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Summary.—The present report deals with the frequency response of pivoted gyratory systems, and in particular, with the response of the ordinary rotor blade, the Hiller servo-blade, and the Young-Bell stabiliser to sinusoidal disturbances caused by pitching oscillations with constant amplitude. Physically, the problem corresponds to a single degree of freedom system excited by beats. The resulting forced oscillations are characterised by the two following phase angles :—

(*a*) a phase angle in the plane of rotation

(*b*) a phase angle of the oscillation of the tip-path plane, where the tip-path plane may be considered as a solid body.

The latter, which is the controlling consideration, depends on the specific damping of the system and the frequency ratio.

In general, the tip-path plane of the systems mentioned above oscillates in two directions, longitudinal and lateral, where both modes of oscillation can be split up into components in phase with the attitude and in phase with the rate of change of attitude. For each individual system explicit formulae are given and the effect of the frequency ratio on the control characteristics of the Bell stabiliser and Hiller servo-blade is shown by vector loci.

1. *Introduction.*—In the investigations of stability problems and transient motions of rotary-wing aircraft, the flapping motion of the rotor blades is generally considered as a sequence of steady conditions. This means the coefficients a_1, b_1 of the flapping angle $\beta (\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi)$ are calculated from a state of steady rotation corresponding to the momentary angular velocity of the helicopter.

In actual practice, however, the helicopter oscillates in pitch and/or roll. The present report checks the 'quasi-static' approximations mentioned above by investigating a rotor which is subjected to pitching oscillations with constant amplitude. For simplicity, it has been assumed that the rotor oscillates about its centre.

The equations of motion for the rotor blade hold good too for the Hiller servo-blade and—if a minor term is omitted—for the Bell stabiliser. It is therefore convenient to deal with the three systems together. The first part of the report gives the analytical solution for each case, and in the second part the well-known vector loci method for the study of servo-mechanisms is applied to the two control devices. Moreover, a physical interpretation of the results is given.

2. *Discussion and Solution of the Equations of Motion.*—The equation of motion for the flapping of a rotor with plain flapping and drag hinges can be written as

$$\ddot{\beta} + 2K\Omega\dot{\beta} + \Omega^2\beta = -2\Omega\dot{\alpha}\sin\psi + \ddot{\alpha}\cos\psi + \underline{2K\Omega\dot{\alpha}\cos\psi} \quad \ldots \quad (1)$$

* R.A.E. Report Aero. 2367, received 18th December, 1950.

In this equation

- β is Flapping angle of the blade
- Ω Angular velocity of the rotor
- K Specific damping (damping/critical damping). $K = \gamma B^4/16$ for an ordinary rotor blade
- γ Inertia number of the blade
- B Tip loss factor
- α Angle of helicopter in pitch, positive nose-up
- ψ Azimuth angle of blade measured from rear position in direction of rotation.

The terms on the right-hand side of equation (1) represent the excitement due to the gyroscopic effect, the excitement due to the mass forces of the angular acceleration and (underlined) the excitement due to the air forces caused by the rate of change of the angle in pitch. It can be easily shown that equation (1) may also be applied to the angular displacement of the Hiller servo-blade, and, if the underlined term is omitted, to the Bell stabiliser. In the case of the Hiller servo-blade, the specific damping is given by

$$K = \frac{\gamma_S}{16}(1 - B_S{}^4) \quad \ldots \quad (2)$$

where γ_S is Inertia number of the servo-blade

B_S Beginning of the profile of Hiller servo-blade in fraction of its radius,

see Ref. 1, Chapter 6. If, as in the case of the Bell stabiliser, the see-saw motion is provided with viscous damping, the quantity K is defined by the equation

$$\text{Damping moment} = 2K\Omega I\dot{\beta} \quad \ldots \quad (3)$$

where I = moment of inertia of the bar about its pivot.

In the following the three gyratory systems are dealt with separately. To avoid reiterations, we shall underline the term which does not apply to the Bell stabiliser. This means that that term has to be omitted if the damping by the air forces is replaced by viscous damping.

(*a*) *Flapping motion of the rotor blade.*

With

$$\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi \quad \ldots \quad (4)$$

$$\dot{\beta} = \sin\psi(a_1\Omega - \dot{b}_1) - \cos\psi(\dot{a}_1 + b_1\Omega) \quad \ldots \quad (5)$$

$$\ddot{\beta} = \sin\psi(-\ddot{b}_1 + 2\Omega\dot{a}_1 + \Omega^2 b_1) + \cos\psi(-\ddot{a}_1 - 2\Omega\dot{b}_1 + \Omega^2 a_1) \quad \ldots \quad (6)$$

equation (1) can be split up into the two following differential equations with constant coefficients

$$+ 2K\Omega^2 a_1 + 2\Omega\dot{a}_1 - 2K\Omega\dot{b}_1 - \ddot{b}_1 = -2\Omega\dot{\alpha} \quad \ldots \quad (7)$$

$$- 2K\Omega^2 b_1 - 2\Omega\dot{b}_1 - 2K\Omega\dot{a}_1 - \ddot{a}_1 = \underline{2K\Omega\dot{\alpha}} + \ddot{\alpha} \quad \ldots \quad (8)$$

For the 'quasi-static' condition (*i.e.*, $\dot{a}_1 = \ddot{a}_1 = \dot{b}_1 = \ddot{b}_1 = 0$), it follows

$$a_1 = -\frac{1}{K}\frac{\dot{\alpha}}{\Omega} \quad \ldots \quad (9)$$

$$b_1 = -\frac{\dot{\alpha}}{\Omega} \quad \ldots \quad (10)$$

where $K \simeq \gamma/16$. Equations (9), (10) are the well-known formulae for the flapping motion of a rotor with uniform angular pitching velocity $\dot{\alpha}$.

Let us consider now a rotor which is subjected to angular oscillations with constant amplitude such as

$$\alpha = \alpha_0 \sin \nu t . \qquad (11)$$

In this case it is convenient to divide a_1 and b_1 into components proportional to the attitude and proportional to the rate of change of attitude :—

$$a_1 = a_{1\alpha}\alpha + a_{1q}\dot{\alpha} \qquad (12)$$

$$b_1 = b_{1\alpha}\alpha + b_{1q}\dot{\alpha} . \qquad (13)$$

Inserting the values a_1, b_1, α and of their differentials in equations (7), (8) and equating coefficients leads to the following equations for the unknowns $a_{1\alpha}$, a_{1q}, $b_{1\alpha}$ and b_{1q} :—

$$+ 2Ka_{1\alpha} - 2\bar{\nu}^2 a_{1q}\Omega + \bar{\nu}^2 b_{1\alpha} + 2K\bar{\nu}^2 b_{1q}\Omega = 0 \qquad (14)$$

$$+ 2a_{1\alpha} + 2Ka_{1q}\Omega - 2Kb_{1\alpha} + \bar{\nu}^2 b_{1q}\Omega = -2 \qquad (15)$$

$$+ \bar{\nu}^2 a_{1\alpha} + 2K\bar{\nu}^2 a_{1q}\Omega - 2Kb_{1\alpha} + 2\bar{\nu}^2 b_{1q}\Omega = -\bar{\nu}^2 \qquad (16)$$

$$- 2Ka_{1\alpha} + \bar{\nu}^2 a_{1q}\Omega - 2b_{1\alpha} - 2Kb_{1q}\Omega = \underline{2K} \qquad (17)$$

where approximately* :—

$$a_{1\alpha} = -\frac{\bar{\nu}^2}{K^2 + \bar{\nu}^2}\left[1 - \left(\frac{K^3}{K^2 + \bar{\nu}^2}\right)^2\right] \qquad (18)$$

$$a_{1q}\Omega = -\frac{K}{K^2 + \bar{\nu}^2} \qquad (19)$$

$$b_{1\alpha} = -\frac{1 \cdot 5K^3\bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2} \qquad (20)$$

$$b_{1q}\Omega = -\frac{K^4}{(K^2 + \bar{\nu}^2)^2} . \qquad (21)$$

In these equations the frequency ratio

$$\bar{\nu} = \frac{\nu}{\Omega} . \qquad (22)$$

The result is represented in Figs. 1 and 2. Fig. 1 shows the components of the longitudinal tilt and Fig. 2 the components of the lateral tilt of the rotor disc in fraction of the amplitude α_0. The curves are plotted against the frequency ratio $\bar{\nu}$, the full lines correspond to an inertia number $\gamma = 12$ and the broken lines to $\gamma = 8$. It is to be seen that the flapping motion depends to a large extent on the frequency ratio $\bar{\nu}$. For the free oscillations of a present-day full-scale helicopter $\bar{\nu} < 0 \cdot 02$. It follows from Figs. 1 and 2 that in this frequency range the components in phase with the attitude are smaller than $0 \cdot 001\alpha_0$, i.e., they are practically zero and can be neglected. This means that the flapping motion of the blade is at any time proportional to the instantaneous rate of change of attitude and that the 'quasi-static' equations (9) and (10) may be used. It is to be seen that, if $\bar{\nu}^2 \ll K^2$, the equations (19) and (21) for the oscillating rotor take the form of equations (9) and (10) corresponding to 'quasi-static' conditions. For the ordinary rotor blade this is generally the case.

