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A Revised Index of Mathematical Tables for Compressible Flow

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1. *Introduction.*—This revision of the original " Index of Mathematical Tables for Compressible Flow " by A. O. L. Atkin (A.R.C. 9893, August, 1946) has been prepared at the request of the Fluid Motion Sub-Committee of the Aeronautical Research Council. It contains information of all the relevant tables known to the author, which can be obtained by workers outside the establishment of origin. The purpose of the index is to make available to workers in the field of compressible flow a reference from which they may trace a tabulation of any function they require, if it exists. It is also hoped that it may help to prevent waste of effort by unnecessary duplication of tables.

The first revision of this Index by the present author in June, 1948, showed that there were in existence tabulations of most of the functions required by workers in this field. These tabulations were, however, scattered throughout a large number of reports, some of which were not easily accessible. As a consequence of this, it was often necessary to use an inferior table or do without. Furthermore, there was no guarantee of the accuracy of the various tables, and there was good reason to be suspicious of some of the tables that would have been the most useful if they had been accurate. It was recommended that a book of collected tables should be prepared, and the Compressible Flow Tables Panel of the Aeronautical Research Council was set up to do this. The book[48], together with a companion volume of graphs, is to be published by the Clarendon Press. In preparing for the book a number of mistakes were found in the tables listed in this Index, but no systematic checks were made. There seems little point in listing here the few mistakes found—it is sufficient to offer a warning.

2. *Classification of the Tables.*—The tables are classified under the following headings:—

A Tables of powers and other simple relations used in compressible flow problems (*e.g.*, powers of $\sqrt{(1 - M^2)}$ and fractional powers).

B Tables of Functions dependent on the elementary physical relationships based on Bernouilli's equation, the equation of state, etc. These are subdivided into tables covering the subsonic, supersonic and both ranges.

C Tables particularly used in the method of characteristics. (Mostly tables with μ or ω as argument.)

D Shock Tables. These are subdivided into tables for normal shocks, oblique shocks and the solutions to the supersonic flow past cones.

E Tables for use in the application of approximate methods other than the hodograph method, and for any special problems.

F Tables for use in the hodograph method.

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The classes are not logically independent, but in practice no confusion should arise. It has not been necessary to enter any table twice.

Each table has been described in its class in the following way. If $F(x)$ is being tabulated as a function of x, x is placed first and underlined as the independent variable. Then $F(x)$ is written, followed by the range of argument tabulated. Figures in brackets indicate the interval of argument, the flanking figures the extremities of the range. Finally the number of significant figures or figures after the decimal point of $F(x)$ is given; in some instances these are followed by R and a number indicating the references in which the table is to be found. Thus

$$\underline{\mu},\ F(\mu),\ 90(\cdot 1)5°,\ 3\,\mathrm{S},\ \mathrm{R}\ 14$$

would mean that in Ref. 14 there is a table in which $F(\mu)$ is tabulated against μ to three significant figures, for μ varying by intervals of 0·1 deg from 90 deg to 5 deg.

Unless otherwise stated the tables have been computed assuming $\gamma = 1 \cdot 400$.

3. *Nomenclature*.—The notation adopted is that which was suggested by the Compressible Flow Tables Panel of the A.R.C. (A.R.C. Report 12,006) and accepted by the Fluid Motion Sub-Committee. The notations used in the various tables referred to have all been translated into this approved notation for the purposes of this Index.

NOTATION

a	Speed of sound
p	Pressure
q	Speed of fluid
r	Distance of streamline from vertex in Prandtl-Meyer expansion
A/A_s	Ratio of cross-sectional area of streamline to that at $M = 1$
C_p	Pressure coefficient
M	Mach number
M_2'	Mach number of flow following a shock moving into still air
R	Reynolds number or gas constant
T	Temperature
β	$\begin{cases}(1 - M^2)^{1/2} & M < 1 \\ (M^2 - 1)^{1/2} & M > 1\end{cases}$
γ	Ratio of specific heats
δ	Deflection of flow in passing through an oblique shock
ζ	Acute angle between streamline and shock
θ	Angle between stream direction and axis
μ	Mach angle
ρ	Density
τ	$(q/q_{\max})^2$
ϕ	Angular co-ordinate measured from the sonic characteristic in the Prandtl-Meyer expansion
ω	Angular deviation of flow referred to direction at the sonic point of the characteristic
Suffixes 1	Conditions before shock
2	Conditions after shock
o	Stagnation conditions
m	Conditions behind shock giving maximum stream deflection
max	Conditions obtainable after efflux to vacuo
s	Sonic conditions

