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Wind-Tunnel Flutter Tests on  
a Model Delta Wing under Fixed and Free  
Root Conditions

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# Wind-Tunnel Flutter Tests on a Model Delta Wing under Fixed and Free Root Conditions

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*Summary.*—Wind-tunnel tests on a model wing of delta plan form are described. Tests have been made with the wing root fixed and also with the root free in pitch and vertical translation. Critical flutter speed and frequency are given for a wide range of variation of wing and fuselage inertia parameters. The results show that, under root-free conditions, body freedom flutter occurs at low values of fuselage pitching moment of inertia, but that at higher values the flutter is similar to that obtained with root-fixed conditions.

1. *Introduction.*—Wind-tunnel flutter tests have previously been made on model wings of straight and swept-back plan forms, both with fixed root conditions by Molyneux<sup>1</sup> (1950) and also with body freedoms in vertical translation and pitch by Jordan and Smith<sup>2</sup> (1950). The present report describes similar work undertaken for wings of delta plan form.

The general advantages of wind-tunnel investigations on models with body freedoms are described in another report by Jordan and Smith<sup>2</sup> (1950). It is there shown that whilst theoretical study of flutter with body freedoms is extremely complicated, the experimental method enables the effects of the various parameters to be readily established. In the particular case of delta wings, the experimental approach has still greater value since theoretical treatment is severely limited by the difficulty of acquiring reliable data on aerodynamic derivatives at the appropriate high taper and low aspect ratios. If the tests are made at low speeds their main limitation is, of course, the exclusion of compressibility effects.

The present report describes low-speed tests which were made on a half-span model delta wing both with fixed-root conditions, and also with body freedoms in pitch and vertical translation, in accordance with the technique described by Jordan and Smith<sup>2</sup> (1950). Variations of wing inertia axis, wing and fuselage centre of gravity, fuselage pitching moment of inertia, and fuselage mass were investigated.

An important feature of these tests is that they are not limited to conditions that are physically possible on an actual aircraft. Thus, for example, some of the tests have spring settings corresponding to negative fuselage inertias (translational or rotational), but the results then obtained enable the operative parts of the curves to be drawn with greater precision, and all parts of the curves can be used to check fundamental theory.

2. *Model Details.*—The wing was of delta plan form with 45 deg sweepback of the leading edge. A half-wing model was used with a span of 45 in., root chord 48 in. and tip chord 3 in. Fig. 1 shows the main structural details and dimensions. A single square-sectioned spar of spruce at 35 per cent of the chord tapered both in width and depth in proportion to wing taper. Uniformly spaced ribs of symmetrical section, RAE 101, having a 10 per cent thickness/chord ratio and constant breadth were glued to the spar parallel to the airflow; the ribs were of composite spruce

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\* R.A.E. Report Structures 89, received 25th January, 1951.

and balsa wood in order to reduce weight and to obtain an inertia axis position for the bare wing at 45 per cent chord.

Paper leading and trailing edges were glued to the wing, and the covering was of silk doped with a solution of Vaseline in chloroform. Movable lead weights were fitted to each rib, enabling three wing inertia axis positions to be obtained, at 40, 45 and 50 per cent of the wing chord.

Stiffness and resonance tests were made on the wing with fixed root, the results of which are given in Table 1 together with further details of the wing. The normal modes for the wing inertia axis at 40 per cent chord are shown in Fig. 2; the normal modes for 45 and 50 per cent wing inertia axis positions were very similar and have not been included.

3. *Test Rig and Method of Test.*—The model was attached at two points on the root rib (Fig. 1) to the supporting rig which permitted body freedoms in vertical translation and in pitch. Weights and springs attached to the framework of the rig enabled fuselage inertia conditions to be represented. Variations in spring stiffness and spring position allowed effective pitching moments of inertia, and mass of the fuselage, to be adjusted to zero or negative values under oscillating conditions, when the springs act as negative masses. For a given position for the wing and fuselage centre of gravity, the pitching moments of inertia were varied by altering the positions of pairs of similar springs and weights, but keeping them symmetrically spaced about the centre of gravity. By this means, the centre of gravity remained fixed for any frequency of oscillation. Frequency was measured by means of a contact make-and-break operating a counter for a given time. A full description of the rig is given by Jordan and Smith<sup>2</sup> (1950).

The wing was mounted vertically in the Royal Aircraft Establishment 5-ft Open-Jet Wind Tunnel. For each test the wing was manually excited in pitch and vertical translation by means of a cord attached to the nose of the fuselage. At the critical flutter speed, tunnel speed and frequency of the oscillation were measured. No safety grab was used in the tests in order to avoid disturbance of the airflow over the wing; a hand-held cord provided a convenient means of arresting the motion.

4. *Range of Tests.*—Three positions of the wing inertia axis were investigated, 40 per cent, 45 per cent and 50 per cent of the chord. At each of these positions three overall centre of gravity positions (wing and fuselage) were obtained, and the effective pitching moment of inertia and effective mass of the fuselage were varied separately in each case.

