REPORT 1212

ANALOG STUDY OF INTERACTING AND NONINTERACTING MULTIPLE-LOOP CONTROL SYSTEMS FOR TURBOJET ENGINES [1]

By GEORGE J. PACK and W. E. PHILLIPS, JR.

SUMMARY

An analog investigation of several turbojet-engine control configurations was made. Both proportional and proportional-plus-integral controllers were studied, and compensating terms for engine interaction were added to the control system. Data were obtained on the stability limits and the transient responses of these various configurations. Analytical expressions in terms of the component transfer functions were developed for the configurations studied, and the optimum form for the compensation terms was determined.

It was found that the addition of the integral term, while making the system slower and more oscillatory, was desirable in that it made the final values of the system parameters independent of source of disturbance and also eliminated droop in these parameters.

Definite improvement in system characteristics resulted from the use of proper compensation terms. At comparable gain points the compensated system was faster and more stable. Complete compensation eliminated engine interaction, permitting each loop to be developed to an optimum point independently.

INTRODUCTION

Turbojet engines with a fixed-area exhaust nozzle do not present too difficult a control problem because only one input variable, fuel flow, is manipulated to maintain desired engine speed or temperature. A single closed-loop system, incorporating overspeed and overtemperature protection along with a schedule of fuel flow to prevent surge on acceleration, will accomplish the necessary control function. When a variable-area exhaust nozzle is added to such an engine, however, the control problem becomes more complex because two input variables are available; these should be so controlled that the engine is at all times operating in a safe and efficient manner. When more than one input variable to an engine is controlled, the resulting system is a multiple-loop configuration. A general discussion of multiple-loop systems with a specific example of an aircraft reciprocating-engine control is given in reference 1.

For the specific case of an engine in which speed and temperature are to be controlled by manipulation of fuel flow and exhaust-nozzle area, two double-loop systems can possibly be employed. In one case, speed can be controlled by exhaust-nozzle area, and temperature can be controlled by fuel flow. In the second system, speed can be controlled by fuel flow, while temperature is controlled by exhaust-nozzle area. A basic characteristic of turbojet engines is that a change in fuel flow or area causes both speed and temperature to change. Therefore, whenever these engine parameters are used in a double-loop control configuration, a disturbance in one loop will introduce an error signal into the other loop. This characteristic will be referred to herein as the interaction effect which exists between the individual control loops in a double-loop system. As a result of such interaction, an unstable system, or one having very oscillatory responses in some regions of control operation, can result even though each loop may be inherently stable when used alone. In order to stabilize a system of this form, it generally becomes necessary to reduce the loop gains or sensitivities; but this is accomplished at the expense of an increase in response time for the complete system.

A general algebraic method of analysis has been applied to the determination of control requirements for multiple-loop engine control systems and is presented in reference 2. It was shown therein that control systems could be designed so as to be noninteracting; that is, each loop in such a configuration can then be considered as acting independently in the combined system. Further analysis also indicates that a noninteracting control system will permit improved stability and faster response than are possible with the current interacting configurations. It was considered important, therefore, to have an understanding of both interacting and noninteracting double-loop systems because the more complex engine types being developed at present, along with the demands for faster responding power plants, necessitate the use of such systems.

For this reason an investigation was initiated at the NACA Lewis laboratory to determine some of the practical aspects of noninteracting systems and to compare these with an interacting configuration. Stability limits and response characteristics were obtained for one basic double-loop system and also for several modifications of the system. An analog computer was used to simulate a current turbojet engine with a variable jet nozzle along with the necessary sensor and servo components of the engine control.

[1] Supersedes NACA TN 3112, "Analog Study of Interacting and Noninteracting Multiple-Loop Control Systems for Turbojet Engines," by George J. Pack and W. E. Phillips, Jr., 1954.

