R. & M. No. 3730

R. & M. No. 3730

MINISTRY OF DEFENCE (PROCUREMENT EXECUTIVE)

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Some Observations on Manoeuvre Stability and Longitudinal Control

By W. J. PINSKER
Aerodynamics Dept., R.A.E., Bedford

LONDON: HER MAJESTY'S STATIONERY OFFICE
1973
PRICE 85p NET

Some Observations on Manoeuvre Stability and Longitudinal Control

By W. J. G. PINSKER
Aerodynamics Dept., R.A.E., Bedford

Reports and Memoranda No. 3730*
March, 1972

Summary

Some of the more important practical implications to pilot's control, flight test analysis, and stability augmentor performance of classical longitudinal stability theory are discussed. The distinction between turns and pull-ups is re-emphasized and the differences quantified. Angular momentum of the engines is shown to make a contribution which can be significant with S.T.O.L. aircraft, being destabilizing in turns in one direction but stabilizing in the opposite direction.

* Replaces R.A.E. Technical Report 72068—A.R.C. 34 020.

LIST OF CONTENTS

1. Introduction

The theory of longitudinal stability and control would appear to be such a well documented and understood subject, that little could be added to the material presented in the many authoritative text books covering aircraft mechanics (e.g. Ref. 1). Indeed this has been the case since an exhaustive and rigorous analysis was established by Gates and Lyon in their classical study[2] in 1944. The principal concepts emerging from their work and that of their predecessors, such as the ideas of manoeuvre margin, static margin and CG margin, and a distinction between turns and pull-ups have become part of the standard equipment of the flight control specialist. However, there are certain refinements contained in the complete formulation of Gates' and Lyons' theory which have received much less attention or have become blurred by the passage of time. This may in part be due to the fact that they were quantitatively insignificant for the aircraft of the day and therefore then of mere academic interest. The one exception, perhaps, was Ref. 3 when, in recognition of the possible pitfalls in a too superficial treatment of manoeuvre stability, an appropriate flight test procedure was proposed for the measurement of longitudinal manoeuvre characteristics. This suggests that tests should only be carried out in pull-ups near level flight in the vertical plane so avoiding the complications of analysis which arise when tests are made in steady turning flight.

However, the phenomena which are recognised to upset or complicate analysis of flight data obtained in turning manoeuvres have consequences to other aspects of flight control as well, especially with the modern aircraft. It would seem appropriate, therefore, to re-examine the implication of Gates' and Lyons' complete theory and to establish the areas where they may be of practical significance. We shall consider in particular the aircraft which derives part of its longitudinal stability from artificial pitch-rate damping.

2. Theoretical Background

The longitudinal control requirements for steady or quasi-steady manoeuvres are defined by equilibrium of pitching moments:

$$-M_\eta \eta = M_0 + M_\alpha \alpha + M_q q + (C - A)rp - H_E r \cos \varepsilon_E - H_E \sin \varepsilon_E \tag{1}$$

when principal inertia axes are used to define r, p, C and A. In addition to the usual aerodynamic terms this equation also allows for the gyroscopic moments generated by the aircraft and by the angular momentum of the engine H_E. H_E is the angular momentum of all engines and is positive for engines rotating clockwise about the direction of flight. ε_E is the inclination of the engine rotor axis with respect to the airframe reference axes in which r and p are measured. (In the following analysis we shall use principal inertia axes.) It should also be noted that H_E will be zero if an even number of engines is installed in 'handed' pairs.

We shall be interested also in cases when pitch damping is augmented by automatic elevator control of the form

$$\eta_S = k_q q. \tag{2}$$

Introducing this term into equation (1) we get

$$-M_\eta \eta_p = M_0 + M_\alpha \alpha + (M_q + M_\eta k_q)q + (C - A)rp - H_E(r \cos \varepsilon_E + p \sin \varepsilon_E), \tag{3}$$

where η_p is now to be treated as that part of the actual elevator deflection commanded by the pilot. Assuming linear control circuit characteristics η_p will be directly proportional to stick force or deflection.

In order to be able to analyse equation (3), we require additional relationships between the motion variables appearing on the right-hand side and a parameter defining the manoeuvre under consideration. Usually normal acceleration is taken as most relevant for longitudinal control, although bank angle ϕ may be of equal interest when discussing steady turns.

The kinematic and dynamic relationships from which the motion variables can be derived in the general case are mathematically extremely involved so that we shall restrict our attention to only two specific manoeuvres in which they become more manageable, namely co-ordinated level flight turns and longitudinal manoeuvres purely in the vertical plane. These happen to be the two classes of greatest practical interest.

It should be noted that of all possible manoeuvres only the turn where the aircraft's path is a helical spiral, can be strictly considered as steady when all physically significant motion parameters, i.e. α, p, q, r, β and speed can be maintained constant. As we shall see later such a condition cannot be satisfied, e.g. in vertical manoeuvres.

Not only is it then impossible to maintain speed but even at constant speed, pitch rate will change when normal acceleration is held constant and *vice versa.* Nevertheless we shall treat manoeuvres in the vertical plane under the assumption that such changes (e.g. those in pitch rate) are slow enough to allow at least portions of the manoeuvre to be treated as quasi-steady.

2.1. Manoeuvres in the Vertical Plane

We shall only consider co-ordinated flight with zero sideslip, and in this case in a purely vertical manoeuvre (pull-up or push over) rates of yaw and roll will be zero. This eliminates the gyroscopic terms from equation (3). If aeroelastic effects are ignored or alternatively if changes in M_0 with normal acceleration due to aeroelasticity are expressed as an equivalent change in M_α, M_0 is irrelevant to manoeuvre stability and will therefore also be ignored.

This then leaves only three terms to be balanced by pilot's control, namely:

$$-M_\eta \eta_p = M_\alpha \alpha + M_q q + M_\eta k_q q \tag{4}$$

where α and η_p are increments from 1 g flight values.

We recognise immediately that in the absence of the last term representing the pitch autostabilizer contribution, the right-hand side of equation (4) is clearly related to the manoeuvre margin of the aircraft. Also we note that artificial pitch rate damping will simply add to the natural m_q contribution and therefore enhance the effective manoeuvre margin.*

In non-dimensional terms equation (4) reads:

$$-m_\eta \eta_p = m_w \alpha + m_q q \frac{l}{V} + m_\eta k_q q. \tag{5}$$

The two motion variables on the right-hand side are related to normal acceleration through

$$\alpha_0 = n \frac{W}{(\partial C_L/\partial_\alpha)(\rho/2V^2S)}, \tag{6}$$

where α_0 is now total effective incidence and n the absolute load factor, and

$$\dot{\gamma} = \frac{g}{V}(n - \cos\gamma) \tag{7}$$

provided $\alpha = \text{const}$, $\dot{\gamma} = q = (g/V)(n - \cos\gamma)$. The latter expression has been plotted in terms of Vq in Fig. 1.

Differentiation of equation (7) with respect to time gives

$$\frac{dq}{dt} = \frac{g}{V}\left(\frac{dn}{dt} + \dot{\gamma}\sin\gamma_0\right). \tag{8}$$

This expression tells us that a manoeuvre in the vertical plane can only be steady, i.e. $dq/dt = 0$ when $dn/dt = 0$, if $\sin\gamma_0 = 0$, that is when $\gamma_0 = 0$ degrees or 180 degrees. In all other cases, the treatment of manoeuvre stability in a quasi-steady sense is strictly not permissible. This observation prompted Lyons Ref. 3 to suggest restricting flight tests for manoeuvre stability to pull-ups near level flight.

