REPORT 1199

A STUDY OF THE PROBLEM OF DESIGNING AIRPLANES WITH SATISFACTORY INHERENT DAMPING OF THE DUTCH ROLL OSCILLATION[1]

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SUMMARY

Considerable interest has recently been shown in means of obtaining satisfactory stability of the Dutch roll oscillation for modern high-performance airplanes without resort to complicated artificial stabilizing devices. One approach to this problem is to lay out the airplane in the earliest stages of design so that it will have the greatest practicable inherent stability of the lateral oscillation. The present report presents some preliminary results of a theoretical analysis to determine the design features that appear most promising in providing adequate inherent stability. These preliminary results cover the case of fighter airplanes at subsonic speeds.

The investigation indicated that it is possible to design fighter airplanes to have substantially better inherent stability than most current designs. Since the use of low-aspect-ratio sweptback wings is largely responsible for poor Dutch roll stability, it is important to design the airplane with the maximum aspect ratio and minimum sweep that will permit attainment of the desired performance. The radius of gyration in roll should be kept as low as possible and the nose-up inclination of the principal longitudinal axis of inertia should be made as great as practicable.

INTRODUCTION

The problem of obtaining satisfactory stability of the Dutch roll oscillation is especially difficult for jet-propelled swept-wing airplanes designed for operation at high speeds and altitudes. The present trend is toward the use of artificial stabilizing devices to provide satisfactory stability since it is usually not possible to modify an existing airplane to provide satisfactory inherent stability. One of the fundamental reasons for the poor inherent stability seems to be that very little consideration is given to dynamic stability in the early stages of design; that is, the basic design of the airplane is determined from other considerations and attempts are made later to improve the dynamic stability by the minor changes in configuration which are then permissible in the design. If such a procedure is continued, all airplanes of this type will probably require artificial stabilizing devices. The armed services and some airplane manufacturers are becoming increasingly concerned over the necessity for using these devices which increase the weight, complexity, and cost of the airplanes. The fact that the use of these devices increases the maintenance problem has been of particular concern to the services.

This concern has led to an increasing interest in means of obtaining satisfactory stability without resort to complicated artificial stabilizing devices. Various methods for accomplishing this aim have been proposed, the most fundamental and perhaps the most promising of which is to alter present design procedures to the extent of giving much more consideration in the early stages of design to features which will lead to better dynamic stability. A study is being made by the National Advisory Committee for Aeronautics to determine the design features which appear most promising in this respect. Some preliminary results of this investigation are included in the present report which covers the case of fighter airplanes at subsonic speeds. The period and damping are the only characteristics of the Dutch roll oscillation considered in detail in the present report.

As a preliminary to the investigation of means of providing inherent stability, a study of the basic causes of the poor stability of modern high-performance fighter airplanes was made. This study included consideration of the effects of increasing relative density and use of sweepback and low aspect ratio. Since the effects of sweep and aspect ratio have not been fully understood because no systematic investigation of their effects has been made, the effects of these factors were analyzed in considerable detail. The results of this analysis are also included in this report.

SYMBOLS

All forces and moments are referred to the stability system of axes which is defined in figure 1.

W	weight of airplane, lb
m	mass of airplane, slugs
S	wing area, sq ft
b	wing span, ft
l	tail length (longitudinal distance from center of pressure of the vertical tail to the center of gravity), ft

[1] Supersedes NACA TN 3035, "A Preliminary Study of the Problem of Designing High-Speed Airplanes With Satisfactory Inherent Damping of the Dutch Roll Oscillation" by John P. Campbell and Marion O. McKinney, Jr., 1953.

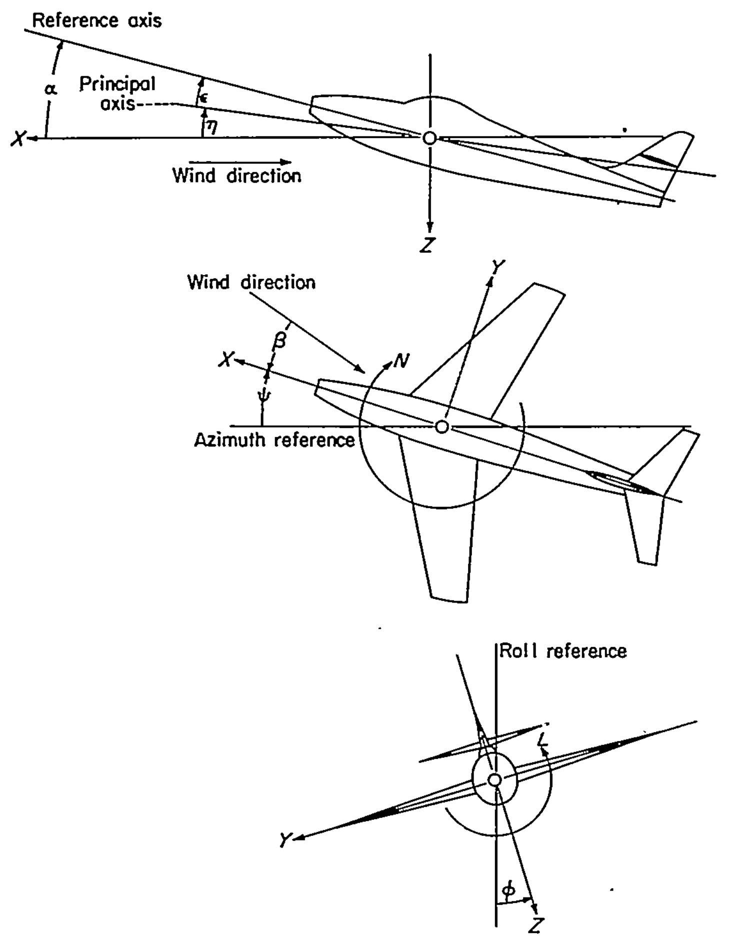


FIGURE 1.—The stability system of axes. Arrows indicate positive directions of moments, forces, and angles. This system of axes is defined as an orthogonal system having the origin at the center of gravity and in which the Z-axis is in the plane of symmetry and perpendicular to the relative wind, the X-axis is in the plane of symmetry and perpendicular to the Z-axis, and the Y-axis is perpendicular to the plane of symmetry. At a constant angle of attack, these axes are fixed in the airplane.

z	tail height (vertical distance from center of pressure of the vertical tail to the center of gravity), ft
A	aspect ratio
Λ	sweepback of wing-quarter-chord line, deg
λ	taper ratio
Γ	geometric dihedral angle, deg
V	true airspeed, ft/sec
v_e	equivalent lateral velocity, ft/sec
M	Mach number
h	pressure altitude, ft
k_{X_0}	radius of gyration about principal longitudinal axis of inertia, ft
k_{Z_0}	radius of gyration about principal normal axis of inertia, ft
$K_{X_0}=k_{X_0}/b$	
$K_{Z_0}=k_{Z_0}/b$	
K_X	radius-of-gyration factor about X-axis, $\sqrt{K_{X_0}{}^2 \cos^2 \eta + K_{Z_0}{}^2 \sin^2 \eta}$
K_Z	radius-of-gyration factor about Z-axis, $\sqrt{K_{Z_0}{}^2 \cos^2 \eta + K_{X_0}{}^2 \sin^2 \eta}$
K_{XZ}	product-of-inertia factor, $(K_{Z_0}{}^2 - K_{X_0}{}^2) \sin \eta \cos \eta$
μ	relative-density factor, $m/\rho Sb$
η	angle of attack of principal longitudinal axis of inertia, deg
ϵ	angle between principal longitudinal axis of inertia and longitudinal body axis, deg
α	angle of attack of longitudinal body axis, deg
ϕ	angle of bank, radians
ψ	angle of yaw, radians
β	angle of sideslip, radians
i_w	wing incidence, deg
ρ	air density, slugs/cu ft
p	rolling velocity, radians/sec
r	yawing velocity, radians/sec
q	dynamic pressure, lb/sq ft
P	period of lateral oscillation, sec
$T_{1/2}$	time to damp to one-half amplitude, sec
Y	lateral force, lb
L	rolling moment, ft-lb
N	yawing moment, ft-lb
C_L	lift coefficient, Lift/qS
C_Y	lateral-force coefficient, Y/qS
C_l	rolling-moment coefficient, L/qSb
C_n	yawing-moment coefficient, N/qSb

$$C_{Y_\beta}=\frac{\partial C_Y}{\partial \beta}$$

$$C_{l_\beta}=\frac{\partial C_l}{\partial \beta}$$

$$C_{n_\beta}=\frac{\partial C_n}{\partial \beta}$$

$$C_{Y_p}=\frac{\partial C_Y}{\partial \left(\frac{pb}{2V}\right)}$$

$$C_{l_p}=\frac{\partial C_l}{\partial \left(\frac{pb}{2V}\right)}$$

$$C_{n_p}=\frac{\partial C_n}{\partial \left(\frac{pb}{2V}\right)}$$

$$C_{Y_r}=\frac{\partial C_Y}{\partial \left(\frac{rb}{2V}\right)}$$

$$C_{l_r}=\frac{\partial C_l}{\partial \left(\frac{rb}{2V}\right)}$$

$$C_{n_r}=\frac{\partial C_n}{\partial \left(\frac{rb}{2V}\right)}$$

METHOD OF ANALYSIS

The period and time to damp to one-half amplitude of the lateral oscillation were the only characteristics of the lateral motion that were considered in the present analysis. These quantities were calculated by the method presented in reference 1. The period and damping requirements from the Air Force and Navy flying-qualities specifications of references 2 and 3 were used as a basis for evaluating the results.

BASIC CONFIGURATIONS STUDIED

In the study of the fundamental causes of the poor stability of modern high-performance airplanes, five basic configurations were considered:

Configuration	Sweepback, deg	Aspect ratio
1	0	6.0
2	30	4.5
3	45	3.0
4	60	1.5
5	0	3.0

These configurations are illustrated by sketches in figure 2 and details of the dimensional and mass characteristics are given in table I. Configurations 2 to 4 were obtained by sweeping back the wings of configuration 1 with appropriate modifications to the tips. In sweeping the wings, the quarter-chord point of the mean aerodynamic chord was kept in the same longitudinal position relative to the body. Although these configurations are part of a systematic family, they are in general typical of present and proposed designs. Configuration 5 was chosen because it represents another trend in the design of high-speed airplanes and because it provides interesting comparisons with two of the other configurations. Comparison of configurations 1 and 5 shows the effect of aspect ratio at 0° sweep and comparison of configurations 3 and 5 shows the effect of sweep at aspect ratio 3. The size of the airplanes was chosen so that the span of the moderately swept wings was representative of that of current fighter airplanes with swept wings.

All the configurations were assumed to have the same fuselage except for minor modifications necessary to accommodate the different tail designs. The size and shape of the fuselage were selected as being representative of many current designs.

The vertical tails for the various configurations had the same value of $C_{Y_{\beta_{tail}}}$. At 0° angle of attack, the center of pressure of the tail for all configurations was the same distance behind and above the center of gravity, which was located at the quarter-chord point of the mean aerodynamic chord.

All the configurations were assumed to have a wing loading of 50 pounds per square foot. The principal longitudinal axis of inertia was assumed to be inclined 2° nose down relative to the longitudinal fuselage axis. These values were selected as being representative of those of current fighter airplanes.

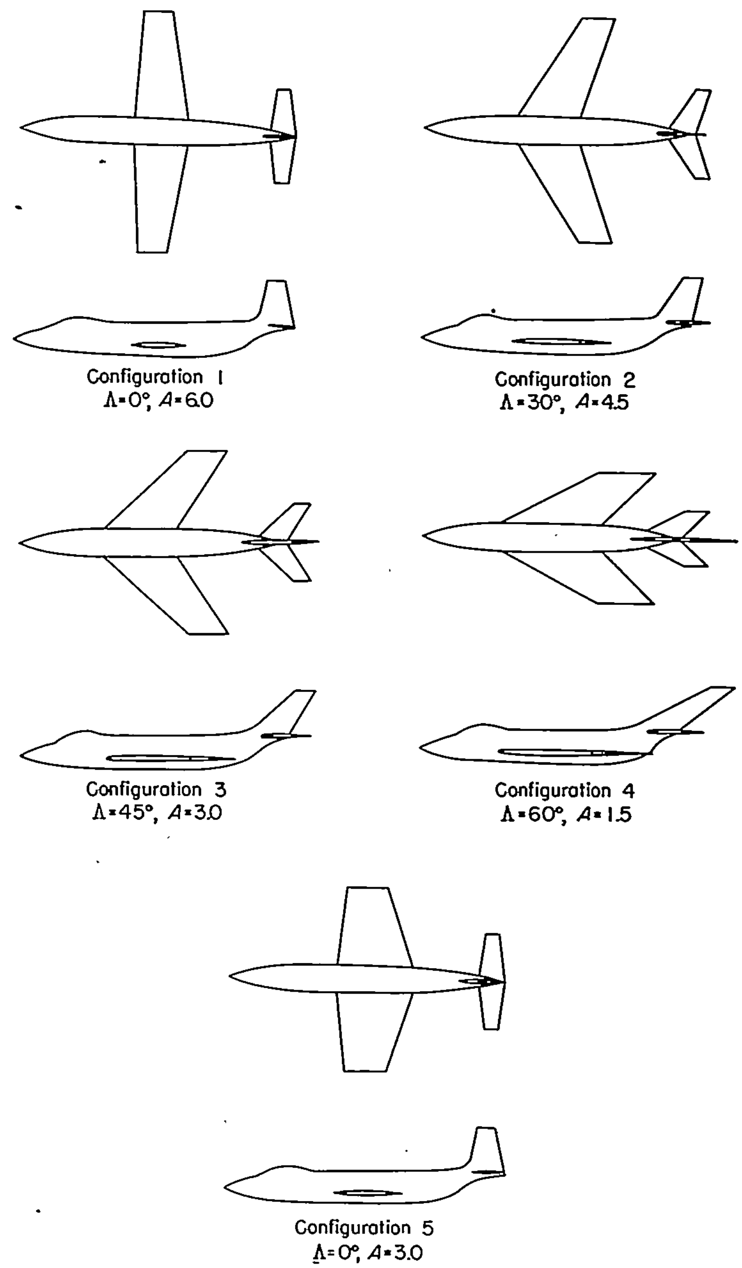


FIGURE 2.—Basic configurations for which calculations were made.