Mass and pitching moment of inertia of the fuselage are expressed throughout the report as percentages of wing mass and moment of inertia, both moments of inertia being calculated about the overall centre of gravity (Table 1). Since changes in frequency affect the effective negative spring masses, it has not been possible to cover the same ranges of mass and pitching moment of inertia in each case, but for all cases the fuselage pitching moments of inertia lie between  $-100$  per cent and  $+200$  per cent of the wing pitching moments of inertia, and the fuselage mass variations are from 16 per cent to 70 per cent of the wing mass.

It was found impossible to obtain actual test results over a small range of effective values of fuselage pitching inertia in the neighbourhood of the zero value. The reasons for this are discussed later in section 5. A sufficiently accurate indication of the results in this range could, however, be obtained by extrapolation (*see* section 5).

5. *Test Results.*—Two distinct types of flutter occurred which may be called 'body-freedom' and 'disturbed-root'. The body-freedom flutter resembled that obtained on the same rig with unswept and swept-back wings, characterised by considerable pitching of the root and wing bending. Although no measurements of the mode were taken, there appeared to be a nodal point at the root in pitch at approximately 30 per cent of the root chord aft of the wing leading edge, and a nodal line on the wing running roughly parallel to the airflow at 70 per cent of the span. The frequencies of body-freedom flutter were between 4.0 and 5.5 cycles per second for all the cases tested.

Disturbed-root flutter resembled fixed-root flutter in the wing mode, but was accompanied by small root displacements mainly in pitch. The wing mode appeared to be primarily a coupling of the normal torsional mode and the first flexural overtone; the frequencies varied between 7.0 and 8.5 cycles per second and were above the appropriate fixed-root flutter frequencies.

The type of flutter encountered changed mainly with the pitching moment of inertia of the fuselage. Body-freedom flutter was generally associated with negative, and disturbed-root flutter with positive, fuselage moments of inertia, although variations of wing inertia axis and overall centre of gravity position affected the value of the fuselage moment of inertia at the change-over point from one type of flutter to the other.

Figs. 3, 4 and 5 show the results for wing inertia axes 40 per cent, 45 per cent and 50 per cent chord respectively. Critical flutter speed and frequency are shown plotted against effective fuselage pitching moment of inertia for overall centre of gravity positions at 50 per cent, 47.5 per cent and 45 per cent of the root chord. The fixed-root flutter speeds and frequencies are also shown on the diagrams for comparison with the disturbed-root flutter results.

The effect of mass on flutter speed is small and in most cases indeterminate, and no attempt has therefore been made to draw separate curves through points of constant mass. Instead, the best curve through all the points obtained has been drawn.

Starting from small or negative values of moment inertia it will be seen that the flutter speed for body-freedom flutter increases with increasing fuselage moment of inertia until a transition point is reached. If the moment of inertia is increased beyond this point disturbed-root flutter occurs, the flutter speed being above the fixed-root flutter speed. A further increase of the moment of inertia reduces the flutter speed, the curve becoming asymptotic to the fixed-root case. The flutter speed is seen to approach the fixed-root speed much more rapidly with increase of moment of inertia in the case of an aft than with a forward wing inertia axis position. At the change-over point a jump in frequency takes place, and the change of mode can be seen. In this region the flutter is a mixture of both types, each type occurring for a few seconds only before being displaced by the other. The frequency change is represented by a discontinuity in the curve, as far as could be ascertained.

It will be seen from Figs. 3, 4 and 5 that a gap occurs in the curves with change of frequency. In order to understand the reasons for this gap let us consider typical curves of flutter speed and frequency plotted against effective pitching moment of inertia,  $I$  (Fig. 6). Suppose that, from a single test, points A and C are established on the speed and frequency curves; then the dotted line through C represents the variation of  $I$  with change of frequency due to the change of equivalent mass of the springs. The dotted line is asymptotic to the  $I$ -axis when the frequency is zero and asymptotic to the line  $I = I_a$  when the frequency is infinite, where  $I_a$  represents the moment of inertia of the fuselage with no springs attached. This spring curve through C cuts the frequency curve at a second point D corresponding to a point B on the speed curve. Points B and D therefore represent a second condition of flutter which may be obtained with exactly the same mass and spring arrangement as A and C. Under test, A and C represent the condition actually obtained, since the flutter speed A is lower than speed B. If the value of  $I_a$  is altered, the values of  $I$  for points A and B move in the same direction, but the values of flutter speed move in opposite directions. The case obtained under test is the point with the lower flutter speed. When the two points have equal flutter speeds, any variation in  $I_a$  will give a lower flutter speed since either A or B must decrease in value. It becomes clear that there is a limiting value of flutter speed dependent on spring stiffness and represented in Fig. 6 by LL'. By optimum variation of both  $I_a$  and spring stiffness the limiting flutter speed is made as large as possible and the gap consequently as small as possible. The wide range of mass and moment of inertia parameters covered by the present tests necessitated the use of a number of sets of springs and it is for this reason that the maximum flutter speeds obtained are not, in all cases, equal on both branches of the curve. In one or two cases the experimental results corresponded to the critical case, represented by LL' (Fig. 6). It is interesting to note that in these cases a mixture of the two types of flutter was observed.