This simplification, however, cannot be applied if higher frequency ratios occur. It will be shown later that the stabilising effect credited to the downwash lag (*see* the model test described in Ref. 2) is partly due to the static stability caused by the term $a_{1\alpha}$ of the flapping motion. Generally speaking, the quantities $a_{1\alpha}$ and $b_{1\alpha}$ increase with

(*a*) an increasing frequency ratio $\bar{\nu}$

(*b*) a decreasing inertia number γ

(*c*) the magnitude of the damping of the pitching motion.

* To get dimensionless terms, a_{1q} and b_{1q} have been multiplied by Ω.

The latter effect, which is not dealt with in the present report, can be easily found if the rotor is subjected to an increasing or decreasing oscillation, *i.e.*, if equation (11) is replaced by

$$\alpha = \alpha_0 e^{\lambda t} \sin \nu t . \qquad (23)$$

In this case there is another parameter, namely the non-dimensional damping

$$\bar{\lambda} = \lambda / \Omega . \qquad (24)$$

In the model test discussed in Ref. 2,

$$\left.\begin{aligned} \gamma &= 8 \cdot 8 \\ \bar{\nu} &= 0 \cdot 147 \\ \bar{\lambda} &= -0 \cdot 0123 \end{aligned}\right\} . \qquad (25)$$

If these figures are applied to a full-scale helicopter with a rotor speed of $\Omega = 25$ per sec, we obtain a period of oscillation of

$$T_0 = \frac{2\pi}{25 \times 0 \cdot 147} = 1 \cdot 7 \text{ sec} \qquad (26)$$

and a time to half-amplitude of

$$T_H = \frac{0 \cdot 69}{25 \times 0 \cdot 0123} = 2 \cdot 2 \text{ sec} . \qquad (27)$$

It is obvious that these times lie outside the normal range of a full-scale helicopter.

The calculation of the flapping motion of the model test mentioned above results in :—

$$\left.\begin{aligned} a_{1\alpha} &= -0 \cdot 063 \\ a_{1q}\Omega &= -1 \cdot 96 \\ b_{1\alpha} &= -0 \cdot 061 \\ b_{1q}\Omega &= -0 \cdot 89 \end{aligned}\right\} . \qquad (28)$$

This means that the longitudinal tilt of the rotor disc has a component of $0 \cdot 063\, e^{\lambda t}\alpha_0$ in counterphase with the attitude and a component of $1 \cdot 96 \times 0 \cdot 147\, e^{\lambda t}\alpha_0 = 0 \cdot 29\, e^{\lambda t}\alpha_0$ in counterphase with the rate of change of attitude. The former has the same effect as an auto-pilot with the control characteristics $\theta_\alpha = 0 \cdot 063$ and by that a decisive effect on the dynamic stability. As shown in Ref. 1 the longitudinal and lateral motion of the Sikorsky R-4 in hovering flight is stabilised by $\theta_\alpha > 0 \cdot 12$ and $\theta_\alpha > 0 \cdot 015$ respectively.

(*b*) *Control characteristics of the Hiller servo-blade*.—If the inertia forces of the main rotor are neglected, *i.e.*, if we assume that the moment of inertia of the main blades about their longitudinal axis is negligibly small in comparison with the moment of inertia of the servo-blades about the pivot, equations (18), (19), (20) and (21) can also be applied directly to the automatic control displacements of the Hiller system. Let the cyclical pitch imposed on the main rotor blade be

$$\vartheta = \vartheta_s \sin \psi + \vartheta_c \cos \psi \qquad (29)$$

where

$$\vartheta_s = -(\theta_\alpha \alpha + \theta_q \dot{\alpha}) \qquad (30)$$

$$\vartheta_c = -(\Gamma_\alpha \alpha + \Gamma_q \dot{\alpha}) \qquad (31)$$

and ψ = azimuth angle of the main rotor. In this case the corresponding quantities are

$$\left.\begin{aligned} &\theta_\alpha \text{ with } -a_{1\alpha} \\ &\theta_q \quad ,, \quad -a_{1q} \\ &\Gamma_\alpha \quad ,, \quad +b_{1\alpha} \\ &\Gamma_q \quad ,, \quad +b_{1q} \end{aligned}\right\} \qquad (32)$$

Considering the fact that due to the small specific damping of the Hiller servo-blade

$$\left(\frac{K^3}{K^2 + \bar{\nu}^2}\right)^2 \ll 1 ,$$

it follows from (32) and equations (18), (19), (20) and (21) :—

$$\theta_\alpha = \frac{\bar{\nu}^2}{K^2 + \bar{\nu}^2} \quad \ldots \quad (33)$$

$$\theta_q \Omega = \frac{K}{K^2 + \bar{\nu}^2} \quad \ldots \quad (34)$$

$$\Gamma_\alpha = -\frac{1 \cdot 5K^3\bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2} \quad \ldots \quad (35)$$

$$\Gamma_q \Omega = -\frac{K^4}{(K^2 + \bar{\nu}^2)^2} \quad \ldots \quad (36)$$

where the specific damping K of the servo-blade is given by equation (2).

(*c*) *Control characteristics of the Bell stabiliser.*—If the gyratory system is damped by viscous damper instead of air forces, the right-hand side of equation (17) becomes zero. Thus, considering (32), the equations (14), (15), (16) and (17) can be rewritten as:—

$$-2K\theta_\alpha + 2\bar{\nu}^2\theta_q\Omega + \bar{\nu}^2\Gamma_\alpha + 2K\bar{\nu}^2\Gamma_q\Omega = 0 \quad \ldots \quad (37)$$

$$-2\theta_\alpha - 2K\theta_q\Omega - 2K\Gamma_\alpha + \bar{\nu}^2\Gamma_q\Omega = -2 \quad \ldots \quad (38)$$

$$-\bar{\nu}^2\theta_\alpha - 2K\bar{\nu}^2\theta_q\Omega - 2K\Gamma_\alpha + 2\bar{\nu}^2\Gamma_q\Omega = -\bar{\nu}^2 \quad \ldots \quad (39)$$

$$+2K\theta_\alpha - \bar{\nu}^2\theta_q\Omega - 2\Gamma_\alpha - 2K\Gamma_q\Omega = 0 \quad \ldots \quad (40)$$

where approximately :—

$$\theta_\alpha = \frac{\bar{\nu}^2}{K^2 + \bar{\nu}^2} \quad \ldots \quad (41)$$

$$\theta_q \Omega = \frac{K}{K^2 + \bar{\nu}^2} \quad \ldots \quad (42)$$

$$\Gamma_\alpha = -\frac{0 \cdot 5K\bar{\nu}^2(K^2 - \bar{\nu}^2)}{(K^2 + \bar{\nu}^2)^2} \quad \ldots \quad (43)$$

$$\Gamma_q \Omega = +\frac{K^2\bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2} . \quad \ldots \quad (44)$$

It should be noted that equations (41), (42), (43) and (44) refer to the linkage ratio $n = 1$ and that for the case $n \neq 1$ the automatic cyclic pitch has to be multiplied by the linkage ratio n = (change of pitch setting of main blade)/(displacement of the bar).

By comparison with the control characteristics of the Hiller system, *see* Table 1, it follows that the longitudinal control displacements of the two devices are identical, The lateral control displacements, however, which result in a coupling between the longitudinal and lateral motion, are distinct. Before going into these questions more thoroughly, we shall give a physical interpretation of the phenomena.