The approximate magnitudes of the radii of gyration for various sweep angles were first determined by averaging the values for a number of current designs. A systematic variation of the radii of gyration with sweep that was in general agreement with these actual values was then set up. This systematic variation which is shown in figure 3 was based on the assumption that the weight distribution along the wing panels remained constant as the sweep of the panels was varied. The assumed weight distribution of the panels was determined from the average weight distribution of several current swept-wing fighter airplanes for which detailed weight data were available.

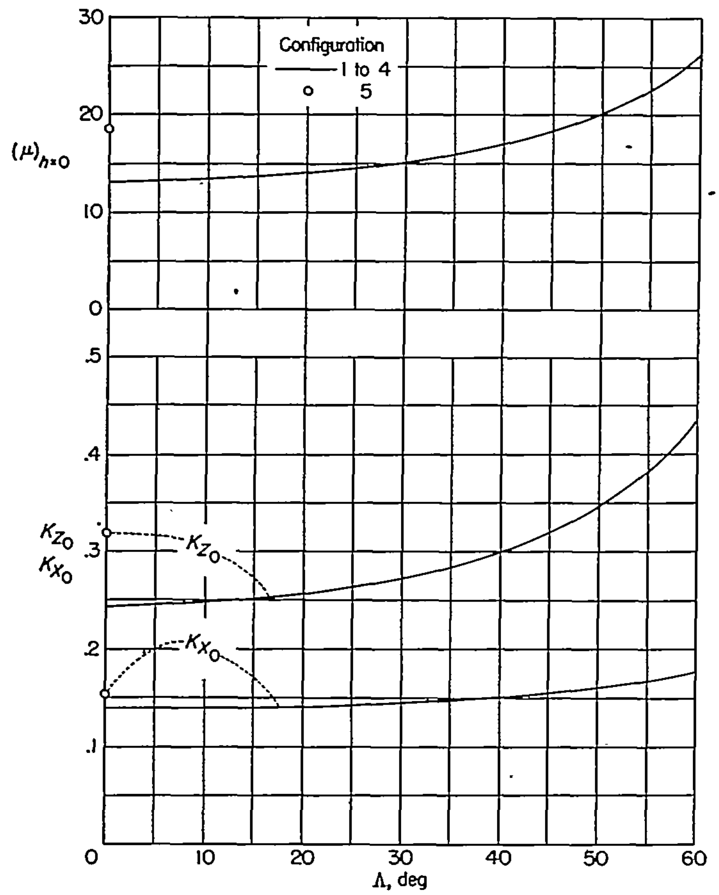


FIGURE 3.—Variation of mass parameters with sweepback.

FLIGHT CONDITIONS

The calculations for both the basic and modified configurations were made for four conditions:

Condition	h, ft	M	C_L
(a)	0	0.75	0.06
(b)	0	.27	.46
(c)	0	.204	.80
(d)	50,000	.75	.46

Conditions (a) and (d) were chosen to show the stability at a high subsonic speed at sea level and at an altitude of 50,000 feet and to show the effect of altitude at a constant Mach number. A Mach number of 0.75 was chosen for these conditions since that was considered about the highest value at which subsonic stability derivatives could be expected to apply for all configurations without compressibility corrections. Condition (b) was chosen for direct comparison with condition (d) to show the effect of altitude at constant lift coefficient where the stability derivatives would be the same. Condition (c) was chosen to show the stability at moderately high lift coefficients with flaps retracted. The lift coefficient of 0.80 used for condition (c) was assumed to represent the highest lift coefficient at which the theoretical variations of the different stability derivatives with lift coefficient were still valid. Above this lift coefficient, flow changes over the wing, fuselage, and tail surfaces often cause the stability derivatives to be greatly different from their theoretical values. For airplanes with thin, highly swept wings or with roughness on the wings, these flow changes might actually occur at lift coefficients below 0.80, but for the purpose of this generalized study it was assumed that the stability derivatives of all configurations would follow theoretical trends up to this lift coefficient. Comparison of conditions (a) to (c) shows the effect of lift coefficient at constant altitude.

All the calculations were made for the condition of level flight at 1 g normal acceleration.

ESTIMATION OF DERIVATIVES

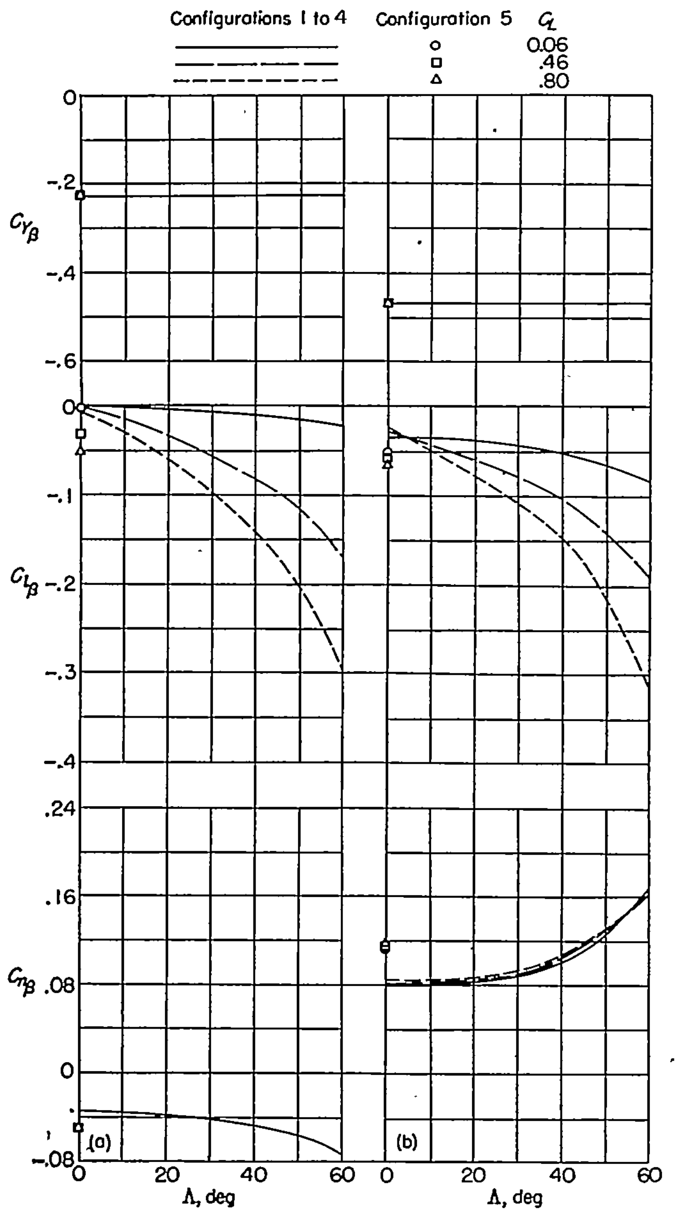
The estimation of the stability derivatives used in the calculations was based on the methods presented in reference 1. Plots showing the variation of the derivatives with sweepback and aspect ratio are shown in figures 4 to 6 for the complete airplanes and for the vertical-tail-off condition. The derivatives for the complete basic configurations are also listed in table II. In some cases, particularly for the wing-fuselage combinations, the estimations were based on experimental data and require some explanation.

Sideslip derivatives.—The value of C_{Y_β} for the vertical-tail-off condition was assumed to be constant at a value of −0.229 per radian ($C_{Y_\beta}=-0.004$ per degree) for all configurations and flight conditions on the basis of experimental data for a number of designs. These data showed no consistent trend for the variation of this factor with sweepback or lift coefficient. As pointed out previously, the vertical tails for all the configurations were designed to give the same value of C_{Y_β}. Since there was assumed to be no variation of $C_{Y_{\beta_{tail}}}$ with angle of attack, the value of C_{Y_β} for the complete airplane was the same for all configurations and flight conditions.

Since the configurations were laid out as midwing designs, the value of $C_{l_{\beta_{tail\,off}}}$ was assumed to be simply the value of $C_{l_{\beta_{wing}}}$. This value and the value of $C_{l_{\beta_{tail}}}$ were determined from the charts and formulas presented in reference 1.

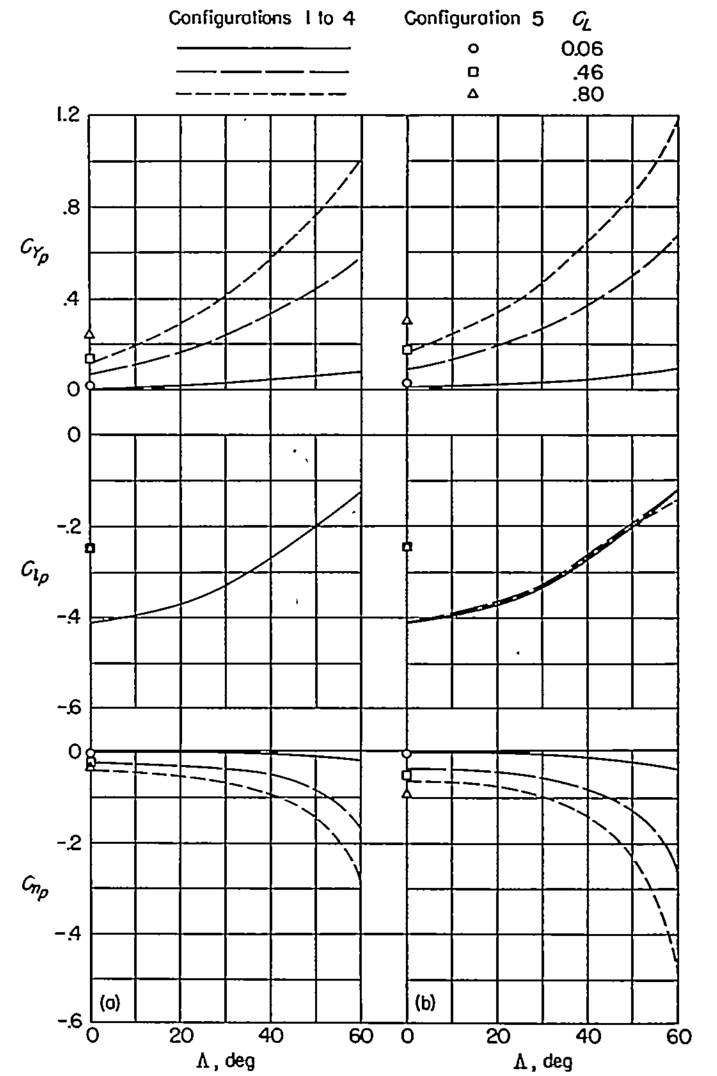
On the basis of experimental data the value of the factor bC_{n_β} for the vertical-tail-off condition was assumed to be constant for all configurations and flight conditions. The magnitude of $C_{n_{\beta_{tail\,off}}}$ therefore varied inversely with wing span. The value of $C_{n_{\beta_{tail}}}$ was calculated from the value of $C_{Y_{\beta_{tail}}}$ by means of the formula given in reference 1.

Rolling derivatives.—The rolling derivatives C_{Y_p}, C_{l_p}, and C_{n_p} were determined by the methods described in reference 1 except that $C_{l_{p_{wing}}}$ was assumed to be constant over the lift-coefficient range at the value given by reference 1 for the zero-lift condition.



(a) Vertical tail off. (b) Complete airplane.

FIGURE 4.—Variation of sideslip stability derivatives with sweepback and lift coefficient.