6. *Discussion.*—6.1. The shape of all the corresponding curves in Figs. 3, 4 and 5 is similar, and the most important effect of variations in the wing inertia axes and overall centre of gravity position is to displace the typical curve along one or both axes. As the wing inertia axis is moved aft the critical flutter speeds are reduced, the reduction being much larger between inertia axes of 45 per cent and 50 per cent than between 40 per cent and 45 per cent. In addition, the curve is displaced in a direction of increasing moment of inertia. Forward movement of the centre of gravity also displaces the curve in a direction of increasing moment of inertia, the increase being largest with the wing inertia axis at 50 per cent chord. It is impossible to deduce that there is any marked change of flutter speed with variation of centre of gravity position.

The frequencies of body freedom flutter remain nearly constant for wing inertia axes at 40 per cent and 45 per cent chord, for all centre of gravity positions, but increase by nearly 25 per cent with wing inertia axis at 50 per cent chord. The frequency variation is similar for disturbed-root flutter and is consistent when considered in conjunction with the fixed-root flutter frequencies. The normal mode frequencies, however, decrease with aft movement of the wing inertia axis, and for wing inertia axis at 45 per cent and 50 per cent chord the fixed-root flutter frequency is above that of the first three normal modes. Although this is unusual, Jordan<sup>3</sup> (1948) has shown that for a forward position of the wing flexural axis the flutter frequency may exceed the frequencies of the uncoupled flexural and torsional modes, and that this is most likely to occur with a wing of low density.

6.2. The tests indicate that for a model delta wing having an inertia distribution and stiffness ratio representative of full-scale practice, there is a possibility of body-freedom symmetric flutter only at small values of fuselage pitching moment of inertia. For large fuselage pitching moments of inertia the critical flutter speed is above the fixed-root flutter speed, the difference being greatest when the wing inertia axis is forward. Since, also, a forward position of the wing inertia axis is associated with body-freedom flutter only in the negative moment of inertia region, it follows that a forward wing inertia axis position combined with an aft centre of gravity position will be the most favourable condition for all real cases.

6.3. On the technique side the tests have shown that improvements are needed in two respects: namely, in the measurement of frequency and in minimising the effects on stiffness, and hence flutter speed, of variations in wind-tunnel temperature and humidity.

Frequency measurement was found to be difficult using the make-and-break contact at the higher frequencies, particularly with small movements of the part of the structure to which the contact breaker was attached.

There is need for an accurate frequency measuring device to deal with frequencies from 0 to 10 cycles per second, particularly important in allowing for spring effect, where a small error in frequency may lead to a large error in moment of inertia, should the springs be far apart.

Temperature and humidity were found to have an important effect on flutter speed (up to 10 per cent) but were, in the main, overcome by making tests, in quick succession while conditions remained constant, and checking a standard case when conditions varied.

In all flutter tests on wood and silk models, ageing of the model occurs and some variation of stiffness distribution may be expected on this account. Here again this may be largely overcome by completing all the tests in as short an overall period of time as possible, and also by preventing violent oscillations during the tests. The present wing appeared to suffer little from ageing after some preliminary fluttering which allowed the glued joints to settle, and surplus glue to work loose.

6.4. In order to correlate the test results with current torsional stiffness requirements, the criterion of A.P. 970<sup>4</sup> has been evaluated for all cases and the results are shown in Fig. 7. It is important to note that the value of  $V$  used here is the actual critical flutter speed and not the design diving speed which would normally be used. For use as a design criterion the values of Fig. 7 would therefore be multiplied by the usual safety factor, which should take account of compressibility effects not included in these tests.

7. *Conclusions.*—(a) Two kinds of flutter have been encountered during these wind-tunnel tests on a model delta wing: body-freedom and disturbed-root. The latter resembles fixed-root flutter in mode except that there are small displacements of the root.

(b) The frequency of the disturbed-root flutter is some 80 per cent to 100 per cent higher than body-freedom flutter frequency and is approximately equal to the fixed-root frequency.

(c) Body-freedom flutter occurs mainly in the region of negative fuselage pitching moment of inertia. The critical flutter speed rises with increasing moment of inertia until a transition point occurs near the zero value. For values of moment of inertia greater than this, the flutter is of disturbed-root type, the critical speed dropping asymptotically to the corresponding fixed-root value.

(d) Variations of wing inertia axis indicate that aft positions give reduced flutter speeds and increase the values of pitching moment of inertia at which the transition point occurs.

