



MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1955

PRICE 7s 6d NET

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
MINISTRY OF SUPPLY

*Reports and Memoranda No 2954**

May, 1952

Summary.—The theories of supersonic flow past slender, smooth, pointed bodies of arbitrary cross-sectional shape, due to Ward¹, and of the flow past slender bodies of revolution with discontinuities in profile slope, due to Lighthill², are applied and extended to calculate first approximations for the aerodynamic forces on bodies of elliptic cross-section with discontinuities in profile slope. Open-nose bodies are included in this class, but only the external forces are considered. The investigation is restricted to bodies the major axes of whose cross-sections are co-planar and whose cross-sections have constant eccentricity.

General expressions are deduced for the wave drag, lift, induced drag, and pitching moments of such bodies. The drag formula bears a marked resemblance to that for the equivalent body of revolution (*i.e.*, the body of revolution with the same axial distribution of cross-sectional area), but the discontinuities introduce a slight difference in one term. The lift formula is identical with that already deduced by Ward for a particular case of the present problem.

The general theory is applied to elliptic cones, and a comparison is made with Squire's solution of this problem³. Numerical results⁴ for the wave drag of bodies of revolution having straight and parabolic profiles are also extended to bodies of elliptic cross-section.

1. *Introduction.*—In Ref. 1 Ward has presented a method of obtaining approximate solutions of the linearized equation of supersonic flow for slender, pointed bodies of arbitrary cross-sectional shape. For Ward's method to be applicable, the bodies must be free from discontinuities in surface slope.

For the special case of bodies of revolution, Lighthill² has extended the slender-body theory to include bodies with discontinuities in surface slope. Open-nose bodies are included in this class[†]. It is apparent from Ref. 2 that discontinuities in slope introduce jumps in pressure which are wholly different in magnitude and form from the pressures encountered on smooth bodies (these pressure jumps are, in fact, two-dimensional expansions or compressions). However, the effects of the discontinuities decay very rapidly along the body, so that when the pressures are integrated over the body, the resulting expressions for drag and lift bear a marked resemblance to the results of the simpler theory for smooth bodies.

In the present paper we consider the supersonic flow past slender bodies of elliptic cross-section for which the major axes of the cross-sections are co-planar and whose cross-sections have constant eccentricity. A velocity potential is found of the general form derived by Ward for smooth bodies. This potential enables us to calculate not only the complete flow about smooth bodies of elliptic cross-section, but, with certain modifications, also the external aerodynamic forces on bodies having discontinuities in slope, provided that these discontinuities are spaced well apart.

* R.A.E. Report Aero. 2466, received 4th September, 1952.

† It is assumed both in Lighthill's and in the present paper that the flow is undisturbed ahead of the nose, *i.e.*, that there is no 'spillage'.

The basis of the method for dealing with the discontinuities is that the general form of Ward's solution may be used to deduce asymptotic expressions for the disturbances from the discontinuities, and if these are handled with sufficient care they yield a first approximation to the aerodynamic forces. The method should be applicable to bodies other than elliptic ones, if the solution is known for smooth bodies of the same cross-sectional shape.

2. *An Outline of Ward's Theory.*—Let (r_1, θ_1, s_1) be a system of cylindrical polar co-ordinates such that an undisturbed stream at infinity, of velocity U , is flowing parallel to the s_1 -axis in the direction of increasing s_1 . Let ϕ be a perturbation potential such that the velocity components in the directions of increasing r_1, θ_1, s_1 are

$$U \frac{\partial \phi}{\partial r_1}, \quad \frac{U}{r_1} \frac{\partial \phi}{\partial \theta_1}, \quad U \left(1 + \frac{\partial \phi}{\partial s_1} \right).$$

Let p be the Heaviside operator with respect to s_1 ; we write $f(p)$ for the operational form of a function $f(s_1)$. The linearized equation of supersonic flow is then

$$\frac{\partial^2 \phi}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial \phi}{\partial r_1} + \frac{1}{r_1^2} \frac{\partial^2 \phi}{\partial \theta_1^2} = \beta^2 p^2 \phi \quad \dots \quad (2.1)$$

where $\beta = \sqrt{M^2 - 1}$. The use of this equation as an approximate form of the exact equation of motion for the problem of flow past slender bodies has been fully investigated and justified by Lighthill^{2,3} and Ward¹.

Consider a pointed body of length $O(1)$ and of maximum thickness t , where t is small compared with unity. The restrictions on the shape of the body are stated by Ward as follows. 'The angle which any tangent plane to the body boundary makes with the undisturbed stream direction must be small and $O(t)$, and the rate of change of this angle along the direction of the body must also be small and $O(t)$. One further restriction on the shape of the body is required in general: this concerns the radius of curvature of any section of the boundary of the body by a plane perpendicular to the stream direction. If d is the diameter of the section then the curvature must be (at most) $O(1/d)$ for all points where the section is convex outwards; there is no restriction at points where the section is concave outwards. This last condition is not always necessary for bodies at zero incidence, but is always required if we wish to calculate the flow at incidence within a known approximation.' The reasons for these restrictions will become apparent from what follows.

If ν_1 denotes the normal to a section of the body in a plane $s_1 = \text{constant}$ (Fig. 1), the boundary condition may be written

$$\frac{\partial \phi}{\partial \nu_1} = \frac{d\nu_1}{ds_1} \left(1 + \frac{\partial \phi}{\partial s_1} \right) \approx \frac{d\nu_1}{ds_1} \quad \dots \quad (2.2)$$

A general solution of (2.1), giving waves travelling outwards and backwards from the axis, is, assuming convergence of the series,

$$\phi = \underline{A}_0 K_0(\beta p r_1) + \sum_{n=1}^{\infty} [A_n(p) \cos n\theta_1 + B_n(p) \sin n\theta_1] \cdot K_n(\beta p r_1), \quad \dots \quad (2.3)$$

where the K_n are modified Bessel functions of the second kind, and the \underline{A}_n and \underline{B}_n are arbitrary functions of p which must be chosen to satisfy the boundary condition.

Near the body r_1 is small and $O(t)$; and by taking the largest two terms of K_0 , and the largest term of each of the other K_n , Ward obtained an approximate perturbation potential ϕ_a , given by

$$\underline{\phi}_a = \mathcal{R} \left[\underline{a}_0 \log z_1 + \underline{b}_0 + \sum_{n=1}^{\infty} \underline{a}_n \cdot z_1^{-n} \right] \quad \dots \quad (2.4)$$

and we write

$$S'(c_i +) - S'(c_i -) = 2\pi ABf(c_i)[f'(c_i +) - f'(c_i -)] = \Delta S'_i,$$

with the convention that $f'(c_0 -) = f'(c_n +) = 0$.

4.2. *The Potential at Zero Incidence.*—The boundary condition (2.2) may now be written for zero incidence,

$$\frac{\partial\phi}{\partial\nu} = \pm \frac{\partial y}{\partial s} \frac{1}{\sqrt{\left\{1 + \left(\frac{\partial y}{\partial x}\right)^2\right\}}}, \quad (\pm \text{ for } y \gtrless 0), \quad \dots \dots \dots (4.2)$$

$$= \frac{ABf(s)f'(s)}{\sqrt{\left\{A^2f^2(s) - \left(1 - \frac{B^2}{A^2}\right)x^2\right\}}}. \quad \dots \dots \dots (4.3)$$

We now make the following transformation :

$$\left. \begin{aligned} x &= \left[\lambda + \frac{(A^2 - B^2)f^2(\sigma)}{4\lambda} \right] \cos \mu \\ y &= \left[\lambda - \frac{(A^2 - B^2)f^2(\sigma)}{4\lambda} \right] \sin \mu \\ s &= \sigma \end{aligned} \right\} \dots \dots \dots (4.4)$$

(We introduce σ because the operations $\partial/\partial s$ and $\partial/\partial\sigma$ will not be the same : the first will imply keeping x, y constant and the second keeping λ, μ constant.) (4.4) is of course the Joukowski transformation

$$z = \zeta + \frac{(A^2 - B^2)f^2(\sigma)}{4\zeta}, \quad \dots \dots \dots (4.5)$$

where $z = x + iy$, and $\zeta = \lambda e^{i\mu}$.

The ellipse (4.1) becomes in the ζ -plane the circle

$$\lambda = \frac{A + B}{2} f(\sigma). \quad \dots \dots \dots (4.6)$$

Since the transformation is conformal in planes $s = \text{constant}$, an element of the normal at the contour in the z -plane will become an element of normal in the ζ -plane and be magnified by the factor $|d\zeta/dz|$. Hence if ν_2 is the normal in the ζ -plane, the boundary condition becomes

$$\frac{\partial\phi}{\partial\nu_2} = \frac{\partial\phi}{\partial\nu} \frac{d\nu}{d\nu_2}, \quad \dots \dots \dots (4.7a)$$

i.e.,

$$\begin{aligned} \frac{\partial\phi}{\partial\lambda} &= \frac{ABf(\sigma)f'(\sigma)}{\sqrt{\left\{A^2f^2(\sigma) - \left(1 - \frac{B^2}{A^2}\right)A^2f^2(\sigma)\cos^2\mu\right\}}} \left| \frac{dz}{d\zeta} \right| \\ &= \frac{S'(\sigma)}{2\pi f(\sigma)\sqrt{(A^2\sin^2\mu + B^2\cos^2\mu)}} \left| \frac{dz}{d\zeta} \right|. \quad \dots \dots \dots (4.7b) \end{aligned}$$

From (4.5) we find that on the body

$$\left| \frac{dz}{d\zeta} \right| = \frac{2}{A + B} \sqrt{(A^2\sin^2\mu + B^2\cos^2\mu)}, \quad \dots \dots \dots (4.8)$$

It may be noted that in our approximation $\partial\varphi_0/\partial\sigma$ is not in fact continuous at $\sigma = c_i$, as was specified above. However, its singularities are the result of discontinuities in $S''(\sigma)$, and not of those in $S'(\sigma)$, and are of the same localized type as those already encountered on a smooth body.

