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Supersonic Flow Past Quasi-Cylindrical Bodies of Almost Circular Cross-Section

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Summary.—The supersonic flow over bodies for which the surface boundary condition may be satisfied on a circular cylinder is considered. The method is based on the linearised small-perturbation theory of supersonic flow. The disturbance velocity potential is obtained as a Fourier series, each term of which contains a certain basic function and the first eleven of these functions are evaluated. The pressure distribution and wave drag have been calculated for some bodies consisting of circular cylinders surmounted by canopies. An extension of the method to solve certain wing-body interference problems is also described.

1. *Introduction.*—In recent years a very considerable amount of work has been done on the supersonic flow over bodies whose geometry is such that the linearised small-perturbation equation for the velocity potential may be used. No general solution has been given for the flow over such bodies, and further restrictions must be imposed on the geometry, leading to various theories, all based initially on the linearised equation. This paper is concerned with an extension of one of these theories, that known as quasi-cylinder theory, which was first discussed by Lighthill[1] and later developed by Ward[2]. These authors considered the flow past a body of revolution which did not differ much from a cylinder. Little other work seems to have been done, except for a paper[3] by Ferrari dealing with quasi-cylinders having minimum wave-drag.

Up till now the term ' quasi-cylinder ' has been used to denote a body which is not only approximately cylindrical in shape, but is also axisymmetrical. In this paper the term ' quasi-cylinder ' is used more generally to denote a body which is only approximately cylindrical in shape. The theory of Refs. 1 and 2 is extended to quasi-cylinders which, though they are not axisymmetrical, are such that the surface boundary condition can be applied on a circular cylinder.

The equation satisfied by the velocity potential is solved by operational methods and the potential is obtained as a Fourier series. Each term of this Fourier series contains a function which is the same for all bodies of the above type. These basic functions are inverses of Laplace transforms involving Bessel functions of imaginary argument and have to be evaluated numerically. The technique used for evaluating these functions is described in a later section and the first eleven are tabulated in Table 1.

The method seems to be especially suited to determining the flow over canopies mounted on circular cylinders, and the examples worked out are bodies of this type.

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In theory, the method can be extended to solve certain wing-body interference problems. In these the wings must be such that the surface boundary condition can be satisfied on a plane, while the body must be a body of the type discussed above and symmetrical about the wing plane. The disturbance velocity potential is obtained as a Fourier series involving the same basic functions as before. Now, however, the functions are needed not only on the cylinder but also on the wing and their computation in the latter case is a much more difficult problem. It is hoped to evaluate them on a high-speed digital computor and to tabulate them in a future paper. There are grounds[4] for believing that some at least of these functions are being tabulated in the U.S.A.

2. *Formulation of the Problem.*—If a body of the type described in the previous section is placed in a supersonic free stream then, provided that the slope at every point of the body, in the direction of the free stream, is small, linearised theory may be used to determine the flow.

In view of the approximation to be made when the boundary conditions are applied, it is desirable to introduce cylindrical polar coordinates, x, r and θ. The x-axis is taken in the direction of the free-stream velocity, and r is the distance from the x-axis (since the body does not depart far from a circular cylinder, the axis of this cylinder may be taken to be the x-axis). x is measured from the mouth of the quasi-cylinder. The velocity of the free stream is U and its Mach number is M. The symbol B denotes $\sqrt{(M^2 - 1)}$. The velocity potential, Φ, of the flow may be written as $\Phi = Ux + U\phi$. ϕ is the reduced disturbance velocity potential due to the presence of the quasi-cylinder, the disturbance velocities being $U\phi_x$, $U\phi_r$, and $(U/r)\phi_\theta$. The linearised approximation for C_p, the pressure coefficient, is

$$C_p = -2\phi_x \,. \qquad (1)$$

It will be assumed that ϕ vanishes upstream of the quasi-cylinder. This is equivalent to assuming that the pre-entry stream tube, whose boundary separates the internal and external flows, is cylindrical.

ϕ satisfies the linearised equation of supersonic flow:

$$B^2\phi_{xx} = \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} \,. \qquad (2)$$

The following boundary conditions must also be satisfied. First, the normal component of the velocity must vanish everywhere on the surface of the quasi-cylinder, whose equation may be written

$$r = R\,[1 + \varepsilon(x/l\,,\,\theta)], \qquad (3)$$

where $\varepsilon(x/l\,,\,\theta) << 1$; l is the length of the cylinder and R has the dimensions of a length. Thus, within the accuracy of the linearised theory,

$$(\phi_r)_{r=R} = \left(\frac{\partial r}{\partial x}\right)_{\text{body}} = \frac{R}{l}\,\varepsilon'\left(\frac{x}{l}\,,\,\theta\right), \qquad (4)$$

where $\varepsilon'(t,\,\theta)$ is written for $\partial\varepsilon(t,\,\theta)/\partial t$. Secondly, since disturbances due to the presence of the body must be confined to the region downstream of the Mach lines emanating from the mouth of the body,

$$\phi \to 0, \quad \text{as} \quad r \to \infty \,. \qquad (5)$$

It remains to find a solution of equation (2) satisfying (4) and (5).

3. *The Operational Solution of the Linearised Equation.*—Using non-dimensional co-ordinates,

$$x' = x/(BR)\,, \qquad (6)$$

$$r' = r/R\,, \qquad (7)$$

equation (2) becomes

$$\phi_{x'x'} = \phi_{r'r'} + \frac{1}{r'}\phi_{r'} + \left(\frac{1}{r'}\right)^2 \phi_{\theta\theta} \,, \qquad (8)$$

with boundary conditions

$$(\phi_{r'})_{r'=1} = \frac{R^2}{l}\,\varepsilon'\left(\frac{BR}{l}x', \theta\right) \qquad (9)$$

and

$$\phi \to 0, \quad \text{as} \quad r' \to \infty \,. \qquad (10)$$

The Laplace transform of a function $f(x')$, written $\bar{f}(p)$, is defined as

$$\bar{f}(p) = \int_0^\infty e^{-px'} f(x')\, dx' \,.$$

The operational form of equation (8) is, therefore[5],

$$p^2\bar{\phi} = \bar{\phi}_{r'r'} + \frac{1}{r'}\bar{\phi}_{r'} + \left(\frac{1}{r'}\right)^2 \bar{\phi}_{\theta\theta} \,, \qquad (11)$$

with boundary conditions

$$(\bar{\phi}_{r'})_{r'=1} = \frac{R^2}{l}\bar{g}(p, \theta) \,, \qquad (12)$$

where

$$g(x', \theta) = \varepsilon'\left(\frac{BR}{l}x', \theta\right) , \qquad (13)$$

and

$$\bar{\phi} \to 0, \quad \text{as} \quad r' \to \infty \,. \qquad (14)$$

The method of separation of variables applied to (11) leads to the well-known solution of this equation,

$$\bar{\phi} = \sum_{n=0}^{\infty}\left[\left\{A_n(p)\, K_n(pr') + C_n(p)\, I_n(pr')\right\}\cos n\theta + \left\{B_n(p)\, K_n(pr') + D_n(p)\, I_n(pr')\right\}\sin n\theta\right] ,$$

where the functions K_n and I_n are Bessel functions of imaginary argument[6], and A_n, B_n, C_n and D_n are arbitrary functions of p.