(e) The position of the overall centre of gravity has no appreciable effect on flutter speed, but forward positions give an increase in the values of pitching moment of inertia at the transition point.

(f) The effect of body mass variations, within the range considered, is small and cannot be determined from the present test results.

(g) The frequencies of both body-freedom and disturbed-root flutter are approximately constant for variations in pitching moment of inertia, mass, and centre of gravity position; there is, however, a rise in frequencies of approximately 25 per cent for the aft position of the wing inertia axis compared with the frequencies at the other positions.

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## REFERENCES

<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	W. G. Molyneux ...	The flutter of swept and unswept wings with fixed-root conditions. R. & M. 2796. January, 1950.
2	P. Jordan and F. Smith ...	Wind-tunnel technique for flutter investigations on swept wings with body freedoms. A.R.C. 13,729. September, 1950.
3	...	A.V.A. Monograph. Reports and Translations No. 1013. March, 1948.
4	...	Ministry of Supply Publication A.P. 970, Chapter 503.

TABLE 1

*Wing Details*

Weight of wing 15.57 lb

Wing density 0.92 lb/cu ft

Wing pitching moments of inertia about overall c.g.

		c.g.: per cent of root chord		
		50	47.5	45
Wing inertia axis: per cent of chord	40	2119 lb/in. <sup>2</sup>	2167 lb/in. <sup>2</sup>	2261 lb/in. <sup>2</sup>
	45	1916 lb/in. <sup>2</sup>	2003 lb/in. <sup>2</sup>	2136 lb/in. <sup>2</sup>
	50	2226 lb/in. <sup>2</sup>	2361 lb/in. <sup>2</sup>	2540 lb/in. <sup>2</sup>

Measured wing stiffnesses at 0.7 span

Torsional stiffness $m_\theta$ .. ..	62 lb ft/radn
<i>(<math>m_\theta</math> measured in the line of flight)</i>	
Flexural stiffness $l_\phi$ .. ..	375 lb ft/radn
Wing flexural axis .. ..	15 per cent chord
Distance $d$ (0.9 semi-span) .. ..	40.5 in.
Mean chord $c_m$ .. ..	25.5 in.

Normal mode frequencies (fixed root)

		1st mode 'Fundamental flexure'	2nd mode '1st flexural overtone'	3rd Mode 'Torsion'
Wing inertia axis: per cent of chord	40	3.3 c.p.s.	7.6 c.p.s.	9.7 c.p.s.
	45	3.3 c.p.s.	5.2 c.p.s.	7.4 c.p.s.
	50	3.3 c.p.s.	4.8 c.p.s.	6.0 c.p.s.

