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Propeller Blade Vibration: Nature and Severity of Vibration at Edgewise Resonance as Influenced by Coupling Effects due to Blade Twist

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Summary.—Reasons for Enquiry.—Information was required as to the stress distribution in propeller blades occurring at edgewise resonance, and the importance of this vibration relative to the other modes.

Range of Investigation.—Tests were carried out on a duralumin-bladed propeller so mounted that the dynamical system was equivalent to an engine and propeller subjected to engine torsional oscillation. The fundamental edgewise vibration and its interaction with the adjacent second overtone flapping vibration was investigated for non-rotating conditions.

Conclusions.—Edgewise resonance is important in so far as the twist of the blade causes unsymmetrical bending on the blade sections. In normal blades this twist results in large deflection in the plane of greatest flexibility, the accompanying stresses being of the order of 70 per cent. of those occurring at the second overtone flapping resonance for the same excitation.

Further Developments.—The effects of blade twist on the vibration of rotating propellers will be examined as opportunity affords.

1.0. *Introduction.*—The vibration of propeller blades in the plane of the chord, usually referred to as edgewise resonant vibration, has not hitherto been considered important as regards the stress in the blade (as distinct from the shank) because of the large resistance to bending in this plane. The investigation described in this report shows, however, that as a result of the twist along the blade, there is associated with edgewise resonance a coupled flapping vibration which can result in important blade stresses. The terms flapping and edgewise vibration are not strictly correct, but are retained because they express respectively, in simple terms, the vibrations controlled by the moment of inertia of the blade section about the chord and about an axis perpendicular to the chord.

2.0. *Analysis of Effect of Twist on Blade Vibration.*—Owing to blade twist, the plane of the bending moment at any section caused by the vibrational inertia forces of the blade beyond that section will be displaced by a small angle from the principal axis. Hence a condition of unsymmetrical bending will exist at each section.

In the case of edgewise resonance in which the motion along the edgewise direction is appreciable, the section shown in Fig. 1 will be subjected to a bending moment M in the direction AA making an angle α with the principal axis OY of the cross-section. In calculating the stresses at the section, the bending moment can be resolved into two components $M \cos \alpha$ and $M \sin \alpha$, in the directions of the two principal axes OY and OZ respectively. The resultant stress f at any point can be obtained by adding algebraically the stresses produced by the bending moments about the principal axis.

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Thus $$f = -M\left[\frac{Y\cos\alpha}{I_Z} + \frac{Z\sin\alpha}{I_Y}\right],$$

where I_Z and I_Y are the moments of inertia about the axes OZ and OY respectively.

The minus sign occurs because the positive bending moments produce compression in the positive Y and Z directions. Equating the expression in the brackets to zero gives the equation of the neutral axis NN which is shown in Fig. 1, making an angle with the Z axis such that

$$\tan\beta = -\frac{Y}{Z} = \frac{I_Z}{I_Y}\tan\alpha\,.$$

For a propeller blade section, $(I_Z/I_Y)\tan\alpha$ is large, hence the neutral axis will approach the principal axis OY. The deflection is perpendicular to the neutral axis NN, hence the blade will bend principally about the OY axis, thus there is a tendency to deflect in the plane of greatest flexibility.

Since the maximum tensile and compressive stresses in the section occur at the points farthest away from the neutral axis, it will be seen that at edgewise resonance the positions of maximum stress will occur at c and d, shown in Fig. 1

Similarly, with flapping resonance, for which the moment of inertia of the section about the chord is important, the plane of the bending moment BB at any section will occur as shown in Fig. 2, with the result that the neutral axis NN will take up the new position as indicated. In this case, however, due to the angle α being large, the neutral axis is only very slightly rotated from the principal axis, and the positions of maximum stress will occur at e and f.

3.0. *Experimental Investigation.*—The above features were made the subject of an experimental investigation on a 10 ft. 10 in. diameter De Havilland variable pitch propeller. The propeller was mounted on a non-rotating torsionally elastic shaft system carrying a mass in which a torsional exciter was incorporated. The dynamical system was thus equivalent to an engine and propeller subjected to engine torsional oscillation, the general characteristics of which have been discussed in R. & M. 1758[1]. The propeller blades were set at a pitch angle of 25·4 deg. at a point 22·5 in. from the tip. Since the accompanying flapping mode occurring with edgewise vibration is influenced by its proximity to the adjacent modes, the torsional mass system was so adjusted that a node occurred at the root of the blades for fundamental edgewise resonance which occurred at 8,800 cycles/min. The criticals of the system adjacent to the edgewise resonance vibration were the first and second overtone flapping vibrations which occurred at 5,700 and 10,100 cycles/min. respectively. It will suffice for the problem in hand to consider only the interaction between the edgewise and second overtone flap, since these are in the closest proximity. Torsional movements were negligible in these tests because the torsional frequency of the blade was well above those examined.

