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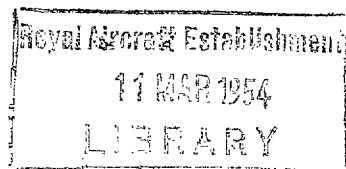
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# The Effect of Sweepback on the Fundamental Derivative Coefficient for Flexural Motion

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*Summary.*—Measurements have been made with new equipment, designed for derivative tests in a  $9 \times 7$  ft tunnel, to determine the effect of sweepback on the derivatives  $\lambda_{\dot{\phi}}$  and  $\lambda_{\phi}$  for a rectangular aerofoil of aspect ratio 6. A numerical reduction was observed in each case, amounting to 15 per cent for  $\lambda_{\dot{\phi}}$  and 20 to 30 per cent for  $\lambda_{\phi}$  over the range  $\omega = 1.0$  to  $1.5$  with a sweepback angle of  $41.3$  deg.

Values of  $\lambda_{\dot{\phi}}$  and  $\lambda_{\phi}$  for the swept model were obtained from measurements relating to oscillation about an axis perpendicular to the leading edge.

A comparison of  $\lambda_{\dot{\phi}}$  and  $\lambda_{\phi}$  with available theoretical results for finite aspect ratio is made and good agreement observed in the case of the former. The less satisfactory agreement with  $\lambda_{\phi}$  is thought to be due to the lower accuracy of the theoretical values.

Some difficulty was experienced in the interpretation of the measurements on the swept aerofoil due to distortion of the model during oscillation. The effect is examined in detail in an appendix and a method of correction devised.

A comparison between measurements of  $\lambda_{\dot{\phi}}$  for the unswept aerofoil and earlier measurements made with the same model by the method of decaying oscillations gives satisfactory agreement.

1. *Introduction.*—The derivative measurements described in this paper were obtained with the equipment constructed at the National Physical Laboratory for the measurement of forces on oscillating models in a  $9 \times 7$  ft wind tunnel. An existing unswept aerofoil was used as a test model during the development of the apparatus. Since measurements of  $\lambda_{\dot{\phi}}$  had already been made on this model by the method of decaying oscillations (R. & M. 2032<sup>1</sup>), results obtained by the two different techniques could be compared.

The swept aerofoil was constructed from an existing unswept model of similar profile, but smaller chord, by addition of suitable tip and root members to give the same aerodynamic chord and aspect ratio as in the unswept case. This resulted in the thickness/chord ratio being reduced to  $\frac{3}{4}$  of the value for the latter.

Only a brief account of the equipment and experimental technique is included here, since a detailed description will be given in a separate report.

2. *Description of Apparatus.*—The method of inextensible forcing is used, and the apparatus includes a mechanical oscillator with a number of forcing stations each consisting of an eccentric fitted to a main driving shaft and connected to a point on the model via a linkage system. A special type of balance incorporated in each linkage system measures the instantaneous reaction at the forcing point due to the aerodynamic forces. Provision has been made for ten forcing stations in the completed equipment, which will enable measurements to be made on a flexible

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model with an imposed mode of deformation. Two stations were available when the present tests were made, one being used for forcing and the other running idle.

Fig. 1 shows a diagrammatic representation of a forcing station. Its position along the shaft is adjustable, the linkage system and balance sliding on I-section steel joists, whilst the eccentric slides along the shaft. Its distance from the shaft can also be altered, but this involves a change in length of the connecting rod between the eccentric and bell-crank. The horizontal lever in the linkage system pivots about point A on the balance to which it transmits the reaction to be measured. Inertial reaction due to the mass of the model and associated fittings is automatically balanced out by the mass M, which is adjusted during a still-air experiment\*. Similarly reactions due to springs tensioning the long and slender driving rod are balanced out by spring S. All pivots transmitting load to the balance are constructed in the form of spring bearings in order to avoid the uncertainties of solid friction.

In the earlier experimental work with the unswept wing a development of the automatic electric balance<sup>2</sup> was employed. This was found to be unsatisfactory at the higher frequencies of oscillation, due to overloading by relatively large vibrational reactions of still higher frequency superimposed on the aerodynamic reactions. These extraneous vibrations are thought to be due partly to tunnel vibration and partly to impulses from ball-bearings. A simpler type of balance was finally adopted in which the measuring element consists of a steel strip to which the reactions are applied longitudinally. The resulting small changes in length are made to produce corresponding changes in the width of an optical slit on which the image of a lamp filament is focussed; the variations in intensity of the emergent light are then converted into voltage variations by means of a photocell.

An attempt was made to use resistance-wire strain-gauges cemented to the surfaces of the steel strip, but the stability was found to be inadequate for measuring the very small strains involved. The strain for a given load is determined by the longitudinal stiffness of the strip, which forms a weak link in the system, a lower limit being set to the stiffness by frequency response requirements. With the strip† used in these experiments the estimated frequency response showed a rise of 1 per cent at 10 c.p.s.

Photographs of the equipment are given in Figs. 3 and 4.

The output from the balance is measured in the a.c. bridge circuit shown in Fig. 2, a.c. reference voltages  $E_1$  and  $E_2$  respectively in phase and in quadrature with the motion of the model are provided by two generators consisting of coils oscillated in magnetic fields by the mechanical oscillator. In the present arrangement, which is temporary, the coils are attached to the vertical links between the bell-cranks and horizontal levers in the two forcing stations, the idle station being set 90 degrees out of phase with the driving station. The voltages  $E_1$ ,  $E_2$  are adjusted by means of voltage dividers and their vector sum balanced against the output  $E$  from the strip balance.

The indicator incorporates a high-gain amplifier followed by a cathode-ray tuning indicator, in which oscillation of the shadow angle is produced when the bridge is unbalanced. The preceding filter is essential for the removal of unwanted frequencies, mainly due to extraneous vibrations, which would obscure the balance point. Since conventional inductance-capacity filters are impracticable at the low frequencies of the tests (1 to 10 c.p.s.), an electronic filter is used, which is in effect a tunable amplifier. Tuning is carried out by adjusting resistive components in a twin-T filter included in a negative feedback loop.

The components of aerodynamic reaction can be calculated from the voltage divider settings when the voltages generated by the oscillating coils and the relation between voltage and load for the balance are known. The latter is determined by static loading, the output voltage being measured with an accurate potentiometer. In the case of the coil voltages, these are first

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\* This automatically subtracts from the reaction measured in the wind a reaction corresponding to the still-air virtual inertia of the model.

† The dimensions of the strip were  $3.0 \times 0.25 \times 0.02$  in.

rectified mechanically by means of relays operated from the main drive. The resulting d.c. voltages are then smoothed in a resistance-capacity filter and measured by a null method using a d.c. potentiometer.

Frequency of oscillation is measured with a special type of stroboscope controlled by an accurate 50-cycle tuning fork.

In the tests to be described the amplitude of oscillation was measured with a travelling microscope focussed on a small ball-bearing attached to the link driving the model and illuminated by a distant lamp.

3. *Dynamic Calibration.*—In order to test the behaviour of the apparatus as a whole, measurements were made of the reaction produced by a mass attached to the link connecting the model to the horizontal lever. The unknown effects of additional aerodynamic inertia and aerodynamic damping associated with this mass were eliminated by making repeat measurements with a mass of the same physical dimensions but much reduced density (wood as compared with steel). If the difference between corresponding components of inertial reaction is denoted by  $\delta R_1$  for the in-phase and  $\delta R_2$  for the out-of-phase component, and  $\delta R_0$  is the calculated reaction, the relations  $\delta R_1/\delta R_0 = 1$  and  $\varepsilon = \tan^{-1} \delta R_2/\delta R_0 = 0$  should be satisfied. Values of  $\delta R_1/\delta R_0$  and  $\varepsilon$  are plotted in Figs. 5a and 5b. For the range of frequency covered the maximum difference between measured and calculated reaction is approximately 2 per cent and the phase is shifted by about  $\frac{1}{4}$  degree. Measurements at lower frequencies were not attempted since a larger mass would have been required to give a measurable reaction. It was thought that this might alter the characteristics of the system.

4. *Description of Models.*—The unswept model was a rectangular symmetrical NACA 0015 aerofoil of 60 in. span and 20 in. chord. The swept model was in effect the same aerofoil sheared back through an angle of  $41.3$  deg and with the thickness/chord ratio reduced by 25 per cent. Both aerofoils were given a rolling oscillation about an axis  $OX$  coincident with the root chord and the wind direction (see Fig. 6). The swept model was also oscillated about axis  $OY$  perpendicular to the leading edge; in which case metal plates for rigidly connecting the outboard to the inboard section were removed, and the latter held fixed by a vertical strut connected to the tunnel structure. Both axes were provided by spring bearings which, in the case of  $OY$ , were housed within the profile. The forcing point was situated at a distance of 40 in. from axis  $OX$  and lay on the inertia axis appropriate to a two-dimensional aerofoil of the same structure. The attachment to the model was in each case by means of an internal spring bearing, which was made universal for the swept wing in order to allow rotation about either axis.

The aerofoils were hollow wooden structures built up from two spars and a set of ribs with a thin wooden cover. Ribs and spars were lightened as far as was consistent with rigidity.

Photographs of the swept aerofoil mounted in the tunnel are given in Figs. 7 and 8.

5. *Definitions.*—The leading edge is taken as reference axis, with a reference centre  $R$  at distance  $l$  from axis  $OX$  (see Fig. 6).

$\phi$  denotes rotation of  $R$  about axis  $OX$  (positive for downward displacement)

$\theta$  denotes change of incidence at  $R$

$y$  denotes distance from  $OX$  of a point on the reference axis

$$\eta = y/l$$

$$l = 0.7s, \text{ where } s \text{ is span}$$

$z(\eta)$ ,  $\theta(\eta)$  denote downward displacement and change of incidence for a point on the reference axis at section  $\eta$

$$z(\eta) = l\phi f(\eta), \theta(\eta) = \theta F(\eta) \text{ where } f(\eta) \text{ and } F(\eta) \text{ are modal functions}$$

The moment about  $OX$  measured at  $R$  is given by \*

$$L = \phi L_\phi + \dot{\phi} L_{\dot{\phi}} + \theta L_\theta + \dot{\theta} L_{\dot{\theta}}.$$

Since in this case  $\phi$  and  $\theta$ , when both present, are oscillatory motions in the same phase, the amplitudes of the components of  $L$  in phase and in quadrature with the motion are given respectively by

$$\begin{aligned} L_1 &= \bar{\phi} L_\phi + \bar{\theta} L_\theta, \\ L_2 &= \dot{\phi} (\bar{\phi} L_{\dot{\phi}} + \bar{\theta} L_{\dot{\theta}}), \end{aligned}$$

the motions being  $\phi = \bar{\phi} e^{j\omega t}$ ,  $\theta = \bar{\theta} e^{j\omega t}$ . In non-dimensional form these become

$$\begin{aligned} l_1 &= -L_1 / \rho V^2 l^3 \bar{\phi} \int_0^{s/l} f^2 d\eta = \lambda_\phi + k\lambda_\theta, \\ l_2 &= -L_2 / \rho V l^3 c \dot{\phi} \bar{\phi} \int_0^{s/l} f^2 d\eta = \lambda_{\dot{\phi}} + k\lambda_{\dot{\theta}} \end{aligned}$$

where

$l$  is air density

$V$  is wind speed

$c$  is chord

$\dot{\phi}$  is circular frequency

$$k = c \bar{\theta} \int_0^{s/l} f F d\eta / l \bar{\phi} \int_0^{s/l} f^2 d\eta$$

and  $\lambda_\phi$ ,  $\lambda_{\dot{\phi}}$ ,  $\lambda_\theta$ ,  $\lambda_{\dot{\theta}}$  are the fundamental equivalent constant leading edge derivative coefficients for the modes under consideration.

