu^\infty \frac{dt}{(t^2 - k^2)\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}} - \frac{\mu}{k^2(k^2 - h^2)} \cdot \frac{k^2 - \nu^2}{\mu^2 - \nu^2} \cdot \sqrt{\left(\frac{\mu^2 - h^2}{\mu^2 - k^2}\right)}\right],$$

$$= C\left[\int_\mu^\infty \frac{dt}{(t^2 - k^2)\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}} - \frac{\mu}{k^2(k^2 - h^2)} \cdot \sqrt{\left(\frac{\mu^2 - h^2}{\mu^2 - k^2}\right)} + \frac{\mu\sqrt{(\mu^2 - k^2)}\sqrt{(\mu^2 - h^2)}}{k^2(k^2 - h^2)(\mu^2 - \nu^2)}\right].$$

As μ tends to k, $\frac{\mu\sqrt{(\mu^2 - k^2)}\sqrt{(\mu^2 - h^2)}}{k^2(k^2 - h^2)(\mu^2 - \nu^2)}$ tends to 0, so that $\partial\Phi/\partial z$ is, in fact, independent of ν and r at the aerofoil, as required. Also, as before,

$$\int_\mu^\infty \frac{dt}{(t^2 - k^2)\sqrt{(t^2 - h^2)(t^2 - k^2)}} - \frac{\mu}{k^2(k^2 - h^2)}\sqrt{\left(\frac{\mu^2 - h^2}{\mu^2 - k^2}\right)}$$

$$= \int_\mu^\infty \frac{d}{dt}\left(\frac{1}{t\sqrt{(t^2 - h^2)}}\right)\frac{dt}{\sqrt{(t^2 - k^2)}} + \frac{1}{\sqrt{(\mu^2 - k^2)}\,.\,\mu\sqrt{(\mu^2 - h^2)}} - \frac{\mu}{k^2(k^2 - h^2)}\sqrt{\left(\frac{\mu^2 - h^2}{\mu^2 - k^2}\right)}$$

by partial integration. Now

$$\lim_{\mu \to k}\left[\frac{1}{\sqrt{(\mu^2 - k^2)}\,.\,\mu\sqrt{(\mu^2 - k^2)}} \frac{\mu}{k^2(k^2 - h^2)}\sqrt{\left(\frac{\mu^2 - h^2}{\mu^2 - k^2}\right)}\right] =$$

$$\lim_{u \to k} \frac{k^2(k^2 - h^2) - \mu^2(\mu^2 - h^2)}{k^2(k^2 - h^2)\mu\sqrt{(\mu^2 - h^2)}\sqrt{(\mu^2 - k^2)}} = 0\,.$$

Hence, at the aerofoil

$$\frac{\partial\Phi}{\partial z} = C\int_k^\infty \frac{d}{dt}\left(\frac{1}{t\sqrt{(t^2 - h^2)}}\right)\frac{dt}{\sqrt{(t^2 - k^2)}}$$

where the integral on the right-hand side is now convergent.

The value of this integral is $\frac{-1}{k(k^2 - h^2)}\; E\left(\frac{h}{k}\right)$, where $E(u)$ is the complete elliptic integral of the second kind, $E(u) = \int_0^{\pi/2}\sqrt{\{1 - u^2\sin^2\phi\}}\,d\phi$. Hence, by (25), $C = -\frac{V\alpha k(k^2 - h^2)}{E\left(\frac{h}{k}\right)}$.

Equation (30) now becomes

$$\Phi = -\frac{V\alpha}{kE\left(\frac{h}{k}\right)}\sqrt{\left(\frac{k^2 - h^2}{n^2}x^2 - k^2y^2\right)}.$$

Recalling that $n = \cot\mu$, $k = \cot\gamma$, $h = \sqrt{(\cot^2\gamma - \cot^2\mu)}$, we finally obtain

$$\Phi = -\frac{V\alpha}{E'(\cot\mu\,.\,\tan\gamma)}\sqrt{(x^2\tan^2\gamma - y^2)}\,, \qquad \ldots \quad \ldots \quad (31)$$

where $E'(u)$ is the complementary complete elliptic integral of the second kind, $E'(u) = \int_0^{\pi/2} \sqrt{\{1 - (1 - u^2) \sin^2 \phi\}}\, d\phi$. According to our choice of co-ordinates, the sign of the square root is to be taken as positive at the top surface of the aerofoil, and as negative at the bottom surface. The magnitude of the pressure difference between top and bottom surfaces is, by (21), and (31)

$$\bar{\Delta p} = \frac{2\rho V^2 \alpha \tan^2 \gamma}{E'(\cot \mu \,.\, \tan \gamma)} \cdot \left| \frac{x}{\sqrt{(x^2 \tan^2 \gamma - y^2)}} \right| . \qquad \ldots \qquad \ldots \qquad (32)$$

2.6. *Lift and Drag.*—The spanwise lift distribution $l(y)$ is obtained by integrating the pressure difference $\bar{\Delta p}$ along the chords of the aerofoil. It will be seen that Φ vanishes at the leading edge, and so $l(y) = \int \bar{\Delta p}\, dx = 2|\Phi(c_0, y)|$, or

$$l(y) = \frac{2\rho V^2 \alpha}{E'(\cot \mu \,.\, \tan \gamma)} \sqrt{(c^2 \tan^2 \gamma - y^2)} . \qquad \ldots \qquad \ldots \qquad \ldots \qquad (33)$$

The total lift L on the aerofoil is obtained by integrating $l(y)$ along the span, from $-\frac{b}{2} = -c \tan \gamma$ to $\frac{b}{2} = c \tan \gamma$,

$$L = \frac{\pi \rho V^2 \alpha c^2 \tan^2 \gamma}{E'(\cot \mu \,.\, \tan \gamma)} . \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (34)$$

