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Calculation of Flutter Derivatives for Wings of General Plan-form

By

DORIS E. LEHRMAN, B.Sc.,
of the Aerodynamics Division, N.P.L.

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Summary.—The vortex-lattice method of calculating flutter derivatives presented in this note is an extension to higher frequencies of the work on stability derivatives reported in R. & M. 2922. The method is a modified form of the scheme outlined in R. & M. 2470 and is suggested as an alternative to the latter method since it gives a simpler routine calculation for wings of general plan-form. Derivatives are calculated for the following wings describing plunging and pitching oscillations:

(a) Delta wings of aspect ratio $A = 1.2$ and 3 and with a taper ratio $1/7$

(b) Arrowhead wing of aspect ratio 1.32 with a taper ratio $7/18$ and angle of sweep of 63.4 deg at quarter-chord.

The results for the delta wing $A = 1.2$ and the arrowhead wing are compared with values of the pitching derivatives obtained in low-speed tests; those for the delta wing $A = 3$ with the values calculated in R. & M. 2841. The comparison indicates that the present method gives reasonable accuracy for low-aspect-ratio wings in incompressible flow; the method may be sufficiently reliable for use with the equivalent wing theory suggested in R. & M. 2855 for the calculation of flutter derivatives in compressible subsonic flow.

1. *Introduction.*—The vortex lattice method of Ref. 2 (R. & M. 2470) has been used to calculate derivatives for a delta wing $A = 3$ and gives satisfactory results for frequencies in the flutter range³ (R. & M. 2841). However, there are some uncertainties in applying that method to a highly tapered wing, since it involves a lift function $C(\omega')$ and chordwise factors $L'_0(k)$, which are functions of the local frequency parameter $\omega' = pc/2V$. If the spanwise variation in $L'_0(k)$ is taken into account, the downwash calculation becomes laborious, and furthermore, because of the limiting form of $C(\omega')$ which involves $\log \omega'$, introduces erroneous effects if ω' is very small at the wing tips. It is not known if it is sufficient to use constant values of $L'_0(k)_m$ and $C(\omega'_m)$ over the span corresponding to the mean frequency parameter ω_m . The present method attempts to avoid some of these difficulties by extending to higher frequencies the modified form of R. & M. 2470, which is used to calculate stability derivatives¹ (R. & M. 2922). For all plan-forms the method leads to a simpler routine than the method of Ref. 2 since the downwash computation, except for the correction for the oscillatory wake, is based on values of the downwash due to a rectangular vortex in steady motion. The present method does, however, involve chordwise factors $L'_0(k)$, which are dependent on the local parameter ω' (see Appendix). It is suggested that these factors may be expressed approximately over a range of ω' values, as polynomials in terms of the mean frequency parameter ω_m . In this case the corresponding downwash is obtained generally in terms of ω_m and it is only necessary to compute the wake correction for each particular value of ω_m by using the oscillatory downwash tables⁷. Except for the wake correction, the calculation reduces to a simple routine which appears suitable for use on a high-speed computing machine.

The method is applied in this note to low-aspect-ratio wings; derivatives are estimated for comparison with measured values and to obtain information on the accuracy of vortex lattice solutions. A reliable method for low-aspect-ratio wings in incompressible flow is required in order to apply the 'equivalent wing' theory of calculating flutter derivatives in compressible subsonic flow; it is shown in Ref. 4 that derivatives for a wing of aspect ratio A oscillating at a frequency f in compressible flow of Mach number M may be calculated by considering a wing of smaller aspect ratio $A\sqrt{(1 - M^2)}$, in incompressible flow, which is oscillating in a different mode at a higher frequency $f/(1 - M^2)$.

LIST OF SYMBOLS AND DEFINITIONS

V	Velocity of flow
x	$= R(y) - \frac{1}{2}c \cos \theta$ $= R(y) + \frac{1}{2}c \xi$ } Definitions of chordwise parameters θ and ξ where $R(y)$ is the mid-chord line
x_l	Leading-edge co-ordinate, $\theta = 0$
x_t	Trailing-edge co-ordinate, $\theta = \pi$
y	$= s\eta$ Definition of spanwise co-ordinate η
$c(y)$	Local chord
c_0	Root chord
c_m	Mean chord
s	Semi-span
S	Area of wing
A	Aspect ratio
$\phi/2\pi$	Frequency
ω	$= 2\omega' = \phi c/V$ Local frequency parameter
ω_m	$= \phi c_m/V$ Mean frequency parameter
$K e^{i\phi t}$	Doublet distribution (discontinuity in velocity potential)
$\Gamma e^{i\phi t}$	Bound velocity
$E e^{i\phi t}$	Free vorticity
$W e^{i\phi t}$	Induced downward velocity
$z' e^{i\phi t}$	Normal downward displacement of point (x, y) on the wing
δL	$= \int_{x_l}^{x_t} \rho V \Gamma dx$. Local lift
δM	$= \int_{x_l}^{x_t} \rho V \Gamma x dx$. Local moment
$L e^{i\phi t}$	$= \int_{-s}^s \int_{x_l}^{x_t} \rho V \Gamma e^{i\phi t} dx dy$. Lift
$M e^{i\phi t}$	Pitching moment about axis $x = 0$ through wing apex $= - \int_{-s}^s \int_{x_l}^{x_t} \rho V \Gamma e^{i\phi t} x dx dy$

LIST OF SYMBOLS AND DEFINITIONS—*continued*

$$\Gamma_0 = 2 \cot \frac{1}{2}\theta$$

$$\Gamma_1 = -2 \sin \theta + \cot \frac{1}{2}\theta + i\omega'[\sin \theta + \frac{1}{2} \sin 2\theta]$$

$$\Gamma_n = -2 \sin n\theta + i\omega' \left[\frac{\sin (n+1)\theta}{n+1} - \frac{\sin (n-1)\theta}{n-1} \right] \text{ when } n \geq 2$$

K_0 is defined by Equations (5) and (6)

$$K_1 = \frac{1}{2}c[\sin \theta + \frac{1}{2} \sin 2\theta]$$

$$K_n = \frac{1}{2}c \left[\frac{\sin (n+1)\theta}{n+1} - \frac{\sin (n-1)\theta}{n-1} \right] \text{ when } n \geq 2$$

$$A_m = (s/c)T_m = s\eta^{m-1}\sqrt{(1-\eta^2)}$$

Definition of Derivatives for Plunging and Pitching Oscillations

(a) Local derivative coefficients † at a spanwise position η , referred to an axis $x = 0$.

$$\frac{\delta L}{\rho V^2 c_m} = [l_z(\eta) + i\omega_m l_z(\eta)]z + [l_\alpha(\eta) + i\omega_m l_\alpha(\eta)]\alpha$$

$$\frac{\delta M}{\rho V^2 c_m^2} = [m_z(\eta) + i\omega_m m_z(\eta)]z + [m_\alpha(\eta) + i\omega_m m_\alpha(\eta)]\alpha$$

where $c_m z$ and α are the amplitudes of the vertical translational and angular displacements of the oscillating wing.

(b) Derivatives † referred to axis $x = 0$:

$$L/(\rho V^2 S) = (l_z + i\omega_m l_z)z + (l_\alpha + i\omega_m l_\alpha)\alpha$$

$$M/(\rho V^2 S c_m) = (m_z + i\omega_m m_z)z + (m_\alpha + i\omega_m m_\alpha)\alpha$$

where $c_m z$ and α are the amplitudes of the vertical translational and angular displacements of the oscillating wing.

(c) Derivatives referred to axis of oscillation $x = h'c_m = hc_0$ are obtained from definitions (a) and (b) by the usual transformation formulae:

$$l'_z = l_z$$

$$l'_\alpha = l_\alpha - h'l_z$$

$$-m'_z = -m_z - h'l_z$$

$$-m'_\alpha = -m_\alpha - h'(l_\alpha - m_z) + h'^2 l_z$$

and similar expressions for the damping derivatives.

† The derivatives $l_z, l_\alpha, -m_z, -m_\alpha$ include the aerodynamic inertia terms.