Since $I_n(pr') \to \infty$ as $r' \to \infty$, while $K_n(pr') \to 0$ as $r' \to \infty$, (14) will be satisfied by writing

$$\bar{\phi} = \sum_{n=0}^{\infty}\left\{A_n(p)\cos n\theta + B_n(p)\sin n\theta\right\} K_n(pr') \qquad (15)$$

The boundary condition (12) requires that

$$\sum_{n=0}^{\infty} p\left\{A_n(p)\cos n\theta + B_n(p)\sin n\theta\right\} K_n'(p) = \frac{R^2}{l}\bar{g}(p, \theta) \,. \qquad (16)$$

This suggests that $\bar{g}(p, \theta)$ and, hence, also $\varepsilon'\{(BR/l)x', \theta\}$ should be expanded as a Fourier series in θ and, writing

$$\varepsilon'\left(\frac{BR}{l}x', \theta\right) = \sum_{n=0}^{\infty}\left\{a_n(x')\cos n\theta + b_n(x')\sin n\theta\right\} , \qquad (17)$$

and

$$\bar{g}(p, \theta) = \sum_{n=0}^{\infty}\left\{\bar{a}_n(p)\cos n\theta + \bar{b}_n(p)\sin n\theta\right\} , \qquad (18)$$

it is found, on comparing equations (16) and (18), that

$$A_n(p) = \frac{R^2}{l}\frac{\bar{a}_n(p)}{pK_n'(p)}, \quad \ldots \quad (19a)$$

$$B_n(p) = \frac{R^2}{l}\frac{\bar{b}_n(p)}{pK_n'(p)}. \quad \ldots \quad (19b)$$

Substitution of (19a) and (19b) into (15) gives

$$\bar{\phi} = \frac{R^2}{l}\sum_{n=0}^{\infty}\left\{\bar{a}_n(p)\cos n\theta + \bar{b}_n(p)\sin n\theta\right\}\frac{K_n(pr')}{pK_n'(p)}. \quad \ldots \quad (20)$$

It is now convenient to introduce certain basic functions, $V_n(x', r')$, defined by their transforms:

$$\bar{V}_n(p, r') = -\frac{K_n(pr')}{pK_n'(p)}, \quad \ldots \quad (21)$$

and (20) may now be written,

$$\bar{\phi} = -\frac{R^2}{l}\sum_{n=0}^{\infty}\left\{\bar{a}_n(p)\cos n\theta + \bar{b}_n(p)\sin n\theta\right\}\bar{V}_n(p, r'). \quad \ldots \quad (22)$$

Application of the product theorem of operational calculus[7] to (22) gives ϕ as

$$\phi = -\frac{R^2}{l}\sum_{n=0}^{\infty}\int_0^{x'} V_n(x' - x_1', r')\left\{a_n(x_1')\cos n\theta + b_n(x_1')\sin n\theta\right\}dx_1'. \quad \ldots \quad (23)$$

In the original coordinates (23) becomes

$$\phi = -\frac{R}{Bl}\sum_{n=0}^{\infty}\int_0^{x} V_n\left(\frac{x - x_1}{BR}, \frac{r}{R}\right)\left\{a_n\left(\frac{x_1}{BR}\right)\cos n\theta + b_n\left(\frac{x_1}{BR}\right)\sin n\theta\right\}dx_1. \quad \ldots \quad (24)$$

Using (1) with (24),

$$C_p = \frac{2R}{Bl}\sum_{n=0}^{\infty}\int_0^{x}\frac{1}{BR}V_n'\left(\frac{x - x_1}{BR}, \frac{r}{R}\right)\left\{a_n\left(\frac{x_1}{BR}\right)\cos n\theta + b_n\left(\frac{x_1}{BR}\right)\sin n\theta\right\}dx_1$$
$$+ \frac{2R}{Bl}\sum_{n=0}^{\infty}V_n\left(0, \frac{r}{R}\right)\left\{a_n\left(\frac{x}{BR}\right)\cos n\theta + b_n\left(\frac{x}{BR}\right)\sin n\theta\right\}, \quad \ldots \quad (25)$$

with $V_n'(t, r/R)$ written for $\partial V(t, r/R)/\partial t$.

After a partial integration (25) becomes

$$C_p = \frac{2R}{Bl}\sum_{n=0}^{\infty}V_n\left(\frac{x}{BR}, \frac{r}{R}\right)\left\{a_n(0)\cos n\theta + b_n(0)\sin n\theta\right\}$$
$$+ \frac{2}{B^2l}\sum_{n=0}^{\infty}\int_0^{x}V_n\left(\frac{x - x_1}{BR}, \frac{r}{R}\right)\left\{a_n'\left(\frac{x_1}{BR}\right)\cos n\theta + b_n'\left(\frac{x_1}{BR}\right)\sin n\theta\right\}dx_1.$$

Hence,

$$(C_p)_{r=R} = \frac{2R}{Bl}\sum_{n=0}^{\infty}V_n\left(\frac{x}{BR}\right)\left\{a_n(0)\cos n\theta + b_n(0)\sin n\theta\right\}$$
$$+ \frac{2}{B^2l}\sum_{n=0}^{\infty}\int_0^{x}V_n\left(\frac{x - x_1}{BR}\right)\left\{a_n'\left(\frac{x_1}{BR}\right)\cos n\theta + b_n'\left(\frac{x_1}{BR}\right)\sin n\theta\right\}dx_1. \quad \ldots \quad (26)$$

In the above equation and throughout the rest of the paper $V_n(t)$ is written for $V_n(t, 1)$.

(26) gives the pressure coefficient on the quasi-cylinder and the drag, D, is given by

$$\frac{D}{\frac{1}{2}\rho U^2} = \int_0^l \int_0^{2\pi} (C_p)_{r=R} \frac{R}{l} \varepsilon' \left(\frac{x}{l}, \theta\right) R \, d\theta \, dx ,$$

where ρ is the density of the free stream. C_D, the drag coefficient based on the cross-sectional area of the circular cylinder $r = R$, is

$$C_D = \frac{1}{\pi l} \int_0^l \int_0^{2\pi} (C_p)_{r=R} \varepsilon' \left(\frac{x}{l}, \theta\right) d\theta \, dx . \qquad (27)$$

Using equations (17), (26) and (27),

$$\begin{aligned} C_D = {} & \frac{4R}{Bl^2} \int_0^l V_0 \left(\frac{x}{BR}\right) a_0 \left(\frac{x}{BR}\right) a_0(0) \, dx \\ & + \frac{4}{B^2 l^2} \int_0^l a_0 \left(\frac{x}{BR}\right) \int_0^x V_0 \left(\frac{x - x_1}{BR}\right) a_0' \left(\frac{x_1}{BR}\right) dx_1 \, dx \\ & + \frac{2R}{Bl^2} \sum_{n=1}^{\infty} \int_0^l V_n \left(\frac{x}{BR}\right) \left\{ a_n \left(\frac{x}{BR}\right) a_n(0) + b_n \left(\frac{x}{BR}\right) b_n(0) \right\} dx \\ & + \frac{2}{B^2 l^2} \sum_{n=1}^{\infty} \int_0^l \int_0^x V_n \left(\frac{x - x_1}{BR}\right) \left\{ a_n \left(\frac{x}{BR}\right) a_n' \left(\frac{x_1}{BR}\right) + b_n \left(\frac{x}{BR}\right) b_n' \left(\frac{x_1}{BR}\right) \right\} dx_1 \, dx , \end{aligned} \qquad (28)$$

a_n and b_n are defined by (17). The V_n are functions which are independent of the particular form of the function $\varepsilon(x/l, \theta)$. Their evaluation as functions of x and n is a numerical problem, the solution of which is discussed in the next section.