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4. TABLES A—*Powers and other Simple Relations*

$\underline{x}$, $\underline{r}$, x^r, x 0·1(·01)1·5, 10, 10^2, 10^3, 10^4, 10^5, 10^6,
r 1/7, 2/7, 5/7; 1/5, 2/5, 7/5; 5/2, 7/2; 5 S, R 48. .. .. .. (1)

$\underline{x}$, $\underline{r}$, x^r, x 0·01(·01)0·99, r 0·001(·001)0·01(·01)0·99, 15 D, R 29. .. .. (2)

$\underline{x}$, $\underline{r}$, x^r, x 1(·01)5·40, $r\ \frac{7}{5}, \frac{5}{7}, \frac{2}{5}, \frac{5}{2}, \frac{7}{2}, \frac{2}{7}$, 4 S or 4 D, R 44. .. .. .. (3)

$\underline{x}$, $\underline{r}$, x^r, x 0·5(·1)1(·2)3(·5)5, 10, $\frac{1}{10}$, r 0(·05)1, 4 to 5 S, R 21. .. .. .. (4)

$\underline{x}$, $x^{1/5}$, $x^{2/5}$, 0·1(·1)8(·2)10, 5 S, R 22. .. .. (5)

$\underline{x}$, $x^{5/2}$, 0(·01)30(·1)99·9, 4 S, R 23. .. .. (6)

$\underline{x}$, $x^{1/7}$, 0·8(·001)1, 5 S, R 15. .. .. (7)

$\underline{x}$, $x^{6/7}$, 0·8(·001)1·2, 5 D, R 15. .. .. (8)

$\underline{x}$, $x^{2/7}$, 0·5(·01)2(·1)6, 7 D, R 51. .. .. (9)

$\underline{r}$, 10^r, π^r, 0·001(·001)1, 15 D, R 29. .. .. (10)

$\underline{M}$, $\underline{r}$, β^r, M 0(·005)1, r −7(1) − 1; 1(1)7, 5 S or 5 D, R 48. .. .. .. (11)

$\underline{M}$, $\underline{r}$, β^r, $\frac{M^2}{1+\beta}$, M 0(·01)1, $r \pm$ 1, 2, 3, 4, 5 S, R 17. .. .. (12)

$\underline{M}$, $\frac{1}{\beta}$, 0(·005)0·1(·001)0·9(·0005)0·96(·0002)0·99(·0001)0·995(var)0·9⁹0,
10 to 7 D, R 30. (13)

$\underline{M}$, β, 0(·0001)1, 8 D, R 31. .. .. (14)

$\underline{x}$, $\underline{r}$, $x^{1/r}$, $x^{(r-1)/r}$, $\sqrt{(1-x^{(r-1)/r})}$, $x^{1/r}\sqrt{(1-x^{(r-1)/r})}$,
x 1(·02)0, r 1(·05)1·8, 4 D, R 49. .. .. (15)

5. TABLES B—*Physical Tables*

(a) *Subsonic Flow Only*

$\underline{M}$, $\frac{p}{p_0}$, $\frac{p_0 - p}{\frac{1}{2}\rho q^2}$, $\frac{p_0}{\frac{1}{2}\rho q^2}$, $\frac{p_0}{p}$, 0(·001)1, 5 or 6 S, R 1. .. .. (16)

$\underline{M}$, $\frac{p}{p_0}$, $\frac{\rho}{\rho_0}$, $\frac{T}{T_0}$, $\frac{A}{A_s}$, $\frac{q}{a_0}$, 0(·001)1, 5 S or 5 D, R 2. .. .. (17)

$\underline{M}$, $\frac{p}{p_0}$, $\frac{\rho}{\rho_0}$, $\frac{T}{T_0}$, $\frac{A_s}{A}$, $\frac{a}{a_0}$, q ft/sec, $\frac{1}{2}\rho q^2$, $\frac{p_0 - p}{\frac{1}{2}\rho q^2}$, $1/\beta$, 0(·01)1, 4 S, R 45. .. .. (18)

$\underline{M}, \frac{p}{p_o}, \frac{p_o}{p}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{a}{a_o}, \frac{q}{a_s}, \frac{A}{A_s}, \frac{p_o - p}{\frac{1}{2}\rho q^2}, \frac{p_o}{\frac{1}{2}\rho q^2}, \frac{\frac{1}{2}\rho q^2}{p_o}, (1 + \gamma M^2)\frac{p}{p_o}$,
0(·01)0·5(·001)1, 5 D, R 48. .. (19)