The basic configuration studied is one in which speed is controlled by fuel flow and temperature is controlled by exhaust-nozzle area. One modification consisted of adding an integral term to each loop of the system, while another modification consisted of adding a term to compensate in part for the interaction characteristic of the engine. Stability limits were determined for these systems. Three different forms of compensation for noninteracting systems were investigated. The investigation was extended to present transient response characteristics of the systems to a step disturbance in set temperature. The engine was assumed to be operating near design speed but at lower than design temperature, and an increase in thrust would be obtained by increasing set temperature. An assumption was also made of linearity in the region of the engine operating point.

COMPUTER AND METHOD

A high-speed electronic analog computer operating at 4800 times real time was used. A number of computational elements of standard form are available, and these can be interconnected by means of plug-in cables. A standard square-wave disturbance voltage with a repetition rate of 60 cycles per second is supplied and, by calibration, its time base represents 20 seconds of real engine time. Solutions are presented on a group of oscilloscopes so that the transient response of several variables due to the applied step disturbance can be observed simultaneously. One of the computational elements is a matrix which is used to simulate the engine. This method is presented in detail in reference 3.

A control simulator component is also available which has the following transfer function, where E_o and E_i are output and input voltages:

$$\frac{E_o}{E_i}=\pm K\left(1+\frac{1}{\tau_i p}\right)\left(1+\tau_0 p\right)\left(\frac{1}{1+\tau_l p}\right)$$

where the gain term K and the integral, derivative, and lag time constants τ_i, τ_0, and τ_l are variable. An added feature is that the integral, derivative, or lag terms can be switched out if required. The computer also contains a number of summing, coefficient, integral, derivative, and lag units along with calibration devices which permit a more accurate setting of the variables and determination of output voltage values. Provisions are also available for photographing the oscilloscope displays.

A high-speed computer of the type used has the advantage that characteristic responses over a broad range of possible control settings of various systems such as shown in figure 1 can be investigated very quickly with minimum effort. (The symbols in fig. 1 and elsewhere are defined in the appendix.) Systems can be quickly changed or modified as required by indicated trends of the investigation.

Stability, in particular, can easily be determined by the following method: With no forcing function or disturbance and with a specific value of temperature loop gain set into the computer, the speed loop gain can be gradually increased from zero until the entire system becomes unstable, as shown by continuous oscillations of all parameters on the oscilloscopes. This procedure can be repeated for a number of values of temperature loop gain over the entire range. A plot of the values of temperature loop gain against speed loop gain at which the system becomes unstable can be made from these data; this curve defines the limits of stability for the configuration. When a disturbance is added to the system, the transient responses of all pertinent parameters can be observed and variations in these responses noted as a function of loop gains.

All engine gain or sensitivity terms used in the simulation were normalized to rated values. Therefore, computer output voltages representing the transients were proportional to a percent of rated value change in all parameters. For the purpose of this report, a 1-percent step disturbance was introduced in set temperature. Speed and temperature droops (which are defined as the deviation in percent of rated value of the parameter in steady state from the desired final value) and maximum excursions (which are defined as the maximum deviations in percent of rated values of the parameters during a transient, measured from the initial starting point) were recorded and plotted as percent deviations on the stability-limit figures. In addition, the time rises (which are defined as the time required to reach maximum excursion) were noted and plotted in a similar manner.

Examination of the resulting maps shows how the transient responses vary as a function of both speed and temperature loop gains and also permits a rapid comparison of the effect on response that can be obtained by modifying the system and by using compensation for the interaction normally found in engines.

SELECTION OF SYSTEMS TO BE INVESTIGATED

Preliminary analysis of interacting and noninteracting systems was conducted to determine the specific configurations to be studied in detail by analog methods.

INTERACTING SYSTEM

A block diagram of the basic double-loop system investigated is shown in figure 1(*a*). The engine, sensors, and controllers have transfer functions symbolized by E, H, and G, respectively.