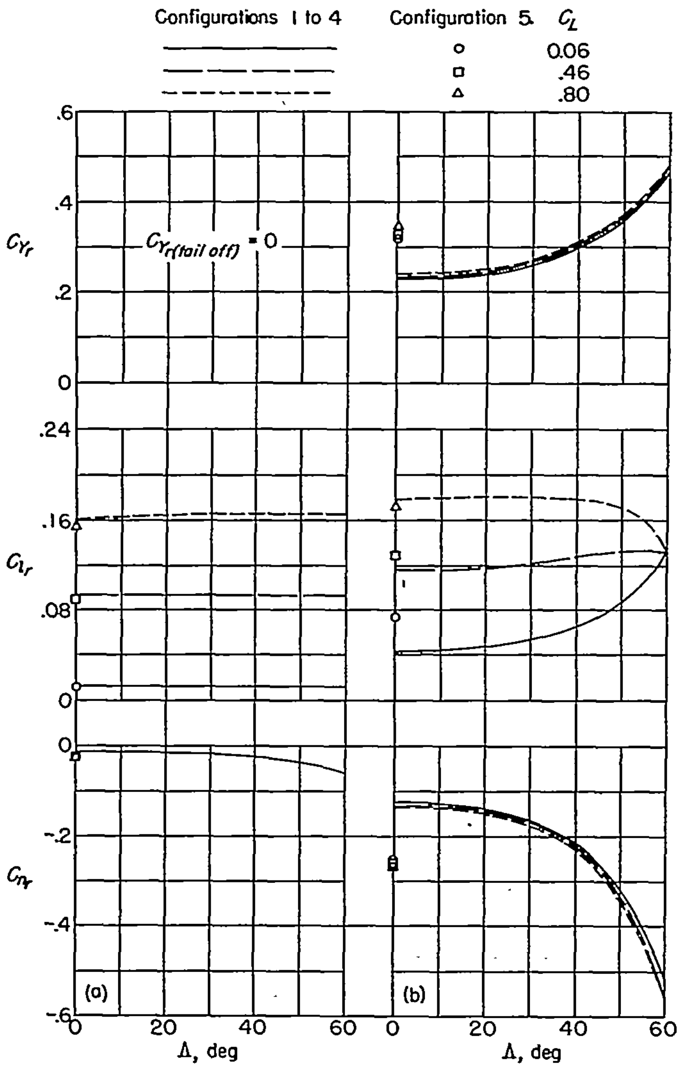
(a) Vertical tail off. (b) Complete airplane.

FIGURE 5.—Variation of rolling stability derivatives with sweepback and lift coefficient.

Yawing derivatives.—The value of $C_{Y_{r_{tail\ off}}}$ was assumed to be zero for all configurations and conditions since experimental data for many wings and wing-fuselage combinations had shown no consistent variation of C_{Y_r} with configuration or lift coefficient. The value of $C_{Y_{r_{tail}}}$ was calculated from the formula presented in reference 1. The values of C_{l_r} were determined by the method of reference 1. A constant value of the factor $b^2C_{n_r}$ for the tail-off condition was assumed for all configurations and flight conditions on the basis of experimental data on a number of configurations so that the magnitude of $C_{n_{r_{tail\ off}}}$ varied inversely with the square of the wing span. These experimental data did not show consistent trends in the variation with configuration or lift coefficient and, since the value of $b^2C_{n_{r_{tail\ off}}}$ is small compared with the value for the complete airplane, the assumption of a constant value of $b^2C_{n_r}$ seemed reasonable. The value of $C_{n_{r_{tail}}}$ was calculated from the equation given in reference 1.

LIMITATIONS OF ANALYSIS

This report presents some preliminary results of a study of the possibility of designing airplanes to have satisfactory inherent dynamic lateral stability. As pointed out previously, these preliminary results cover only the case of fighter airplanes at subsonic speeds and cover only the period and damping of the lateral oscillation. The calculated period and damping of the oscillation for the basic and modified configurations are compared with the Air Force and



(a) Vertical tail off. (b) Complete airplane.

FIGURE 6.—Variation of yawing stability derivatives with sweepback and lift coefficient.

Navy period and damping requirements. The authors realize that these requirements are not adequate in some cases and that other factors, such as the ratio of roll to yaw, should be considered in a comprehensive analysis. Although these additional factors are not considered in detail in this preliminary analysis, they are discussed briefly with regard to the effects of some of the mass and aerodynamic parameters.

A few comments are required on the applicability of the calculated data presented in this report to actual airplanes of similar configuration before these results are discussed in detail. The reader should bear in mind that small changes in some of the important stability derivatives can have a significant effect on dynamic stability and that such changes might result unpredictably from apparently minor changes in design. These calculations are intended to show the general trends in the effects of the various design factors covered and are not intended for use in predicting the stability of specific airplane designs which are superficially similar to these configurations.

One reason that the stability of these hypothetical configurations might be very different from the stability of actual airplanes is that the theoretical values of the wing contributions to the stability derivatives were assumed to be accurate for the entire range of lift coefficients covered by the calculations (C_L=0.06 to 0.80). Actually, this assumption may be far from correct at the higher lift coefficients for airplanes of practical construction, particularly for those having thin, highly swept wings. There is evidence from experimental data on such designs that the values of the derivatives C_{l_β}, C_{n_p}, and C_{l_r} may diverge from the theoretical variation with lift coefficient at moderate lift coefficients (C_L near 0.4) and be greatly different—perhaps even have a different sign—at a lift coefficient of 0.8.

RESULTS AND DISCUSSION

CAUSES OF INADEQUATE DUTCH ROLL STABILITY

The causes of the poor dynamic lateral stability of modern high-performance fighter airplanes must be established before a reasonable approach can be made to the problem of designing such airplanes to have satisfactory inherent stability. The first part of the present analysis therefore treats the stability of the series of basic configurations which are representative of present-day airplane designs with emphasis on the determination of the reasons that the dynamic lateral stability of these airplanes is generally worse than that of World War II fighter airplanes which had lower relative density, less sweep, and higher aspect ratio. The results of the calculations made for this part of the analysis are presented in tables II and III and figures 7 to 9.

Effect of sweepback and aspect ratio.—The data of figure 7 show that at the low lift coefficient (C_L=0.06) the period and damping were about the same for all the configurations. At the higher lift coefficients, however, the damping became worse and the period became shorter as the sweepback was increased and the aspect ratio reduced simultaneously in the manner representative of present-day design practice (configurations 1 to 4). Comparison of the data for configurations 1, 3, and 5 shows that both of these factors were responsible for this reduction in stability. There was some reduction in stability when the aspect ratio alone was reduced (configurations 1 and 5) and there was a greater reduction when sweepback alone was increased (configurations 3 and 5).

Examination of figures 3 to 6 gives some indication of the causes of the detrimental effects of increasing sweepback and reducing aspect ratio on dynamic stability. These figures show that, of the mass parameters and stability derivatives which generally have an important effect on dynamic stability, the values of μ, K_X, K_Z, C_{l_β}, C_{l_p}, and C_{n_p} are changed in the adverse direction by sweepback for configurations 1 to 4, whereas the values of C_{n_β} and C_{n_r} are changed in the favorable direction. These figures also show that the same effects are caused, but to a lesser degree, by a reduction in aspect ratio (configurations 1 and 5). The changes in the mass parameters and C_{n_β} and C_{n_r} are almost entirely caused by the

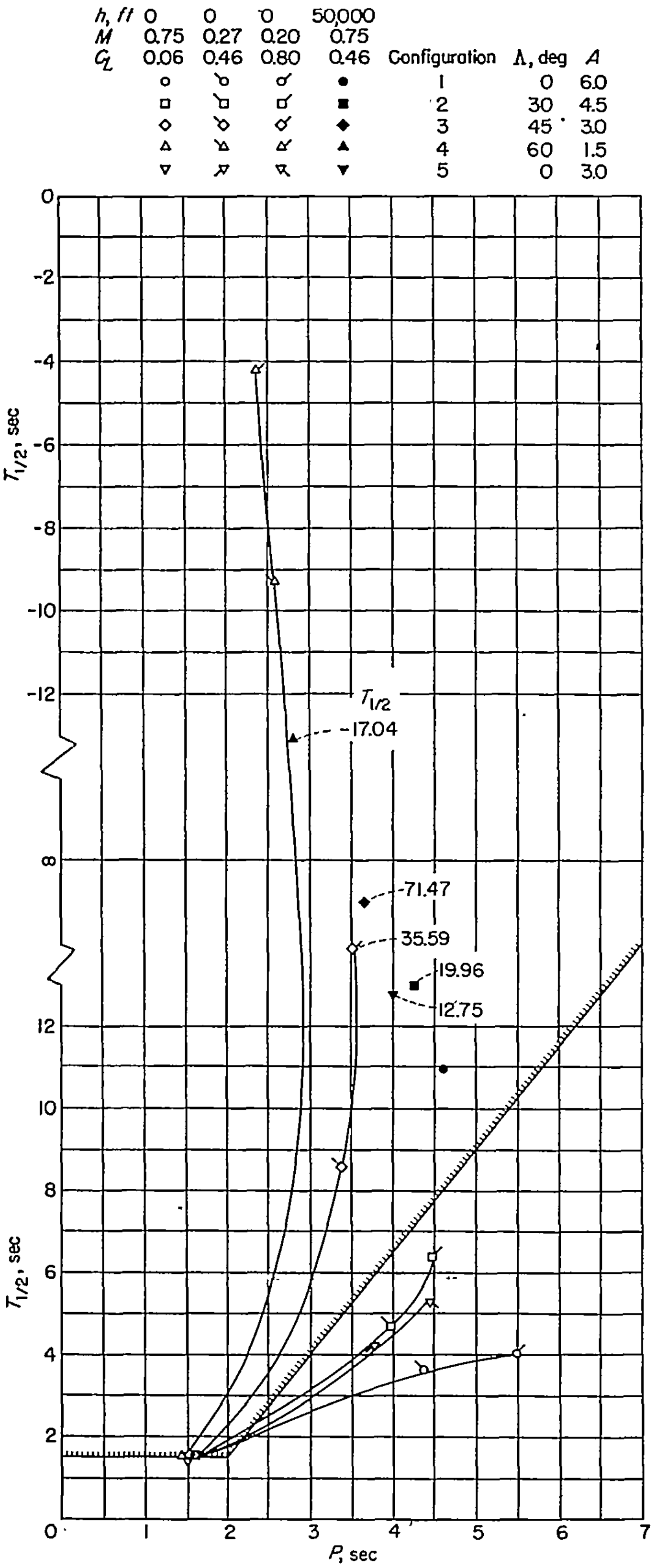


FIGURE 7.—Stability of basic configurations. Hatched boundary is period-damping requirement of references 2 and 3.

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reduction in the span on which the nondimensional form of these factors is based; the changes in C_{l_p} are caused by the change in aspect ratio; the changes in C_{n_p} are caused by the changes in sweep and in the span on which the coefficient is based; and the changes in C_{l_β} are caused by the change in sweep, aspect ratio, and the span on which the coefficient is based.

Effect of mass parameters and individual stability derivatives.—Figure 8 and table III present the results of calculations made to determine whether the mass parameters or any of the stability derivatives discussed in the preceding paragraph were predominantly responsible for the decrease in stability as sweepback was increased and aspect ratio reduced. These calculations were made for only the high-aspect-ratio, unswept and the 45° swept-wing configurations (configurations 1 and 3). Although only one flight condition was considered (C_L=0.46; h=0 ft), the results obtained are believed to be indicative, at least at moderate and high lift coefficients, of the effect of independently changing the mass parameters or the individual stability derivatives for one of these configurations to the values for the other configuration. The results of these calculations show that, when either the mass parameters or one of the stability derivatives C_{l_β}, C_{l_p}, or C_{n_p} for configuration 3 was changed to the value for configuration 1, the stability of configuration 3 became almost as good as that of configuration 1. When the value of one of these factors for configuration 1 was changed to the value for configuration 3, the stability did not generally become much worse. It is clearly evident from these results that it is very difficult to generalize on the effects of these stability parameters. No one factor is the cause of the reduction in stability as the sweep is increased and aspect ratio reduced. Changes in any one of several derivatives, however, resulted in substantial improvements in the stability of the swept-wing configuration.

Some of the data in figure 8 can be used to illustrate why the elimination of the propeller makes the stability of jet airplanes worse than that of propeller-driven airplanes. Experimental data have shown that the propeller provides a substantial increase in damping in yaw $-C_{n_r}$ and, in many cases, a reduction in static directional stability C_{n_β}. The results of figure 8 show that for the unswept configuration both of these changes provide an improvement in the period-damping relationship (that is, a reduction in time to damp and an increase in period).