$\underline{M}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{a}{a_o}, \frac{A_s}{A}, \frac{q}{q_{\max}}, \frac{q}{q_s}, \frac{q}{a_o}$, 0(·01)1, 4 S, R 25. .. .. .. (20)

$\underline{M}, \frac{\frac{1}{2}\rho q^2}{p_o}, \frac{T}{T_s}, \frac{a}{a_s}, \frac{\rho}{\rho_s}, \frac{A_s}{A}$, 0(·05)1, 4 D, R 8. .. .. (21)

$\underline{\tau}(\gamma = 1{\cdot}405), \frac{q}{q_s}, M, \frac{\rho}{\rho_o}, \frac{p}{p_o}$, 0(·02)0·40, 4 D, R 35. .. .. (22)

$\underline{\frac{p_o - p}{p}}, M$, 0(·001)0·11(·002)0·3(·005)0·895, 3 S, R 15. .. .. (23)

$\underline{\frac{p}{p_o}}, \underline{M}, C_p, \frac{p}{p_o}$ 1(·01)0·2, M 0·05(·05)0·6(·02)0·8(·1)1, 3 S, R 48. .. .. (24)

$\underline{M}, C_{ps}$, 0(·01)1, 5 S, R 48. .. .. (25)

$\underline{q}, (p_o - p), p_o, q$ is given in m.p.h., knots and km.p.h., the pressures in atmospheres and lb/sq ft, R 18. .. .. (26)

(b) *Tables Covering Subsonic and Supersonic Speeds*

$\underline{M}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{\frac{1}{2}\rho q^2}{p}, \frac{\frac{1}{2}\rho^2 q}{p_o}, \frac{A}{A_s}, \frac{q}{a_o}, \frac{q}{a_s}, \frac{T}{T_o}, \frac{\rho q T_o^{1/2}}{p}, (1 + \gamma M^2)\frac{p}{p_o}$, 0(·01)5,
5 D, R 44. .. .. .. (27)

$\underline{M}, \frac{T}{T_o}, \frac{p}{p_o}, \frac{a}{a_o}, \frac{q}{a_o}, \frac{A}{A_s}, \frac{\rho q}{\rho_o a_o}, \frac{\frac{1}{2}\rho q^2}{p_o}, \frac{\rho}{\rho_o}$, 0(·001)0·05(·01)0·8(·001)1·2(·01)2(·1)5(1)25,
3–5 S, R 28. .. .. .. (28)

$\underline{M}, \frac{R}{R_s}$, for $T_o = 200(100)500, 1000, 2000$ °K, 0(·1)5, 3 S, R 48. .. .. .. (29)

$\underline{M}, \frac{q}{q_s}, \frac{A}{A_s}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{(1 + \gamma M^2)}{(1 + \gamma)}\frac{pA}{p_s A_s}, \frac{pA}{p_o A_s}$, 0(·01)3(·1)5(1)10, 5 S, R 49. (30)

$\underline{M}, \frac{p}{p_o}, \frac{T}{T_o}, \frac{\rho}{\rho_o}, \frac{A_s}{A}, \frac{q}{q_{\max}}, \mu, \omega$, 0(·01)4, 4 S, R 50. .. .. (31)

$\underline{\frac{p}{p_o}}, M$, 1(·001)0, 4 S, R 48. .. .. (32)

$\underline{\frac{p}{p_o}}, \frac{p_o}{p}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{q}{q_{\max}}, \frac{\rho q}{\rho_o q_{\max}}$, 1(·001)0·96(·002)0·90(·005)0·20(·002)0·07(·001)
0·04(·0005)0·01(·0002)0·004(·0001)0, 5 D, R 49. .. .. .. (33)

$\underline{\frac{p}{p_o}}, \frac{A}{A_s}, M, q$ (ft/sec), 1(·001)0·04(·0002)0·001, 3 to 5 S, R 44. .. .. .. (34)

$\underline{\frac{p}{p_o}}\ (\gamma = 1{\cdot}405), M, \frac{q}{q_s}, \frac{\rho q}{\rho_s a_s}$, 1 (about 40 irregular intervals) 0, 3 D, R 37. .. (35)

$\underline{\tau}, \frac{q}{q_{\max}}, \frac{\rho}{\rho_o}, \frac{p}{p_o}, M, \frac{\rho q}{\rho_o q_{\max}}, \frac{p_o - p}{\frac{1}{2}\rho q^2}$, 0(·01)1, 5 D or 5 S, R 40 and R 48. .. .. (36)