Sufficiently close to level flight, when $\cos\gamma = 1$, equation (7) takes the form

$$q = \frac{g}{V}(n - 1) = \frac{g}{V}\Delta n \tag{9}$$

---

* This is of course only true if this signal is not transientized. With washout the steady manoeuvre stability of this aircraft will not be so affected.

and interpreting α as incremental from the trimmed condition, equation (6) becomes

$$\Delta\alpha = \Delta n \frac{W}{(\partial C_L/\partial\alpha)(\rho/2V^2S)}, \tag{10}$$

and with these expressions equation (5) can be transformed into

$$-m_\eta \Delta n_p = \Delta n \left(\frac{\partial C_m}{\partial C_L} + \frac{m_q}{\mu} + \frac{m_\eta}{\mu}\frac{V}{l}k_q\right) C_{L_0}, \tag{11}$$

with $C_{L_0} = W/(\rho/2V^2S)$ = trimmed level flight C_L and $\Delta\eta_p$ the increment in pilot-applied elevator deflection.

The term in brackets is the effective manoeuvre margin H_m; without artificial pitch damping ($k_q = 0$) it reduces to the basic aerodynamic manoeuvre margin of the aircraft $((\partial C_m/\partial C_L) + m_q/\mu)$. Equation (11) is only valid as a small perturbation approximation near level flight and this applies equally to any deductions we wish to make with respect to such quantities as elevator angle per g or stick force per g.

In manoeuvres starting *from* or taking the aircraft *to* a pitch attitude significantly different from zero, we must exercise caution. Also we must distinguish between these two cases. From Fig. 1 or equation (7) we note that as pitch attitude increases up to 180 degrees the pitch rate associated with a given normal acceleration increases, having its maximum in inverted flight. As a consequence the contribution to manoeuvre stability from m_q and also from k_q increases and an appropriately larger elevator angle has to be applied by the pilot to maintain a given value of normal acceleration. With equation (7) for q, the pitching moment equation then takes the general form:

$$-m_\eta \Delta n_p = C_{L_0}\left\{\frac{\partial C_m}{\partial C_L}(n-1) + (n - \cos\gamma)\left(\frac{m_q}{\mu} + k_q\frac{m_\eta}{\mu}\frac{V}{l}\right)\right\}. \tag{12}$$

Of course the difference from the simple linearized expression given by equation (11) is significant only if the pitch damping contributions are not negligible by comparison to the static margin $\partial C_m/\partial C_L$ i.e. equation (11) is valid generally if:

$$\left(\frac{m_q}{\mu} + \frac{m_\eta}{\mu}k_q\frac{V}{l}\right) \ll \frac{\partial C_m}{\partial C_L}. \tag{13}$$

The particular effects considered in equation (12) are only important if we consider large perturbation manoeuvres from trimmed level flight. Small perturbation manoeuvres on the other hand from any trimmed steady condition would still appear to be governed by the conventional manoeuvre margin, since the rate of change of q with Δn is the same for all γ (*see* Fig. 1). However, we recall equation (8) which suggests that such manoeuvres are not strictly conceivable as quasi-steady unless $\sin\gamma_0 = 0$.

We note from equation (8) that with n = const there will be a pitching acceleration proportional to pitch rate $q = \dot{\gamma}$ i.e.

$$\frac{dq}{dt} = \frac{g}{V} q \sin\gamma_0. \tag{14}$$

We can account for the associated inertia reaction by adding to the basic pitching moment equation (5) a term

$$-\dot{q}\frac{B}{\rho/2V^2Sl} = q i_B \mu \left(\frac{l}{V}\right)^2 \frac{g}{V}\sin\gamma_0$$

giving

$$-m_\eta \Delta\eta_p = \Delta n C_{L_0}\left(\frac{\partial C_m}{\partial C_L} + \frac{m_q}{\mu} + \frac{m_\eta}{\mu}\frac{l}{V}k_q - i_B\frac{l_g}{V^2}\sin\gamma_0\right). \tag{15}$$

The terms in the bracket then represent a general form of an apparent manoeuvre margin valid for manoeuvres which only involve small changes in flight path from any initial datum value γ_0. It is readily seen that the aircraft inertia term can only become significant at very low speed and is stabilising in climbing flight and *vice versa*.

2.2. Steady Turns

The only form of non-rectilinear flight that can be treated as strictly steady is the steady turn in which the aircraft's path forms a helical spiral about an axis aligned with the gravity vector. The level flight turn is a particular case of this general family of manoeuvres. Since in a turning manoeuvre the lateral motion parameters r and p are also involved, equation (3) must be considered in full. To do so we require the relevant kinematic relationships between all the variables involved. As this leads to elaborate algebra, this analysis is presented in the Appendix. Here we repeat only the final results.

If γ is the inclination of the flight path with respect to the horizontal, the resultant angular velocity Ω of the turn (measured in earth axes) is

$$\Omega = \pm\frac{g}{V}\sqrt{\left(\frac{n}{\cos\gamma}\right)^2 - 1}. \tag{16}$$

The flight path bank angle ϕ_a, i.e. the angle through which the aircraft has to be rolled about the flight path vector is given by

$$\sin\phi_a = \sqrt{1 - \left(\frac{\cos\gamma}{n}\right)^2}, \tag{17}$$

and the corresponding ordinary bank angle, which for clear identification we shall describe as Euler-bank angle, is

$$\tan\phi = \frac{\sin\phi_a}{\cos\alpha}\,\frac{1}{\cos\phi_a - \tan\gamma\tan\alpha}. \tag{18}$$

The three components of aircraft angular velocity measured in body axes, which are assumed to be at an incident α with respect to the flight path are:

$$q = \frac{g}{V}\left(n - \frac{\cos^2\gamma}{n}\right) \tag{19}$$

$$p_B = -\frac{g}{V}\left(\pm\sqrt{n^2 - \cos^2\gamma}\right)\left(\tan\gamma\cos\alpha + \sin\alpha + \frac{\cos\gamma}{n}\right) \tag{20}$$

$$r_B = \frac{g}{V}\left(\pm\sqrt{n^2 - \cos^2\gamma}\right)\left(\frac{\cos\gamma}{n}\cos\alpha - \tan\gamma\sin\alpha\right). \tag{21}$$

The product $r_B p_B$ required for the inertia cross coupling terms is

$$p_B r_B = -\left(\frac{g}{V}\right)^2 (n^2 - \cos^2\gamma)\left\{\frac{\sin\gamma}{n}(\cos^2\alpha - \sin^2\alpha) + \sin\alpha\cos\alpha\left(\frac{\cos^2\gamma}{n^2} - \tan^2\gamma\right)\right\}. \tag{22}$$