For oscillation about axis  $OX$ ,  $\theta$  is absent and the mode is linear. The modes† for oscillation about axis  $OY$  are shown in Fig. 9, the corresponding value of  $k$  being 0.2975.

6. *Apparatus Damping and Inertia Corrections.*—Let  $\bar{R}_1$  and  $\bar{R}_2$  denote the measured in-phase and in-quadrature reactions at the forcing station. For oscillations in still air it was found impracticable to reduce  $\bar{R}_1$  exactly to zero by adjustment of the mass  $M$  and spring  $S$  referred to in section 2. The small residual value was therefore measured at each frequency and applied as a correction to the value obtained with the tunnel running, the corrected in-phase reaction being denoted by  $R_1$ . It should be noted that stiffness derivatives measured with this equipment represent the total in-phase reaction due to aerodynamic forces in the wind stream less a reaction corresponding to still-air virtual inertia.

Values of  $\bar{R}_2$  for oscillation in still air are shown plotted against frequency in Fig. 10. Since these curves tend asymptotically to zero with frequency, it is clear that hysteresis damping in spring bearings and tensioning springs was negligible, as this would produce a constant out-of-phase reaction independent of frequency. The reaction was thus regarded as due solely to still-air damping on the aerofoil and driving linkage. It was clear from the geometry of the system that the latter would be small compared with the former, and since the total still-air value was never greater than about 10 per cent of the value obtained with the tunnel running, the apparatus damping was taken to be negligible. The out-of-phase reaction with the tunnel running is denoted by  $R_2$ .

7. *Guard Interference.*—The driving rod and tensioning wire shown in Fig. 1 were shielded from the action of the wind by streamlined tubular guards. In the early tests, made with the automatic electric balance, the guards carried horizontal streamlined plates faced with rubber in order to protect the surface of the model in case of a breakage in the linkage system. The

\* This is derived from the actual measured moment by the principle of virtual work.

† The mode  $F$  introduces a discontinuity in the aerofoil section between  $Y$  and  $OX$ .



interference was determined by making measurements with dummy guards and plates situated a short distance inboard of the forcing station, the same distance outboard, and in both positions simultaneously. The results were found to be additive, and the effect of the guards at the forcing station was taken to be equal to the mean of the effects at the inboard and outboard positions.

On changing to the steel-strip balance the horizontal plates were removed, as they had been found to have a large effect on  $\lambda_\phi$ . The method of correction described above could not now be applied, since the effects were no longer additive. The method finally used was to fit telescopic guards at the forcing station and determine the effect of altering the distance of the ends of the guards from the surface of the model. At a fair distance from the model the effect was found to be small and linear with position, whilst close to the surface large non-linear effects were obtained. The latter were taken to result from disturbed flow near the model, and the former from exposure to the wind forces of varying lengths of driving rod and tensioning wire. By extrapolation of the linear range to the surface of the model, results could be obtained free from both interference effects. Corrections to results obtained with the guards in the normal working position are shown in Figs. 11 and 12.  $\Delta\lambda_\phi$  was found to be independent of wind speed  $V$ , and  $\Delta\lambda_\phi$  independent of frequency parameter  $\omega$  over the ranges covered. No corrections were measured for oscillation of the swept wing about axis  $OY$ .

8. *Measurements on the Unswept Aerofoil.*—Values of  $\lambda_\phi$  and  $\lambda_i$  for wind speeds ranging from 40 to 210 ft/sec are shown plotted against  $\omega$  in Figs. 13 to 16 and tabulated in Tables 1 and 2. The Reynolds number ranges from  $0.42 \times 10^6$  to  $2.20 \times 10^6$ , and if due allowance is made for experimental scatter, it appears that scale effect is negligible. The amplitude of oscillation  $\bar{\phi}$  was approximately 0.5 deg and the mean incidence 0 deg for all the tests made with the inexorable forcing apparatus.

Figs. 13 and 14 compare results obtained with the steel-strip balance and the automatic electric balance. Earlier results for  $\lambda_\phi$  obtained by the method of decaying oscillations (R. & M. 2032) are also included. On the whole the agreement is satisfactory, the greatest discrepancy occurring in the measurements made with the electric balance at small values of  $\omega$  and the highest wind speed. It is possible that the discrepancy is due to unreliable guard interference corrections, since the electric balance results were obtained with the earlier arrangement of guards carrying horizontal plates adjacent to the model.

9. *Measurements on the Swept Aerofoil.*—9.1. *Effects of Model Distortion.*—Values of  $\lambda_i$  and  $\lambda_\phi$  calculated from measured reactions for oscillation about axis  $OX$ , in the same way as for the unswept aerofoil, are shown in Figs. 17 and 18\*. These curves exhibit a marked effect due to wind speed, and since no scale effect had been observed in the unswept case, distortion of the model was suspected.

An approximate determination of the mode of deformation was made by means of a small concave mirror attached to the upper surface of the model at points on a reference axis parallel to the leading edge and passing through the forcing point. The chosen points were roughly at the tip, three-quarter span, mid-span, and quarter-span positions. A beam of light reflected from the mirror passed through a hole in the tunnel roof and was brought to a focus on a translucent paper screen to produce a Lissajous figure which was traced in with pencil and subsequently analysed.

The results of these tests showed that on the reference axis chosen, the slope perpendicular to  $OX$  was linear, whilst a torsional mode existed which in general involved a spanwise variation of phase. Modal functions  $F_1$  and  $F_2$  are shown in Fig. 19 as functions of the spanwise co-ordinate  $\eta$  which in this case is made unity at the forcing point. The change of incidence at section  $\eta$  is now given by

$$\bar{\theta}(\eta) = F_1\bar{\theta}_1 + jF_2\bar{\theta}_2$$

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\* These results are uncorrected for guard interference, which explains the failure of the  $\lambda_\phi$ -curves to pass through the origin.



10. *Measurements of  $C_l$ .*—For these tests the incidence of the model was altered by rotation about an axis perpendicular to  $OX$  and passing through the forcing point. The rolling moment about  $OX$  was determined for each incidence from measurements of the reaction at the forcing point for a number of wind speeds, and the result corrected for model distortion in the manner already described for the oscillatory case\*.

A curve of  $C_l$  plotted against  $\theta$  is given in Fig. 25, where  $C_l$  is defined by the relation  $L = \frac{1}{2}\rho V^2 s^2 c C_l$ . A theoretical curve for 45-deg sweepback is included for comparison, based on solution 13 of R. & M. 2596<sup>3</sup> by Falkner.

11. *Comparison with Theory.*—Mean experimental curves of  $\lambda_{\dot{\delta}}$  and  $\lambda_{\delta}$  are compared with theoretical results in Figs. 26 to 29. Values calculated by Miss D. E. Lehrian (Current Paper No. 51<sup>4</sup>) using a general method for any plan form due to W. P. Jones (R. & M. 2470<sup>5</sup>) are included in Figs. 26 and 27. This method is based on a combination of a theoretical treatment by Jones and a form of Falkner's scheme for the calculation of normal induced velocity due to simple doublet distributions. Its main advantage in comparison with earlier methods is a considerable reduction in the computational labour involved, particularly for wings of constant chord.

The agreement with experiment in the case of  $\lambda_{\dot{\delta}}$  is very good for  $\omega > 0.5$ , both for the unswept and swept aerofoils. The values for  $\lambda_{\delta}$  show best agreement for the swept aerofoil, the results for the unswept aerofoil being low. This may be due to inaccuracies in the calculations, since the separation of  $\lambda_{\delta}$  from the still-air virtual inertia term involves a small difference of two large quantities.

A comparison is made in Figs. 28 and 29 with earlier results calculated by W. P. Jones (R. & M. 2142<sup>6</sup>) using an approximate method for unswept rectangular wings. The agreement here is satisfactory in the case of the damping, and is better for the stiffness derivative  $\lambda_{\delta}$  if the symmetrical modes are compared. It is clear from a consideration of the image system in the tunnel walls that this is the appropriate mode.

Curves of theoretical two-dimensional derivative coefficients are also included in the figures. Over the range  $\omega = 0.5$  to 1.5 the experimental values of  $\lambda_{\dot{\delta}}$  for the unswept and swept aerofoils are roughly 66 per cent and 56 per cent respectively of the two-dimensional values. For the same range of  $\omega$  the value of  $\lambda_{\delta}$  varies from 50 per cent to 80 per cent of the two-dimensional value approximately in the case of the unswept aerofoil and from 30 per cent to 60 per cent for the swept aerofoil.

The curves of  $\lambda_{\dot{\delta}}$  and  $\lambda_{\delta}$  in Figs. 23 and 24 are compared with two-dimensional theory. In this case no theoretical values for finite aspect-ratio and the appropriate mode were available.

12. *Conclusions.*—The effect of sweepback on the derivatives  $\lambda_{\dot{\delta}}$  and  $\lambda_{\delta}$  is to produce a reduction in magnitude, which in the present case amounts to 15 per cent for damping and 20 to 30 per cent for stiffness over the range  $\omega = 1.0$  to 1.5.

The agreement with theory is satisfactory for  $\lambda_{\dot{\delta}}$ , both as regards the effect of aspect-ratio and sweepback. It is possible that the agreement for  $\lambda_{\delta}$  would be improved if more accurate calculations were made.

An important result of the tests on the swept wing is the demonstration of the serious effect which model distortion can have on the measurements. It is emphasized that the possibility of errors from this cause should always be borne in mind in derivative measurements when an apparent scale effect is observed.

An incidental result of the tests on the unswept wing is the comparison of measurements of  $\lambda_{\dot{\delta}}$  with earlier measurements made by the method of decaying oscillations. Satisfactory agreement is obtained.

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\* It is clear that the error in the reaction due to twist is proportional to the latter, which is itself proportional to aerodynamic loading, *i.e.*, to  $V_{\infty}$ . This leads to a relation between reaction and wind speed identical with (1) in section 9.1.