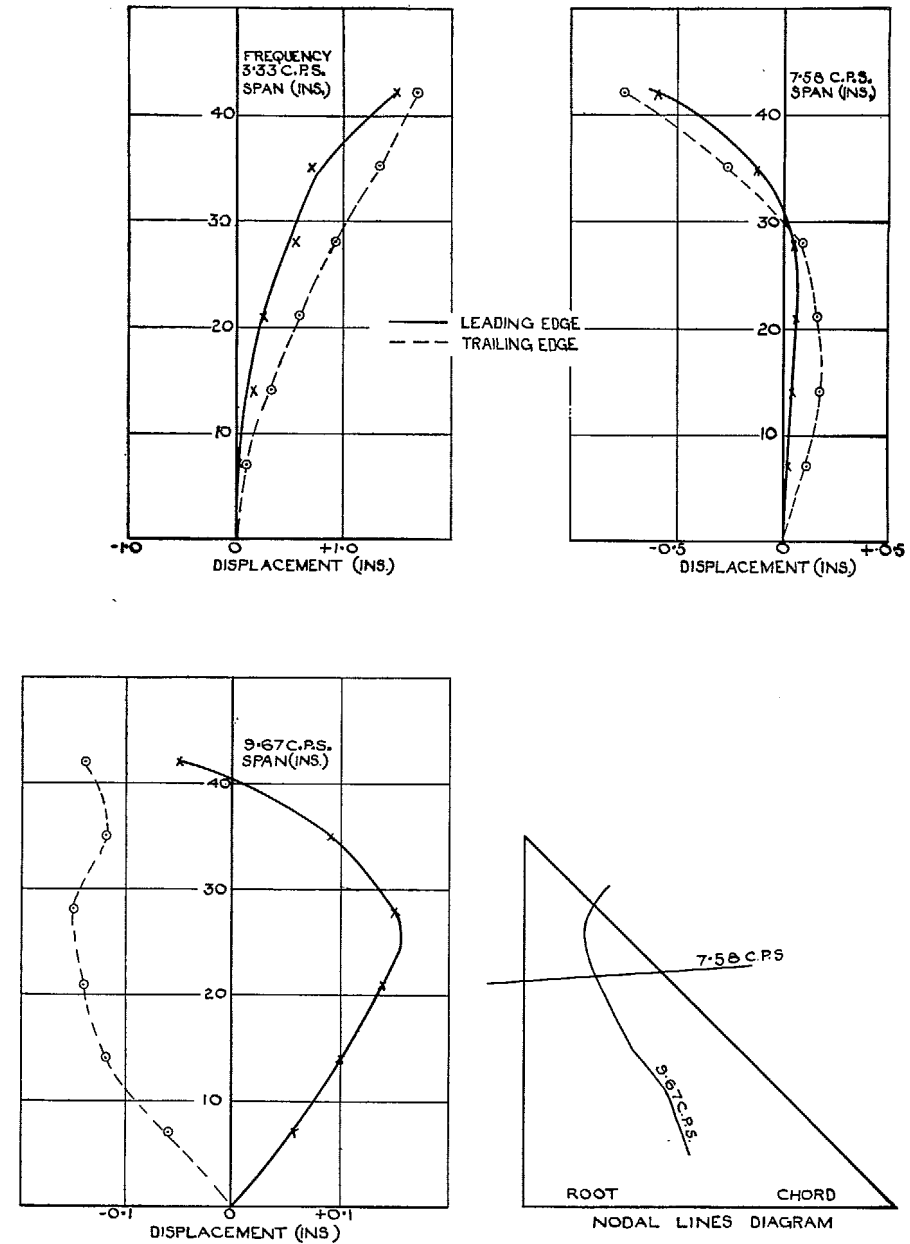
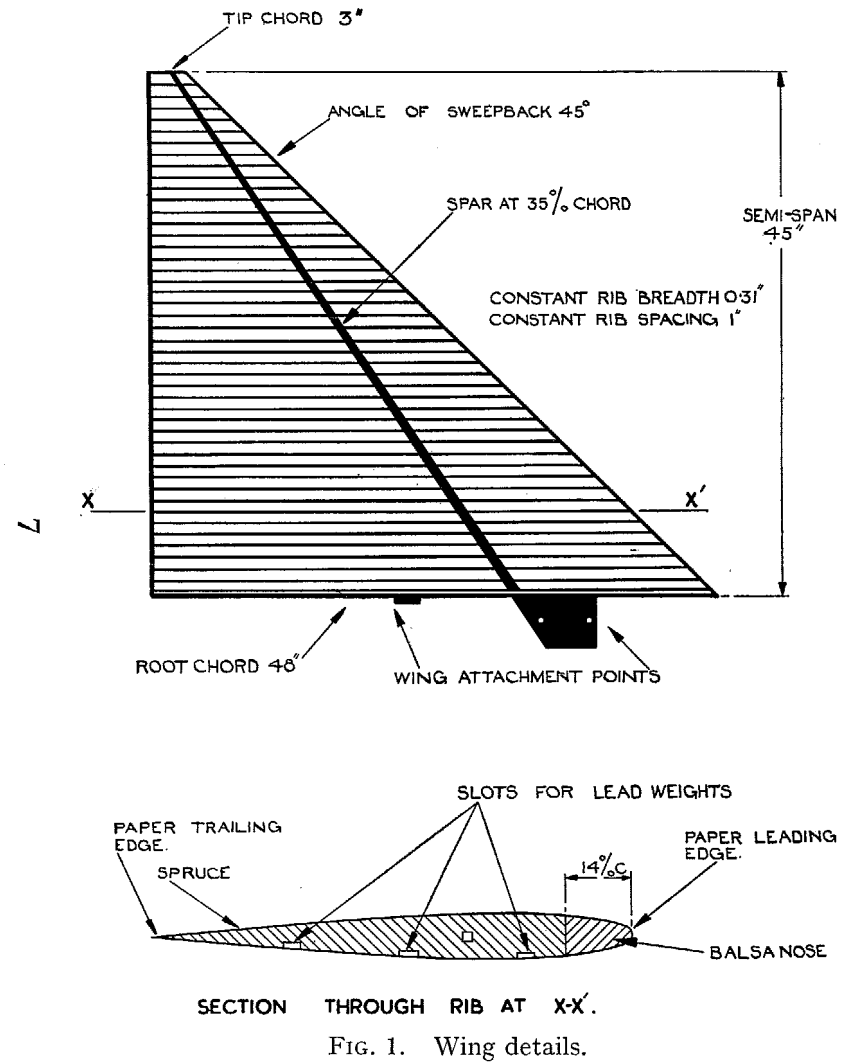


FIG. 2. Wing normal modes: fixed root inertia axis: 40 per cent chord.