Significant system transfer functions have been derived and are presented herein with the added substitution that the product of all terms in each simple loop is characterized by one symbol. That is, the product of the speed loop terms, H_1, G_1, and E_1, is replaced by L_N, while the product of terms in the temperature loop, H_2, G_2, and E_4, is replaced by L_T. A third loop is formed in this configuration that includes the interacting engine terms and therefore is called the interaction loop. This loop consists of H_1, G_1, E_2, H_2, G_2, and E_3. The product of all these terms is indicated by L_X in subsequent discussion.

System transfer functions are

$$\frac{N}{N_s}=\frac{\frac{1}{H_1}[L_N(1+L_T)-L_X]}{(1+L_N)(1+L_T)-L_X} \quad (1)$$

$$\frac{N}{T_s}=\frac{G_2E_3}{(1+L_N)(1+L_T)-L_X} \quad (2)$$

$$\frac{T}{T_s}=\frac{\frac{1}{H_2}[L_T(1+L_N)-L_X]}{(1+L_N)(1+L_T)-L_X} \quad (3)$$

$$\frac{T}{N_s}=\frac{G_1E_2}{(1+L_N)(1+L_T)-L_X} \quad (4)$$

The stability of the system can be determined from analysis of the denominator of these transfer functions, which when set equal to zero is the characteristic equation of the system. Further examination of this equation, however, indicates that, if the interaction loop term L_X were made zero, then the system would behave as two independent single-loop systems.

NONINTERACTING SYSTEM

A completely noninteracting system can be derived by adding two new elements to the control configuration as shown in figure 1(b). The purpose of X is to add a function of temperature error to a function of speed error so that the resulting change in fuel flow compensates for the speed change resulting from the action of temperature error on exhaust-nozzle area. Therefore, with a properly chosen value of X, no speed error will be evident when a change in controlled engine temperature is required by manipulation of set temperature. Another element Y can be added to the system in a similar manner so that speed error will have no effect on temperature when set speed is varied.

The following transfer functions for the system shown in figure 1(b) have been derived:

$$\frac{N}{N_s}=\frac{\frac{1}{H_1}[L_{N'}(1+L_{T'})-L_{X'}]}{(1+L_{N'})(1+L_{T'})-L_{X'}} \quad (5)$$

$$\frac{N}{T_s}=\frac{G_1E_1X+G_2E_3}{(1+L_{N'})(1+L_{T'})-L_{X'}} \quad (6)$$

$$\frac{T}{T_s}=\frac{\frac{1}{H_2}[L_{T'}(1+L_{N'})-L_{X'}]}{(1+L_{N'})(1+L_{T'})-L_{X'}} \quad (7)$$

$$\frac{T}{N_s}=\frac{G_1E_2+YG_2E_4}{(1+L_{N'})(1+L_{T'})-L_{X'}} \quad (8)$$

In these equations $L_{N'}$ is equal to $H_1(G_1E_1+YG_2E_3)$, which is the product of all terms in the speed loop where now a parallel feed path exists through G_1E_1 and YG_2E_3. Similarly, $L_{T'}$ is equal to $H_2(G_2E_4+XG_1E_2)$ with the parallel feed being through G_2E_4 and XG_1E_2. The interaction loop is given by $L_{X'}$, which is $H_1H_2(G_1E_2+YG_2E_4)(G_2E_3+XG_1E_1)$. Two parallel feed paths are evident in this loop.