2.1. *Calculation of the Downwash W'_{0m} .*—The doublet distribution $K'_0 A_m$ is of constant strength over the wake in the chordwise direction and may be replaced by a lattice of rectangular vortices as in steady motion. If the continuous chordwise distribution K'_0 is represented by k' vortices of strength $cL'_0(k)$, $k = 1, 2, \dots, k'$, then a typical rectangular vortex at a chordwise position $(2k - 1)c/2k'$ and spanwise position $\eta_j s$ is of strength

$$cL'_0(k) A_m(\eta_j) = sL'_0(k) T_m(\eta_j), \quad \dots \quad (10)$$

where $T_m = \eta^{m-1} \sqrt{1 - \eta^2}$. The factors $L'_0(k)$ are chosen on a two-dimensional basis, as described in the Appendix and give the correct value K'_0 at the trailing edge. Values of $L'_0(k)/\pi$ corresponding to $k' = 6$ and $k' = 2$ are tabulated in Table 1, with modified second differences, for a range of values of the local frequency parameter $\omega = 2\omega' = \rho c/V$. The downwash W'_{0m} at a collocation point (x_1, y_1) is then evaluated by summation; the critical downwash tables⁶ for steady motion give the downwash F at a point (X^*, Y^*) † due to a rectangular vortex of width $2s_1$ and strength $4\pi s_1$, so that W'_{0m} is obtained as

$$W'_{0m} = \frac{s}{4s_1} \sum_{\eta_j} \sum_k \left(\frac{L'_0(k)}{\pi} T_m(\eta_j) F \right) \dots \quad (11)$$

For a wing of general plan-form there is a spanwise variation in the parameter ω' and therefore in the factors $L'_0(k)$ for each position η_j of the lattice. It is suggested that the spanwise variation of these factors may be allowed for in the calculation (11) by expressing $L'_0(k)$ as polynomials in ω' . For example, if the factors are known for a range of values of ω' , it is possible to fit polynomials

$$L'_0(k) = L'_a(k) + i\omega' L'_b(k) + \omega'^2 L'_c(k) + i\omega'^3 L'_d(k), \quad \dots \quad (12)$$

where L'_a, L'_b, \dots are real numbers, to give a good approximation over a range $\omega'_1 \leq \omega' \leq \omega'_2$ say. Then since

$$\omega' = \omega_m \left(\frac{c}{2c_m} \right) = \omega_m f(|\eta|), \quad \dots \quad (13)$$

it follows that the downwash W'_{0m} can be expressed generally in terms of ω_m as

$$W'_{0m} = W'_{am} + i\omega_m W'_{bm} + \omega_m^2 W'_{cm} + i\omega_m^3 W'_{dm}, \quad \dots \quad (14)$$

where $W'_{am}, W'_{bm}, W'_{cm}, W'_{dm}$ are the downwash values due to a lattice of rectangular vortices of strength

$$\begin{aligned} & sL'_a(k) T_m(\eta_j), \\ & sL'_b(k) f(|\eta_j|) T_m(\eta_j), \\ & sL'_c(k) f^2(|\eta_j|) T_m(\eta_j), \\ & sL'_d(k) f^3(|\eta_j|) T_m(\eta_j) \text{ respectively.} \end{aligned}$$

It should be noted that the factors L'_a, L'_b, \dots are applicable, provided

$$\omega'_1 \leq \omega_m f(|\eta_j|) \leq \omega'_2 \quad \dots \quad (15)$$

for all positions η_j across the wing span and that the downwash W'_{0m} can be calculated from (14) for any value ω_m satisfying this condition. The range (ω'_1, ω'_2) of the polynomial representation (12) could be extended by considering a polynomial of higher order.

†The co-ordinates (X^*, Y^*) give the position of collocation point relative to the rectangular vortex in terms of semi-width s_1 , where the positive X axis extends upstream.

2.2. *Calculation of the Downwash W''_{0m} .*—The downwash W''_{0m} at a collocation point (x_1, y_1) is calculated by using the lattice scheme of Ref. 2. The doublet distribution $K''_0 A_m$, as defined by (8), is replaced by narrow doublet strips of width $2s_1$ in the spanwise direction, which extend downstream from $x = x_1$ to $x = \infty$ and are of strength

$$K_0(\pi)[e^{-ip(x-x_1)/V} - 1]A_m(\eta_j).$$

The downwash W''_{0m} is then evaluated for a particular value of the mean frequency parameter ω_m by using the steady tables of Ref. 6, and the tables of Ref. 7† which give the downwash W_c at a point (t, n) due to an oscillating doublet strip of width $2s_1$ which extends from $X = 0$ to $X = \infty$ and is of strength $s_1 \exp(-ipX/V) = s_1 \exp(-i\bar{\omega}t)$. Hence

$$W''_{0m} = \frac{s}{4s_1} \sum_{\eta_j} \frac{K_0(\pi)}{\pi c} T_m(\eta_j) [4\pi W_c - F], \quad \dots \dots \dots \quad (16)$$

where $K_0(\pi)$ is defined by equation (7) and is a function of the local frequency parameter ω' .

For wings of general plan-form, there is a spanwise variation of $K_0(\pi)$ with ω' . If the polynomial representation suggested in section 2.1 is used to calculate W''_{0m} , then the downwash W''_{0m} may be calculated by using in equation (16)

$$K_0(\pi) = c \sum_{k=1}^k L'_0(k), \quad \dots \dots \dots \quad (17)$$

where $L'_0(k)$ and ω' are defined by (12) and (13) with the condition (15).

3. *Present Application of the Method: Results and Comparisons.*—In the present note, aerodynamic derivatives are calculated for three wings describing plunging and pitching oscillations for various values of the mean frequency parameter ω_m . The results are obtained from solutions in which :

(a) Distributions (1) to (3) are limited to two chordwise terms $n = 0$ and 1, and three spanwise terms corresponding to symmetrical motion $m = 1, 3$ and 5

(b) A 6-chordwise \times 21-spanwise lattice with the corrector vortices‡ is used for the downwash calculation as in Ref. 8 ; the 6-chordwise vortices are reduced to 2 when any strip η_j is at a distance $\geq 10s_1$ from the collocation point (x_1, y_1)

(c) The six collocation points are placed on the 1/2 and 5/6 chord-lines at spanwise positions $\eta = 0.2, 0.6, 0.8$.

The polynomial representation suggested in section 2.1 is used for the calculation of the downwash W''_{0m} . The factors $L_0(k)$ are represented by the polynomials of equation (12) for a frequency parameter range $0 \leq \omega' \leq 0.4$ and, for convenience, the factors $L'_a(k)$ and $L'_b(k)$ are given the same values as those used in the $\omega_m \rightarrow 0$ method of Ref. 1. The factors $L'_c(k)$ and $L'_d(k)$ are then determined by a least-squares method so that the polynomial representation of $L'_0(k)$ gives an accuracy to within 1 or 2 per cent of the true $L'_0(k)$ values for the range $0 \leq \omega' \leq 0.4$. The factors L'_a, L'_b, L'_c, L'_d are tabulated in Table 2 for the 6 chordwise lattice $k = 1, 2, \dots, 6$ and the reduced 2 chordwise lattice $k = 1, 2$. The values of the downwash W''_{nm} computed for the three wings are given in Tables 3(a) to 3(c).

† The tables are available for values of the parameter $\bar{\omega} = (s_1/2s)\omega_m A$ equal to 0.01 (0.01) 0.04 (0.02) 0.08, 0.09 0.12, 0.16, 0.18, 0.24 ; the co-ordinates (t, n) give the position of the collocation point relative to the doublet strip, in terms of the semi-width s_1 , where positive t axis extends downstream.

‡ The corrector vortices are neglected in the calculation of W''_{0m} .

3.1. *Local Derivative Coefficients.*—For the delta wings $A = 1.2$ and $A = 3$ and the arrowhead wing $A = 1.32$, values of the local derivative coefficients are obtained at spanwise positions $\eta = 0$ (0.2) 1.0 by use of the definitions given in the list of symbols. The local lift and moment for a wing of aspect ratio A are given by

$$\frac{\delta L}{\rho V^2 c_m} = \frac{\pi A}{2} \left[D_0 + \frac{i\omega'}{4} D_1 \right]$$

$$\frac{\delta M}{\rho V^2 c_m^2} = -\frac{\bar{m}c}{c_m} \left(\frac{\delta L}{\rho V^2 c_m} \right) + \frac{\pi A c}{16 c_m} \left[2D_0 + \left(1 + \frac{i\omega'}{4} \right) D_1 \right]$$

where $x = R(y) = \bar{m}c$ is the mid-chord line

$$\omega' = \omega_m c / 2c_m = \omega_m f(|\eta|)$$

$$D_n = \sum_m \eta^{m-1} C_{nm} \sqrt{1 - \eta^2}, \quad m = 1, 3, 5$$

and the coefficients C_{nm} are for a particular value of the frequency parameter ω_m . In the solution for $\omega_m \rightarrow 0$ only first-order terms in frequency are retained as in Ref. 1.

For each wing, values of the local derivative coefficients are tabulated for one axis position $x = hc_0$. Values for the delta wing $A = 3$ are given in Table 4 for the axis $hc_0 = 0.556c_0$ and mean frequency parameter values $\omega_m = 0, 0.26, 0.40$ and 0.53 . The values for the delta wing $A = 1.2$ with axis $0.556c_0$ and $\omega_m = 0, 0.33, 0.67$ and those for the arrowhead wing $A = 1.32$ with axis $0.738c_0$ and $\omega_m = 0, 0.30, 0.61$ are given in Tables 5 and 6 respectively.

3.2. *Delta Wing of Aspect Ratio 3, Taper Ratio 1/7.*—Derivatives are calculated for two axis positions, $hc_0 = 0$ and $0.556c_0$, for values of the mean frequency parameter $\omega_m = 0.26, 0.40$ and 0.53 . The results are tabulated in Table 7 and the pitching derivatives for the axis $0.556c_0$ are plotted in Figs. 1a and 1b. Values of stability and flutter derivatives previously obtained by vortex lattice methods^{1,3} are quoted in Table 4 and used in drawing the graphs in Fig. 1. The correlation between the results is satisfactory and indicates the relative accuracy of the present method and the methods of Refs. 1 and 2. From Fig. 1, which covers the frequency parameter range $0 \leq \omega_m \leq 0.8$, it can be seen that the first-order theory with wake correction of Ref. 1 gives a good idea of the rate of change of the derivatives with ω_m but becomes less accurate as ω_m increases to the higher values. The discrepancies between present results and those of Ref. 3 are less than 2 per cent and are probably due to differences between the present method and that of Ref. 2 and the assumptions made in the actual application of the method. Values of the pitching derivatives calculated by the Multhopp-Garner method⁹ for $\omega_m \rightarrow 0$ and the experimental values of $-m_a$ obtained at the National Physical Laboratory by Bratt for low frequencies are also shown in Fig. 1.