Comparing (4.14) with the boundary condition (4.9) we deduce that the boundary condition for φ_{0i} is

$$\frac{\partial\varphi_{0i}}{\partial\lambda} = \frac{\Delta S'_i}{2\pi\lambda}, \quad (\text{on the body, for } \sigma > c_i). \quad \dots \dots \dots (4.15)$$

Thus the asymptotic expression for φ_{0i} is

$$\varphi_{0i} \sim \frac{\Delta S'_i}{2\pi} \log \lambda + b_{0i}(\sigma), \quad \dots \dots \dots (4.16)$$

where $b_{0i}(\sigma)$ must be evaluated from the original definition (2.5), since we now have $a_{0i}(c_i) \neq 0$. (4.16) then becomes

$$\varphi_{0i} \sim \frac{\Delta S'_i}{2\pi} \log \frac{\beta\lambda}{2(\sigma - c_i)}, \quad \dots \dots \dots (4.17)$$

and this expression becomes an increasingly good approximation for $\sigma - c_i > O(t)$, (section 3 and Appendix). Thus the asymptotic value of the complete potential is, for $C_i < \sigma < c_{i+1}$,

$$\phi_0 \sim \frac{1}{2\pi} \left\{ S'(\sigma) \log \frac{\beta\lambda}{2} - \sum_{j=0}^i \Delta S'_j \log (\sigma - c_j) - \int_{c_0}^{\sigma} S''(u) \log (\sigma - u) du \right\}, \quad \dots (4.18)$$

where the \sim here denotes that (4.18) becomes a valid approximation some distance behind each of the discontinuities on the body. Comparing this with the potential for a smooth body (4.10), we note that the two expressions are very similar, but that the b_{0i} have introduced a new term.

4.3. The Incidence Potential.—For the flow at incidence we consider incidences α_1 and α_2 applied in the x_1s_1 - and y_1s_1 -planes respectively (Fig. 3), and we assume that α_1 and α_2 are at most $O(t)$. The relation between the co-ordinates with respect to the wind axes (x_1, y_1, s_1), and the co-ordinates with respect to the body axes (x, y, s), is then given by

$$\begin{aligned} z &= z_1 + s_1\alpha e^{i\eta} + O(t^3) \\ s &= s_1 + O(t^2), \quad \dots \dots \dots (4.19) \end{aligned}$$

where $\alpha = \sqrt{(\alpha_1^2 + \alpha_2^2)}$ and $\eta = \tan^{-1}(\alpha_2/\alpha_1)$.

We take as our complete velocity potential

$$\begin{aligned} \Phi &= [U \cos \alpha \cdot s + U \phi_0] + [U \sin \alpha (x \cos \eta + y \sin \eta) + U \phi_1^{(1)}] \\ &= U \cos \alpha \cdot s + U \phi_0 + U \phi_1^{(2)}, \quad \dots \dots \dots (4.20) \end{aligned}$$

where $\phi_1^{(2)}$ is more than a perturbation potential, for it includes the uniform cross-flow at infinity. The complete boundary condition is then

$$\frac{\partial}{\partial\nu} (\phi_0 + \phi_1^{(2)}) = \frac{d\nu}{ds} \left[\cos \alpha + \frac{\partial}{\partial s} (\phi_0 + \phi_1^{(2)}) \right], \quad \dots \dots \dots (4.21)$$

so that the boundary condition for $\phi_1^{(2)}$ (we shall hereafter drop the superscript (2)) is

$$\frac{\partial\phi_1}{\partial\nu} = O + O(t^3 \log t), \quad \dots \dots \dots (4.22)$$

the order term being $O(t^2)$ immediately behind a discontinuity in slope. Hence for smooth, pointed bodies a suitable potential is given by

$$\phi_{a1} = \mathcal{R}\alpha \left[\zeta e^{-i\eta} + \frac{(A+B)^2 f^2(\sigma)}{4\zeta} e^{i\eta} \right] \quad \dots \quad (4.23)$$

and

$$\phi_a = \mathcal{R} \left\{ \frac{S'(\sigma)}{2\pi} \log \zeta + b_0(\sigma) + \alpha \left[\zeta e^{-i\eta} + \frac{(A+B)^2 f^2(\sigma)}{4\zeta} e^{i\eta} \right] \right\} \quad \dots \quad (4.24)$$

Although it is written in terms of the co-ordinates with respect to the body axes, (4.24) is of the required form (2.4) for ϕ_a , because by (4.19) the expression inside the curly brackets is an analytic function of z_1 as well as of ζ or z , except for negligible terms of $O(t^4)$; and also

$$b_0(\sigma) = b_0(s_1) + O(t^4).$$

For a body with discontinuities we could again divide the potential ϕ_1 into continuous and discontinuous parts, but since the term b_0 occurs only in conjunction with the zero-incidence potential, it is immediately apparent that the asymptotic form of ϕ_1 will in this case be an expression identical to (4.23).

5. *The Pressure Coefficient.*—We have, writing q for total velocity,

$$C_p = 1 - \frac{q^2}{U^2} + O \left[\left(1 - \frac{q^2}{U^2} \right)^2 \right],$$

and by (4.20) this becomes

$$C_p = \alpha^2 - 2 \left(\frac{\partial \phi_0}{\partial s} + \frac{\partial \phi_1}{\partial s} \right) - \left(\frac{\partial \phi_0}{\partial \nu} + \frac{\partial \phi_1}{\partial \nu} \right)^2 - \left(\frac{\partial \phi_0}{\partial \tau} + \frac{\partial \phi_1}{\partial \tau} \right)^2 + O \left[\left(\frac{\partial \phi}{\partial s} \right)^2 \right] \quad \dots \quad (5.1)$$

For smooth, pointed bodies we may use the approximate forms ϕ_{a0} , ϕ_{a1} , and this expression is sufficiently accurate for all points on and near to the body.

For bodies with discontinuities in slope (5.1) may be used on the body to predict the aerodynamic forces. It is true that behind points of discontinuity the O -term is only $O(t^2)$, but as these regions are only of length $O(t)$ the effect on lift and drag is $O(t^4)$ and $O(t^5)$, respectively, and therefore negligible. Similarly in the term $(\partial \phi / \partial \tau)^2$ we may use the asymptotic forms of the previous section, although these are not good approximations immediately behind points of discontinuity*. $\partial \phi / \partial \nu$ is of course known exactly on the body from the boundary condition.

It remains to consider the term $-2\partial \phi / \partial s$, which is $O(t)$ immediately behind a discontinuity and will have to be handled with some care. We cannot simply substitute the asymptotic value of $\partial \phi / \partial s$, for two reasons: (i) the lift and drag contributions of the true $\partial \phi / \partial s$ in the regions where the asymptotic form is invalid are $O(t^3)$ and $O(t^4)$ respectively, *i.e.*, of the same order as the total lift and drag; and (ii) the singularities of the asymptotic form at $s = c_i$ are non-integrable (*see* (4.18)).

We now transform (5.1) into a more convenient form. From (4.18) and (4.22) it follows that on the body

$$\frac{\partial \phi_0}{\partial \mu} \sim 0, \quad \text{i.e., } \frac{\partial \phi_0}{\partial \tau} \sim 0; \quad \frac{\partial \phi_1}{\partial \nu} = 0, \quad \text{i.e., } \frac{\partial \phi_1}{\partial \lambda} = 0,$$

so that on the body (5.1) becomes in terms of the variables (λ, μ, σ)

$$C_p = C_{p0} + C_{p1},$$

where

$$C_{p0} = -2 \left(\frac{\partial \phi_0}{\partial \sigma} + \frac{\partial \phi_0}{\partial \lambda} \frac{\partial \lambda}{\partial s} \right) - \left(\frac{\partial \phi_0}{\partial \lambda} \right)^2 \left| \frac{d\zeta}{dz} \right|^2, \quad \dots \quad (5.2)$$

$$C_{p1} = \alpha^2 - 2 \left(\frac{\partial \phi_1}{\partial \sigma} + \frac{\partial \phi_1}{\partial \mu} \frac{\partial \mu}{\partial s} \right) - \left(\frac{1}{\lambda} \frac{\partial \phi_1}{\partial \mu} \right)^2 \left| \frac{d\zeta}{dz} \right|^2. \quad \dots \quad (5.3)$$

* These arguments can also be developed more rigorously by the method of the Appendix.

Differentiating the first two equations of the transformation (4.4) partially with respect to s , substituting the value of λ on the body, and solving for $\partial\lambda/\partial s$, $\partial\mu/\partial s$, we obtain

$$\frac{\partial\lambda}{\partial s} = f'(\sigma) \frac{A+B}{2} \left(1 - \frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} \right), \quad \dots \quad (5.4)$$

$$\frac{\partial\mu}{\partial s} = \frac{f'(\sigma)}{f(\sigma)} \cot \mu \left(1 - \frac{B^2}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} \right). \quad \dots \quad (5.5)$$

These expressions are $O(t)$ and $O(1)$, respectively, so that in (5.2) and (5.3) only $\partial\phi_0/\partial\sigma$ and $\partial\phi_1/\partial\sigma$ are ever $O(t)$, the terms $(\partial\phi_0/\partial\lambda)(\partial\lambda/\partial s)$ and $(\partial\phi_1/\partial\mu)(\partial\mu/\partial s)$ being always $O(t^2)$. Accordingly we may use the asymptotic forms for these latter terms; (in the case of $\partial\phi_0/\partial\lambda$ the exact and asymptotic forms are of course identical on the body by virtue of the boundary condition).

It is convenient to combine the last two terms of (5.2) at this stage. From (4.8), (4.9) and (5.4), we have on the body

$$\begin{aligned} -2 \frac{\partial\phi_0}{\partial\lambda} \frac{\partial\lambda}{\partial s} &= -\frac{2S'(\sigma)}{\pi(A+B)f(\sigma)} f'(\sigma) \frac{A+B}{2} \left(1 - \frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} \right), \\ &\quad - \left(\frac{\partial\phi_0}{\partial\lambda} \right)^2 \left| \frac{d\xi}{dz} \right|^2 = -\frac{S'^2(\sigma)}{\pi^2(A+B)^2 f^2(\sigma)} \frac{(A+B)^2}{4(A^2 \sin^2 \mu + B^2 \cos^2 \mu)}. \end{aligned}$$

Hence

$$C_{p0} = -\frac{2\partial\phi_0}{\partial\sigma} + ABf'^2(\sigma) \left(\frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} - 2 \right). \quad \dots \quad (5.6)$$

It may be of interest to note that while there is no circumferential velocity on a smooth pointed body of elliptic cross-section at zero incidence, or at some distance behind a discontinuity on any elliptic body [see equations (4.10), (4.18)], a circumferential pressure gradient does exist. The effect of this gradient is of course to curve the streamlines on the surface of the body.