4. *Evaluation of the Basic Functions*.—The inversion formula* for a function $f(x)$, the transform of which is $\bar{f}(p)$, is[8]

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{f}(p) \, e^{px} \, dp , \qquad (29)$$

the integration being along a line from $c - i\infty$ to $c + i\infty$ such that all the poles of $\bar{f}(p)$ lie to the left of this line. Subject to this requirement the value of c is arbitrary. (29) may be used to derive a formula for $V_n(x, r)$,

$$V_n(x, r) = -\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{K_n(pr)}{pK_n'(p)} e^{px} \, dp , \qquad (30)$$

where n is a positive integer. $K_n(pr)$ and $K_n'(p)$ both have a branch point at $p = 0$ and this introduces a complication into the evaluation of the line integral. Apart from this it is also necessary to know where the zeros of $K_n'(p)$ lie. The general result, for the zeros of $K_\nu'(p)$, where ν is not necessarily an integer, is[9] that $K_\nu'(p)$ has all its zeros to the left of the imaginary axis and the number of zeros is the nearest even integer to $\nu + \frac{1}{2}$ (the only exception to this is when $\nu + \frac{1}{2}$ is an odd integer; in this case the number of zeros is $\nu + \frac{1}{2}$). Thus $K_0'(p)$ has no zero, $K_1'(p)$ and $K_2'(p)$ each have two zeros, $K_3'(p)$ and $K_4'(p)$ each have four zeros, and so on. The zeros are symmetrically placed about the real axis (with one zero lying on this axis when $\nu + \frac{1}{2}$ is an odd integer). For the moment it will be assumed that these zeros are known.

* Strictly speaking the variables x and r used throughout this section should be written x' and r' for consistency with section 3, but the primes have been omitted for simplicity.

The integral in equation (30) can be evaluated integrating $V_n(p, r)$ round the contour of Fig. 1. Suppose there are $2m$ zeros of $K_n'(p)$, occurring at $p = \alpha_i$, i going from 1 to $2m$ inclusive, then the residues of $\bar{V}_n(p, r)\, e^{px}$ are

$$-\frac{K_n(r\alpha_i)\exp x\alpha_i}{\alpha_i K_n''(\alpha_i)}, \qquad (i + 1, 2, \ldots, 2m)\,.$$

As the radius of Γ tends to infinity the behaviour of $K_n(p)$ for large values of p shows that the integral round Γ vanishes. As the radius of γ tends to zero the behaviour of $K_n(p)$ for small values of p shows that the integral round γ vanishes. Hence,

$$V_n(x, r) = -\frac{1}{2\pi i}\int_{-\infty}^{0}\frac{e^{-px}K_n(rp\, e^{i\pi})\,dp}{pK_n'(p\, e^{i\pi})} - \frac{1}{2\pi i}\int_{0}^{-\infty}\frac{e^{-px}K_n(rp\, e^{-\pi i})\,dp}{pK_n'(p\, e^{-\pi i})} - \sum_{i=0}^{2m}\frac{\exp x\alpha_i K_n(r\alpha_i)}{\alpha_i K_n''(\alpha_i)}\,. \qquad (31)$$

Now[10],

$$K_n(p\, e^{\pm\pi i}) = e^{\mp n\pi i}K_n(p) \mp \pi i I_n(p)\,, \qquad (32a)$$

$$-K_n'(p\, e^{\pm\pi i}) = e^{\mp n\pi i}K_n'(p) \mp \pi i I_n'(p)\,. \qquad (32b)$$

From the differential equation for $K_n(p)$,

$$K_n''(\alpha i) = -\frac{1}{\alpha_i}K_n'(\alpha_i) + \left(1 + \frac{n^2}{\alpha_i^{\,2}}\right)K_n(\alpha_i)\,. \qquad (33)$$

Substitution of (32a), 32b) and (33) in (31) gives, after some manipulation,

$$V_n(x, r) = -\sum_{i=1}^{2m}\frac{\alpha_i K_n(r\alpha_i)\exp\alpha_i x}{(\alpha_i^{\,2} + n^2)K_n(\alpha_i)} + (-1)^n\int_0^\infty\frac{e^{-px}}{p}\,\frac{K_n(pr)I_n'(p) - I_n(pr)K_n'(p)}{[K_n'(p)]^2 + \pi^2[I_n'(p)]^2}\,dp\,. \qquad (34)$$

This function has to be evaluated for a range of values of x, r and n in order to solve wing-body interference problems in which the potential away from the body is required. In the second term the integrand tends to zero fairly quickly as p increases, and all the functions required are well tabulated. The series, however, involves K_n of complex argument. No tables are available for K_n when the argument is complex and the amount of time required to work out $K_n(r\alpha_i)$ on a desk machine for, say, eleven values of n and a sufficient range of values of r ($r = 1$ to 11, say), would be very great.

However, for problems in which the potential on the body only is required, V_n is needed for $r = 1$ only, and (34) is considerably simplified when r is put equal to unity. Since[10] the Wronskian of I_n and K_n is equal to $-1/p$, i.e., $K_n(p)I_n'(p) - K_n'(p)I_n(p) = 1/p$, (34) becomes, on putting $r = 1$,

$$V_n(x) = -\sum_{i=1}^{2m}\frac{\alpha_i\exp\alpha_i x}{(\alpha_i^{\,2} + n^2)} + (-1)^n\int_0^\infty\frac{e^{-px}}{[K_n'(p)]^2 + \pi^2[I_n'(p)]^2}\,\frac{dp}{p^2}\,. \qquad (35)$$

The integral is not very troublesome to evaluate. For $n > 0$ it is small compared with the series. Further, $K_n'(p)$ and $I_n'(p)$ are easily obtained in terms of $K_{n-1}(p)$, $K_n(p)$, $I_{n-1}(p)$ and $I_n(p)$. The last four functions are well tabulated[11]. The series involves exponential and trigonometrical functions and the zeros of $K_n'(p)$. The position of the zeros may be found approximately by interpolating between the zeros of $K_{n+1/2}'(p)$; ($K_{n+1/2}'(p)$ can be expressed as [10]

$$K_{n+1/2}'(p) = -\left(\frac{\pi}{2p}\right)^{1/2}e^{-p}\left\{1 + \frac{(2n+1)!}{n!\,2^{n+1}}p^{-(n+1)} + \sum_{s=1}^{n}\frac{(n^2 + s^2 + n)(n + s - 1)!}{s!\,(n - s + 1)!\,(2p)^s}\right\}$$

and the zeros are obtained by solving an algebraic equation of the $(n + 1)$th order). If the first approximation to a zero of $K_n'(p)$ is p_0, say, then for small δ,

$$K_n'(p_0 + \delta) \sim K_n'(p_0) + \delta K_n''(p_0) ,$$

and a better approximation is

$$p_1 = p_0 - \frac{K_n'(p_0)}{K_n''(p_0)} .$$

This process can be repeated as often as necessary.

