R. & M. No. 2626
(10,509)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

An Experimental Investigation on the Flutter Characteristics of a Model Flying Wing

By

N. C. LAMBOURNE, B.Sc.,
of the Aerodynamics Division, N.P.L.

LONDON : HER MAJESTY'S STATIONERY OFFICE
1952

PRICE 6*s* 6*d* NET

An Experimental Investigation on the Flutter Characteristics of a Model Flying Wing

By

N. C. LAMBOURNE, B.Sc., of the Aerodynamics Division, N.P.L.

Reports and Memoranda No. 2626
April, 1947

Summary.—This report describes some preliminary experimental work that has been carried out in an attempt to gain information on the flexural-torsional flutter characteristics of flying wing types of aircraft. Tests were made with two flexible tip-to-tip models:—

A Rectangular plan form.

B Cranked and tapered plan form.

The method of supporting the models in the wind tunnel allowed certain bodily freedoms to be present either singly or simultaneously, and measurements were made of critical speeds and frequencies, and in a few cases the flutter motion was analysed by means of cinematograph records. The experimental results are in no way conclusive and cannot be directly applied to full-scale problems, but they do point to some of the difficulties in the treatment of the flutter of flying wings. Further, the difficulties encountered during the flutter tests themselves lead to suggested modifications in the technique of providing a model in a wind tunnel with the bodily freedoms appropriate to free flight conditions.

1. *General Introduction*.—In the problem of wing flutter of conventional single-engined aeroplanes it is usually assumed that the critical speed that will be met in practice is only slightly different from that which would be obtained were the fuselage immobile. Frazer and Duncan[1] (1931) investigated theoretically the effect of fuselage mobility on binary flexural-aileron flutter for one particular aeroplane that was typical of practice at that time. Their conclusions were that the critical speed for longitudinal-symmetrical flutter differs little from that which is obtained when the fuselage is regarded as immobile, and that the critical speed for lateral-anti-symmetrical flutter is considerably higher than that corresponding to a fixed fuselage. The same authors also give one calculation to show that freedom of the fuselage in roll raises the critical speed for anti-symmetrical flutter of the ternary flexural-torsional-aileron type. More recently W. P. Jones[2] (1944) has examined the case of anti-symmetrical flutter of a large transport aeroplane when the ailerons are mass underbalanced. He treats, *inter alia*, the cases of binary flexural-aileron flutter and of ternary flexural-torsional-aileron flutter, and concludes that the critical speeds for both types are raised by the introduction of rolling mobility.

As far as pure flexural-torsional flutter is concerned the effect of fuselage mobility in roll has been investigated theoretically by Pugsley, Morris and Naylor[3] (1939). They conclude that for a single-engined aeroplane anti-symmetrical flutter is the least probable in practice, and they point out that in symmetrical flutter the vertical motion of the fuselage is likely to be small since the associated mass is large compared with the wings. Similarly, since the pitching moment of inertia of the fuselage is large compared with that of the wings, it may also be concluded that the pitching motion of the fuselage will be small during symmetrical flexural-torsional flutter.

Published with the permission of the Director, National Physical Laboratory.

Although most of the foregoing evidence refers to flutter involving aileron movements, it has been usual to regard the effect of fuselage mobility on the flexural-torsional flutter speed as negligible, at least with single-engined aeroplanes. Valuable experimental investigations with half-span model wings rigidly held at their roots have been carried out on this basis. Not only were the experiments able to provide controls on critical speed calculations, but the results could, with a considerable measure of truth, be directly applied on a basis of dynamical similarity to the flutter of actual aircraft.

In the case of an aeroplane with wing engines, where up to roughly three-quarters of the total weight may be located in the wings, the mobility of the fuselage becomes important. Experimentation using model cantilever wings rigidly attached at the root is still able to provide controls on calculations, but the results of the experiments are no longer directly applicable to the flutter which might occur in flight. With the advent of the flying wing type of aircraft in which there is a relatively uniform distribution of mass over a single lifting surface a more general approach to the flutter problem must be sought. The instabilities which may occur in flight can no longer be conveniently separated into those that do not involve structural distortions (*i.e.*, the bodily oscillations dealt with by classical aircraft stability theory) and those that only involve structural distortions and control surface movements (*i.e.*, flutter).

The flutter problem of practical flying wings is also complicated by the sweep back (or forward) which will be present for the purposes of attaining both 'rigid body' stability and high speeds.

The investigation to be described was concerned mainly with the problem of mounting a model flying wing in a wind tunnel so as to provide satisfactory allowances for true bodily freedoms which are present in flight. In the main the system of supporting the model was designed on the lines suggested by Frazer,[4] the models themselves consisted of flexible wings attached to a central body which could be supported in either of two manners for separate study of the symmetrical and the anti-symmetrical types of oscillations. The two interchangeable suspension systems provided the following alternative sets of bodily freedoms (*see* Fig. 1):—

(i) *Appropriate to symmetrical flutter.*

Pitching.

Vertical translation.

(ii) *Appropriate to anti-symmetrical flutter.*

Rolling.

Yawing.

Lateral translation.

Tests were carried out with two tip-to-tip models* (*see* Fig. 2) as follows:—

(A) Rectangular.

(B) Tapered and cranked.

The experiments with A were intended primarily as a simple approach to the more general problem.

* Tests with a further model (tapered without sweep back) were originally intended, but on the completion of the work with the other two models it was not considered worthwhile to proceed with the construction of the third.

The original experimental programme was ambitious in including provision for the variation of the stiffness and mass loading of the wings. In practice, however, the adjustments necessary to match the stiffnesses of the port and starboard wings were so tedious that no attempt was made to vary the wing stiffnesses once these matching adjustments were complete; nor was any attempt made to vary the wing inertias, and the flutter tests all relate to the effect of the freedoms of the central body.

2. *Description of Model Wings*.—The plan forms of the models are shown in Fig. 2. Each wing (root to tip) consisted of two separate parts, an inner and an outer bay, the inner bay being common to both the rectangular and the cranked model. The internal wing structure was designed to provide easily definable stiffness properties, and was of course not representative of actual practice. The general layout of the rectangular wings is shown in Fig. 3, whilst the outer bays of the cranked wings were constructed on similar lines except that the spars were swept back. Wooden spars at the leading and trailing edges of the inner bay were independently hinged by means of small ball bearings to an aluminium box (the central body) which is described in section 3. The spars were also connected to the central body by flat steel strips, C and D, which acted as torsion springs and constrained the hinging of the spars. The leading and trailing edge spars of the outer bay were similarly hinged to the outer ribs of the inner bay and were constrained by the steel strips A and B. The spars themselves were T-sectioned, quite stiff in bending but flexible in torsion, so that, whilst the flexure of the wing was controlled directly by the 4 springs A, B, C and D, wing torsion took place by the differential hinging of the leading edge spars. Thus, the torsional stiffness of the wing was dependent not only on the 4 springs but also on the characteristics of the wooden structure itself. Provision was also made for cross-connection of the leading and trailing edges by flat steel strips E and F at the outer sections of both bays, so that the torsional stiffness of the wing could be adjusted independently of the flexural stiffness. The leading and trailing-edge spars were also cross-connected at a number of points by light wooden ribs parallel to the longitudinal axis of the complete model and defining the aerodynamic sections. The inner and outer bays were separately covered with vaseline-doped silk and access to the springs was gained by removal of portions of the coverings (*see* Fig. 4).

3. *Description of Support System*.—The two interchangeable support systems shown in Fig. 1 have been described in outline in section 1. The apparatus as arranged for the case of symmetrical flutter and with all the possible spring attachments is shown diagramatically in Fig. 5. Rod AB was mounted in ball bearings bb attached to the walls of the wind tunnel. Rod CD, which was integral with AB, was arranged along the wind axis and supported by springs S_v. A light aluminium box which formed the central body to which the wings were attached, could pitch about an axis through the ball bearings P,P; these small bearings were carried by a fitting E clamped to a sleeve R at the upstream end of rod CD, (*see* also Fig. 6). This sleeve was itself mounted on small ball bearings and could turn about its own axis to provide a rolling freedom for the model. During tests involving symmetrical flutter this last degree of freedom was of course locked. The wing-body combination then had two possible bodily freedoms as follows:—

1 Pitching about PP.

2 Pitching about AB (approximately a freedom in vertical translation and subsequently referred to as such).

Independent clamps were provided so that either or both of these freedoms could be eliminated, and the majority of the tests described in this report refer to pitching freedom about PP only. In this case the central body was supported in the horizontal position by the pitching springs as shown in the diagram. It may be noted that if both the freedoms had been present simultaneously a cross stiffness would have been introduced by the pitching springs.

It was possible to fix the pitching axis at various positions behind the leading edge of the model and the pitching inertia could be varied by masses clamped to rod F. Fig. 7 shows the aluminium box with one wing attached. During the flutter tests the box was shielded by a plywood cover, and Fig. 8 shows the complete apparatus mounted in the wind tunnel.

For the case of anti-symmetrical flutter it was intended to keep the plane of the wings horizontal with the supporting apparatus reorientated as follows:—

1 Axis PP vertical, to form the axis of yawing.

2 Axis AB vertical, to provide approximately lateral translation.

3 Rolling freedom unlocked.

The experimental difficulties encountered during the tests with freedoms appropriate to symmetrical flutter led to the conclusion that the alternative arrangement of the apparatus would not be satisfactory for tests involving anti-symmetrical flutter. In fact, rolling was the only freedom appropriate to anti-symmetrical flutter that was used.

4. *Experiments with Rectangular Wings.—Elastic Stiffness and Flexibility Coefficients.*—The elastic properties were determined by means of flexibility coefficients referring to points 1, 2, 3, 4 on each wing (*see* Table 1).

From these measurements the flexural and torsional stiffnesses and the positions of the flexural centres were obtained on the assumption that the chordwise sections do not distort. The loads were applied to the spars themselves, and the vertical displacements of needles attached to the spars near the loading points were measured by means of micrometer heads. During these measurements the central body was clamped as effectively as possible and equal loads were applied at corresponding positions in both the port and starboard wings, so that any deflection due to the slight residual flexibility in roll was eliminated. There was, however, some movement of the central body in the vertical plane, and displacements were also measured at two additional points 5, 6, so that corrections could be applied to the wing deflections. It was found that creep occurred after a load had been applied, and to obtain consistent sets of coefficients the wings were allowed to settle for 4 minutes before readings were taken.

Although the port and starboard wings were constructed in the same manner and similar flat steel springs were fitted in each, it was found at the outset that the two wings had vastly different stiffnesses. Much time was spent in altering the stiffnesses of the steel springs by modifying either their width or thicknesses until the wings had approximately equal stiffnesses.