## APPENDIX I

### *Errors in Measurements of $L_\phi$ and $L_{\dot{\phi}}$ Due to Torsion*

The determinantal equation for the system used in making the measurements may be written

$$\begin{vmatrix} \sigma_\phi - \dot{p}^2 I_\phi - L_\phi - j\dot{p}L_{\dot{\phi}} + (r_1 + jr_2), & \sigma_1 - \dot{p}^2 P - L_\theta - j\dot{p}L_{\dot{\theta}} \\ \sigma_1 - \dot{p}^2 P - M_\phi - j\dot{p}M_{\dot{\phi}}, & \sigma_\theta - \dot{p}^2 I_\theta - M_\theta - j\dot{p}M_{\dot{\theta}} \end{vmatrix} = 0, \dots \dots \quad (2)$$

where  $I_\phi, I_\theta, \sigma_\phi, \sigma_\theta$  are inertias and direct elastic stiffnesses appropriate to the  $\phi$  and  $\theta$ -freedoms, and  $P, \sigma$  are product of inertia and elastic cross-stiffness respectively. The conventional notation is used for the derivative coefficients.  $r_1$  and  $r_2$  represent the in-phase and out-of-phase components of the forcing reaction per unit amplitude. The measured reactions  $R_1$  and  $R_2$  of section 9(i) are given in terms of these by

$$\left. \begin{aligned} R_1 &= (r_1 - r_1')\bar{\phi}, \\ R_2 &= r_2\bar{\phi} \end{aligned} \right\} \dots \dots \dots \dots \dots \dots \quad (3)$$

where  $r_1'$  relates to oscillation in still air.

If it is assumed that for oscillation in still air the aerodynamic reactions are negligible compared with the elastic and inertial reactions, it follows from (2) that the still-air forcing reaction  $r_1'$  is given by

$$\begin{vmatrix} \sigma_\phi - \dot{p}^2 I_\phi + r_1', & \sigma_1 - \dot{p}^2 P \\ \sigma_1 - \dot{p}^2 P, & \sigma_\theta - \dot{p}^2 I_\theta \end{vmatrix} = 0. \dots \dots \dots \dots \dots \dots \quad (4)$$

$r_1'$  exceeds the reaction which would be obtained with the  $\theta$ -freedom absent by an amount  $\delta r_1'$  where

$$r_1' - \delta r_1' = -(\sigma_\phi - \dot{p}^2 I_\phi), \dots \dots \dots \dots \dots \dots \quad (5)$$

thus from (4)

$$\begin{vmatrix} \delta r_1', & \sigma_1 - \dot{p}^2 P \\ \sigma_1 - \dot{p}^2 P, & \sigma_\theta - \dot{p}^2 I_\theta \end{vmatrix} = 0. \dots \dots \dots \dots \dots \dots \quad (6)$$

The method of measurement described in the main text leads to the relations

$$r_1 - r_1' = L_\phi + \delta L_\phi, \dots \dots \dots \dots \dots \dots \quad (7)$$

$$r_2 = \dot{p}(L_{\dot{\phi}} + \delta L_{\dot{\phi}}), \dots \dots \dots \dots \dots \dots \quad (8)$$

where  $\delta L_\phi, \delta L_{\dot{\phi}}$  are the errors in the derivative measurements due to the unwanted freedom. Substitution from (5), (7) and (8) in (2) leads to

$$\begin{vmatrix} \delta L_\phi + \delta r_1' + j\dot{p}\delta L_{\dot{\phi}}, & \sigma_1 - \dot{p}^2 P - L_\theta - j\dot{p}L_{\dot{\theta}} \\ \sigma_1 - \dot{p}^2 P - M_\phi - j\dot{p}M_{\dot{\phi}}, & \sigma_\theta - \dot{p}^2 I_\theta - M_\theta - j\dot{p}M_{\dot{\theta}} \end{vmatrix} = 0, \dots \dots \dots \quad (9)$$

which together with (6) enables the errors to be determined. These may be written in the form

$$\begin{aligned} \delta l_\phi + j\omega\delta l_{\dot{\phi}} &= \frac{\left(\frac{\sigma_1}{\rho V^2 l^3} - \omega^2 \frac{P}{\rho c^2 l^3} - l_\theta - j\omega l_{\dot{\theta}}\right) \left(\frac{\sigma_1}{\rho V^2 l^3} - \omega^2 \frac{P}{\rho c^2 l^3} - m_\phi - j\omega m_{\dot{\phi}}\right)}{\frac{\sigma_\theta}{\rho V^2 l^3} \left(1 - \frac{\dot{p}^2}{\dot{p}_\theta^2}\right) - m_\theta - j\omega m_{\dot{\theta}}} \\ &\quad - \frac{\left(\frac{\sigma_1}{\rho V^2 l^3} - \omega^2 \frac{P}{\rho c^2 l^3}\right)^2}{\frac{\sigma_\theta}{\rho V^2 l^3} \left(1 - \frac{\dot{p}^2}{\dot{p}_\theta^2}\right)} \dots \dots \dots \dots \dots \quad (10) \end{aligned}$$

where  $l_\phi, l_{\dot{\phi}}, m_\theta, m_{\dot{\theta}}$ , etc., are non-dimensional forms of the derivative coefficients, and  $p_\theta$  is the natural circular frequency for torsion with the  $\phi$ -motion suppressed.

If  $p$  is small compared with  $p_\theta$ , and the direct torsional aerodynamic moments are small compared with the corresponding elastic moment, the right-hand side of (10) has the common denominator  $\sigma_\theta/\rho V^2 l^3$ . Hence if the elastic cross-stiffness  $\sigma_1$  is zero

$$\delta l_\phi + j\omega \delta l_{\dot{\phi}} = \frac{\rho V^2 l^3}{\sigma_\theta} \left[ (l_\theta + j\omega l_{\dot{\theta}}) (m_\phi + j\omega m_{\dot{\phi}}) + \frac{\omega^2 P}{\rho c^2 l^3} \left\{ (l_\theta + m_\phi) + j\omega (l_{\dot{\theta}} + m_{\dot{\phi}}) \right\} \right]. \quad (11)$$

Thus for constant  $\omega$  the errors in the measurements of the non-dimensional in-phase and out-of-phase aerodynamic reactions are proportional to  $V^2$ .

It follows therefore, that with constant  $\omega$  the measured reactions  $R_1$  and  $R_2$  will be of the form

$$\left. \begin{aligned} R_1 &= (L_\phi + \delta L_\phi) \bar{\phi} = (a_1 + b_1 x) x \\ R_2 &= p(L_{\dot{\phi}} + \delta L_{\dot{\phi}}) \bar{\phi} = (a_2 + b_2 x) x \end{aligned} \right\}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

where  $x = V^2$ . These expressions are of the same form as (1) in section 9.1.

The spanwise variation of phase mentioned in section 9.1 appears at first sight to complicate the problem. It may be shown, however, that the relations (12) still hold. For the purpose of setting up the dynamical equations,  $\theta$  is replaced by independent displacements  $\theta_1$  and  $\theta_2$  associated with the modal functions  $F_1$  and  $F_2$  of Fig. 19 and the problem treated as for three degrees of freedom. A solution of the form

$$\left. \begin{aligned} \phi &= \bar{\phi} e^{jpt}, \\ \theta_1 &= \bar{\theta}_1 e^{jpt}, \\ \theta_2 &= j\bar{\theta}_2 e^{jpt}, \end{aligned} \right\} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

is introduced, where  $\bar{\phi}, \bar{\theta}_1, \bar{\theta}_2$  are real.

From the resulting equations it may be seen that if the torsional aerodynamic moments relating to  $\theta_1$  are small compared with the corresponding elastic and inertial reactions, and  $p$  is small compared with  $p_\theta$ , which now represents the natural frequency for the  $\theta_1$  motion, then

$$\bar{\theta}_1 \propto p^2. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (14)$$

Also, if in addition the direct torsional damping moment relating to  $\theta_1$  is small compared with the torsional moment due to rate of roll, *i.e.*,

$$pM_{\dot{\theta}_1} \bar{\theta}_1 \ll pM_{\dot{\phi}} \bar{\phi}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

then for constant  $\omega$

$$\bar{\theta}_2 \propto pV \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (16)$$

Condition (15) will be satisfied if  $\bar{\theta}_1$  is very small. It was verified for the conditions of the experiment by using measured amplitudes, and derivatives based on two-dimensional theory. The relations (14) and (16) agree with the experimental results given in 9.1, the only difference being that the experimental relation for  $\bar{\theta}_2$  is independent of  $\omega$ . This implies that  $M_{\dot{\phi}}$  in (15) is constant over the range of  $\omega$  covered.

It follows from (14) and (16) that for constant  $\omega$

$$\frac{\bar{\theta}_1}{\bar{\theta}_2} \propto \frac{p}{V} \propto \omega = \text{a constant.} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (17)$$

This implies that for a given  $\omega$  the problem may be regarded as involving two degree of freedom only, and hence the expressions (12) are applicable.

## APPENDIX II

### *Approximate Calculation of Errors*

The three-degree of freedom treatment outlined in Appendix I leads to the equations

$$\left. \begin{aligned} \delta L_{\phi} &= L_{\theta_1} \frac{\bar{\theta}_1}{\phi} - p L_{\dot{\theta}_2} \frac{\bar{\theta}_2}{\phi} \\ p \delta L_{\dot{\phi}} &= (p^2 P_2 + L_{\theta_2}) \frac{\bar{\theta}_2}{\phi} + p L_{\dot{\theta}_1} \frac{\bar{\theta}_1}{\phi} \end{aligned} \right\}, \dots \dots \dots \dots \dots \dots \dots \quad (18)$$

where  $L_{\theta_1}$ ,  $L_{\dot{\theta}_1}$ ,  $L_{\theta_2}$ ,  $L_{\dot{\theta}_2}$  are the derivative coefficients relating to flexural moments produced by  $\theta_1$  and  $\theta_2$ -motions, and  $P_2$  is the product of inertia between the  $\theta_2$  and  $\phi$ -motions. Values for the derivatives were calculated on the basis of two-dimensional theory using the modal functions given in Fig. 19. A value for  $P_2$  was obtained from the approximate relation

$$p^2 P_2 \bar{\theta}_2 = p M_{\phi} \bar{\theta}_1, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (19)$$

which results from the assumptions leading to (14) and (16) in Appendix I. The derivative  $M_{\phi}$  in (19) relates to the  $\theta_1$ -motion, and it was calculated on the same basis as those in (18).

Values of  $\delta \lambda_{\phi}$  and  $\delta \lambda_{\dot{\phi}}$  calculated by substituting measured amplitudes in (18) are tabulated below and compared with measured values of the errors. These are obtained as differences between corrected and uncorrected measurements of the derivatives.

$\omega$	$\delta \lambda_{\phi}/V^2$		$\delta \lambda_{\dot{\phi}}/V^2$	
	Calculated	Measured	Calculated	Measured
0.4	$2.30 \times 10^{-6}$	$2.45 \times 10^{-6}$	$7.50 \times 10^{-6}$	$6.13 \times 10^{-6}$
1.0	10.29   ,,	15.9     ,,	13.16   ,,	10.9     ,,
1.6	23.75   ,,	37.0     ,,	26.16   ,,	13.9     ,,

It seems clear from these results that the distortion of the model would be expected to produce effects of the same order of magnitude as those observed. The differences between calculated and measured values of the errors are no doubt due to the very approximate nature of the data used in the calculations.