$\underline{\frac{q}{a_s}}, \frac{p}{p_s}, \frac{\rho}{\rho_s}, \frac{T}{T_s}, \frac{a}{a_s}, M, \frac{A_s}{A}$, 0(·01)2·44 and $\sqrt{6}$, 5 D, R 48. .. .. .. (37)

$\underline{\frac{q}{a_s}}, \mu$, 1(·005)1·5(·01)2·44 and $\sqrt{6}$, 5 S, R 48. .. .. (38)

$\underline{\frac{q}{q_{\max}}}, \frac{\rho}{\rho_o}, \frac{p}{p_o}, \frac{T}{T_o}, \frac{a}{a_o}, M, \frac{\rho q}{\rho_o q_{\max}}, \frac{p_o - p}{\frac{1}{2}\rho q^2}$, 0(·01)1, 4 or 5 D, R 40. .. .. (39)

$\underline{\frac{\rho}{\rho_o}}, \frac{q}{q_{\max}}, \frac{p}{p_o}, \frac{T}{T_o}, \frac{a}{a_o}, M, \frac{\rho q}{\rho_o q_{\max}}, \frac{p_o - p}{\frac{1}{2}\rho q^2}$, 0(·01)1, 5 D or 5 S, R 40. .. (40)

(c) *Supersonic Flow Only*

$\underline{M}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{A}{A_s}, \frac{a}{a_o}, \frac{\rho q^2}{\rho_s {q_s}^2}, q, \frac{1}{2}\rho q^2, 1/\beta, \mu$ (2 D), ω (2 D), 1(·01)10,
4 S, R 45. .. .. (41)

$\underline{M}, (\gamma = 1{\cdot}4$ and $1{\cdot}27), X, \frac{dX}{d\gamma}$, (Separate tables for $X = \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{A}{A_s}, \frac{\frac{1}{2}\rho q^2}{p_o}$),
1(·01)5, 5 and 3 S, R 44. .. .. .. (42)

$\underline{M}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{a}{a_o}, \frac{q}{a_s}, \frac{A}{A_s}, \frac{p}{\frac{1}{2}\rho q^2}, \frac{\frac{1}{2}\rho q^2}{p_o}, (1 + \gamma M^2)\frac{p}{p_o}, \mu$, 1(·01)5,
5 S or D, R 48. .. (43)

$\underline{M}, \frac{p}{p_o}, \frac{d}{d\gamma}\left(\frac{p}{p_o}\right), \frac{\frac{1}{2}\rho q^2}{p_o}, \frac{d}{d\gamma}\left(\frac{\frac{1}{2}\rho q^2}{p_o}\right)$, 0(·1)5, 5 and 3 D, R 48. .. .. .. (44)

$\underline{M}, M^2, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{a}{a_o}, \frac{A_s}{A}, \frac{\frac{1}{2}\rho q^2}{p}, \frac{\frac{1}{2}\rho q^2}{p_o}, \beta, \frac{1}{\beta}, \omega, \mu$,
1(·01)3·5(·1)5(1)10(5)20, 100, ∞, 4 to 5 S, R 25. .. .. (45)

$\underline{M}, \frac{p}{p_o}, \frac{\rho}{\rho_o}, \frac{T}{T_o}, \frac{A}{A_s}, \frac{q}{a_o}, \mu$, 1(·01)3, 5 to 6 S, R 3. .. .. (46)

$\underline{M}, \frac{\rho}{\rho_s}, \frac{p}{p_s}, \frac{a}{a_s}, \frac{q}{q_s}, \mu, \omega$, 1(·05)2(·1)4, 4 or 5 S, R 7. .. .. (47)

$\frac{p}{p_o}$ ($\gamma = 1 \cdot 4$ and $1 \cdot 27$), $\frac{A}{A_s}$, $\frac{d}{d\gamma}\left(\frac{A}{A_s}\right)$, $0 \cdot 530(\cdot 002)0 \cdot 06(\cdot 001)0 \cdot 035$, 4 and 2 D, R 44. .. (48)

$\frac{p}{p_o}$, M, $\frac{dM}{d\gamma}$, $0 \cdot 530(\cdot 002)0 \cdot 21(\cdot 001)0 \cdot 05(\cdot 0005)0 \cdot 014(\cdot 0002)0 \cdot 0092$, 3 D, R 44. (49)

$\frac{p}{p_o}$, M, $\frac{dM}{d\gamma}$, $0 \cdot 53(\cdot 01)0$, 3 D, R 48. .. .. (50)