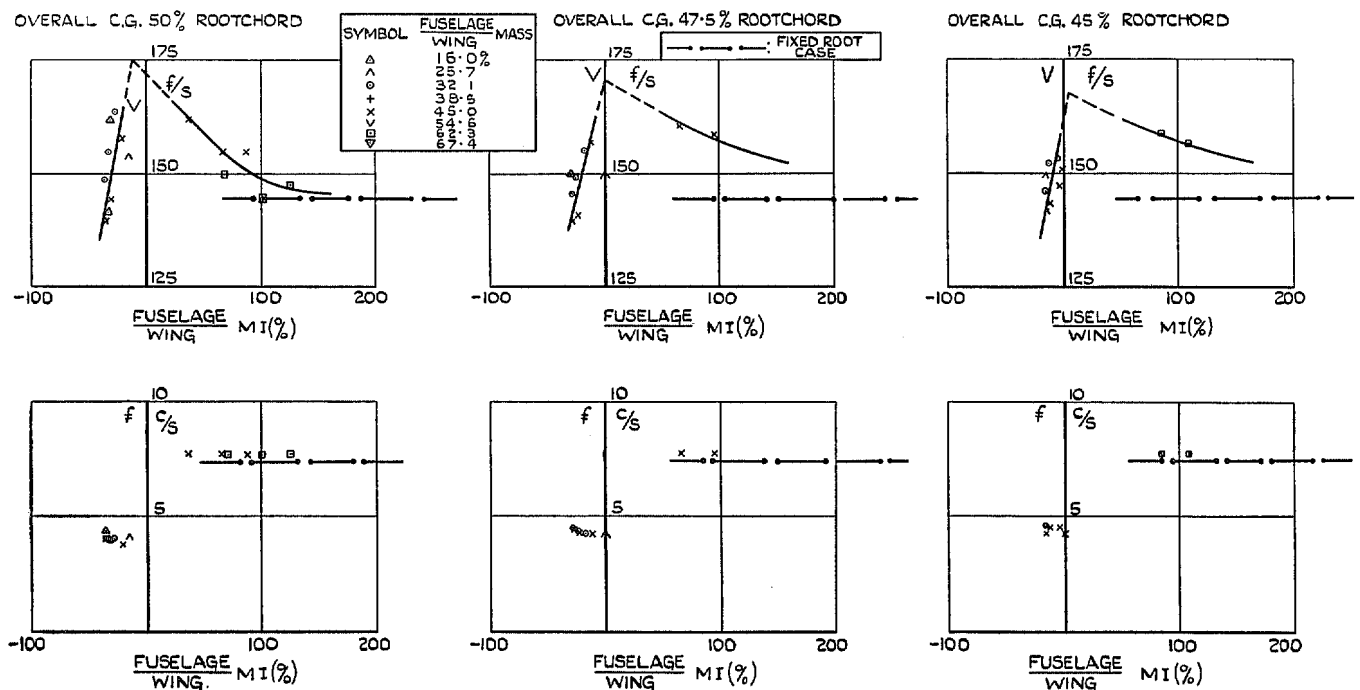


FIG. 3. Wing inertia axis: 40 per cent chord. Flutter speed and frequency plotted against the ratio of fuselage to wing moment of inertia.