3.3. *Delta Wing of Aspect Ratio 1.2, Taper Ratio 1/7.*—Derivatives are calculated for three axis positions $hc_0 = 0, 0.431c_0$ and $0.556c_0$ for values of $\omega_m = 0.33$ and 0.67 , and the results are tabulated in Table 8. Pitching derivatives for the two axis positions $0.431c_0$ and $0.556c_0$ are plotted in Figs. 2a to 2d. Values of the stability derivatives for $\omega_m \rightarrow 0$, calculated by the vortex lattice method¹ and the Multhopp-Garner method⁹, are also shown in Fig. 2. Experimental values of the pitching derivatives have been obtained in low-speed tests at the N.P.L.¹⁰ for the axis positions $0.431c_0$ and $0.556c_0$ and these results are plotted in Figs. 2a to 2d. The values shown in Fig. 2 are for zero mean incidence and the tests show no amplitude effects. The derivatives were measured for frequency parameter values $\omega_m = 0.06$ to 0.60 and were approximately constant over this range.

3.4. *Arrowhead-Wing Aspect Ratio 1.32, Taper Ratio 7/18, $\frac{1}{4}$ -Chord Angle = 63.4 deg.*—Derivatives are calculated for three axis positions, $hc_0 = 0, 0.613c_0$ and $0.738c_0$ for values $\omega_m = 0.30$ and 0.61 , and the results are given in Table 9. The pitching derivatives for axis

positions $0.613c_0$ and $0.738c_0$ are graphed in Figs. 3a to 3d ; the values of the stability derivatives from Refs. 1 and 9 are also plotted in Fig. 3. Low-speed tests made on this wing at the N.P.L.¹⁰ give values of the pitching derivatives for these axis positions ; the values for zero mean incidence are plotted in Figs. 3a to 3d.

Concluding Remarks.—Comparison of the vortex lattice results and measured values of the derivatives indicates that the present method as applied in this note using a 21×6 lattice, gives reasonable accuracy for the calculation of flutter derivatives for low-aspect-ratio wings describing plunging and pitching oscillations in incompressible flow. It is noted that the results obtained in tests made at the Royal Aircraft Establishment for a delta wing $A = 3$ with body¹¹ at low values of the frequency parameter ω_m , are also in quite good agreement with the calculated values of the pitching derivatives. The method can be applied to wings oscillating in elastic modes, although it may then be necessary to use a finer lattice and more collocation points in the calculation.

Flutter derivatives for a wing in compressible flow may be calculated by applying the theory of Ref. 4 and using the present method to calculate the downwash values on the 'equivalent wing' in incompressible flow. If the original wing oscillates at a high frequency then it is apparent, from the relations given in the Introduction, that the frequency of oscillation of the equivalent wing could be very high. With this in view, the chordwise factors $L'_0(k)$ are tabulated in Table 1 of this note for very large values of the frequency parameter $\omega = 2\omega'$. It can be seen that these factors vary considerably when $\omega > 2$ and the use of polynomials in ω' to represent $L'_0(k)$, as suggested in section 2.1, then becomes impractical. Hence, if the equivalent wing is tapered and the equivalent frequency is high, the only practical way to allow for the spanwise variation of these factors is to use the set of values $L'_0(k)$, at each strip η_j of the lattice, which corresponds to the local frequency parameter value ω_j .

The downwash distribution on the equivalent wing has to satisfy a complicated tangential flow condition and it is suggested in Ref. 4 that if this condition is represented to first-order accuracy in frequency, then the simplified calculation should give results of reasonable accuracy for all practical purposes. The values of flutter derivatives for a finite wing would probably be sufficiently accurate for a Mach number up to about 0.75, but it is doubtful if the approximation would be good enough when high Mach number and high frequency are considered simultaneously. It is hoped that some information on this point will be obtained from calculations which are now in progress for rectangular wings.

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APPENDIX

Calculation of the Chordwise Factors $L'_0(k)$

The chordwise doublet distribution K'_0 is replaced by k' vortices of strength $cL'_0(k)$, $k = 1, 2 \dots k'$, placed at positions $(2k - 1)c/2k'$, $k = 1, 2 \dots k'$, from leading edge of the chord. The factors $L'_0(k)$ are chosen so that the downwash value at each position kc/k' , $k = 1, 2 \dots (k' - 1)$, as calculated by two-dimensional theory to correspond to the k' vortices, is equal to the downwash W'_0 corresponding to the continuous 2-dimensional distribution K'_0 . These conditions give $(k' - 1)$ equations and the condition

$$\sum_{k=1}^{k'} c L'_0(k) = K'_0(\pi)$$

provides the k th equation needed to determine the factors.

The downwash W'_0 is evaluated, for any position $x_1 = c(1 + \xi_1)/2$ from leading edge of the chord, as follows. It is known that

$$2\pi W'_0 = - \int_0^\infty \frac{1}{x - x_1} \frac{\partial K'_0}{\partial x} dx,$$

therefore

$$2\pi W'_0 = - \int_0^\infty \frac{1}{x - x_1} \frac{\partial K_0}{\partial x} dx + \int_c^\infty \frac{1}{x - x_1} \frac{\partial K''_0}{\partial x} dx \quad \text{by (8)}$$

$$\equiv 2\pi(W_0 - W''_0) \text{ say (18)}$$

The downwash W_0 corresponding to a bound vorticity distribution $\Gamma_0 = 2 \cot \frac{1}{2}\theta$, and therefore to the doublet distribution K_0 is given in Ref. 5 (R. & M. 2117, Appendix I), as

$$W_0 = - \frac{1}{2}\pi(i\omega' e^{-i\omega'\xi_1}[H_1^{(2)}(\omega') + iH_0^{(2)}(\omega')]), \quad \text{. (19)}$$

where $\omega' = pc/2V$ and $H_0^{(2)}$, $H_1^{(2)}$ are Hankel functions. In terms of Bessel functions of first and second kind

$$H_n^{(2)}(\omega') = J_n(\omega') - iY_n(\omega'). \quad \text{. (20)}$$

The downwash value W''_0 , corresponding to the doublet distribution K''_0 over the wake, is deduced from equation (18) by use of equations (7) and (8).

$$\begin{aligned}
 W''_0 &= - \int_1^\infty \frac{K_0(\pi)}{\pi c(\xi - \xi_1)} \frac{\partial}{\partial \xi} [e^{-i\omega'(\xi-1)} - 1] d\xi \\
 &= \frac{K_0(\pi)}{c\pi} \left\{ i\omega' e^{i\omega'} \int_1^\infty \frac{e^{-i\omega'\xi}}{\xi - \xi_1} d\xi \right\} \\
 &= i\omega' e^{-i\omega'\xi_1} [J_0(\omega') - iJ_1(\omega')] \int_u^\infty \frac{e^{-it}}{t} dt, \quad \dots \dots \dots (21)
 \end{aligned}$$

where $\alpha = \omega'(1 - \xi_1)$.

Since $-1 < \xi_1 < 1$, α is greater than 0 and the integral in (21) can be written as

$$\int_u^\infty \frac{e^{-it}}{t} dt = -\text{Ci}(\alpha) - i \left\{ \frac{1}{2}\pi - \text{Si}(\alpha) \right\}, \quad \dots \dots \dots (22)$$

where $\text{Ci}(\alpha)$ is the cosine integral $-\int_u^\infty \frac{\cos t}{t} dt$,

$\text{Si}(\alpha)$ is the sine integral $\int_0^\alpha \frac{\sin t}{t} dt$.

It follows from equations (18) to (22) that the downwash W''_0 at a point ξ_1 is

$$\begin{aligned}
 W''_0 &= -i\omega' e^{-i\omega'\xi_1} [\{Y_0(\omega') - iY_1(\omega')\} \\
 &\quad + \{J_0(\omega') - iJ_1(\omega')\} \{-\text{Ci}(\alpha) + i\text{Si}(\alpha)\}] \quad \dots \dots \dots (23)
 \end{aligned}$$

where $\alpha = \omega'(1 - \xi_1)$.

The value $W''_0(\xi_1)$ is computed for values of the frequency parameter ω' by using tables† of the Bessel functions J_0 , J_1 , Y_0 , Y_1 and of the sine and cosine integrals $\text{Si}(\alpha)$ and $\text{Ci}(\alpha)$.

Values of $L'_0(k)/\pi$, $k = 1, 2 \dots 6$, for six vortices positioned at $1/12, 3/12 \dots 11/12$ chord are tabulated in Table 1(a) for values of the frequency parameter $\omega = 2\omega' = 0, 2, 4$ and in Table 1(b) for values $\omega = 2, 4, 6$. Values of the modified second differences δ^2 are tabulated for use with the Everett interpolation formula $f(n) = (1-n)f_0 + nf_1 + E''_0(n)\delta_0^2 + E''_1(n)\delta_1^2$ where n is the fraction of the interval of tabulation; δ_0^2 and δ_1^2 are modified second differences. The Everett coefficients :

$$E''_0 = - \frac{n(n-1)(n-2)}{3!}$$

$$E''_1 = \frac{(n+1)n(n-1)}{3!}$$

are tabulated in Tables XXI and XXVI of 'Interpolation and Allied Tables' published by H.M.S.O.