(d) *Other Physical Quantities*

T, a, 300(2)698 °K, 4 to 5 S, R 28. .. .. (51)

T, a, 150(10)350 °K, 4 S, R 44. .. .. (52)

T, R_s/lp_o where l is in ft and p_o is in lb wt/sq in., 150(50)2000 °K, 3 to 4 S, R 48. (53)

6. TABLES C—*Characteristic Tables*

The following relations should be noted in the interpretation of these tables:

$$\omega = f\left(\frac{\pi}{2}\right) - f(\mu)$$

$$\frac{p}{p_o} = \frac{g(\mu)}{g(\mu_o)}$$

$$\frac{\rho}{\rho_o} = \frac{h(\mu)}{h(\mu_o)}$$

$$G(\mu) = \left(1 + \frac{1}{5}\operatorname{cosec}^2 \mu\right)^3$$

$$F(\mu) = \frac{(\cot \mu)^{1/2}}{(1 + \frac{1}{5}\operatorname{cosec}^2 \mu)^{7/4}}.$$

It should also be noted that Busemann's pressure number P, used in German reports, is equivalent to $(1000 - \omega)$.

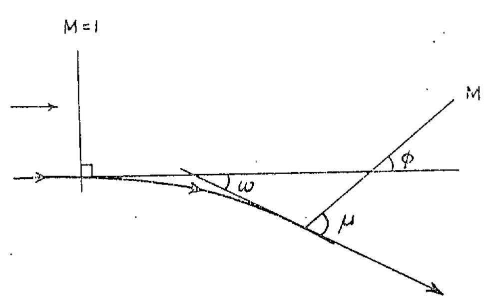

FIG. 1.

$\underline{\omega}$, μ, M, $\frac{q}{a_s}$, $\frac{q}{q_{\max}}$, $\frac{p}{p_s}$, $\frac{\rho}{\rho_s}$, $\frac{T}{T_s}$, $\frac{a}{a_s}$, $\frac{A_s}{A}$, $\frac{r_s}{r}$, 0(·5)100(1)130°. 5 S or D, R 48. .. (54)

$\underline{\omega}$, μ, M, 0(·001)0·05°; 0(·01)1°; 0(·1)30°, 5 S, R 48. .. .. (55)

$\underline{\omega}$, μ, 0(·001)0·3(·01)5(·1)60(1)130°, 3 D, R 16. .. .. (56)

$\underline{\omega}$, μ, M, 0(·1)30°, 7 S, R 5. .. .. (57)

$\underline{\omega}$, μ, $\frac{p}{p_o}$, 0(1)53°, 3 and 4 S, R 11. .. .. (58)

$\underline{\omega}(\gamma = 1{\cdot}405)$, $\frac{p}{p_o}$, $\frac{q}{q_s}$, M, $\frac{T}{T_o}$, $\frac{A_s}{A}$, 0(1)53°, 3 D, R 6. .. .. (59)

$\underline{\omega}$, μ, M, $\frac{q}{q_s}$, $\frac{p}{p_o}$, $\frac{\rho}{\rho_o}$, $\frac{A}{A_s}$, $(\mu - \omega)$, 0(·5)90°, 5 to 6 S, R 24 and R 49. .. .. (60)

$\underline{\omega}$, M, $\frac{p}{p_s}$, μ, 0(1)106°, 3 to 5 S, R 50. .. .. (61)

$\underline{\mu}$, ω, ϕ, M, $\frac{q}{a_s}$, $\frac{p}{p_s}$, $\frac{\rho}{\rho_s}$, $\frac{T}{T_s}$, $\frac{a}{a_s}$, $\frac{A_s}{A}$, $\frac{r_s}{r}$, 90(1)0°, 5 S, R 48. .. .. .. (62)

$\underline{\mu}$, $f(\mu)$, 90(5)45(1)25(1·25)20(2·5)0°, 4 S, R 12. .. .. (63)

$\underline{\mu}$, $-\log_{10} g(\mu)$, 90(5)30(1)15, 10, 0°, 4 D, R 12. .. .. (64)

$\underline{\mu}$, $f(\mu)$, $g(\mu)$, $\{g(\mu)\}^{5/7}$, 90(1)0°, 4 S, R 20 and R 13. .. .. (65)

$\underline{\mu}$, $f(\mu)$, $G(\mu)\sin\mu$, 90(1)20°, 4 to 6 S, R 14. .. .. (66)

$\underline{\mu}$, $f(\mu)+\mu$, $f(\mu)-\mu$, $G(\mu)$, $F(\mu)$, $F(\mu)/2\cos\mu$, 4 to 5 S, R 13. .. .. .. (67)