Effect of relative-density factor.—The relative-density factor of modern high-performance fighter airplanes is generally greater than that of older types because of increases in wing loading and operational altitude and because of the use of low-aspect-ratio wings. The effect of increasing the relative-density factor on stability can be seen in figure 7 by a comparison of the sea-level and altitude conditions. These results show that an increase in altitude had a detrimental

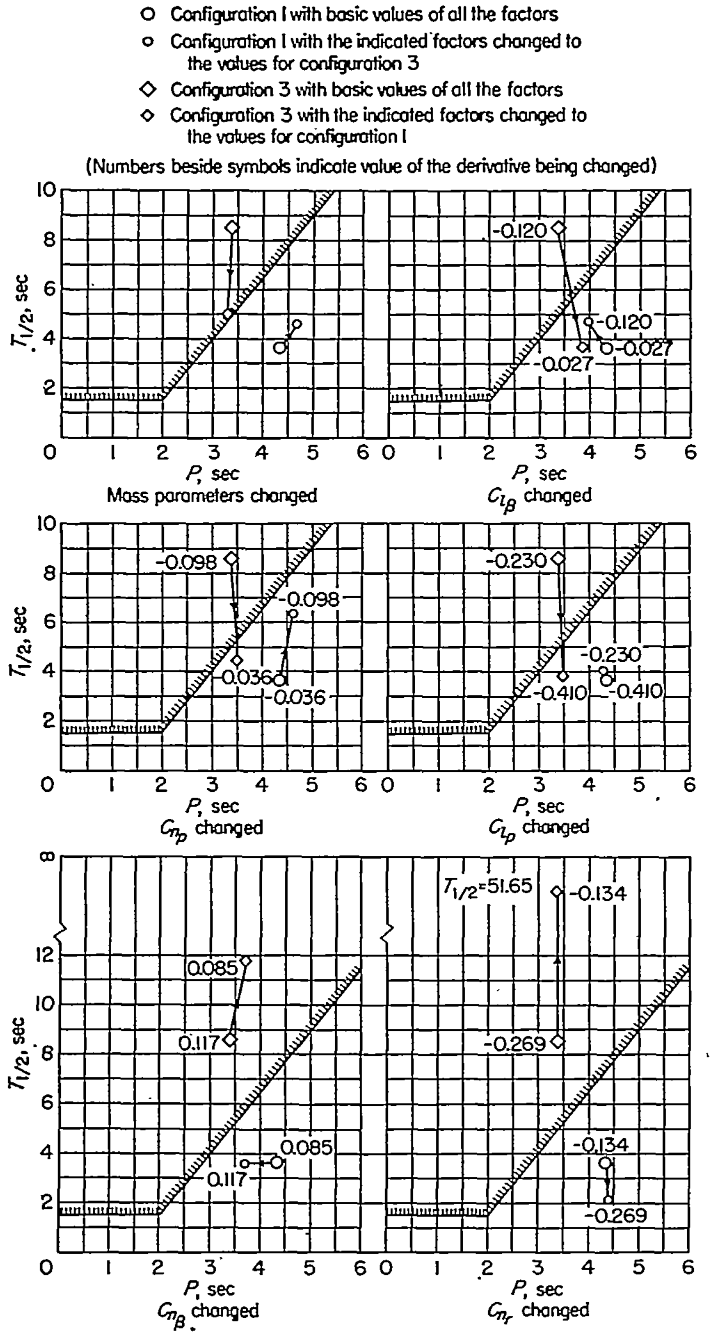
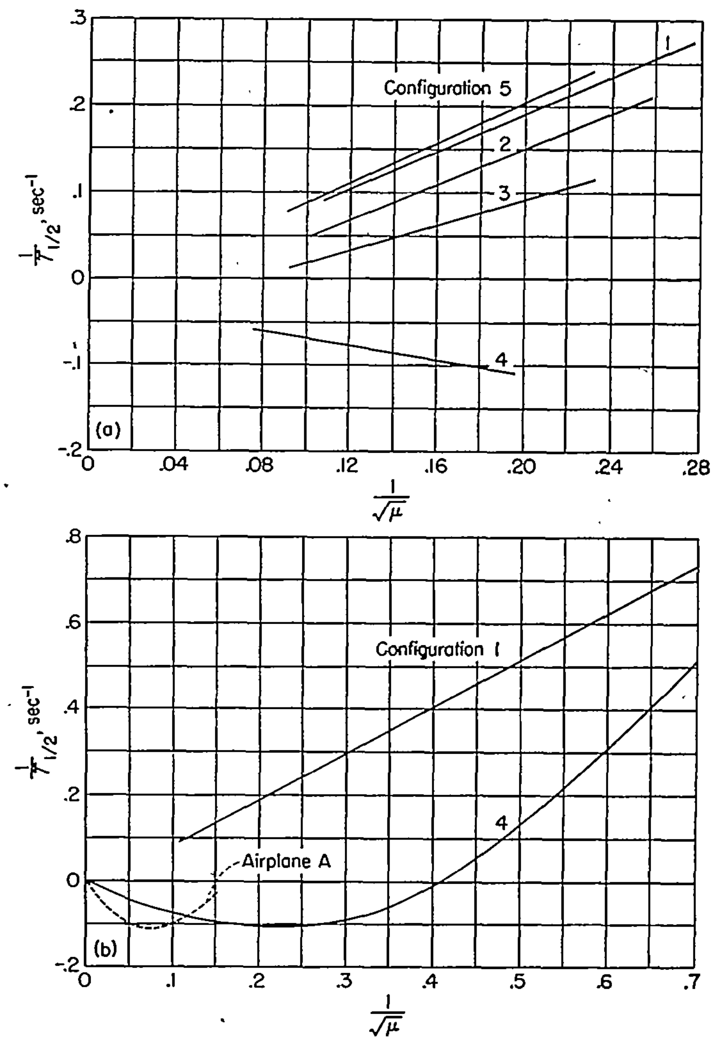


FIGURE 8.—Effect of the differences in mass parameters and individual stability derivatives on stability of configurations 1 and 3. $C_L=0.46$; $h=0$ feet.



(a) Basic configurations. Data taken from table II for altitudes of 0 and 50,000 feet.

(b) Configurations 1 and 4 for an extended range of μ, and an actual airplane for altitudes from sea level to infinity.

FIGURE 9.—Variation of damping with relative-density factor. $C_L=0.46$.

effect on the stability of all configurations when compared at a constant Mach number ($M=0.75$). An increase in altitude at a constant lift coefficient also had a detrimental effect for all configurations except configuration 4 where the airplane was unstable at sea level. This effect of increasing μ for a configuration which is unstable would generally be expected since an airplane is neutrally stable when the relative-density factor is infinite. This result is illustrated in figure 9. Figure 9 (a) which was taken from the data of table II shows that neutral stability is approached as μ is increased. The data are presented in terms of $\frac{1}{T_{1/2}}$ and $\frac{1}{\sqrt{\mu}}$ since $\frac{1}{T_{1/2}}$ is a direct measure of damping and since for a one-degree-of-freedom oscillation the value of $\frac{1}{T_{1/2}}$ would vary directly with the value of $\frac{1}{\sqrt{\mu}}$. The three-degree-of-freedom data of figure 9 (a) appear as straight lines since only two points (the end points of these lines) were available from the calculations. These end points were taken from the 0- and 50,000-foot-altitude conditions at a lift coefficient 0.46. The fact that $\frac{1}{T_{1/2}}$ does not necessarily vary directly with $\frac{1}{\sqrt{\mu}}$ for a three-degree-of-freedom motion, however, is illustrated in figure 9 (b) where the variation is shown for an extended range of μ for configurations 1 and 4 and for another configuration indicated as airplane A. The results for airplane A were included to show that this nonlinear variation of $\frac{1}{T_{1/2}}$ with $\frac{1}{\sqrt{\mu}}$, which shows up for configuration 4 only when values of μ below the normal range are considered, can occur in the range of normal values of μ for some airplanes.

MEANS OF IMPROVING DUTCH ROLL STABILITY

On the basis of the preceding results regarding the causes of inadequate Dutch roll stability, an analysis has been carried out to determine means of improving this stability.

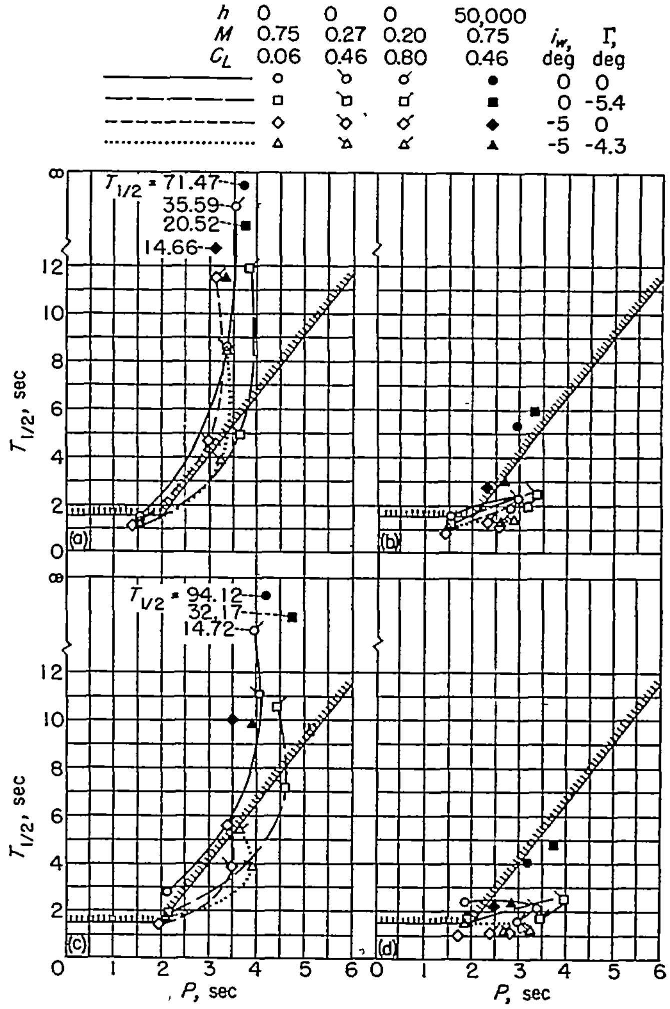
Factors that can be changed.—If it is assumed that the wing loading is determined from performance considerations, there are three mass factors that can be changed to improve dynamic lateral stability—the inclination of the principal axis of inertia, the radius of gyration in roll, and the radius of gyration in yaw. An increase in the nose-upward inclination of the principal axis of inertia increases the beneficial effect of the product of inertia as described in references 4 and 5. A reduction in the radius of gyration in roll is beneficial, particularly when the principal axis is inclined nose upward. Changing the radius of gyration in yaw might or might not have a beneficial effect on the stability depending upon many related factors, the inclination of the principal axis of inertia in particular. If the principal axis is inclined nose up relative to the stability axis, increasing the radius of gyration in yaw might be beneficial since the favorable product-of-inertia effect would tend to offset the normally adverse effect of increasing the radius of gyration.

Five of the aerodynamic stability derivatives generally have an important effect on dynamic lateral stability: C_{l_β}, C_{n_β}, C_{l_p}, C_{n_p}, and C_{n_r}. The derivative C_{l_β} can easily be changed independently of the others by varying the geometric dihedral. The derivatives C_{l_p} and C_{n_p}, however, cannot be changed appreciably by geometric changes other than major changes in the wing plan form. The two derivatives C_{n_β} and C_{n_r} can be changed simultaneously by varying the size of the vertical tail but they cannot conveniently be varied an appreciable amount independently of each other. The changes in stability that result from varying these derivatives simultaneously by changing the tail size tend to offset each other. An increase in tail size increases $-C_{n_r}$ and thereby increases the damping but the accompanying increase in C_{n_β} reduces the period. On a plot such as figure 7, this simultaneous reduction in time to damp and period tends to shift a point parallel to the period-damping boundary given by the flying-qualities requirements for periods greater than 2 seconds. The effect of changing the size of the vertical tail should be studied for any particular design, however, since it offers possibilities for improving stability in some cases.