The lift coefficient C_L, based on surface area, is given by $C_L = \frac{L}{\frac{1}{2}\rho V^2 S}$, $S = c^2 \tan \gamma$, or

$$C_L = \frac{2\pi \alpha \tan \gamma}{E'(\cot \mu \,.\, \tan \gamma)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (35)$$

The longitudinal component of the pressure integral is given by

$$D_p = L\alpha = \frac{\pi \rho V^2 \alpha^2 c^2 \tan^2 \gamma}{E'(\cot \mu \,.\, \tan \gamma)} . \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (36)$$

However, the induced drag D_i (defined as the total drag associated with the lift, section 1.3) will not in general be equal to D_p but will be rather smaller than that quantity. In fact, by equations (31) and (32), the longitudinal component of induced velocity, and hence the pressure difference, both tend to infinity at the leading edge. As in subsonic flow, (compare Ref. 5), this indicates the presence of a suction force whose longitudinal component D_S acts in a direction opposite to that of the longitudinal component of the pressure integral, D_p. It follows from the nature of this force that the contribution to it of any particular element of the leading edge depends only on the local conditions (*e.g.*, the local trend to infinity of $\partial\Phi/\partial x$) and not on conditions elsewhere in the field.

Let dl be an element of the leading edge of the aerofoil, and assume dl to be yawed at an angle β (Fig. 6). Let x_0, y_0 be the co-ordinates of the midpoint of dl, and dy the length of its lateral projection. Assume that on approaching dl longitudinally against the direction of flow, $\partial\Phi/\partial x$ is given by

$$\frac{\partial \Phi}{\partial x} = \frac{\bar{C}}{\sqrt{x_0 - x}} + \ldots \text{ (finite terms)}$$

on the upper surface. It is then shown in Appendix III that the longitudinal component dD_S of the suction force contributed by dl equals

$$dD_S = \bar{C}^2 \pi \rho \sqrt{(\tan^2 \beta - \cot^2 \mu)}\, dy , \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (37)$$

where μ is the Mach angle, as before. The total suction force is then obtained by integrating dD_S across the span of the aerofoil. For the Delta wing,

$$\bar{C} = \frac{V\alpha\sqrt{(y_0 \tan\gamma)}}{\sqrt{2}E'(\cot\mu\,.\,\tan\gamma)}\,, \qquad \beta = \pm\left(\frac{\pi}{2} - \gamma\right).$$

Hence

$$D_S = \frac{\pi\rho V^2\alpha^2c^2 \tan^3\gamma\,\sqrt{(\cot^2\gamma - \cot^2\mu)}}{2\,[E'(\cot\mu\,.\,\tan\gamma)]^2}\,, \qquad \ldots \quad \ldots \quad \ldots \quad (38)$$

and

$$D_i = D_p - D_S = \frac{\pi\rho V^2\alpha^2c^2 \tan^2\gamma}{E'(\cot\mu\,.\,\tan\gamma)} - \frac{\pi\rho V^2\alpha^2c^2\tan^3\gamma\,\sqrt{(\cot^2\gamma - \cot^2\mu)}}{2[E'(\cot\mu\,.\,\tan\gamma)]^2}\,. \quad \ldots \quad \ldots \quad (39)$$

Let C_{Di} be the induced drag coefficient based on surface area, $D_i = C_{Di}\,.\,\frac{1}{2}\rho V^2 S$. Then

$$C_{Di} = \frac{\pi\alpha^2\tan\gamma}{[E'(\cot\mu\,.\,\tan\gamma)]^2}\left[2E'(\cot\mu\,.\,\tan\gamma) - \tan\gamma\sqrt{\cot^2\gamma - \cot^2\mu}\right] \quad \ldots \quad \ldots \quad (40)$$

and, observing that the aspect ratio A equals $4\tan\gamma$,

$$\frac{C_{Di}}{C_L{}^2/\pi A} = 2E'(\cot\mu\,.\,\tan\gamma) - \tan\gamma\,\sqrt{(\cot^2\gamma - \cot^2\mu)}\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

This completes the justification of the data given in section 1.3 in relation to Delta wings whose apex semi-angle is smaller than the given Mach angle.

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APPENDIX I

The Second Differential Parameter in a Pseudo-orthogonal System

In this appendix we shall use the notation of the tensor calculus, including Einstein's convention. Accordingly, we replace x_1, x_2, x_3 and y_1, y_2, y_3 as used in para. 2.1 by x^1, x^2, x^3, and y^1, y^2, y^3 respectively.

Let $g_{i\nu}$ be the fundamental covariant tensor of the quadratic differential form $(dx^1)^2 - (dx^2)^2 - (dx^3)^2$, *i.e.*,

$$[g_{i\nu}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{, so that } g = |g_{i\nu}| = 1 \, .$$

If the x^j are rectangular cartesian co-ordinates, and the y^j form a pseudo-orthogonal system, then by (5), the above tensor is given by

$$(\bar{g}_{i\nu}] = \begin{bmatrix} h_1^2 & 0 & 0 \\ 0 & -h_2^2 & 0 \\ 0 & 0 & -h_3^2 \end{bmatrix} , \bar{g} = |\bar{g}_{i\nu}| = h_1^2 h_2^2 h_3^2 \, ,$$

in the y^j co-ordinates.