Values of $L'_0(k)/\pi$, $k = 1, 2$ for two vortices positioned at $\frac{1}{4}$ and $\frac{3}{4}$ chord, are also tabulated with modified second differences in Table 1(c). These factors are used for the reduced lattice as indicated in section 3(b).

† British Association Mathematical Tables, Vol. VI, Tables of Bessel Functions. Bureau of Standards: Tables of Sine, Cosine and Exponential Integrals.

TABLE 1(a)

Factors $L'_0(k) = 1, 2, \dots, 6$, for Six Chordwise Vortices

Positioned at $1/12, 3/12, \dots, 11/12$ chord

Values of $L'_0(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 1$				$k = 2$				$k = 3$			
	Re	δ^2	Im	δ^2	Re	δ^2	Im	δ^2	Re	δ^2	Im	δ^2
0	0.45117	-30	0	0	0.20508	-140	0	0	0.13672	-295	0	0
0.2	0.45102	-30	-0.01143	2	0.20438	-140	-0.02023	7	0.13525	-291	-0.02532	31
0.4	0.45057	-29	-0.02284	4	0.20228	-138	-0.04038	20	0.13087	-289	-0.05033	60
0.6	0.44983	-29	-0.03421	3	0.19880	-138	-0.06034	25	0.12361	-279	-0.07474	87
0.8	0.44880	-28	-0.04555	6	0.19394	-134	-0.08004	37	0.11356	-271	-0.09827	120
1.0	0.44749	-27	-0.05683	7	0.18774	-132	-0.09938	42	0.10081	-256	-0.12061	144
1.2	0.44591	-26	-0.06804	8	0.18022	-129	-0.11829	54	0.08550	-242	-0.14154	170
1.4	0.44407	-25	-0.07917	7	0.17141	-125	-0.13667	57	0.06778	-224	-0.16071	194
1.6	0.44198	-23	-0.09023	8	0.16135	-121	-0.15447	67	0.04783	-202	-0.17797	219
1.8	0.43966	-22	-0.10121	8	0.15008	-117	-0.17160	74	0.02586	-180	-0.19305	235
2.0	0.43712	-22	-0.11211	8	0.13764	-110	-0.18799	81	0.00209	-156	-0.20578	255

δ^2 are modified second differences (see Appendix)

TABLE 1(a)—continued

Values of $L'_0(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 4$				$k = 5$				$k = 6$			
	Re	δ^2	Im	δ^2	Re	δ^2	Im	δ^2	Re	δ^2	Im	δ^2
0	+0.09766	-478	0	0	+0.06836	-679	0	0	+0.04102	-889	0	0
0.2	0.09528	-471	-0.02884	68	0.06498	-672	-0.03128	126	0.03662	-863	-0.03217	200
0.4	0.08820	-458	-0.05700	135	0.05493	-637	-0.06131	247	0.02366	-800	-0.06235	394
0.6	0.07656	-434	-0.08381	201	0.03855	-582	-0.08888	363	+0.00277	-696	-0.08862	570
0.8	0.06059	-404	-0.10862	259	+0.01638	-511	-0.11284	468	-0.02502	-558	-0.10924	719
1.0	0.04059	-363	-0.13084	319	-0.01087	-421	-0.13215	556	-0.05833	-386	-0.12273	837
1.2	+0.01697	-317	-0.14989	367	-0.04230	-313	-0.14593	631	-0.09547	-198	-0.12792	914
1.4	-0.00981	-264	-0.16528	413	-0.07685	-200	-0.15344	685	-0.13456	+11	-0.12404	956
1.6	-0.03922	-205	-0.17656	446	-0.11338	-74	-0.15414	718	-0.17354	219	-0.11069	947
1.8	-0.07067	-139	-0.18339	476	-0.15064	+57	-0.14770	732	-0.21033	429	-0.08795	898
2.0	-0.10351	-76	-0.18548	496	-0.18733	+188	-0.13399	719	-0.24286	+622	-0.05631	805

δ^2 are modified second differences (see Appendix)

TABLE 1(b)

*Factors $L'_0(k) = 1, 2, \dots, 6$, for Six Chordwise Vortices
 Positioned at $1/12, 3/12, \dots, 11/12$ chord
 Values of $L'_0(k)/\pi$*

$\omega = \frac{\rho c \cdot V}{V}$	$k = 1$				$k = 2$				$k = 3$			
	Rl	δ^2	Im	δ^2	Rl	δ^2	Im	δ^2	Rl	δ^2	Im	δ^2
2.0	0.43712	-87	-0.11211	32	+0.13764	-444	-0.18799	324	+0.00209	-628	-0.20578	1019
2.4	0.43139	-82	-0.13368	25	0.10949	-406	-0.21829	368	-0.04988	-416	-0.22346	1134
2.8	0.42483	-81	-0.15500	20	0.07729	-359	-0.24491	413	-0.10598	-189	-0.22988	1204
3.2	0.41745	-85	-0.17612	16	0.04150	-312	-0.26741	449	-0.16395	+50	-0.22434	1232
3.6	0.40921	-98	-0.19708	13	+0.00259	-264	-0.28542	486	-0.22142	287	-0.20656	1213
4.0	0.40000	-104	-0.21790	16	-0.03895	-212	-0.29858	515	-0.27603	519	-0.17672	1157
4.4	0.38974	-117	-0.23855	28	-0.08260	-157	-0.30659	545	-0.32547	739	-0.13539	1054
4.8	0.37832	-124	-0.25892	39	-0.12781	-94	-0.30916	566	-0.36756	937	-0.08358	919
5.2	0.36567	-126	-0.27889	59	-0.17396	-31	-0.30607	589	-0.40032	1115	-0.02265	744
5.6	0.35177	-120	-0.29827	77	-0.22041	+39	-0.29711	599	-0.42199	1258	+0.04566	539
6.0	0.33668	-112	-0.31688	96	-0.26646	+117	-0.28217	603	-0.43114	+1369	+0.11931	309

δ^2 are modified second differences (see Appendix)

TABLE 1(b)—continued

Values of $L'_0(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 4$				$k = 5$				$k = 6$			
	Rl	δ^2	Im	δ^2	Rl	δ^2	Im	δ^2	Rl	δ^2	Im	δ^2
2.0	-0.10351	-300	-0.18548	+1992	-0.18733	+753	-0.13399	+2904	-0.24286	+2513	-0.05631	+3262
2.4	-0.17073	+262	-0.17475	2036	-0.25383	1759	-0.08544	2554	-0.28766	3801	+0.02954	2037
2.8	-0.23534	818	-0.14393	1930	-0.30302	2611	-0.01195	1867	-0.29549	4507	0.13495	+362
3.2	-0.29184	1335	-0.09407	1682	-0.32658	3195	+0.07975	+928	-0.25954	4493	0.24362	-1520
3.6	-0.33513	1775	-0.02763	1302	-0.31879	3448	0.18043	-181	-0.18002	3716	0.33729	-3292
4.0	-0.36087	2102	+0.05164	819	-0.27720	3221	0.27922	-1334	-0.06454	2271	0.39876	-4666
4.4	-0.36582	2305	0.13896	+256	-0.20308	2817	0.36481	-2410	+0.07279	+343	0.41472	-5398
4.8	-0.34799	2356	0.22877	-343	-0.10139	1980	0.42666	-3287	0.21321	-1775	0.37812	-5337
5.2	-0.30688	2255	0.31514	-950	+0.01963	881	0.45618	-3869	0.33610	-3773	0.28963	-4448
5.6	-0.24350	2001	0.39208	-1519	0.14918	+375	0.44769	-4075	0.42205	-5330	+0.15797	-2829
6.0	-0.16036	+1611	+0.45396	-2018	0.27493	-1660	+0.39918	-3880	+0.45595	-6190	-0.00104	-684

δ^2 are modified second differences (see Appendix)

TABLE 1(c)

*Factors $L'_0(k)$, $k = 1, 2$, for Two Chordwise Vortices
 Positioned at $\frac{1}{4}$ and $\frac{3}{4}$ Chord*

Values of $L'_0(k)/\pi$

$\omega = \frac{\rho c}{V}$	$k = 1$				$k = 2$			
	Re	δ^2	Im	δ^2	Re	δ^2	Im	δ^2
0	+0.75000	-454	0	0	+0.25000	-2055	0	0
0.2	0.74774	-447	-0.05760	42	0.23979	-2019	-0.09167	+393
0.4	0.74101	-442	-0.11478	85	0.20952	-1913	-0.17943	775
0.6	0.72988	-425	-0.17112	124	0.16025	-1735	-0.25949	1130
0.8	0.71451	-405	-0.22623	156	0.09374	-1501	-0.32832	1448
1.0	0.69510	-382	-0.27978	191	+0.01233	-1206	-0.38276	1716
1.2	0.67188	-353	-0.33144	214	-0.08106	-870	-0.42014	1929
1.4	0.64513	-328	-0.38097	235	-0.18309	-501	-0.43835	2078
1.6	0.61511	-295	-0.42816	249	-0.29009	-109	-0.43591	2159
1.8	0.58213	-269	-0.47287	257	-0.39816	+293	-0.41202	2162
2.0	0.54646	-244	-0.51502	262	-0.50331	689	-0.36663	2102
2.0	0.54646	-969	-0.51502	1049	-0.50331	2774	-0.36663	8478
2.4	0.46803	-801	-0.59151	1031	-0.68927	5724	-0.21458	7128
2.8	0.38144	-721	-0.65775	977	-0.81915	8026	+0.00703	4817
3.2	0.28750	-719	-0.71419	940	-0.87045	9392	0.27562	+1848
3.6	0.18624	-796	-0.76112	964	-0.82980	9663	0.56215	-1422
4.0	+0.07696	-909	-0.79823	1090	-0.69455	8806	0.83465	-4583
4.4	-0.04138	-1007	-0.82422	1339	-0.47306	6938	1.06218	-7264
4.8	-0.16965	-1029	-0.83660	1712	-0.18357	4309	1.21849	-9153
5.2	-0.30796	-911	-0.83169	2180	+0.14820	+1232	1.28504	-10058
5.6	-0.45505	-610	-0.80490	2695	0.49215	-1917	1.25292	-9901
6.0	-0.60784	-83	-0.75122	3176	+0.81744	-4782	+1.12358	-8751