$\underline{\sqrt{\left(\frac{A}{A_s}\right)}}$, μ, 1(·01)4·9, 4 S, R 19. .. .. (68)

$\underline{\omega}$, μ(2D), M(3 to 5 S), 0(·5)130°, R 28. .. .. (69)

7. TABLES D—*Shock Tables*

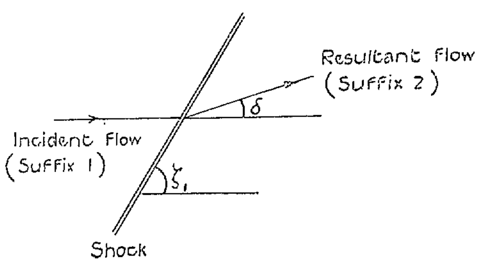

FIG. 2.

(a) *Normal Shocks*

$\underline{M_1}, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_2}{p_{1o}}, \frac{p_1}{p_{2o}}$, 1(·01)5, 6 S or 5 D, R 48. .. .. .. (70)

$\underline{M_1}, M_2, M_2'$, 1(·01)5, 5 D, R 48. .. .. (71)

$\underline{M_1}, M_2, \frac{dM_2}{d\gamma}, \frac{p_{2o}}{p_{1o}}, \frac{d}{d\gamma}\left(\frac{p_{2o}}{p_{1o}}\right)$, 1(·1)5, 5 and 3 D, R 48. .. .. (72)

$\underline{M_1}, \mu, M_2, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_1}{p_{2o}}, \frac{q_2}{q_1}, \frac{a_2}{a_1}, \frac{\Delta S}{R}$, 1(·01)5, 5 D, R 44. .. .. (73)

$\underline{M_1}$, ($\gamma = 1\cdot4$ and $1\cdot27$), $X, \frac{dX}{d\gamma}\left(X = M_2, \frac{p_2}{p_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_{2o}}{p_1}\right)$, 1(·01)5, 5 and 3 S, R 44. .. (74)

$\underline{M_1}, M_2, \frac{p_1}{p_{1o}}, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{a_2}{a_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_2}{p_{2o}}, \frac{p_2}{p_{1o}}, \frac{q_1}{a_s}, \frac{q_1}{a_{1o}}, \frac{q_1}{q_{\max}}$, 1(·01)3·5(·1)5(1)10(5)20, 100, ∞, 4 S, R 25. .. .. (75)

$\underline{M_1}, M_2, \frac{p_{2o}}{p_{1o}}, \frac{p_2}{p_1}, \frac{p_2}{p_{1o}}, \frac{q_2}{q_1}$, 1(·01)3, 5 S, R 4. .. .. (76)

$\underline{M_1}, M_2, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{p_{2o}}{p_{1o}}, \frac{q_2}{q_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_1}, \frac{a_2}{a_1}$, 1(·01)2(·1)5(1)25, 5 S, R 28. .. .. (77)

$\underline{M_1}, \frac{p_2}{p_1}, \frac{p_2}{p_{1o}}, \frac{p_{2o}}{p_{1o}}, \frac{\rho_2}{\rho_1}, M_2, \frac{q_2}{q_1}$, 1(·01)10, 4 S, R 45. .. .. (78)

$\underline{M_1}, M_2, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_{2o}}{p_1}$, 1(·01)3(·5)5(1)10, 5 S, R 49. .. .. .. (79)

$\underline{M_1}, M_2, \frac{p_2}{p_1}, \frac{p_{2o}}{p_{1o}}$, 1(·01)4, 4 S, R 50. .. .. (80)

$\underline{M_1}, M_2, \frac{p_1}{p_2}, \frac{p_2}{p_1}, \frac{p_{2o}}{p_{1o}}, \frac{p_1}{p_{2o}}$, 1(·1)2, 4 D, R 8. .. .. (81)

$\underline{M_1}$ ($\gamma = 1\cdot405$), $\frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_{1o}}$, 1(·05)5, 4 S, R 38. .. .. (82)

$\underline{\frac{q_1}{a_s}}, \frac{q_2}{a_s}, \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{2o}}{p_{1o}}$, 1(·01)2·44 and $\sqrt{6}$, 5 S, R 48. .. .. (83)

$\underline{\frac{p_1}{p_{2o}}}, M_1$, 0·528(·001)0·03, 4 S, R 48. .. .. (84)

$\underline{\frac{p_{2o}}{p_{1o}}}, M_1$, 1(·001)0·03, 4 S, R 48. .. .. (85)