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2	J. B. Bratt and K. C. Wight ..	The Development of an Automatic Electric Balance for Research on Aerodynamic Stability. <i>Phil. Mag.</i> , Ser. 7, Vol. XXXV, p. 588. September, 1944.
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5	W. P. Jones .. .. .	The Calculation of Aerodynamic Derivative Coefficients for Wings of any Plan Form in Non-Uniform Motion. R. & M. 2470. December, 1946.
6	W. P. Jones .. .. .	Theoretical Air-load Derivative Coefficients for Rectangular Wings. R. & M. 2142. February, 1943.

TABLE I  
*Values of  $\lambda_{\dot{\phi}}$  for the Unswept Aerofoil*

$V = 40$ ft/sec		$V = 80$ ft/sec		$V = 160$ ft/sec		$V = 210$ ft/sec	
$\omega$	$\lambda_{\dot{\phi}}$	$\omega$	$\lambda_{\dot{\phi}}$	$\omega$	$\lambda_{\dot{\phi}}$	$\omega$	$\lambda_{\dot{\phi}}$
0.268	1.401	0.132	1.199	0.066	1.560	0.050	1.478
0.536	1.321	0.263	1.482	0.132	1.561	0.100	1.562
0.803	1.256	0.397	1.401	0.198	1.502	0.150	1.543
1.071	1.220	0.527	1.360	0.264	1.498	0.200	1.520
1.339	1.193	0.659	1.322	0.329	1.454	0.249	1.512
1.604	1.171	0.790	1.279	0.400	1.448	0.304	1.486
1.871	1.160	0.922	1.250	0.460	1.434	0.355	1.475
2.138	1.117	1.054	1.228	0.532	1.395	0.405	1.456
2.406	1.097	1.186	1.227	0.596	1.375	0.451	1.430
2.673	1.075	1.318	1.205	0.663	1.353	0.500	1.428

TABLE 2  
*Values of  $\lambda_{\phi}$  for the Unswept Aerofoil*

$V = 40$ ft/sec		$V = 80$ ft/sec		$V = 160$ ft/sec		$V = 210$ ft/sec	
$\omega$	$\lambda_{\phi}$	$\omega$	$\lambda_{\phi}$	$\omega$	$\lambda_{\phi}$	$\omega$	$\lambda_{\phi}$
0.268	0.036	0.132	0.036	0.066	0.030	0.050	0.018
0.536	0.155	0.263	0.067	0.132	0.040	0.100	0.037
0.803	0.238	0.397	0.112	0.198	0.061	0.150	0.051
1.071	0.290	0.527	0.154	0.264	0.079	0.200	0.063
1.339	0.374	0.659	0.202	0.329	0.105	0.249	0.083
1.604	0.471	0.790	0.231	0.400	0.127	0.304	0.095
1.871	0.559	0.922	0.279	0.460	0.147	0.355	0.115
2.138	0.671	1.054	0.314	0.532	0.173	0.405	0.137
2.406	0.824	1.186	0.363	0.596	0.202	0.451	0.151
2.673	0.961	1.318	0.411	0.663	0.214	0.500	0.168

TABLE 3  
*Values of  $\lambda_{\phi}$  and  $\lambda_{\dot{\phi}}$  for the Swept Aerofoil*

$\omega$	$\lambda_{\phi}$	$\lambda_{\dot{\phi}}$	$\omega$	$\lambda_{\phi}$	$\lambda_{\dot{\phi}}$
0	0				
0.1	0.005	1.245	0.9	0.199	1.071
0.2	0.013	1.228	1.0	0.223	1.051
0.3	0.038	1.199	1.1	0.241	1.029
0.4	0.056	1.170	1.2	0.264	1.017
0.5	0.075	1.159	1.3	0.290	1.009
0.6	0.123	1.139	1.4	0.330	1.038
0.7	0.150	1.123	1.5	0.371	1.019
0.8	0.180	1.105	1.6	0.402	0.990



**TABLE 4**  
*Values of  $l_1$  and  $l_2$  for the Swept Aerofoil (Axis OY)*

$\omega$	$l_1$	$l_2$	$\omega$	$l_1$	$l_2$
0	0.514		0.9	0.653	
0.1	0.520	1.470	1.0	0.676	↑
0.2	0.525	↑	1.1	0.709	Constant ↓
0.3	0.526	Constant	1.2	0.737	
0.4	0.542	↓	1.3	0.746	
0.5	0.551		1.4	0.853	
0.6	0.597		1.5	0.894	
0.7	0.627		1.6	0.912	
0.8	0.652				

**TABLE 5**  
*Values of  $\lambda_0$  and  $\lambda_{\theta}$  for the Swept Aerofoil*

$\omega$	$\lambda_0$	$\lambda_{\theta}$	$\omega$	$\lambda_0$	$\lambda_{\theta}$
0	1.773	0.735	0.9	1.525	1.322
0.1	1.720	0.826	1.0	1.514	1.393
0.2	1.713	0.911	1.1	1.511	1.454
0.3	1.697	0.978	1.2	1.521	1.518
0.4	1.667	1.049	1.3	1.555	1.558
0.5	1.633	1.120	1.4	1.616	1.568
0.6	1.612	1.187	1.5	1.687	1.562
0.7	1.575	1.258	1.6	1.767	1.552
0.8	1.552				

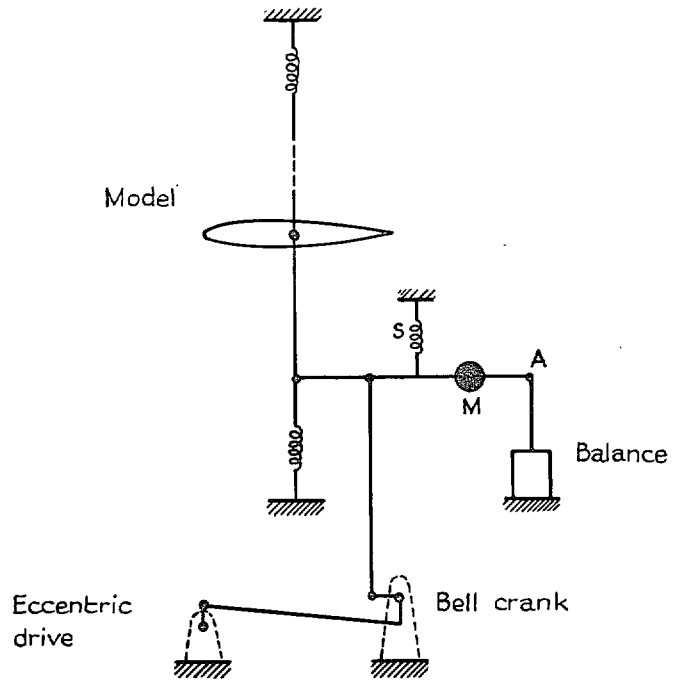


FIG. 1. Diagram of forcing station.

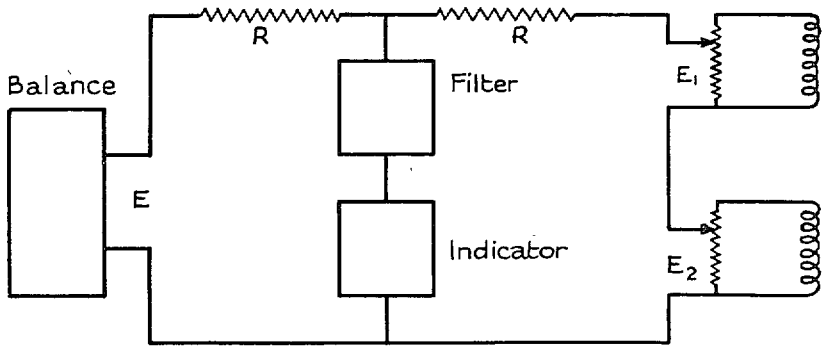


FIG. 2. Alternating current bridge circuit.

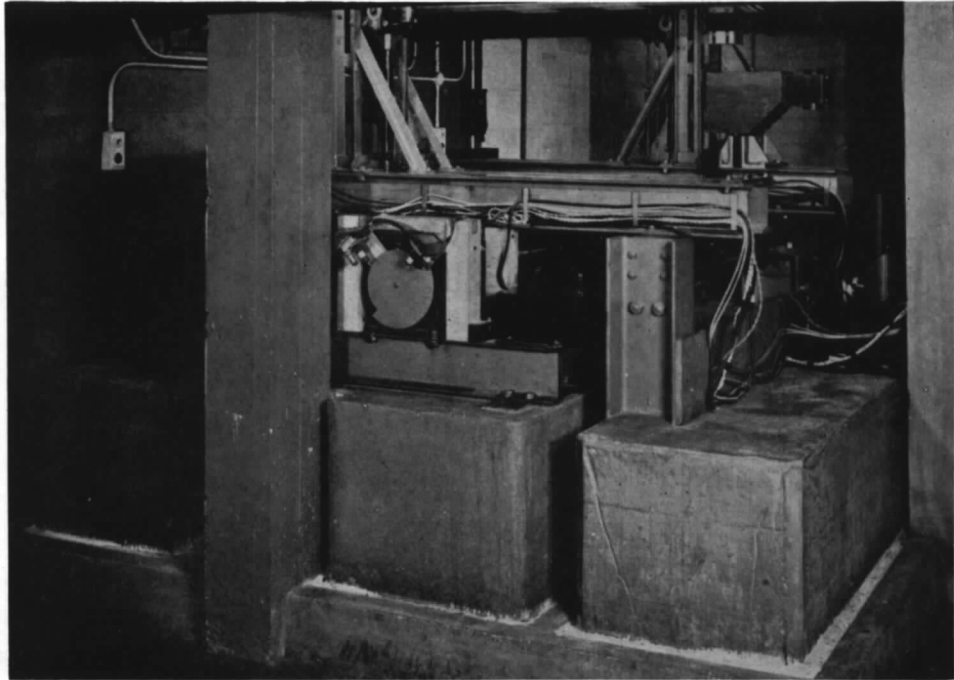


FIG. 3. Mechanical oscillator.

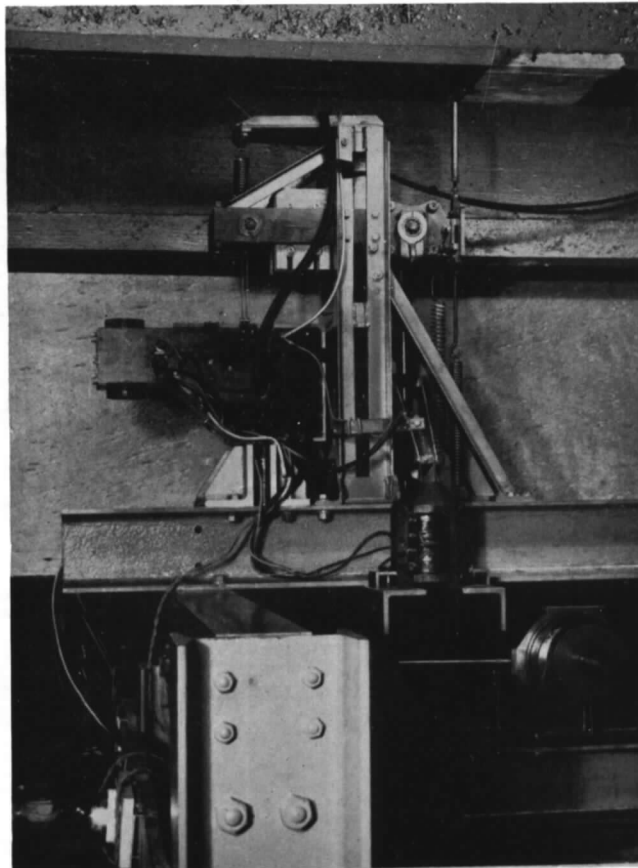


FIG. 4. Forcing station with balance.