The method of carrying out these adjustments was firstly to remove the outer bays, and modify springs C and D (*see* Fig. 3) by trial and error until the flexural stiffnesses and positions of the flexural centre at sections (3, 4) were approximately the same for both wings. It was then found that the torsional stiffnesses of the inner bays were also in reasonable agreement, and no alteration of spring F was necessary. The outer bays were then attached and springs A and B were modified until the flexural stiffnesses and positions of the flexural centres as measured at sections (1, 2) were the same for both wings. It was again found that the torsional stiffnesses measured at these sections were in agreement, and no alteration of spring E was necessary.

The final elastic properties of the two wings are contained in Table 1, where a_{ij} is the deflection at position i due to unit load at position j. The measuring and loading positions did not quite coincide and they are given in Table 2.

A more convenient picture of the final elastic state of the wings is provided by Table 3 which gives the flexural and torsion stiffnesses and the positions of the flexural centre for sections (1, 2) and (3, 4).

5. *Inertial Properties and Natural Frequencies.*—As a means of comparing the inertial properties of the port and starboard wings, each wing in turn was forced sinusoidally through a weak spring connected to the appropriate tip rib whilst the central body was held as firmly as possible. The reasonance frequencies were as follows:—

	Port	*Starboard*
1st resonance (mainly flexure)	4·27 c.p.s.	4·06 c.p.s.
2nd resonance (mainly torsion)	8·17 c.p.s.	8·22 c.p.s.

It should be noted that since a metal fitting and a spring were attached to each wing during these measurements, the above frequencies are not the natural frequencies applicable to the conditions of the flutter tests; they merely serve to provide a comparison of the oscillatory characteristics of the two wings. This comparison was considered to be satisfactory and no adjustment of the mass properties was made.

In order to determine approximately the natural frequencies of the wings without attachments the central body was forced inexorably in pitching. The measured frequencies which are applicable to the "body clamped" condition were as follows:—

	Port	*Starboard*
1st resonance (mainly flexure)	3·21 c.p.s.	3·17 c.p.s.
2nd resonance (mainly torsion)	6·82 c.p.s.	6·92 c.p.s.

The nodal lines at the second resonance were found to be approximately at 0·5 chord aft of the leading edges of both wings.

6. *Flutter Experiments.*—The critical speed for flutter was in each case measured by finding the lowest speed at which the system would continue to oscillate after a disturbance. The disturbance was initiated by padded levers which were normally clear of the wing, but could be operated to strike either or both wings. Clamps were also provided so that the wings might be held.

As a preliminary experiment all the freedoms of the central body were locked, and the critical speed and frequency were measured for each wing in turn whilst the other was held in its clamp. The following results were obtained:—

	Port	*Starboard*
Critical speed (ft/sec)	43·0	42·9
Flutter frequency (c.p.s.)	4·68	4·98

With the central body locked, but with both wings free, independent oscillations of both wings were obtained at the following critical speeds and frequencies:—

	Port	*Starboard*
Critical speed (ft/sec)	42·7	42·4
Flutter frequency (c.p.s.)	4·66	4·94

A more detailed description of these tests may be of interest. When the wind speed was adjusted to 42·7 ft/sec and the port wing was disturbed flutter of this wing occurred. At the same time a small amplitude oscillation was picked up by the starboard wing due to some slight freedom of the central body. This forced oscillation of the starboard wing was, however, not sufficient to initiate growing flutter oscillations. Similarly, when the starboard wing was disturbed at a wind speed of 42·4 ft/sec it fluttered whilst the port wing picked up a small oscillation in sympathy. If both wings were disturbed simultaneously at the higher wind speed, they fluttered at their different frequencies, and a small amount of beating was noticeable in the amplitudes.

When this experiment was repeated on later occasions it was sometimes found that the wings, instead of fluttering independently oscillated symmetrically; on no occasion was anti-symmetrical flutter observed. Whether or not independent flutter occurred seemed to depend very critically on the effectiveness of the clamping of the central body.

The critical speed for wing divergence appeared to be only slightly above the critical speed for flutter, but no accurate determination of the divergence speed was made.

Effect of Bodily Freedoms.—6.1. *Pitching Freedom.*—The central body was allowed freedom to pitch about an axis PP 0·2-chord aft of the leading edge, which was the most forward position that could be obtained with the apparatus. It was found that the system became statically unstable almost as soon as the wind stream was started and remained so at least up to 50 ft/sec; this suggested that the aerodynamic centre was forward of the 0·2-chord position. It was thought at first that this might be caused by the aerodynamic moment due to the rod and inertia weight which projected forward of the central body, but tests with these removed showed that the wing-body combination was itself aerodynamically unstable. Therefore, it was necessary to provide some stability in pitching, and springs were attached to the central body from above and below. Critical speeds and frequencies were measured for variations of both the pitching inertia of the central body I_α and the pitching stiffness m_α. In Fig. 9, V_c and f_c are plotted against I_α/Mc^2 for one value of m_α, where M is the sum of the masses of the port and starboard wings alone. The diagram shows that as I_α is increased from zero the critical speed for symmetrical flutter falls gradually and then rises rapidly. With further increase of pitching inertia symmetrical flutter was replaced by anti-symmetrical flutter, the speed and flutter frequency being almost the same as that corresponding to the 'central body clamped' condition. An explanation of the presence of this anti-symmetrical flutter is contained in the Appendix.

Theoretically it is possible, on the assumption of simple harmonic motion of the central body, to combine the two variables I_α and m_α into a single parameter which may be called the effective pitching inertia of the central body and is defined by

$$I_\alpha' = I_\alpha - m_\alpha/4\pi^2 f_c^2$$

where f_c is the frequency of symmetrical flutter. The parameter I_α' has more significance from the point of view of free flight conditions and Fig. 10 shows the results plotted on this basis. No results could be obtained to the right of the dotted line due to the presence of anti-symmetrical flutter as already mentioned, but the diagram does suggest that with a practical value of the pitching inertia (*i.e.*, a positive value of the effective inertia) the critical speed is raised when a pitching freedom is introduced. However, a similar conclusion could not be drawn from the tests with the cranked wings (*see* section 8.1).

6.2. *Rolling Freedom.*—The wing-body combination was allowed to roll under the constraint of springs attached to the forward cross bar (*see* Fig. 5) but pitching was prevented. Some spring constraint was necessary to maintain rolling stability, and although various values of the rolling stiffness were tried, only symmetrical flutter occurred, and in each case the critical speed and frequency corresponded to the 'body clamped' condition.

The negative result of this experiment indicates that in the presence of a rolling freedom the critical speed for anti-symmetrical flutter is above that corresponding to a fixed body.

6.3. *Pitching and Rolling Simultaneously.*—In the previous experiment with a rolling freedom it was found that the first critical speed encountered corresponded to symmetrical flutter, so that the critical speed corresponding to anti-symmetrical flutter could not be reached. In an endeavour to eliminate this unwanted symmetrical flutter, the pitching inertia of the central body was firstly adjusted until symmetrical flutter was absent when the rolling freedom was locked. In other words the pitching inertia was adjusted to correspond to a value of I_α'/Mc^2 to the right of the dotted line in Fig. 10. Anti-symmetrical flutter then occurred at 42 ft/sec. The rolling freedom was then unlocked, but unfortunately divergence of the system occurred at a speed only slightly above 42 ft/sec due to the fact that the pitching stiffness was necessarily low to secure a positive value for the effective pitching inertia.

It was hoped that this experiment might have shown that, with the body freedoms of pitching and rolling, the critical speeds for both symmetrical and anti-symmetrical flutter would be considerably higher than the critical speed corresponding to the body rigidly held. However, since divergence occurred no evidence on this point could be obtained.

Experiments with Cranked Wings.

7. *Elastic Stiffness and Flexibility Coefficients.*—The flat steel springs that had controlled the stiffness of the outer bays of the rectangular model were removed and re-embodied in the outer bays of the cranked model. The elastic characteristics were again obtained in terms of flexibility coefficients measured at 4 points and the values are given in Tables 4 and 5. The flexural and torsional stiffnesses referring to sections (1, 2) and (3, 4) are given in Table 6, whilst Fig. 11 shows the wing distortion due to couples applied at these sections. The curves showing the torsional mode and the position of the centre of twist along the span, which are given in the diagram, were obtained by a consideration of the geometry of the structure on the assumption that the spars do not bend but simply hinge about their inboard ends. The actual torsional mode is therefore unlikely to have as high a curvature as is shown in the diagram.

It is necessary to emphasise again that the structure of the model was not representative of practice, but was adopted to simplify possible flutter speed calculations in which the elastic distortion of the wing might be defined approximately in terms of 4 co-ordinates. Although the distortion of the model is therefore unlikely to be strictly comparable with that of practice, it is thought that the twisting properties of the model as shown by the spanwise variations of the centres of twist may be reasonable approximations to those of a practical structure of the same plan form.

8. *Flutter Frequencies.*—The resonance frequencies of the cranked model were not measured, and initially the critical speeds of the individual wings with the central body locked were found to differ by only a small amount. This difference was brought to zero by attaching a lead weight to the tip of one wing, and it was then found that with the central body clamped as securely as possible symmetrical flutter occurred at 77·4 ft/sec, 7·85 c.p.s. The fact that symmetrical and not independent flutter of the two wings occurred indicates that there was some slight residual flexibility of the central body.

Effect of Bodily Freedoms.—8.1. *Pitching Freedom*.—In a similar experiment to that with the rectangular wings, the central body was allowed to pitch about on axis $0{\cdot}2c_0$ behind the leading edge, but in this case, due to the sweepback, the system was statically stable in the wind stream without the addition of springs. However, springs were attached to the central body so that a systematic series of critical speed and frequency measurements could be made with various values of the pitching inertia I_α and the pitching stiffness m_α. As before, in the presentation of the results these two variables are combined in a single parameter I_α' ($\equiv I_\alpha - m_\alpha/4\pi^2 f_c^2$), the effective pitching inertia of the central body. A similar series of tests was also carried out with the pitching axis of the central body at $0{\cdot}39c_0$, and in this case the system was only just statically stable in the wind. The main results for both positions of the pitching axis are shown in Fig. 12. The results are also replotted on a much smaller scale in Fig. 13 to include a point corresponding to a large value of I_α' which was obtained by means of an inertia lever connected to the central body.

These results differ considerably from those obtained with the rectangular wing (Fig. 10). In the present case (Figs. 12, 13), there is no rapid increase of the critical speed in the neighbourhood of $I_\alpha' = 0$, and the results indicate that the critical speed tends to a finite asymptote and the frequency to zero as I_α' tends to ∞. It is noticed that the values of the critical speed and frequency for high values of I_α' are considerably lower than the values obtained when the central body is clamped. At first sight this appears to be in conflict with the generally accepted principle that from the point of view of flutter a fuselage of infinite mass is equivalent to one of zero mobility, but the results are not seen in their true light without some indication of the type of flutter motion which occurred.

Cinematograph records of the flutter motion were obtained for the following three conditions of the central body:—

Central body clamped.

Central body free to pitch about $0{\cdot}2c_0$, corresponding to point A in Fig. 12.

Central body free to pitch about $0{\cdot}2c_0$, corresponding to point B in Fig. 12.