Modifications considered.—In the study of means of improving the Dutch roll stability of modern high-speed fighter airplanes, configurations 3 and 5 were chosen as basic configurations from which to work since they were considered representative of proposed high-speed designs. Five modifications to each of these basic airplanes were considered:

(1) K_{X_0} reduced to 0.65 times the basic value
(2) K_{Z_0} increased to 1.41 times the basic value
(3) K_{Z_0} increased and K_{X_0} reduced simultaneously to 1.25 and 0.65 times the basic values, respectively
(4) i_w changed from 0° to −5°
(5) Γ adjusted to give zero C_{l_β} at a lift coefficient of 0.06

These changes were considered separately and in various



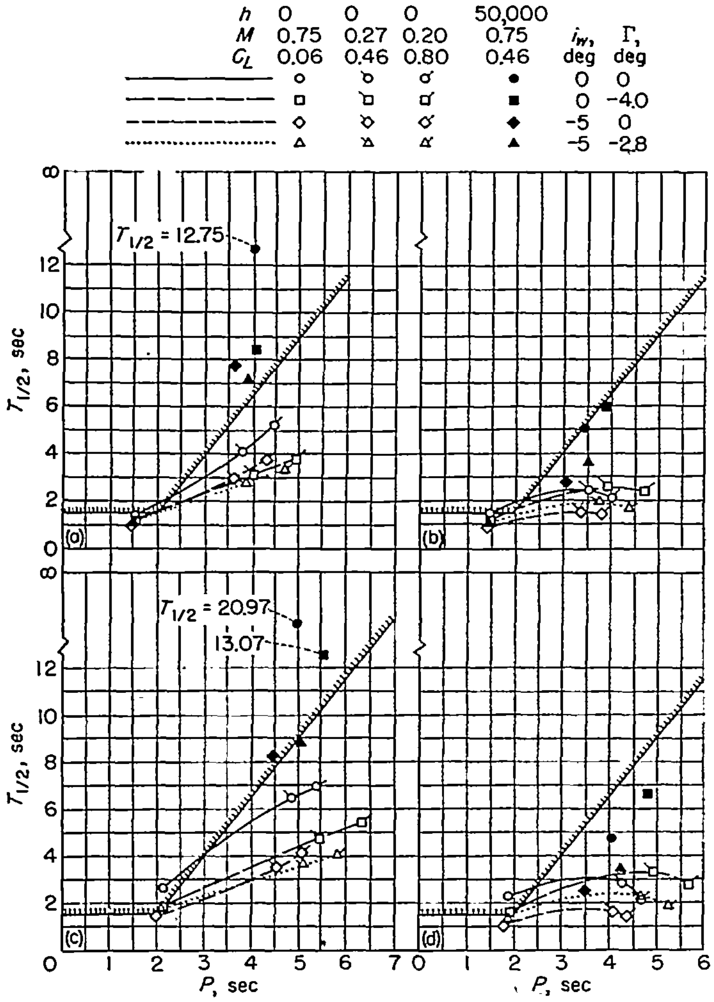
(a) Basic mass characteristics. (b) Reduced K_{X_0}.
(c) Increased K_{Z_0}. (d) Reduced K_{X_0} and increased K_{Z_0}.

FIGURE 10.—Stability of modified configuration derived from configuration 3.

combinations. The modifications should not be considered as practicable changes that can be made to improve the stability of an existing airplane. They are intended only to show what factors should be considered in the early design stages and to illustrate the improvements in inherent stability that can be obtained by designing for stability. The results of the calculations made for this part of the analysis are presented in table IV and figures 10 and 11.

Effect of radii of gyration.—The designer is concerned with the radii of gyration about axes which are fixed in the airplane and approximately coincide with the body and wing axes. For this reason the modified configurations were established by changing the radius-of-gyration factors about the principal axes of inertia K_{X_0} and K_{Z_0}. The radii of gyration used in equations of motion in stability work, however, are usually referred to the stability axes. The effects of the changes in K_{X_0} and K_{Z_0} are therefore analyzed in terms of the effects of K_X, K_Z, and K_{XZ}.

The magnitudes of the changes in K_{X_0} and K_{Z_0} assumed for the modified configurations were determined from the following considerations. In order to obtain the maximum bene-



(a) Basic mass characteristics. (b) Reduced K_{X_0}.
(c) Increased K_{Z_0}. (d) Reduced K_{X_0} and increased K_{Z_0}.

FIGURE 11.—Stability of modified configuration derived from configuration 5.

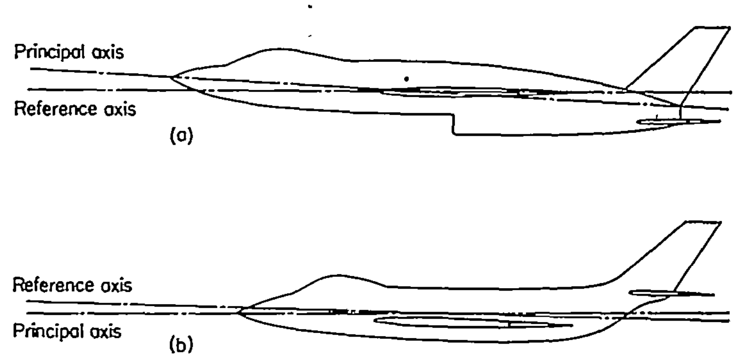
ficial effect from the inertia changes, the value of K_{X_0} was made as small as practicable. A study of moments of inertia of a number of current and proposed designs indicated that a value of K_{X_0} of 0.0100 (0.65 times the basic value of 0.0154) was probably the minimum value that could be obtained on a practical airplane. The determination of the value of K_{Z_0} for the modified configuration was not so straightforward because the direction in which K_{Z_0} should be changed to give a beneficial effect is not always the same. Since increasing K_{Z_0} is generally beneficial from an overall standpoint, however, only increases were considered in this analysis. Since there is no definite maximum value to which K_{Z_0} can be increased, two relatively large values (1.25 and 1.41 times the basic value) were chosen to illustrate the effect of varying K_{Z_0}. These values are in line with the general trend toward increased K_{Z_0} which results from the use of very long fuselages in the latest designs.

A reduction in the radius-of-gyration factor in roll K_{X_0} improved the stability in almost every case for both configurations 3 and 5 as shown in figures 10 and 11. The only exceptions were the two cases in which K_{X_0} was reduced for the basic configurations at a lift coefficient of 0.06. In these cases the principal axis was inclined nose down relative to the stability axes so that the effect of the product of inertia was unfavorable, and evidently the adverse effect of increasing the product-of-inertia factor was greater than the favorable effect of reducing K_X.

There was no consistent effect of increasing the radius-of-gyration factor in yaw K_{Z_0} alone either for configuration 3 or configuration 5. As shown in figures 10 and 11 there was generally an adverse effect of increasing K_{Z_0} for the low lift coefficients and a favorable effect at the high lift coefficients. This result can be explained by the following reasoning: At low angles of attack the increase in the product-of-inertia factor K_{XZ} which resulted from an increase in K_{Z_0} caused either a small favorable or unfavorable effect depending on the inclination of the principal axis, but in neither case did this effect offset the adverse effect of increasing the value of K_Z. At high angles of attack the effect of the product-of-inertia factor was always favorable and was generally greater than the adverse effect of the greater value of K_Z.

When K_{X_0} was reduced and K_{Z_0} was increased simultaneously, the stability at the moderate and high angles of attack was even better than it was when K_{X_0} was reduced by itself. At the low angle of attack, however, the stability was worse than it was for the basic configuration or the configuration with reduced K_{X_0}. This result is illustrated in figures 10 and 11 for both configurations 3 and 5. This simultaneous change in both the radii of gyration seems somewhat better than a reduction in K_{X_0} alone since it is more effective for the high-altitude condition and since the adverse effect on the stability at the low angle of attack can be counteracted by other means as is shown subsequently.

Effect of inclination of principal axis.—There are a number of ways that the inclination of the principal axis relative to the stability axis can be changed by changing the design of an airplane. A simple change in wing incidence was the method considered in the present analysis. The sketch of figure 12 illustrates another way in which it can be done. This figure shows the profile of a configuration in which the weight in the rear of the airplane is kept as low as practicable.



(a) Modified design.
(b) Configuration 3.

FIGURE 12.—Illustration of profile of an airplane designed to have positive inclination of the principal longitudinal axis of inertia and comparison with profile of configuration 3 which is representative of many designs.

The engine is located low in the rear of the airplane behind an underslung inlet and the horizontal tail is mounted low at the rear of the fuselage. The forward part of the fuselage is located as high as possible without increasing the frontal area of the fuselage. The midsection of the fuselage has a narrow oval cross section about the same width as the engine so that this distribution of the weight in the fuselage can be accomplished without increasing the frontal area of the fuselage. The profile of configuration 3, which is representative of a current trend in design, is shown in figure 12 for comparison. The modified design would have a principal-axis inclination ϵ of 2° or 3° nose up relative to the wing chord instead of 2° or 3° nose down as would be the case for an airplane of the type represented by configuration 3. This would give a change in the inclination of the principal axis of inertia of about 5° which is the same as would be obtained with the simple 5° change in wing incidence assumed in the calculations for the modified configurations. The method of changing the inclination in this analysis is not important except that it indicates how the tail contributions to the stability derivatives were changed.

The results presented in figures 10 and 11 show that the use of 5° negative wing incidence to increase the nose-up inclination of the principal axis had either a favorable effect or no significant effect on the lateral stability for all the radius-of-gyration and dihedral conditions covered in the calculations. The favorable effect of negative wing incidence was particularly significant at the low-lift-coefficient condition ($C_L=0.06$) where it made all the conditions satisfactory which were otherwise marginal or unsatisfactory.

Effect of dihedral.—The amount of negative geometric dihedral covered in the calculations was limited to the amount required to give zero effective dihedral ($C_{l_\beta}=0$) at the low-lift-coefficient condition since the use of greater negative geometric dihedral would probably make the airplane uncomfortable to fly at the low angles of attack where the effective dihedral would be negative.

The effect of negative geometric dihedral is shown by figures 10 and 11 to vary from a slight favorable effect to no significant effect at the moderate and high lift coefficients. In many of these cases the use of negative dihedral caused the time to damp to increase but, because of the accompanying increase in period, the stability did not appear to become less satisfactory with respect to the flying-qualities damping requirement indicated by the boundaries in figures 10 and 11. At the low lift coefficients the use of negative geometric dihedral had a favorable effect when the wing incidence was 0° and an adverse effect when the wing incidence was −5°. The conditions under which varying the dihedral can be expected to have a favorable effect on stability can be determined from the expression

$$C_{n_p}-2C_LK_Z^2-C_{l_p}\frac{K_{XZ}}{K_X^2}$$

as explained in reference 6. Negative values of this quantity indicate that the use of negative geometric dihedral will reduce the time to damp for the oscillation. This test will not work in every case, however, since its derivation involved a number of simplifications and generalizations. Examination of the expression shows that the sum of the first two terms will almost always be negative since C_{n_p} is usually negative and $-2C_LK_Z^2$ is always negative. Since C_{l_p} is always negative for practical flight conditions and K_X^2 is always positive, the sign of the third term will always be the same as the sign of K_{XZ}. When K_{XZ} is positive and of relatively large magnitude (that is, when the principal axis of inertia is inclined nose up relative to the flight path) and the value of K_X is low, the third term will have a large positive value which will usually mean that the effect of using negative dihedral will be unfavorable. Since these mass characteristics (large positive value of K_{XZ} and small value of K_X) are desirable from the standpoint of oscillatory stability, the use of negative dihedral may be unfavorable for a design in which the mass characteristics have been made as favorable as possible. The effect of dihedral angle, however, should be studied for each particular airplane configuration.

Effect of modifications on roll-to-yaw ratio and control.—It has been fairly well established that a pilot's opinion of the acceptability of a lateral oscillation is influenced by the ratio of roll to yaw which has been expressed in terms of ϕ/ψ, ϕ/β, and ϕ/v_e by various investigators. Although no definite requirement has been generally accepted, it seems evident that increasing the ratio of roll to yaw makes the lateral oscillation more objectionable. Some of the modifications covered in the present study which improved the stability from the standpoint of the present Air Force and Navy flying-qualities requirement would have an adverse effect from the standpoint of roll-to-yaw ratio. Either reductions in the rolling radius of gyration K_{X_0} or increases in the yawing radius of gyration K_{Z_0} would increase the ratio of roll to yaw. On the other hand, the use of negative geometric dihedral would reduce the ratio of roll to yaw. Whether or not reasonable changes in the radii of gyration or dihedral would have a large effect on the flying qualities because of their effect on the ratio of roll to yaw is a subject for further study.

Another factor to be considered is the effect of the modifications on the adverse yaw caused by a rolling acceleration and consequently on the adverse rolling moments caused by the adverse yaw. An increase in the nose-upward inclination of the principal axes will cause an increase in the adverse yaw in rolls.

APPLICATION OF RESULTS TO ACTUAL AIRPLANES

The foregoing analysis has brought out a number of factors that should be considered in designing an airplane so that it will have the best inherent stability that it is practicable to obtain. Some of these factors will probably conflict with factors that appear desirable from some other standpoint. It is up to the designer in any particular case, then, to weigh all the facts and decide on the relative merits of these design features for his particular application. The application of the results of the analysis to the problem of designing airplanes so that they will have satisfactory inherent dynamic lateral stability is discussed in the following paragraphs.

Wing plan form.—One of the principal facts brought out by this analysis is that the use of low-aspect-ratio and sweptback wings has a very detrimental effect on dynamic lateral stability. Within the limits permitted by high-speed performance requirements, the use of unswept wings of higher aspect ratio (about 6) is very desirable. The next most desirable wings appear to be an unswept wing of low aspect ratio similar to that of configuration 5 or a wing of moderate sweep similar to that of configuration 2.