The corresponding contravariant tensors are

$$[g^{i\nu}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{and} \quad [\bar{g}^{i\nu}] = \begin{bmatrix} 1/h_1^2 & 0 & 0 \\ 0 & -1/h_2^2 & 0 \\ 0 & 0 & -1/h_3^2 \end{bmatrix} .$$

The expression $\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i}\left(\sqrt{g}\, g^{i\nu} \frac{\partial \Phi}{\partial x^\nu}\right)$ is called the second differential parameter of the (arbitrary) scalar function Φ. This expression is known to be an absolute scalar, *i.e.*,

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i}\left(\sqrt{g}\, g^{i\nu} \frac{\partial \Phi}{\partial x^\nu}\right) = \frac{1}{\sqrt{\bar{g}}} \frac{\partial}{\partial y^i}\left(\sqrt{\bar{g}}\, \bar{g}^{i\nu} \frac{\partial \Phi}{\partial y^\nu}\right) .$$

In the particular case here under consideration, this equation becomes

$$\frac{\partial^2 \Phi}{\partial (x^1)^2} - \frac{\partial^2 \Phi}{\partial (x^2)^2} - \frac{\partial^2 \Phi}{\partial (x^3)^2} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial y^1}\left(\frac{h_2 h_3}{h_1} \cdot \frac{\partial \Phi}{\partial y^1}\right) - \frac{\partial}{\partial y^2}\left(\frac{h_1 h_3}{h_2} \frac{\partial \Phi}{\partial y^2}\right) - \frac{\partial}{\partial y^3}\left(\frac{h_1 h_2}{h_3} \frac{\partial \Phi}{\partial y^3}\right)\right] .$$

which is equivalent to equation (6).

APPENDIX II

Calculation of Some Partial Derivatives

Using the notation of section 2.1 we require expressions for $\partial g_j/\partial x_k$, $j = 1, 2, 3$, $k = 1, 2, 3$, in terms of y_1, y_2, y_3. We have

$$dx_j = \sum_{k=1}^{3} a_{jk}\, dy_k$$

where $a_{jk} = \dfrac{\partial f_j}{\partial y_k}$. Solving for dy_k,

$$dy_k = \sum_{j=1}^{3} A_{kj}\, dx_j$$

where $[A_{kj}]$ is the inverse matrix of $[a_{jk}]$, $[A_{kj}] = [a_{jk}]^{-1}$. On the other hand, evidently $A_{kj} = \dfrac{\partial g_k}{\partial x_j}$.

Now let the x_j be rectangular cartesian co-ordinates, and the y_j any system of pseudo-orthogonal co-ordinates. Using the relations of pseudo-orthogonality we find, by direct (matrix) multiplication that

$$\begin{bmatrix} \dfrac{\partial f_1}{\partial y_1}/h_1^2 & -\dfrac{\partial f_2}{\partial y_1}/h_1^2 & -\dfrac{\partial f_3}{\partial y_1}/h_1^2 \\ -\dfrac{\partial f_1}{\partial y_2}/h_2^2 & \dfrac{\partial f_2}{\partial y_2}/h_2^2 & \dfrac{\partial f_3}{\partial y_2}/h_2^2 \\ -\dfrac{\partial f_1}{\partial y_3}/h_3^2 & \dfrac{\partial f_2}{\partial y_3}/h_3^2 & \dfrac{\partial f_3}{\partial y_3}/h_3^2 \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\partial f_1}{\partial y_1} & \dfrac{\partial f_1}{\partial y_2} & \dfrac{\partial f_1}{\partial y_3} \\ \dfrac{\partial f_2}{\partial y_1} & \dfrac{\partial f_2}{\partial y_2} & \dfrac{\partial f_2}{\partial y_3} \\ \dfrac{\partial f_3}{\partial y_1} & \dfrac{\partial f_3}{\partial y_2} & \dfrac{\partial f_3}{\partial y_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The second of the above matrices is identical with $[a_{jk}]$ so that the first must be $[A_{kj}]$. Hence

$$\frac{\partial g_1}{\partial x_1} = \frac{\partial f_1}{\partial y_1}/h_1^2 \qquad \frac{\partial g_1}{\partial x_2} = -\frac{\partial f_2}{\partial y_1}/h_1^2 \qquad \frac{\partial g_1}{\partial x_3} = -\frac{\partial f_3}{\partial y_1}/h_1^2$$

$$\frac{\partial g_2}{\partial x_1} = -\frac{\partial f_1}{\partial y_2}/h_1^2 \qquad \frac{\partial g_2}{\partial x_2} = \frac{\partial f_2}{\partial y_2}/h_2^2 \qquad \frac{\partial g_2}{\partial x_3} = \frac{\partial f_1}{\partial y_2}/h_2^2$$

$$\frac{\partial g_3}{\partial x_1} = -\frac{\partial f_1}{\partial y_3}/h_3^2 \qquad \frac{\partial g_3}{\partial x_2} = \frac{\partial f_2}{\partial y_3}/h_3^2 \qquad \frac{\partial g_3}{\partial x_3} = \frac{\partial f_3}{\partial y_3}/h_3^2 .$$

These are the required expressions.

APPENDIX III

Calculation of the Suction Force

As in section 2.6, let dl be an element of the leading edge of the aerofoil, yawed at an angle β. Let x_0, y_0 be the co-ordinates of the midpoint of dl, and dy the length of its lateral projection. As the suction force depends only on local conditions, we may modify the boundary conditions elsewhere at pleasure. Accordingly, we may assume that dl forms part of an infinite straight leading edge (Fig. 6). It is, therefore, sufficient for oui purposes to calculate the suction force at the leading edge of a yawed infinite flat plate. The type of flow is again assumed to be such that

$$\frac{\partial \Phi}{\partial x} = \frac{\bar{C}}{\sqrt{(x_0 - x)}} + \ldots \text{ (finite terms)} .$$

The free-stream velocity is $-V$.

According to a now well-established argument (*e.g.*, Ref. 6) the flow round an infinite aerofoil yawed at an angle β to the free-stream direction is the resultant of (i) a uniform field of flow parallel to the leading edge at a velocity $-V \sin \beta$ and (ii) a two-dimensional field of flow at a free-stream velocity $-V \cos \beta$ in planes normal to the leading edge. Field (i) does not affect the dynamic reactions at the aerofoil, so that we may confine our attention to Field (ii). Since Field (ii) can produce no reactions in a direction parallel to the leading edge, it follows that

$$dD_S = \cos \beta \, dD'_S , \qquad \ldots \qquad (42)$$

where dD'_S is the suction foice produced by dl against the direction of the free stream of Field (ii).