For each of these cases the film records were analysed to give the motions of three representative wing sections:—

Wing root.

Section at junction of rectangular and swept back portions.

Tip section.

From the results of these analyses Figs. 16, 17, 18 have been prepared to illustrate the flutter motions. In this method of presentation a half-span wing is regarded as moving across the page from left to right with a speed proportional to the critical speed V_c, and the displacements of the three reference sections are shown at intervals during the cycle. The distance travelled across the page during a half cycle is proportional to $V_c/2f_c$ and is thus inversely proportional to the frequency parameter $\omega = 2\pi f_c c/V_c$ in order to make the motions of the wing sections more obvious the amplitudes are exaggerated but the correct amplitude ratios have been preserved.

With the central body clamped (Fig. 16) the flutter, of course, consists solely of flexural-torsional motion. At the tip section the torsional motion lags approximately $\pi/4$ behind the flexural motion*, this agrees with the results of Lambourne and Weston[5] (1944) for a straight tapered wing. There are no large torsional phase changes along the span.

* Based on the usual convention that downward flexure and nose-up torsion are positive deflections.

When the central body is free to pitch (Fig. 17) the three reference sections as drawn in the diagram, remain very nearly parallel to one another throughout the diagram, thus indicating that there is little torsion along the span. In other words, the wing appears to behave as a rigid body as far as torsion is concerned, and the oscillation is of a flexural-pitching type.

The third diagram (Fig. 18) refers to a higher pitching inertia of the central body, and is similar to the previous diagram except that the frequency parameter ω is considerably lower. There is, however, an indication that some torsion is taking place.

In Fig. 17 the pitching motion lags approximately $\pi/2$ behind the flexural motion, whilst in Fig. 18 this phase lag is increased to approximately $3\pi/4$.

It is concluded that the right-hand asymptotic values in Figs. 12, 13 correspond to flutter which is mainly composed of flexure and pitching motions. In the hypothetical case $I_{\alpha}' \to \infty$ two possible types of flutter may occur. The one involves pitching motion and is associated with zero frequency, whilst the other does not involve pitching motion and is identical in character with the flutter (*i.e.*, pure flexural-torsional) occurring in the body clamped condition. The experimental results indicate that in the former case although the frequency becomes evanescent as $I_{\alpha}' \to \infty$ the critical speed remains definite and for the present model at least this critical speed is lower than that corresponding to flexural-torsional flutter.

The question as to whether or not a flexural-pitching oscillation would occur with an actual aircraft in flight is obviously intimately bound up with the classical (*i.e.*, rigid aircraft) longitudinal stability characteristics, and in this connection two factors which may be important in influencing this type of oscillation are:—

(i) The additional bodily freedom in vertical translation which would be present.

(ii) The pitching damping due to the tail. (This would be present in the case of a conventional aircraft).

It is likely that the latter would be a decisive factor in preventing this type of oscillation as far as conventional aircraft are concerned.

8.2. *Low Wind Speed Oscillation.*—With the pitching axis at $0{\cdot}2c_0$ and the system mass balanced and without spring constraints an oscillation was observed at a low wind speed (about 8 ft/sec). The wings appeared to behave as rigid bodies and the oscillation was thought to be due to an instability in a single degree of freedom (pitching). The system had a slight amount of gravitational stability in still air and the aerodynamic oscillation occurred at approximately the natural frequency (0·2 c.p.s.). As the wind speed was raised the amplitude of the oscillation increased, reached a maximum, and then decreased till eventually the system became stable again. The range of wind speed for instability was from 6 to 12 ft/sec and appeared to be independent of the natural frequency which was varied between 0·2 and 1·0 c.p.s. by altering the inertia of the system.

This was not a resonance phenomenon associated with the rotation of either the fan or the windmill of the wind tunnel, since there was no correlation between their rotational speeds and the frequency of the oscillation. The oscillation, in fact, appeared to be attributed to the presence of negative aerodynamic pitching damping.

From the formulae for the aerodynamic damping deduced by vortex sheet theory and given by W. P. Jones[6] it can be shown that for very low values of the frequency parameter and for certain positions of the axis the pitching damping is negative. The curves of Fig. 14 show the relations between the frequency parameter ω and the position of the pitching axis when the aerodynamic damping is zero both for the case of a two-dimensional wing and for the present cranked wing, the results for the latter having been deduced by strip theory. According to these curves negative damping is not to be expected for the range of frequency parameters ($\omega_0 = 0{\cdot}14$ to $0{\cdot}70$) appropriate to the observed oscillation. However, the theory assumes a high

Reynolds number, whereas the observed oscillation occurred at $R=6\times10^4$ approximately. It is, in fact, quite probable that the phenomenon was associated with a transition of airflow occurring at some critical Reynolds number. Although this oscillation is not considered to have any importance as far as practice is concerned, it is hoped to carry out further investigation with a rigid wing.

8.3. *Vertical Translation and Pitching Freedoms Simultaneously.*—The central body was allowed to pitch about the $0{\cdot}2c_0$ axis position without spring constraint whilst the horizontal supporting rod was unclamped and constrained by weak springs to turn about the downstream axis (*see* Figs. 1, 5). This freedom of the supporting rod provided the wing body combination with a second freedom which approximated to vertical translation, and for convenience the spring stiffness associated with this freedom is referred to hereinafter as the stiffness of the model in vertical translation.

Initially the natural frequency of the system in vertical translation was 0·7 c.p.s. In the first test the central body was balanced about the pitching axis by adjustment of the mass on the forward rod, and it was found that an oscillatory instability occurred at a relatively low wind speed, and the frequency coincided approximately with the natural frequency of the system in vertical translation. There was no obvious structural distortion of the wings, the wing-body combination appearing to move as a rigid body; the motion of the system was mainly confined to vertical translation, the pitching motion of the wings being almost imperceptible. Unfortunately, with the method of support used pure vertical translation alone could not be provided so that it was impossible to test whether the oscillation would actually occur with only a single degree of freedom. Certainly clamping of the pitching freedom (*i.e.*, about $0{\cdot}2c_0$) eliminated the instability, but then the wing combination was capable of performing not pure vertical translation but pitching about a rearward axis.

The 'pitching mass balance' of the wing-body system could be varied by altering the position of the mass on the rod projecting forward of the central body (*see* Fig. 5). In still air the wings were then pitched either up or down since no spring stiffness was present, but, provided the unbalance was not too great, they were brought very nearly horizontal by the aerodynamic moment. The variation of the critical speed with pitching mass balance is shown in Fig. 15, and it is noticed that for some positions of mass over-balance there are three critical speeds, whilst the frequency remains approximately constant. No results to the right or left of the curve could be obtained because the aerodynamic pitching moment was not sufficient to bring the unbalanced wing-body combination clear of the stops.

The vertical translation stiffness was increased until the natural frequency of the system in vertical translation was 1·9 c.p.s.; the observed instability did involve some distortion of the wings, but the critical speed appeared to be unaltered by variation of the mass balance. These results are also shown in Fig. 15, which in addition, includes the value of the critical speed when the vertical translation freedom is locked.

On the assumption that structural distortion of the wings is unnecessary for the occurrence of the instability, two alternative explanations are offered. The first is that only vertical translation is involved and therefore the phenomenon is similar to that described in section 8.2; in other words it is a case of negative damping in vertical translation. The second and more plausible possibility is that the oscillation involved both pitching and vertical translation. On this assumption curve ABC in Fig. 15 probably corresponds to the oscillatory instability that would occur with a rigid wing provided with the two degrees of bodily freedom, (vertical translation and pitching), and curve CD to an oscillation involving wing distortion in addition.

The results of these experiments indicate that the addition of vertical translational freedom to a system already possessing pitching freedom lowers the wind speed at which instability occurs. It might be objected that in these experiments the vertical translation freedom was spring constrained. However, since the effective mass of a corresponding system free from spring constraint is proportional to $1-(f_0/f)^2$ (where f_0 is the natural frequency and f the frequency of the oscillation), the effective mass of the central body may be regarded as approximately zero for the lower and negative for the higher vertical translation stiffness.

These experiments clearly illustrate the difficulties connected with support systems providing several bodily freedoms and indicate the possible close relationship between the problems of flutter and aircraft stability.

8.4. *Rolling Freedom.*—Symmetrical flutter occurred at 83 ft/sec* both when all the bodily freedoms were locked and when the central body was free to roll without spring constraint. During the latter experiment, due to a slight aerodynamic asymmetry, the wings actually rotated continuously about the roll axis at a very slow rate, but this does not affect the result. Springs were then attached to constrain the model in rolling and, for each of the stiffness values tried, symmetrical flutter again occurred at the same wind speed as before.

These tests show that with the introduction of rolling freedom the critical speed for anti-symmetrical flutter is raised above that corresponding to the body-locked condition. In the Appendix it is shown that with a tip-to-tip model the critical speed for one type of flutter (*i.e.*, symmetrical or anti-symmetrical) may be masked by the onset of the other. Although, owing to the presence of symmetrical flutter, the critical speed for anti-symmetrical flutter could not be observed directly, an attempt was made to deduce that speed by removal of one wing. The critical speed measurement for the remaining wing in the body locked condition was repeated and the speed found to be 85 ft/sec. The rolling freedom was then unlocked so that the wing hung vertically in the tunnel like a pendulum. No instability occurred, although the wind speed was raised to just over 100 ft/sec, and from the behaviour of the wing, which appeared to be highly damped, this speed was judged to be well below the critical speed for flutter. This test confirms that the critical speed is raised considerably by the introduction of rolling freedom.

9. *General Conclusions.—On the Problem of the Flutter of Flying Wings.*—The results of the experiments are in no way conclusive but they do provide evidence to support the following:—

1. The lowest critical speed that will be met in practice will refer to symmetrical flutter.
2. The critical speed calculated on the assumption of immobility of the centre section is likely to be above the critical speed when pitching of the centre section is included.

On the experimental technique.

3. Some simplification of the scope of the experiments is necessary and it would be preferable to make separate investigations under the following headings:
   (i) The influence of bodily freedoms on the flutter of a few models representative of present and possible future practice.
   (ii) The influence of sweep (back and forward), elastic stiffnesses, and inertias for cantilever wings (roots fixed).
4. In experiments involving bodily freedoms there are distinct advantages in using a half-span model and providing the necessary freedoms at the root (*see* the Appendix).

10. *Acknowledgements.*—Acknowledgements are due to Mr. C. Scruton who initiated the design and construction of the model and supporting apparatus, to Mr. C. J. Davis for assistance in many of the tests, and to Miss N. Belgrave who helped to carry out the analysis of the film records of the flutter motion.

* The critical speed corresponding to the body-clamped condition varied slightly from day to day due to slight changes in the wing stiffnesses.