(b) *Oblique Shocks*

$\underline{\zeta_1}$, $\underline{\delta}$, $\frac{\rho_2}{\rho_1}$, ζ_1 1(1)76°, δ 1(1)30°, 4 S, R 20 and R 49. .. .. (86)

$\underline{\zeta_1}$, $\underline{\delta}$, M_1, ζ_1 15(1)76°, δ 1(1)30°, 4 S, R 20 and R 49. .. .. (87)

$\underline{\zeta_1}$, $\underline{\delta}$, M_2, ζ_1 24(1)76°, δ 1(1)30°, 4 S, R 20 and R 49. .. .. (88)

$\underline{\zeta_1}$, $\underline{\delta}$, $\frac{p_2}{p_1}$, ζ_1 1(1)76°, δ 1(1)30°, 4 S, R 20 and R 49. .. .. (89)

$\underline{M_1}$, $\underline{\zeta_1}$, δ, M_2, $\frac{p_2}{p_1}$, $\frac{\rho_1}{\rho_2}$, ΔS, M_1 1·05(·05)4, ζ_1 at 3° intervals, 4 S, R 46 and R 50. (90

$\underline{M_1}$, $\underline{\delta}$, ζ_1, M_2, M_1 1·05(·05)4, δ at ½° or 1° intervals, 4 S, R 46. .. .. .. (91)

$\underline{\zeta_1}$, $\underline{M_1}$, ($\gamma = 1\cdot405$), δ, ζ 90(2)0°, M 1·1(·1)2(·2)4(var)10, ∞, 2 D, R 38. .. .. (92)

$\underline{\zeta_1}$, $\underline{\delta}$, M_1, M_2, $\frac{p_2}{p_1}$, $\frac{p_2 - p_1}{\frac{1}{2}\rho_2 q_2^2}$, ζ 8(1)30(2)70°, δ 0(1)15°, 5 D, R 26. .. .. .. (93)

$\underline{M_1}$, $\underline{\frac{p_2}{p_1}}$, $\frac{p_2 - p_1}{\frac{1}{2}\rho_1 q_1^2}$, M_1 0(·05)0·1(·25)2(·5)8, $\frac{p_2}{p_1}$ 0(·05)0·1(·1)2(·2)10,
5 D or more, R 26. .. (94)

[($p_2/p_1 < 1$ indicates an isentropic expansion and the table is calculated on this assumption.]

$\underline{M_1}$, δ_s, δ_m, 1(·01)2·25, 2·3(·1)3(var.)20, ∞, 2 D, R 20 and R 49. .. .. .. (95)

$\underline{M_1}$, δ_m, 1(·01)10, 2 D, R 45. .. .. (96)

$\underline{M_1}$, δ_m, δ_s, 1(·01)2(·1)5(1)20, 2 D, R 48. .. .. (97)

$\underline{M_1}$, ζ_m, ζ_s, δ_m, δ_s, M_{2m}, 1(·1)4, 4 S, R 46 and R 50. .. .. (98)

$\underline{\delta_m}$, M_1, M_2, $\frac{p_2}{p_1}$, $\frac{\rho_2}{\rho_1}$, $\frac{p_{2o}}{p_{1o}}$, ζ, 0(·5)1, 2, 2·5, 5(5)45, 3 or 4 D, R 27. .. .. (99)

N.D.R.C. A.M.P. have done some computations as can be seen in R 39. They have also done a large number of calculations on multiple shock systems .. .. (100)

(c) *Cone Flow*

Suffix c indicates condition on cone,

w indicates condition just behind shock.

$\underline{\frac{q_c}{q_{\max}}}$, $\underline{\theta_c}$, ($\gamma = 1\cdot405$), $\frac{p_w}{p_1}$, $\frac{p_1}{p_{1o}}$, $\frac{p_{2o}}{p_{1o}}$, M_1, $\frac{p_c - p_1}{\rho_1 U^2}$, $\theta_c = 10°, 20°, 30°$

$\frac{q_c}{q_{\max}}$ 0·3(·1)0·9 approx., about 3 S, R 41. .. (101)

$\underline{\dfrac{q_c}{q_{max}}}$, $\underline{\theta_c}$ $(\gamma = 1\cdot405)$, θ_w, M_1, $\dfrac{T_c}{T_w}$, $\dfrac{p_c}{p_w}$, $\dfrac{\rho_c}{\rho_w}$, $\dfrac{T_w}{T_1}$, $\dfrac{p_w}{p_1}$, $\dfrac{\rho_w}{\rho_1}$, K_D,