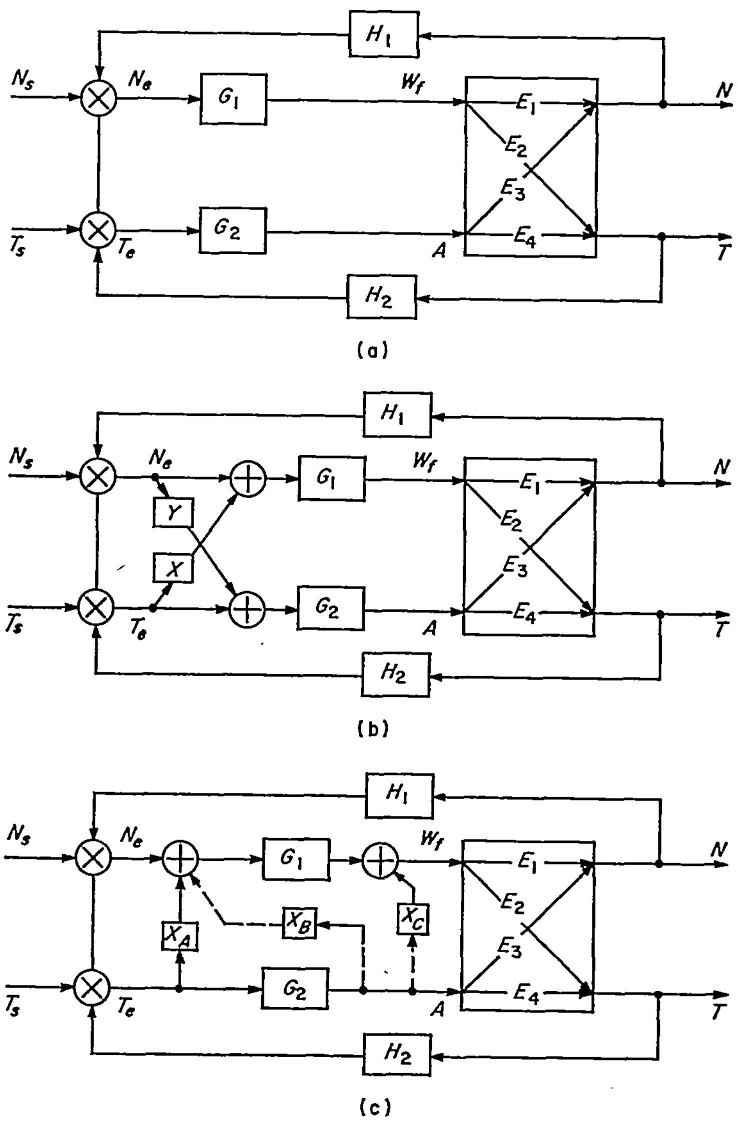
The interaction loop $L_{X'}$ is equal to zero if either X or Y has the following values:

$$X=-\frac{G_2E_3}{G_1E_1} \quad (9)$$

$$Y=-\frac{G_1E_2}{G_2E_4} \quad (10)$$

Substituting equations (9) and (10) into the characteristic equation for the noninteracting system results in the following expression:

$$(1+L_{N'})(1+L_{T'})-L_{X'}=\left[1+L_N\left(1-\frac{E_2E_3}{E_1E_4}\right)\right]\left[1+L_T\left(1-\frac{E_2E_3}{E_1E_4}\right)\right]$$



(a) Basic system.
(b) System with complete compensation.
(c) Three forms of partial compensation.

FIGURE 1. Block diagrams of systems studied.

This expression indicates that the multiple-loop system can be considered to consist of two independent loops. The characteristic equations of these loops are

$$1+L_N\left(1-\frac{E_2E_3}{E_1E_4}\right)=0 \text{ and } 1+L_T\left(1-\frac{E_2E_3}{E_1E_4}\right)=0$$

System instability can occur only when one of the loops is unstable by itself.

Further examination of the characteristic equation for the compensated system shows that only one compensating term X or Y is necessary to make the interaction loop equal to zero. However, the transient responses of a system with only one added element will be different from those of a system with both X and Y added. As an example of this, consider only an X element added to the system. Speed will not be affected by a disturbance in set temperature even though the system will act to minimize the temperature error by causing the exhaust-nozzle area to change. A disturbance in set speed, however, will cause the temperature to deviate from its initial value as well as cause the speed to change and thereby minimize the speed error. The resulting temperature error, however, will not cause an additional change in speed because of the influence of the X term. With the compensating Y element in place, temperature would not be affected by a change in set speed.