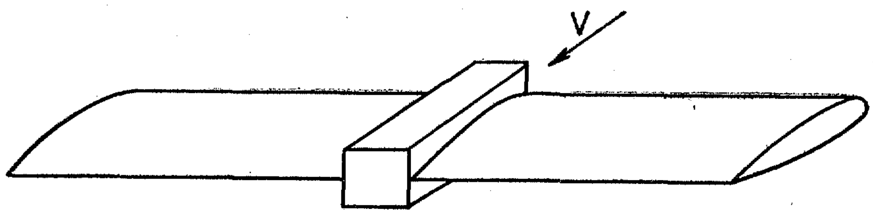
REFERENCES

No.	*Author*	*Title, etc.*
1	R. A. Frazer and W. J. Duncan ..	The Flutter of Monoplanes, Biplane and Tail Units. R. & M. 1255. 1931.
2	W. P. Jones	Antisymmetrical Flutter of a larger Transport Aeroplane. R. & M. 2363. July, 1944.
3	A. G. Pugsley, J. Morris and G. A. Naylor ..	The effect of Fuselage Mobility in Roll upon Wing Flutter. R. & M. 2009. October, 1939.
4	R. A. Frazer	Note on the Flutter of Flying Wings. A.R.C. 6247. 1942. (Unpublished.)
5	N. C. Lambourne and D. Weston ..	An Experimental Investigation of the Effect of Localised Masses on the Flutter of a Model Wing. R. & M. 2533. April, 1944.
6	W. P. Jones	Summary of Formulae and Notation used in Two-Dimensional Derivative Theory. R. & M. 1958. 1942.

APPENDIX

The use of Tip-to-tip Models

Consider the oscillations of a tip-to-tip model in which the two wings have identical elastic and inertial properties, and are attached to a central body.



Then the possible types of flutter oscillations may be divided into two classes by the character of the modes of distortion of the wings.

(a) *Oscillations with symmetrical modes.* The motions of the port and starboard wings will be in phase, and the corresponding amplitudes will be equal. There will be no rolling motion of the central body, but there will, in general, be motions in pitching and vertical translation.

(b) *Oscillations with anti-symmetrical modes.* In this case the amplitudes of the port and starboard wings will be equal but the motions will be in anti-phase. There will be rolling of the central body with the possibility of yawing and lateral translation, but no motion in pitching or vertical translation.

If the central body could be locked quite rigidly and if aerodynamic interaction between the two wings were negligible, the motion of the port and starboard wings would be wholly independent. Under these hypothetical conditions flutter of both wings would develop at the same speed V and the same frequency f but the phase difference between the two wings would be arbitrary.

Next suppose the central body to be given a freedom in vertical translation. It is then possible for the system to flutter at a new critical speed V_v and frequency f_v in a mode comprising symmetrical distortion of the wings and vertical translation of the body. The system is also capable of fluttering in an anti-symmetrical mode in which there is no motion of the central body, and this flutter would be identical in character with that which would occur if the body were rigidly held, except that the phase difference between the port and starboard motions is π and not arbitrary. The critical speed and frequency for the anti-symmetrical flutter will be V and f respectively, the same as for the body-held condition.

In an experiment with a system that has a number of critical speeds each denoting the onset of a certain type of unstable oscillation, the only oscillation that will be observed will be the one corresponding to the lowest critical speed. Hence, if vertical translation is the sole bodily freedom, the observed flutter will be either symmetrical or anti-symmetrical according as V_v is less than or greater than V. Similarly, if only rolling freedom is allowed the observed flutter will be either anti-symmetrical or symmetrical according as V_r is less than or greater than V, where V_r is the critical speed for anti-symmetrical flutter when rolling freedom is present.

The above remarks have been based on the assumption that the wings have identical elastic and inertial properties, but in practice this will not be strictly true. However, provided the characteristics of the wings are not widely different it will still be possible to classify an oscillation as symmetrical or anti-symmetrical, and the fact remains that for any condition of the central body there will in general exist at least one critical speed referring to the onset of symmetrical flutter, and at least one corresponding to anti-symmetrical flutter. Again, in a practical experiment in which the bodily freedoms are clamped there will still exist, due to the clamping not being absolute, a slight amount of elastic coupling between the wings, and the flutter which occurs may no longer be independent but may be either symmetrical or anti-symmetrical.

It may be difficult to arrange an experiment to determine the influence of a parameter on one particular type of flutter. If, for instance, an attempt is made to find the effect of a certain parameter on the critical speed for anti-symmetrical flutter, and the only body freedom allowed is rolling then any flutter at a speed above the critical speed for the body locked condition will *not* be obtained due to the occurrence of symmetrical flutter. One method of overcoming this difficulty might be to provide the bodily freedoms appropriate to symmetrical flutter and to adjust the inertias and stiffnesses corresponding to these freedoms so that the critical speed for symmetrical flutter is raised sufficiently high. However, even apart from the experimental difficulties of providing several bodily freedoms simultaneously, it may still be difficult or impossible to eliminate the unwanted type.

These difficulties were in fact encountered in the flutter tests described in this report. When the body was allowed the single freedom in pitching, the critical speeds for symmetrical flutter were (except in one case) found to be lower than the critical speed for the body-clamped condition, and the influence of the pitching inertia of the body on the critical speed for symmetrical flutter could be investigated. On the other hand when rolling was the only body freedom provided, only symmetrical flutter corresponding to the body clamped condition occurred and no information could be obtained on the critical speed for anti-symmetrical flutter.

For separate investigation of the two types of flutter, it would be preferable to use a half-span model and to provide the appropriate freedoms at the root. In this case if, for instance, only rolling freedom is provided at the root, the flutter which occurs *can only* correspond to the anti-symmetrical type. Similarly, if freedom in pitching and vertical translation are provided the observed flutter can be regarded as symmetrical. Provided the wing is built out from a wall of the wind tunnel, on the theory of images, the air flow of the tests would correctly represent symmetrical flutter. The flow appropriate to anti-symmetrical flutter would, however, not be obtained, but this disadvantage is not considered to be serious.

Flexibility Coefficients for Rectangular Wings

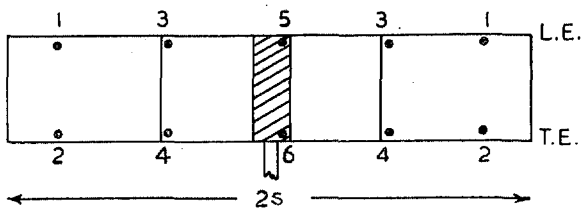


TABLE 1

Values of a_{ij} (in./lb)

where a_{ij} is deflection at position i due to unit load at position j

	Port				Starboard			
	i				i			
j	1	2	3	4	1	2	3	4
1	2·693	0·903	0·494	0·247	2·891	1·305	0·503	0·271
2	0·938	4·400	0·238	1·072	1·231	5·150	0·263	0·926
3	0·497	0·210	0·193	0·064	0·512	0·247	0·182	0·060
4	0·230	1·152	0·069	0·523	0·271	0·979	0·069	0·439

Positions 1 and 2 at 0·81s. Positions 3 and 4 at 0·32s.

TABLE 2

Values of l *and* m (in.)

l distance of loading point from leading edge.
m distance of measuring point from leading edge.

	Port				Starboard			
i or j	1	2	3	4	1	2	3	4
l	0·20	11·45	0·25	11·95	0·20	11·50	0·25	12·05
m	0·20	11·75	0·60	11·70	0·20	11·70	0·55	11·75

TABLE 3

Stiffness and Flexural Centre Positions for Rectangular Wings

	Port	*Starboard*
Section (1, 2) 0·81s.		
*Flexural stiffness	5·74 lb/ft	4·97 lb/ft
Torsional stiffness	2·06 lb ft/radn	1·97 lb ft/radn
Position of flexural centre with respect to leading edge	0·33c	0·29c
Section (3, 4) 0·32s.		
*Flexural stiffness	72·6 lb/ft	83·2 lb/ft
Torsional stiffness	18·6 lb ft/radn	22·4 lb ft/radn
Position of flexural centre with respect to leading edge	0·23c	0·25c

* The flexural stiffness is here defined as the load which when applied at the flexural centre causes unit linear displacement.

Flexibility Coefficients for Cranked Wings

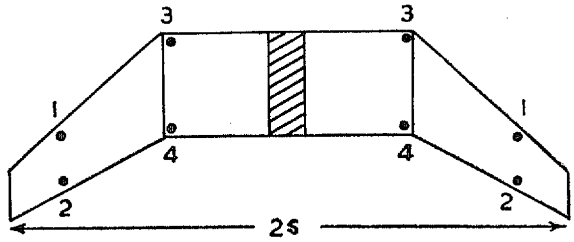


TABLE 4

Values of a_{ij} (in./lb)

where a_{ij} is deflection at position i due to unit load at position j

	Port				Starboard			
	i				i			
j	1	2	3	4	1	2	3	4
1	2·685	3·096	0·447	0·942	3·020	3·247	0·534	0·909
2	3·016	4·536	0·346	1·334	3·197	4·670	0·329	1·258
3	0·393	0·264	0·286	0·121	0·482	0·327	0·295	0·133
4	0·927	1·310	0·141	0·605	0·867	1·205	0·135	0·545

Positions 1 and 2 at 0·72s. Positions 3 and 4 at 0·32s.

TABLE 5

Values of l and m (in.)

where l = distance of loading point from leading edge.
m = distance of measuring point from leading edge.

	Port				Starboard			
i or j	1	2	3	4	1	2	3	4
l	10·90	18·80	0·50	11·90	10·82	19·31	0·41	12·01
m	10·65	18·85	0·50	11·90	10·80	18·82	0·55	11·60

TABLE 6

Stiffnesses for Cranked Wings

	Port	*Starboard*
Section (1, 2) 0·72s.		
*Flexural stiffness	4·68 lb/ft	4·02 lb/ft
†Torsional stiffness	4·87 lb ft/radn	4·55 lb ft/radn
Section (3, 4) 0·32s.		
*Flexural stiffness	48·4 lb/ft	48·1 lb/ft
Torsional stiffness	17·21 lb ft/radn	18·65 lb ft/radn

* The flexural stiffness is here defined as the load which applied at the flexural centre causes unit linear displacement.
† This refers to a section in a fore-and-aft plane.

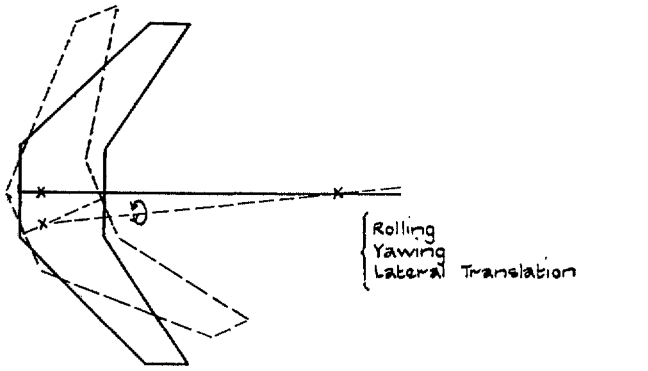


FIG. 1. Suspension systems.
Upper diagram—Symmetrical flutter.
Lower diagram—Antisymmetrical flutter.