3. *Physical Interpretation of the Phenomena.*—It is shown in the Appendix that the mass forces due to the angular acceleration, *i.e.*, the second term on the right-hand side of equation (1), can be neglected. This means that the equation of motion for a gyratory system provided with viscous damper may be simplified to

$$\ddot{\beta} + 2K\Omega\dot{\beta} + \Omega^2\beta = -2\Omega\dot{\alpha}\sin\Omega t . \quad \ldots \quad (45)$$

If this system is subjected to angular oscillations with constant amplitude as given by equation (11), the above equation of motion can be written as

$$\ddot{\beta} + 2K\Omega\dot{\beta} + \Omega^2\beta = -2\Omega\nu\alpha_0 \cos \nu t \sin \Omega t$$

$$= -\Omega\nu\alpha_0(\sin \nu_1 t + \sin \nu_2 t) \quad \ldots \quad (46)$$

where the two frequencies $\nu_{1,2}$ are given by the following equations:—

$$\nu_1 = \Omega + \nu = \Omega(1 + \bar{\nu}) \quad \ldots \quad (47)$$

$$\nu_2 = \Omega - \nu = \Omega(1 - \bar{\nu}) \,. \quad \ldots \quad (48)$$

Equation (46) represents the equation of motion of a single degree of freedom system excited by beats. As the undamped natural frequency of the system is identical with its angular velocity Ω, the ratios (frequency of forced oscillation)/(undamped natural frequency) are equal to $(1 + \bar{\nu})$ and $(1 - \bar{\nu})$ respectively.

It is known from vibration analysis (*see* any text book on vibrations, for instance Ref. 5) that the forced vibrations of equation (46) can be written as

$$\beta = \beta_1 \sin(\nu_1 t - \Phi_1) + \beta_2 \sin(\nu_2 t - \Phi_2) \,. \quad \ldots \quad (49)$$

In these equations

$\beta_{1,2}$ = amplitudes of the forced oscillations,

$\Phi_{1,2}$ = phase angles,

where the index $_1$ refers to the excitement with the frequency ν_1 and the index $_2$ to that with the frequency ν_2. It has already been mentioned that for the full-scale helicopter $\bar{\nu} \ll 1$. This means that approximately $\beta_1 = \beta_2$. The quantities Φ_1, Φ_2 which depend on the specific damping of the system and on the ratio (forced frequency)/(undamped natural frequency) can be taken from the well-known phase angle curves of a single degree of freedom system, *see* Fig. 3. The curves are plotted for $K = 0$, $0 \cdot 03$, and $0 \cdot 75$; they correspond to (*a*) an undamped system, (*b*) a Bell or Hiller system with a following time of approximately 3 sec and (*c*) to an ordinary rotor blade with $\gamma = 12$.

With $\beta_1 = \beta_2 = \beta_3$ equation (49) reads as

$$\beta = \beta_3\{\sin(\nu_1 t - \Phi_1) + \sin(\nu_2 t - \Phi_2)\} \,. \quad \ldots \quad (50)$$

On the other hand, the forced oscillations of equation (46) can be written as

$$\beta = \beta_0 \sin(\Omega t - \psi_1) \cos(\nu t - \varphi) \quad \ldots \quad (51)$$

where ψ_1 is Phase angle in the plane of rotation

φ is Phase angle of the tilt of the tip-path plane.

If we consider the tip-path plane as a solid body with the two degrees of freedom longitudinal and lateral tilt, the former determines the azimuth angle of the oscillation of the tip-path plane, and the latter its time lag. Equation (51) can be transformed into

$$\beta = \tfrac{1}{2}\beta_0\{\sin(\nu_1 t - \psi_1 - \varphi) + \sin(\nu_2 t - \psi_1 + \varphi)\} \,. \quad \ldots \quad (52)$$

By comparison with equation (50) it follows that

$$-\Phi_1 = -\psi_1 - \varphi \quad \ldots \quad (53)$$

$$-\Phi_2 = -\psi_1 + \varphi \,. \quad \ldots \quad (54)$$

This means $\psi_1 = \frac{1}{2}(\Phi_1 + \Phi_2)$ .. .. (55)

$\varphi = \frac{1}{2}(\Phi_1 - \Phi_2)$ .. .. (56)

where $\Phi_{1,2}$ can be taken from Fig. 3. For small frequency ratios $\bar{\nu}$

$$\Phi_1 + \Phi_2 \simeq 180 \text{ deg}$$

and by that $\psi_1 \simeq 90$ deg,

i.e., the tip-path plane of the system described by equation (45) oscillates in the fore-and-aft direction.

With regard to the time lag of the tilt of the tip-path plane, the following statements can be made. From equation (56) and Fig. 3 it follows that φ increases with

(*a*) an increasing frequency ratio $\bar{\nu}$, and

(*b*) a decreasing specific damping K.

In the case of the ordinary rotor blade ($K = 0 \cdot 75$) the phase angle φ is negligibly small, *i.e.*, we have quasi-static conditions. For the slightly damped Hiller or Bell system ($K = 0 \cdot 03$), however, the phase angle φ is appreciable. This means that such control devices apply both control displacements in phase with the attitude and in phase with the rate of change of attitude.

It should be noted that the above remarks on the phase angles ψ_1 and φ apply also to the other two excitement terms on the righthand side of equation (1). These excitements (viz., $\ddot{\alpha} \cos \psi$ and $2K\Omega\dot{\alpha} \cos \psi$) have their maxima and minima at $\psi = 0$ deg and 180 deg respectively and result therefore in a lateral oscillation of the tip-path plane. This means they give a coupling of the longitudinal motion with the lateral motion where the phase angle φ depends again on the specific damping of the system and the frequency ratio $\bar{\nu}$.

4. *Representation of the Automatic Control Displacements of the Hiller and Bell Systems by Vector Loci.*—Analogous to equations (7) and (8), the equations of motion for the longitudinal and lateral control displacements of the Hiller and Bell systems can be written as

$$\ddot{\vartheta}_s + 2K\Omega\dot{\vartheta}_s - 2\Omega\dot{\vartheta}_c - 2K\Omega^2\vartheta_c = -\ddot{\alpha} - \underline{2K\Omega\dot{\alpha}} \quad .. \quad .. \quad (57)$$

$$2\Omega\dot{\vartheta}_s + 2K\Omega^2\vartheta_s + \ddot{\vartheta}_c + 2K\Omega\dot{\vartheta}_c = -2\Omega\dot{\alpha}\,. \quad .. \quad .. \quad (58)$$

If the system is subjected to oscillations with constant amplitude as given by equation (11), the disturbances on the right-hand side of the above equations change sinusoidally and any term in either equation (57) or equation (58) can be represented by a vector rotating with the angular velocity ν.

Formally, this is done by replacing

$$\alpha \text{ by } \alpha e^{i\nu t}$$

$$\vartheta_s \text{ by } \vartheta_s e^{i\nu t}$$

$$\vartheta_c \text{ by } \vartheta_c e^{i\nu t}$$

where ϑ_s and ϑ_c are now unknown complex numbers. In vector representation equations (57) and (58) read :—

$$\vartheta_s(-\bar{\nu}^2 + 2K\bar{\nu}i) - \vartheta_c(2K + 2\bar{\nu}i) = +\alpha(\bar{\nu}^2 - \underline{2K\bar{\nu}i}) \quad .. \quad .. \quad (59)$$

$$\vartheta_s(2K + 2\bar{\nu}i) + \vartheta_c(-\bar{\nu}^2 + 2K\bar{\nu}i) = -\alpha 2\bar{\nu}i\,. \quad .. \quad .. \quad .. \quad (60)$$

These vector equations hold good for both the Hiller servo-blade and the Bell stabiliser, in the latter case the underlined term in equation (59) has again to be omitted.