Let x', y', z' be a new system of co-ordinates, obtained by turning the x, y, z system round the z-axis through an angle β, *i.e.*,

$$x' = x \cos \beta - y \sin \beta ,$$
$$y' = x \sin \beta + y \cos \beta ,$$
$$z' = z .$$

The linearised equation for the velocity potential is

$$\cot^2 \mu \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial z^2} = 0 .$$

For the primed co-ordinates, this equation is carried into

$$(\cot^2 \mu \cos^2 \beta - \sin^2 \beta) \frac{\partial^2 \Phi}{\partial z'^2} + (\cot^2 \mu \sin^2 \beta - \cos^2 \beta) \frac{\partial' \Phi}{\partial y'^2}$$
$$+ 2(\cot^2 \mu - 1) \cos \beta \sin \beta \frac{\partial^2 \Phi}{\partial x' \partial y'} - \frac{\partial^2 \Phi}{\partial z'^2} = 0 .$$

Now in Field (ii) both the total velocity potential and the induced velocity potential satisfy $\partial \Phi / dy' = 0$ so that the above equation becomes

$$q^2 \frac{\partial^2 \Phi}{\partial x'^2} + \frac{\partial^2 \Phi}{\partial z'^2} = 0 , \qquad \ldots \qquad (43)$$

where

$$q^2 = \sin^2 \beta - \cot^2 \mu \cos^2 \beta .$$

(43) is, in fact, the linearised compressible flow equation corresponding to the component free-stream velocity in the direction of the x'-axis (Mach number $\tilde{M} = \sqrt{(1 - q^2)} = M \cos \beta$).

It was assumed above that the total velocity in longitudinal direction is of the form

$$\frac{\bar{C}}{\sqrt{(x_0 - x)}} + \ . \ . \ . \ \text{(finite — } i.e., \text{ bounded-terms)}.$$

This is the resultant of the longitudinal component of Field (i), which is bounded, and of Field (ii). Hence the component velocity of Field (ii) in the direction of the x'-axis is of the form

$$\frac{\bar{C}}{\cos \beta \sqrt{(x_0 - x)}} + \ . \ . \ . \ \text{(bounded terms)} \ ,$$

or

$$\frac{\tilde{C}}{\sqrt{(x'_0 - x')}} + \ . \ . \ . \ \text{(bounded terms)} \ ,$$

where x'_0 is the x'-co-ordinate of the midpoint of dl (*see* Fig. 6) and $\tilde{C} = \dfrac{\bar{C}}{\sqrt{(\cos \beta)}}$.

Again, it follows from the character of the suction force as depending only on a local singularity, that the suction force per unit length in the direction of the x'-axis (σ, say) depends only on ρ, $\tilde{C}$, and the parameter q (representing the Mach number $\tilde{M} = M \cos \beta$). To calculate it, we consider the special case of an infinite flat plate of constant chord width $\tilde{C}$ at an incidence $\tilde{\alpha}$ in a uniform stream of velocity $\tilde{V}$, the corresponding Mach number being $\tilde{M} < 1$. It is known that, by linearised theory, the longitudinal induced velocity at the plate is given by $v = \dfrac{\tilde{\alpha}}{q} \tilde{V} \sqrt{\dfrac{\tilde{c} + x'}{x'}}$, where the leading edge of the plate is at $x' = 0$, and its trailing edge at $x' = -\tilde{c}$. Hence, in the notation used above, $\tilde{C} = \dfrac{\tilde{\alpha}}{q} \tilde{V} \sqrt{\tilde{c}}$. Also, the total pressure per unit length of the span is given by $\tilde{L} = \pi \rho \tilde{c} \tilde{V}^2 \dfrac{\tilde{\alpha}}{q}$. Now the pressure acts in a direction normal to the surface so that there is a backward component of magnitude $\tilde{L}\tilde{\alpha} = \pi \rho \tilde{c} \tilde{V}^2 \dfrac{\tilde{\alpha}^2}{q} = \pi q \rho \tilde{C}^2$ per unit length. As there can be no resultant drag in two-dimensional potential flow (*see* Ref. 9 for compressible fluid flow), it follows that the suction force exactly balances the above backward component, or

$$\sigma = \pi q \rho \tilde{C}^2 . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (44)$$

For $q = 1$, this formula was first given by Grammel (compare Ref. 5).

It follows from the character of the suction force that (44) holds not only for the case for which it has been established, but also for any other case with equal q, p, $\tilde{C}$. In the particular circumstances in which we are interested $\tilde{C} = \dfrac{\bar{C}}{\cos \beta}$, $q = \sin^2\beta - \cot^2 \mu \cos^2 \beta$, and so

$$\sigma = \pi \rho q \bar{C}^2 \sqrt{(\tan^2 \beta - \cot^2 \mu)} . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (45)$$

This is the result stated in section 2.6.