4.0. *Observations and Results.*—4.1. *Strain Gauge Observations.*—Strain elements were attached to the blade on the curved face along the line of maximum thickness and around the circumference at the root, in accordance with the usual procedure used in strain-gauging propeller blades. The strain elements used were of the capacity type in which the change in capacity corresponding to the vibration strain was measured by a high-frequency carrier-wave method. Figs. 3 and 4 show typical stress distribution curves along the blade for edgewise and second overtone flap resonance. It will be noticed that, except that the points of zero stress occur farther along the blade towards the tip with the second overtone vibration, the flapping curves for edgewise and second overtone vibration are similar in form and comparable in magnitude. There is thus no apparent evidence of edgwise vibration, or its effects, excepting at the root, from the stress curves taken in this manner. Hence the reason for the neglect of edgewise vibration, and also for the possible misinterpretation of strain-gauge results taken by the standard method.

4.2. *Determination of Neutral Axis of Section from Strain-gauge Measurements.*—By measuring the stress distribution across the blade at selected sections, the angular displacements of the neutral axis from the principal axis in edgewise vibration may be determined. The sections chosen for these additional measurements should be those at which the maximum stresses occur due to the flapping vibration, and in this case, as seen from Figs. 3 and 4, these occur at 9 in. and 28·5 in. from the tip.

At these sections four strain elements were placed longitudinally at the positions shown in Figs. 5 and 8, and these sufficed to give the necessary information to determine the neutral axis from the stress distribution. This axis is obtained by drawing a line through a centroid of the section so as to conform with the measured stresses. At edgewise resonance, the angle β which the neutral axis makes with the principal axis OZ for the section 28·5 in. from the tip is 87 deg., and with $I_Z/I_Y = 74{\cdot}5$ the angle α which the plane of bending makes with the principal axis OY is 14 deg. 24 min., as shown in Fig. 5.

At flapping resonance the measured stresses at this section are given in Fig. 6, from which the corresponding plane of bending and neutral axis are derived. It will be seen that the neutral axis is only slightly rotated from the principal axis OZ, and that the plane of bending is practically perpendicular to the principal axis OY.

In a similar manner the neutral axes and planes of bending moment are derived for the 9-in. section for edgewise and flapping resonance from the measured stresses given on Figs. 7 and 8. At this section, the neutral axes for both edgewise and flapping resonance practically coincide with the principal axis although the planes of bending moment for both cases are different.

4.3. *Amplitude Measurements.*—In order to check the results at edgewise resonance, the vertical and horizontal movements of the blade were measured every 3 in. along the blade from the tip to some distance beyond the sections considered. This was done by means of a small capacity element built as a seismic unit. As this instrument gave only relative values, it was necessary to measure the phase and absolute movements at the tip. These were taken by recording on a moving film camera the corresponding reflections from two pairs of cylindrical mirrors which were located at the tip, one pair being situated along the chord and the other perpendicular to the chord. In addition, the motion of an illuminated spot on the tip was viewed and measured with a telescope, the results of which confirmed those recorded by the film camera.

The results obtained are plotted in Fig. 9, which shows the movement at the tip and the vertical and horizontal movements along the blade. It will be seen that the motion of the blade at edgewise resonance comprises a fundamental edgewise movement and a large flapping action conforming somewhat to that which occurs at second overtone flapping resonance. The two movements attain their maximum values simultaneously and the condition is one of resonance. This is revealed by the tip movement observations, and is in agreement with the theoretical investigations in Ref. 2.

4.4. *Determination of Neutral Axis of Section from Amplitude Measurements.*—Since the curves of Fig. 9 give the maximum amplitudes during the vibration, the corresponding inertia loading on the blade in the two planes can be determined, from which the corresponding bending moments along the blade can be obtained. Due to the twist of the blade, the horizontal and vertical bending moments require to be resolved about the principal axes for each section. The results of these calculations are shown plotted in Fig. 10. Thus at the 28·5-in. section, the bending moments about the major and minor principal axis are 2,744 and — 653 lb. in. respectively. Combining these vectors on Fig. 5 it will be seen that the resultant practically coincides with the plane of bending moment derived from the stress measurements. Further, the stress distribution corresponding to the bending moments derived from the amplitude measurements agrees roughly with that obtained directly by strain measurement. It can therefore be taken that the stress distribution, the position of the neutral axis and the plane of bending are as shown in Fig. 5.

In flapping resonance at the 28·5-in. section as shown in Fig. 6, the neutral axis is only very slightly displaced from the principal axis OY, and hence the plane of bending is practically perpendicular to the principal axis. The motion in the edgewise direction was practically zero.

Similar agreement is obtained with the section at 9 in. from the tip, as shown in Figs. 7 and 8. In this case the neutral axes are only slightly displaced from the principal axes.