δ^2 are modified second differences (see Appendix)

TABLE 2

*Approximate Representation of the Chordwise Factors
 $L'_0(k)$ for the Frequency Parameter Range $0 \leq \omega' \leq 0.4$;*

$$L'_0(k) \equiv L'_a + i\omega' L'_b + \omega'^2 L'_c + i\omega'^3 L'_d$$

Factors for 6 Vortices at 1/12, 3/12 . . . 11/12 Chord

k	L'_a	L'_b	L'_c	L'_d
1	0.4512π	-0.1143π	-0.0148π	0.0026π
2	0.2051π	-0.2025π	-0.0696π	0.0149π
3	0.1367π	-0.2537π	-0.1448π	0.0499π
4	0.0976π	-0.2895π	-0.2318π	0.1123π
5	0.0684π	-0.3149π	-0.3252π	0.2051π
6	0.0410π	-0.3251π	-0.4133π	0.3244π

Factors for 2 Vortices at $\frac{1}{4}$ and $\frac{3}{4}$ Chord

k	L'_a	L'_b	L'_c	L'_d
1	0.7500π	-0.5767π	-0.2219π	0.0696π
2	0.2500π	-0.9233π	-0.9776π	0.6396π

TABLE 3

Values of the Downwash* W_{nm} at Collocation Points (ξ, η) for Frequency Parameter Values ω_m

TABLE 3(a)

Delta Wing $A = 3$ (taper ratio 1/7)

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$\omega_m = \frac{\rho c_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0.26	(0, 0.2)	2.545252 - i0.082822	-0.181656 + i0.031994	-0.109239 + i0.020431	+0.645743	-0.025716	-0.013964
	(0, 0.6)	2.542664 + i0.295211	+1.393250 - i0.040106	+0.372680 - i0.015113	0.869258	+0.324655	+0.082044
	(0, 0.8)	2.453213 + i0.407252	+2.636882 + i0.023664	+1.893298 - i0.029642	+1.003016	+0.712047	+0.456151
	(0.6, 0.2)	2.388822 - i0.385899	-0.366899 + i0.071600	-0.175960 + i0.040352	-0.126059	-0.033161	-0.009490
	(0.6, 0.6)	2.625207 + i0.095922	+1.291855 - i0.142992	+0.210472 - i0.036674	-0.180251	-0.066988	-0.044739
	(0.6, 0.8)	2.695214 + i0.280461	+2.767484 - i0.111123	+1.896697 - i0.123987	-0.205678	-0.099858	-0.068765
0.40	(0, 0.2)	2.621509 - i0.163755	-0.171779 + i0.041998	-0.104626 + i0.027997	+0.645743	-0.025716	-0.013964
	(0, 0.6)	2.669031 + i0.391140	+1.409604 - i0.069545	+0.378071 - i0.026546	0.869258	+0.324655	+0.082044
	(0, 0.8)	2.573240 + i0.563594	+2.663459 + i0.026227	+1.903663 - i0.048366	+1.003016	+0.712047	+0.456151
	(0.6, 0.2)	2.419571 - i0.634190	-0.350550 + i0.098306	-0.167116 + i0.056366	-0.126059	-0.033161	-0.009490
	(0.6, 0.6)	2.763677 + i0.077810	+1.299078 - i0.225010	+0.213229 - i0.059278	-0.180251	-0.066988	-0.044739
	(0.6, 0.8)	2.831878 + i0.362419	+2.790724 - i0.178361	+1.902114 - i0.190797	-0.205678	-0.099858	-0.068765

*Values computed using a 21 x 6 lattice (see section 3).

TABLE 3(a)—continued
 Delta Wing $A = 3$ (taper ratio 1/7)

$\omega_m = \frac{\rho c_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0.53	(0, 0.2)	2.709425 - i0.276618	-0.161046 + i0.048922	-0.099448 + i0.034464	+0.645743	-0.025716	-0.013964
	(0, 0.6)	2.823964 + i0.455474	+1.428525 - i0.103370	+0.384006 - i0.039381	0.869258	+0.324655	+0.082044
	(0, 0.8)	2.717751 + i0.689617	+2.696466 + i0.022756	+1.916463 - i0.069598	+1.003016	+0.712047	+0.456151
	(0.6, 0.2)	2.433302 - i0.919008	-0.332643 + i0.120171	-0.156829 + i0.070250	-0.126059	-0.033161	-0.009490
	(0.6, 0.6)	2.928592 + i0.014488	+1.303425 - i0.312299	+0.214633 - i0.083504	-0.180251	-0.066988	-0.044739
	(0.6, 0.8)	2.996106 + i0.407105	+2.817651 - i0.252979	+1.907173 - i0.260532	-0.205678	-0.099858	-0.068765

*Values computed using a 21 × 6 lattice (see section)

TABLE 3(b)

Delta Wing $A = 1.2$ (taper ratio 1/7)

$\omega_m = \frac{\rho c_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0.3	(0, 0.2)	2.112126 - i0.314994	-0.190966 + i0.009856	-0.095695 + i0.012854	+0.376591	-0.049566	-0.018831
	(0, 0.6)	1.949935 + i0.174801	+1.230396 - i0.115602	+0.322158 - i0.050133	0.527714	+0.188470	+0.020035
	(0, 0.8)	1.552378 + i0.372245	+2.167702 - i0.035688	+1.603480 - i0.077761	+0.647579	+0.475148	+0.290190
	(0.6, 0.2)	1.798222 - i0.592607	-0.443860 + i0.066115	-0.198334 + i0.039002	-0.035192	-0.035352	-0.013248
	(0.6, 0.6)	1.916705 + i0.003309	+1.030227 - i0.215753	+0.085418 - i0.063831	-0.034863	-0.023507	-0.032319
	(0.6, 0.8)	1.900147 + i0.243745	+2.336401 - i0.185945	+1.616846 - i0.182231	-0.024056	+0.004907	-0.005391
0.6	(0, 0.2)	2.101135 - i0.663741	-0.189025 + i0.016308	-0.091486 + i0.023826	+0.376591	-0.049566	-0.018831
	(0, 0.6)	2.174305 + i0.266866	+1.231967 - i0.239939	+0.317546 - i0.102952	0.527714	+0.188470	+0.020035
	(0, 0.8)	1.797448 + i0.663994	+2.204600 - i0.084402	+1.610081 - i0.160178	+0.647579	+0.475148	+0.290190
	(0.6, 0.2)	1.579886 - i1.193350	-0.429191 + i0.126825	-0.183584 + i0.073309	-0.035192	-0.035352	-0.013248
	(0.6, 0.6)	2.152709 - i0.117903	+0.983320 - i0.435760	+0.064299 - i0.127723	-0.034863	-0.023507	-0.032319
	(0.6, 0.8)	2.181827 + i0.378005	+2.348403 - i0.386502	+1.597716 - i0.367748	-0.024056	+0.004907	-0.005391

TABLE 3(c)

Arrowhead Wing A = 1.32

$\omega_m = \frac{pc_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0.303	(0, 0.2)	2.482843 - i0.202426	-0.093039 - i0.003335	-0.070750 + i0.008337	+0.508700	-0.046908	-0.016548
	(0, 0.6)	1.795205 + i0.231324	+1.249513 - i0.102610	+0.349294 - i0.046877	0.681309	+0.207564	+0.019803
	(0, 0.8)	1.251431 + i0.364138	+2.004976 - i0.060076	+1.494930 - i0.095898	+0.697997	+0.470449	+0.274126
	(0.6, 0.2)	2.155778 - i0.492366	-0.343441 + i0.034328	-0.156266 + i0.025548	-0.142266	-0.059275	-0.017160
	(0.6, 0.6)	1.972589 + i0.031702	+1.119696 - i0.215699	+0.146579 - i0.068215	-0.071080	-0.085286	-0.074402
	(0.6, 0.8)	1.777549 + i0.236414	+2.306562 - i0.229443	+1.617099 - i0.216950	+0.042384	-0.006498	-0.023228
0.606	(0, 0.2)	2.539712 - i0.448094	-0.092402 - i0.010293	-0.067778 + i0.014866	0.508700	-0.046908	-0.016548
	(0, 0.6)	2.024731 + i0.386421	+1.254423 - i0.213611	+0.345821 - i0.096258	0.681309	+0.207564	+0.019803
	(0, 0.8)	1.497862 + i0.642022	+2.037023 - i0.133855	+1.496510 - i0.196248	+0.697997	+0.470449	+0.274126
	(0.6, 0.2)	2.059323 - i1.019561	-0.338042 + i0.064539	-0.147713 + i0.048060	-0.142266	-0.059275	-0.017160
	(0.6, 0.6)	2.210724 - i0.055103	+1.077825 - i0.436377	+0.126270 - i0.136590	-0.071080	-0.085286	-0.074402
	(0.6, 0.8)	2.073506 + i0.348347	+2.304622 - i0.474958	+1.583621 - i0.436332	+0.042384	+0.006498	-0.023228