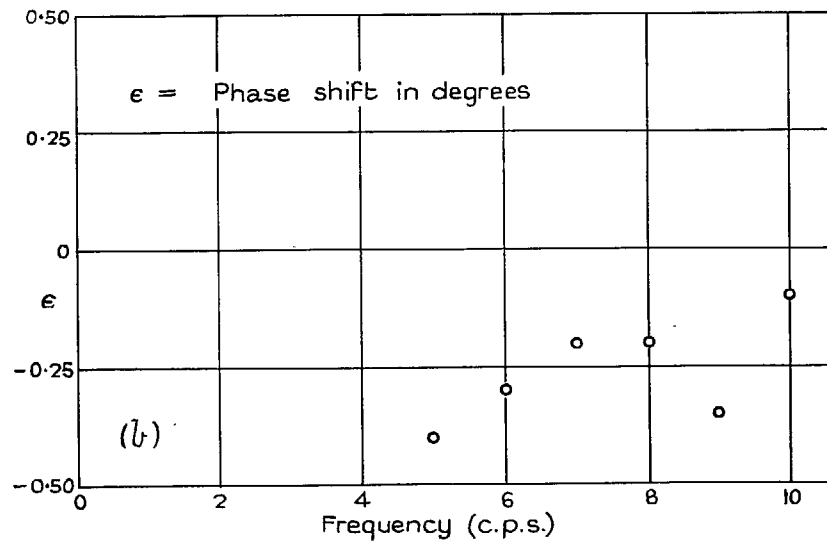
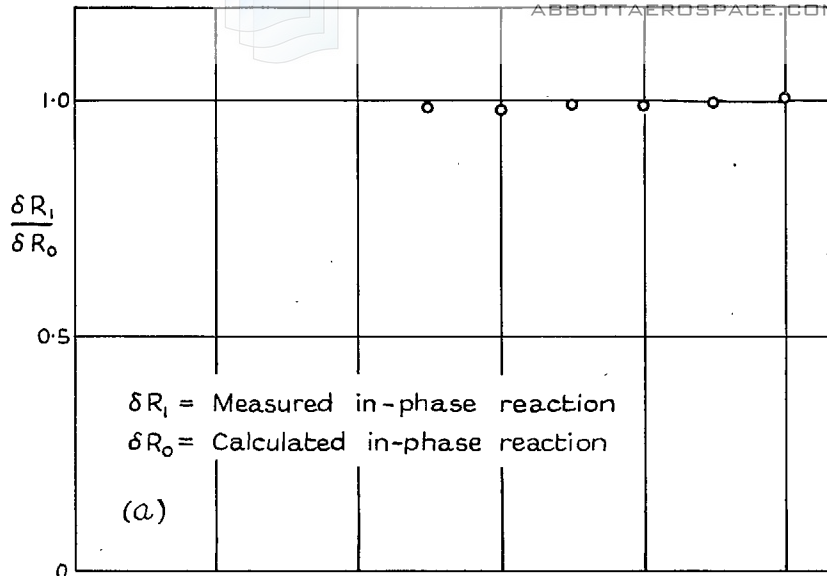


FIG. 5. Dynamic calibration.

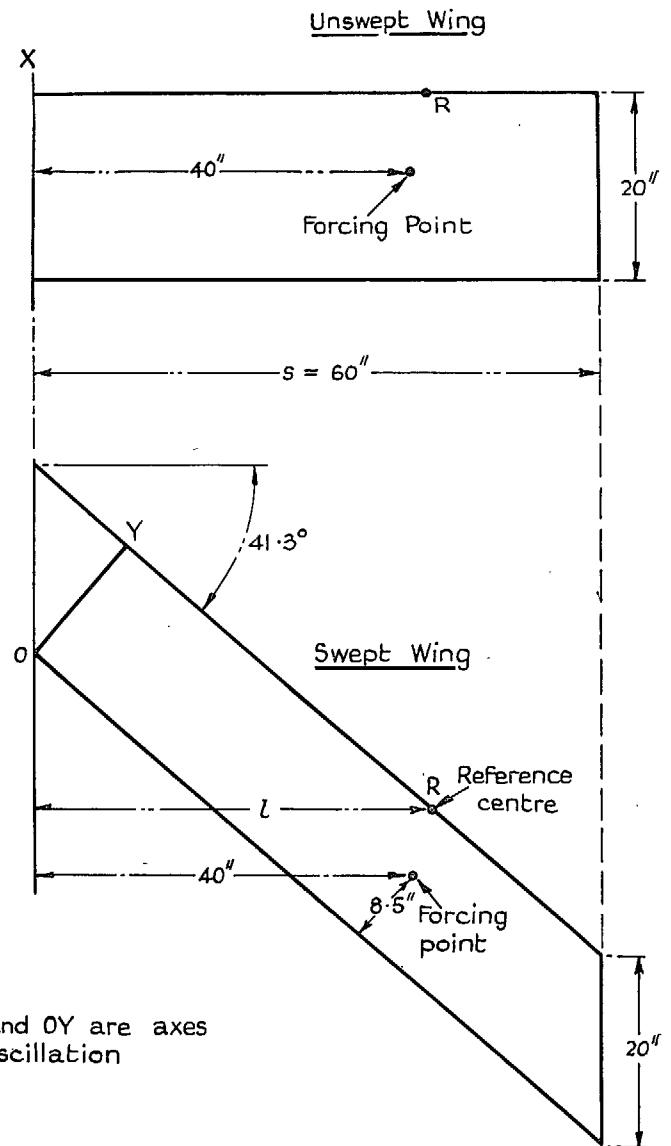


FIG. 6. Planform of aerofoils.



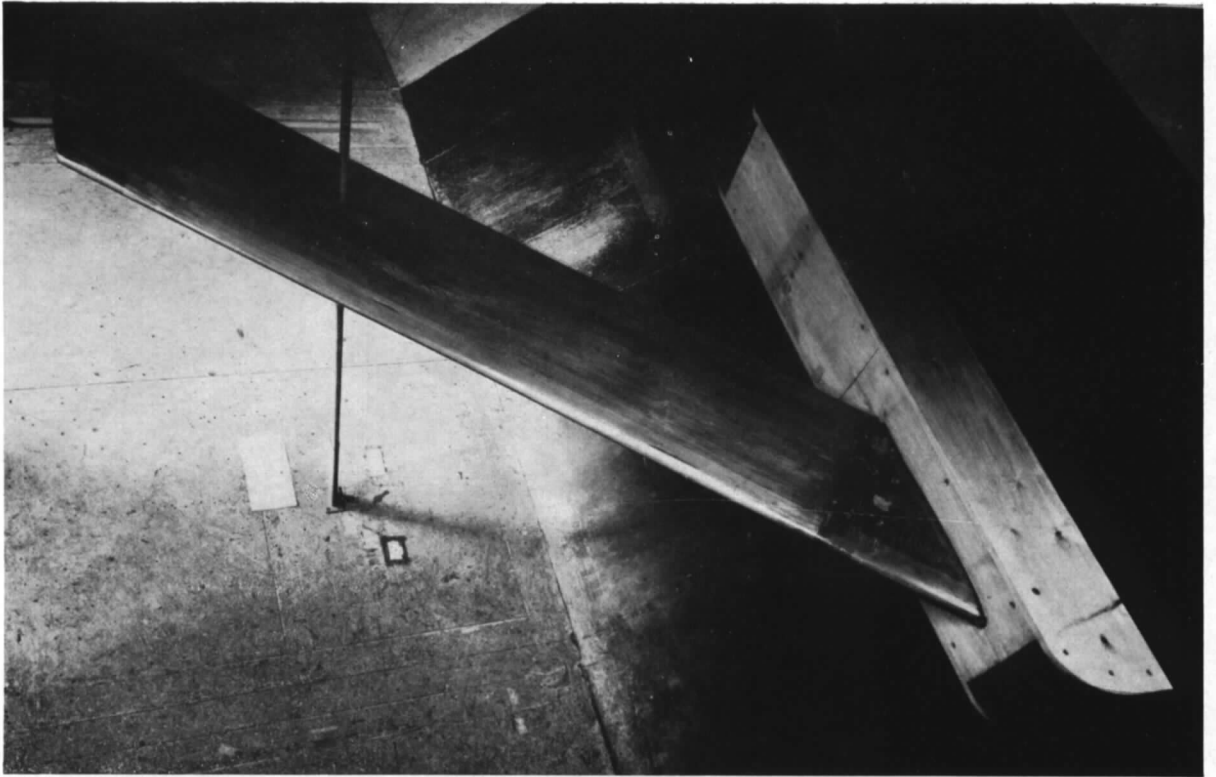


FIG. 7. View of swept wing from below.

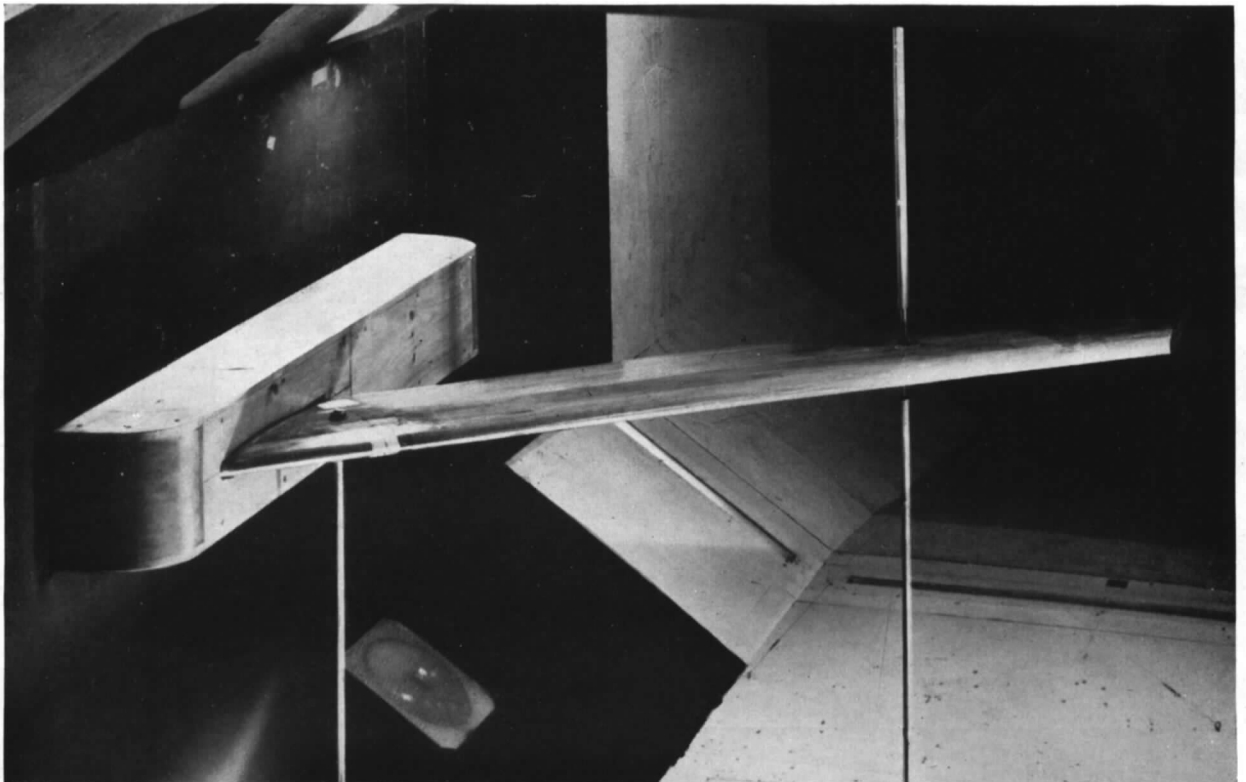


FIG. 8. View of swept wing from above.