An engine control system may not require the complexity of complete compensation for both temperature and speed error interaction. Compensation for the effect of temperature error on speed should be sufficient because normal engine operation is usually at top speed, where speed is held constant and thrust variations are made by changing temperature only. If the compensating element is exactly as specified by equation (9), the compensation is complete and no speed disturbance results during a transient from a set temperature change. However, because speed variation within certain limits can be tolerated, the compensation element need not be so complex as indicated by equation (9). The analysis reported herein is based on the use of only a gain term for the compensating element instead of one having all the necessary dynamic terms indicated by equation (9).

Figure 1(c) is presented to show three possible positions of the compensating term in a control configuration. The complete forms of X for the three positions can be derived and are

$$X_A=-\frac{G_2E_3}{G_1E_1},\ X_B=-\frac{E_3}{G_1E_1},\ X_C=-\frac{E_3}{E_1}$$

These expressions indicate that the compensating element will have different required characteristics depending on the function of temperature and speed errors considered. By using only a gain term in the compensating element, partial compensation to different degrees is achieved with X_A and X_B, while X_C supplies complete compensation to the system.

SPECIFIC SYSTEMS INVESTIGATION

Figure 2 shows a block diagram of the systems investigated as set up on the computer by using the method of reference 3. Component gains and time constants were chosen to be representative of current devices, and specific values are shown on the figure.

The engine has a time constant of 1.75 seconds at the operating point chosen, which was (based on design values) 96-percent speed, 86-percent temperature, 67-percent fuel flow, and 98-percent exhaust-nozzle area, where turbine-exit area is defined as 100 percent. The total exhaust-nozzle area range is 75 to 133 percent.

The speed sensor was simulated by a first-order lag having a time constant of 0.05 second, while the temperature sensor was assumed to be a thermocouple with a nominal time constant of 1 second.

The fuel-flow servo was represented by two lags in series, each having a time constant of 0.10 second. The exhaust-nozzle-area servo, which in practice is a much slower device, was considered to consist of a 0.3-second time-constant lag in series with a 0.15-second time-constant lag. The system was calibrated in such a manner that loop gains could be read directly from dial settings.

The first system investigated consisted of the basic configuration where speed is controlled by fuel flow and temperature is controlled by exhaust-nozzle area, with proportional control in both loops. This system was then modified by the addition of an integral term to each loop. Integral action results in elimination of droop that is characteristic of proportional controls. The integral time constant was chosen to be equal to the engine time constant. A third system studied consisted of adding a gain term to the basic system to compensate for the effect of temperature error on speed. The compensating element X_A was used and, as mentioned before, this element provides only partial compensation. The fourth system investigated used both the integral and compensating terms of the previous configurations.

In considering the gross effect of loop gains, it becomes apparent that sensitivity of control is related to this gain. A high loop gain results in high sensitivity and rapid recovery to an imposed disturbance. The transient responses of this system, however, become more oscillatory as loop gain is increased. At some value of this term, depending on the dynamics involved, the entire system can become unstable, at which point a self-sustained oscillation will occur, as shown in reference 4. Preliminary investigations were conducted to determine the effect on stability limits of the three forms of compensation X_A, X_B, and X_C.

STABILITY LIMITS

Figure 3 presents stability limits obtained with the basic system and also with each of the three gain compensation terms X_A, X_B, and X_C. In these data, speed loop gain K_N has the same significance whether or not the compensation term is used. When compensation is used, the gain of the temperature loop is actually that computed from $L_{T'}$, which is

$$L_T\left(1-\frac{E_2E_3}{E_1E_4}\right)$$

These data, however, are plotted for comparison purposes on the basis of the simple temperature loop L_T, which has a gain symbolized by K_T.