Radii of gyration.—It also appears highly desirable to keep the radius-of-gyration factor in roll K_{X_0} as low as possible. This feature appears particularly important if a highly swept wing is used. For example, it appeared to be impossible to make configuration 3 satisfactory unless K_{X_0} were reduced. The use of a longer fuselage to accommodate items normally located in the wings might be slightly beneficial if the principal axis of inertia is inclined nose up relative to the flight path.

Inclination of principal axis of inertia.—The inclination of the principal axis of inertia is also a very important factor, particularly for obtaining satisfactory stability at low angles of attack. For this reason the use of high horizontal tails and vertical tails located on a boom over the jet exit are definitely undesirable from the standpoint of dynamic stability. Every effort should be made to design the airplane to take advantage of the large favorable effect of a more nose-up inclination of the principal axis by designing the airplane so that the weight forward is located high and the weight rearward is located low relative to the wing chord plane.

Dihedral and tail area.—The use of a reasonable amount of negative geometric dihedral would probably not have a large effect on the dynamic lateral stability but this modification should be considered since it may improve the stability in some cases and may also be helpful by reducing the adverse rolling moments which result from adverse yaw in an aileron roll. The effect of dihedral should be investigated for each airplane design. Similarly the effect of vertical-tail area is not immediately obvious and should be investigated for each particular design in an effort to determine the optimum size from considerations of both stability and control.

CONCLUSIONS

On the basis of the present theoretical analysis to determine the design features that appear most promising in providing inherent Dutch roll stability, the following conclusions were drawn for the case of fighter airplanes at subsonic speeds:

1. The stability of the Dutch roll oscillation of modern high-speed fighter airplanes is less satisfactory than that of older types of fighter airplanes such as those used in World War II because of the use of low-aspect-ratio sweptback wings and because of the higher wing loadings and operating altitudes. The unfavorable effect of the use of low-aspect-ratio sweptback wings was caused mainly by the increase in the relative density μ, the effective dihedral $-C_{l_\beta}$, and the yawing moment due to rolling $-C_{n_p}$, and the decrease in the damping in roll $-C_{l_p}$ which resulted from the change from the older type of unswept wings of higher aspect ratio.

2. It is possible to design high-performance fighter airplanes to have substantially better inherent stability of the Dutch roll oscillation than that of most current fighter designs. It is important to design the airplane with the maximum aspect ratio and minimum sweep that will permit attainment of the desired performance. For a given configuration the radius of gyration in roll should be kept as low as possible and the nose-up inclination of the principal longitudinal axis of inertia should be made as great as practicable. The optimum dihedral angle and vertical-tail area should be selected on the basis of a study of the stability and control of the particular airplane design.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *August 19, 1953.*

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TABLE I

DIMENSIONAL AND MASS CHARACTERISTICS OF BASIC CONFIGURATIONS

Configuration	1	2	3	4	5
Wing:					
Λ, deg	0	30	45	60	0
A	6.0	4.5	3.0	1.5	3.0
λ	0.5	0.5	0.5	0.5	0.5
b, ft	50.0	43.4	35.4	25.0	35.4
S, sq ft	417	417	417	417	417
Vertical tail:					
Λ, deg	0	30	45	60	0
A	1.64	1.50	1.36	1.17	1.64
λ	0.5	0.5	0.5	0.5	0.5
S, sq ft	45.5	50.1	55.0	64.1	45.5
$(l)_{\alpha=0°}$, ft	24.5	24.5	24.5	24.5	24.5
$(z)_{\alpha=0°}$, ft	7.2	7.2	7.2	7.2	7.2
Mass characteristics:					
W, lb	20,833	20,833	20,833	20,833	20,833
W/S, lb/sq ft	50	50	50	50	50
$(\mu)_{h=0}$	13.0	15.0	18.4	20.0	18.4
K_{X_0}	0.1400	0.1448	0.1540	0.1708	0.1540
K_{Z_0}	0.243	0.272	0.320	0.435	0.318
ϵ, deg	−2	−2	−2	−2	−2

TABLE II

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS FOR BASIC CONFIGURATIONS

[$\Gamma=0°$; $i_w=0°$]

Configuration	Λ, deg	A	Flight conditions: h, ft	M	C_L	α, deg	η, deg	Mass parameters: μ	K_X^2	K_Z^2	K_{XZ}	Stability derivatives: C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	Results, Aperiodic mode: $T_{1/2}$, sec	$T_{1/2}$, sec	Oscillatory mode: P, sec	$T_{1/2}$, sec
1	0	6.0	0	0.75	0.06	0.71	−1.29	13	0.0196	0.0592	−0.00090	−0.4660	−0.0332	0.0806	0.0116	−0.4110	−0.0040	0.2323	0.0444	−0.1288	863.0	0.1015	1.037	1.455
			0	.27	.46	5.41	3.41	13	.0197	.0591	.00234	−.4660	−.0269	.0829	.0906	−.4098	−.0358	.2370	.1163	−.1335	−40.1	.284	4.347	3.028
			0	.204	.80	9.41	7.41	13	.0203	.0585	.00606	−.4660	−.0231	.0853	.1569	−.4096	−.0632	.2417	.1775	−.1383	−17.0	.370	5.480	4.008
			50,000	.75	.46	5.41	3.41	85	.0197	.0591	.00234	−.4660	−.0269	.0829	.0905	−.4098	−.0358	.2370	.1163	−.1335	−101.5	.713	4.594	10.919
2	30	4.5	0	.75	.06	.82	−1.18	15	.0210	.0738	−.00110	−.4660	−.0444	.0881	.0353	−.3323	−.0075	.2607	.0533	−.1634	212.0	.134	1.625	1.514
			0	.27	.46	6.30	4.30	15	.0213	.0735	.00395	−.4660	−.0799	.0904	.2879	−.3299	−.0567	.2654	.1237	−.1686	110.0	.364	3.960	4.695
			0	.204	.80	10.96	8.96	15	.0223	.0725	.00816	−.4660	−.1068	.0928	.4666	−.3301	−.0990	.2702	.1799	−.1740	122.0	.472	4.488	0.345
			50,000	.75	.46	6.30	4.30	98	.0213	.0735	.00395	−.4660	−.0799	.0904	.2879	−.3299	−.0567	.2654	.1237	−.1686	280.0	.890	4.275	19.960
3	45	3.0	0	.75	.06	1.03	−.97	18.4	.0237	.1025	−.00138	−.4660	−.0574	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2557	78.6	.207	1.538	1.500
			0	.27	.40	7.94	5.94	18.4	.0245	.1017	.00806	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2089	19.0	.511	3.353	8.560
			0	.204	.80	13.80	11.80	18.4	.0270	.0992	.01580	−.4660	−.1728	.1191	.7425	−.2325	−.1712	.3413	.1766	−.2757	15.8	.028	3.541	35.587
			50,000	.75	.40	7.94	5.94	120.5	.0245	.1017	.00806	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2089	48.4	1.212	3.692	71.472
4	60	1.5	0	.75	.06	1.58	−.42	26	.0313	.1890	−.00110	−.4660	−.0837	.1613	.0883	−.1260	−.0343	.4693	.1343	−.5940	20.0	.430	1.466	1.568
			0	.27	.40	12.10	10.10	26	.0360	.1842	.02720	−.4660	−.1880	.1634	.0765	−.1224	−.2065	.4835	.1317	−.5531	7.8	.745	2.016	−9.128
			0	.204	.80	21.05	19.05	26	.0477	.1723	.04850	−.4660	−.3149	.1634	1.1763	−.1448	−.4039	.4835	.1244	−.5531	7.9	.817	2.381	−4.151
			50,000	.75	.46	12.10	10.10	170.5	.0360	.1842	.02720	−.4660	−.1880	.1634	.0765	−.1224	−.2065	.4835	.1317	−.5531	20.0	1.775	2.842	−17.036
5	0	3.0	0	.75	.06	.91	−1.09	18.4	.0237	.1012	−.00149	−.4660	−.0497	.1120	.0227	−.2489	−.0070	.3271	.0743	−.2557	110.0	.1975	1.532	1.418
			0	.27	.40	6.98	4.98	18.4	.0243	.1006	.00673	−.4660	−.0594	.1168	.1703	−.2462	−.0530	.3365	.1280	−.2089	243.0	.532	3.775	4.141
			0	.204	.80	12.13	10.13	18.4	.0261	.0983	.01340	−.4660	−.0671	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−127.0	.080	4.453	5.291
			50,000	.75	.40	6.98	4.98	120.5	.0243	.1006	.00673	−.4660	−.0594	.1168	.1703	−.2462	−.0530	.3365	.1280	−.2089	620.0	1.330	4.021	12.746

TABLE III

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS TO DETERMINE EFFECTS OF MASS PARAMETERS AND INDIVIDUAL STABILITY DERIVATIVES

Basic configuration	Λ, deg	A	Changes from basic configuration	i_w, deg	Γ, deg	Flight conditions: h, ft	M	C_L	α, deg	γ, deg	Mass parameters: μ	K_X^2	K_Z^2	K_{XZ}	Stability derivatives: C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	Results, aperiodic mode: $T_{1/2}$, sec	$T_{1/2}$, sec	Results, oscillatory mode: P, sec	$T_{1/2}$, sec
1	0	6	Derivatives changed one at a time to the values for configuration 3. Derivatives changed are underlined.	0	0	0	0.27	0.46	7.94	5.94	18.4	0.0245	0.1017	0.00606	−0.4660	−0.0269	0.0829	0.0006	−0.4098	−0.0358	0.2370	0.1103	−0.1335	−41.6	0.340	4.661	4.604
									5.41	3.41	13	.0197	.0601	.00234	−.4660	−.1200	.0829	.0006	−.4098	−.0358	.2370	.1103	−.1335	45.8	.377	3.966	4.713
									5.41	3.41	13	.0197	.0601	.00234	−.4660	−.0269	.1168	.0006	−.4098	−.0358	.2370	.1103	−.1335	−33.4	.283	3.695	3.596
									5.41	3.41	13	.0197	.0601	.00234	−.4660	−.0269	.0829	.0006	−.2301	−.0358	.2370	.1103	−.1335	−24.1	.462	4.287	4.048
									5.41	3.41	13	.0197	.0601	.00234	−.4660	−.0269	.0829	.0006	−.4098	−.0978	.2370	.1103	−.1335	−30.3	.324	4.646	6.309
									5.41	3.41	13	.0197	.0601	.00234	−.4660	−.0269	.0829	.0006	−.4098	−.0358	.2370	.1103	−.2089	−101.5	.263	4.307	2.109
3	45	3	Derivatives changed one at a time to the values for configuration 1. Derivatives changed are underlined.	0	0	0	.27	.46	5.41	3.41	13	.0197	.0601	.00234	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3305	.1303	−.2089	17.1	.401	3.280	5.007
									7.94	5.94	18.4	.0245	.1017	.00606	−.4660	−.0269	.1168	.4275	−.2301	−.0978	.3305	.1303	−.2089	−27.7	.576	3.573	3.679
									7.94	5.94	18.4	.0245	.1017	.00606	−.4660	−.1200	.0829	.4275	−.2301	−.0978	.3305	.1303	−.2089	12.9	.501	3.716	11.786
									7.94	5.94	18.4	.0245	.1017	.00606	−.4660	−.1200	.1168	.4275	−.4098	−.0978	.3305	.1303	−.2089	27.2	.341	3.479	3.817
									7.94	5.94	18.4	.0245	.1017	.00606	−.4660	−.1200	.1168	.4275	−.2301	−.0358	.3305	.1303	−.2089	16.0	.508	3.503	4.424
									7.94	5.94	18.4	.0245	.1017	.00606	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3305	.1303	−.1335	411.0	.507	3.357	51.649

TABLE IV

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS FOR DETERMINING MEANS OF IMPROVING DUTCH ROLL STABILITY

(a) Basic configuration 3 ($\Lambda=45°$; $A=3$)