4.1. *Hiller Servo-Blade.*—For the control displacements of the Hiller servo-blade it follows from equations (59) and (60):—

$$\frac{\vartheta_s}{\alpha} = \frac{\begin{vmatrix} \bar{\nu}^2 - 2K\bar{\nu}i & -(2K + 2\bar{\nu}i) \\ -2\bar{\nu}i & -\bar{\nu}^2 + 2K\bar{\nu}i \end{vmatrix}}{\begin{vmatrix} -\bar{\nu}^2 + 2K\bar{\nu}i & -(2K + 2\bar{\nu}i) \\ 2K + 2\bar{\nu}i & -\bar{\nu}^2 + 2K\bar{\nu}i \end{vmatrix}} \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

$$\frac{\vartheta_c}{\alpha} = \frac{\begin{vmatrix} -\bar{\nu}^2 + 2K\bar{\nu}i & \bar{\nu}^2 - 2K\bar{\nu}i \\ 2K + 2\bar{\nu}i & -2\bar{\nu}i \end{vmatrix}}{\begin{vmatrix} -\bar{\nu}^2 + 2K\bar{\nu}i & -(2K + 2\bar{\nu}i) \\ 2K + 2\bar{\nu}i & -\bar{\nu}^2 + 2K\bar{\nu}i \end{vmatrix}} \quad \ldots \quad \ldots \quad \ldots \quad (62)$$

For the *longitudinal* control displacement:—

$$\mathrm{Re}\left(\frac{\vartheta_s}{\alpha}\right) = \frac{AC - BD}{C^2 + D^2} \quad \ldots \quad \ldots \quad (63)$$

$$\mathrm{Im}\left(\frac{\vartheta_s}{\alpha}\right) = -\frac{BC + AD}{C^2 + D^2} \quad \ldots \quad \ldots \quad (64)$$

where Re (ϑ_s/α) and Im (ϑ_s/α) denote the real and imaginary parts of (ϑ_s/α) respectively and the coefficients A, B, C, D are given by

$$A = 4\bar{\nu}^2(1 + K^2) - \bar{\nu}^4 \quad \ldots \quad \ldots \quad (65)$$

$$B = 4K\bar{\nu}(1 - \bar{\nu}^2) \quad \ldots \quad \ldots \quad (66)$$

$$C = 4K^2 + \bar{\nu}^4 - 4\bar{\nu}^2(1 + K^2) \quad \ldots \quad \ldots \quad (67)$$

$$D = 4K\bar{\nu}(2 - \bar{\nu}^2)\,. \quad \ldots \quad \ldots \quad (68)$$

The corresponding equations for the *lateral* control displacement of the Hiller servo-blade are:—

$$\mathrm{Re}\left(\frac{\vartheta_c}{\alpha}\right) = \frac{FD - EC}{C^2 + D^2} \quad \ldots \quad \ldots \quad (69)$$

$$\mathrm{Im}\left(\frac{\vartheta_c}{\alpha}\right) = \frac{CF + DE}{C^2 + D^2} \quad \ldots \quad \ldots \quad (70)$$

where C, D are again given by equations (67), (68) and

$$E = 2K\bar{\nu}^2 \quad \ldots \quad \ldots \quad (71)$$

$$F = 4K^2\bar{\nu}\,. \quad \ldots \quad \ldots \quad (72)$$

The meaning which has to be attached to these results is that the control displacements ϑ_s, ϑ_c consist of two components, one (the real part) in phase with the attitude α, and another (the imaginary part) in phase with the rate of change of attitude. We shall deal with these questions more thoroughly in connection with the vector loci.

4.2. *Bell Stabiliser.*—In the same manner, the Bell stabiliser is investigated. If the underlined term in equation (59) is omitted, the following results are obtained :—

$$\text{Re}\left(\frac{\vartheta_s}{\alpha}\right) = \frac{CG - DH}{C^2 + D^2} \qquad (73)$$

$$\text{Im}\left(\frac{\vartheta_s}{\alpha}\right) = -\frac{CH + DG}{C^2 + D^2} \qquad (74)$$

$$\text{Re}\left(\frac{\vartheta_c}{\alpha}\right) = \frac{CJ}{C^2 + D^2} \qquad (75)$$

$$\text{Im}\left(\frac{\vartheta_c}{\alpha}\right) = -\frac{DJ}{C^2 + D^2} \qquad (76)$$

where

$$G = \bar{\nu}^2(4 - \bar{\nu}^2) \qquad (77)$$

$$H = 2K\bar{\nu}(2 - \bar{\nu}^2) \qquad (78)$$

$$J = 2K\bar{\nu}^2 . \qquad (79)$$

The connection between the vector representation and the control characteristics θ_α, θ_q, Γ_α, Γ_q for both systems (Hiller and Bell) is given by

$$\text{Re}\left(\frac{\vartheta_s}{\alpha}\right) = -\theta_\alpha \qquad (80)$$

$$\text{Im}\left(\frac{\vartheta_s}{\alpha}\right) = -\theta_q\Omega \times \bar{\nu} = -\theta_q\nu \qquad (81)$$

$$\text{Re}\left(\frac{\vartheta_c}{\alpha}\right) = -\Gamma_\alpha \qquad (82)$$

$$\text{Im}\left(\frac{\vartheta_c}{\alpha}\right) = -\Gamma_q\Omega \times \bar{\nu} = -\Gamma_q\nu . \qquad (83)$$

Equations (63), (64), (69), (70) and (73), (74), (75), (76) represent the exact solution for the forced oscillations of the two control devices. It has been found, however, that the approximate solutions of section 2 (*see* Table 1) agree very closely with the exact ones.

4.3. *Vector Loci.*—We shall deal now with the representation of the above results by vector loci. For the reader who is not familiar with this kind of representation, some introductory remarks are given. *See* also Ref. 4.

The vector loci show the output (magnitude and phase) of a servo-mechanism for a given sinusoidal input as a function of the frequency ν. In graphical studies, however, great confusion would arise if all the output-vectors were plotted. Therefore only the line joining the tips of all output-vectors is drawn and individual values of frequency are marked along it. The direction in which the output-vector moves with increasing frequency is indicated by an arrow.

Let us take the case of a hypothetical auto-pilot governing the pitching motion of an aircraft as an example, *see* Fig. 4. The input vector α (given by the amplitude of the angular oscillation in pitch) is plotted horizontally in the positive direction of the real axis.. As differentiation of a vector is equivalent to a multiplication of the length of the vector by ν and a forward rotation through 90 deg, the velocity vector $\dot{\alpha}$ shows in the positive direction of the imaginary axis. The output vector δ, i.e., the control displacement, changes in magnitude and phase with the frequency ν, where for the frequency $\nu_j (j = 1, 2, 3 \ldots)$ the magnitude equals δ_j.

Another, and for our purpose more convenient, interpretation is that the control displacement may be divided into components in phase with the attitude and in phase with the rate of change of attitude. If the attitude vector is plotted horizontally as mentioned above, the former is identical with the real, and the latter with the imaginary part of the complex number δ.

In Figs. 5 to 7 the corresponding vector loci for the longitudinal and lateral control displacements of the Hiller servo-blade and Bell stabiliser are shown. The output-vector represents the quantities (ϑ_s/α) and (ϑ_c/α) respectively, or, in other words, the cyclical pitch imposed on the main blades due to a pitching oscillation with the amplitude $\alpha_0 = 1$. The specific damping $K = 0 \cdot 03$, *i.e.*, for an angular velocity of $\Omega = 25$ per sec the following time amounts to

$$T_f = \frac{2 \cdot 3}{K\Omega} = 3 \text{ sec}$$

where T_f is defined as the time in which an angular displacement of the system about its pivot is reduced to a tenth of the initial value.

The figures 0, 0·01, 0·02, etc., indicated along the vector loci refer to the frequency ratio $\bar{\nu}$. As already mentioned, for a present-day full-scale helicopter $0 < \bar{\nu} < 0 \cdot 02$.

Fig. 5 represents the longitudinal control displacement due to pitching motion and can be applied to both the Hiller and Bell system. It will be seen that the vector loci is a semi-circle with the radius 0·5 about the point $(-0 \cdot 5 + 0i)$ as centre. For $\bar{\nu} = 0$, the control displacement is equal to zero ; and for $\bar{\nu} = \infty$, ϑ_s is equal to and in counterphase with the attitude α. One can easily verify that the shape and scale of the vector loci is independent of the quantity K, and that for another specific damping K only the accompanying $\bar{\nu}$-figures along the semicircle change. It follows from equations (33) and (34) that, if the ratio $\bar{\nu}/K$ is constant, a series of conditions are obtained resulting in the same longitudinal control displacement. For example, the two conditions $\bar{\nu} = 0 \cdot 01$, $K = 0 \cdot 03$ and $\bar{\nu} = 0 \cdot 02$, $K = 0 \cdot 06$ give the same control displacement ϑ_s, namely

$0 \cdot 1\alpha_0$ in counterphase with the attitude, and

$0 \cdot 3\alpha_0$ in counterphase with the rate of change of attitude.