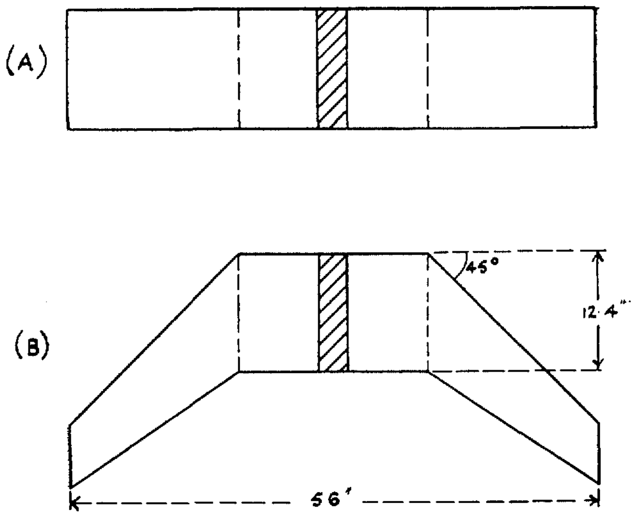


FIG. 2. Plan forms of model.

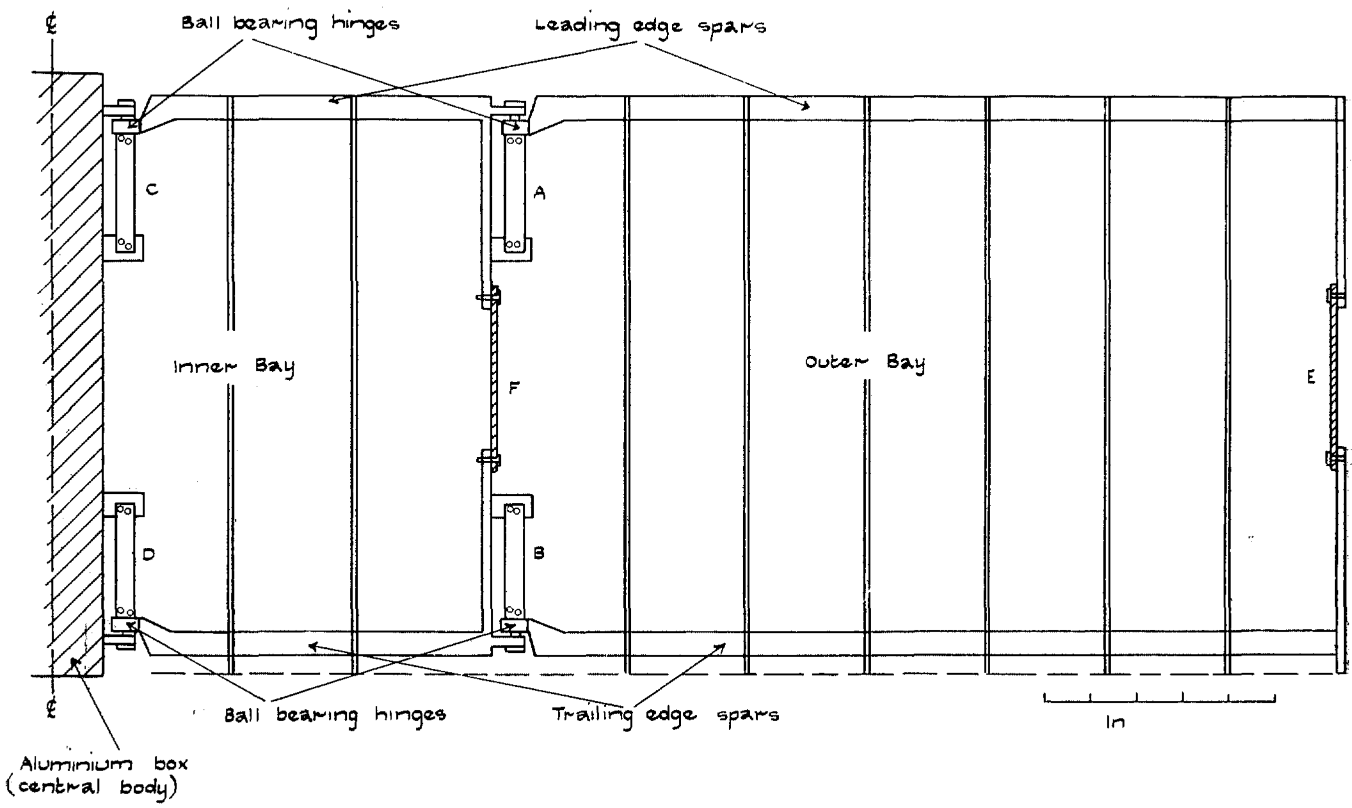


FIG. 3. Wing construction (showing flat steel torque springs A B C D E F).

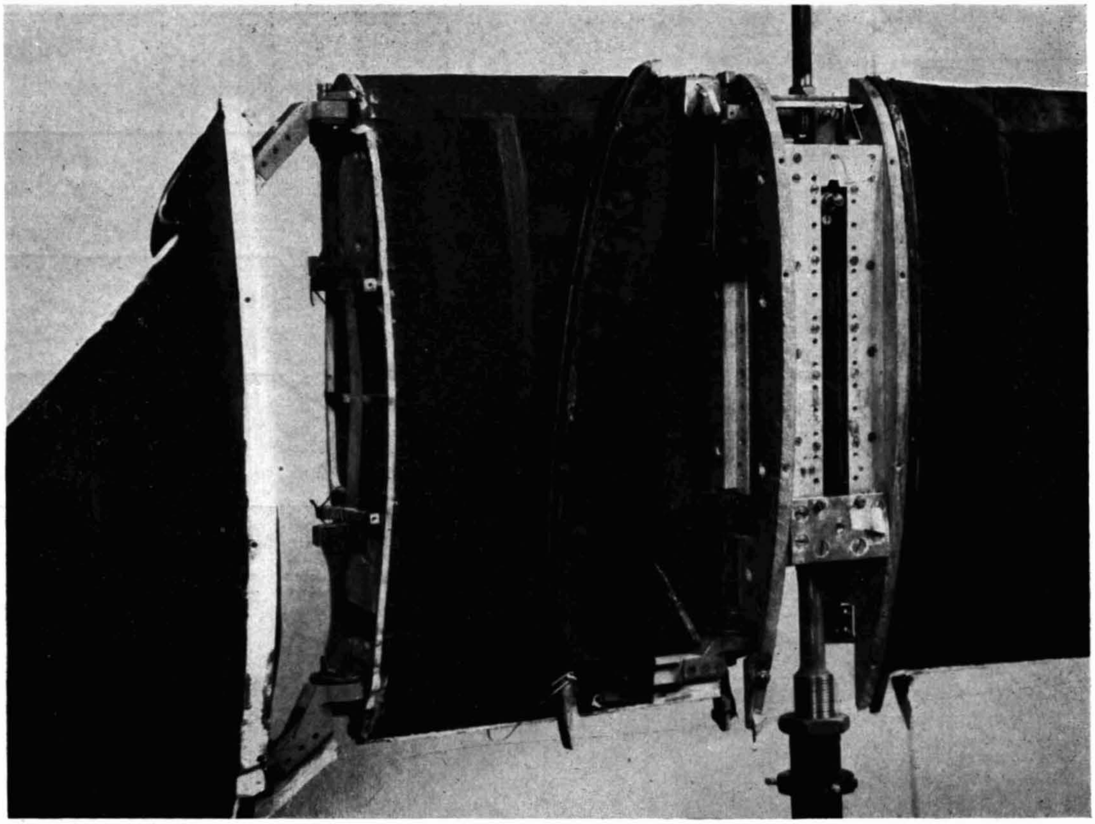


FIG. 4. Cranked wings (covering partly removed to show the internal springs).

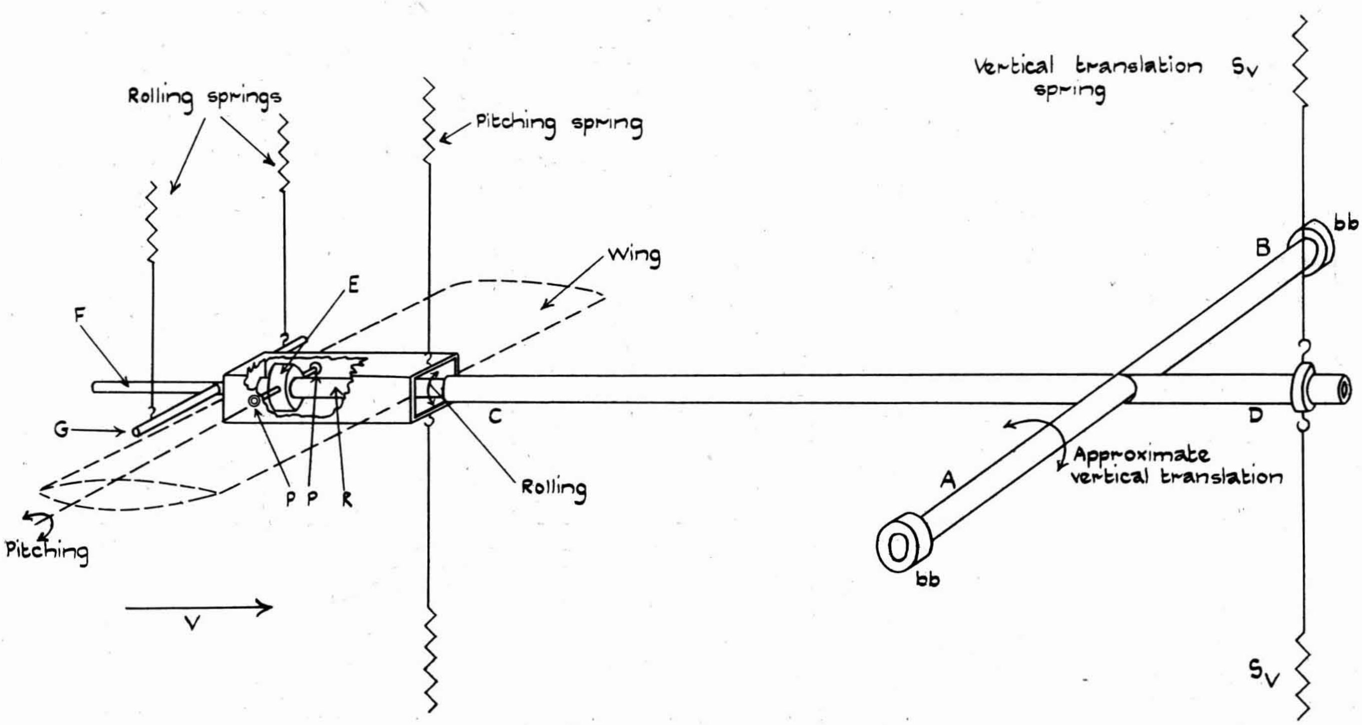


FIG. 5. Scheme of suspension.