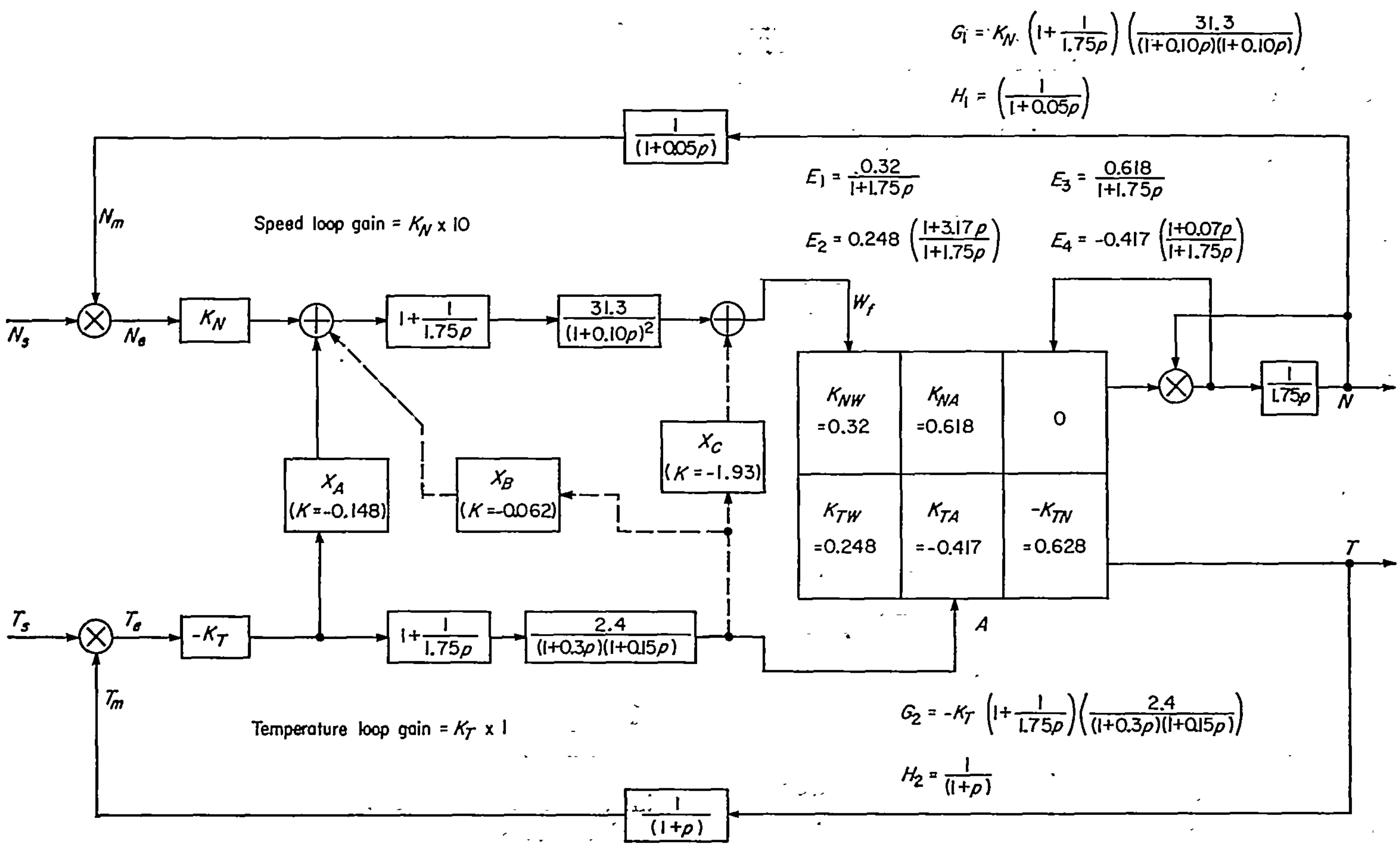


Figure 2.—Block diagram of system on analog computer.