5.0. *Comparison of Blade Stresses Due to Edgewise and Flapping Movements.*—The division of stresses due to edgewise and flapping movements at edgewise resonance can be obtained by calculating the stresses along the blade due to the component bending moments given in Fig. 10. As the point A (Fig. 11) on the face of the blade at the leading edge is the most highly stressed point under vibration and working load, the division of stresses for this point have been examined and are plotted in Fig. 11. The stress ratio due to each effect can be seen at once. Curve I shows the stress at A due to pure edgewise bending on each section, and is approximately that which would occur if there were no twist in the blade. Curve II is the stress at A due to the flapping bending moment introduced by the coupling effect of the twist along the blade. Curve III is the combined stress at A due to both effects. The measured stresses obtained from the strain gauge at position B at the 9-in. and 28·5-in. sections are also given for comparison. These stresses are somewhat lower in magnitude than those calculated for position A, due mainly to the difference in location, but they are sufficiently close to confirm the general nature of the combined stress curve. The curve may also be compared with that shown in Fig. 4 showing the stress variation along the blade at the points of maximum thickness. This again confirms that the stress at edgewise resonance is mainly due to the accompanying bending action in the flap direction.

It is also to be noted that the stresses induced at edgewise resonance are of considerable magnitude, and an idea as to their relative value is obtained by making comparison with those occurring at second overtone flapping resonance Since the excitation on the experimental system varies as the square of the speed, and the edgewise and second overtone flappng resonance occurs at 8,800 and 10,100 cycles/min. respectively, the second overtone flap values require to be reduced in proportion to a base of equivalent excitation. Fig. 12 shows the measured values at the 9-in. and 28·5-in. section reduced in this manner. It will be seen that at the 9-in. section the maximum stress for edgewise resonance is 72·5 per cent. of that occurring at second overtone flapping resonance, and at the 28·5-in. section the ratio is 62 per cent. Clearly, therefore, edgewise resonance in propeller blades is a critical of some importance.

6.0. *Possible Effects of Rotation.*—With the rotating propellers the resonant frequencies will be raised due to the stiffening effect of centrifugal force by an amount dependent on the mode of vibration and mass distribution along the blade. From available data on speed coefficients it appears that the second overtone flap resonance will diverge from the fundamental edgewise with increasing rotational speed, whereas the first overtone flap resonance will approach it and under certain conditions will even coincide. Due to the coupled effects of twist such a condition might be quite serious, and it is proposed to examine this feature as opportunity affords.

7.0. *Conclusions.*—Observations made on a non-rotating propeller, have shown that, owing to blade twist, edgewise resonance causes unsymmetrical bending of the blade with vibrational coupling which gives large deflections in the plane of greatest flexibility. The unsymmetrical bending increases along the blade from the tip and the sum total of its effect at each section is reflected at the tip mainly as a flapping vibration. The accompanying stresses are considerable; for the case examined they are about 70 per cent. of those occurring at the second overtone flapping resonance for the same excitation.

Since the stresses induced at edgwise resonance are mainly due to the coupled flapping vibration, the stress distribution along the blade as obtained by the standard strain gauge method does not reveal the vibration as being fundamentally an edgewise critical, but indicates it as being of a flapping nature approximating the adjacent flapping mode. The stress distribution at the root will, however, give the necessary clue.

For the type of blade section considered, the extreme point on the face at the leading edge is the most highly stressed under vibration and working load both for edgewise and flapping resonance. It is significant that the starting point of fatigue failures which have occurred usually take place at that point. It is recommended that in future observations on this type of section an additional strain gauge be placed at that point.

8.0. *Further Developments.*—Due to the stiffening effect of rotation it appears to be possible for the first-order flapping resonance to coincide with the fundamental edgewise resonance and with the coupled effects of twist. Such a condition might be serious. It is proposed to examine this feature as opportunity affords.

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No.	*Author*	*Title, etc.*
1	B. C. Carter	The Vibration of Airscrew Blades with Particular Reference to their Response to Harmonic Torque Impulses in the Drive. R. & M. 1758 (1937).
2*	J. Morris and I. T. Minhinnick	The Calculation of the Natural Frequencies and Modes of Vibration of Airscrews, with Special Reference to Twist. A.R.C. 5069 (January, 1941).

* This work has been amplified later by :—

	J. Morris and J. W. Head	Coupled Flexural Vibrations of the Blades of a Propeller and Torsional Vibration of an Engine Crankshaft System. R. & M. 2011 (October, 1942).

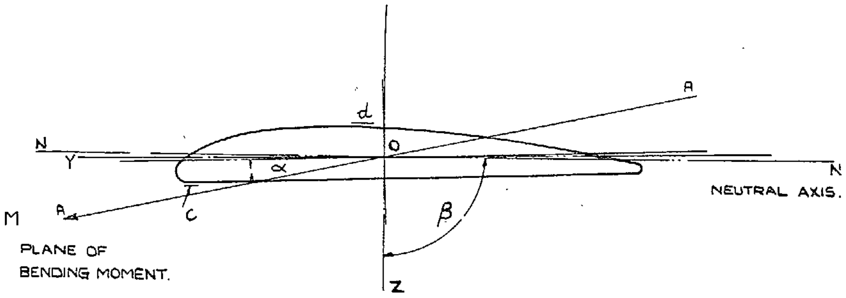


FIG. 1. Edgewise Vibration.