In level flight with $\gamma = 0$ these reduce to

$$\sin\phi_a = \sin\phi = \sqrt{1 - \frac{1}{n^2}}, \tag{23}$$

$$q = \frac{g}{V}\left(n - \frac{1}{n}\right), \tag{24}$$

$$p_B = -\frac{g}{V}\sin\alpha\left(\pm\sqrt{1-\frac{1}{n^2}}\right), \tag{25}$$

$$r_B = \frac{g}{V}\cos\alpha\left(\pm\sqrt{1-\frac{1}{n^2}}\right) \tag{26}$$

and

$$p_B r_B = -\left(\frac{g}{V}\right)^2 \sin\alpha\cos\alpha\left(1-\frac{1}{n^2}\right). \tag{27}$$

If $\alpha = 0$ the body axes angular velocities p_B and r_B become identical to wind axes values p_a and r_a, q is as given by equation (19) and

$$p_B = p_a = -\frac{g}{V}(\pm\sqrt{n^2-\cos^2\gamma})\tan\gamma, \tag{28}$$

$$r_B = r_a = \frac{g}{V}(\pm\sqrt{n^2-\cos^2\gamma})\frac{\cos\gamma}{n} \tag{29}$$

and

$$p_B r_B = p_a r_a = -\left(\frac{g}{V}\right)^2\left(n - \frac{\cos^2\gamma}{n}\right)\sin\gamma$$

$$= -\frac{g}{V}q_B\sin\gamma. \tag{30}$$

Where alternative signs ($\pm$) are indicated in these equations, the positive root is associated with turns to starboard and the negative sign with turns to port. In Figs. 2–5 these relationships are presented graphically. The results for roll and yaw rate according to equations (20) and (21) are complex functions of α and γ and only the special cases with $\alpha = 0$ and $\gamma = 0$ respectively have been selected for presentation in Figs. 4 and 5. The values shown refer to starboard turns, they change sign for corresponding port turns.

Perhaps the most generally important of these results is the relationship between pitch rate and normal acceleration shown in Fig. 2, or more particularly, the difference between this and the corresponding linear relationship applying in purely vertical manoeuvres (Fig. 1). These differences are most marked within the range of small normal acceleration increments and for better clarity they are compared at an enlarged scale in Fig. 6. We note that at a given value of n the consequent pitch rate in a turning manoeuvre is always greater than that resulting from a purely vertical manoeuvre. It can be readily shown that near the origin the rate of change of q with n for turning manoeuvres is exactly twice that observed in pull-ups. The relative difference between the two conditions tends to diminish with increasing n.

If we consider now the particular condition of level flight turns, we can write equation (3) as:

$$-m_\eta \eta_p = C_{Lo}\left\{\frac{\partial C_m}{\partial C_L}(n-1) + \left(n-\frac{1}{n}\right)\left[\frac{m_q}{\mu} + \frac{m_\eta}{\mu}\frac{V}{l}k_q - (i_C - i_A)\frac{g}{V}\frac{l}{V}\sin\alpha\cos\alpha\right]\right.$$

$$\left. -\frac{H_E}{mlV}\left(\pm\sqrt{1-\frac{1}{n^2}}\right)\cos\alpha\cos\varepsilon_E(1-\tan\alpha\tan\varepsilon_E)\right\}. \tag{31}$$

Only in V/S.T.O.L. designs with lifting engines installed in unconventional alignment can ε_E be large enough to justify $\cos\varepsilon_E \neq 1$, normally ε_E will be small and since the range of incidence of practical interest is also small we can simplify equation (31) to

$$-m_\eta \eta_p = C_{Lo}\left\{\frac{\partial C_m}{\partial C_L}(n-1) + \left(n-\frac{1}{n}\right)\left[\frac{m_q}{\mu} + \frac{m_\eta}{\mu}\frac{V}{l}k_q - \frac{i_C - i_A}{n}\frac{g}{V}\frac{l}{V}\sin\alpha\right] - \frac{H_E}{mlV}\left(\pm\sqrt{1-\frac{1}{n^2}}\right)\right\}. \tag{32}$$

All the contributions to equations (31) and (32) are independent of the direction of the turn with the exception of the engine momentum term H_E which therefore must be expected to be stabilizing or destabilizing depending on the sense of the turn manoeuvre.

An observation of more general significance is that of the terms considered, only that associated with the C.G. margin $\partial C_m/\partial C_L$ is linear with respect to n. As a consequence the simple concept of relating pilot's control η_p directly to the manoeuvre margin of the aircraft is not valid in turning manoeuvres. As with a similar situation described in the earlier discussion of vertical manoeuvres, these nonlinear effects are only of practical significance, if the terms other than $\partial C_m/\partial C_L$ in equation (31) are not numerically negligible by comparison with it. This will normally be true for the m_q contribution and—if applicable—for the autostabilizer term k_q. The gyroscopic terms associated with the aircraft inertia distribution $(i_C - i_A)$ and with engine momentum H_E can be significant only at low speeds, since they vary with $1/V^2$ or $1/V$ respectively. They could therefore play an important role in S.T.O.L. aircraft. The $(i_C - i_A)$ term is also proportional to $\sin\alpha$, the incidence of the principal inertia axis which will always be small in the type of manoeuvre under consideration.

We shall now discuss the practical significance of the relationships derived in this section, in particular we shall consider pilot control, flight testing and automatic control.

3. Pilot's Control

The immediate consequence of the results of the above analysis is that the concept of a simple and linear relationship between the aircraft manoeuvre margin and such control parameters as stick force per g is strictly only valid within a very restricted range of manoeuvres, namely those in the vertical plane close to normal level flight. In every other case, such as in banked turns and in manoeuvres involving large pitch attitude, more sophisticated analysis is required to predict elevator control requirements. The additional terms as defined in equations (12), (15) and (21) are additive, with the exception of the engine gyroscopic reactions H_E in (21) which can have either sign. Hence the amount of pilot's control required to maintain such manoeuvres will generally be greater than that suggested by simple linear analysis based on the concept of the manoeuvre margin.

Manoeuvres involving large pitch attitude changes belong to the realm of aerobatics. Of more general interest, however, are turning manoeuvres where due to the larger pitch rates associated with a given Δn, the pitch damping contribution will be amplified and lead to an apparent increase in manoeuvre stability. The magnitude of this effect depends on the proportion of the basic aircraft manoeuvre margin that is provided by pitch damping. This is illustrated in Fig. 7. The range covered in this diagram extends to the extremes when pitch damping gives more than 100 per cent of the manoeuvre margin H_m i.e. to cases where the aerodynamic C.G. margin is negative. It is also seen that these effects become less powerful as n increases, i.e. they are more important within the modest manoeuvre envelope of a transport aircraft than for fighter aircraft.