It should be noted that this rule may only be applied to the *longitudinal* control displacement.

Figs. 6 and 7 give the lateral control displacement ϑ_c of the Hiller servo-blade and Bell stabiliser due to the pitching motion. It will be seen that the lateral control displacements of the two control devices are distinct. For $\bar{\nu} = 0 \cdot 02$, for instance, ϑ_c is approximately

$\pm\, 0 \cdot 005\alpha_0$ in the case of the Bell stabiliser and

$\pm\, 0 \cdot 015\alpha_0$ in the case of the Hiller system.

These lateral control displacements give a coupling of the longitudinal motion with the lateral motion and can therefore be used to compensate an existing coupling between these two motions.

5. *Conclusions.*—A gyratory system subjected to pitching oscillations about its centre is excited by

(*a*) the gyroscopic effect of the rotating masses,

(*b*) air forces due to the angular velocity of the pitching motion (this excitement disappears if the system is provided with viscous damper), and

(*c*) mass forces due to the angular acceleration.

As shown in the Appendix, the latter effect can be neglected.

If there is no 'delta three' effect, *i.e.*, if the undamped natural frequency of the flapping or see-saw motion is equal to the angular speed of the system, the phase angle in the plane of rotation is approximately 90 deg. This means that excitement (*a*) gives predominantly a longitudinal and excitement (*b*) predominantly a lateral oscillation of the tip-path plane, where the tip-path plane may be considered as a solid body having the two degrees of freedom, longitudinal and lateral tilt. It can be seen that the tip-path plane follows the disturbances with a certain time lag where the phase angle is given by

$$\tan \varphi = \frac{\bar{\nu}}{K}\left[1 - \left(\frac{K^3}{K^2 + \bar{\nu}^2}\right)^2\right]. \quad \ldots \quad (84)$$

In this equation $\bar{\nu}$ = frequency ratio of the pitching motion and K = specific damping of the system.

The phase angle φ has the effect that one component of the longitudinal or lateral tilt of the tip-path plane is out of phase with the rate of change of attitude, or, in other words, in phase with the attitude. For the longitudinal motion of the rotary wing aircraft it means that there is a kind of static stability which, as shown in Ref. 1, is an essential condition for the dynamic stability.

The phase angle φ of the rotor blade of a full-scale rotary wing aircraft is generally negligibly small, *i.e.*, the flapping motion of the ordinary rotor blade is at any time proportional to the instantaneous rate of change of attitude and may therefore be considered as a sequence of steady conditions. However, the component in phase with the attitude can no longer be neglected if higher frequency ratios $\bar{\nu}$ occur. One can easily verify that the stabilising effect credited to the downwash lag (*see* Ref. 2) is partly due to the static stability caused by the flapping motion in phase with the attitude.

For the Bell stabiliser and Hiller servo-blade the specific damping K is much smaller than that of the ordinary rotor blade. This means that even for small frequency ratios $\bar{\nu}$ appreciable phase angles occur. Or, in other words, the said control devices apply both automatic control displacements in phase with the attitude and in phase with the rate of change of attitude.

LIST OF SYMBOLS

α	Angle in pitch, positive nose-up, radn. $\alpha = \alpha_0 \sin \nu t$
α_0	Amplitude of pitching oscillation, radn
ν	Circular frequency of pitching oscillation, per sec
$\bar{\nu}$	Frequency ratio, $\bar{\nu} = \nu/\Omega$
Ω	Angular velocity of gyratory system, per sec
ν_1, ν_2	Frequencies, per sec $\nu_1 = \Omega + \nu = \Omega(1 + \bar{\nu})$; $\nu_2 = \Omega - \nu = \Omega(1 - \bar{\nu})$
I	Moment of inertia of gyratory system about its pivot, ft lb sec^2
ψ	Azimuth angle, radn, measured in sense of rotation from the down-wind position
β	Angular displacement of the gyratory system, radn. For the ordinary rotor blade $\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi$, where $a_1 = a_{1\alpha}\alpha + a_{1q}\dot{\alpha}$; $b_1 = b_{1\alpha}\alpha + b_{1q}\dot{\alpha}$

K	Specific damping of gyratory system (damping/critical damping), *i.e.*, the damping moment $M_D = 2KI\Omega\dot{\beta}$. For the ordinary rotor blade: $K = \gamma B^4/16$. For the Hiller servo-blade: $K = (\gamma_S/16)(1 - B_S^4)$
B	Tip loss factor of an ordinary rotor blade, $B \simeq 0\cdot98$
B_S	Beginning of the profile of Hiller servo-blade in fraction of R_S
γ	Inertia number of blade
γ_S	Inertia number of a single Hiller servo-blade, $\gamma_S = R_S^4 c_S a_S \rho/(I/2)$
R_S	Radius of Hiller servo-rotor, ft
c_S	Chord of Hiller servo-blade, ft
a_S	Lift-curve slope of Hiller servo-blade
ρ	Density of air, lb sec^2 ft^{-4}
ϑ_s, ϑ_c	Cyclical pitch, radn: $\vartheta_s = -(\theta_\alpha \alpha + \theta_q \dot{\alpha})$; $\vartheta_c = -(\Gamma_\alpha \alpha + \Gamma_q \dot{\alpha})$
ψ_1	Phase angle in plane of rotation, radn: $\psi_1 = \frac{1}{2}(\Phi_1 + \Phi_2) \simeq 90$ deg
q	Rate of change of attitude, radn per sec, $q = \dot{\alpha}$
φ	Phase angle of the oscillation of the tip-path plane, radn: $\varphi = \frac{1}{2}(\Phi_1 - \Phi_2)$
Φ_1	Phase angle of an ordinary single degree of freedom system with the undamped natural frequency Ω, a specific damping K and excited by the frequency $\nu_1 = \Omega(1 + \bar{\nu})$, radn
Φ_2	The same if excited by the frequency $\nu_2 = \Omega(1 - \bar{\nu})$, radn
λ	Damping factor of an increasing or decreasing oscillation such as $\alpha = \alpha_0 e^{\lambda t} \sin \nu t$, per sec
$\bar{\lambda}$	Non-dimensional damping factor, $\bar{\lambda} = \lambda/\Omega$
t	Time, sec
T_0	Period of oscillation, sec
T_H	Time to half amplitude, sec
T_f	Following time, time to reduce the angular displacement of a gyratory system to a tenth of its initial value, sec: $T_f = 2\cdot3/K\Omega$

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APPENDIX I

*Effect of the Angular Acceleration on the Longitudinal Tilt of the Rotor Disc**

In the following the effect of the angular acceleration (mass forces) on the longitudinal tilt of the rotor disc is investigated. If only the disturbances due to $\ddot{\alpha}$ are considered, equations (7), (8) read

$$+ 2K\Omega^2 a_1 + 2\Omega\dot{a}_1 - 2K\Omega\dot{b}_1 - \ddot{b}_1 = 0 \quad \ldots \quad (85)$$

$$- 2K\Omega\dot{a}_1 - \ddot{a}_1 - 2K\Omega^2 b_1 - 2\Omega\dot{b}_1 = \ddot{\alpha} . \quad \ldots \quad (86)$$

Or, as vector equations:—

$$a_1(2K + 2\bar{\nu}i) + b_1(\bar{\nu}^2 - 2K\bar{\nu}i) = 0 \quad \ldots \quad (87)$$

$$a_1(\bar{\nu}^2 - 2K\bar{\nu}i) - b_1(2K + 2\bar{\nu}i) = - \bar{\nu}^2\alpha . \quad \ldots \quad (88)$$

From equations (87), (88) it follows that

$$\left(\frac{a_1}{\alpha}\right)_{\text{acc}} = \bar{\nu}^3\left(\frac{2KD - \bar{\nu}C}{C^2 + D^2} + i\,\frac{2KC + \bar{\nu}D}{C^2 + D^2}\right) \quad \ldots \quad (89)$$

where the quantities C, D are given by equations (67), (68) and the notation $(a_1/\alpha)_{\text{acc}}$ indicates that only the acceleration term on the right-hand side of equation (1) is considered. The physical interpretation of equation (89) is that one component (the real part) is in phase with the attitude α and another component (the imaginary part) in phase with the rate of change of attitude $\dot{\alpha}$. If higher orders of $\bar{\nu}$ than $\bar{\nu}^4$ are neglected, the following approximations can be used:—

The component in phase with the attitude $\Big\} = \dfrac{0 \cdot 75K^2\bar{\nu}^4}{(K^2 + \bar{\nu}^2)^2}\alpha_0$, ... (90)

the component in phase with the rate of change of attitude $\Big\} = \dfrac{0 \cdot 5K^3\bar{\nu}^3}{(K^2 + \bar{\nu}^2)^2}\alpha_0$ . .. . (91)

* It should be noted that the present report deals only with the *mass* forces due to angular acceleration and that the still somewhat dubious effect of the downwash lag is neglected. However, at the time being some tests with oscillating rotors are in progress at the Royal Aircraft Establishment and it is to be hoped that these tests give some information on this subject.