6. The Drag at Zero Incidence.—The drag integral may be written

$$\frac{D}{\frac{1}{2}\rho U^2} = \int_{c_0}^{c_n} ds \oint C_{p0} \frac{dv}{ds} d\tau \quad \dots \quad (6.1)$$

where

$$\frac{dv}{ds} = \frac{\partial y}{\partial s} \frac{1}{\sqrt{\left\{ 1 + \left(\frac{\partial y}{\partial x} \right)^2 \right\}}} = \frac{S'(s)}{2\pi\sqrt{\{A^2 f^2(s) - x^2\}}} \frac{1}{\sqrt{\left\{ 1 + \left(\frac{\partial y}{\partial x} \right)^2 \right\}}}, \quad \dots \quad (6.2)$$

and

$$d\tau = dx \sqrt{\left\{ 1 + \left(\frac{\partial y}{\partial x} \right)^2 \right\}}. \quad \dots \quad (6.3)$$

By substituting these expressions into (6.1) and writing $Af(\sigma) \cos \mu$ for x on the body, we obtain

$$\frac{D}{\frac{1}{2}\rho U^2} = \frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_0^{2\pi} C_{p0} d\mu. \quad \dots \quad (6.4)$$

We now consider the contributions of the various terms in C_{p0} , (5.6). From $-2\partial\phi_0/\partial\sigma$ we have, by (4.13),

$$-\frac{1}{\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \left[S''(\sigma) \log \frac{\beta(A+B)f(\sigma)}{4} - \int_{u=c_0}^{\sigma} \log(\sigma-u) dS''(u) \right]. \quad (6.5)$$

From each $-2\partial\varphi_{0i}/\partial\sigma$ we have a term

$$-\frac{1}{\pi} \int_0^{2\pi} d\mu \int_{c_i}^{c_n} S'(\sigma) \frac{\partial\varphi_{0i}}{\partial\sigma} d\sigma \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.6)$$

We cannot substitute our asymptotic value for $\partial\varphi_{0i}/\partial\sigma$ here, for the reasons already noted; however, we may proceed as follows. Let

$$\int_{c_i}^{\sigma} \frac{\partial\varphi_{0i}}{\partial\sigma} d\sigma = F_i(\sigma, \mu), \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.7)$$

where $\partial\varphi_{0i}/\partial\sigma$ represents the true value, and the integral is taken along a line $\mu = \text{constant}$ on the body. Then integrating (6.6) by parts, and noting that $F_i(c_i, \mu) = 0$ by (6.7), we obtain

$$-\frac{1}{\pi} \int_0^{2\pi} d\mu \left[- \int_{\sigma=c_i+}^{c_n+} F_i(\sigma, \mu) dS'(\sigma) \right] \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.8)$$

Now

$$\varphi_{0i}(\sigma, \mu) = \varphi_{0i}(c_i, \mu) + \int_{c_i}^{\sigma} \frac{\partial\varphi_{0i}}{\partial\sigma} d\sigma + \int_{c_i}^{\sigma} \frac{\partial\varphi_{0i}}{\partial\lambda} \frac{d\lambda}{d\sigma} d\sigma,$$

where the true (as opposed to the asymptotic) value of $\varphi_{0i}(c_i, \mu)$ is zero (section 3), and on the body, by (4.15) and (4.6),

$$\frac{\partial\varphi_{0i}}{\partial\lambda} = \frac{\Delta S_i'}{\pi(A+B)f(\sigma)},$$

$$\frac{d\lambda}{d\sigma} = \frac{A+B}{2} f'(\sigma).$$

Hence

$$F_i(\sigma, \mu) = \int_{c_i}^{\sigma} \frac{\partial\varphi_{0i}}{\partial\sigma} d\sigma = \varphi_{0i}(\sigma, \mu) - \int_{c_i}^{\sigma} \frac{\Delta S_i' f'(\sigma)}{2\pi f(\sigma)} d\sigma$$

by (4.17)

$$\sim \frac{\Delta S_i'}{2\pi} \left[\log \frac{\beta(A+B)f(\sigma)}{4(\sigma-c_i)} - \int_{c_i}^{\sigma} d \log f(\sigma) \right]$$

$$= \frac{\Delta S_i'}{2\pi} \log \frac{\beta(A+B)f(c_i)}{4(\sigma-c_i)} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.9)$$

It is shown in the Appendix that if we substitute this asymptotic expression into (6.8) the error is $O(\epsilon^2)$. We have in fact overcome the difficulty by working with the asymptotic form of φ_{0i} , instead of that for $\partial\varphi_{0i}/\partial\sigma$, and by applying the true value of φ_{0i} at $\sigma = c_i$. We now have for the drag contribution of the $-2\partial\varphi_{0i}/\partial\sigma$ terms

$$-\frac{1}{\pi} \sum_{i=0}^{n-1} \Delta S_i' \left[S'(c_i+) \log \frac{\beta(A+B)f(c_i)}{4} + \int_{\sigma=c_i+}^{c_n+} \log(\sigma-c_i) dS'(\sigma) \right] \quad (6.10)$$

From the last term in C_{p0} , (5.6), we have

$$\begin{aligned}
 & \frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_0^{2\pi} ABf'^2(\sigma) \left[\frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} - 2 \right] d\mu \\
 &= -\frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) \cdot ABf'^2(\sigma) \cdot 2\pi d\sigma \\
 &= -\frac{1}{2\pi} \int_{c_0}^{c_n} S'^2(\sigma) \frac{d}{d\sigma} \log f(\sigma) d\sigma \\
 &= -\frac{1}{2\pi} \sum_{i=0}^{n-1} [S'^2(c_{i+1}-) \log f(c_{i+1}) - S'^2(c_i+) \log f(c_i)] \\
 &\quad + \frac{1}{\pi} \int_{c_0}^{c_n} S'(\sigma) S''(\sigma) \log f(\sigma) d\sigma \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.11)
 \end{aligned}$$

The drag is the sum of (6.5), (6.10) and (6.11). (6.5) may be written

$$\begin{aligned}
 & -\frac{1}{\pi} \left\{ \int_{c_0}^{c_n} S'(\sigma) S''(\sigma) \log f(\sigma) d\sigma + \log \frac{\beta(A+B)}{4} \sum_{i=0}^{n-1} \frac{1}{2} [S'^2(c_{i+1}-) - S'^2(c_i+)] \right. \\
 & \quad \left. - \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_{u=c_0-}^{\sigma} \log(\sigma-u) dS''(u) \right\} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.12)
 \end{aligned}$$

The first terms of (6.10) and (6.11), and the second term of (6.12) combine to give

$$-\frac{1}{2\pi} \sum_{i=0}^n (\Delta S'_i)^2 \log \frac{\beta(A+B)f(c_i)}{4}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.13)$$

and the remainder of the drag is

$$-\frac{1}{\pi} \sum_{i=0}^{n-1} \Delta S'_i \int_{\sigma=c_i+}^{c_{n+}} \log(\sigma-c_i) dS'(\sigma) + \frac{1}{\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_{u=c_0-}^{\sigma} \log(\sigma-u) dS''(u) \quad \dots \quad \dots \quad (6.14)$$

Reversing the order of integration in the last term, and then integrating twice by parts gives

$$\begin{aligned}
 & \frac{1}{\pi} \int_{u=c_0-}^{c_{n+}} dS''(u) \int_u^{c_n} \log(\sigma-u) S'(\sigma) d\sigma \\
 &= -\frac{1}{\pi} \int_{c_0}^{c_n} S''(u) du \int_{\sigma=u}^{c_{n+}} \log(\sigma-u) dS'(\sigma) \\
 &= -\frac{1}{\pi} \left\{ \int_{c_0}^{c_n} S''(u) du \int_u^{c_n} \log(\sigma-u) S''(\sigma) d\sigma \right. \\
 & \quad \left. + \sum_{i=1}^n \Delta S'_i \int_{c_0}^{c_i} S''(u) \log(c_i-u) du \right\} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6.15)
 \end{aligned}$$

The last term in (6.15) combines with part of the first term in (6.14) to give

$$-\frac{1}{\pi} \sum_{i=0}^n \Delta S_i' \int_{c_0}^{c_n} S''(\sigma) \log |\sigma - c_i| d\sigma, \dots \dots \dots \dots \dots (6.16)$$

and the remainder of the first term in (6.14) is

$$-\frac{1}{\pi} \sum_{i=0}^{n-1} \Delta S_i' \sum_{j=i+1}^n \Delta S_j' \log (c_j - c_i) \dots \dots \dots \dots \dots (6.17)$$

Hence, collecting terms, we finally have

$$\begin{aligned} \frac{D}{\frac{1}{2}\rho U^2} &= \frac{1}{2\pi} \int_{c_0}^{c_n} \int_{c_0}^{c_n} S''(\sigma) S''(u) \log \frac{1}{|\sigma - u|} d\sigma du + \frac{1}{\pi} \sum_{i=0}^n \Delta S_i' \int_{c_0}^{c_n} S''(\sigma) \log \frac{1}{|\sigma - c_i|} d\sigma \\ &+ \frac{1}{\pi} \sum_{i=0}^{n-1} \Delta S_i' \sum_{j=i+1}^n \Delta S_j' \log \frac{1}{c_j - c_i} + \frac{1}{2\pi} \sum_{i=0}^n (\Delta S_i')^2 \log \frac{4}{\beta(A + B)f(c_i)} + O(t^5), \dots (6.18a) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2\pi} \int_{\sigma=c_0-}^{c_n+} \int_{u=c_0-}^{c_n+} \log \frac{1}{|\sigma - u|} dS'(\sigma) dS'(u) \\ &+ \frac{1}{2\pi} \sum_{i=0}^n (\Delta S_i')^2 \log \frac{4}{\beta(A + B)f(c_i)} + O(t^5), \dots \dots \dots \dots (6.18b) \end{aligned}$$

where the asterisk denotes the 'finite part of' the double Stieltjes integral (cf. Ref. 2, equation (37)). (6.18) is extremely similar to the expression for the drag of the equivalent body of revolution (*i.e.*, the body of revolution which has the same area distribution $\hat{S}(s)$), the only difference being in the last term. The drag of the body of revolution is greater by an amount

$$\frac{1}{2\pi} \sum_{i=0}^n (\Delta S_i')^2 \log \frac{A + B}{2\sqrt{AB}}, \dots \dots \dots \dots \dots (6.19)$$

which is always positive and independent of Mach number. However, if A and B are approximately equal, so that

$$\frac{A}{B} = 1 + O(t^{1/2}),$$

then

$$\log \frac{A + B}{2\sqrt{AB}} = O(t),$$

so that the difference in drag of the two bodies in this case is only $O(t^5)$, which is negligible to the order of accuracy of the theory.