Another case needing careful consideration is the S.T.O.L. aircraft operating at low speed with substantial rotating engine machinery, i.e. large H_E. This term changes sign with the direction of the turn. We illustrate by a numerical example. Assume an aircraft with 90 000 kg mass (200 000 lb weight) having two fan engines of the type used on the airbus project, $H_E = 430\,000\ \mathrm{kg\,m^2\,s^{-1}}$ (320 000 lb ft s) angular momentum. Using the mean chord as the reference length $l = 6{\cdot}5$ m, at $V = 80$ kn the resulting change in the effective manoeuvre margin—depending on the direction of the turn—is a function of applied normal acceleration:

normal g	ΔH_m *due to* H_E	
1·1	$\pm 7{\cdot}4\%$	of reference chord
1·25	$\pm 4{\cdot}3\%$	
1·5	$\pm 2{\cdot}7\%$	

If the engines are rotating clockwise in the direction of flight $(H_E > 0)$ the + sign is associated with turns to starboard and *vice verse.* In a turn to port the aircraft will appear—as far as elevator control is concerned—as if it had its manoeuvre margin reduced by the amounts shown in the table above. In gentle manoeuvres this this may mean that the stick has to be pushed forward to stabilize a banked turn. In turns to starboard the opposite effect would occur, substantial stick pull being required for quite modest manoeuvres. At a typical approach incidence the contribution in this case from the aircraft inertia term $(i_C - i_A)$ would be of the order of $\Delta H_m = +2$ per cent, always in the stabilizing sense, reducing to about +1 per cent for more severe manoeuvres. It should be noted that these contributions to what we defined as an *effective* manoeuvre margin are only relevant in the context of control of steady manoeuvres, they do *not* affect the proper manoeuvre margin defining dynamic longitudinal stability. The arguments developed in this report are *not* relevant to stability as such.

4. Flight Test Analysis

The area in which the implications of the results given in Section 2 of this Report are perhaps most widely appreciated is that of flight testing for the determination of longitudinal manoeuvre stability. In Ref. 3 it is suggested that complications and possible errors in the analysis of flight data are best avoided by making measurements in straight pull-ups close to horizontal flight. On the other hand, steady turns are clearly more attractive in allowing stabilized flight to be maintained for a substantial period of time, thereby offering a more secure basis for data acquisition. The technique is perfectly permissible if the analysis takes full account of the terms on the right-hand side of equation (3). It will be necessary, however, in this case to have prior knowledge of the aerodynamic derivative m_q as well as of the inertia parameters $(i_C - i_A)$. In modern aircraft-design practice it can generally be assumed that this data will be known at the flight test stage with an accuracy sufficient for the purpose of such analysis.

One may note that the difference between the proper manoeuvre margin, $((\partial C_m/\partial C_L) + m_q/\mu)$, and the more complex and nonlinear relationship defining elevator control in steady turns tend to diminish with increasing g, so that they can perhaps be ignored when testing near the extremes of the manoeuvre envelope of fighter type aircraft. Within the range of g's appropriate for transport aircraft, such short cuts are, however, not permissible.

5. Automatic Control

The kinematic and dynamic effects analysed in this Report have important consequences to automatic control. One instance when this was encountered is reported in Ref. 4, discussing the development of a take-off director. A flight director can be seen as equivalent in general principle to an automatic control, using the human pilot as a servo actuator, and in this sense the design requirements for a director are identical to those of a fully automatic system. The pitch control channel of the device described in Ref. 4 was designed as a pitch rate demand system, using pitch attitude, air speed, horizontal acceleration and vertical velocity signals to direct the pilot towards a safe and efficient climbout. When an aircraft banks either inadvertently or in a deliberate manoeuvre, a pitch rate is generated according to equation (17) without there being any change in pitch attitude. This would be sensed by the director and give rise to a nose-down control demand. In order to avoid this undesirable response, a bank compensation term was provided, designed to subtract $\Delta q = (g/V) \tan \phi \sin \phi$ (or rather an approximation to this law) from the measured aircraft pitch rate. This technique is well-known to the autopilot designer.

The same kinematic phenomenon also affects the operation of the pitch demand of a stability augmentation system, although here the consequences are of an entirely different nature and not so widely appreciated. Pitch rate is the feedback signal most commonly used to improve short period dynamics. Pitch rate may be used directly (untransientized) to drive the elevator or it may be transientized through an appropriately-chosen washout circuit. In some designs both types of signal are used simultaneously. What we want to discuss here applies strictly to an untransientized pitch damping signal, although the presence of a transientized signal will also have some effect in manoeuvres maintained only for short periods. If pitch rate damping is applied through a control law

$$\eta_S = k_q q, \tag{33}$$

and the autostabilizer is allowed authority over $\pm\eta_L^0$ elevator, the system will saturate if pitch rate exceeds

$$q \geqslant \frac{\eta_L}{k_q}. \tag{34}$$

We see from Fig. 6 that in steady flight, a given pitch rate is associated with different values of normal acceleration depending on the nature of the manoeuvre. In steady turns, such a system will saturate at much lower values of normal accelerations than in a straight pull-up. Let us consider a pitch damper operating with a gain of 1 degree elevator per degree/second pitch rate, i.e. $k_q = 1$, having authority over ± 2 degree elevator. It will saturate therefore when $q > 2$ degrees/second. From Fig. 6 we note that at $V = 400$ kn ($qV = 800$) this device will saturate at $n = 1{\cdot}43$ g in turns but at $n = 1{\cdot}72$ in vertical manoeuvres. The corresponding limits at $V = 200$ kn would be 1·2 g and 1·37 g respectively. Therefore in turning manoeuvres the pilot will lose the benefit of the autostabilizer much sooner than in vertical manoeuvres. This can be embarrassing if the aircraft depends critically on stability augmentation for safe handling and is only marginally controllable in its unaided condition. If the difference between vertical and turning flight is not properly anticipated an aircraft equipped with a pitch autostabilizer having just sufficient authority to cover its manoeuvre envelope in plain vertical manoeuvres, assuming $q = (g/V)(n - 1)$, will find itself insufficiently covered during turns.

The early saturation in turns is due to the fact that the autostabilizer reacts to total pitch rate, only part of which is associated with longitudinal motion proper, i.e. change in q with n at ϕ = const. The excess elevator it applies in response to the additional pitch rate generated by bank angle is unnecessary, providing, in fact, an excess in manoeuvre stability. Assuming $\cos\theta = 1$ we can calculate this 'false' pitch rate as

$$\Delta q = \frac{g}{V}\left\{\left(n - \frac{1}{n}\right) - (n - 1)\right\} = \frac{g}{V}\left(1 - \frac{1}{n}\right) = \frac{g}{V}(1 - \cos\phi). \quad (35$$

With equation (26) this can be reduced to

$$\Delta q = \tan\left(\frac{\phi}{2}\right) r. \quad (36$$

If Δq, generated from the terms defined above, is subtracted from measured pitch rate to provide a correctec input to the pitch damper, the autostabilizer will then respond only to proper longitudinal motion and saturate at the same value of n irrespective of the nature of the manoeuvre.

6. Dynamic Stability

We had already made the observation that the phenomena discussed here as relevant to manoeuvre stability do not influence directly dynamic longitudinal stability, or more specifically the role of the manoeuvre margin in the short period oscillation. Dynamic stability analysis is concerned with small perturbation disturbances within the plane of symmetry of the aircraft. In this plane, increments in normal accelerations (or incidence) will always command a linearly proportional increment in pitch rate (i.e. $\Delta q = (g/V)\Delta n$) without change in yaw or roll rate; in consequence the classical manoeuvre margin as defined in equation (11) always applies.

It may in fact be said that the manoeuvre margin H_m, popularly thought of as a universal longitudinal control parameter, is much more closely related to dynamic stability and that its relationship with elevator control is far less direct.