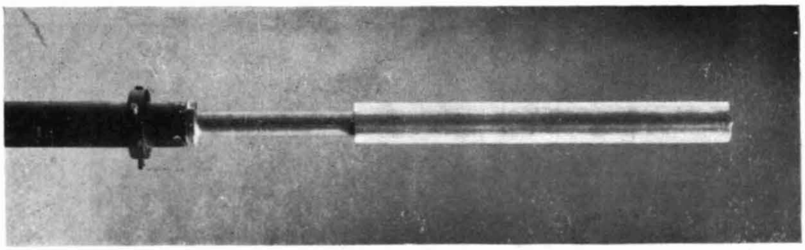


Fig. 6. Sleeve providing axis of roll.

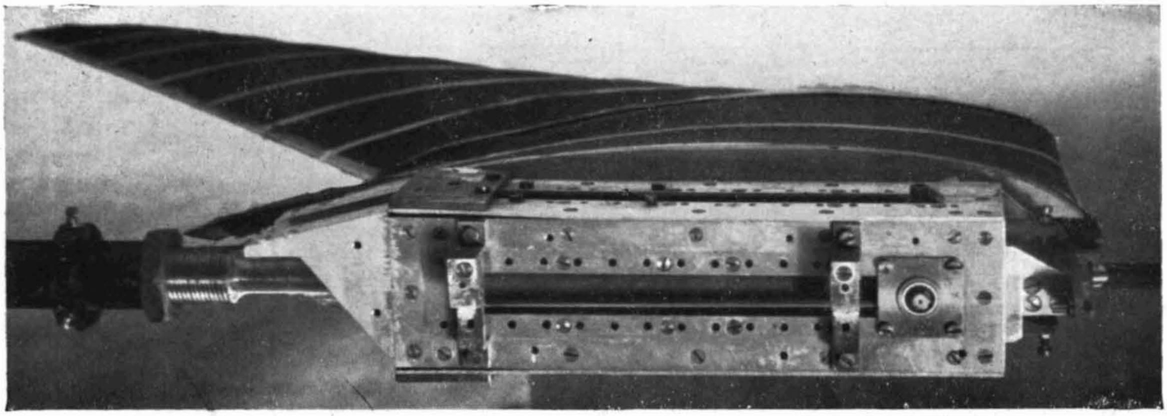


Fig. 7. The central body (without covering) with one cranked wing attached.

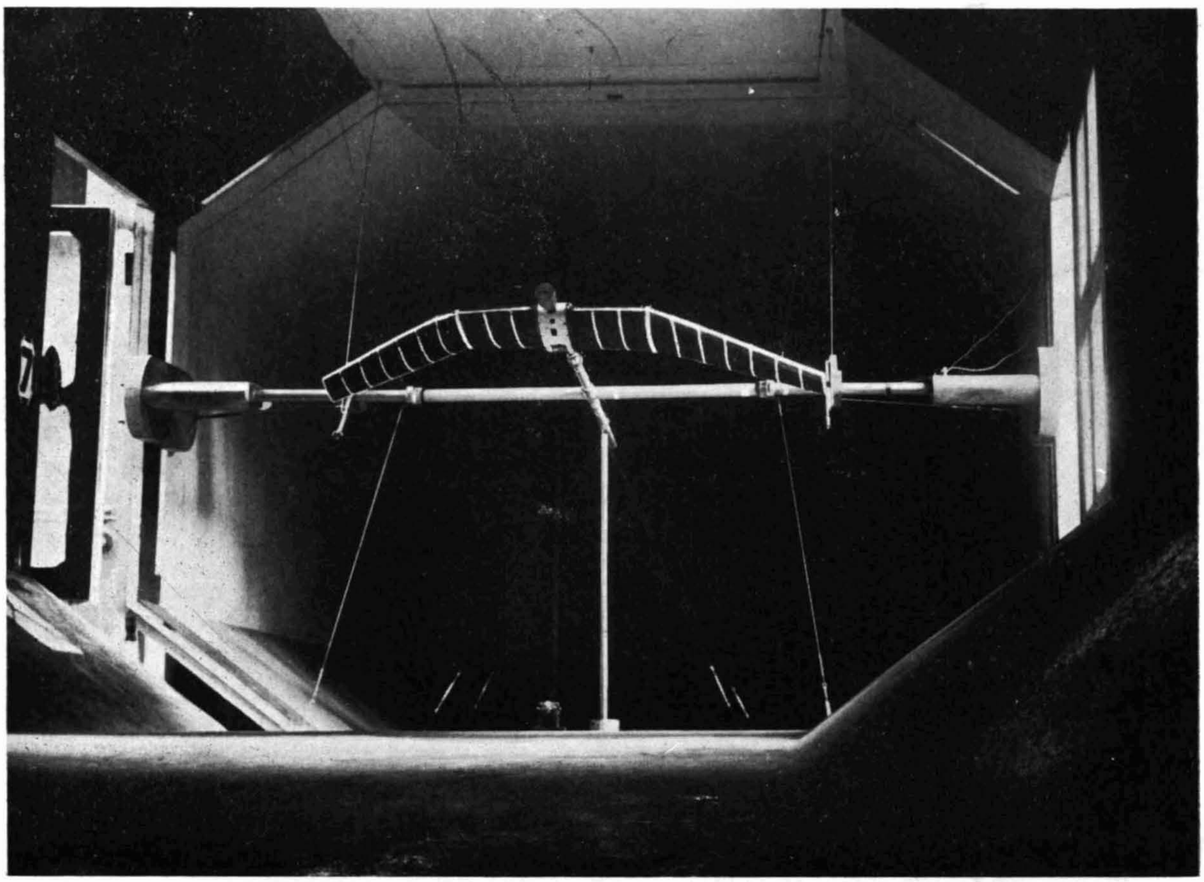


Fig. 8. Cranked wing mounted in the tunnel with freedoms in pitching and vertical translation.

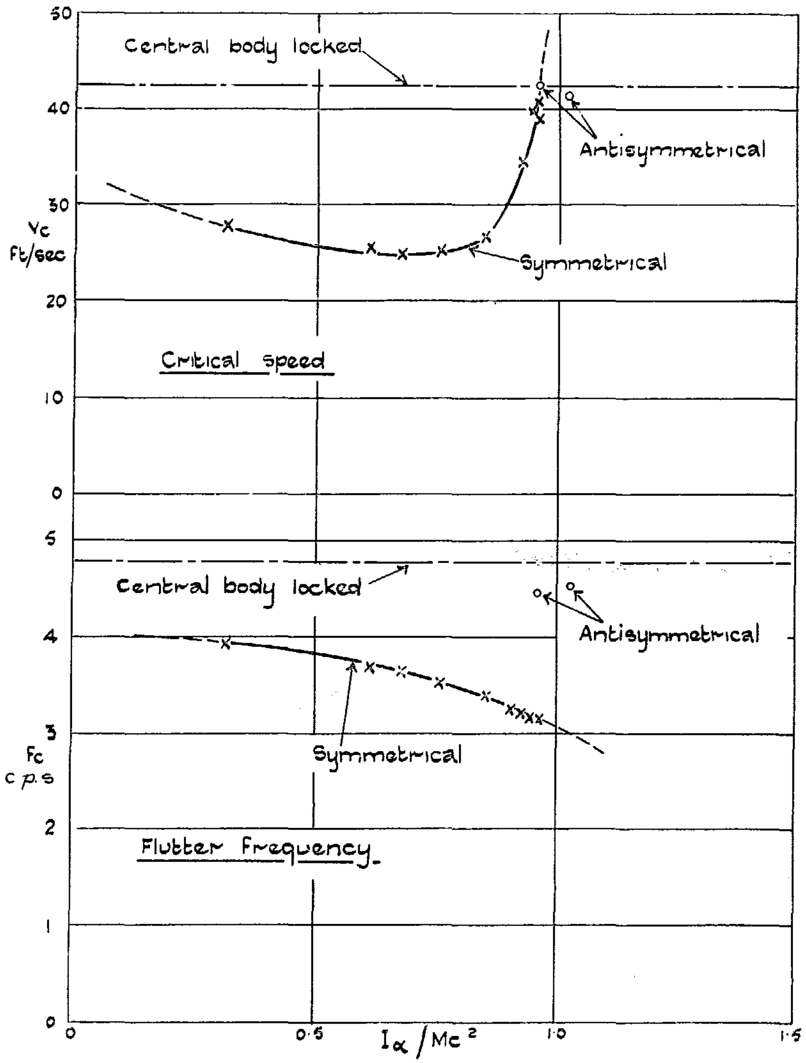


FIG. 9. Rectangular wings central body free to pitch. Variation of critical speed and frequency with pitching inertia of central body. (Pitching axis 0·2 chord, Pitching stiffness $m_\alpha = 20{\cdot}4$ lb ft/radn, M = mass of wings).

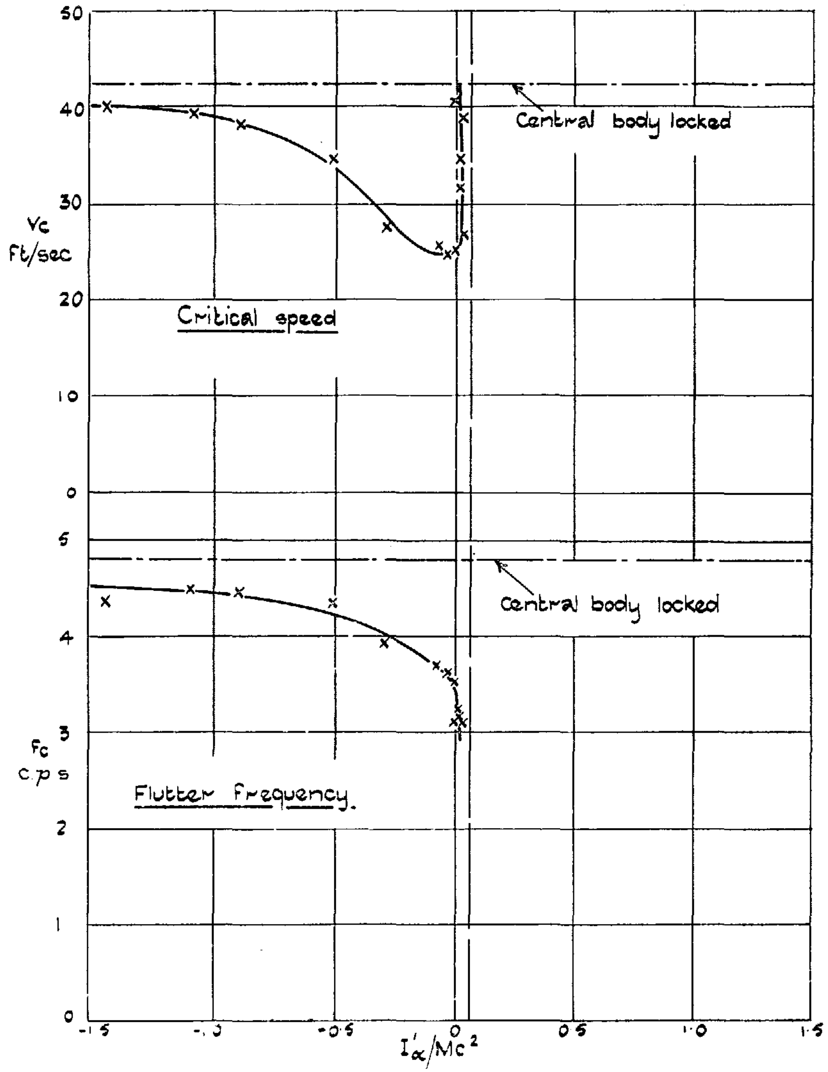


FIG. 10. Rectangular wings. Central body free to pitch about 0·2c. Variation of critical speed and frequency with effective pitching inertia of central body. ($I_\alpha' = I_\alpha - m_\alpha/4\pi^2 f_c^2$).