Once the α_i have been obtained the problem of computing $V_n(x)$ from (35) is merely tedious. Writing $\alpha_i = -\beta_i + i\gamma_i$, and remembering that $-\beta_i - i\gamma_i$ is also a root of $K_n'(p)$, the formula for $V_n(x)$ becomes

$$V_n(x) = 2 \sum_{i=1}^{m} \frac{\beta_i(\beta_i^2 + n^2 + \gamma_i^2) \cos \gamma_i x + \gamma_i(n^2 - \beta_i^2 - \gamma_i^2) \sin \gamma_i x}{(\beta_i^2 + n^2 - \gamma_i^2)^2 + 4\beta_i^2\gamma_i^2} \exp(-\beta_i x)$$
$$+ (-1)^n \int_0^\infty \frac{e^{-px}}{[K_n'(p)]^2 + \pi^2[I_n'(p)]^2} \frac{dp}{p^2} . \qquad (36)$$

There is one check on the computing: this is the fact that, for all n, $V_n(0) = 1$. This result is not obvious from (35), but can be seen quite easily by considering the operational form of $V_n(x)$. From (20) this is

$$\bar{V}_n(p) = -\frac{K_n(p)}{pK_n'(p)} .$$

The asymptotic expansion of $K_n(p)$ is well known[12]; it is

$$K_n(p) \sim \left(\frac{\pi}{2p}\right)^{1/2} e^{-p} \left(1 + \frac{4n^2 - 1}{8p} + \ldots\right) .$$

Hence,

$$\bar{V}_n(p) \sim \frac{1}{p} - \frac{1}{2p^2} - \ldots \qquad (37)$$

The asymptotic expansion of the transform of a function corresponds to a Taylor expansion of its inverse[13], and, inverting (37),

$$V_n(x) = 1 - \tfrac{1}{2}x - \ldots \qquad (38)$$

This proves the above statement that $V_n(0) = 1$. For all the eleven functions worked out $(n = 0, 1, \ldots, 10)$, this check was satisfied to five places of decimals.

$V_0(x)$ and $V_1(x)$ have been tabulated previously. $V_0(x)$ is the same function as the $U(x)$ of Ref. 1, and $V_1(x)$ is the same function as the $V(x)$ of Ref. 2. No significant difference was found between the results of this paper and those of Refs. 1 and 2.

$V_n(x)$ is tabulated in Table 1 for eleven values of n and for a range of values of x, (0 to 20). Two of the functions, $V_2(x)$ and $V_6(x)$, are shown graphically in Fig. 2. The roots of $K_n'(p)$ for $n = 1$ to 10 inclusive, are tabulated in Table 2.

5. *Some Applications of the Method.*—The theory developed in previous sections can be applied to quasi-cylinders with cross-sections which do not depart far from circles.

An application was made to a configuration consisting of a circular cylinder surmounted by a canopy. The canopy consisted of a body of revolution formed by two tangent ogives placed back to back with their axes of revolution coincident with the top of the cylinder. The ratio of canopy height to the radius of the cylinder was 0·5, and the length of the canopy was five times the radius of the cylinder. An identical canopy was mounted symmetrically on the bottom of the cylinder. The drag was calculated for a Mach number of 1·4. At this Mach number there is some interference between the two canopies and so the flow over a single canopy was also determined in order to estimate the effect of this interference on the drag.

The results obtained by using the method described in section 3 are shown in Figs. 3 and 4. The drag coefficient (based on the total canopy frontal area) came to 0·247 when both canopies were present. This compares not unfavourably with the experimental value of 0·30 obtained when the drag of this configuration was measured by the free-flight model technique[14]. When only one canopy was present the theoretical drag coefficient (based on canopy frontal area) came to 0·282, an increase of 14 per cent on the value for the first case. Fig. 3 shows the distribution along the x-axis of the drag loading, *i.e.*, of $\frac{1}{R}\int_0^{2\pi} (C_p)_{r=R}\, sR\, d\theta$; the drag is obtained from this by multiplying by R and integrating with respect to x from $x = 0$ to $x = l$. Here, s is the slope at a point on the configuration in the x-direction and R is the radius of the circular cylinder on which the boundary conditions were satisfied. R was taken as 1·021 times the radius of the circular cylinder on which the canopy was surmounted, giving the helpful value of 5 for the value of l/BR in the application of (28). Fig. 4 shows the distribution along the body of the lift loading, *i.e.*, of $-\frac{1}{R}\int_0^{2\pi} (C_p)_{r=R}\, R\, d\theta$, when only one canopy is present. The disturbance due to the presence of the canopy almost dies out in a distance equal to one length of the canopy. The theoretical lift coefficient and pitching moment about the nose of the canopy are so small that they cannot be accurately determined by the approximate theory used.

It was not found possible to give examples of pressure distributions over the configurations. The pressure coefficient is given by (26) and the drag by (28). Inspection of these two equations will show that the series for the drag converges more rapidly than the series for the pressure coefficient, due to the presence of an extra $a_n(x/BR)$ in the former series. In fact, eleven terms are not enough to give even an approximation to the pressure coefficient, while they are quite sufficient for the drag.

The experimental work mentioned above also gives results for canopies with windscreens; these canopies are derived by a geometrical process from the above basic canopy. It is hoped to apply the technique of this paper to such canopies in a later note.

6. *Wing-Body Interference*.—The preceding work can be extended to solve certain wing-body interference problems. The wings must be such that the boundary condition can be satisfied on a plane, while the body must be of the type discussed in this paper and symmetrical about the wing plane. This plane may be taken to be the plane $\theta = 0$, $\theta = \pi$.

It is assumed that the potential due to the wings alone has already been determined by linearised wing theory. For simplicity in calculating the velocity potential due to the wing alone, the wings are assumed to be continued through the body to meet on the axis (if any other assumption is made with regard to the extent of the wings, the only difference is in the values of the separate potentials ϕ_W, ϕ_I of equation (39) below, the value of ϕ remaining the same).

The velocity potential, Φ, of the flow may be written as $\Phi = Ux + U\phi$, ϕ being the disturbance velocity potential due to the presence of the wing-body combination. ϕ is written as the sum of two potentials,

$$\phi = \phi_W + \phi_I , \quad \ldots \quad (39)$$

where ϕ_W is the potential due to the wings alone and ϕ_I may be termed the interference potential due to the presence of the quasi-cylinder. ϕ_W is assumed to be known and ϕ_I is taken as a potential of the form of equation (15) with the cosine terms only appearing since the body is assumed to be symmetrical about the plane $\theta = 0$, $\theta = \pi$. It follows that ϕ satisfies the potential equation (8) and the boundary condition at infinity (14). The boundary condition on the wing is automatically satisfied by symmetry.