TABLE 4

*Local Derivative Coefficients referred to an Axis Position $0.556c_o$
 for the Delta Wing $A = 3$ describing Plunging and Pitching Oscillations*

ω_m	η	$l_z(\eta)$	$l_z(\eta)$	$l_a(\eta)$	$l_a(\eta)$	$-m_z(\eta)$	$-m_z(\eta)$	$-m_a(\eta)$	$-m_a(\eta)$
$\rightarrow 0$	0	0	1.9972	1.9972	1.5864	0	-1.0021	-1.0021	0.3997
	0.2	0	1.9489	1.9489	1.4744	0	-0.5602	-0.5602	0.4820
	0.4	0	1.8028	1.8028	1.1529	0	-0.1409	-0.1409	0.4474
	0.6	0	1.5498	1.5498	0.7394	0	+0.2067	+0.2067	0.3304
	0.8	0	1.1460	1.1460	0.3891	0	+0.4045	+0.4045	0.2068
	1.0	0	0	0	0	0	0	0	0
0.26	0	-0.0520	1.9466	1.9695	1.7133	-0.0109	-0.9763	-0.9971	0.3353
	0.2	-0.0450	1.8929	1.9068	1.5969	-0.0155	-0.5443	-0.5568	0.4461
	0.4	-0.0255	1.7457	1.7534	1.2719	-0.0155	-0.1364	-0.1432	0.4383
	0.6	-0.0023	1.4972	1.5035	0.8509	-0.0083	+0.2000	+0.1973	0.3460
	0.8	+0.0112	1.1034	1.1108	0.4769	+0.0014	+0.3894	+0.3907	0.2377
	1.0	0	0	0	0	0	0	0	0
0.4	0	-0.1282	1.9132	1.9587	1.7650	-0.0207	-0.9579	-1.0028	0.3219
	0.2	-0.1123	1.8516	1.8771	1.6439	-0.0329	-0.5322	-0.5593	0.4405
	0.4	-0.0684	1.7011	1.7124	1.3179	-0.0346	-0.1330	-0.1482	0.4376
	0.6	-0.0160	1.4554	1.4639	0.8961	-0.0202	+0.1944	+0.1877	0.3523
	0.8	+0.0166	1.0696	1.0815	0.5131	+0.0001	+0.3774	+0.3787	0.2507
	1.0	0	0	0	0	0	0	0	0
0.53	0	-0.2432	1.8813	1.9540	1.8020	-0.0307	-0.9392	-1.0157	0.3103
	0.2	-0.2147	1.8087	1.8461	1.6747	-0.0551	-0.5192	-0.5656	0.4355
	0.2	-0.1371	1.6537	1.6655	1.3481	-0.0605	-0.1294	-0.1559	0.4364
	0.6	-0.0441	1.4114	1.4184	0.9281	-0.0380	+0.1885	+0.1754	0.3564
	0.8	+0.0167	1.0341	1.0486	0.5400	-0.0044	+0.3648	+0.3647	0.2600
	1.0	0	0	0	0	0	0	0	0

TABLE 5

*Local Derivative Coefficients referred to an Axis Position $0.556c_0$
 for the Delta Wing $A = 1.2$ describing Plunging and Pitching Oscillations*

ω_m	η	$l_z(\eta)$	$l_z(\eta)$	$l_a(\eta)$	$l_a(\eta)$	$-m_z(\eta)$	$-m_z(\eta)$	$-m_a(\eta)$	$-m_a(\eta)$
$\rightarrow 0$	0	0	1.0336	1.0336	1.1638	0	-0.4249	-0.4249	0.2748
	0.2	0	1.0139	1.0139	1.1183	0	-0.2190	-0.2190	0.3356
	0.4	0	0.9511	0.9511	0.9511	0	-0.0309	-0.0309	0.3258
	0.6	0	0.8323	0.8323	0.7078	0	0.1229	0.1229	0.2741
	0.8	0	0.6237	0.6237	0.4425	0	0.2098	0.2098	0.2082
	1.0	0	0	0	0	0	0	0	0
0.33	0	-0.0662	1.0321	1.0472	1.1792	-0.0044	-0.4254	-0.4444	0.2705
	0.2	-0.0620	1.0057	1.0104	1.1292	-0.0113	-0.2188	-0.2302	0.3342
	0.4	-0.0466	0.9396	0.9364	0.9610	-0.0146	-0.0322	-0.0392	0.3257
	0.6	-0.0261	0.8211	0.8159	0.7185	-0.0123	0.1203	0.1155	0.2756
	0.8	-0.0089	0.6139	0.6112	0.4510	-0.0065	0.2063	0.2036	0.2108
	1.0	0	0	0	0	0	0	0	0
0.67	0	-0.2689	1.0403	1.0986	1.1927	-0.0197	-0.4288	-0.5061	0.2739
	0.2	-0.2509	0.9934	1.0102	1.1287	-0.0470	-0.2177	-0.2647	0.3393
	0.4	-0.1902	0.9158	0.9011	0.9562	-0.0590	-0.0334	-0.0618	0.3269
	0.6	-0.1098	0.7969	0.7728	0.7171	-0.0501	0.1155	0.0959	0.2746
	0.8	-0.0407	0.5923	0.5785	0.4507	-0.0276	0.1987	0.1868	0.2102
	1.0	0	0	0	0	0	0	0	0

TABLE 6

*Local Derivative Coefficients referred to an Axis Position 0.738c₀
 for the Arrowhead Wing A = 1.32 describing Plunging and Pitching Oscillations*

ω_m	η	$l_z(\eta)$	$l_z(\eta)$	$l_a(\eta)$	$l_a(\eta)$	$-m_z(\eta)$	$-m_z(\eta)$	$-m_a(\eta)$	$-m_a(\eta)$
→ 0	0	0	1.0064	1.0064	0.7600	0	-0.6059	-0.6059	0.0089
	0.2	0	0.9977	0.9977	0.7556	0	-0.3631	-0.3631	0.1109
	0.4	0	0.9628	0.9628	0.7112	0	-0.1385	-0.1385	0.1751
	0.6	0	0.8743	0.8743	0.6189	0	+0.0626	+0.0626	0.2187
	0.8	0	0.6771	0.6771	0.4615	0	+0.1980	+0.1980	0.2335
	1.0	0	0	0	0	0	0	0	0
0.30	0	-0.0415	0.9998	1.0130	0.7740	0.0080	-0.6012	-0.6124	0.0024
	0.2	-0.0394	0.9885	0.9963	0.7689	-0.0002	-0.3593	-0.3653	0.1071
	0.4	-0.0312	0.9519	0.9526	0.7248	-0.0068	-0.1366	-0.1400	0.1730
	0.6	-0.0195	0.8630	0.8586	0.6324	-0.0092	+0.0620	+0.0583	0.2190
	0.8	-0.0082	0.6666	0.6620	0.4726	-0.0072	+0.1950	+0.1911	0.2366
	1.0	0	0	0	0	0	0	0	0
0.61	0	-0.1700	0.9901	1.0413	0.7804	0.0334	-0.5938	-0.6379	0.0027
	0.2	-0.1615	0.9705	1.0000	0.7734	0	-0.3519	-0.3756	0.1085
	0.4	-0.1301	0.9280	0.9289	0.7310	-0.0268	-0.1330	-0.1464	0.1736
	0.6	-0.0847	0.8375	0.8173	0.6416	-0.0375	+0.0601	+0.0453	0.2200
	0.8	-0.0398	0.6432	0.6220	0.4827	-0.0308	+0.1881	+0.1718	0.2391
	1.0	0	0	0	0	0	0	0	0

TABLE 7

*Derivatives of the Delta Wing $A = 3$
 for Plunging and Pitching Oscillations*

hc_0	ω_m	l_z	l_z'	l_α	l_α'	$-m_z$	$-\dot{m}_z$	$-m_\alpha$	$-m_\alpha'$	Remarks
0	$\rightarrow 0$	0	1.539	1.539	2.423	0	+1.414	+1.414	2.623	Ref. 1
	0.13	-0.003	1.533	1.534	2.462	-0.005	1.407	1.404	2.654	
	0.26	-0.015	1.521	1.516	2.501	-0.023	1.389	1.371	2.685	
	Present method	0.26	-0.017	1.490	1.483	2.480	-0.025	1.367	1.346	2.675
		0.40	-0.048	1.452	1.422	2.485	-0.066	1.331	1.269	2.682
		0.53	-0.099	1.413	1.339	2.475	-0.131	1.293	1.163	2.674
		0.80	-0.316	1.346	1.039	2.520	-0.375	+1.234	+0.821	2.712
0.556c ₀	$\rightarrow 0$	0	1.539	1.539	0.926	0	-0.083	-0.083	0.346	Ref. 1
	0.13	-0.003	1.533	1.537	0.970	-0.002	-0.085	-0.087	0.341	
	0.26	-0.015	1.521	1.531	1.021	-0.009	-0.090	-0.096	0.340	
	Present method	0.26	-0.017	1.490	1.499	1.030	-0.008	-0.082	-0.088	0.342
		0.40	-0.048	1.452	1.469	1.072	-0.019	-0.082	-0.096	0.345
		0.53	-0.099	1.413	1.435	1.100	-0.035	-0.082	-0.106	0.346
		0.80	-0.316	1.346	1.347	1.211	-0.068	-0.076	-0.125	0.333

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*Solutions with factors L'_0 and $C(\omega')$ variable with ω across wing span.