It is apparent from the symmetrical form of (6.18) that the 'reverse' of any elliptic body has the same drag as the original body.

7. *The Forces and Moments at Incidence.*—7.1. *The Lateral Forces.*—The incidence problem is somewhat simpler than the drag problem for two reasons. Firstly, the cross-flow potential (4.23) gives, without further modifications, an asymptotic expression for the true potential when discontinuities are present; and secondly, in the integrals below for the lateral forces and pitching moments the continuous function $f(\sigma)$ appears where previously we had $S'(\sigma)$.

We have for the lateral forces in the directions of increasing x and y ,

$$\begin{aligned} \frac{X}{\frac{1}{2}\rho U^2} &= \int_{c_0}^{c_n} ds \left[\int_{-Bf(s)}^{Bf(s)} (-C_{p1}|_{x>0}) dy + \int_{-Bf(s)}^{Bf(s)} (C_{p1}|_{x<0}) dy \right] \\ &= -B \int_{c_0}^{c_n} f(\sigma) d\sigma \int_0^{2\pi} C_{p1} \cos \mu d\mu, \quad \dots \dots \dots \quad (7.1) \end{aligned}$$

$$\frac{Y}{\frac{1}{2}\rho U^2} = -A \int_{c_0}^{c_n} f(\sigma) d\sigma \int_0^{2\pi} C_{p1} \sin \mu d\mu. \quad \dots \dots \dots \quad (7.2)$$

Our potential is, from (4.23),

$$\phi_1 \sim \alpha \left[\lambda + \frac{(A+B)^2 f^2(\sigma)}{4\lambda} \right] \cos(\mu - \eta), \quad \dots \dots \dots \quad (7.3)$$

where the \sim here denotes that (7.3) becomes a valid approximation some distance behind each of the discontinuities on the body. We have shown that this is in adequate form for all the terms in C_{p1} , (5.3), except $-2\partial\phi_1/\partial\sigma$.

The term α^2 , and the term $-(\partial\phi_1/\lambda\partial\mu)^2 |d\zeta/dz|^2$, which occurs in the drag of an infinite elliptic cylinder in incompressible flow, will not contribute anything to X and Y .

The contribution to X of $-2\partial\phi_1/\partial\sigma$ may be written

$$2B \int_0^{2\pi} \cos \mu d\mu \int_{c_0}^{c_n} \frac{\partial\phi_1}{\partial\sigma} f(\sigma) d\sigma,$$

and if

$$\int_{c_0}^{\sigma} \frac{\partial\phi_1}{\partial\sigma} d\sigma = G(\sigma, \mu),$$

(the integral being taken along a line $\mu = \text{constant}$ on the body), this may be written

$$2B \int_0^{2\pi} \cos \mu d\mu \left[f(c_n) G(c_n, \mu) - \int_{c_0}^{c_n} G(\sigma, \mu) f'(\sigma) d\sigma \right]. \quad \dots \dots \dots \quad (7.4)$$

Now since $\partial\phi_1/\partial\lambda = 0$ on the body

$$G(\sigma, \mu) = \phi_1(\sigma, \mu) - \phi_1(c_0, \mu). \quad \dots \dots \dots \quad (7.5)$$

We have shown (section 3) that a perturbation potential tends to zero as $\sigma \rightarrow c_0$, but our ϕ_1 includes the uniform cross-flow; hence at $\sigma = c_0$ it must represent a uniform cross-flow in the entire z -plane:

$$\begin{aligned} \phi_1(c_0, \mu) &= \Re(\alpha z e^{-i\eta}) \\ &= \Re \left\{ \alpha \left[\zeta + \frac{(A^2 - B^2)f^2(\sigma)}{4\zeta} \right] e^{-i\eta} \right\} \\ &= \alpha f(c_0) [A \cos \mu \cos \eta + B \sin \mu \sin \eta] \quad \text{on the body.} \end{aligned}$$

Hence $G(\sigma, \mu) \sim \alpha [(A+B)f(\sigma) \cos(\mu - \eta) - f(c_0)(A \cos \mu \cos \eta + B \sin \mu \sin \eta)], \dots \quad (7.6)$

and as before we may substitute this into (7.4); the error will be $O(t^4)$. (7.4) now becomes

$$\begin{aligned} 2B \int_0^{2\pi} \cos \mu \cdot \alpha \left[\frac{A+B}{2} (f^2(c_n) + f^2(c_0)) \cos(\mu - \eta) - f^2(c_0)(A \cos \mu \cos \eta + B \sin \mu \sin \eta) \right] d\mu \\ = \pi \alpha \cos \eta \cdot B [(A+B)f^2(c_n) - (A-B)f^2(c_0)]. \quad \dots \dots \dots \quad (7.7) \end{aligned}$$

The term $-2(\partial\phi_1/\partial\mu)(\partial\mu/\partial s)$ in C_{p1} contributes to X , by (5.5) and (7.3),

$$\begin{aligned} & -2B \int_{c_0}^{c_n} f(\sigma) d\sigma \int_0^{2\pi} \alpha(A+B) \sin(\mu-\eta) \cdot f'(\sigma) \cot \mu \left[1 - \frac{B^2}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} \right] \cdot \cos \mu d\mu \\ & = -2\pi \alpha \cos \eta \cdot B(A-B) \int_{c_0}^{c_n} f(\sigma) f'(\sigma) d\sigma \\ & = -\pi \alpha \cos \eta \cdot B(A-B) [f^2(c_n) - f^2(c_0)] \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.8) \end{aligned}$$

Adding (7.7) and (7.8) we have

$$\begin{aligned} \frac{X}{\frac{1}{2}\rho U^2} & = 2\pi B^2 \alpha \cos \eta \cdot f^2(c_n) + O(t^4) \\ C_X & = \frac{X}{\frac{1}{2}\rho U^2 S(c_n)} = 2 \frac{B}{A} \alpha \cos \eta = 2 \frac{B}{A} \alpha_1 + O(t^2) \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.9) \end{aligned}$$

Similarly

$$C_Y = 2 \frac{A}{B} \alpha \sin \eta = 2 \frac{A}{B} \alpha_2 + O(t^2) \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.10)$$

This result is the same as that obtained by Ward¹ for the particular case of a smooth, pointed body of elliptic cross-section ending in a cylinder. X and Y have been derived as forces normal to the body axis, but they are equally 'lift' forces to the order of accuracy shown.

7.2. *The Induced Drag.*—The induced drag is given by

$$\frac{D_i}{\frac{1}{2}\rho U^2} = \frac{\sin \alpha}{\frac{1}{2}\rho U^2} (X \cos \eta + Y \sin \eta) + \frac{\cos \alpha}{2\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_0^{2\pi} C_{p1} d\mu.$$

To our order of accuracy this may be written

$$\frac{D_i}{\frac{1}{2}\rho U^2} = \frac{X}{\frac{1}{2}\rho U^2} \alpha_1 + \frac{Y}{\frac{1}{2}\rho U^2} \alpha_2 + \frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_0^{2\pi} C_{p1} d\mu \quad \dots \quad \dots \quad \dots \quad (7.11)$$

The α^2 term in C_{p1} , (5.3), contributes to the integral in (7.11)

$$\alpha^2 [S(c_n) - S(c_0)] \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.12)$$

Considerations of symmetry show that there is no contribution from $-2(\partial\phi_1/\partial s)$, (*i.e.*, from the whole second term in (5.3)), and from the last term in C_{p1} we have, by (4.8) and (7.3),

$$\begin{aligned} & -\frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) d\sigma \int_0^{2\pi} 4 \alpha^2 \sin^2(\mu-\eta) \cdot \frac{(A+B)^2}{4(A^2 \sin^2 \mu + B^2 \cos^2 \mu)} d\mu \\ & = -\frac{1}{2\pi} \int_{c_0}^{c_n} S'(\sigma) \cdot 2\pi \alpha^2 \left(1 + \frac{B}{A} \cos^2 \eta + \frac{A}{B} \sin^2 \eta \right) d\sigma \\ & = -[S(c_n) - S(c_0)] \left[\alpha^2 + \frac{B}{A} \alpha_1^2 + \frac{A}{B} \alpha_2^2 \right] \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.13) \end{aligned}$$

Adding (7.12), (7.13) and the contributions of the lateral forces, we have

$$\frac{D_i}{\frac{1}{2}\rho U^2} = [S(c_n) + S(c_0)] \left[\frac{B}{A} \alpha_1^2 + \frac{A}{B} \alpha_2^2 \right] + O(t^5) \quad \dots \quad \dots \quad \dots \quad \dots \quad (7.14)$$

7.3. *The Pitching Moments.*—For convenience we now take the x - and y -axes in the plane of the nose, writing $c_0 = 0$. The moments about these axes in the sense tending to decrease the applied incidence are then, respectively,

$$\frac{M_x}{\frac{1}{2}\rho U^2} = -A \int_0^{c_n} \sigma f(\sigma) d\sigma \int_0^{2\pi} C_{p1} \sin \mu d\mu, \quad \dots \dots \dots (7.15)$$

$$\frac{M_y}{\frac{1}{2}\rho U^2} = -B \int_0^{c_n} \sigma f(\sigma) d\sigma \int_0^{2\pi} C_{p1} \cos \mu d\mu. \quad \dots \dots \dots (7.16)$$