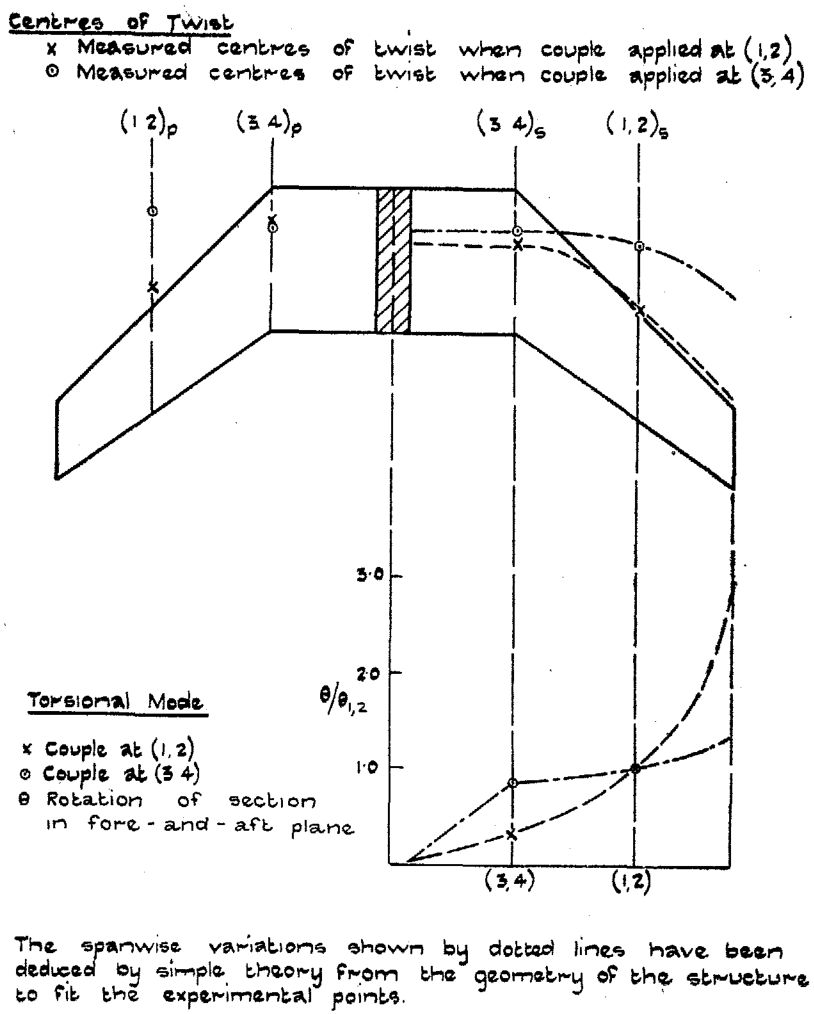


FIG. 11. Distortion of the model cranked wing due to a couple (central body held rigidly).

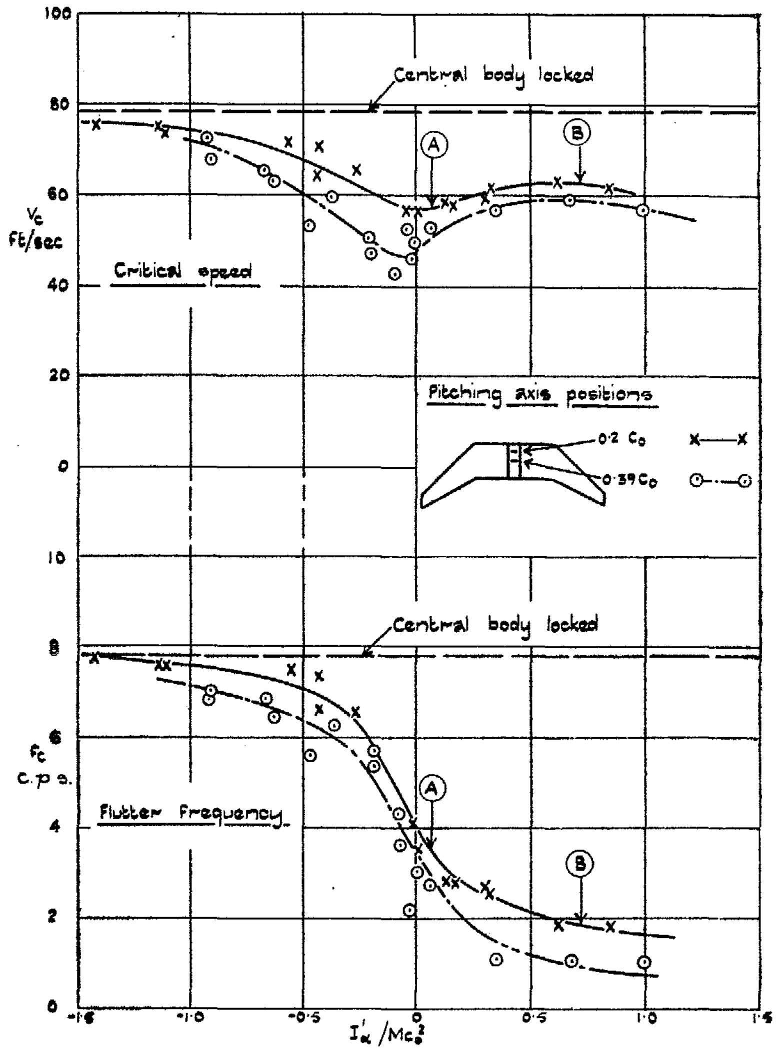


FIG. 12. Cranked wings. Central body free to pitch. Variation of critical speed and frequency with effective pitching inertia of central body. $I_\alpha' = I_\alpha - m_\alpha/4\pi^2 f_c^2$.

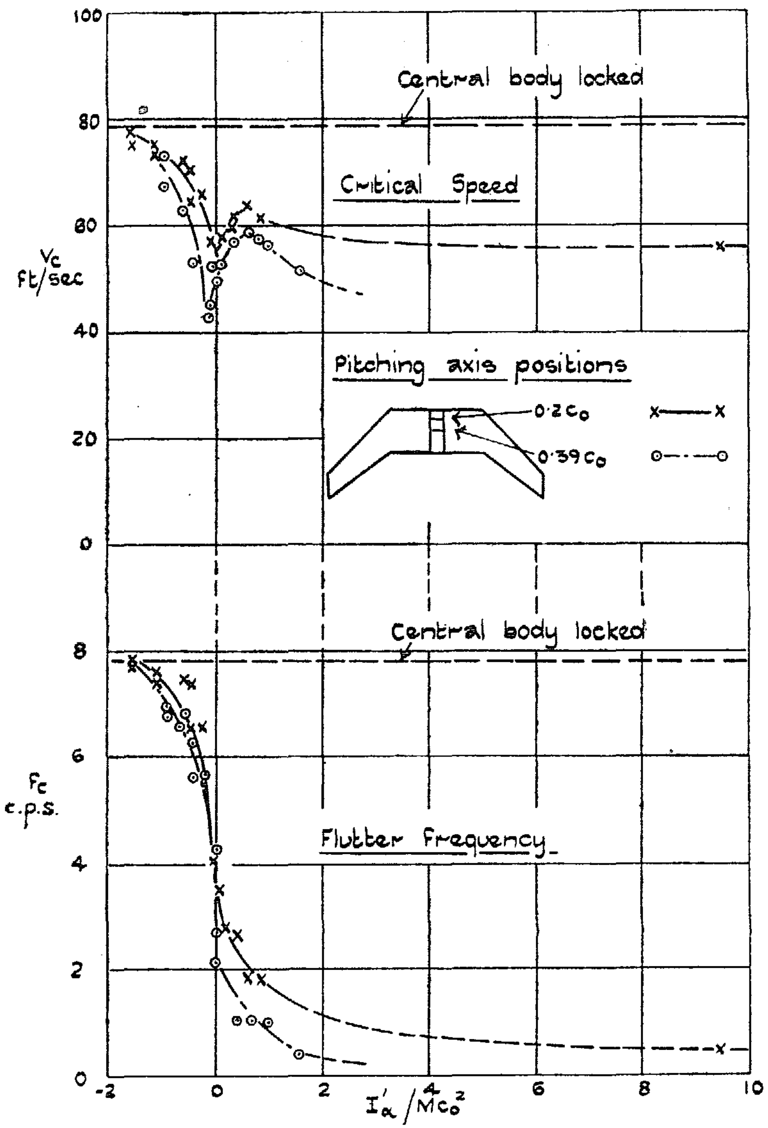


FIG. 13. Cranked wings. Central body free to pitch. Variation of critical speed and frequency with effective pitching inertia of central body. $I_\alpha' = I_\alpha - m_\alpha/4\pi^2 f_c^2$.