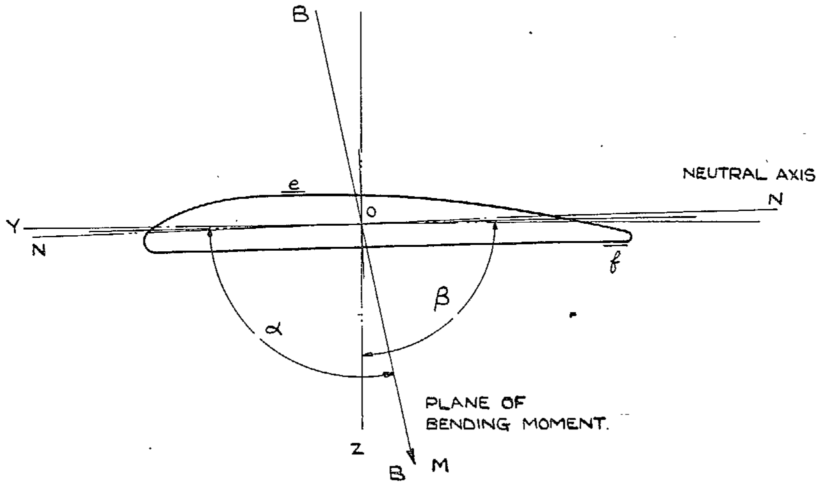


FIG. 2. Flapping Vibration. The Plane of Bending and Neutral Axis at Sections of a Twisted Blade.

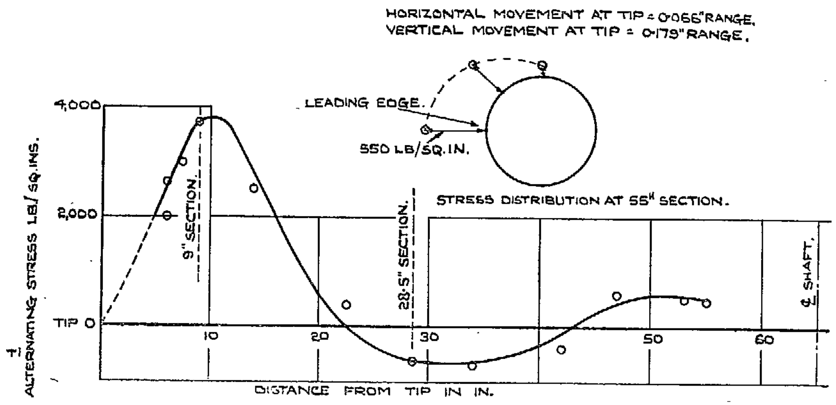


FIG. 3. Stress Distribution along Blade for Fundamental Edgewise Resonance.—Frequency 8,800 cycles/min, Excitation 3,200 lb/in.

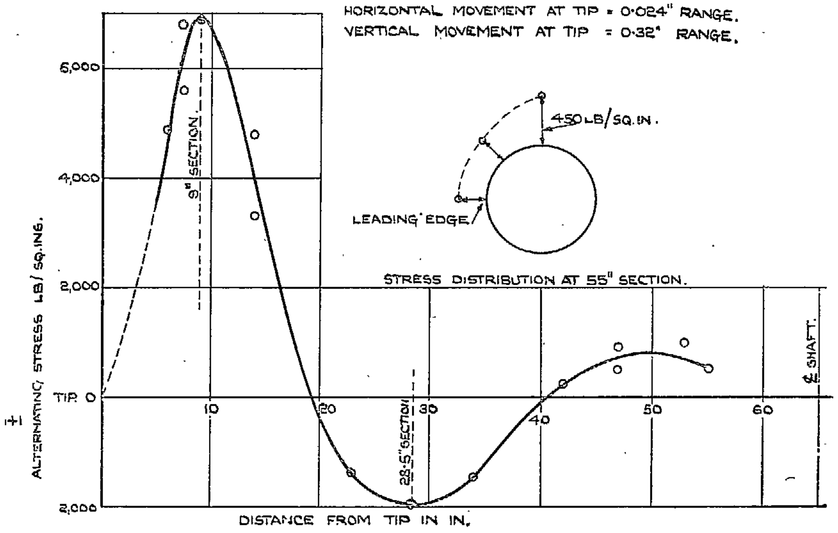


FIG. 4. Stress Distribution along Blade for Second Overtone Flap Resonance.—Frequency 10,100 cycles/min, Excitation 4,200 lb/in.