The integration is of course very similar to that for the lateral forces. From $-2\partial\phi_1/\partial\sigma$ we have the contribution to M_y (cf. (7.4))

$$2B \int_0^{2\pi} \cos \mu d\mu \left\{ c_n f(c_n) G(c_n, \mu) - \int_0^{c_n} G(\sigma, \mu) [f(\sigma) + \sigma f'(\sigma)] d\sigma \right\},$$

and upon substitution for G from (7.6) this becomes

$$\begin{aligned} & 2B \int_0^{2\pi} \cos \mu \cdot \alpha \frac{A+B}{2} \cos(\mu - \eta) \left[c_n f^2(c_n) - \int_0^{c_n} f^2(\sigma) d\sigma \right] d\mu \\ &= \pi \alpha \cos \eta \cdot B(A+B) \left[c_n f^2(c_n) - \int_0^{c_n} f^2(\sigma) d\sigma \right]. \quad \dots \dots \dots (7.17) \end{aligned}$$

From $-2(\partial\phi_1/\partial\mu)(\partial\mu/\partial s)$ we have [cf. (7.8)]

$$\begin{aligned} & -2\pi \alpha \cos \eta B(A-B) \int_0^{c_n} \sigma f(\sigma) f'(\sigma) d\sigma \\ &= -\pi \alpha \cos \eta B(A-B) \left[c_n f^2(c_n) - \int_0^{c_n} f^2(\sigma) d\sigma \right]. \quad \dots \dots \dots (7.18) \end{aligned}$$

Hence, adding (7.17) and (7.18), we obtain

$$\frac{M_y}{\frac{1}{2}\rho U^2} = 2\pi \alpha \cos \eta \cdot B^2 \left[c_n f^2(c_n) - \int_0^{c_n} f^2(\sigma) d\sigma \right]. \quad \dots \dots \dots (7.19)$$

Hence

$$\frac{M_y}{\frac{1}{2}\rho U^2} = 2 \frac{B}{A} \alpha_1 \left[(c_n - c_0) S(c_n) - \int_{c_0}^{c_n} S(\sigma) d\sigma \right] + O(t^4), \quad \dots \dots \dots (7.20)$$

$$\frac{M_x}{\frac{1}{2}\rho U^2} = 2 \frac{A}{B} \alpha_2 \left[(c_n - c_0) S(c_n) - \int_{c_0}^{c_n} S(\sigma) d\sigma \right] + O(t^4). \quad \dots \dots \dots (7.21)$$

Thus the X and Y forces act through the same point. The distance from the nose of the centre of pressure (or, in this case, of the aerodynamic centre), expressed as a fraction of body length, is given by

$$\begin{aligned} h &= 1 - \frac{1}{(c_n - c_0) S(c_n)} \int_{c_0}^{c_n} S(\sigma) d\sigma + O(t) \quad \dots \dots \dots (7.22) \\ &= 1 - \frac{\text{mean cross-section area}}{\text{base area}}. \end{aligned}$$

8. *Applications.*—8.1. *The Restriction on Section Curvature.*—In order to satisfy Ward's general restriction on the curvature of the cross-section we originally specified that both the major- and minor-axis parameters, A and B , be $O(t)$. In practice it may sometimes be desirable to consider bodies whose elliptic cross-section is very flat, that is, to let B be $O(t^2)$.

This is wholly justifiable for smooth bodies at zero incidence, since the circumferential velocity is zero in this case. We might also expect it to be permissible for bodies with discontinuities in slope at zero incidence, since at the actual points of maximum curvature symmetry requires zero circumferential velocity. However, in this case no rigorous justification is possible by the methods of the present paper, because we have no detailed knowledge of the flow immediately behind points of discontinuity.

The flat elliptic body at incidence has already been discussed by Ward¹ for the case of smooth bodies. Ward suggested that although the flow could no longer be calculated within a known approximation, the solutions obtained by his method gave a first approximation for sufficiently small angles of attack, since the forces and moments tend to a finite limit as $B \rightarrow 0$. This argument may be extended to the more general class of bodies considered here.

8.2. *The Elliptic Cone at Zero Incidence.*—For the particular problem of an elliptic cone we may write

$$f(s) = s, \quad S(s) = \pi AB s^2, \quad S''(s) = 2\pi AB. \quad \dots \dots \dots (8.1)$$

From equations (5.6) and (4.11), we then have for the pressure coefficient on the cone at zero incidence

$$\begin{aligned} C_{p0} &= -\frac{1}{\pi} \left[S''(\sigma) \log \frac{\beta\lambda}{2} - \int_{u=0}^{\sigma} \log(\sigma - u) dS''(u) \right] \\ &\quad + ABf'^2(\sigma) \left[\frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} - 2 \right] \\ &= AB \left[2 \log \frac{4}{\beta(A+B)} + \frac{AB}{A^2 \sin^2 \mu + B^2 \cos^2 \mu} - 2 \right]. \quad \dots \dots (8.2) \end{aligned}$$

This pressure coefficient is plotted for a range of ratios B/A in Fig. 4. It may be noted that as the eccentricity increases the pressure gradient over most of the ellipse first increases and then decreases, and that a pressure peak appears at the end of the major axis or 'leading edge'.

The drag coefficient is, by (6.18),

$$C_D = AB \left[2 \log \frac{4}{\beta(A+B)} - 1 \right]. \quad \dots \dots \dots (8.3)$$

In Ref. 5, Squire has obtained an exact solution of the linearized equation for flat elliptic cones, the major axis of his ellipse being finite and the minor axis infinitesimal. Squire's boundary condition is that of a wing problem, and may be written

$$\frac{\partial \phi}{\partial y} = \frac{\partial y}{\partial s},$$

where the corresponding form of our boundary condition is

$$\frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial x} \frac{\partial y}{\partial x} = \frac{\partial y}{\partial s}.$$

Nevertheless we would expect the two solutions to be similar for the common problem of a cone having A of $O(t)$ and B of $O(t^2)$, and this is in fact the case.

Squire's corrected solution consists of two parts as follows :

- (i) A constant pressure over the elliptic cone, which in our notation is given by

$$C_p = 2AB \frac{K\sqrt{1 - A^2\beta^2} - E\sqrt{1 - A^2\beta^2}}{1 - A^2\beta^2}, \quad \dots \dots \dots (8.4)$$

where K and E are complete elliptic integrals of the first and second kinds.

- (ii) A drag contribution from the singularity at the leading edge. Expressed as force per unit length normal to the leading edge, this is

$$\frac{F_n}{\frac{1}{2}\rho U^2} = \pi r_c \frac{A^2}{(A^2 + 1) \sqrt{\left\{1 - (\beta^2 + 1) \frac{A^2}{A^2 + 1}\right\}}}, \quad \dots \dots \dots (8.5)$$

where r_c is the radius of curvature of the leading edge.

In (i), when A is $O(t)$ we may expand the elliptic integrals in series, writing

$$K\sqrt{1 - A^2\beta^2} = \log \frac{4}{A\beta} + O(t^2 \log t),$$

$$E\sqrt{1 - A^2\beta^2} = 1 + O(t^2 \log t),$$

and so we obtain, for B of $O(t^2)$,

$$C_p = AB \left(2 \log \frac{4}{A\beta} - 2 \right) + O(t^5 \log t). \quad \dots \dots \dots (8.6)$$

In Fig. 5 this pressure coefficient is compared with that predicted by the theory of this paper (equation (8.2)). It is interesting to note that the two values agree well over most of the surface, and that, instead of Squire's singularity, the theory of this paper predicts a finite pressure peak at the leading edge. This peak is in fact $O(t^2)$, whereas the lower pressures are $O(t^3 \log t)$.

In (ii) above we have

$$r_c = \frac{B^2}{A} s + O(t^5). \quad \dots \dots \dots (8.7)$$

Hence we deduce

$$\Delta C_D = AB + O(t^5),$$

and

$$C_D = AB \left(2 \log \frac{4}{A\beta} - 1 \right) + O(t^5 \log t). \quad \dots \dots \dots (8.8)$$

The two expressions (8.3) and (8.8) for the drag coefficient are equal to the lowest order, for B of $O(t^2)$, the difference being $O(t^4 \log t)$.

Similar agreement may be obtained for the lift of the elliptic cone when incidence is applied in the ys -plane. It is already well known that the expression for the lift of a delta wing reduces to that given by (7.10) when the aspect ratio becomes small.

8.3. Conical and Parabolic Forebodies and Afterbodies.—Ref. 6 presented curves of the drag predicted by slender body theory for certain families of bodies of revolution. These included curves of the drag of forebodies and afterbodies of straight and parabolic profile; the afterbodies were assumed to be situated behind an infinitely long parallel portion, so that by the reversibility property mentioned in section 6, forebodies and afterbodies which are the 'reverse' of one another have equal drag.

It is now a simple matter to extend these results to slender bodies of elliptic cross-section with the same area distribution $S(s)$ as the bodies of revolution in question.

Let R_0 and R_1 denote the radii at the initial and final sections of a forebody of circular cross-section, and let l be the length. Then we have forebodies of straight profile

$$\begin{aligned} \Delta S_0' &= 2\pi R_0 \frac{R_1 - R_0}{l}, \\ \Delta S_1' &= -2\pi R_1 \frac{R_1 - R_0}{l}; \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \end{aligned} \quad (8.9)$$

and for bodies of parabolic profile,

$$\Delta S_0' = 4\pi R_0 \frac{R_1 - R_0}{l} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8.10)$$

It follows from equation (6.19) that the drag of the equivalent bodies of elliptic cross-section is that of the bodies of revolution plus the following term :

for bodies of straight profile,

$$\Delta C_D \left(\frac{l}{R_1} \right)^2 = \Delta C_D \left(\frac{\pi l^2}{S_1} \right) = -2 \left(1 + \frac{S_0}{S_1} \right) \left(1 - \sqrt{\frac{S_0}{S_1}} \right)^2 \log \frac{1 + B/A}{2\sqrt{B/A}}, \quad \dots \quad \dots \quad (8.11)$$

and for bodies of parabolic profile,

$$\Delta C_D \left(\frac{\pi l^2}{S_1} \right) = -8 \frac{S_0}{S_1} \left(1 - \sqrt{\frac{S_0}{S_1}} \right)^2 \log \frac{1 + B/A}{2\sqrt{B/A}} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8.12)$$

These correction terms are plotted in Fig. 6: comparison with the values of Ref. 6 shows that in general they are a very small proportion of the drag of the body.