TABLE 8

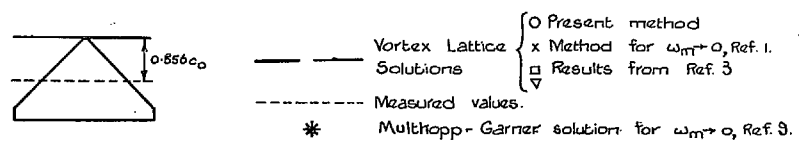
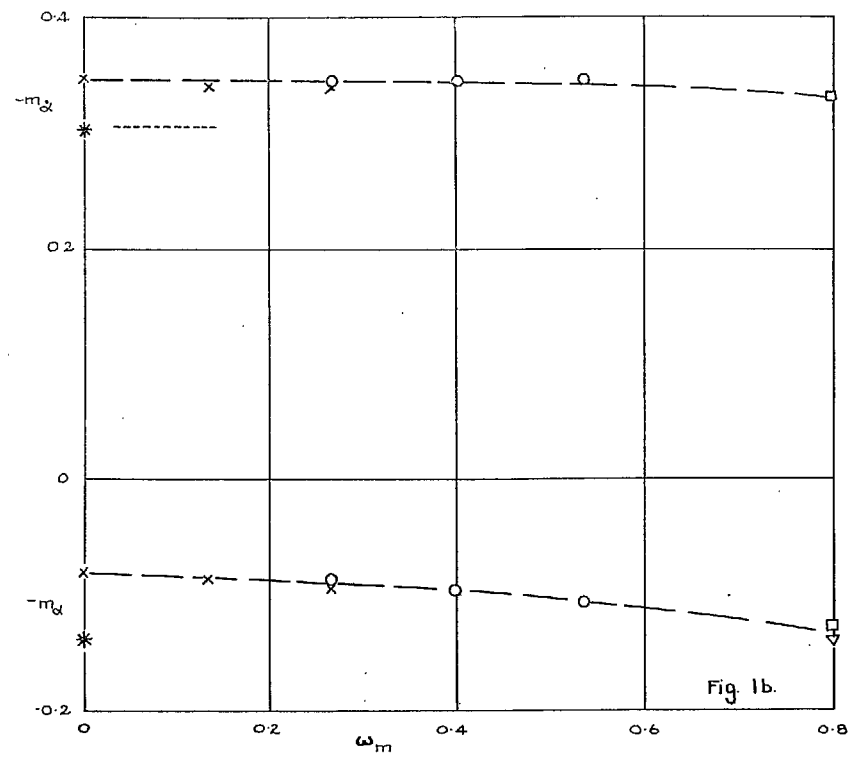
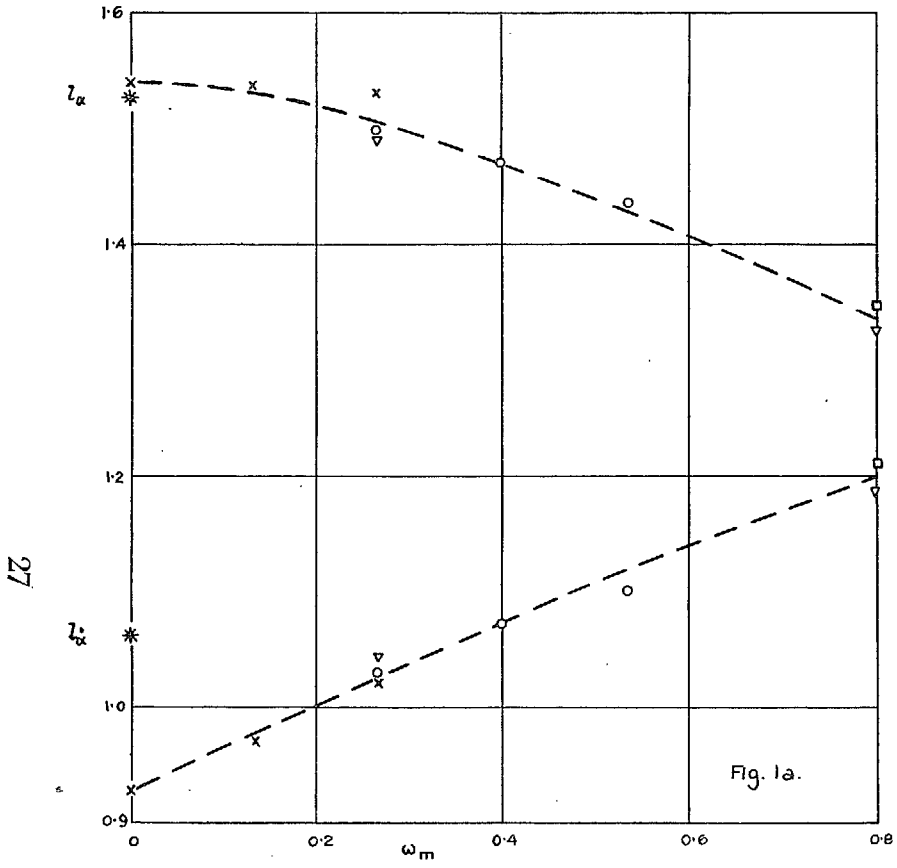
*Derivatives of the Delta Wing $A = 1.2$
 for Plunging and Pitching Oscillations*

hc_0	ω_m	l_z	l_z	l_a	l_a	$-m_z$	$-m_z$	$-m_a$	$-m_a$	Remarks
0	$\rightarrow 0$	0	0.815	0.815	1.571	0	0.784	0.784	1.789	Ref. 1
	0.33	-0.036	0.805	0.771	1.571	-0.044	0.774	0.724	1.788	Present method
	0.67	-0.146	0.788	0.644	1.554	-0.182	0.753	0.545	1.768	Present method
0.431 c_0	$\rightarrow 0$	0	0.815	0.815	0.956	0	0.170	0.170	0.476	Ref. 1
	0.33	-0.036	0.805	0.798	0.964	-0.017	0.166	0.155	0.477	Present method
	0.67	-0.146	0.788	0.754	0.959	-0.072	0.159	0.114	0.477	Present method
0.556 c_0	$\rightarrow 0$	0	0.815	0.815	0.778	0	-0.008	-0.008	0.268	Ref. 1
	0.33	-0.036	0.805	0.805	0.788	-0.010	-0.010	-0.017	0.269	Present method
	0.67	-0.146	0.788	0.786	0.787	-0.040	-0.014	-0.043	0.270	Present method

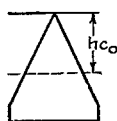
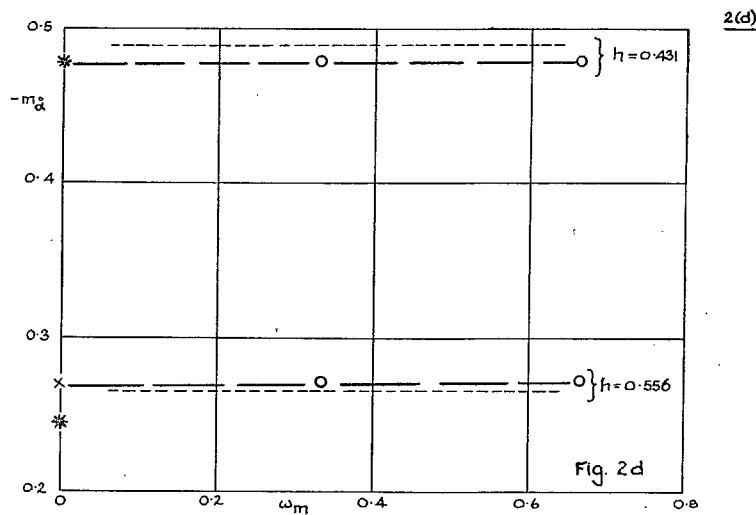
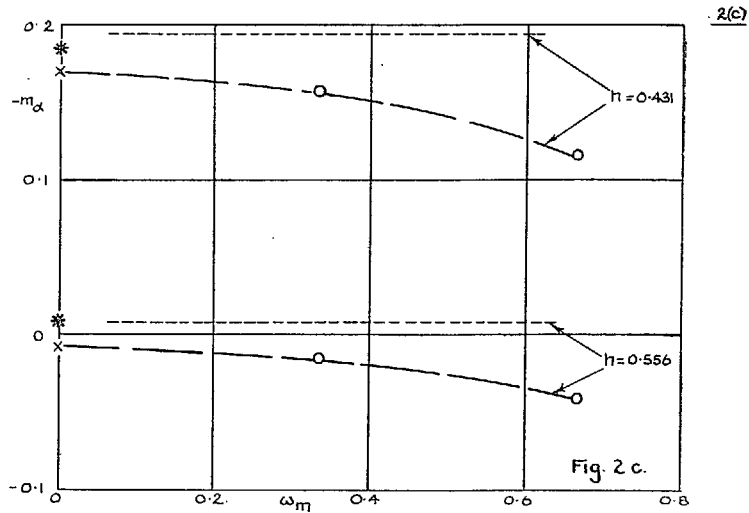
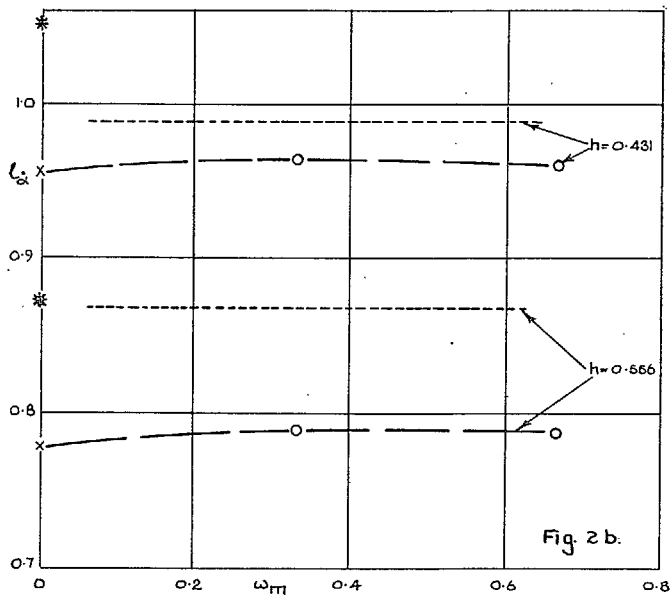
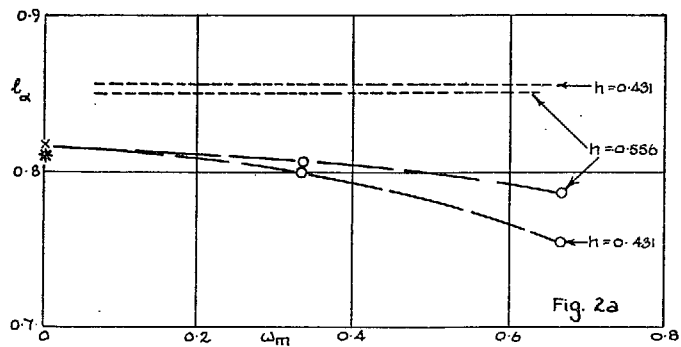
TABLE 9