7. Conclusions

The practical implications of classical manoeuvre stability theory (Ref. 2) are re-examined. Particular attention is given to certain requirements contained in Gates' and Lyon's theory, which are usually ignored and may become significant for instance with S.T.O.L. designs. In addition the case is considered when artificial pitch damping is used to supplement the natural longitudinal stability of the aircraft. In this study the distinction in control between turning manoeuvres and pull-ups in the vertical plane is re-emphasized and the differences are quantified. An effect particularly significant for S.T.O.L. flight is that generated by the angular momentum of the engines which changes sign with the direction of the turn. In aircraft flying with a small manoeuvre margin turns to starboard may require the pilot to push the stick for coordination in sustained manoeuvres, whereas in the equivalent turn to port an unexpectedly large amount of pull may be needed.

Implications of these findings are discussed in relation to pilot's control, flight test analysis and automatic control design.

None of the effects discussed have any influence on the stability of the short period oscillations.

LIST OF SYMBOLS

A	Aircraft inertia in roll
B	Aircraft inertia in pitch
C	Aircraft inertia in yaw
C_L	Lift coefficient
$C_{L_0} = \dfrac{W}{\rho/2 V^2 S}$	Level flight lift coefficient
$C_m = \dfrac{M}{\rho/2 V^2 S l}$	Pitching moment coefficient
C_{m_0}	Zero incidence pitching moment coefficient
g	Earth gravity
H_E	Rotary engine momentum—positive for clockwise rotation seen in the direction of flight
H_m	Manoeuvre margin
$i_A = \dfrac{A}{ml^2}$	Aircraft inertia parameter in roll
i_B	Aircraft inertia parameter in pitch
i_c	Aircraft inertia parameter in yaw
k_q	Pitch damper gain
L	Lift
l	Aircraft reference length
m	Aircraft mass
M	Pitching moment
M_0	Zero incidence pitching moment
$M_\alpha = \dfrac{\partial M}{\partial \alpha}$	Static longitudinal stability
$M_q = \dfrac{\partial M}{\partial q}$	Pitch damping
$M_\eta = \dfrac{\partial M}{\partial \eta}$	Elevator power
$m_w = \dfrac{\partial C_m}{\partial \alpha}$, $m_q = \dfrac{\partial C_m}{\partial (ql/V)}$, $m_\eta = \dfrac{\partial C_m}{\partial \eta}$	Non-dimensional pitching moment derivatives
n	Normal acceleration load factor
p	Rate of roll
q	Rate of pitch
r	Rate of yaw
S	Wing reference area

V	True airspeed
V_H	Component of airspeed in the horizontal plane
W	Aircraft weight
α	Incidence of principal inertia axis
γ	Flight path inclination
η	Elevator angle
η_p	Elevator angle commanded by the pilot
η_S	Elevator angle commanded by the auto-stabilizer
ϕ	Euler bank angle (body axes)
ϕ_a	flight path axes bank angle
θ	Pitch attitude
θ_0	Datum pitch attitude
$\mu = \frac{2m}{\rho l S}$	Relative density
ρ	Air density
ε_E	Incidence of the engine rotor axis with respect to the longitudinal aircraft reference axis

Suffices

B	Body fixed axes, normally principal inertia axes
a	Flight path axes
o	Initial flight condition

REFERENCES

No.	Author(s)	Title, etc.
1	B. Etkin	*Dynamics of flight, stability and control.* John Wiley & Sons Inc. New York 1965.
2	S. B. Gates and H. M. Lyon	A continuation of longitudinal stability and control analysis. Part 1 general theory. A.R.C. R & M. 2027 (1944).
3	D. J. Lyons	An examination of the technique of the measurement of the longitudinal manoeuvring characteristics of an aeroplane, and a proposal for a standardized method. A.R.C. R. and M. 2597 (1947).
4	C. O. O'Leary and J. N. Cannel ..	Flight tests of a pitch rate take-off and overshoot director on a Vulcan aircraft. R.A.E. Technical Report 68136 (1968), A.R.C. 30815.
5	H. R. Hopkin	A scheme of notations and nomenclature for aircraft dynamics and associated aerodynamics, Part 2—Basic notation and nomenclature. A.R.C. R. and M. 3562, Part 2 (1970).

APPENDIX

Kinematics of Steady Turning Flight

We consider a steady helical turn with the resultant angular velocity Ω and flight path inclination γ.

The angular velocity components about the three aircraft axes using the aerodynamic (flight path axes) system are

$$p_a = -\Omega \sin \gamma, \tag{A–1}$$

$$q_a = \Omega \sin \phi_a \cos \gamma \tag{A–2}$$

and

$$r_a = \Omega \cos \phi_a \cos \gamma. \tag{A–3}$$

Since in the equations of motion we found it more convenient to use principal inertia axes we have to transform these rates to principal inertia axes, being at an incidence α with regard to flight path axes, hence

$$p_B = p_a \cos \alpha - r_a \sin \alpha = -\Omega(\sin \gamma \cos \alpha + \cos \phi_a \cos \gamma \sin \alpha), \tag{A–4}$$

$$q_B = q_a = \Omega \sin \phi_a \cos \gamma \tag{A–5}$$

and

$$r_B = r_a \cos \alpha + p_a \sin \alpha = \Omega(\cos \phi_a \cos \gamma \cos \alpha - \sin \gamma \sin \alpha). \tag{A–6}$$

We now require a relationship between Ω and normal acceleration and/or bank angle.

The centrifugal force Z is

$$Z = mR\Omega^2 \tag{A–7}$$

where R is the radius of the helical flight path.

The relationship between R and Ω involves speed, but the speed V_H required is the component of airspeed V in the horizontal plane, i.e. in a plane normal to the axis of the helix. Since

$$V_H = V \cos \gamma$$

and

$$R = \frac{V}{\Omega} \cos \gamma \tag{A–8}$$

hence

$$Z = mV\Omega \cos \gamma. \tag{A–9}$$

The centripetal component of lift can only be meaningfully derived by considering the flight path bank angle ϕ_a rather than ϕ and we get then

$$Z = L \sin \phi_a. \tag{A–10}$$

Since

$$L \cos \phi_a = W \cos \gamma$$

$$\frac{L}{W} = n = \frac{\cos \gamma}{\cos \phi_a}. \tag{A–11}$$

Equating equations (A–7) and (A–10) we get

$$mV\Omega \cos\gamma = L \sin\phi_a$$

or after division by W

$$\frac{V}{g}\Omega \cos\gamma = n \sin\phi_a.$$

Hence

$$\Omega = n\frac{\sin\phi_a}{\cos\gamma}\frac{g}{V}. \qquad \text{(A–12)}$$

We can eliminate n by introducing (A–11) to get

$$\Omega = \frac{g}{V}\tan\phi_a, \qquad \text{(A–13)}$$

or alternatively if we are more interested in normal acceleration as the primary parameter with

$$\sin\phi_a = \pm\sqrt{1-\cos^2\phi_a} = \pm\sqrt{1-\frac{\cos^2\gamma}{n^2}} \qquad \text{(A–14)}$$

we can write

$$\Omega = (\pm\sqrt{n^2-\cos^2\gamma})\frac{1}{\cos\gamma}\frac{g}{V} \qquad \text{(A–15)}$$

and remind ourselves that the positive root relates to positive bank, i.e. turns to starboard and *vice versa*.