Changes from basic configuration	i_w, deg	Γ, deg	Flight conditions: h, ft	M	C_L	α, deg	η, deg	Mass parameters: μ	$K_X{}^2$	$K_Z{}^2$	K_{XZ}
Reduced i_w	−5	0	0	0.75	0.06	6.03	4.03	18.4	0.0241	0.1021	0.00650
			0	.27	.46	12.94	10.94	18.4	.0265	.0997	.01470
			0	.204	.80	18.80	16.80	18.4	.0303	.0959	.02180
			50,000	.75	.46	12.94	10.94	120.5	.0265	.0997	.01470
Reduced Γ	0	−5.4	0	.75	.06	1.03	−.97	18.4	.0237	.1025	−.00133
			0	.27	.46	7.94	5.94	18.4	.0245	.1017	.00806
			0	.204	.80	13.80	11.80	18.4	.0270	.0992	.01580
			50,000	.75	.46	7.94	5.94	120.5	.0245	.1017	.00800
Reduced i_w and Γ	−5	−4.3	0	.75	.06	6.03	4.03	18.4	.0241	.1021	.00600
			0	.27	.46	12.94	10.94	18.4	.0265	.0997	.01470
			0	.204	.80	18.80	16.80	18.4	.0303	.0959	.02180
			50,000	.75	.46	12.94	10.94	120.5	.0265	.0997	.01470
Reduced K_{X_0}	0	0	0	.75	.06	1.03	−.97	18.4	.0100	.1025	−.00100
			0	.27	.46	7.94	5.94	18.4	.0110	.1015	.00950
			0	.204	.80	13.80	11.80	18.4	.0139	.0985	.01850
			50,000	.75	.46	7.94	5.94	120.5	.0110	.1015	.00950
Reduced i_w and K_{X_0}	−5	0	0	.75	.06	6.03	4.03	18.4	.0105	.1021	.00640
			0	.27	.46	12.94	10.94	18.4	.0133	.0992	.01720
			0	.204	.80	18.80	16.80	18.4	.0177	.0948	.02560
			50,000	.75	.46	12.94	10.94	120.5	.0133	.0992	.01720
Reduced Γ and K_{X_0}	0	−5.4	0	.75	.06	1.03	−.97	18.4	.0100	.1025	−.00160
			0	.27	.46	7.94	5.94	18.4	.0110	.1015	.00060
			0	.204	.80	13.80	11.80	18.4	.0139	.0988	.01850
			50,000	.75	.46	7.94	5.94	120.5	.0110	.1015	.00960
Reduced i_w, Γ and K_{X_0}	−5	−4.3	0	.75	.06	6.03	4.03	18.4	.0106	.1021	.00040
			0	.27	.46	12.94	10.94	18.4	.0133	.0992	.01720
			0	.204	.80	18.80	10.80	18.4	.0177	.0948	.02560
			50,000	.75	.46	12.04	10.94	120.5	.0133	.0992	.01720

Changes from basic configuration	Stability derivatives: C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	Results, Aperiodic mode: $T_{1/2}$, sec	$T_{1/2}$, sec	Results, Oscillatory mode: P, sec	$T_{1/2}$, sec
Reduced i_w	−0.4660	−0.0468	0.1168	0.0946	−0.2305	−0.0347	0.3306	0.0587	−0.2690	90.2	0.226	1.438	1.126
	−.4660	−.1260	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	19.2	.560	3.033	4.538
	−.4660	−.1926	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	10.2	.670	3.142	11.434
	−.4660	−.1260	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	40.0	1.387	3.102	14.664
Reduced Γ	−.4660	0	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2557	−57.0	.218	1.537	1.242
	−.4660	−.0626	.1168	.4275	−.2301	−.0078	.3365	.1303	−.2689	161.0	.849	3.660	4.927
	−.4660	−.1154	.1191	.7425	−.2325	−.1712	.3413	.1766	−.2757	81.0	.660	3.855	11.908
	−.4660	−.0026	.1168	.4275	−.2301	−.0078	.3365	.1303	−.2689	−2,510.0	1.320	3,766	20.515
Reduced i_w and Γ	−.4660	0	.1168	.0946	−.2305	−.0347	.3306	.0587	−.2690	−71.5	.222	1.499	1.146
	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	47.8	.585	3.274	3.880
	−.4660	−.1466	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	22.5	.698	3.367	8.446
	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	131.5	1.440	3.337	11.485
Reduced K_{X_0}	−.4660	−.0674	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2557	78.7	.069	1.517	1.601
	−.4660	−.1200	.1168	.4275	−.2301	−.0078	.3306	.1303	−.2689	10.2	.278	2.989	2.331
	−.4660	−.1728	.1191	.7425	−.2325	−.1712	.3413	.1766	−.2757	15.9	.482	2.709	1.907
	−.4660	−.1200	.1168	.4275	−.2301	−.0078	.3365	.1303	−.2689	40.0	.735	2.989	5.382
Reduced i_w and K_{X_0}	−.4660	−.0468	.1168	.0946	−.2305	−.0347	.3306	.0587	−.2690	96.3	.100	1.420	.864
	−.4660	−.1260	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	19.4	.353	2.536	1.232
	−.4660	−.1926	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	16.4	.535	2.322	1.819
	−.4660	−.1260	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	49.5	.902	2.375	2.724
Reduced Γ and K_{X_0}	−.4660	0	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2567	−56.8	.092	1.537	1.225
	−.4660	−.0626	.1168	.4275	−.2301	−.0074	.3365	.1303	−.2689	101.5	.273	3.338	2.420
	−.4660	−.1154	.1194	.7425	−.2325	−.1712	.3413	.1766	−.2757	31.4	.418	3.190	1.938
	−.4660	−.0026	.1168	.4275	−.2301	−.0074	.3365	.1303	−.2689	−2,520.0	.715	3.326	5.943
Reduced i_w, Γ and K_{X_0}	−.4660	0	.1168	.0946	−.2306	−.0347	.3360	.0687	−.2690	−71.5	.094	1.503	1.085
	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2780	48.2	.330	2.875	1.373
	−.4660	−.1469	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2780	23.9	.520	2.509	1.350
	−.4660	−.0792	1191	.5362	−.2318	−.1331	.3412	.1204	−.2780	123.0	.940	2.681	2.911

TABLE IV—Continued

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS FOR DETERMINING MEANS OF IMPROVING DUTCH ROLL STABILITY

(a) Basic configuration 3 (Λ=45°; A=3).—Concluded

Changes from basic configuration	i_w, deg	Γ, deg	Flight conditions: h, ft	M	C_L	α, deg	η, deg	Mass parameters: μ	K_X^2	K_Z^2	K_{XZ}	Stability derivatives: C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	Results, Aperiodic mode: $T_{1/2}$, sec	Results, Aperiodic mode: $T_{1/2}$, sec	Oscillatory mode: P, sec	Oscillatory mode: $T_{1/2}$, sec
Increased K_{Z_0}	0	0	0	0.75	0.06	1.03	−0.97	18.4	0.0238	0.2000	−0.00290	−0.4660	−0.0574	0.1120	0.0659	−0.2338	−0.0128	0.3271	0.0745	−0.2557	80.5	0.200	2.120	2.808
			0	.27	.46	7.94	5.94	18.4	.0255	.1981	.01820	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	23.4	.551	4.068	11.030
			0	.204	.80	13.80	11.80	18.4	.0311	.1926	.03530	−.4660	−.1728	.1191	.7425	−.2325	−.1712	.3413	.1766	−.2757	21.1	.734	3.967	14.720
			50,000	.75	.46	7.94	5.94	120.5	.0255	.1981	.01820	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	59.6	1.325	4.229	94.121
Reduced i_w and increased K_{Z_0}	−5	0	0	.75	.06	6.03	4.03	18.4	.0246	.1991	.01240	−.4660	−.0458	.1168	.0946	−.2305	−.0347	.3366	.0587	−.2690	93.3	.238	1.999	1.417
			0	.27	.46	12.94	10.94	18.4	.0300	.1937	.03290	−.4660	−.1250	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	23.4	.682	3.502	3.835
			0	.204	.80	18.80	16.80	18.4	.0384	.1853	.04580	−.4660	−.1928	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	21.3	.874	3.376	5.610
			50,000	.75	.46	12.94	10.94	120.5	.0300	.1937	.03290	−.4660	−.1250	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	60.0	1.730	3.508	10.059
Reduced Γ and increased K_{Z_0}	0	−5.4	0	.75	.06	1.03	−.97	18.4	.0238	.2000	−.00290	−.4660	0	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2557	−57.0	.219	2.143	1.909
			0	.27	.46	7.94	5.94	18.4	.0255	.1981	.01820	−.4660	−.0626	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	187.5	.573	4.617	7.101
			0	.204	.80	13.80	11.80	18.4	.0311	.1926	.03530	−.4660	−.1154	.1191	.7425	−.2326	−.1712	.3413	.1766	−.2757	40.5	.750	4.438	10.512
			50,000	.75	.46	7.94	5.94	120.5	.0255	.1981	.01820	−.4660	−.0626	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	−2,940.0	1.385	4.745	32.167
Reduced i_w and Γ and increased K_{Z_0}	−5	−4.3	0	.75	.06	6.03	4.03	18.4	.0246	.1991	.01240	−.4660	0	.1168	.0946	−.2305	−.0347	.3366	.0587	−.2690	−72.0	.226	2.082	1.704
			0	.27	.46	12.94	10.94	18.4	.0300	.1937	.03290	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	56.1	.665	3.919	3.794
			0	.204	.80	18.80	16.80	18.4	.0384	.1853	.04580	−.4660	−.1460	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	30.8	.876	3.602	5.354
			50,000	.75	.46	12.94	10.94	120.5	.0300	.1937	.03290	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	144.5	1.720	3.915	9.811
Increased K_{Z_0} and reduced K_{X_0}	0	0	0	.75	.06	1.03	−.97	18.4	.0100	.1600	−.00254	−.4660	−.0574	.1120	.0659	−.2338	−.0128	.3271	.0745	−.2557	79.8	.089	1.879	2.409
			0	.27	.46	7.94	5.94	18.4	.0116	.1576	.01550	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	21.8	.296	3.414	2.180
			0	.204	.80	13.80	11.80	18.4	.0163	.1539	.03020	−.4660	−.1728	.1191	.7425	−.2325	−.1712	.3413	.1766	−.2757	19.1	.500	2.989	1.571
			50,000	.75	.46	7.94	5.94	120.5	.0116	.1576	.01550	−.4660	−.1200	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	55.5	.839	3.209	4.068
Reduced i_w, increased K_{Z_0} and reduced K_{X_0}	−5	0	0	.75	.06	6.03	4.03	18.4	.0108	.1593	.01040	−.4660	−.0458	.1168	.0946	−.2305	−.0347	.3366	.0587	−.2690	97.5	.101	1.755	.975
			0	.27	.46	12.94	10.94	18.4	.0154	.1546	.02790	−.4660	−.1250	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	21.9	.415	2.799	1.064
			0	.204	.80	18.80	16.80	18.4	.0225	.1475	.04160	−.4660	−.1928	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	19.4	.670	2.408	1.127
			50,000	.75	.46	12.94	10.94	120.5	.0154	.1546	.02790	−.4660	−.1250	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	56.0	1.248	2.506	2.292
Reduced Γ, increased K_{Z_0}, and reduced K_{X_0}	0	−5.4	0	.75	.06	1.03	−.97	18.4	.0100	.1600	−.00254	−.4660	0	.1120	.0559	−.2338	−.0128	.3271	.0745	−.2557	−56.9	.092	1.918	1.648
			0	.27	.46	7.94	5.94	18.4	.0116	.1576	.01550	−.4660	−.0626	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	177.0	.260	2.985	2.568
			0	.204	.80	13.80	11.80	18.4	.0163	.1539	.03020	−.4660	−.1154	.1191	.7425	−.2326	−.1712	.3413	.1766	−.2757	37.0	.468	3.495	1.720
			50,000	.75	.46	7.94	5.94	120.5	.0116	.1576	.01550	−.4660	−.0626	.1168	.4275	−.2301	−.0978	.3365	.1303	−.2689	−2,760.0	.785	3.779	4.851
Reduced i_w and Γ, increased K_{Z_0}, and reduced K_{X_0}	−5	−4.3	0	.75	.06	6.03	4.03	18.4	.0108	.1593	.01040	−.4660	0	.1168	.0946	−.2305	−.0347	.3366	.0587	−.2690	−71.5	.095	1.673	1.426
			0	.27	.46	12.94	10.94	18.4	.0154	.1546	.02790	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	53.4	.389	3.281	1.249
			0	.204	.80	18.80	16.80	18.4	.0226	.1475	.04150	−.4660	−.1460	.1191	.9110	−.2384	−.2158	.3412	.1893	−.2750	26.1	.640	2.701	1.163
			50,000	.75	.46	12.94	10.94	120.5	.0154	.1546	.02790	−.4660	−.0792	.1191	.5362	−.2318	−.1331	.3412	.1294	−.2750	130.2	1.180	2.891	2.459

TABLE IV—Continued

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS FOR DETERMINING MEANS OF IMPROVING DUTCH ROLL STABILITY

(b) Basic configuration 5 ($\Lambda=0°$; $A=3$)