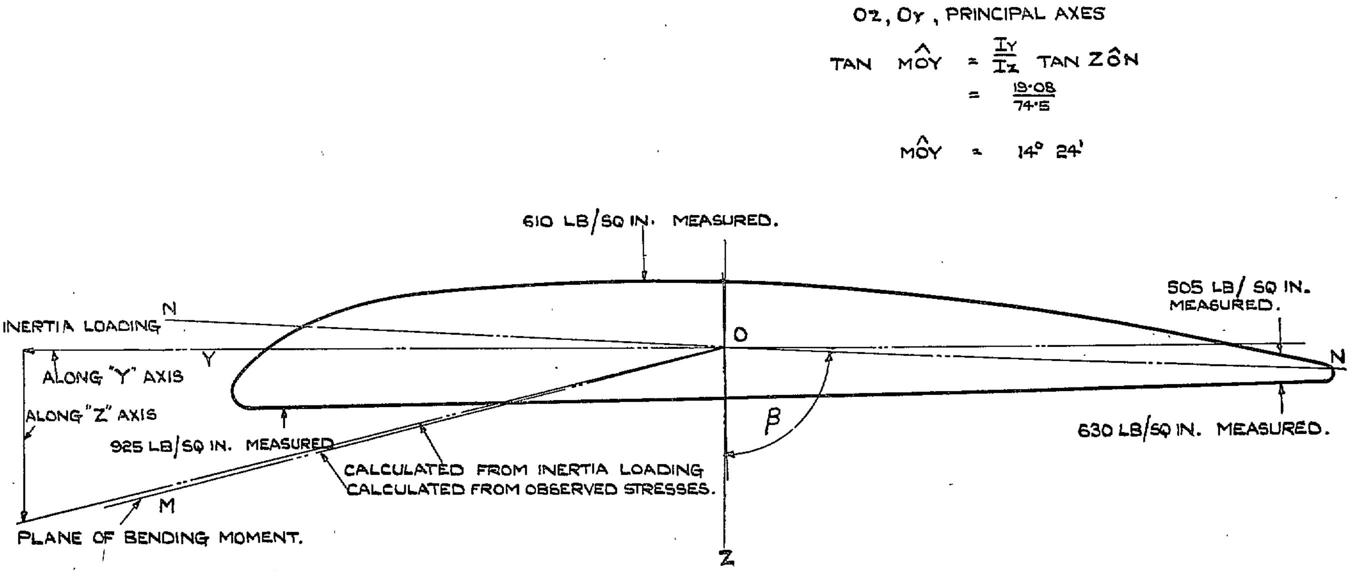


FIG. 5. The Plane of Bending and Neutral Axis from Observed Stresses and Check of Plane of Bending from Inertia Loading.

Section $28\frac{1}{2}$-in. from tip. Edgewise vibration frequency 8,800 per min. Strain elements located as indicated.

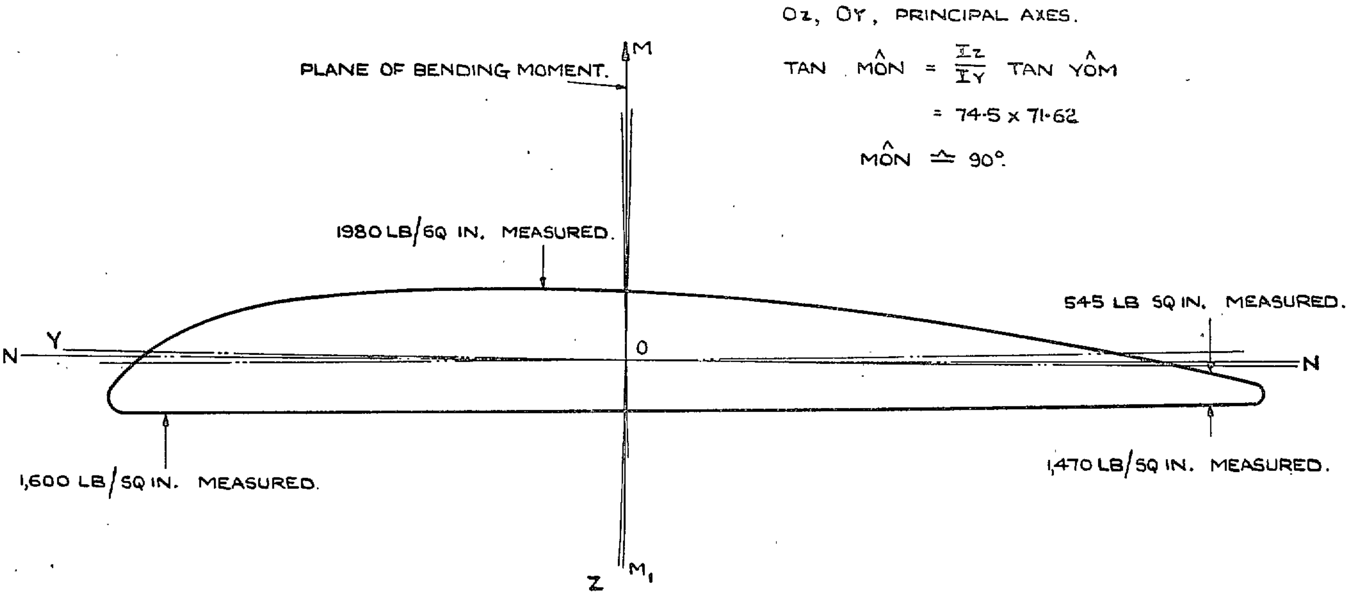


FIG. 6. The Plane of Bending and Neutral Axis from Observed Stresses.