This leaves one more boundary condition, that on the surface of the quasi-cylinder, to be satisfied. Using the notation of section 3, this condition is

$$(\phi_r)_{r=R} = \frac{R}{l}\,\varepsilon'\left(\frac{x}{l}\,,\,\theta\right),$$

or

$$\left(\frac{\partial \phi_I}{\partial r}\right)_{r=R} = \frac{R}{l}\,\varepsilon'\left(\frac{x}{l}\,,\,\theta\right) - \left(\frac{\partial \phi_W}{\partial r}\right)_{r=R}. \qquad \ldots \quad (40)$$

From this point the analysis proceeds as in section 3. The right-hand side is expanded as a Fourier series in θ, and ϕ_I is obtained as

$$\phi_I = -\frac{R}{Bl}\sum_{n=0}^{\infty}\int_0^x V_n\left(\frac{x - x_1}{BR}\,,\,\frac{r}{R}\right) c_n\left(\frac{x_1}{BR}\right) dx_1 \cos n\theta\,, \qquad \ldots \quad (41)$$

where the c_n are defined by

$$\frac{R^2}{l}\,\varepsilon'\left(\frac{x}{l}\,,\,\theta\right) - R\left(\frac{\partial \phi_W}{\partial r}\right)_{r=R} = \frac{R^2}{l}\sum_{n=0}^{\infty} c_n\left(\frac{x}{BR}\right)\cos n\theta\,. \qquad \ldots \quad (42)$$

C_p is determined as in section 3 if required over the quasi-cylinder alone. To find the value of C_p on the wing it is necessary to tabulate $V_n(x, r)$ as a function of n, x and r. It was stated in section 4 that the evaluation of V_n in this general case was beyond the power of a computer using a desk machine. A short note by Mersman[4], however, suggests that some of these functions may have been worked out in the U.S.A. No further details are known at the moment.

The problem of wing-body interference in combinations of the above type has also been treated by Nielsen[15].

7. *Conclusions*.—The linearised theory of supersonic flow has been used to formulate the problem of flow past certain quasi-cylindrical bodies and to determine the velocity potential on the surface of such bodies. The quasi-cylinders are not necessarily axisymmetrical but must be such that the surface boundary condition can be applied on a circular cylinder. The disturbance velocity potential is obtained as a Fourier series, each term of which involves a certain basic function. The first eleven of these functions are tabulated in Table 1 and it has not so far been necessary to go beyond this number.

The method is particularly suitable for the determination of the flow over a circular cylinder surmounted by a canopy, and has been applied to such a body. The theoretical value obtained for the drag is in fair agreement with experiment.

It is also shown in principle how the method can be extended to solve certain wing-body interference problems.

(71643) A*

LIST OF SYMBOLS

$A_n(p)$		Arbitrary function of p
$a_n(x')$		Fourier coefficient defined in (17)
B	=	$\sqrt{(M^2 - 1)}$
$B_n(p)$		Arbitrary function of p
$b_n(x')$		Fourier coefficient defined in (17)
C_D		Drag coefficient based on a suitable area
$C_n(p)$		Arbitrary function of p
C_p		Pressure coefficient
c		Defined after equation (29)
$c_n(x/BR)$		Fourier coefficient defined in (43)
D		Drag
$D_n(p)$		Arbitrary function of p
$g(x', \theta)$	=	$\varepsilon'\left(\frac{BR}{l} x', \theta\right)$
$h(x/l)$		Defined in (39)
$I_n(x)$		Bessel function of imaginary argument of the first kind
$K_n(x)$		Bessel function of imaginary argument of the second kind
l		Length of quasi-cylinder
M		Mach number of free stream
m		$2m$ is the number of zeros of $K_n'(p)$
p		Variable of Laplace transform (*cf.* section 3)
R		Radius of the circular cylinder on which the surface boundary condition is satisfied
r		Radial co-ordinate in cylindrical polar co-ordinates
r'	=	r/R
s		Slope in the x-direction at a point of the canopy of section 5
U		Velocity of free stream
$V_n(x, r)$		Inverse of $-\dfrac{K_n(pr)}{pK_n'(p)}$
$V_n(x)$		Inverse of $-\dfrac{K_n(p)}{pK_n'(p)}$; i.e., $V_n(x, 1)$
x		Axial co-ordinate in cylindrical polar co-ordinates
x'	=	x/BR
x_1		Variable of integration
α_i		Position of zero of $K_n'(p)$
β_i		$-\beta_i$ is the real part of α_i

LIST OF SYMBOLS—*continued*

γ_i	$\pm\gamma_i$ is the imaginary part of α_i
$\varepsilon(x/l, \theta)$	A function always small compared with unity
θ	Angular co-ordinate in cylindrical polar co-ordinates
ρ	Density of free stream
Φ	Total velocity potential
ϕ	Disturbance velocity potential
ϕ_I	Defined after equation (40)
ϕ_W	Defined after equation (40)

A Laplace transform of a function is denoted by a bar placed over the symbol for the function.