Changes from basic configuration	i_w, deg	Γ, deg	Flight conditions					Mass parameters				Stability derivatives									Results			
																					Aperiodic mode		Oscillatory mode	
			h, ft	M	C_L	α, deg	η, deg	μ	K_X^2	K_Z^2	K_{XZ}	C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	$T_{1/2}$, sec	$T_{1/2}$, sec	P, sec	$T_{1/2}$, sec
Reduced i_w	−5	0	0	0.75	0.06	5.91	3.91	18.4	0.0241	0.1008	0.00626	−0.4660	−0.0350	0.1157	0.0516	−0.2455	−0.0275	0.3340	0.0550	−0.2665	182.0	0.212	1.456	1.112
			0	.27	.46	11.98	9.98	18.4	.0260	.0989	.01290	−.4660	−.0440	.1193	.2006	−.2463	−.0753	.3416	.1064	−.2760	−52.1	.569	3.612	3.009
			0	.204	.80	17.13	15.13	18.4	.0290	.0959	.01960	−.4660	−.0524	.1197	.3370	−.2492	−.1131	.3424	.1503	−.2770	−50.7	.762	4.269	3.803
			50,000	.75	.46	11.98	9.98	120.5	.0260	.0969	.01390	−.4660	−.0440	.1193	.2006	−.2402	−.0753	.3416	.1064	−.2760	−1,116.0	1.460	3.610	7.806
Reduced Γ	0	−4	0	.75	.06	.91	−1.09	18.4	.0237	.1012	−.00140	−.4660	0	.1120	.0327	−.2489	−.0070	.3371	.0743	−.2567	−60.9	.206	1.528	1.234
			0	.27	.46	6.98	4.98	18.4	.0243	.1006	.00673	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	−17.0	.558	4.089	3.171
			0	.204	.80	12.13	10.13	18.4	.0261	.0988	.01340	−.4660	−.0174	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−12.4	.712	4.906	3.882
			50,000	.75	.46	6.98	4.98	120.5	.0243	.1006	.00673	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	−43.0	1.403	4.069	8.406
Reduced i_w and Γ	−5	−2.8	0	.75	.06	5.91	3.91	18.4	.0241	.1008	.00520	−.4660	0	.1157	.0516	−.2455	−.0275	.3340	.0550	−.2665	−81.1	.210	1.498	1.147
			0	.27	.46	11.98	9.98	18.4	.0260	.0989	.01320	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−21.5	.585	3.885	2.800
			0	.204	.80	17.13	15.13	18.4	.0290	.0959	.01950	−.4660	−.0174	.1197	.3370	−.2492	−.1131	.3424	.1503	−.2770	−15.4	.760	4.059	3.384
			50,000	.75	.46	11.98	9.98	120.5	.0260	.0980	.01320	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−54.4	1.485	3.860	7.221
Reduced K_{X_0}	0	0	0	.75	.06	.91	−1.09	18.4	.0100	.1012	−.00180	−.4660	−.0497	.1120	.0297	−.2489	−.0070	.3271	.0743	−.2557	118.5	.094	1.517	1.512
			0	.27	.46	6.98	4.98	18.4	.0107	.1005	.00790	−.4660	−.0694	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	242.0	.249	3.504	2.491
			0	.204	.80	12.13	10.13	18.4	.0128	.0948	.01570	−.4660	−.0671	.1191	.2981	−.2462	−.0919	.3418	.1714	−.2757	−128.0	.356	4.003	2.125
			50,000	.75	.46	6.98	4.98	120.5	.0107	.1005	.00790	−.4660	−.0694	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	620.0	.675	3.484	5.121
Reduced i_w and K_{X_0}	−5	0	0	.75	.06	5.91	3.91	18.4	.0104	.1008	.00620	−.4660	−.0350	.1157	.0516	−.2455	−.0275	.3340	.0550	−.2665	182.0	.091	1.449	.903
			0	.27	.46	11.98	9.98	18.4	.0127	.0984	.01560	−.4660	−.0440	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−4,560.0	.276	3.433	1.889
			0	.204	.80	17.13	15.13	18.4	.0162	.0950	.02290	−.4660	−.0524	.1197	.3370	−.2492	−.1131	.3424	.1503	−.2770	−69.5	.414	3.861	1.491
			50,000	.75	.46	11.98	9.98	120.5	.0127	.0984	.01560	−.4660	−.0440	.1193	.2006	−.2402	−.0753	.3416	.1064	−.2760	−1,140.0	.838	3.116	2.850
Reduced Γ and K_{X_0}	0	−4	0	.75	.06	.91	−1.09	18.4	.0100	.1012	−.00180	−.4660	0	.1120	.0227	−.2489	−.0070	.3371	.0743	−.2567	−60.7	.065	1.537	1.218
			0	.27	.46	6.98	4.98	18.4	.0107	.1005	.00790	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	−16.8	.244	3.998	2.629
			0	.204	.80	12.13	10.13	18.4	.0128	.0984	.01570	−.4660	−.0174	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−12.2	.333	4.756	2.428
			50,000	.75	.46	6.98	4.98	120.5	.0107	.1005	.00790	−.4660	−.0097	.1168	.1703	−.2452	−.0630	.3365	.1280	−.2689	−42.5	.629	3.941	6.025
Reduced i_w, Γ, and K_{X_0}	−5	−2.8	0	.75	.06	5.91	3.91	18.4	.0104	.1008	.00620	−.4660	0	.1157	.0516	−.2455	−.0275	.3340	.0550	−.2665	−80.3	.086	1.502	1.096
			0	.27	.46	11.98	9.98	18.4	.0127	.0984	.01560	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2700	−21.2	.280	3.813	1.971
			0	.204	.80	17.13	15.13	18.4	.0162	.0950	.02290	−.4660	−.0174	.1197	.3270	−.2492	−.1131	.3424	.1503	−.2770	−15.3	.372	4.440	1.775
			50,000	.75	.46	11.98	9.98	120.5	.0127	.0984	.01500	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3410	.1064	−.2700	−54.0	.735	3.531	3.031

TABLE IV—Concluded

CONDITIONS FOR WHICH CALCULATIONS WERE MADE, DERIVATIVES USED IN CALCULATIONS, AND RESULTS OF CALCULATIONS FOR DETERMINING MEANS OF IMPROVING DUTCH ROLL STABILITY

(b) Basic configuration 5 ($\Lambda=0°$; $A=3$)—Concluded

Changes from basic configuration	i_w, deg	Γ, deg	Flight conditions					Mass parameters				Stability derivatives									Results			
			h, ft	M	C_L	α, deg	η, deg	μ	K_X^2	K_Z^2	K_{XZ}	C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}	Aperiodic mode $T_{1/2}$, sec	Aperiodic mode $T_{1/2}$, sec	Oscillatory mode P, sec	Oscillatory mode $T_{1/2}$, sec
Increased K_{X_0}	0	0	0	0.75	0.06	0.91	−1.09	18.4	0.0238	0.1999	−0.00340	−0.4660	−0.0497	0.1120	0.0227	−0.2489	−0.0070	0.3271	0.0743	−0.2557	121.0	0.195	2.140	2.622
			0	.27	.46	6.98	4.98	18.4	.0250	.1967	.01530	−.4660	−.0594	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	280.0	.535	4.860	6.494
			0	.204	.80	12.13	10.13	18.4	.0292	.1945	.03040	−.4660	−.0071	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−169.2	.718	5.402	6.966
			50,000	.75	.46	6.98	4.98	120.5	.0250	.1967	.01530	−.4660	−.0594	.1168	.1703	−.2452	−.0530	.3305	.1280	−.2689	720.0	1.345	4.945	20.974
Reduced i_w and increased K_{X_0}	−5	0	0	.75	.06	5.91	3.91	18.4	.0245	.1992	.01200	−.4660	−.0350	.1157	.0616	−.2455	−.0275	.3340	.0650	−.2065	186.0	.220	1.906	1.467
			0	.27	.46	11.98	9.98	18.4	.0290	.1947	.03010	−.4660	−.0440	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−524.0	.626	4.569	3.518
			0	.204	.80	17.13	15.13	18.4	.0357	.1880	.04440	−.4660	−.0594	.1197	.3270	−.2492	−.1131	.3424	.1503	−.2770	−87.0	.835	5.107	4.005
			50,000	.75	.46	11.98	9.98	120.5	.0290	.1947	.03010	−.4660	−.0440	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−1,310.0	1.080	4.476	8.275
Reduced Γ and increased K_{X_0}	0	−4	0	.75	.06	.91	−1.09	18.4	.0238	.1999	−.00340	−.4660	0	.1120	.0227	−.2489	−.0070	.3271	.0743	−.2557	−60.8	.200	2.143	1.912
			0	.27	.46	6.98	4.98	18.4	.0250	.1967	.01530	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	−18.0	.567	5.488	4.700
			0	.204	.80	12.13	10.13	18.4	.0292	.1945	.03040	−.4660	−.0174	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−14.4	.729	6.346	5.404
			50,000	.75	.46	6.98	4.98	120.5	.0250	.1967	.01530	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	−45.5	1.410	5.539	13.074
Reduced i_w and Γ and increased K_{X_0}	−5	−2.8	0	.75	.06	5.91	3.91	18.4	.0245	.1992	.01200	−.4660	0	.1157	.0616	−.2455	−.0275	.3340	.0650	−.2605	−81.5	.210	2.090	1.727
			0	.27	.46	11.98	9.98	18.4	.0290	.1947	.03010	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3410	.1004	−.2760	−22.2	.603	5.141	3.068
			0	.204	.80	17.13	15.13	18.4	.0357	.1880	.04440	−.4660	−.0174	.1197	.3270	−.2492	−.1131	.3424	.1503	−.2770	−18.2	.810	5.834	4.014
			50,000	.75	.46	11.98	9.98	120.5	.0290	.1947	.03010	−.4660	−.0090	.1193	.2006	−.2462	.0753	.3416	.1004	−.2760	−58.2	1.560	5.065	8.741
Increased K_{X_0} and reduced K_{X_0}	0	0	0	.75	.06	.91	−1.09	18.4	.0101	.1599	−.00288	−.4660	−.0497	.1120	.0227	−.2480	−.0070	.3271	.0743	−.2557	120.0	.085	1.894	2.205
			0	.27	.46	−0.98	4.98	18.4	.0111	.1589	.01305	−.4660	−.0594	.1168	.1703	−.2452	−.0530	.3305	.1280	−.2689	268.0	.252	4.200	2.803
			0	.204	.80	12.13	10.13	18.4	.0146	.1554	.02590	−.4660	−.0671	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2787	−149.0	.398	4.675	2.105
			50,000	.75	.46	6.98	4.98	120.5	.0111	.1589	.01305	−.4660	−.0594	.1168	.1703	−.2452	−.0530	.3365	.1280	−.2689	681.0	.718	4.055	4.732
Reduced i_w, increased K_{X_0}, and reduced K_{X_0}	−5	0	0	.75	.06	5.91	3.91	18.4	.0107	.1593	.01020	−.4660	−.0350	.1157	.0510	−.2455	−.0275	.3340	.0550	−.2605	184.0	.092	1.808	1.078
			0	.27	.40	11.98	9.98	18.4	.0145	.1555	.02560	−.4660	−.0440	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2700	−498.0	.292	4.104	1.650
			0	.204	.80	17.13	15.13	7.4	.0202	.1498	.03770	−.4660	−.0594	.1197	.3270	−.2492	−.1131	.3424	.1503	−.2770	−80.0	.440	4.411	1.425
			50,000	.75	.40	11.98	9.98	120.5	.0145	.1555	.02560	−.4660	−.0440	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−1,242.0	.975	3.508	2.512
Reduced Γ, increased K_{X_0}, and reduced K_{X_0}	0	−4	0	.75	.06	.91	−1.09	18.4	.0101	.1599	−.00288	−.4660	0	.1120	.0227	−.2489	−.0070	.3271	.0743	−.2557	−60.7	.067	1.917	1.654
			0	.27	.40	6.98	4.98	18.4	.0111	.1589	.01305	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3305	.1280	−.2689	−14.4	.242	4.938	3.338
			0	.204	.80	12.13	10.13	18.4	.0140	.1554	.02590	−.4660	−.0174	.1191	.2981	−.2462	−.0919	.3413	.1714	−.2757	−13.4	.334	5.707	2.795
			50,000	.75	.46	6.98	4.98	120.5	.0111	.1589	.01306	−.4660	−.0097	.1168	.1703	−.2452	−.0530	.3305	.1280	−.2689	−44.0	.648	4.827	6.666
Reduced i_w and Γ, increased K_{X_0}, and reduced K_{X_0}	−5	−2.8	0	.75	.06	5.91	3.91	18.4	.0107	.1593	.01020	−.4660	0	.1157	.0616	−.2455	−.0275	.3340	.0650	−.2665	−81.1	.088	1.885	1.456
			0	.27	.46	11.98	9.98	18.4	.0145	.1556	.02560	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−22.4	.201	4.677	2.289
			0	.204	.80	17.13	15.13	18.4	.0202	.1498	.03770	−.4660	−.0174	.1197	.3270	−.2492	−.1131	.3424	.1503	−.2770	−17.0	.334	5.267	1.862
			50,000	.75	.46	11.98	9.98	120.5	.0145	.1556	.02560	−.4660	−.0090	.1193	.2006	−.2462	−.0753	.3416	.1064	−.2760	−56.8	.798	4.246	3.420