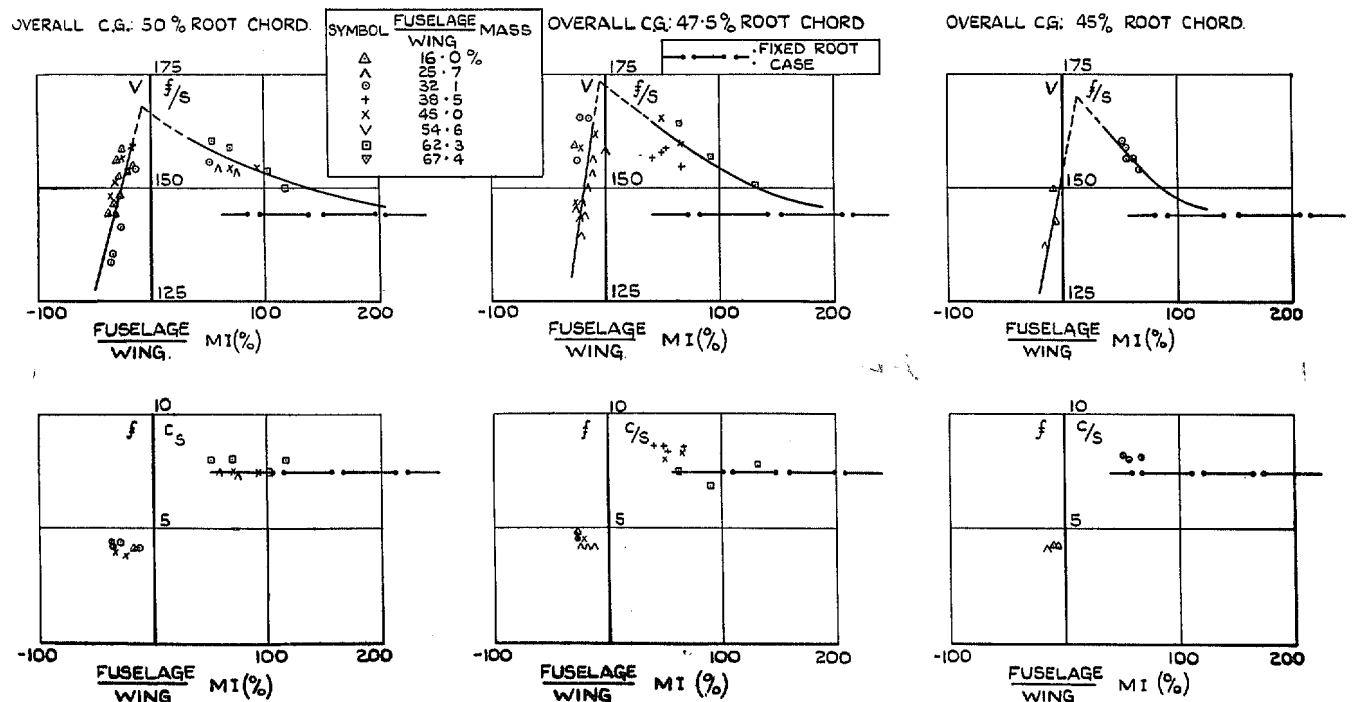


FIG. 4. Wing inertia axis: 45 per cent chord. Flutter speed and frequency plotted against the ratio of fuselage to wing moment of inertia.

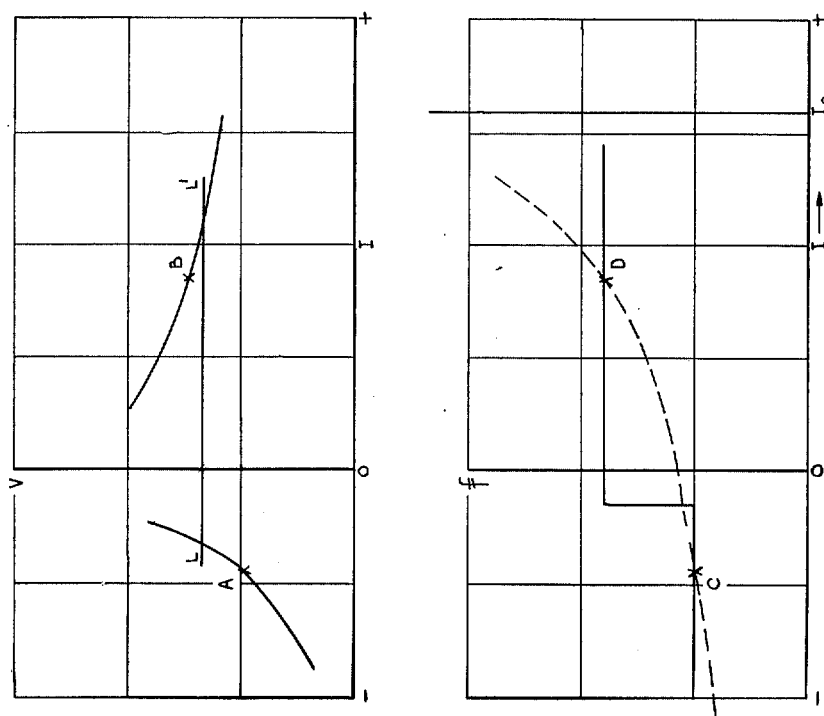
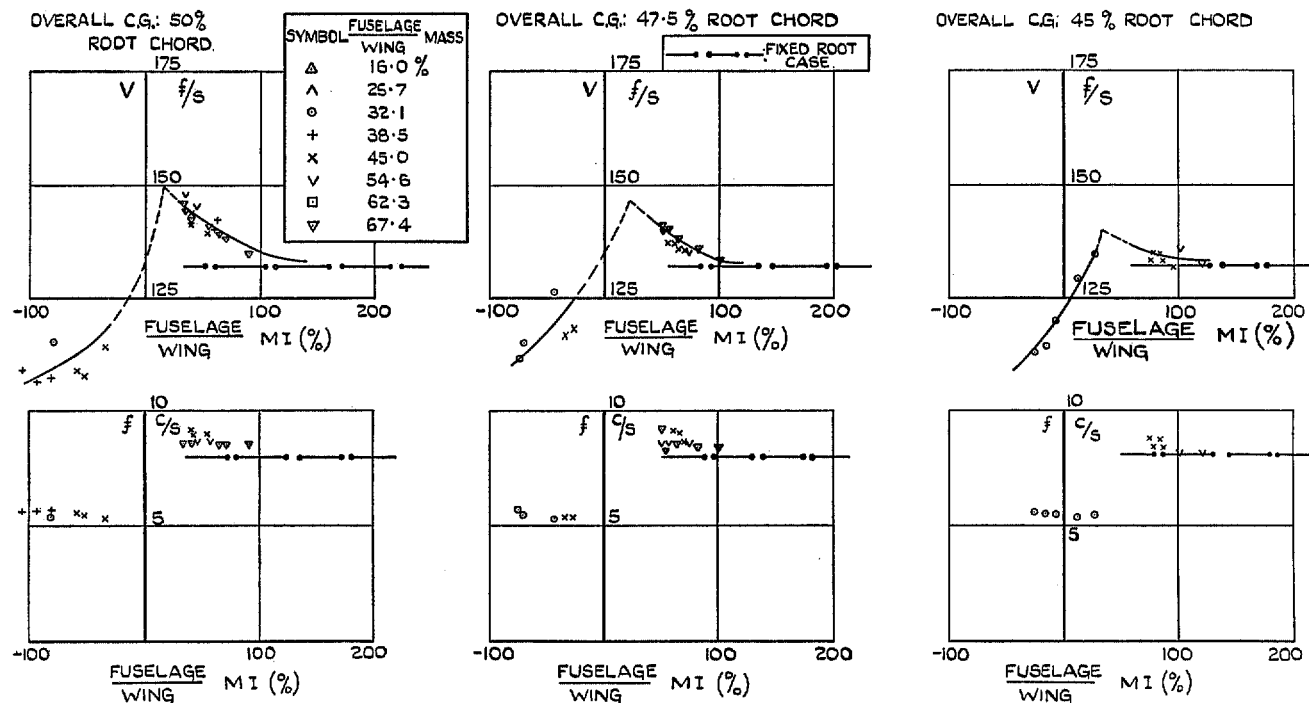