$\dfrac{q_c}{q_{max}}$ takes 32 values between $0\cdot175$ and $0\cdot99551$, θ_c $5(2\cdot5)25(5)50°$, 5 D, R 32. (102)

$\underline{\theta_c}$, $\underline{M_1}$, $\dfrac{q_c}{q_{max}}$, θ_w, θ_c $5(2\cdot5)25(5)50°$, M_1 $1\cdot05(\cdot05)4$, 5 D, R 32. .. .. .. (103)

R 32 also contains the whole solution of the flow round the cones for the intervals listed. .. .. (104)

8. TABLES E—*Special Methods (not Hodograph)*

(a) *Ackeret-Busemann—Approximate Theory*

C_1 and C_2 are the first two coefficients in the expansion of p/p_o as a power series in the deflection

$$C_1 = 2/\sqrt{(M^2 - 1)}$$

$$C_2 = \tfrac{1}{2}\{(M^2 - 2)^2 + \gamma M^4\}/(M^2 - 1)^2$$

$\underline{M}$, C_1, C_2, $1\cdot02(\cdot02)3\cdot5(\cdot1)5(1)10(5)20$, 100, 3 D, R 25. .. .. (105)

$\underline{M}$, C_1, C_2, $1\cdot10(\cdot02)1\cdot60$, 3 D, R 42. .. .. (106)

$\underline{M}$, C_1, C_2, $1(\cdot1)1\cdot6(\cdot2)3(1)6$, 8, 10, 100, 4 S, R 27. .. .. (107)

(In this table the values given for C_3, b_3 are incorrect.)

9. TABLES F—*Hodograph Tables*

$$F_n(\tau) = F(a_n, b_n;\; n+1;\; \tau)$$

$$\text{where } a_n + b_n = n - \frac{1}{\gamma - 1} \text{ and } a_n b_n = -\frac{n(n+1)}{2(\gamma - 1)}$$

$$\psi_n(\tau) = \tau^{n/2} F_n(\tau)$$

$$F_{-n}(\tau) = F(n + a_{-n}, n + b_{-n};\; 1 + n;\; \tau)\,.$$

In all these Tables n and τ are the independent variables.

$F_n(\tau)$, $F'_n(\tau)$, $\psi_n(\tau)$, $\psi'_n(\tau)$, n $1(1)15$, τ $0(\cdot02)0\cdot5$, 7 S (approx.), R 34. .. .. (108)

R 33 contains some tables of coefficients for use in solving the flow through a channel. (109)

$(\gamma = 1\cdot405)$, $F_{-n}(\tau)$, n $-10\cdot5(1)10\cdot5$, τ 0, $0\cdot10(\cdot02)0\cdot14(\cdot01)0\cdot16(\cdot005)$ $0\cdot20(\cdot01)0\cdot30(\cdot02)0\cdot50$, 5 D, R 36. .. (110)

There are a few errors in this Table, indicated in R 35. A number of other functions are also tabulated.

$(\gamma = 1\cdot405)$ $F(-a_{-n}, -b_{-n};\ n+1;\ \tau)$, $F_{-n}(\tau)$, $F(-a_{-n}, -b_{-n};\ n;\ \tau)$,
n 1(1)13 and $-$ 10·5(1)0·5, τ 0(·01)0·1; also $n = 2$, $\tau = 0(\cdot02)0\cdot4$
6 D or 6 S, R 35. .. .. (111)

$(\gamma = 1\cdot405)$ $2\tau\ F'(-a_{-n}, -b_{-n};\ n+1;\ \tau)/n$
n $-$ 9·5(1)0·5, τ 0·10, 0·12, 0·14(·01)0·3(·02)0·4, 4 to 5 S, R 35. .. .. (112)

R 35 also contains a few other tables of hypergeometric functions. A number of hypergeometric functions are tabulated in R 43 and R 47 for $\gamma = 1\cdot405$. .. (113)

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ABBREVIATIONS

A.R.C.	Aeronautical Research Council. Great Britain.
C.S.I.R.	Council for Scientific and Industrial Research. Australia.
N.A.C.A.	National Advisory Committee for Aeronautics. U.S.A.
N.D.R.C., A.M.P.	National Defense Research Council, Applied Mathematics Panel. U.S.A.
R.A.E.	Royal Aircraft Establishment. Great Britain.
W.P.A., M.T.P.	Works Projects Administration, Mathematical Tables Panel. U.S.A.
R. & M.	Reports and Memoranda Series. A.R.C.
T.N.	Technical Note.

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