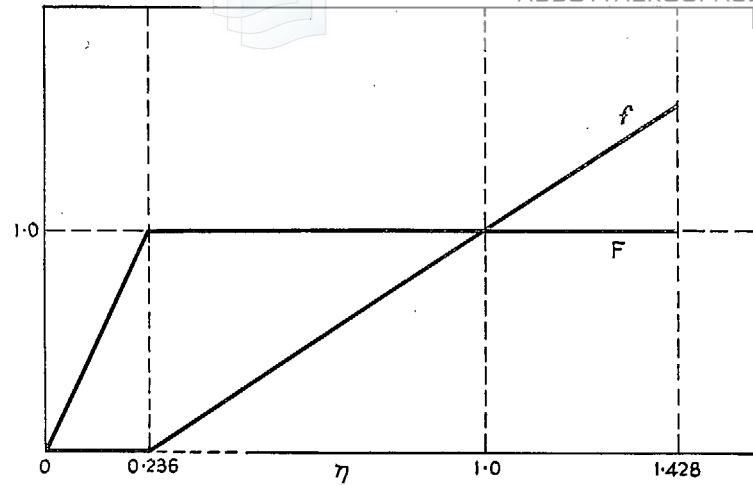


FIG. 9. Modal functions for oscillation about  $OY$  (swept aerofoil).

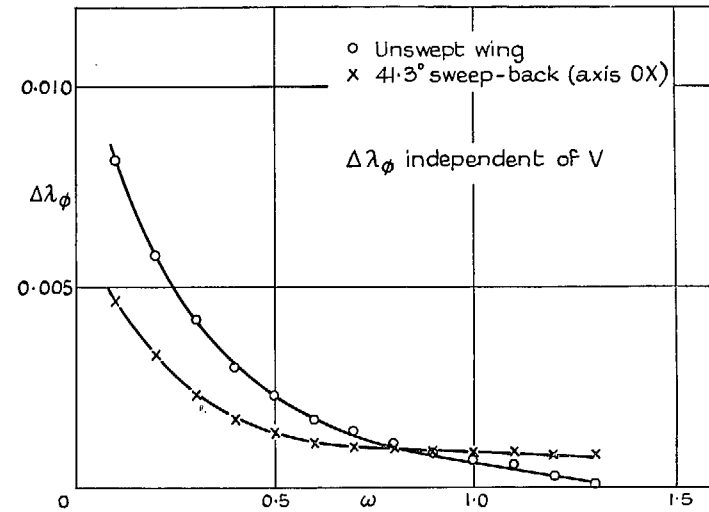


FIG. 11. Corrections to  $\lambda_\phi$  for guard interference.

17

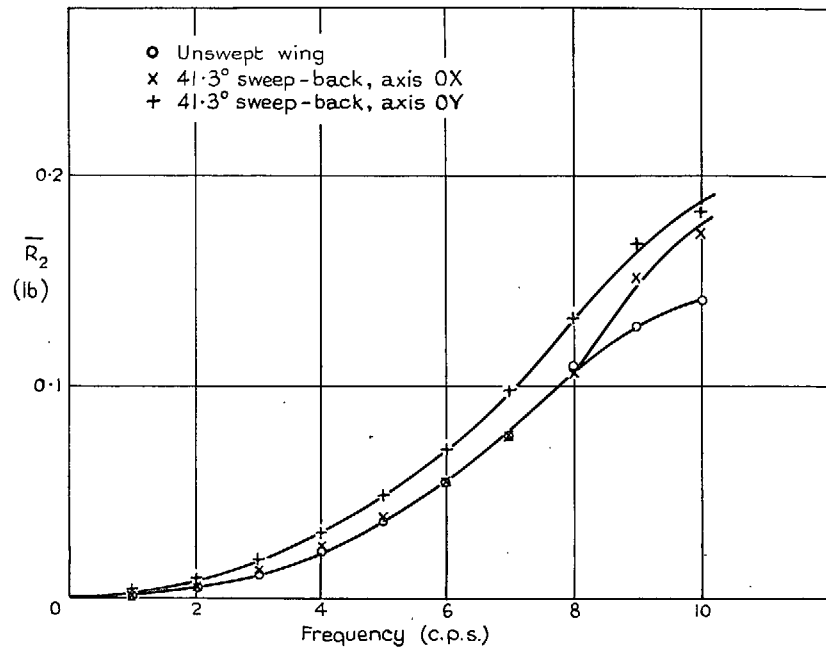


FIG. 10. Dependence of still air damping on frequency.

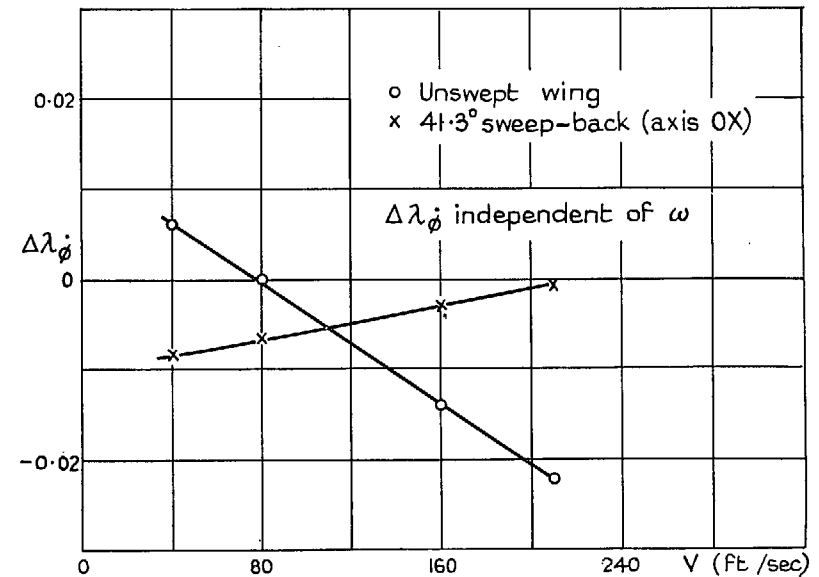


FIG. 12. Corrections to  $\lambda_\phi$  for guard interference.

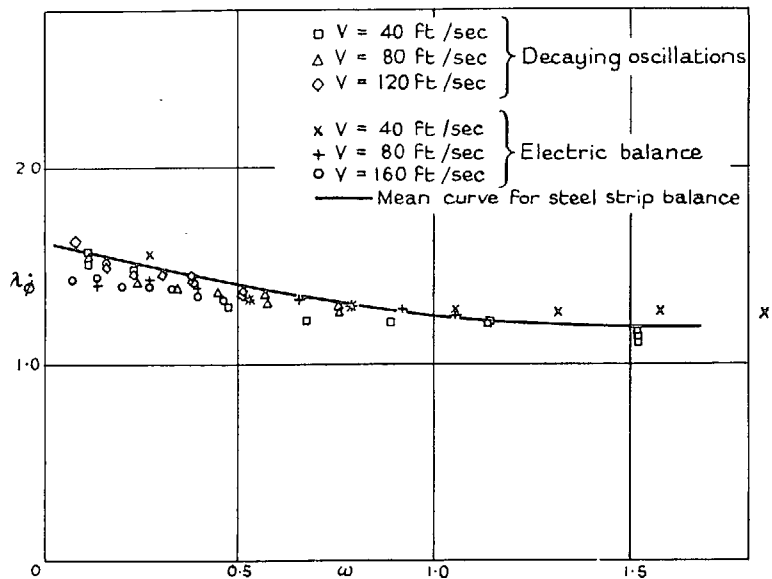


FIG. 13. Comparison of steel-strip and electric balances with decaying oscillation method.

18

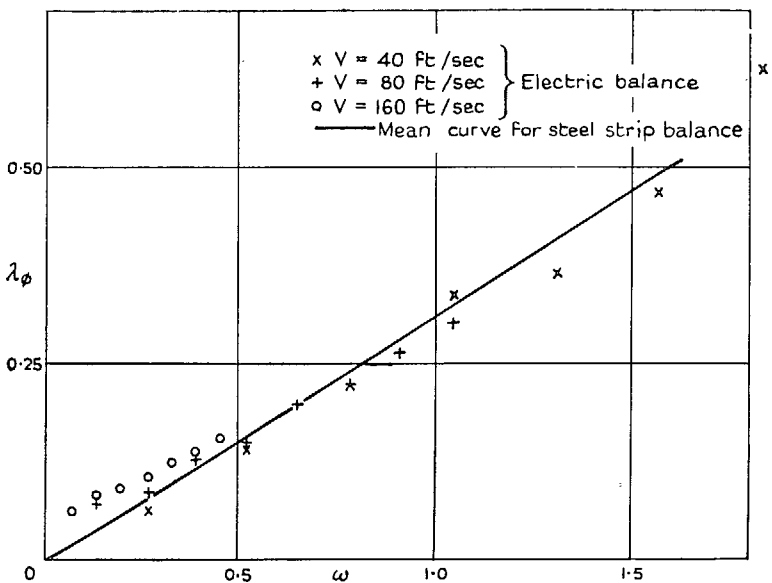


FIG. 14. Comparison of steel-strip and automatic electric balances.