Introducing (A–15) into (A–5) we get finally

$$q_B = \frac{g}{V}(\pm\sqrt{n^2-\cos^2\gamma})\sin\phi_a. \qquad \text{(A–16)}$$

The kinematic relationship between the flight path bank angle ϕ_a and the Euler bank angle ϕ is given in Ref. 5 for $\beta = 0$ as

$$\tan\phi = \frac{\sin\phi_a}{\cos\alpha}\frac{1}{\cos\phi_a - \tan\gamma\tan\alpha}. \qquad \text{(A–17)}$$

For level flight turns with $\gamma = 0$ this simplifies to

$$\tan\phi = \frac{\tan\phi_a}{\cos\alpha}.$$

Introducing equation (A–17) into (A–16) gives

$$q_B = \frac{g}{V}\left(n - \frac{\cos^2\gamma}{n}\right)\cos\gamma. \qquad \text{(A–18)}$$

An equivalent process applied to equation (A–4) leads to an expression for roll rate

$$p_B = -\frac{g}{V}(\pm\sqrt{n^2-\cos^2\gamma})\left(\tan\gamma\cos\alpha + \sin\alpha\frac{\cos\gamma}{n}\right) \qquad \text{(A–19)}$$

and for yaw rate we obtain

$$r_B = \frac{g}{V}(\pm\sqrt{n^2 - \cos^2 \gamma})\left(\frac{\cos \gamma}{n} \cos \alpha - \tan \gamma \sin \alpha\right) \tag{A–20}$$

We shall also require for the aircraft inertia reaction terms the product of p and r,

$$p_B r_B = -\left(\frac{g}{V}\right)^2 (n^2 - \cos^2 \gamma)\left\{\frac{\sin \gamma}{n}(\cos^2 \alpha - \sin^2 \alpha) + \sin \alpha \cos \alpha\left(\frac{\cos^2 \gamma}{n^2} - \tan^2 \gamma\right)\right\}. \tag{A–21}$$

For the special case of level turns, $\gamma = 0$ and

$$q_B = \frac{g}{V}\left(n - \frac{1}{n}\right),$$

$$p_B = -\frac{g}{V}\left(\pm\sqrt{1 - \frac{1}{n^2}}\right) \sin \alpha,$$

$$r_B = \frac{g}{V}\left(\pm\sqrt{1 - \frac{1}{n^2}}\right) \cos \alpha$$

and

$$p_B r_B = -\left(\frac{g}{V}\right)^2 \left(1 - \frac{1}{n^2}\right) \sin \alpha \cos \alpha. \tag{A–22}$$

On the other hand assuming $\alpha = 0$

$$q_B = \frac{g}{V}\left(n - \frac{\cos^2 \gamma}{n}\right)$$

$$p_B = -\frac{g}{V}(\pm\sqrt{n^2 - \cos^2 \gamma}) \tan \gamma$$

$$r_B = \frac{g}{V}(\pm\sqrt{n^2 - \cos^2 \gamma})\frac{\cos \gamma}{n}$$

$$p_B r_B = -\frac{g}{V}\left(n - \frac{\cos^2 \gamma}{n}\right) \sin \gamma. \tag{A–23}$$

Finally we may be interested in the relationship between bank angle ϕ_a and normal acceleration which from (A–11) can be written as

$$\cos \phi_a = \frac{\cos \gamma}{n}. \tag{A–24}$$

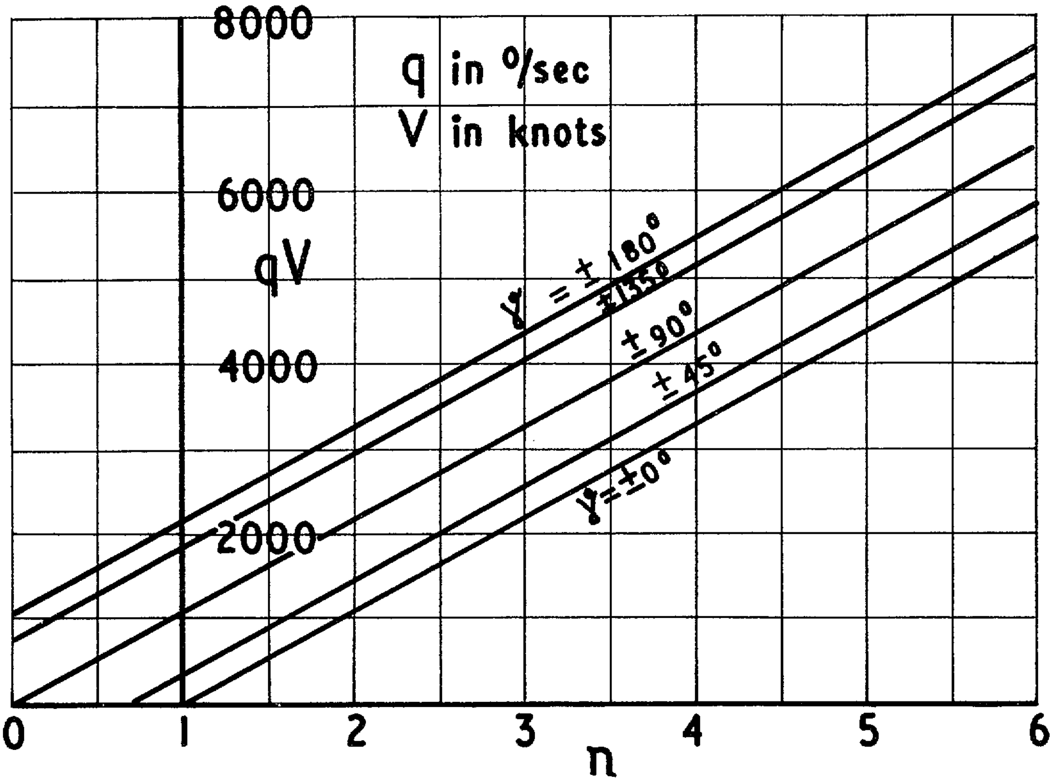

FIG. 1. Pitch rate in quasi-steady longitudinal manoeuvres in the vertical plane.

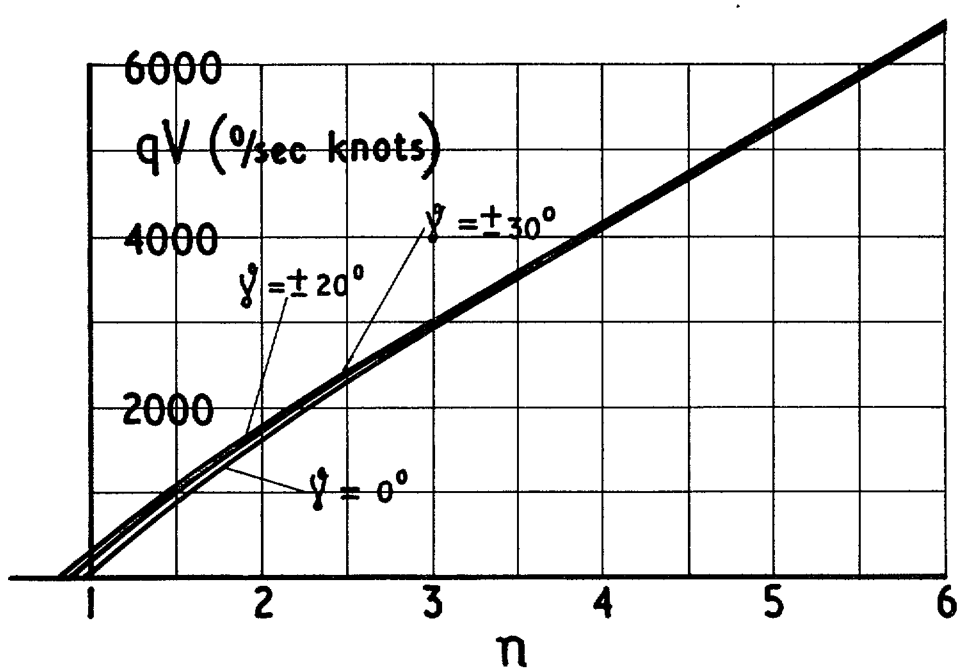

FIG. 2 Pitch rate in steady turning flight.