8.4. *The Interference Effect of a Forebody on an Afterbody.*—It was demonstrated in Ref. 6 that when one considers the drag of a body consisting of a forebody, a parallel mid-portion and an afterbody, it is often convenient to calculate the drag as the sum of the following three components :

- (i) The forebody drag.
- (ii) The 'principal afterbody drag' (denoted by D_{A1}), which is the drag that the afterbody would have if it were situated behind an infinitely long parallel portion.
- (iii) The interference drag (denoted by D_{A2}) due to the effect of the forebody on the afterbody.

Curves of interference drag for a number of conical and parabolic bodies of revolution were given in Ref. 6.

For all cases where the distance between successive discontinuities in slope is large with respect to the thickness of the body, the interference drag is unaffected by the change from a circular to an elliptic cross-section. For such bodies the interference drag may be deduced from the drag formula (6.18a) by subtracting the forebody and principal afterbody drags from the drag of the complete body. We denote the ends of the parallel portion by $s = c_k$ and c_{k+1} , and the result is :

$$\begin{aligned} \frac{D_{A2}}{\frac{1}{2}\rho U^2} &= \frac{1}{\pi} \int_{c_0}^{c_k} S''(\sigma) d\sigma \int_{c_{k+1}}^{c_n} S''(u) \log \frac{1}{u - \sigma} du \\ &+ \frac{1}{\pi} \sum_{i=0}^k \Delta S_i' \int_{c_{k+1}}^{c_n} S''(\sigma) \log \frac{1}{\sigma - c_i} d\sigma + \frac{1}{\pi} \sum_{i=k+1}^n \Delta S_i' \int_{c_0}^{c_k} S''(\sigma) \log \frac{1}{c_i - \sigma} d\sigma \\ &+ \frac{1}{\pi} \sum_{i=0}^k \Delta S_i' \sum_{j=k+1}^n \Delta S_j' \log \frac{1}{c_j - c_i} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8.13) \end{aligned}$$

This is identical with equation (27) of Ref. 6, and is independent of Mach number.

When discontinuities in slope occur at both ends of the parallel portion, and the length of that portion is only of the order of the thickness of the body, we have no sufficiently accurate expression for the interference drag of the elliptic body, for we have no function corresponding to the exact form of $U_1(x)$ in Ref. 6. However, if this case should prove of real interest, the difficulty could be partially overcome as follows.

The value of the interference drag when the length of the parallel portion is zero may again be deduced from (6.18a). (The appropriate expression is given below, and in this case the interference drag is a function both of Mach number and of the eccentricity of the ellipse.) It should now be possible to guess fairly accurately the variation of interference drag with the length of the parallel portion, using as a guide the known initial values (8.14), the unique curve given by (8.13) for long parallel portions, and the known variation for the equivalent body of revolution (cf. Figs. 8 and 11, Ref. 6).

To evaluate the interference drag when the length of the parallel portion is zero, we let $\Delta S_{k-}'$ denote the final discontinuity in $S'(s)$ of the forebody alone (according to the convention of section 4.1), and $\Delta S_{k+}'$ the initial discontinuity of the afterbody alone. Then when the two are joined there is a discontinuity in $S'(s)$ at $s = c_k$ given by

$$\Delta S_k' = \Delta S_{k-}' + \Delta S_{k+}' ,$$

and the interference drag is found to be

$$\begin{aligned} \frac{D_{A2}}{\frac{1}{2}\rho U^2} &= \frac{1}{\pi} \int_{c_0}^{c_k} S''(\sigma) d\sigma \int_{c_k}^{c_n} S''(u) \log \frac{1}{u - \sigma} du \\ &+ \frac{1}{\pi} \sum_{i=0}^{k-} \Delta S_i' \int_{c_k}^{c_n} S''(\sigma) \log \frac{1}{\sigma - c_i} d\sigma + \frac{1}{\pi} \sum_{i=k+}^n \Delta S_i' \int_{c_0}^{c_k} S''(\sigma) \log \frac{1}{c_i - \sigma} d\sigma \\ &+ \frac{1}{\pi} \sum_{i=0}^{k-1} \Delta S_i' \sum_{j=k+1}^n \Delta S_j' \log \frac{1}{c_j - c_i} \\ &+ \frac{1}{\pi} \Delta S_{k-}' \sum_{i=k+1}^n \Delta S_j' \log \frac{1}{c_i - c_k} + \frac{1}{\pi} \Delta S_{k+}' \sum_{i=0}^{k-1} \Delta S_i' \log \frac{1}{c_k - c_i} \\ &+ \frac{1}{\pi} (\Delta S_{k-}') (\Delta S_{k+}') \log \frac{4}{\beta(A + B)f(c_k)} \dots \dots \dots \dots \dots (8.14) \end{aligned}$$

Because the last term of (6.18a) has contributed the last term of this expression, the interference drag is now a function both of Mach number and of the eccentricity of the ellipse.

NOTATION

a_n	Coefficients in Ward's approximate potential ϕ_a , equation (2.4)
A	Parameter defining the semi-major-axis of the elliptic cross-section
b_0	Function of s_1 in Ward's approximate potential ϕ_a , equation (2.4)
B	Parameter defining the semi-minor-axis of the elliptic cross-section
c_i	Values of s at which discontinuities occur
C_p	Pressure coefficient $(p - p_0)/\frac{1}{2}\rho U^2$
D	Drag

$f(s)$	Function defining the variation with s of the major and minor axes of the ellipse
F_i	See equation (6.7)
G	See equation (7.5)
M	Mach number of the free stream
M_x, M_y	Pitching moments about the x - and y -axes, respectively, in the plane of the nose
p	Heaviside operator with respect to s_1
(r_1, θ_1, s_1)	Cylindrical polar co-ordinates with respect to the 'wind axes'
\Re	'Real part of'
S	Cross-section area
t	Maximum thickness of body
(x, y, s)	Cartesian co-ordinates with respect to the body axes
(x_1, y_1, s_1)	Cartesian co-ordinates with respect to the 'wind axes'
X, Y	Lateral forces in the direction of increasing x and y
$z, (z_1) =$	$x + iy, (x_1 + iy_1)$
α	Total incidence
α_1, α_2	Incidences in the x_1s_1 - and y_1s_1 -planes, respectively
β	$\sqrt{(M^2 - 1)}$
γ	Euler's constant
ζ	Complex variable $\lambda e^{i\eta}$ of transformed (circle) plane
$\eta =$	$\tan^{-1} \alpha_2/\alpha_1$
(λ, μ, σ)	Elliptic-type co-ordinates with respect to the body axes, see equation (4.4)
$\nu, (\nu_1)$	Normal to the body contour in a plane $s = \text{constant}$ ($s_1 = \text{constant}$)
ρ	Density of the free stream
$\tau, (\tau_1)$	Circumferential variable in a plane $s = \text{constant}$ ($s_1 = \text{constant}$)
ϕ	Exact perturbation potential
ϕ_a	Ward's approximate form of perturbation potential, equation (2.4)
ϕ_b, ϕ_c	Errors of ϕ_a , see Appendix
ϕ_0, ϕ_{a0}	Zero-incidence (drag) potential
ϕ_1, ϕ_{a1}	Cross-flow (lift) potential
φ_0, φ_{0i}	Components of ϕ_0 , see section 4.2

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APPENDIX

The Effect of Discontinuities in the Source and Multi-source Strengths and their Derivatives.

A.1. *The Error of the Asymptotic Solutions.*—In this section we examine the accuracy of the slender body approximation as a solution of the linearized problem when there are discontinuities in the source and multi-source strengths $a_n(s_1)$. We let ϕ_a denote the asymptotic slender-body solution; we let $(\phi_a + \phi_b)$ denote the exact solution of the linearized equation with the source and multi-source strengths of the slender-body solution (i.e., $(\phi_a + \phi_b)$ is defined by (2.3), with the A_n and B_n derived from (2.5) and from the solution of the harmonic problem); and we let $(\phi_a + \phi_b + \phi_c)$ denote the exact solution of the linearized equation which satisfies the boundary condition (2.2) on the surface of the body. We consider values at a point for which the series (2.3) and (2.4) converge, and assume that the resultant orders of magnitude are valid everywhere on and near the body.