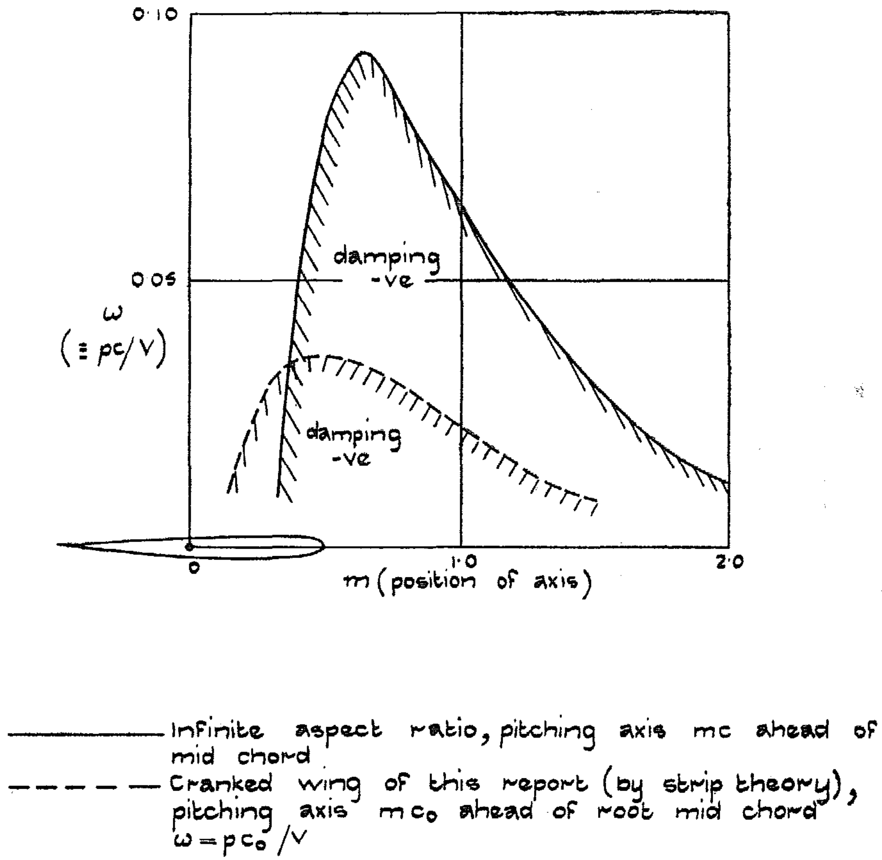


FIG. 14. Theoretical aerodynamic pitching damping. Values of frequency parameter ω and position of pitching axis for which damping is zero, calculated from two-dimensional vortex sheet theory (R. & M. 1958).

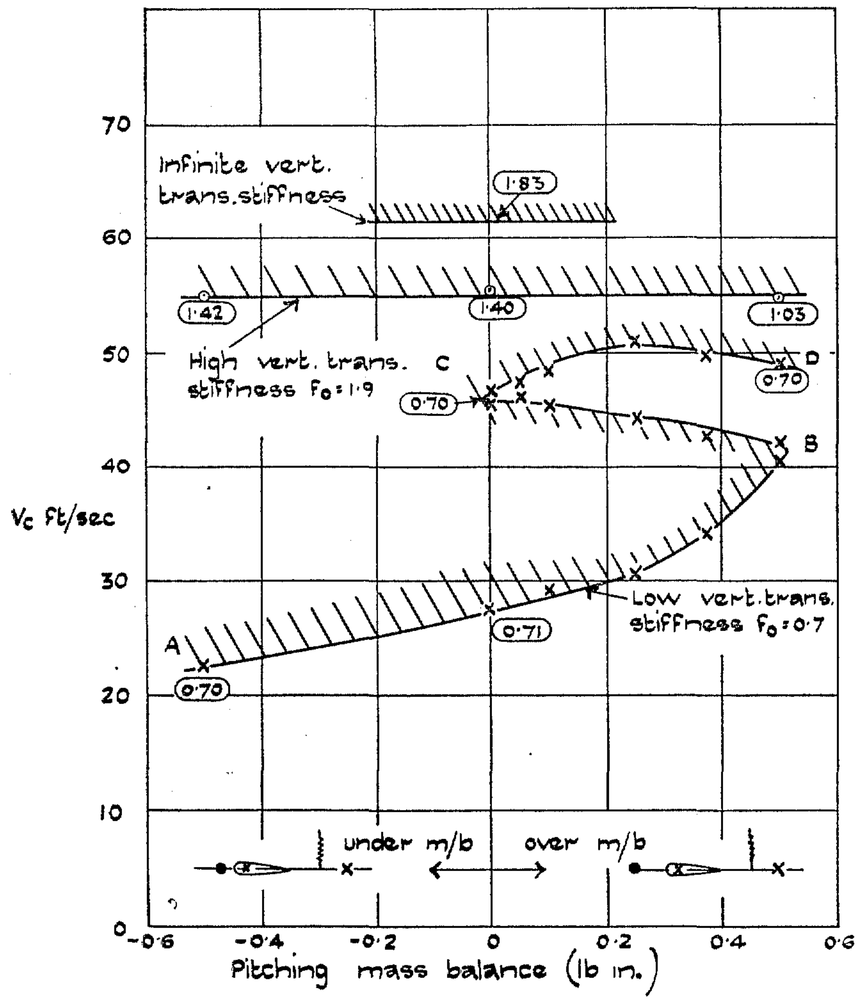


FIG. 15. Cranked wings. Freedoms in pitch (about $0{\cdot}2c_0$) and vertical translation (spring constrained). Variation of critical speed with pitching mass balance for 3 values of vertical translation stiffness. Frequencies in c.p.s. are given by the encircled numbers.

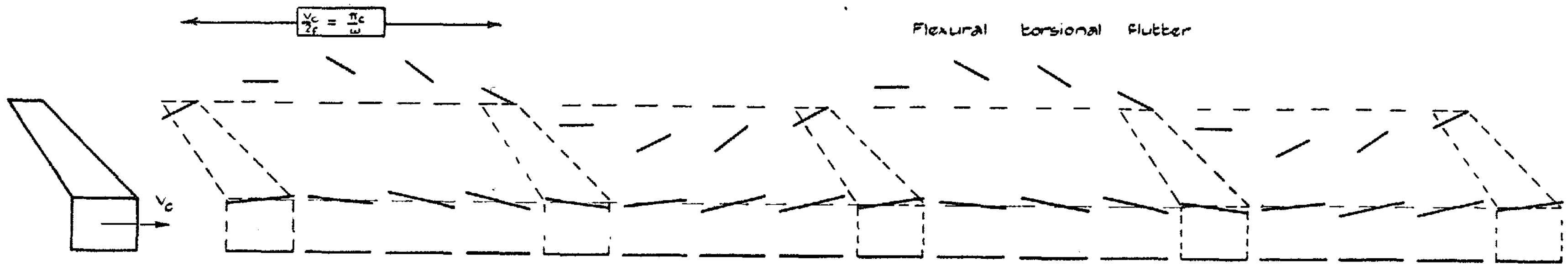


FIG. 16. Central body-clamped.

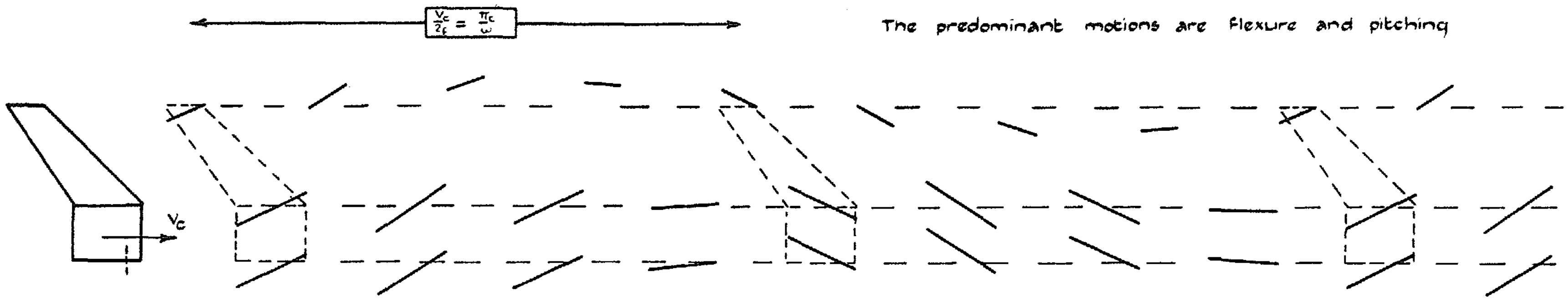


Fig. 17. Central body free to pitch about $0{\cdot}2c_0$. (Low pitching inertia, see point A on Fig. 12).

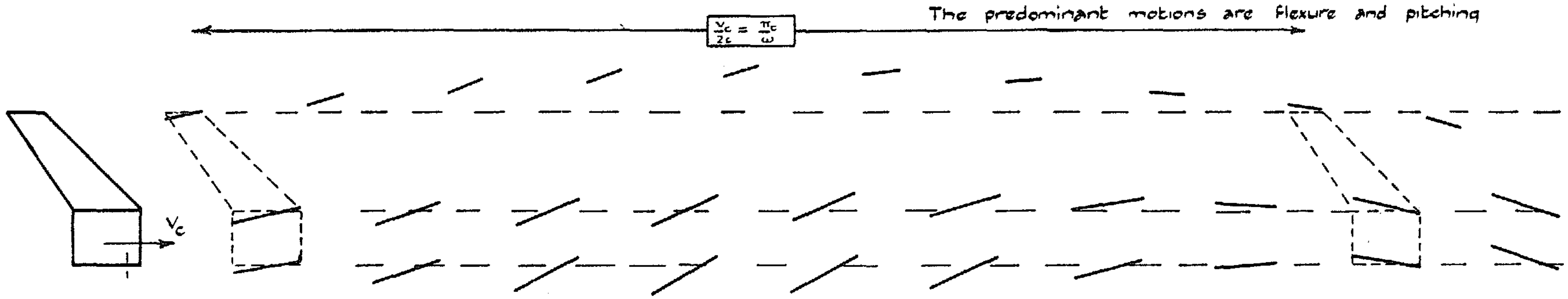


Fig. 18. Central body free to pitch about $0{\cdot}2c_0$. (High pitching inertia, see point B on Fig. 12).

Flutter modes obtained from cinematograph records.

Ps. 13027. Wt. 15. K9. 7/51. (H.P.W.—34374). Gp. 1008.

PRINTED IN GREAT BRITAIN

R. & M. No. 2626
(10,509)
A.R.C. Technical Report

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)—

1934–35 Vol. I. Aerodynamics. *Out of Print.*
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40*s.* (40*s.* 8*d.*)

1935–36 Vol. I. Aerodynamics. 30*s.* (30*s.* 7*d.*)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30*s.* (30*s.* 7*d.*)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s.* (50*s.* 10*d.*)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 10*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s.* (61*s.*)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50*s.* (51*s.*)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s.* (30*s.* 9*d.*)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (50*s.* 11*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 2*d.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s.* (51*s.*)

Certain other reports proper to the 1940 *volume will subsequently be included in a separate volume.*

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933–34	1*s.* 6*d.* (1*s.* 8*d.*)
1934–35	1*s.* 6*d.* (1*s.* 8*d.*)
April 1, 1935 to December 31, 1936.	4*s.* (4*s.* 4*d.*)
1937	2*s.* (2*s.* 2*d.*)
1938	1*s.* 6*d.* (1*s.* 8*d.*)
1939–48	3*s.* (3*s.* 2*d.*)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 R. & M. No. 2600. 2*s.* 6*d.* (2*s.* 7½*d.*)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936—June 30, 1939	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 4½*d.*)
July 1, 1939—June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 1½*d.*)
July 1, 1945—June 30, 1946.	R. & M. No. 2050.	1*s.* (1*s.* 1½*d.*)
July 1, 1946—December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 4½*d.*)
January 1, 1947—June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 4½*d.*)

Prices in brackets include postage.

Obtainable from

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller

S.O. Code No. 23-2626