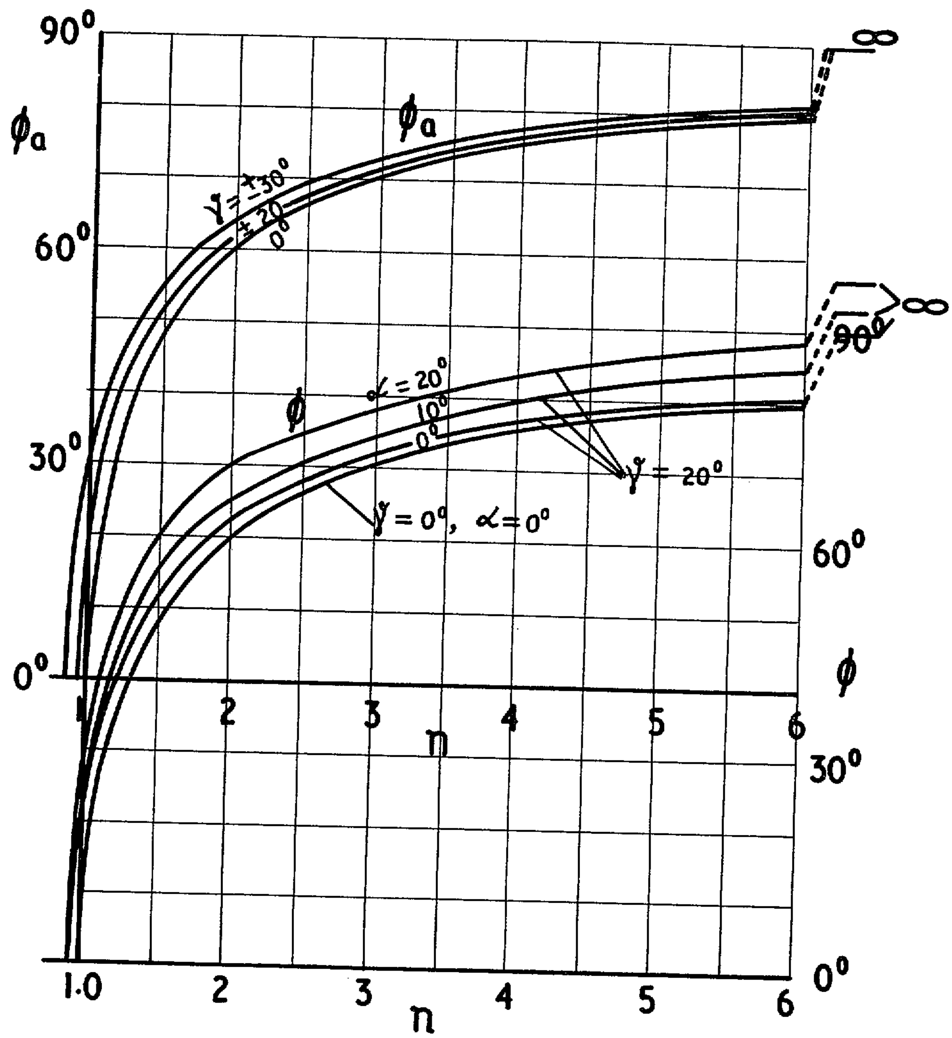

FIG. 3. Flight path bank angle ϕ_a and Euler bank angle ϕ versus n in steady turns.

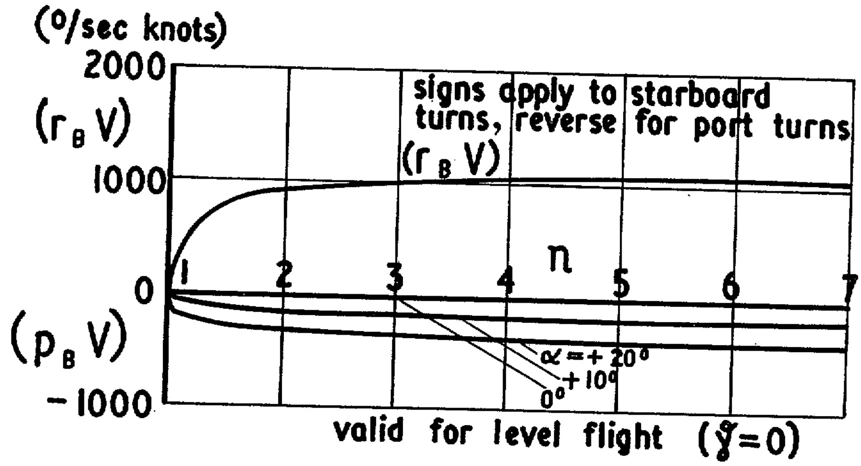

FIG. 4. Body axis yaw and roll rate in level flight steady turns.