APPENDIX IV

Solutions in Terms of Lamé Functions

The differential equation of a pseudo-harmonic in hyperboloido-conal co-ordinates is (compare section 2.2, equation (15)),

$$(\mu^2 - \nu^2)\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) - \sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\frac{\partial}{\partial\mu}\left(\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\frac{\partial\Phi}{\partial\mu}\right)$$

$$- \sqrt{\{(\nu^2 - h^2)(k^2 - \nu^2)\}}\frac{\partial}{\partial\nu}\left(\sqrt{\{(\nu^2 - h^2)(k^2 - \nu^2)\}}\frac{\partial\Phi}{\partial\nu}\right) = 0$$

We try to find solutions of the form $\Phi = r^n \Psi$, where Ψ is a function of μ and ν only. On substitution in the above equation we have

$$n(n+1)(\mu^2 - \nu^2)\Psi - \sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\frac{\partial}{\partial\mu}\left(\sqrt{\{(\mu^2 - h^2)(\mu^2 - k^2)\}}\frac{\partial\Psi}{\partial\nu}\right)$$

$$- \sqrt{\{(\nu^2 - h^2)(k^2 - \nu^2)\}}\frac{\partial}{\partial\nu}\left(\sqrt{\{(\nu^2 - h^2)({}^2k - \nu^2)\}}\frac{\partial\Psi}{\partial\nu}\right) = 0 . \quad .. \quad (46)$$

Next, we assume Ψ to be of the form $\Psi = G(\mu)\ H(\nu)$. The differential equation (46) now becomes

$$H(\nu)\left[n(n+1)\mu^2 G(\mu) - \mu(2\mu^2 - h^2 - k^2)\frac{dG}{d\mu} - (\mu^2 - h^2)(\mu^2 - k^2)\frac{d^2G}{d\mu^2}\right]$$

$$- G(\mu)\left[n(n+1)\nu^2 H(\nu) - \nu(2\nu^2 - h^2 - k^2)\frac{dH}{d\nu} - (\nu^2 - h^2)(\nu^2 - k^2)\frac{d^2H}{d\nu^2}\right] = 0 .$$

In order that this equation should be satisfied it is required that

$$\frac{1}{G(\mu)}\left[n(n+1)\mu^2 G(\mu) - \mu(2\mu^2 - h^2 - k^2)\frac{dG}{d\mu} - (\mu^2 - h^2)(\mu^2 - k^2)\frac{d^2G}{d\mu^2}\right]$$

$$= \frac{1}{H(\nu)}\left[n(n+1)\nu^2 H(\nu) - \nu(2\nu^2 - h^2 - k^2)\frac{dH}{\partial\nu} - (\nu^2 - h^2)(\nu^2 - k^2)\frac{d^2H}{d\nu^2}\right] ,$$

$$= \text{const.} = p(h^2 + k^2), \text{ say,}$$

where p is an arbitrary constant. It follows that $G(\mu)$ has to satisfy the differential equation

$$[n(n+1)\mu^2 - p(h^2 + k^2)]\ G(\mu) - \mu(2\mu^2 - h^2 - k^2)\frac{dG}{d\mu} - (\mu^2 - h^2)(\mu^2 - k^2)\frac{d^2G}{d\mu^2} = 0 , \quad .. \quad (47)$$

with an exactly similar equation for $H(\nu)$.

Equation (47) is Lamé's equation (compare Ref. 7). For given n, p can be determined in $(2n + 1)$ different ways, so that $G(\mu)$ is of one of the following four forms

$$K(\mu) = (a_0\mu^n + a_1\mu^{n-2} + \ldots) ,$$
$$L(\mu) = \sqrt{|\mu^2 - h^2|}\ (a_0\mu^{n+1} + a_1\mu^{n-3} + \ldots)$$
$$M(\mu) = \sqrt{|\mu^2 - k^2|}\ (a_0\mu^{n-1} + a_1\mu^{n-3} + \ldots) ,$$
$$N(\mu) = \sqrt{|\mu^2 - h^2|}\ \sqrt{|\mu^2 - k^2|}\ (a_0\mu^{n-2} + a_1\mu^{n-4} + \ldots) ,$$

where the expressions $a_0\mu^n + a_1\mu^{n-2}$, $a_0\mu^{n-1} + a_1\mu^{n-3}$, $a_0\mu^{n-1} + a_1\mu^{n-3} + \ldots$, $a_0\mu^{n-2} + a_1\mu^{n-4} + \ldots$, are all polynomials in μ.

Thus, for $n = 0$, the only solution of the above-mentioned type is (except for a constant factor)

$$E_0^1(\mu) = 1 \; .$$

For $n = 1$, there are three independent solutions,

$$E_1^1(\mu) = \mu, \qquad E_1^2(\mu) = \sqrt{|\mu^2 - k^2|}, \qquad E_1^3(\mu) = \sqrt{|\mu^2 - k^2|} \; .$$

Assume that $E_n^m(\mu)$ has already been determined for given n and for an appropriate p. Then a second solution of Lamé's equation is given by

$$F_n^m(\mu) = E_n^m(\mu) \int_\mu^\infty \frac{dt}{[E_n^m(t)^2] \sqrt{|(t^2 - h^2)(t^2 - k^2)|}} \; .$$

Thus

$$F_0'(\mu) = \int_\mu^\infty \frac{dt}{\sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}} \; ,$$

$$F_1^1(\mu) = \mu \int_\mu^\infty \frac{dt}{t^2 \sqrt{\{(t^2 - h^2)(t^2 - k^2)\}}} \; ,$$

$$F_1^2(\mu) = \sqrt{(\mu^2 - h^2)} \int_\mu^\infty \frac{dt}{(t^2 - h^2)^{3/2} \sqrt{(t^2 - k^2)}} \; ,$$

$$F_1^3(\mu) = \sqrt{(\mu^2 - k^2)} \int_\mu^\infty \frac{dt}{\sqrt{(t^2 - h^2)}(t^2 - k^2)^{3/2}} \; .$$

From the above particular solutions of Lamé's equation we then obtain 'normal' pseudo-harmonics of the form $r^n G(\mu) H(\nu)$. For instance,

$$\Phi = \frac{C}{k \sqrt{(k^2 - h^2)}} r F_1^3(\mu) E_1^3(\nu)$$

is the solution used in the body of the report.