Section $28\frac{1}{2}$-in. from tip. Flapping vibration frequency 10,100 per min. Strain elements located as indicated.

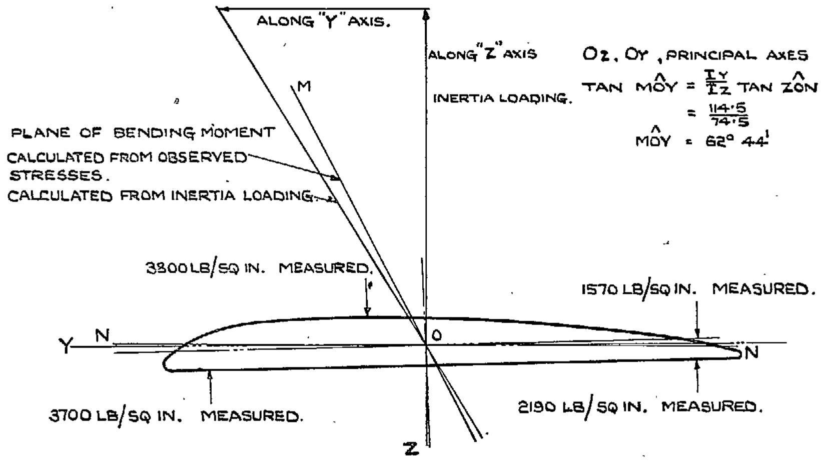


FIG. 7. The Plane of Bending and Neutral Axis from Observed Stresses and Check of Plane of Bending from Inertia Loading.

Section 9-in. from tip. Edgewise vibration frequency 8,800 per min. Strain elements located as indicated.

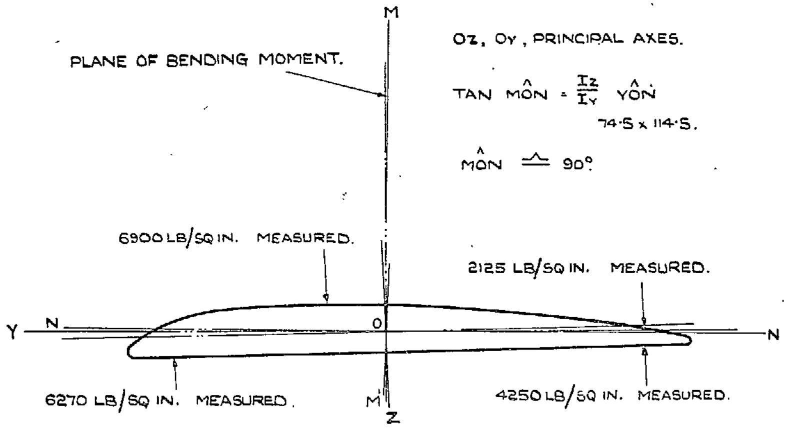


FIG. 8. The Plane of Bending and Neutral Axis from Observed Stresses.

Section 9-in. from tip. Flapping vibration frequency 10,100 per min. Strain elements located as indicated.