In these equations α_0 = amplitude of the oscillation in pitch. For comparison, the total tilt $(a_1/\alpha_0)_{\text{total}}$ of the rotor disc and the tilt $(a_1/\alpha_0)_{\text{acc}}$ due to the angular acceleration $\ddot{\alpha}$ have been calculated, *see* Table 2.

In these examples $\gamma = 12$ and $\bar{\nu} = 0{\cdot}02$, $0{\cdot}06$, and $0{\cdot}10$. It is to be seen that within the range investigated, the acceleration term of equation (1) contributes less than 2 per cent to the total tilt of the rotor disc, *i.e.*, the enforced oscillations due to the angular acceleration can be neglected. This result does not agree with that of Ref. 3. It should be noted that the physical interpretation for the longitudinal or lateral tilt of the rotor disc given in Ref. 3 is somewhat misleading: The 'acceleration derivative' $\partial a_1/\partial \dot{q}$, for instance, includes both

(*a*) the enforced longitudinal oscillation due to the angular acceleration, and

(*b*) the longitudinal component out of phase with $\dot{\alpha}$ and introduced by the phase angle φ in the response of the rotor disc to gyroscopic effect.

This means the term $\partial a_1/\partial \dot{q}$ is not a true 'acceleration' derivative. The same applies to the 'acceleration' derivative $\partial b_1/\partial \dot{q}$ of Ref. 3.

TABLE 1

Response of Gyratory Systems to Oscillations with Constant Amplitude such as $\alpha = \alpha_0 \sin \nu t$

Rotor blade $K = \dfrac{\gamma B^4}{16}$; $\bar{\nu} = \dfrac{\nu}{\Omega}$	Hiller servo-blade $K = \dfrac{2 \cdot 3}{T_f \Omega}$; $\bar{\nu} = \dfrac{\nu}{\Omega}$	Bell stabiliser $K = \dfrac{2 \cdot 3}{T_f \Omega}$; $\bar{\nu} = \dfrac{\nu}{\Omega}$
$a_{1\alpha} = -\dfrac{\bar{\nu}^2}{K^2 + \bar{\nu}^2}\left[1 - \left(\dfrac{K^3}{K^2 + \bar{\nu}^2}\right)^2\right]$ $\doteqdot \dfrac{\bar{\nu}^2}{K^2}(1 - K^2)$	$\theta_\alpha = -a_{1\alpha} = +\dfrac{\bar{\nu}^2}{K^2 + \bar{\nu}^2}\left[1 - \left(\dfrac{K^3}{K^2 + \bar{\nu}^2}\right)^2\right]$ $\doteqdot + \dfrac{\bar{\nu}^2}{K^2 + \bar{\nu}^2}$	$\theta_\alpha = + \dfrac{\bar{\nu}^2}{K^2 + \bar{\nu}^2}$
$a_{1q}\Omega = -\dfrac{K}{K^2 + \bar{\nu}^2}$ $\doteqdot -\dfrac{1}{K}$	$\theta_q \Omega = -a_{1q}\Omega = +\dfrac{K}{K^2 + \bar{\nu}^2}$	$\theta_q \Omega = + \dfrac{K}{K^2 + \bar{\nu}^2}$
$b_{1\alpha} = -\dfrac{1{\cdot}5K^3\bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2}$ $\doteqdot -\dfrac{1{\cdot}5\bar{\nu}^2}{K}$	$\Gamma_\alpha = b_{1\alpha} = -\dfrac{1{\cdot}5K^3\bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2}$	$\Gamma_\alpha = \dfrac{-\frac{1}{2}K\bar{\nu}^2(K^2 - \bar{\nu}^2)}{(K^2 + \bar{\nu}^2)^2}$
$b_{1q}\Omega = -\dfrac{K^4}{(K^2 + \bar{\nu}^2)^2}$ $\doteqdot -1$	$\Gamma_q \Omega = b_{1q}\Omega = -\dfrac{K^4}{(K^2 + \bar{\nu}^2)^2}$	$\Gamma_q \Omega = + \dfrac{K^2 \bar{\nu}^2}{(K^2 + \bar{\nu}^2)^2}$

TABLE 2

Effect of the Angular Acceleration on the Longitudinal Tilt of the Rotor Disc

$\gamma = 12$		$\bar{\nu} = 0{\cdot}02$	$\bar{\nu} = 0{\cdot}06$	$\bar{\nu} = 0{\cdot}10$
$\left(\frac{a_1}{\alpha_0}\right)_{\text{total}}$	Component in phase with α	$-0{\cdot}4 \times 10^{-3}$	$-3{\cdot}7 \times 10^{-3}$	$-10{\cdot}5 \times 10^{-3}$
	Component in phase with $\dot{\alpha}$	-28×10^{-3}	-86×10^{-3}	-140×10^{-3}
$\left(\frac{a_1}{\alpha_0}\right)_{\text{acc}}$	Component in phase with α	$+0{\cdot}2 \times 10^{-6}$	$+20 \times 10^{-6}$	$+150 \times 10^{-6}$
	Component in phase with $\dot{\alpha}$	$+6 \times 10^{-6}$	$+152 \times 10^{-6}$	$+692 \times 10^{-6}$

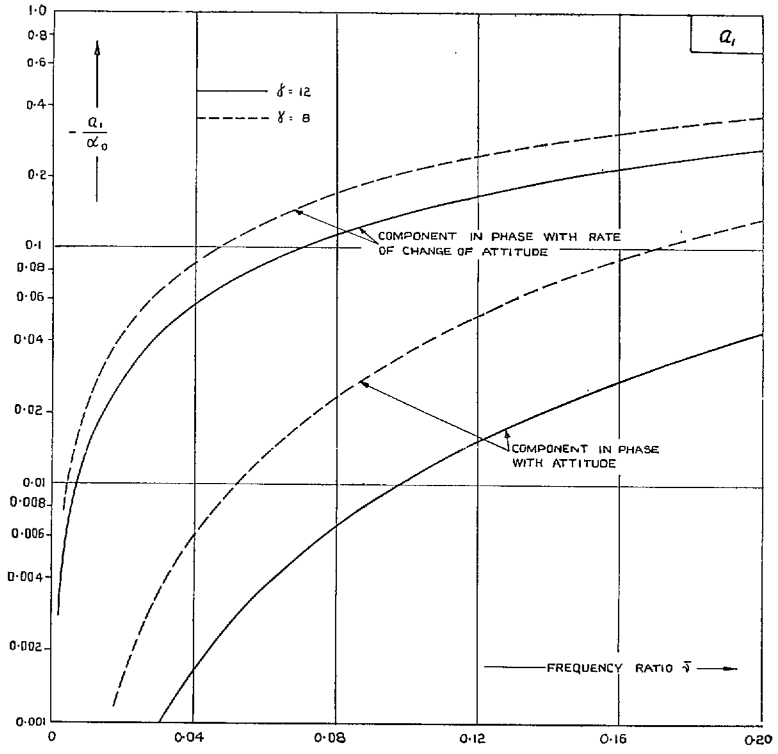

FIG. 1. Longitudinal tilt of the rotor disc.