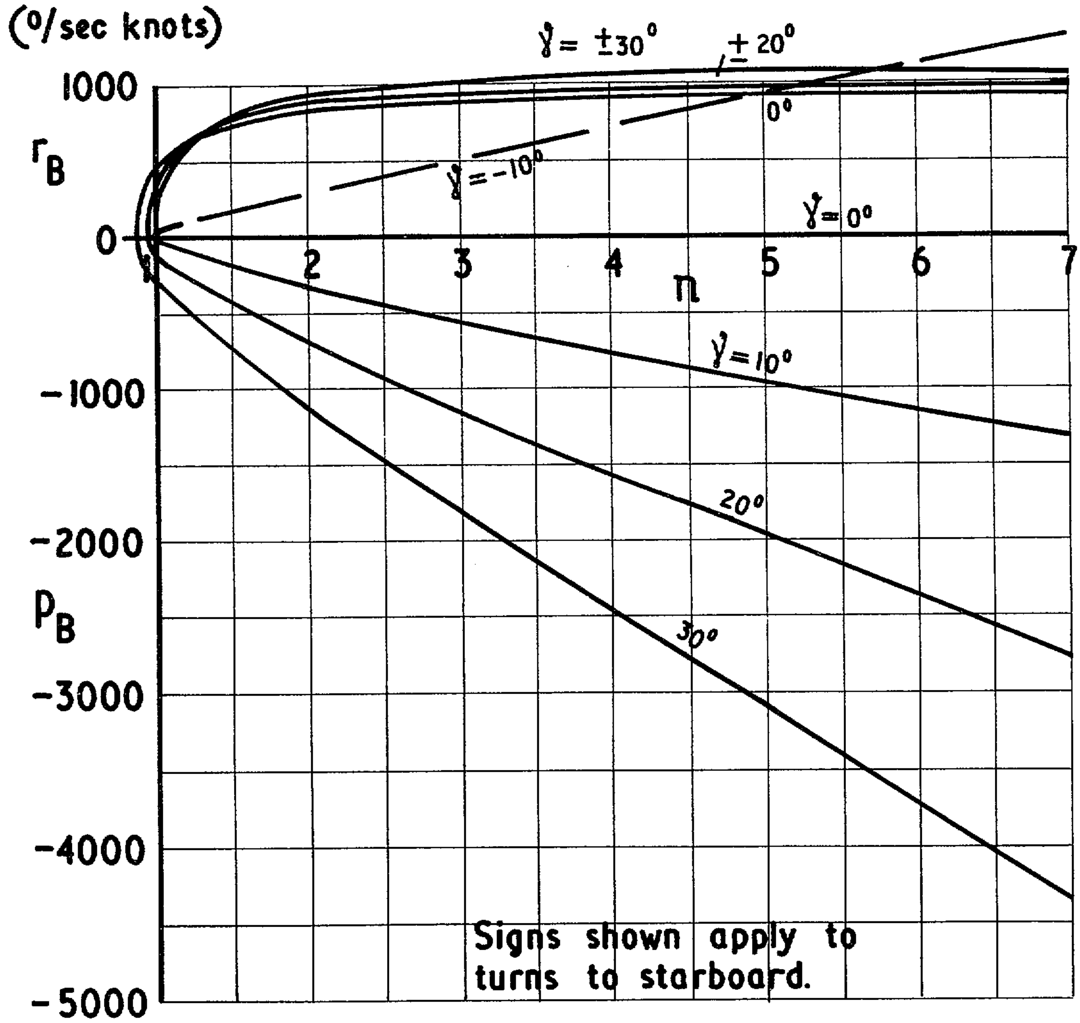

FIG. 5. Yaw and roll rate in steady turns as a function of glide path angle γ and normal acceleration load factor n. Results are body axes rates if $\alpha = 0$ or rates in stability axes irrespective of α.

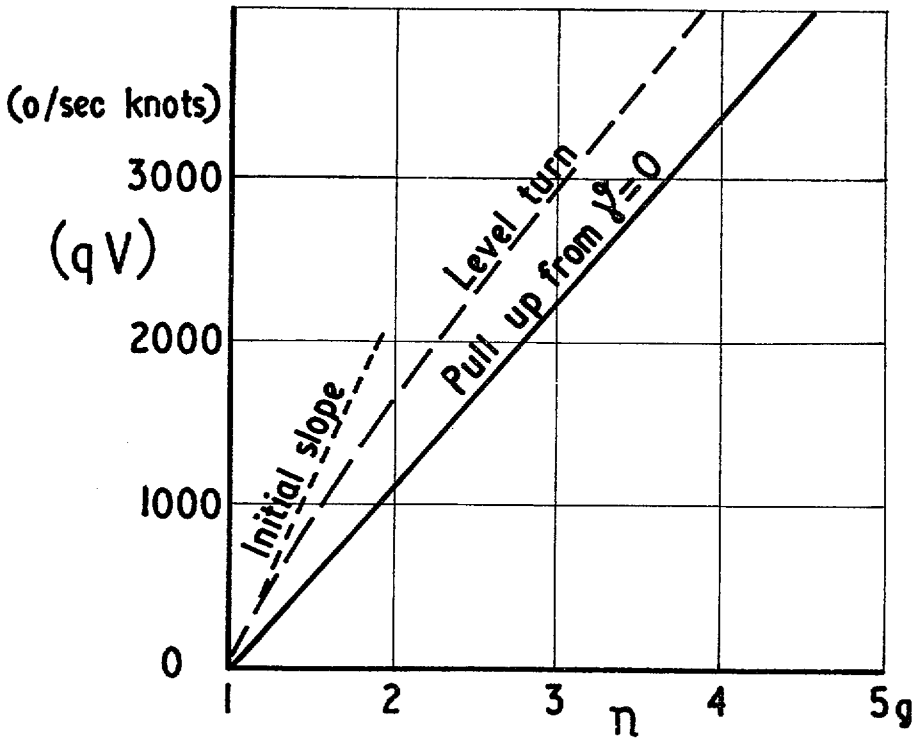

FIG. 6. Comparison of pitch rate $q(n)$ in turns and straight pull ups.

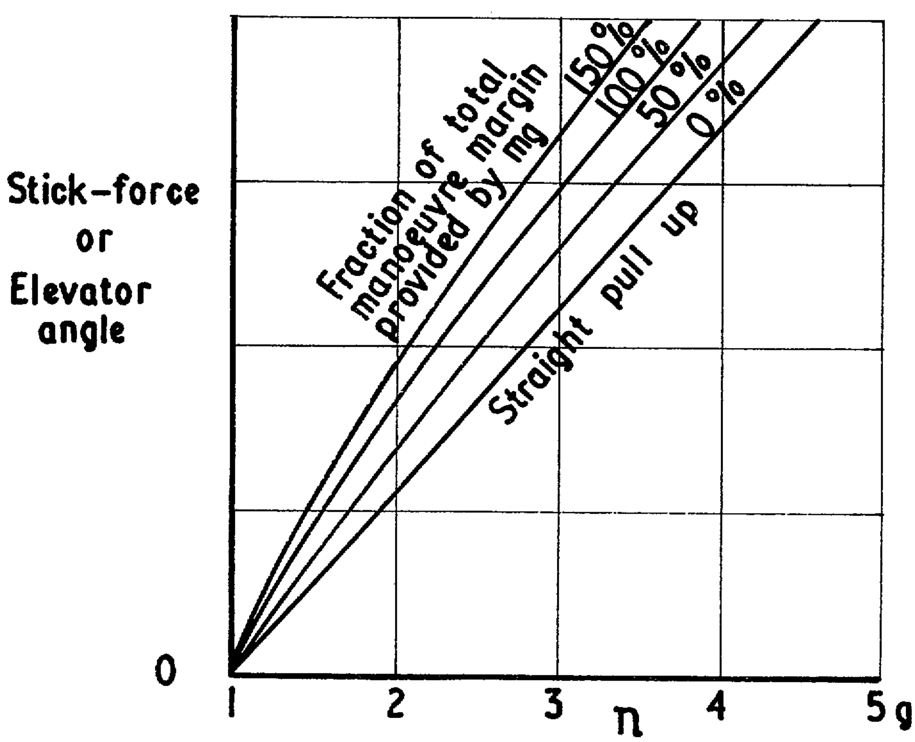

FIG. 7. Effect on elevator control characteristics of pitch rate kinematics in turning manoeuvres.

Printed in England for Her Majesty's Stationery Office by J. W. Arrowsmith Ltd, Bristol BS3 2NT
Dd. 505715 K5 9/73

HER MAJESTY'S STATIONERY OFFICE

Government Bookshops

49 High Holborn, London WC1V 6HB
13a Castle Street, Edinburgh EH2 3AR
41 The Hayes, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
Wine Street, Bristol BS1 2BQ
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY

Government publications are also available through booksellers

R. & M. No. 3730
ISBN 0 11 470823 1