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TABLE 1

The Functions $V_n(x)$

x	$V_0(x)$	$V_1(x)$	$V_2(x)$	$V_3(x)$	$V_4(x)$	$V_5(x)$
0	1·00000	+1·00000	+1·00000	+1·00000	+1·00000	+1·00000
0·1	0·95182	0·94947	0·94248	0·93087	0·91474	0·89420
0·2	0·90703	0·89827	0·87226	0·82975	0·77196	0·70058
0·3	0·86533	0·84689	0·79275	0·70638	0·59333	0·46076
0·4	0·82646	0·79576	0·70710	0·57044	0·40096	0·21717
0·5	0·79016	0·74522	0·61820	0·43087	0·21488	+0·00561
0·6	0·75621	0·69560	0·52859	0·29549	+0·05119	—0·14962
0·7	0·72442	0·64715	0·44048	0·17066	—0·07900	—0·23771
0·8	0·69461	0·60006	0·35572	+0·06116	—0·16988	—0·26056
0·9	0·66663	0·55451	0·27581	—0·02985	—0·22065	—0·23005
1·0	0·64034	0·51063	0·20193	—0·10074	—0·23466	—0·16409
1·0	0·64034	0·51063	0·20193	—0·10074	—0·23466	—0·16409
1·2	0·59230	0·42829	+0·07536	—0·18243	—0·18008	—0·00316
1·4	0·54960	0·35356	—0·02052	—0·19460	—0·07254	+0·09988
1·6	0·51150	0·28661	—0·08593	—0·15855	+0·02643	0·10487
1·8	0·47737	0·22737	—0·12375	—0·09849	0·08112	+0·04535
2·0	0·44671	0·17560	—0·13853	—0·03559	0·08574	—0·01935
2·2	0·41907	0·13089	—0·13563	+0·01537	0·05587	—0·04892
2·4	0·39408	0·09275	—0·12046	0·04709	+0·01492	—0·03882
2·6	0·37141	0·06066	—0·09801	0·05880	—0·01761	—0·00963
2·8	0·35080	0·03404	—0·07253	0·05434	—0·03221	+0·01442
3·0	0·33201	+0·01232	—0·04729	0·03977	—0·02955	0·02108
3·2	0·31483	—0·00507	—0·02462	0·02142	—0·01661	0·01288
3·4	0·29909	—0·01867	—0·00594	+0·00438	—0·00189	+0·00017
3·6	0·28464	—0·02900	+0·00812	—0·00812	+0·00840	—0·00791
3·8	0·27133	—0·03653	0·01755	—0·01490	0·01190	—0·00833
4·0	0·25906	—0·04170	0·02276	—0·01633	0·00965	—0·00369
4·2	0·24772	—0·04490	0·02445	—0·01384	+0·00452	+0·00140
4·4	0·23721	—0·04650	0·02346	—0·00920	—0·00054	0·00377
4·6	0·22746	—0·04680	0·02061	—0·00410	—0·00363	0·00277
4·8	0·21840	—0·04607	0·01671	+0·00023	—0·00426	+0·00080
5·0	0·20996	—0·04457	0·01240	0·00312	—0·00304	—0·00109
5·2	0·20209	—0·04248	0·00818	0·00441	—0·00110	—0·00163
5·4	0·19473	—0·03998	0·00443	0·00434	+0·00059	—0·00101
5·6	0·18785	—0·03721	+0·00136	0·00337	0·00146	—0·00003
5·8	0·18139	—0·03430	—0·00096	0·00199	0·00148	+0·00061
6·0	0·17533	—0·03133	—0·00251	+0·00062	0·00092	0·00065
6·2	0·16964	—0·02838	—0·00337	—0·00044	+0·00021	+0·00029
6·4	0·16428	—0·02552	—0·00367	—0·00106	—0·00033	—0·00010
6·6	0·15922	—0·02278	—0·00353	—0·00126	—0·00056	—0·00029
6·8	0·15444	—0·02019	—0·00310	—0·00113	—0·00050	—0·00024
7·0	0·14993	—0·01778	—0·00249	—0·00080	—0·00026	—0·00006
7·2	0·14566	—0·01556	—0·00182	—0·00040	—0·00001	+0·00008
7·4	0·14162	—0·01353	—0·00116	—0·00005	+0·00016	0·00013
7·6	0·13778	—0·01170	—0·00058	+0·00020	0·00021	0·00008
7·8	0·13413	—0·01006	—0·00011	0·00033	0·00016	+0·00004
8·0	0·13067	—0·00860	+0·00025	0·00034	+0·00007	—0·00005
8·2	0·12738	—0·00731	0·00048	0·00028	—0·00002	—0·00005
8·4	0·12424	—0·00619	0·00061	0·00018	—0·00007	—0·00002
8·6	0·12124	—0·00521	0·00064	0·00007	—0·00007	+0·00001
8·8	0·11838	—0·00437	0·00061	+0·00002	—0·00005	0·00002
9·0	0·11565	—0·00365	+0·00053	—0·00007	—0·00002	+0·00002

TABLE 1—*continued*

x	$V_0(x)$	$V_1(x)$	$V_2(x)$	$V_3(x)$	$V_4(x)$	$V_5(x)$
9·2	0·11304	−0·00304	+0·00043	−0·00010	+0·00001	+0·00001
9·4	0·11054	−0·00253	0·00032	−0·00009	0·00003	−0·00001
9·6	0·10815	−0·00211	0·00021	−0·00007	0·00003	−0·00001
9·8	0·10586	−0·00176	0·00012	−0·00004	0·00002	+0·00000
10·0	0·10366	−0·00148	0·00004	−0·00001	0·00000	+0·00000
10·0	0·10366	−0·00148	+0·00004	−0·00001	+0·00000	
10·5	0·09853	−0·00099	−0·00007	+0·00003	−0·00001	
11·0	0·09388	−0·00073	−0·00008	0·00002	+0·00000	
11·5	0·08965	−0·00061	−0·00005	0·00000	+0·00000	
12·0	0·08578	−0·00056	−0·00001	+0·00000		
12·5	0·08222	−0·00055	+0·00001			
13·0	0·07895	−0·00056	0·00002			
13·5	0·07593	−0·00056	0·00001			
14·0	0·07313	−0·00056	0·00001			
14·5	0·07052	−0·00054	0·00000			
15·0	0·06809	−0·00052	+0·00000			
15·5	0·06582	−0·00050				
16·0	0·06370	−0·00047				
16·5	0·06172	−0·00043				
17·0	0·05985	−0·00040				
17·5	0·05810	−0·00037				
18·0	0·05644	−0·00034				
18·5	0·05488	−0·00031				
19·0	0·05340	−0·00029				
19·5	0·05200	−0·00026				
20·0	0·05067	−0·00024				

Beyond $x = 20$, $V_0(x) = \frac{1}{z} + \frac{1}{z^3}(2 \log 2z - 2)$. (Ref. 16)

Beyond $x = 20$, $V_1(x) = -\frac{2}{z^3} + \frac{24}{z^5}\left(\log 2z - \frac{31}{12}\right)$. (Ref. 2)

TABLE 1—*continued*

x	$V_6(x)$	$V_7(x)$	$V_8(x)$	$V_9(x)$	$V_{10}(x)$
0	+1·00000	+1·00000	+1·00000	+1·00000	+1·00000
0·1	0·86941	0·84055	0·80783	0·77151	0·73179
0·2	0·61766	0·52558	0·42694	+0·32451	+0·22115
0·3	0·31698	+0·17087	+0·03114	−0·09407	−0·19790
0·4	+0·03863	−0·11647	−0·23348	−0·30288	−0·32145
0·5	−0·16442	−0·27156	−0·30499	−0·26815	−0·17739
0·6	−0·26676	−0·28431	−0·21339	−0·08721	+0·04974
0·7	−0·27121	−0·19114	−0·04710	+0·09481	0·17824
0·8	−0·20240	−0·05279	+0·09827	0·17410	0·14662
0·9	−0·09632	+0·07086	0·16218	0·13284	+0·02041
1·0	+0·01080	0·14024	0·13627	0·02579	−0·08749
1·0	0·01080	0·14024	+0·13627	+0·02579	−0·08749
1·2	0·13026	+0·09682	−0·03246	−0·10162	−0·04715
1·4	+0·09259	−0·04008	−0·08568	−0·00459	+0·06698
1·6	−0·00863	−0·07611	+0·00173	+0·05842	+0·00118
1·8	−0·06368	−0·01579	0·05032	−0·00247	−0·03776
2·0	−0·04444	+0·03755	+0·00949	−0·03298	+0·01328
2·2	+0·00427	+0·02825	−0·02722	+0·00416	+0·01605
2·4	0·03054	−0·00814	−0·01121	+0·01831	−0·01326
2·6	+0·02130	−0·02086	+0·01340	−0·00388	−0·00400
2·8	−0·00196	−0·00563	+0·00929	−0·01002	+0·00881
3·0	−0·01452	+0·00954	−0·00580	+0·00304	−0·00103
3·2	−0·01014	+0·00811	−0·00658	+0·00540	−0·00449
3·4	+0·00093	−0·00160	+0·00198	−0·00218	+0·00223
3·6	0·00690	−0·00560	+0·00422	−0·00287	+0·00166
3·8	+0·00482	−0·00186	−0·00027	+0·00147	−0·00185
4·0	−0·00044	+0·00240	−0·00250	+0·00150	−0·00024
4·2	−0·00328	+0·00230	−0·00036	−0·00096	+0·00111
4·4	−0·00229	−0·00027	+0·00137	−0·00076	−0·00028
4·6	+0·00021	−0·00150	+0·00050	+0·00061	−0·00051
4·8	0·00156	−0·00059	−0·00069	+0·00038	+0·00034
5·0	+0·00109	+0·00059	−0·00043	−0·00038	+0·00016
5·2	−0·00010	+0·00064	+0·00031	−0·00018	−0·00025
5·4	−0·00074	−0·00003	+0·00031	+0·00023	+0·00000
5·6	−0·00052	−0·00040	−0·00011	+0·00008	+0·00014
5·8	+0·00005	−0·00018	−0·00020	−0·00014	−0·00005
6·0	0·00035	+0·00014	+0·00002	−0·00004	−0·00006
6·2	+0·00025	0·00018	0·00008	+0·00008	+0·00005
6·4	−0·00002	+0·00000	+0·00001	+0·00001	+0·00001
6·6	−0·00017	−0·00010	−0·00007	−0·00005	−0·00003
6·8	−0·00012	−0·00006	−0·00002	+0·00000	+0·00000
7·0	+0·00001	+0·00004	+0·00004	0·00003	+0·00002
7·2	0·00008	0·00005	+0·00002	+0·00000	−0·00001
7·4	+0·00006	+0·00000	−0·00002	−0·00002	−0·00001
7·6	−0·00001	−0·00003	−0·00002	+0·00000	+0·00001
7·8	−0·00004	−0·00002	+0·00001	0·00001	0·00000
8·0	−0·00003	+0·00001	0·00001	0·00000	+0·00000
8·2	+0·00000	0·00001	0·00000	+0·00000	
8·4	0·00002	+0·00000	+0·00000		
8·6	0·00001	−0·00001			
8·8	+0·00000	+0·00000			
9·0	−0·00001	+0·00000			
9·2	−0·00001				
9·4	+0·00000				
9·6	+0·00000				