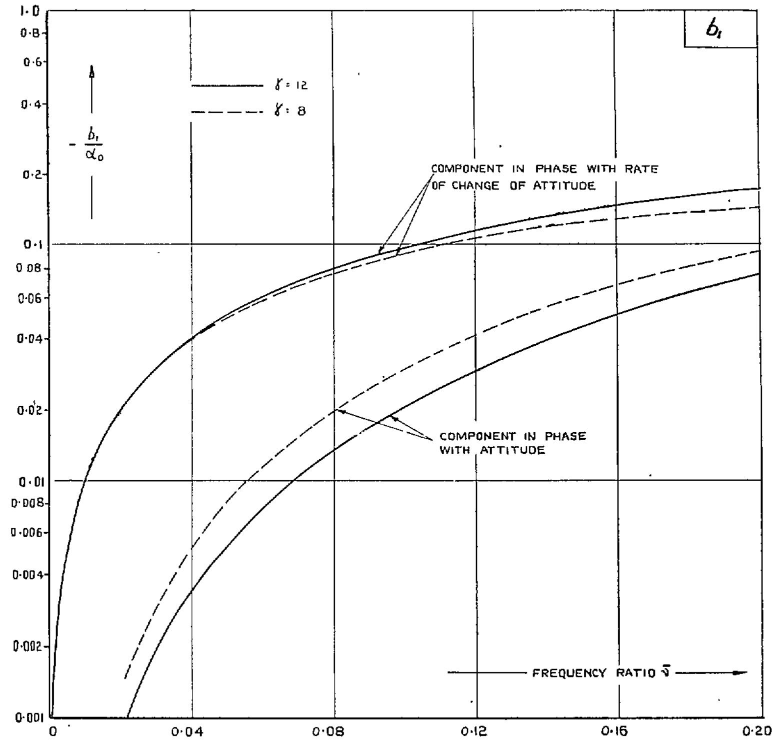

FIG. 2. Lateral tilt of the rotor disc.

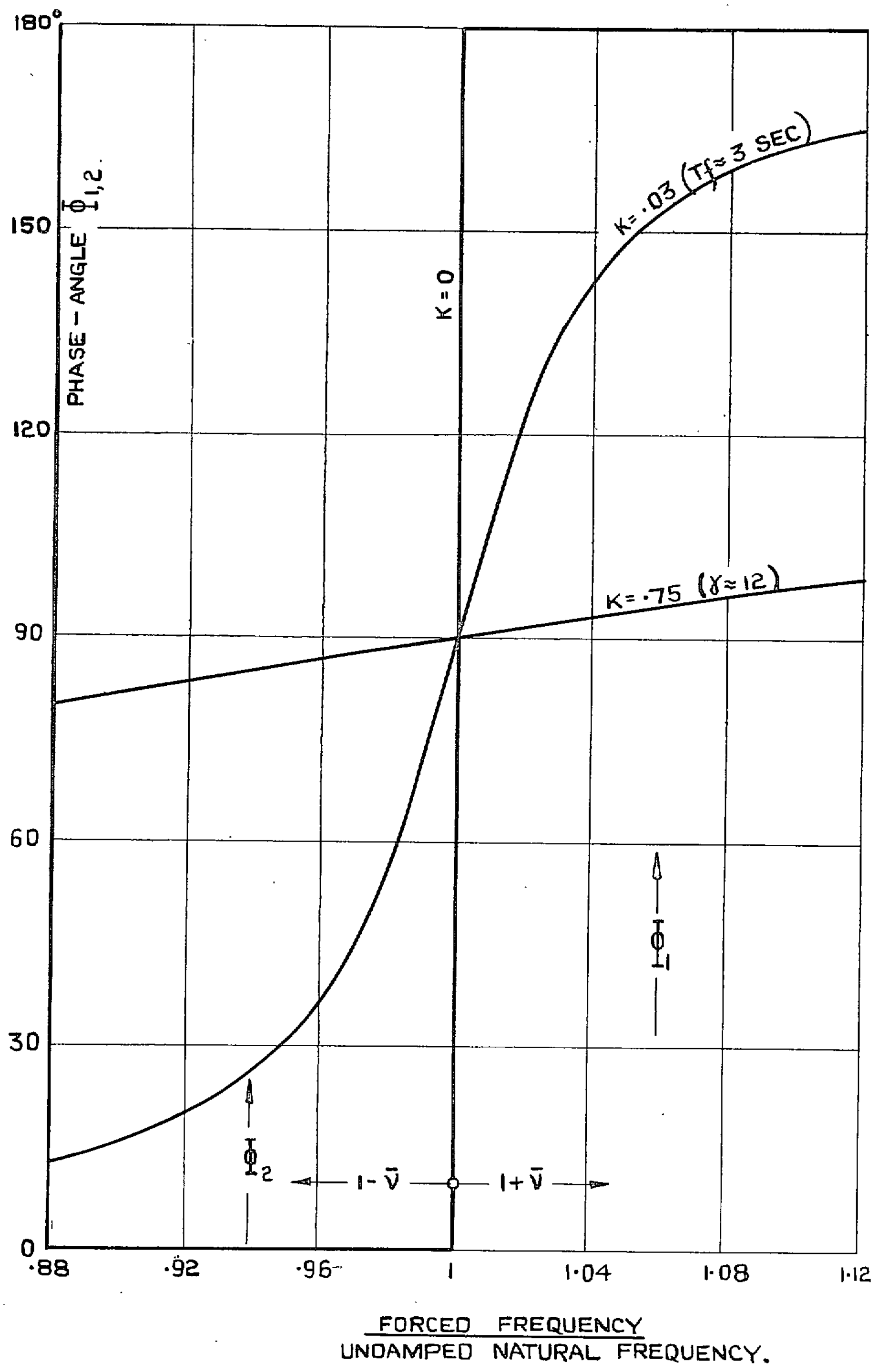

FIG. 3. Phase angle as a function of the frequency.

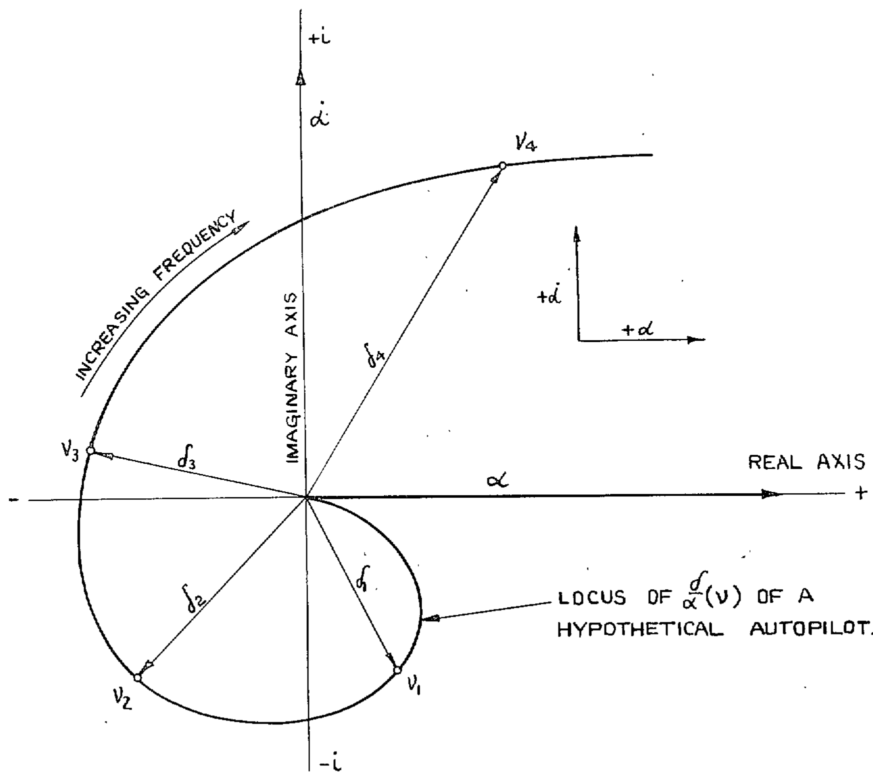

FIG. 4. Development of vector locus for a hypothetical auto-pilot.