Writing $a_n(s_1) = a_{nR}(s_1) + ia_{nI}(s_1)$ we have

$$\phi_a + \phi_b = -\underline{a}_0(p)K_0(\beta pr_1) + \sum_{n=1}^{\infty} \frac{2}{(n-1)!} \left(\frac{\beta p}{2}\right)^n [\underline{a}_{nR}(p) \cos n\theta_1 + \underline{a}_{nI}(p) \sin n\theta_1] K_n(\beta pr_1), \quad (A.1)$$

and

$$\phi_a = \underline{a}_0(p) \left(\gamma + \log \frac{\beta pr_1}{2} \right) + \sum_{n=1}^{\infty} [\underline{a}_{nR}(p) \cos n\theta_1 + \underline{a}_{nI}(p) \sin n\theta_1] r_1^{-n}. \quad \dots \dots \dots (A.2)$$

We interpret the first (source) terms in these expressions by standard methods, using the product theorem in the form

$$\underline{f}(p) \underline{g}(p) \doteq \int_{u=0-}^{s_1} f(s_1 - u) dg(u).$$

The second (doublet) terms are interpreted by differentiating the source terms and their interpretations with respect to x or y and replacing a_0 by a_{1R} or a_{1I} . Subsequent terms may be found similarly. There result

$$\phi_a + \phi_b = - \int_{u=0-}^{s_1 - \beta r_1} \cosh^{-1} \frac{s_1 - u}{\beta r_1} da_0(u) + \frac{\cos \theta_1}{r_1} \int_{u=0-}^{s_1 - \beta r_1} \frac{s_1 - u}{\sqrt{\{(s_1 - u)^2 - \beta^2 r_1^2\}}} da_{1R}(u) + \dots, \quad (A.3)$$

and

$$\phi_a = - \int_{u=0-}^{s_1} \log \frac{2(s_1 - u)}{\beta r_1} da_0(u) + \frac{\cos \theta_1}{r_1} \int_{u=0-}^{s_1} da_{1R}(u) + \dots \quad (A.4)$$

Hence from a discontinuity in the $a_n(s_1)$ at $s_1 = c_i$ we get

$$[A(\phi_a + \phi_b)]_i = - \cosh^{-1} \frac{s_1 - c_i}{\beta r_1} (Aa_0)_i + \frac{\cos \theta_1}{r_1} \frac{s_1 - c_i}{\sqrt{\{(s_1 - c_i)^2 - \beta^2 r_1^2\}}} (Aa_{1R})_i + \dots, \quad (A.5)$$

and

$$(A\phi_a)_i = - \log \frac{2(s_1 - c_i)}{\beta r_1} (Aa_0)_i + \frac{\cos \theta_1}{r_1} (Aa_{1R})_i + \dots \quad (A.6)$$

Subtracting, we see that there exists some positive finite number C_1 such that

$$(A\phi_b)_i \text{ is } O\left[\frac{t^4}{(s_1 - c_i)^2}\right] \text{ for } s_1 - c_i \geq C_1 t. \quad (A.7)$$

ϕ_c is closely related to ϕ_b . By definition ϕ_c is an exact solution of the linearized equation, tending to zero as $r_1 \rightarrow \infty$, such that

$$\frac{\partial \phi_c}{\partial r_1} = - \frac{\partial \phi_b}{\partial r_1} \text{ on the body.} \quad (A.8)$$

Accordingly we assume that

$$(A\phi_c)_i \text{ is } O\left[\frac{t^4}{(s_1 - c_i)^2} \log \frac{t}{s_1 - c_i}\right] \text{ for } s_1 - c_i \geq C_1 t, \quad (A.9)$$

where the log term has been included because with it $\partial \phi_c / \partial r_1$ may still be $O[t^3 / (s_1 - c_i)^2]$, as is required by (A.8). This assumption is confirmed for the particular case of a body of revolution by Ward's quasi-cylinder solution⁷.

In the region $s_1 - c_i \leq C_1 t$, $(A\phi_a)_i$ is $O\{t^2 \log [t / (s_1 - c_i)]\}$, and by the arguments of section 2 $[A(\phi_a + \phi_b + \phi_c)]_i$ is $O(t^2)$, so that we may write

$$[A(\phi_b + \phi_c)]_i \text{ is } O\left[t^2 \log \frac{t}{s_1 - c_i}\right] \text{ for } s_1 - c_i \leq C_1 t. \quad (A.10)$$

We now examine the case where the $a_n(s_1)$ themselves are continuous, but the derivatives $a_n'(s_1)$ are discontinuous. The integrals in (A.3) and (A.4) are then ordinary Riemann integrals, and integrating these by parts we have

$$\begin{aligned} \phi_a + \phi_b = & - \int_{u=0-}^{s_1 - \beta r_1} \left[(s_1 - u) \cosh^{-1} \frac{s_1 - u}{\beta r_1} - \sqrt{\{(s_1 - u)^2 - \beta^2 r_1^2\}} \right] da_0'(u) \\ & + \frac{\cos \theta_1}{r_1} \int_{u=0-}^{s_1 - \beta r_1} \sqrt{\{(s_1 - u)^2 - \beta^2 r_1^2\}} da_{1R}'(u) + \dots, \quad (A.11) \end{aligned}$$

It may be noted that ϕ_a is continuous in this case, so that the forms (A.16) and (A.18) are wholly equivalent even when $(\phi_b + \phi_c)$ is neglected. Now since the distance between successive discontinuities is $O(1)$, and l_1 is $O(1)$, $(\phi_b + \phi_c)$ is $O(t^4 \log^2 t)$ at $\sigma = l_1$ and 0.

Also $\partial\phi/\partial\lambda$ is known from the boundary condition. Hence the dominant term in the error due to neglecting $(\phi_b + \phi_c)$ in (A.18) is

$$\int_0^{l_1} [\phi_b(\sigma, \mu) + \phi_c(\sigma, \mu)] S''(\sigma) d\sigma = O \left\{ \int_0^{c_2 t} t^3 |S''(\sigma)| d\sigma + \int_{c_2 t}^{l_1} \frac{t^4}{\sigma} \log^2 \frac{t}{\sigma} |S''(\sigma)| d\sigma \right\} \\ = O(t^6 \log^3 t) \dots \dots \dots (A.19)$$

A.3. *The Error Due to the Asymptotic Expressions for F_i and G .*—Finally we justify the substitution of the asymptotic form of F_i into the equation (6.8) of the main text. The $a_n(s_i)$ themselves are now discontinuous. Essentially we have to justify the neglect of $(\phi_b + \phi_c)$ in the integral

$$\int_{\sigma=l_1+}^{l_1+} (\phi_a + \phi_b + \phi_c) dS'(\sigma) .$$

The error due to this step is

$$O \left\{ \int_{\sigma=l_1+}^{c_1 t} t^2 \left| \log \frac{t}{\sigma} \right| |dS'(\sigma)| + \int_{c_1 t}^{l_1+} \frac{t^4}{\sigma^2} \log^2 \frac{t}{\sigma} |dS'(\sigma)| \right\} . \dots \dots (A.20)$$

If $(c_{i+1} - c_i)$ is always $O(1)$, as we have specified, the discontinuities in $S'(\sigma)$ will contribute only to the second integral of this expression, and their contributions will be $O(t^6 \log^2 t)$. The remainder of (A.20) is

$$O \left\{ t^4 \left[\int_0^{l_1} \log \frac{t}{\sigma} d\sigma + \int_{l_1}^{c_1 t} \log \frac{\sigma}{t} d\sigma \right] + t^6 \int_{c_1 t}^{l_1} \frac{1}{\sigma^2} \log^2 \frac{t}{\sigma} d\sigma \right\} = O(t^5) \dots \dots (A.21)$$

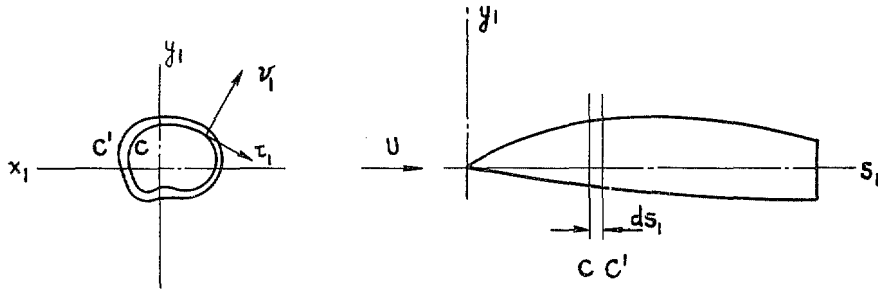


FIG. 1. Notation.

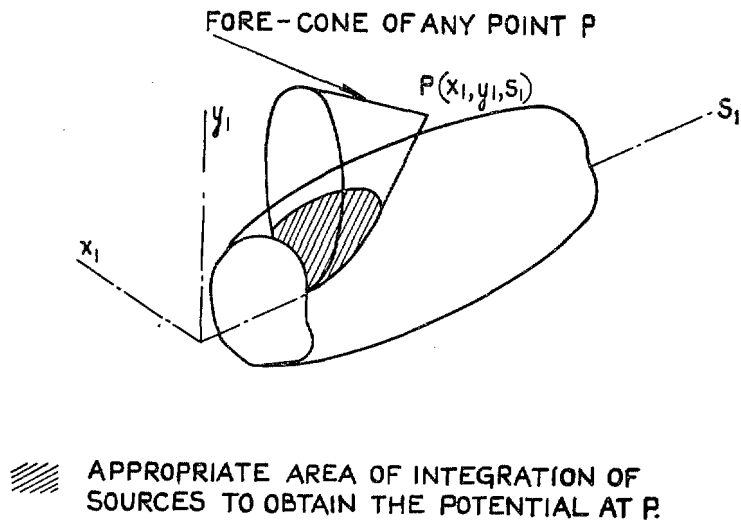


FIG. 2. Area of integration of sources.

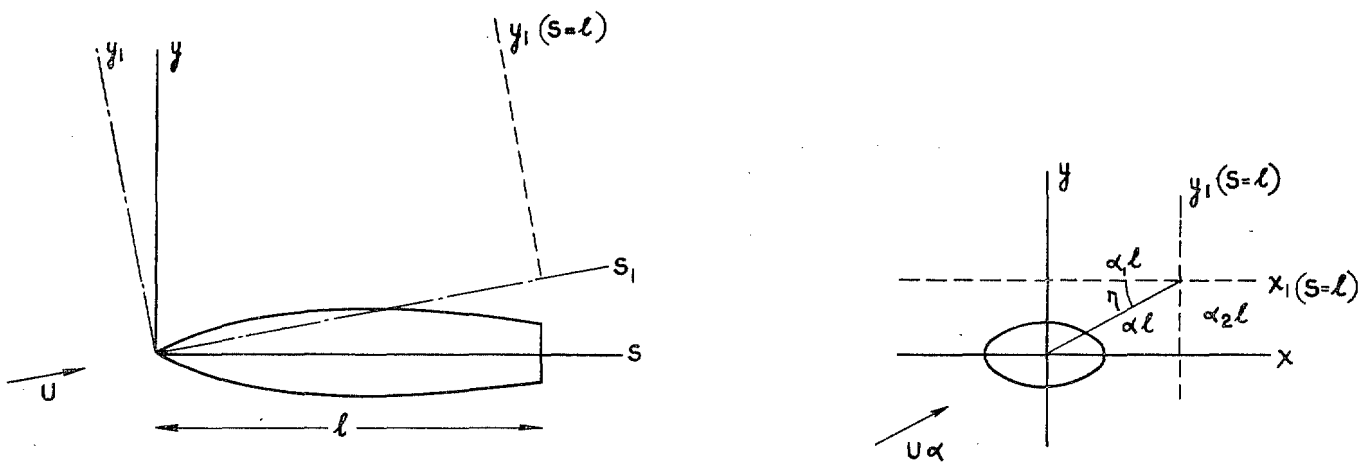


FIG. 3. Body and wind axes.