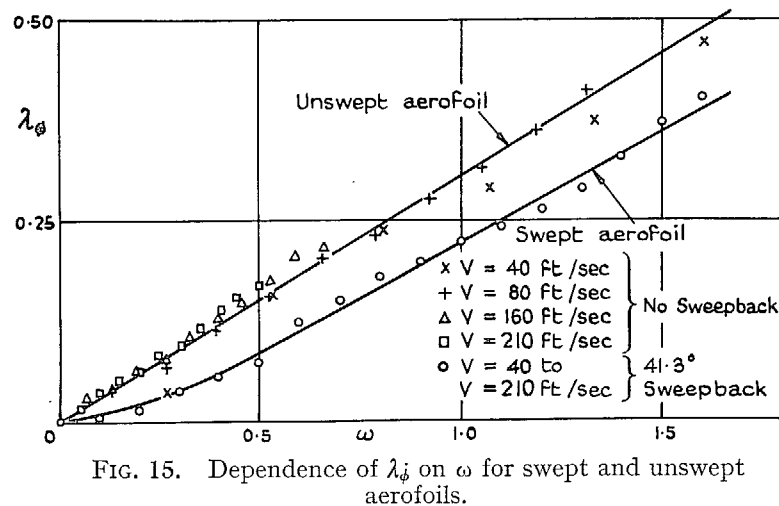


FIG. 15. Dependence of  $\lambda_\phi$  on  $\omega$  for swept and unswept aerofoils.

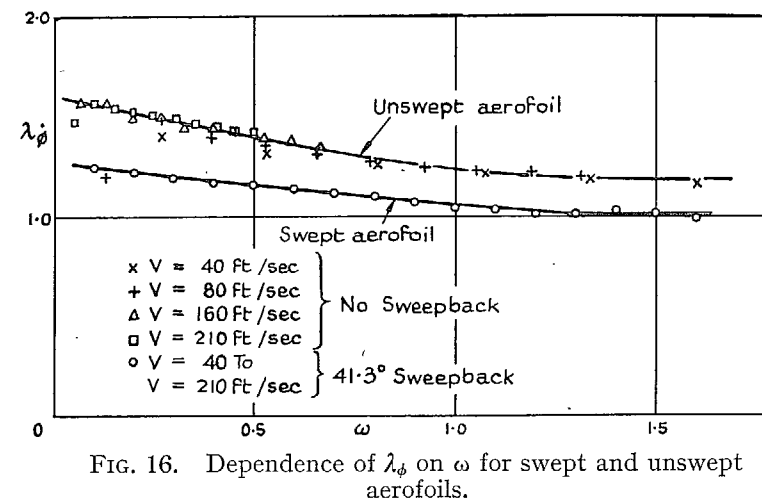


FIG. 16. Dependence of  $\lambda_\phi$  on  $\omega$  for swept and unswept aerofoils.

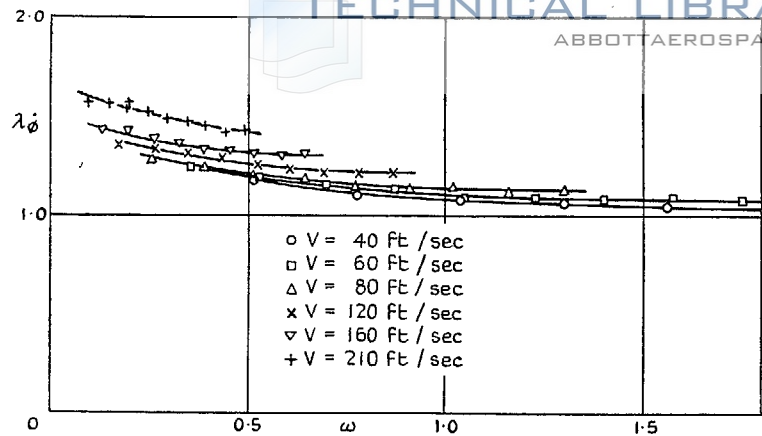


FIG. 17. Effect of model distortion on  $\lambda_\phi$  (swept aerofoil).

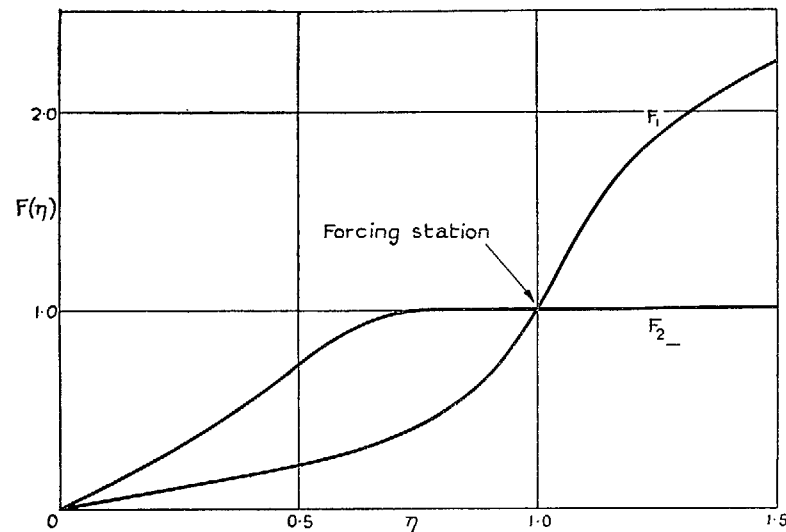


FIG. 19. Distortion modes for swept aerofoil (axis  $OX$ ).

19

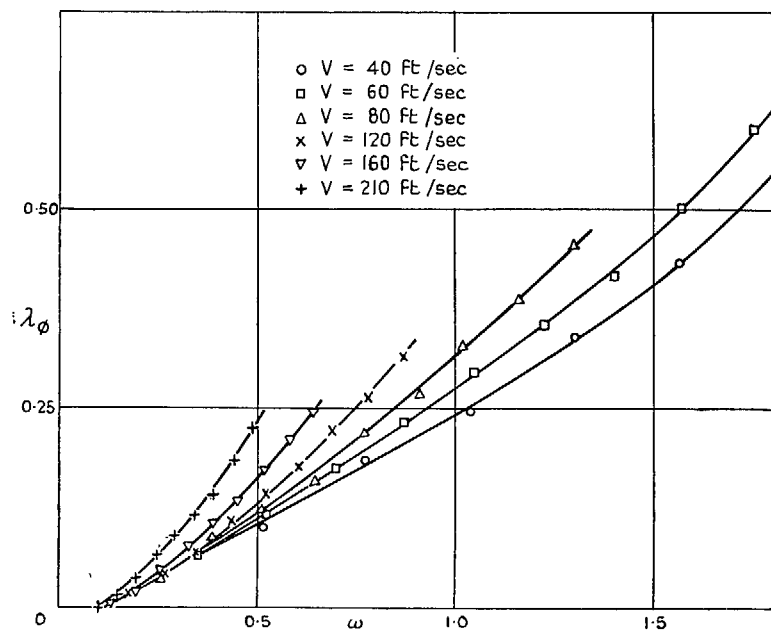


FIG. 18. Effect of model distortion on  $\lambda_\phi$  (swept aerofoil).

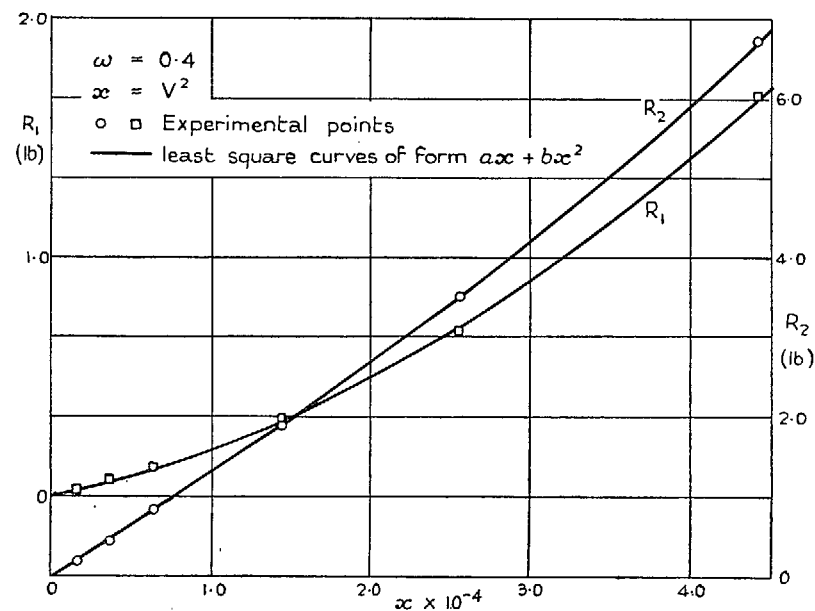


FIG. 20. Measured reactions for constant  $\omega$  (swept aerofoil—axis  $OX$ ).



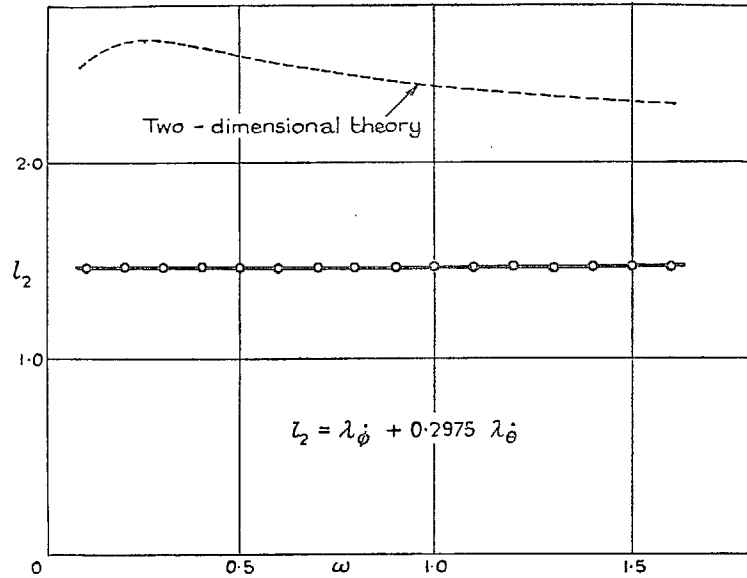


FIG. 21. Dependence of  $l_2$  on  $\omega$  (swept aerofoil—axis  $OY$ ).

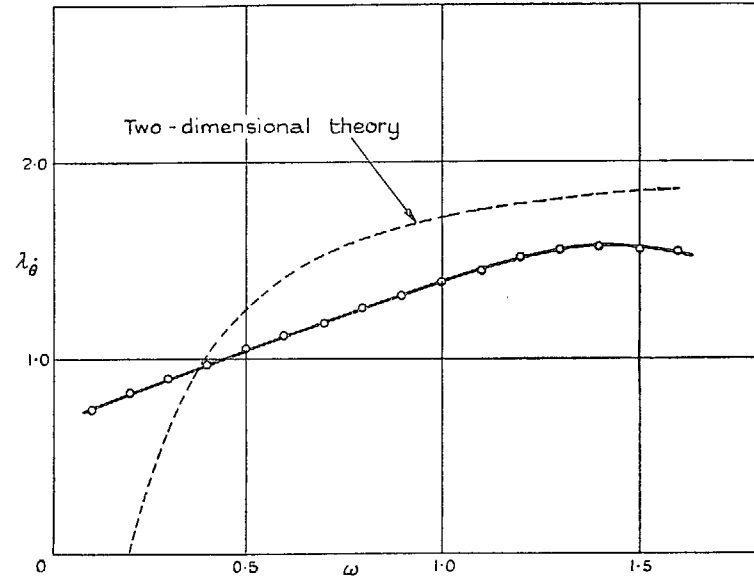


FIG. 23. Dependence of  $\lambda_\theta$  on  $\omega$  (swept aerofoil—axis  $OY$ ).