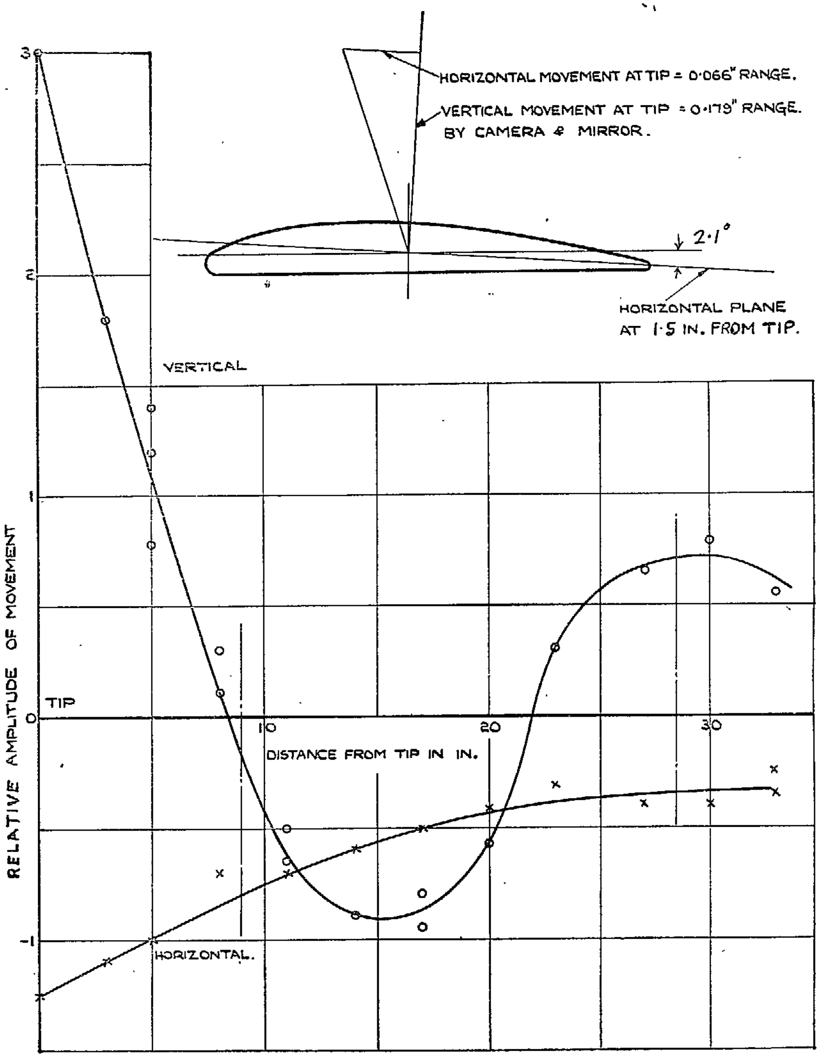


FIG. 9. Movement of Blade for Edgewise Resonance. Frequency 8,800 cycles per min.

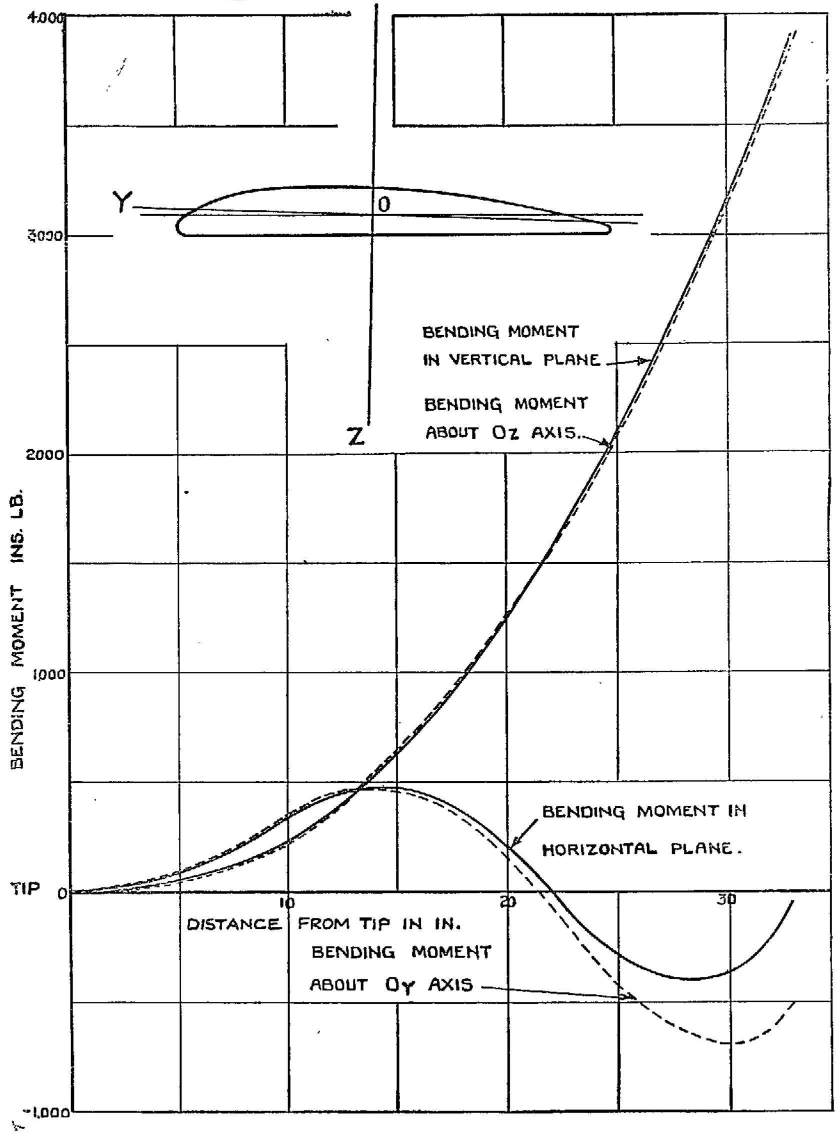


FIG. 10. Bending Moments on Blade Sections Due to Edgewise Resonance Calculated from Inertia Loading.