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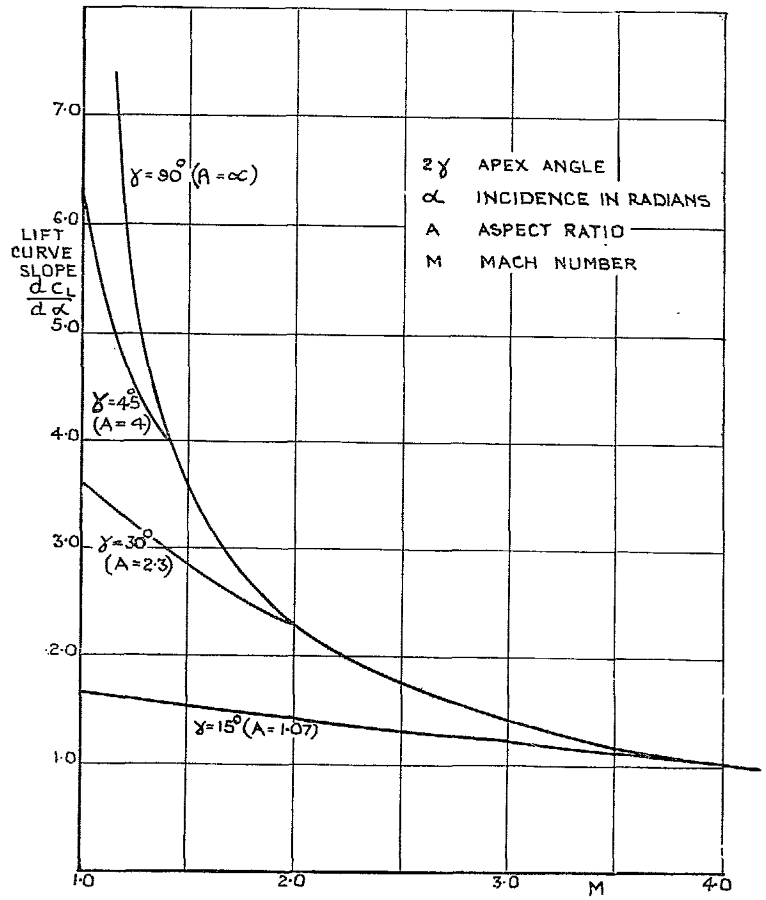
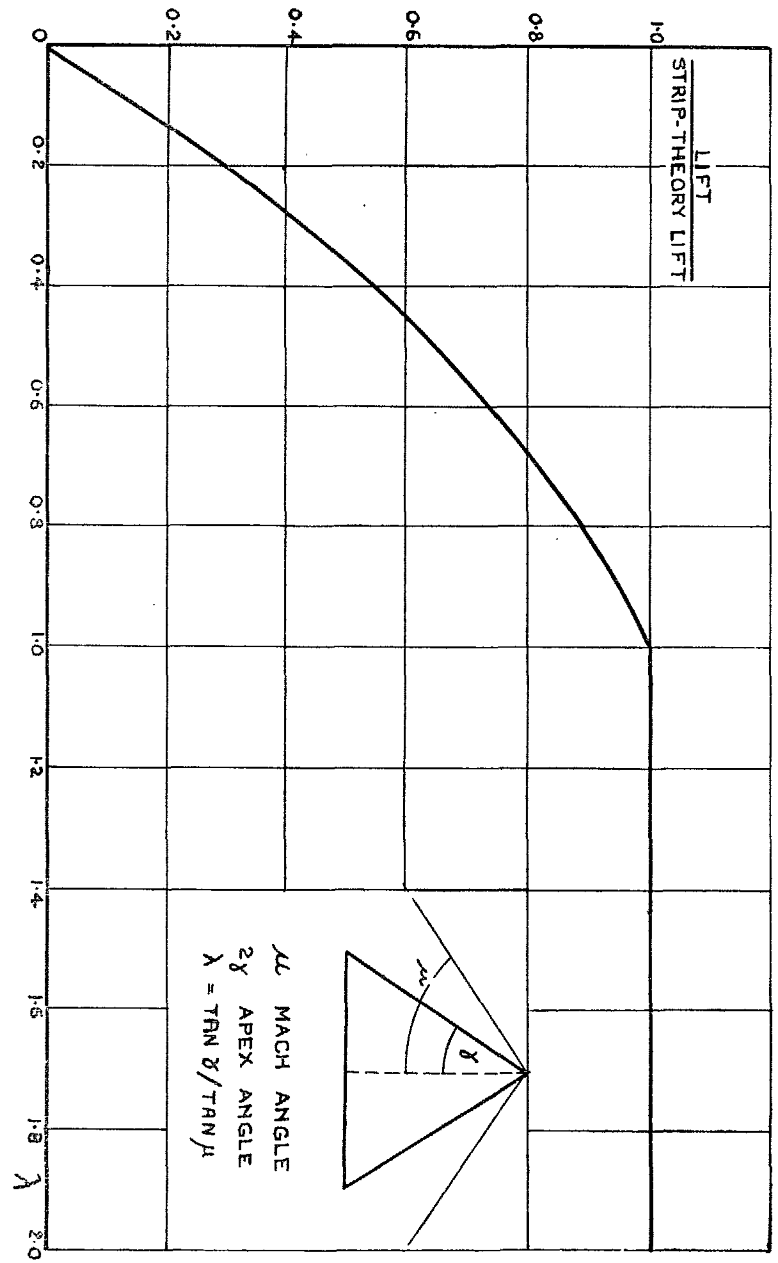


FIG. 2. Variation of the lift coefficient of a Delta wing with Mach number for various apex angles.

FIG. 1. Lift of a flat Delta wing.