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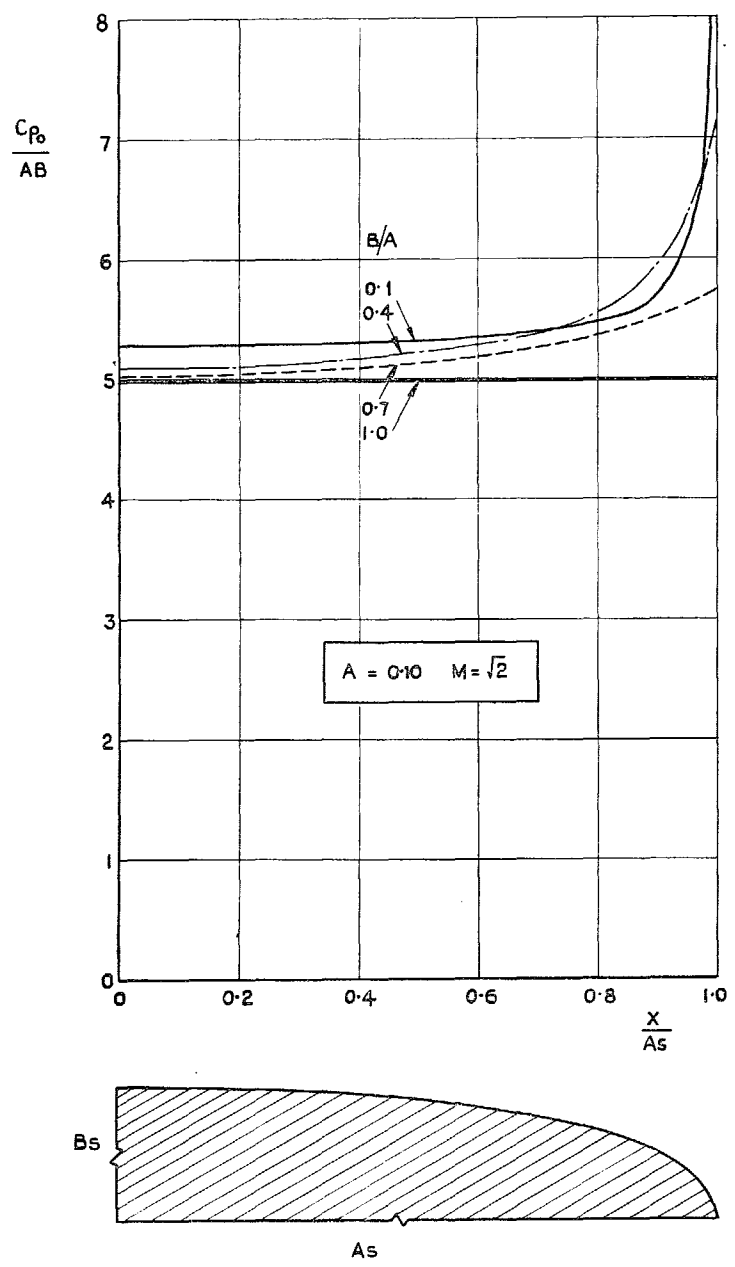


FIG. 4. Examples of the pressure distribution on elliptic cones.

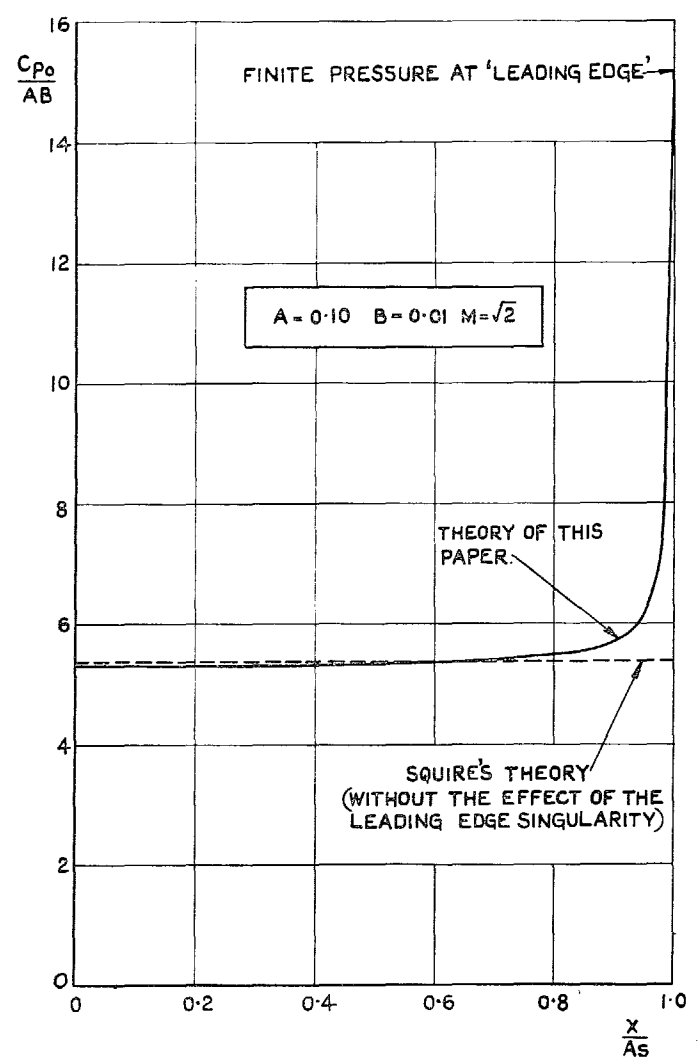


FIG. 5. Comparison with Squire's theory of the pressure on a flat elliptic cone.

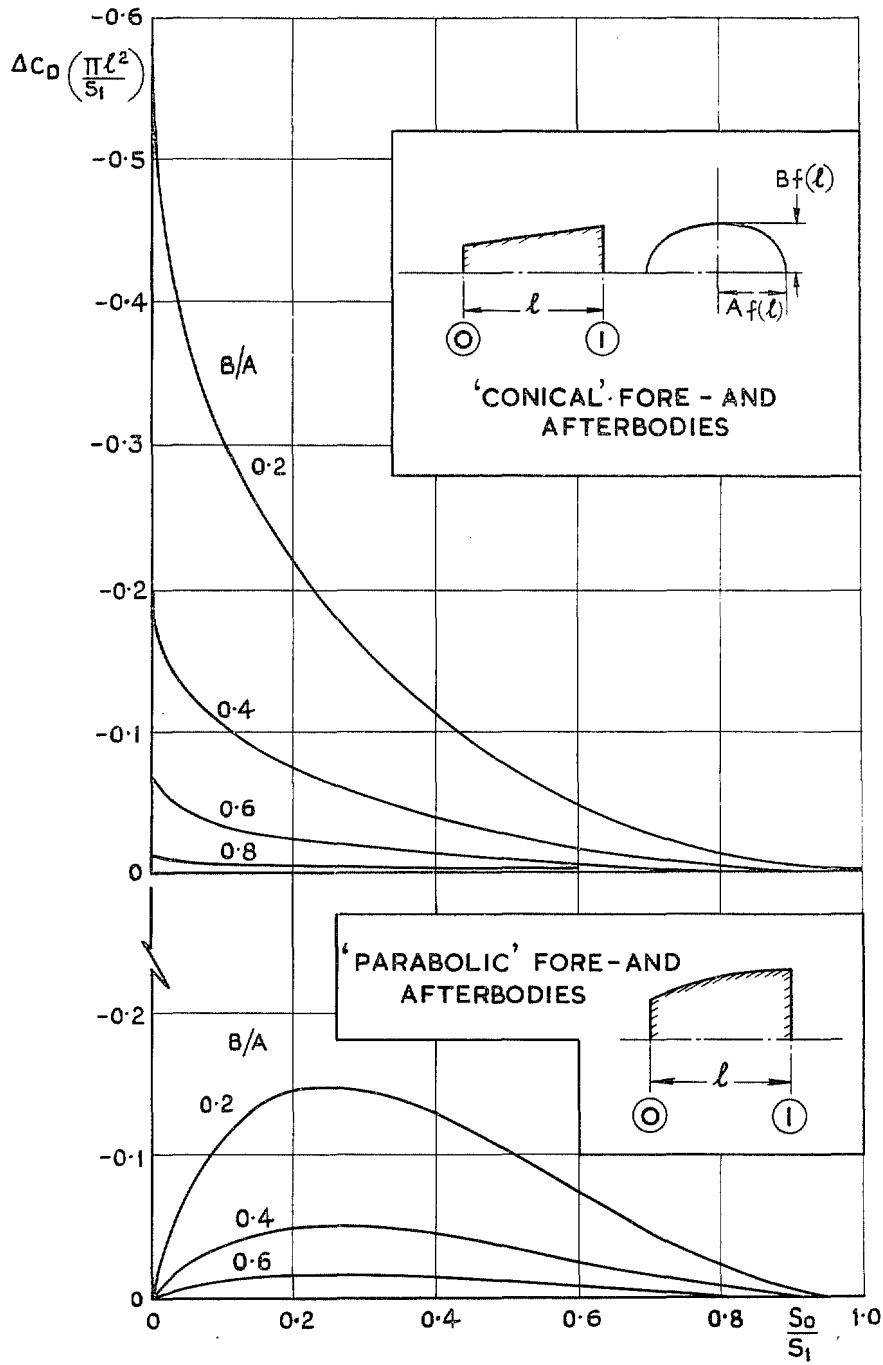


FIG. 6. Differences between the drags of bodies of elliptic and circular cross-section.

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