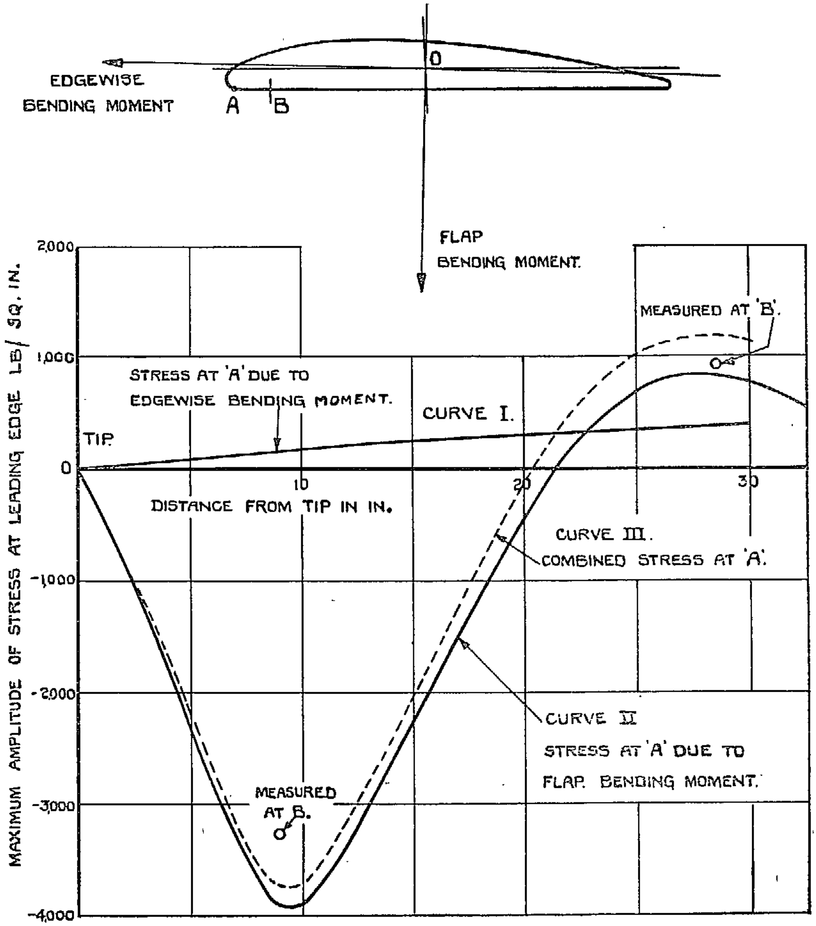
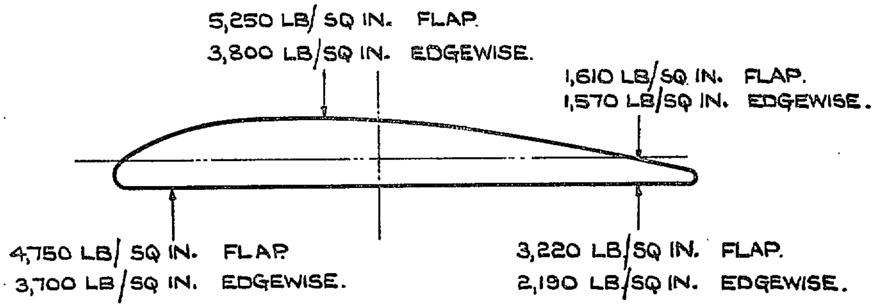
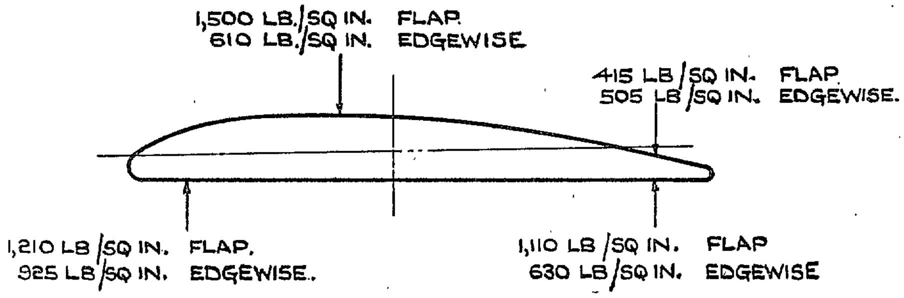


FIG. 11. Maximum Stress at Leading Edge Due to Edgewise Resonance Calculated from Inertia Loading.



SECTION 9 IN. FROM TIP.



SECTION 28·5 IN. FROM TIP.

STRESSES BROUGHT TO EQUIVALENT EXCITATION.

FIG. 12. Stresses Obtained for Flap Resonance Multiplied by $(8{,}800/10{,}100)^2$ for Comparison with Stresses Obtained for Edgewise Resonance.

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