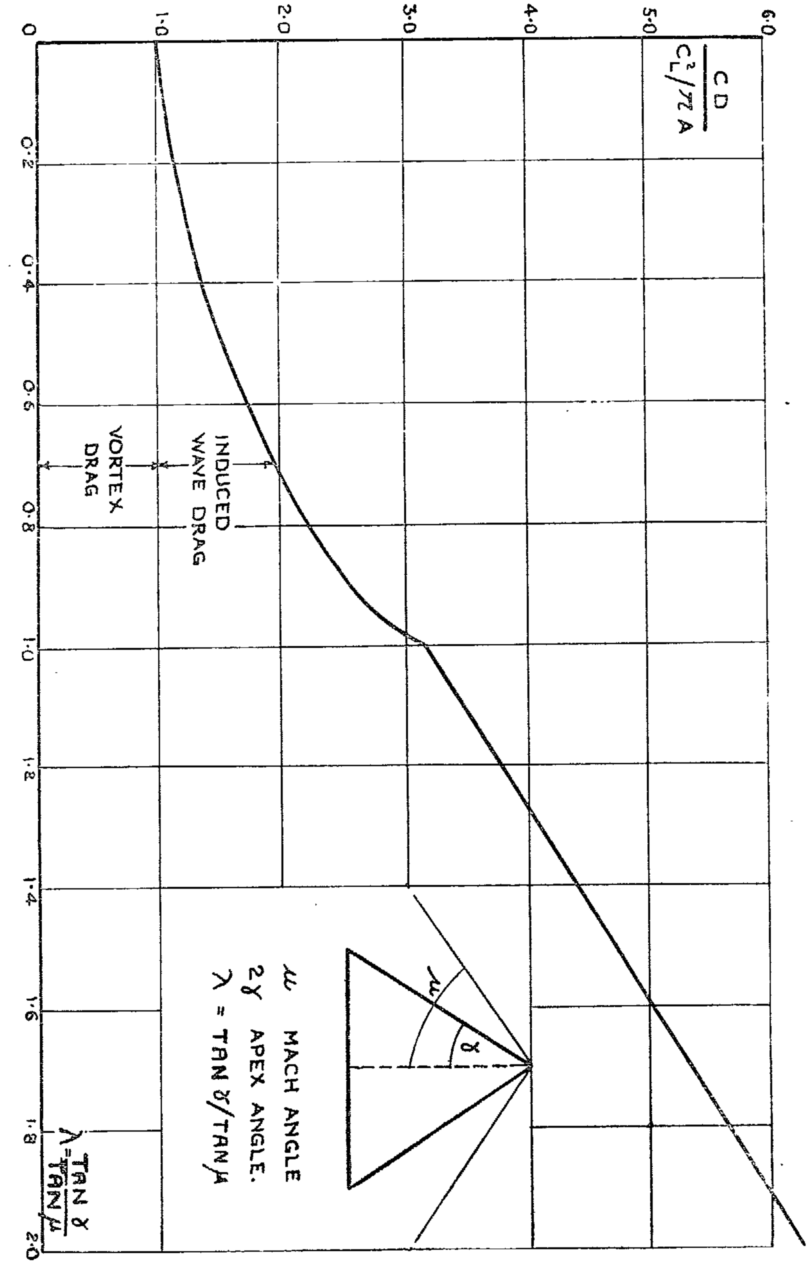


FIG. 3. Induced drag of a flat Delta wing.

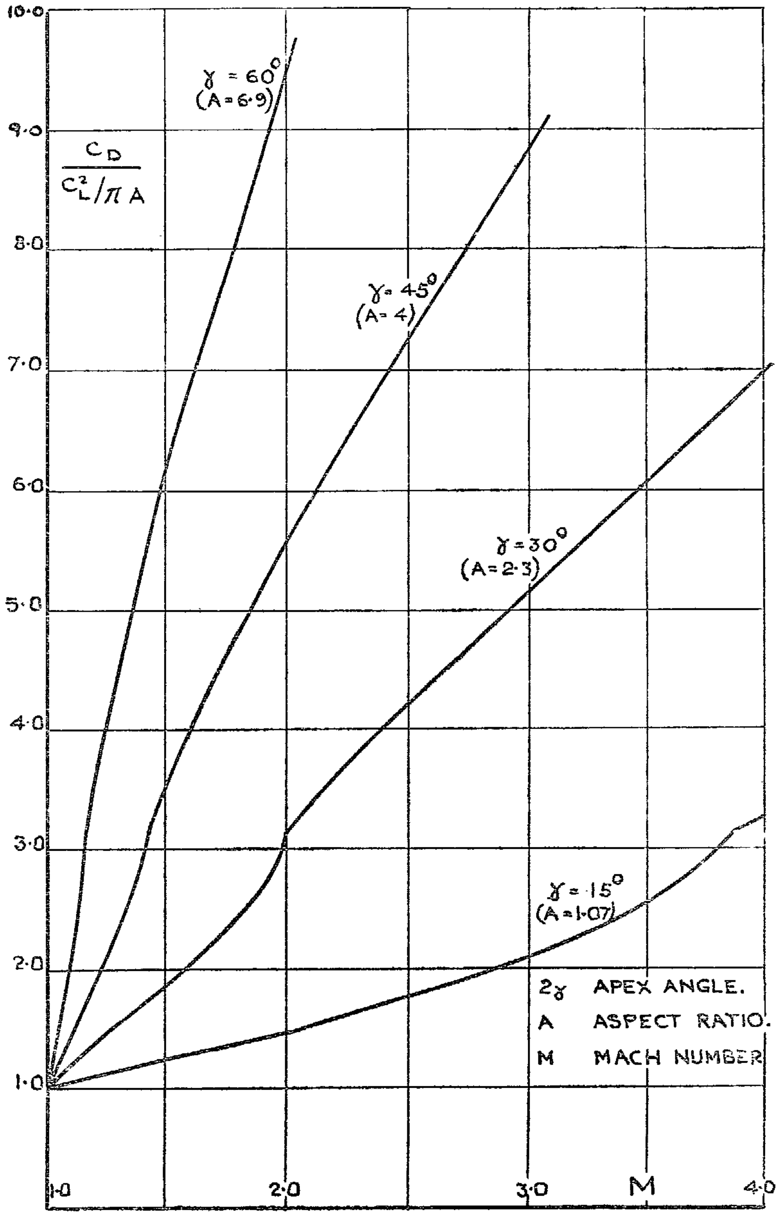


FIG. 4. Variation of the induced drag of a Delta wing with Mach number for various apex angles.