*Derivatives of the Arrowhead Wing $A = 1.32$
 for Plunging and Pitching Oscillations*

hc_0	ω_m	l_z	l_z	l_a	$l_{\dot{a}}$	$-m_z$	$-m_z$	$-m_a$	$-m_{\dot{a}}$	Remarks
0	$\rightarrow 0$	0	0.833	0.833	1.491	0	+0.795	+0.795	1.653	Ref. 1
	0.30	-0.024	0.823	0.799	1.493	-0.030	0.785	0.750	1.655	Present method
	0.61	-0.101	0.802	0.697	1.478	-0.125	0.764	0.615	1.641	Present method
0.613 c_0	$\rightarrow 0$	0	0.833	0.833	0.756	0	0.060	0.060	0.284	Ref. 1
	0.30	-0.024	0.823	0.820	0.766	-0.009	0.059	0.053	0.286	Present method
	0.61	-0.101	0.802	0.786	0.769	-0.036	+0.056	+0.031	0.288	Present method
0.738 c_0	$\rightarrow 0$	0	0.833	0.833	0.606	0	-0.090	-0.090	0.165	Ref. 1
	0.30	-0.024	0.823	0.824	0.618	-0.004	-0.089	-0.094	0.164	Present method
	0.61	-0.101	0.802	0.805	0.625	-0.018	-0.089	-0.107	0.165	Present method



FIGS. 1a and 1b. Derivative coefficients for pitching oscillations of delta wing, $A = 3$ referred to the axis position $0.556c_0$.

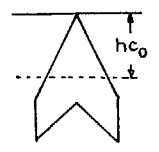
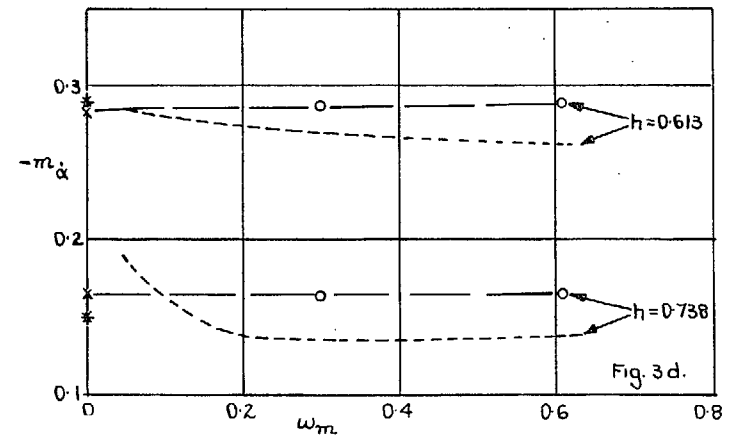
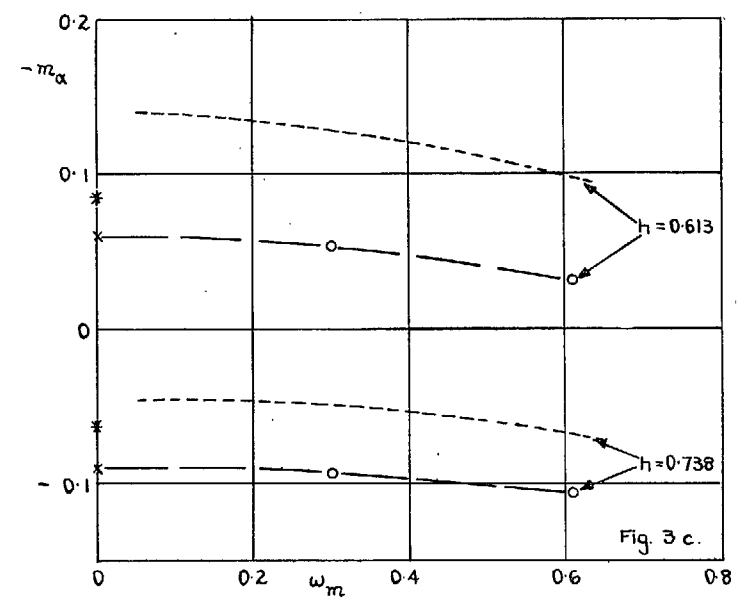
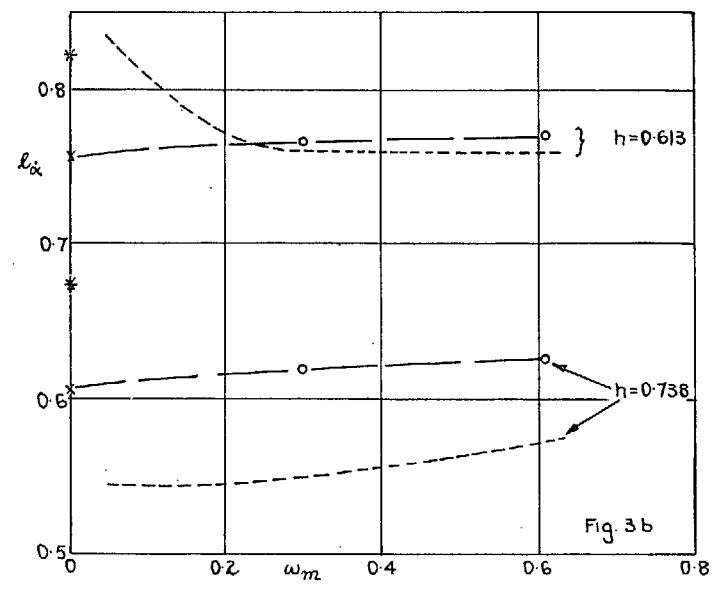
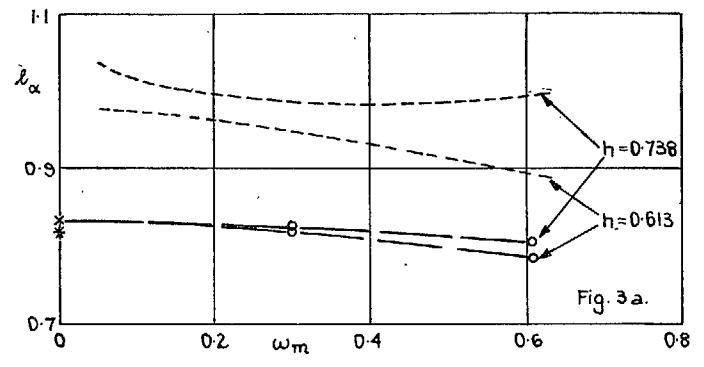


— Vortex Lattice Solutions { x Method for $\omega_m \rightarrow 0$, Ref. 1.
 o Present method.
 - - - Measured values, Ref. 10.
 * Multhopp-Garner solution for $\omega_m \rightarrow 0$, Ref. 9.

Figs. 2a to 2d. Derivative coefficients for pitching oscillations of delta wing $A = 1.2$, referred to axis positions hc_0 .

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— Vortex lattice Solutions
 * Mulhopp-Garner solution for $\omega_m \rightarrow 0$, Ref. 9
 o Present method
 - - - Measured values, Ref. 10

Figs. 3a to 3d. Derivative coefficients for pitching oscillations of arrowhead wing, $A = 1.32$, referred to axis positions hc_0 .

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