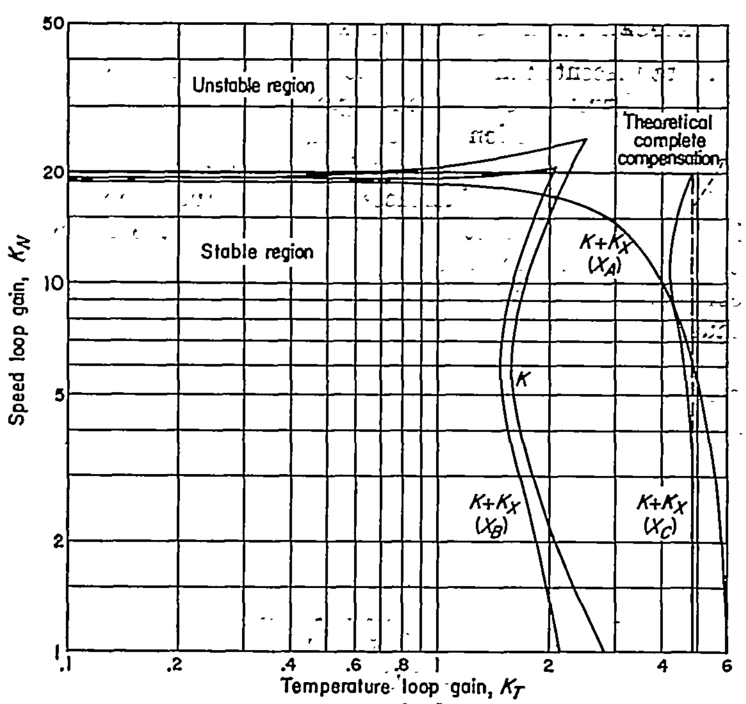


FIGURE 3.—Comparison of stability limits. No interaction compensation and three partial compensation methods with proportional control.

These conditions account for the coincidence of the stability limits of all systems in the high K_N and low K_T region where the system is predominately a single speed - fuel-flow loop and for the divergence of points in the region of low K_N and high K_T where the system is predominately a single temperature - area loop.

Examination of the curve showing the stability limit with no compensation (curve K) indicates that the interaction loop has a severe effect on the stability of the system in the high-temperature-loop-gain region and acts to reduce the gain that this loop could tolerate if it were operating alone. The stability limit for the system with X_C, in which case only a gain term provides complete compensation, shows that when the product of the terms of the interaction loop is zero the stability limit approaches the theoretical limit. The result is that each loop is independent of the other up to the single-loop stability limit. The slight deviation of the derived limit, obtained with the analog, from the theoretical limit can be attributed to minor inaccuracies in adjustment of the compensating gain and too small dynamic terms associated with the computer elements.

The system employing X_A is shown to be more effective than that using X_B and therefore was used as the compensation form for subsequent work. The term X_B, in fact, decreased the stability limit below that with no compensation. The variations in effect of X_A and X_B can be explained if the necessary forms of these cross-overs for dynamic compensation are considered. With gain compensation only, X_A is much closer to complete dynamic compensation than is X_B. Compensation of the form X_C, while considered better than X_A or X_B, is unfortunately not useful on a real engine system because it is impractical to vary fuel flow as a function of exhaust-nozzle area without introducing additional dynamics to the system.

Figure 4 shows the stability limits obtained for the four configurations investigated. These data show that, when compared with a simple proportional control system (curve K), the addition of the integral term $\left(\text{curve } K\left(1+\frac{1}{\tau p}\right)\right)$ compresses the stability limit over the entire region. The addition of compensation to the proportional control system (curve $K+K_X$) expands the limit in the region of high temperature loop gains. The addition of an integral term to the system with compensation $\left(\text{curve } K\left(1+\frac{1}{\tau p}\right)+K_X\right)$ compresses the stability limit to a small extent, but a significant improvement is still evident when compared with the limit curve for the proportional-plus-integral configuration.