TABLE 2

The Roots of $K_n'(p)$

The following table gives all the solutions of $K_n'(-\alpha \pm i\beta) = 0$, for $n = 1$ to 10 inclusive. $K_0'(p)$ has no zero.

n	α	β	n	α	β
1	0·64355	0·50118	8	1·36941	7·16673
2	0·83455	1·43444	8	3·60872	4·92519
3	0·96756	2·37386	8	4·67839	3·07327
3	1·98162	0·44080	8	5·19993	1·36046
4	1·07279	3·32208	9	1·42667	8·13578
4	2·44093	1·32259	9	3·82205	5·84153
5	1·16125	4·27689	9	5·02798	3·96284
5	2·80372	2·21193	9	5·69438	2·18088
5	3·30981	0·43637	9	5·96253	0·43478
6	1·23832	5·23662	10	1·47973	9·10691
6	3·10823	3·10944	10	4·01755	6·76252
6	3·83945	1·31040	10	5·34531	4·85738
7	1·30706	6·20015	10	6·13751	3·05917
7	3·37302	4·01418	10	6·54610	1·30462
7	4·28713	2·18909			
7	4·63644	0·43517			

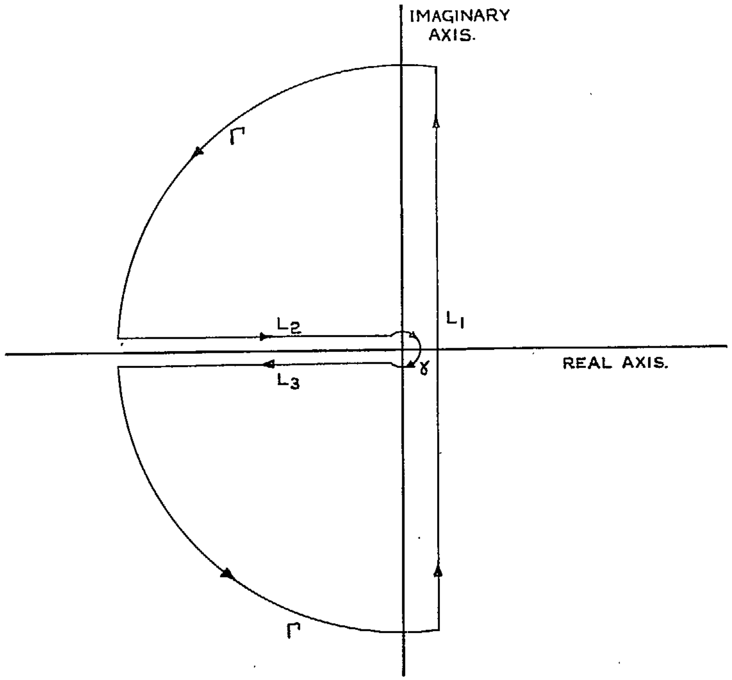

FIG. 1. Contour described in section 4.

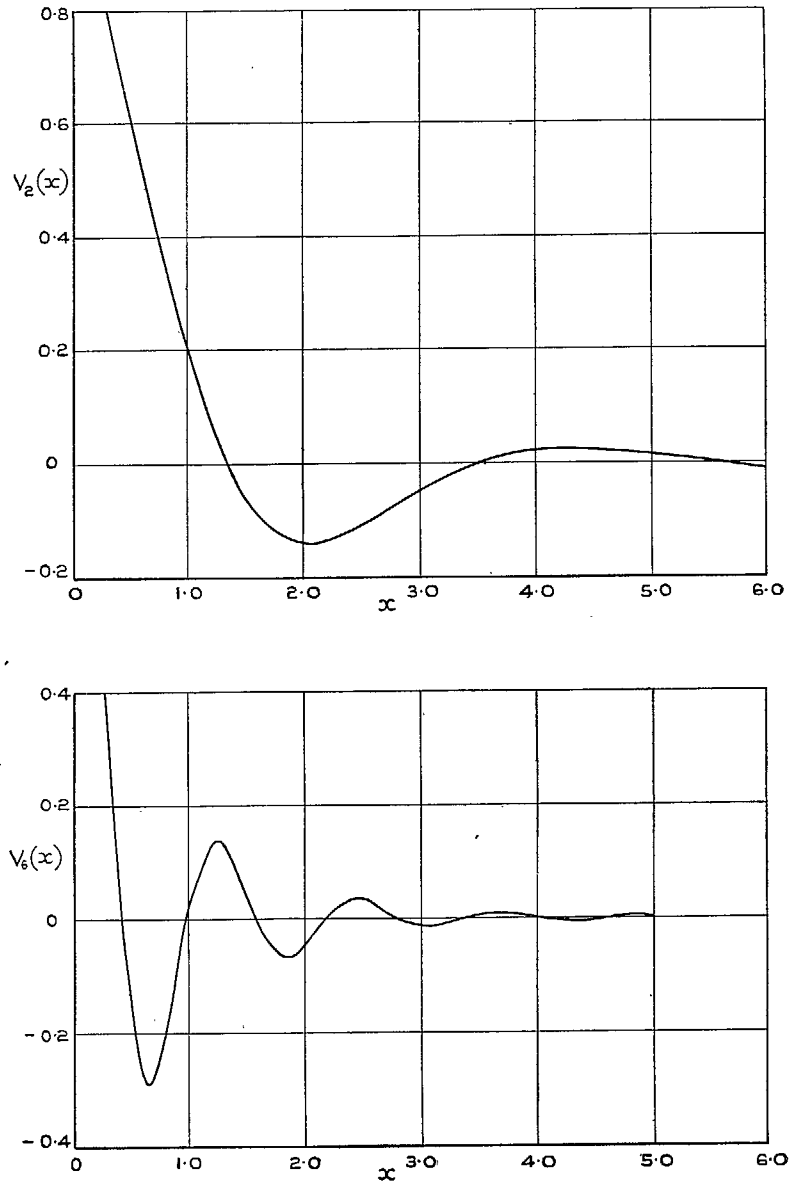

FIG. 2. Examples of the basic functions discussed in section 4.