FIG. 6. Typical curves of flutter speed and frequency plotted against the ratio of fuselage to wing moment of inertia.

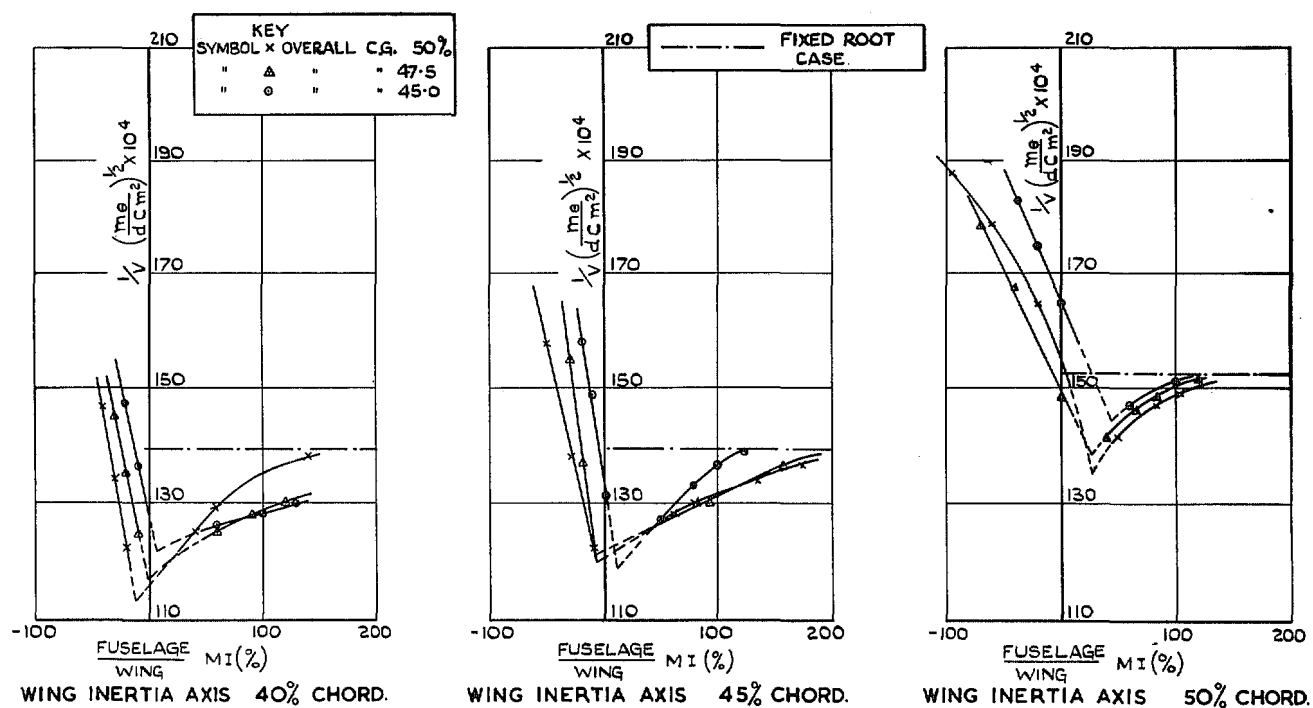


FIG. 7.  $(1/v)(m_0/dc^2_m)^{1/2}$  plotted against the ratio of fuselage to wing moment of inertia.

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