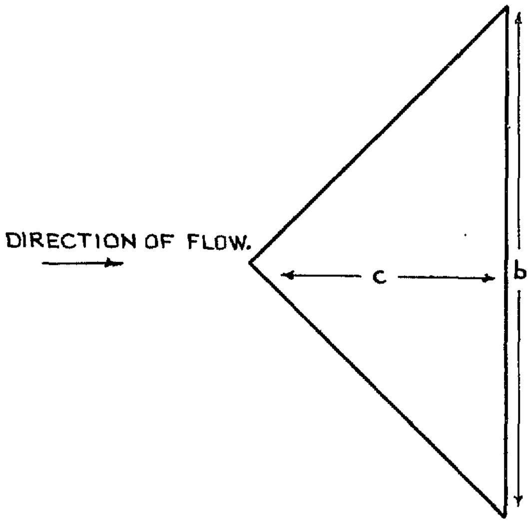


FIG. 5.

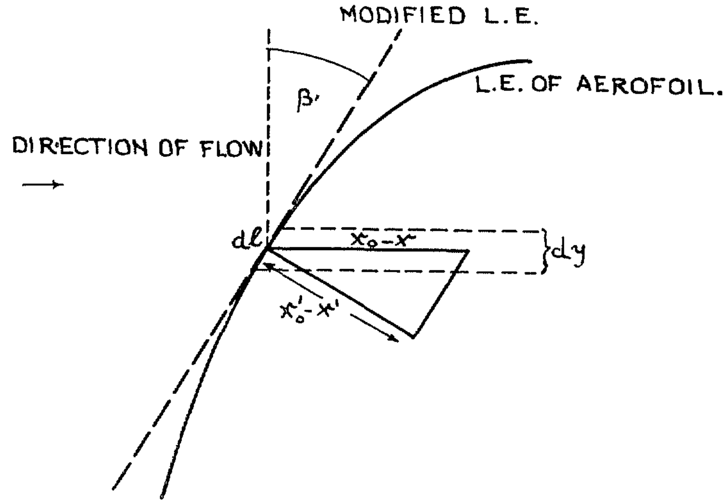


FIG. 6.

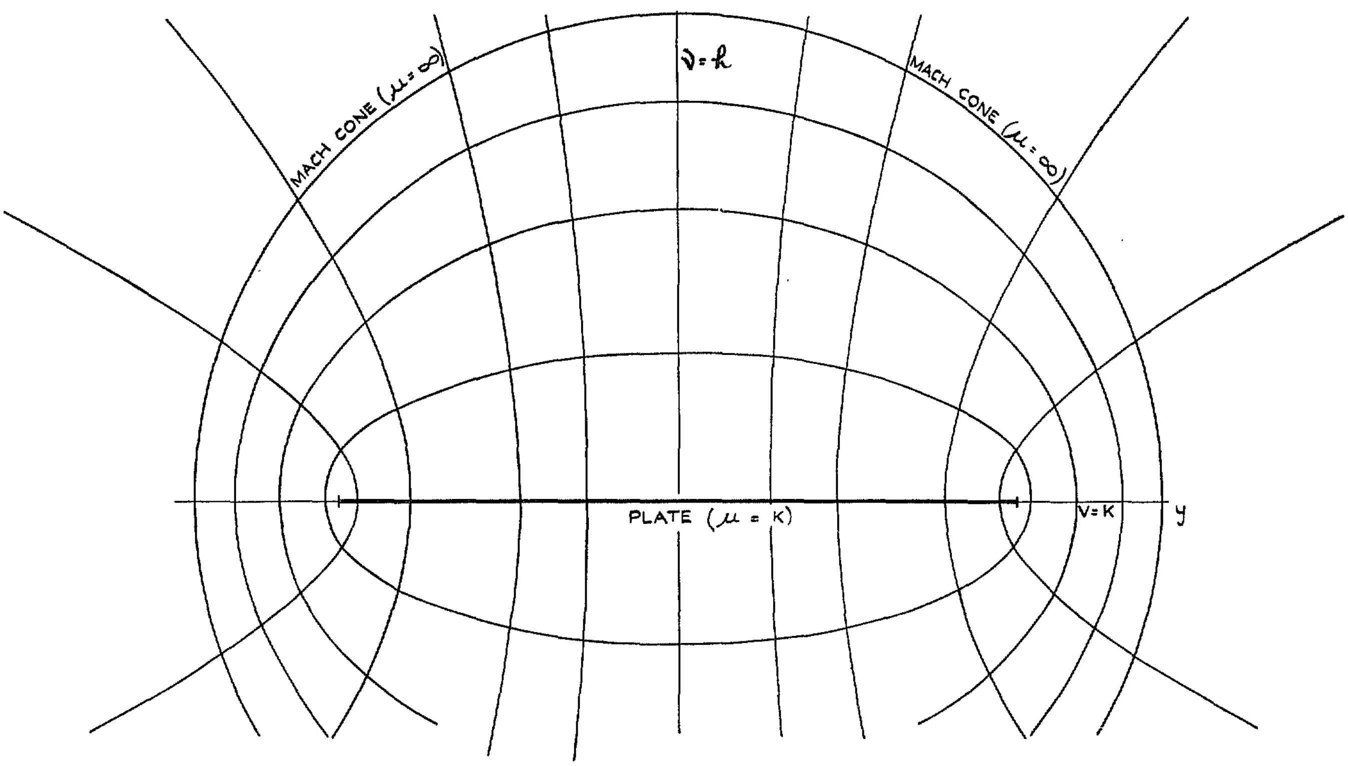


FIG. 7. Families, μ = const. (ellipses) and ν = const. (hyperbolae) in a plane x = const.

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