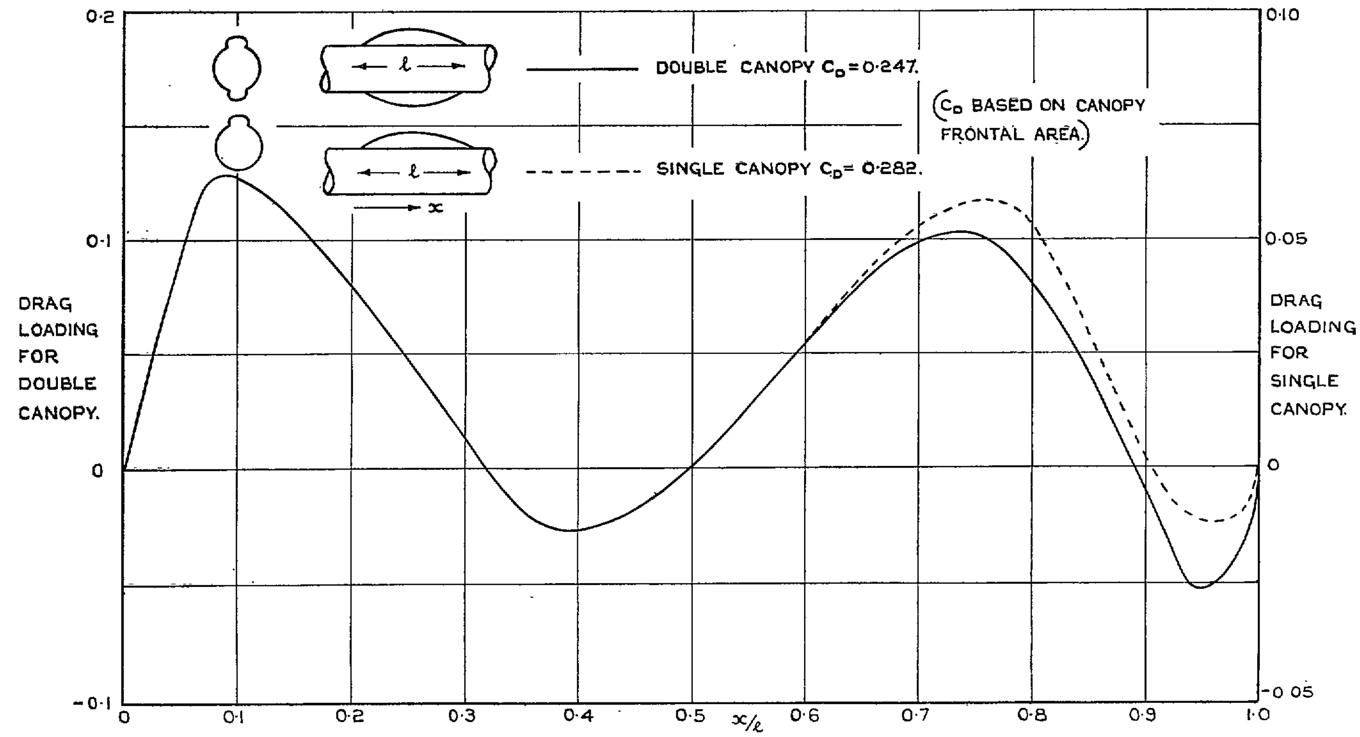

FIG. 3. Drag loading over the bodies of section 6 at $M = 1 \cdot 4$ (Drag loading defined in section 5).

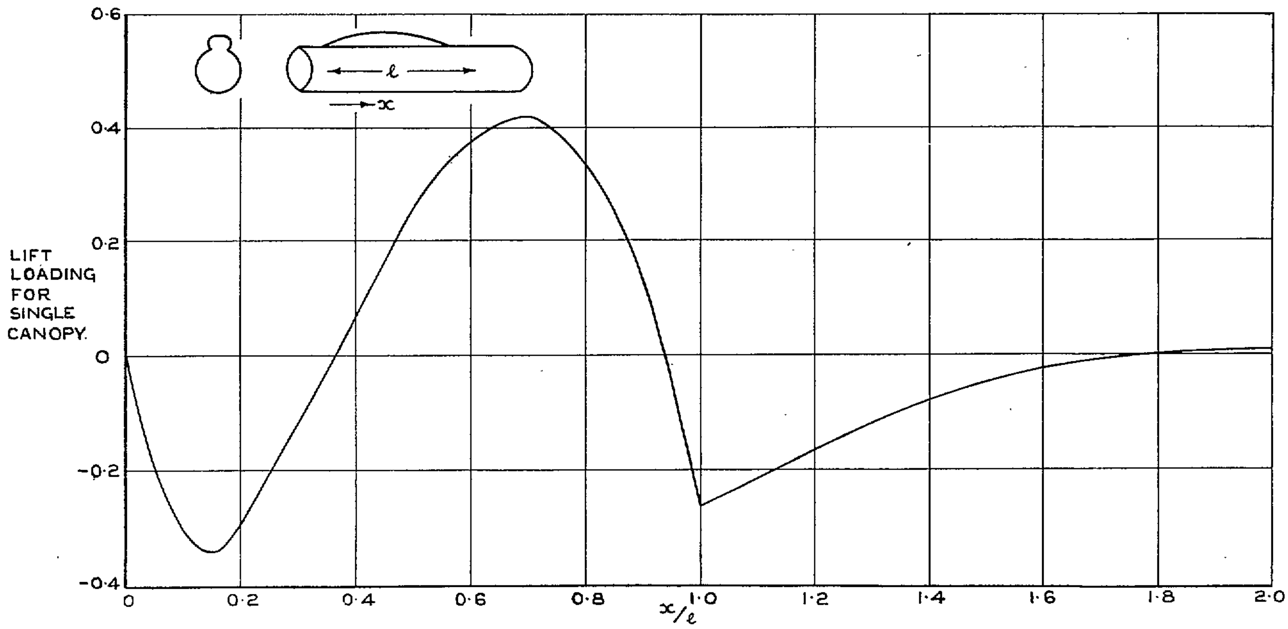

FIG. 4. Lift loading over the body of section 6 at $M = 1 \cdot 4$ (Lift loading defined in section 5).

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