20

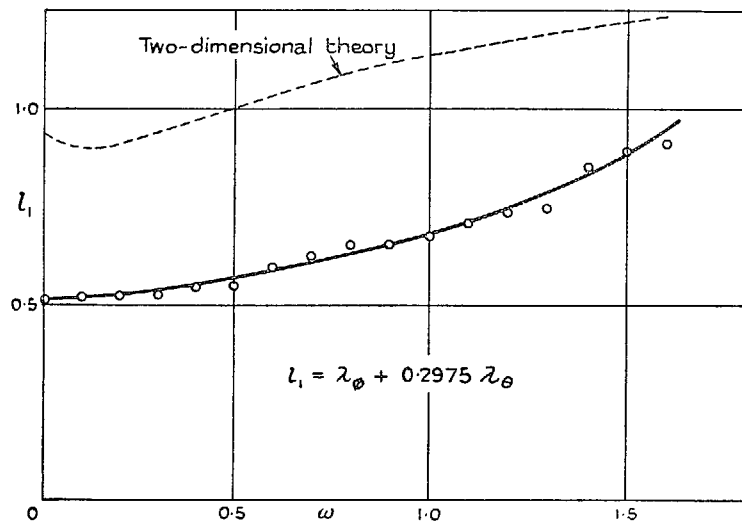


FIG. 22. Dependence of  $l_1$  on  $\omega$  (swept aerofoil—axis  $OY$ ).

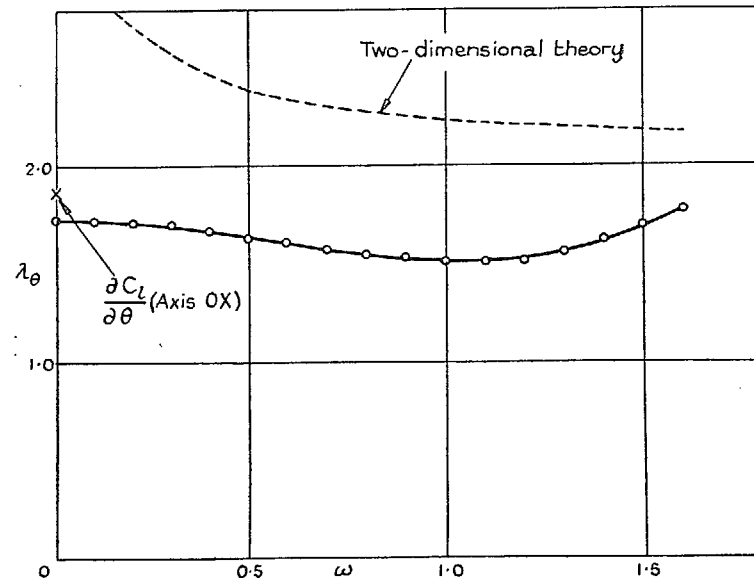


FIG. 24. Dependence of  $\lambda_\theta$  on  $\omega$  (swept aerofoil—axis  $OY$ ).

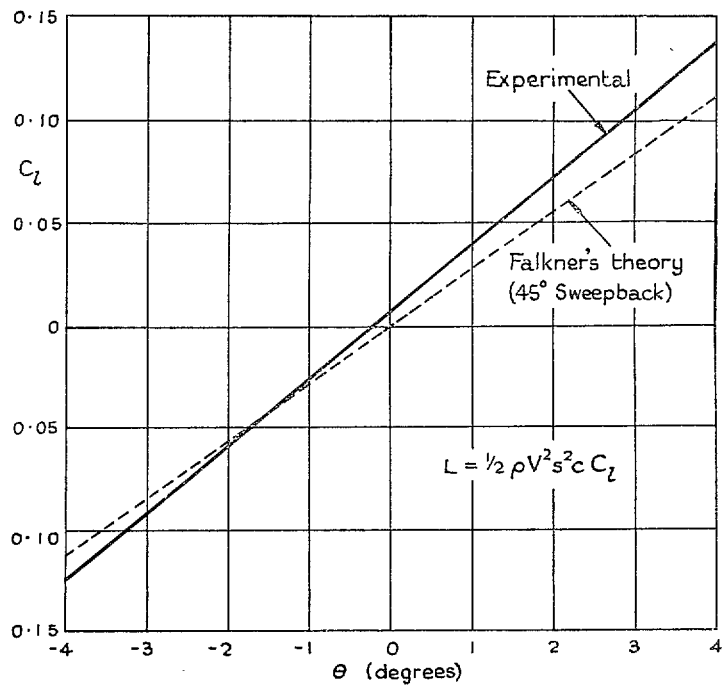


FIG. 25. Variation of  $C_l$  with  $\theta$  (swept aerofoil—axis  $OX$ ).

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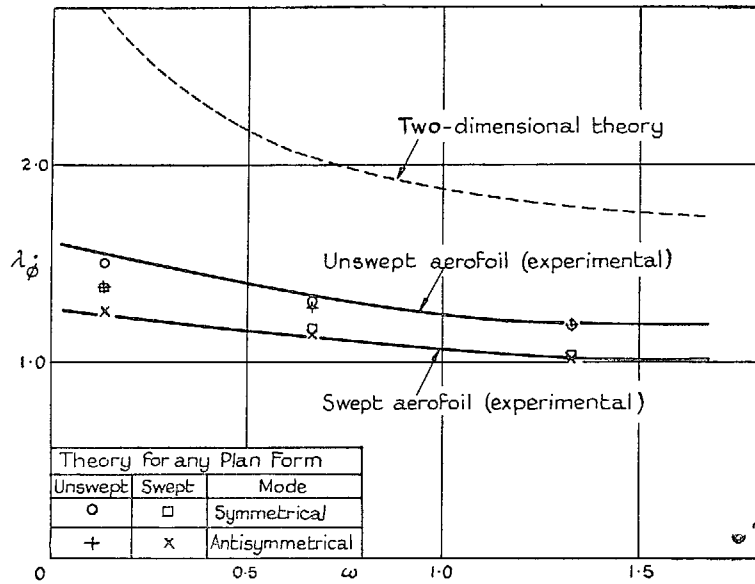


FIG. 26. Comparison with theory for  $\lambda_\phi$ .

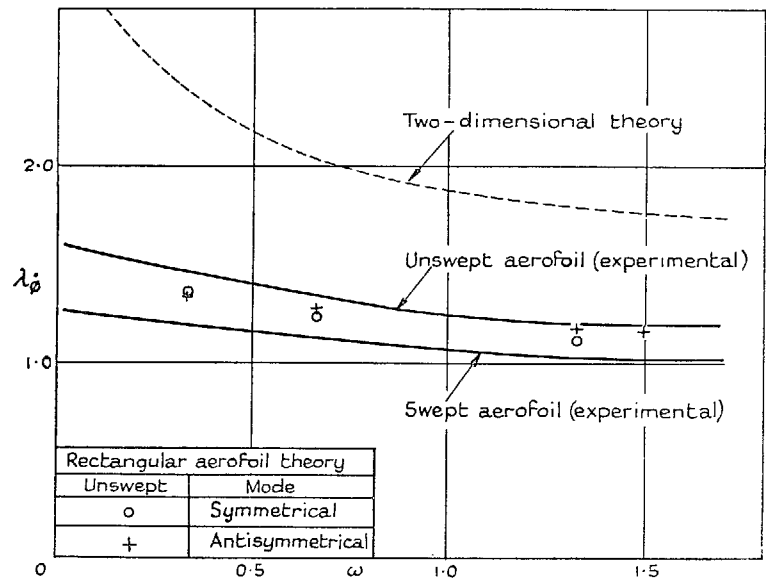


FIG. 28. Comparison with theory for  $\lambda_\phi$ .

22

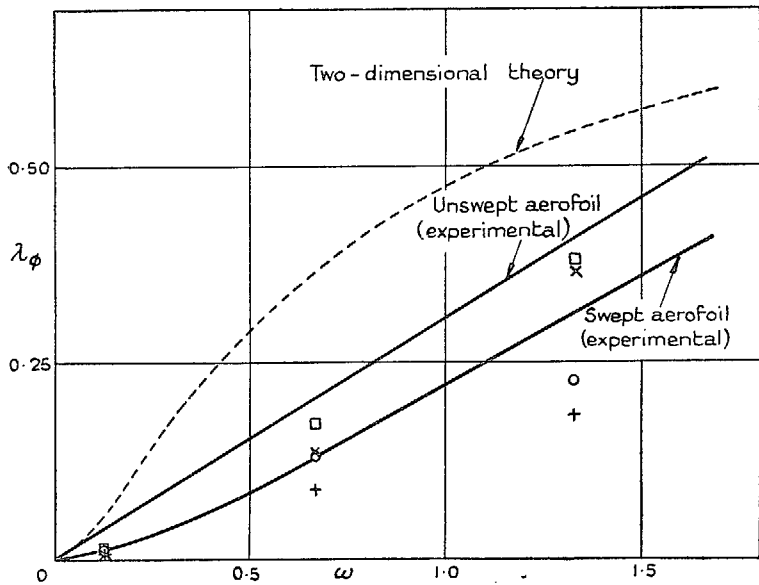


FIG. 27. Comparison with theory for  $\lambda_\phi$ .

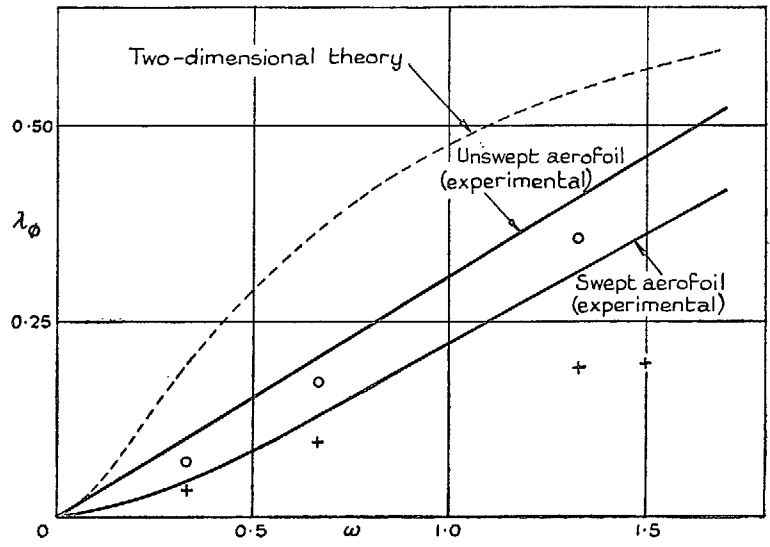


FIG. 29. Comparison with theory for  $\lambda_\phi$ .

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