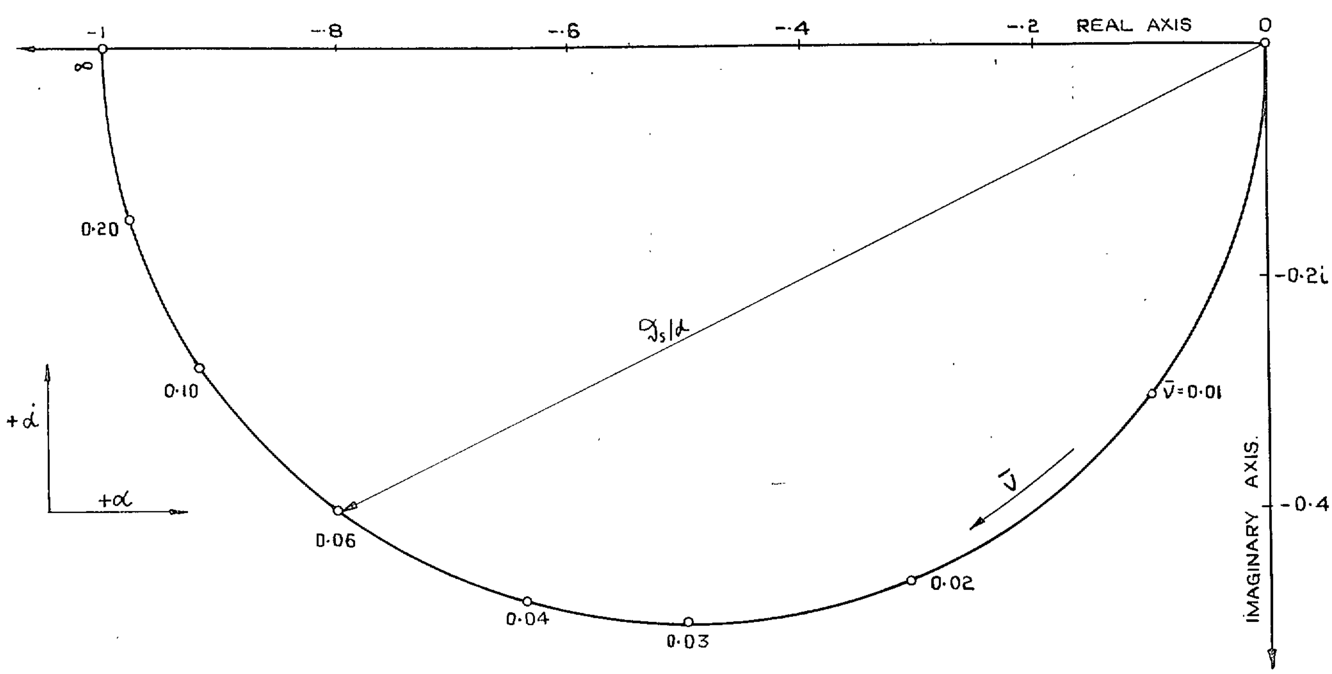

FIG. 5. Longitudinal control displacement due to pitching motion (Hiller servo-blade or Bell stabiliser, $K = 0 \cdot 03$).

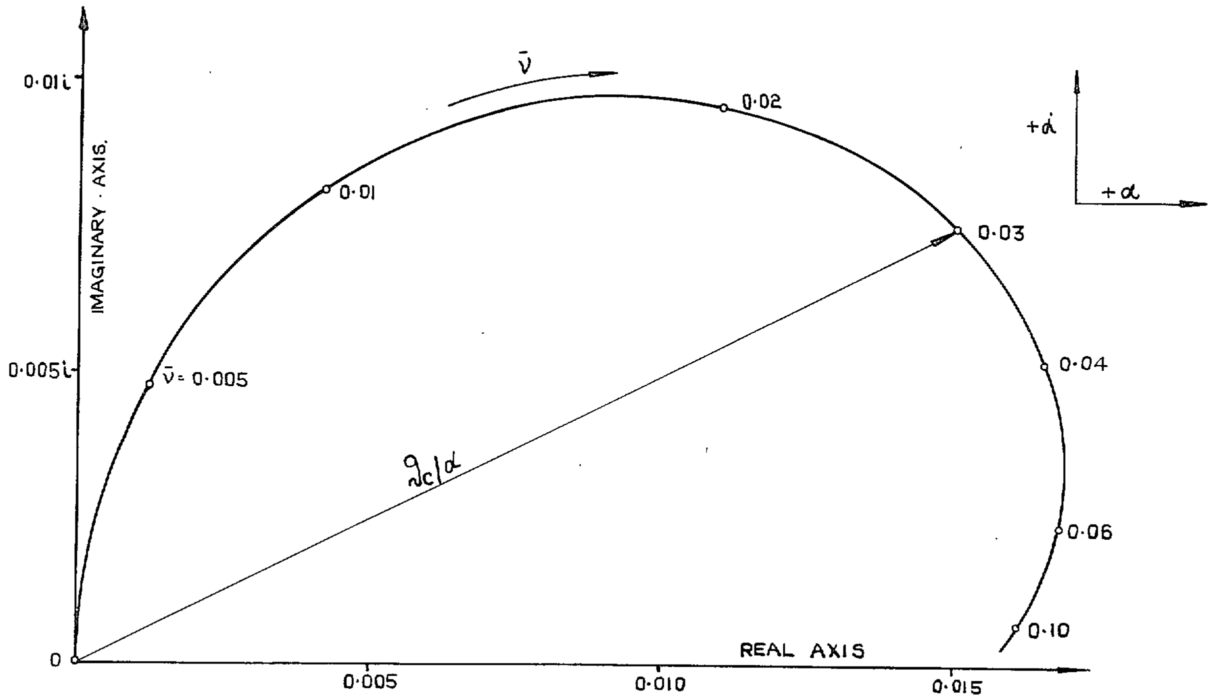

FIG. 6. Lateral control displacement due to pitching motion (Hiller servo-blade, $K = 0 \cdot 03$).

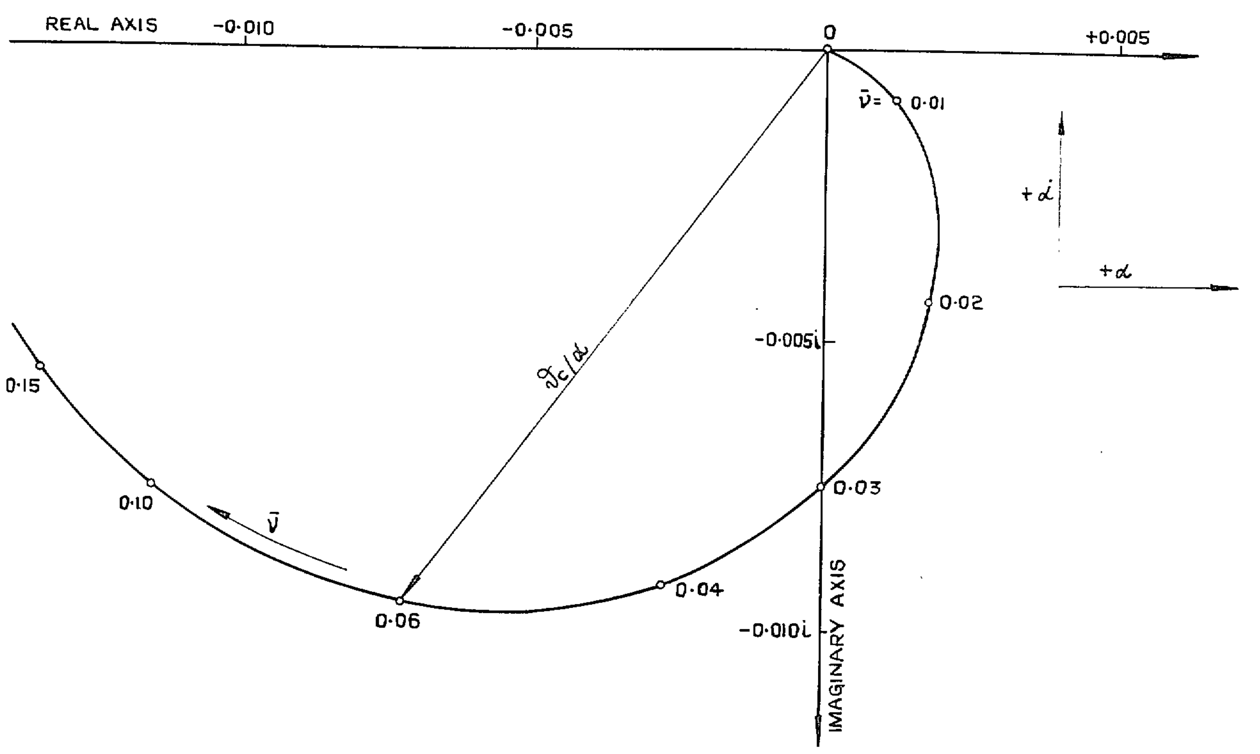

FIG. 7. Lateral control displacement due to pitching motion (Bell stabiliser, $K = 0 \cdot 03$).

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