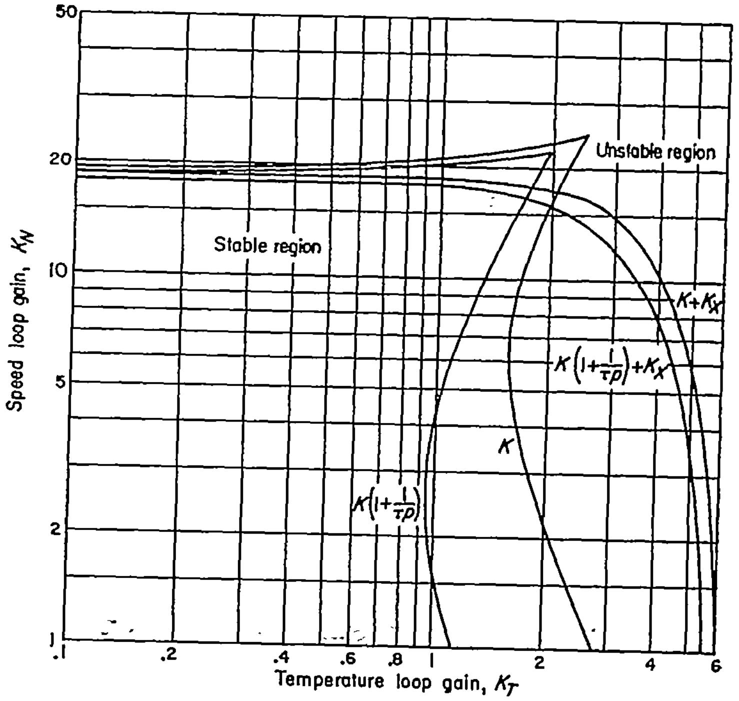


FIGURE 4.—Comparison of stability limits obtained with four configurations studied in detail. The term K_X denotes partial compensation X_A.

TRANSIENT STUDIES

Knowledge of stability limits is not sufficient to characterize a system from all points of view. The reaction of a system to some disturbance must be determined, especially with relation to engine safety, speed of response, and nature of error in all pertinent engine parameters during transient operation. Transient characteristics of the four systems having the stability limits presented in figure 4 were therefore investigated. In all cases data were obtained by introducing a step disturbance in set temperature. This disturbance was considered to be a 1-percent change in required temperature, where sea-level rated temperature (absolute) is assumed to be 100 percent. Data were taken at numerous operating points in the stable region of each system. Maximum speed and temperature excursion were recorded. Engine safety as related to overspeed and overtemperature can be determined from an examination of the maximum excursion data.

The time in seconds for the engine to reach maximum speed excursion after start of transient was also recorded. From these data, a general indication of the speed of responses can be obtained.

For the proportional control system, speed and temperature droops were also noted.

These data for the various systems are plotted as contour lines on their respective stability limit maps. This presentation permits evaluation of the effect that either loop has on the other and also enables comparison of the systems investigated. Contour lines are not extended to the stability limit line because the systems become too oscillatory and critical to adjustment in the region close to the limit. In addition to these data, photographs of transient responses of actual temperature T_a, measured temperature T_m, speed N, fuel flow W_f, and area A were taken at a number of operating points. On the photographs of the transients, amplitude sensitivity of set temperature disturbance represents 1-percent change. The same amplitude sensitivity applies to all traces.

PROPORTIONAL AND PROPORTIONAL-PLUS-INTEGRAL CONTROLS

Data presented in figure 5 show that for both the proportional and proportional-plus-integral controls maximum speed excursion is a function of speed and temperature loop gains. In both systems, speed excursion decreases as speed loop gain is increased and increases as temperature loop gain is increased. These facts can be explained by the following considerations: High speed loop gains result in a sensitive control, so that small off-speed signals during a transient cause large correcting signals which tend to decrease the speed overshoot. However, with increasing values of temperature loop gain, the gain or sensitivity of the interaction loop also increases. Therefore, a small temperature-error signal during the transient introduces a large opposing signal into the speed loop, which results in a corresponding increase in speed excursion.

The system with integral added produces a slightly greater speed overshoot during the transient at comparable operating points than does the proportional control. However, the advantage of this system is that no steady-state error or droop exists regardless of loop gains.

The magnitude of change in droop in the proportional system is shown in figure 6. These values were calculated from a consideration of equation (1) and were also derived by analog methods. Droop follows the same trends as does speed excursion in that it decreases with speed loop gain but increases with temperature loop gain.

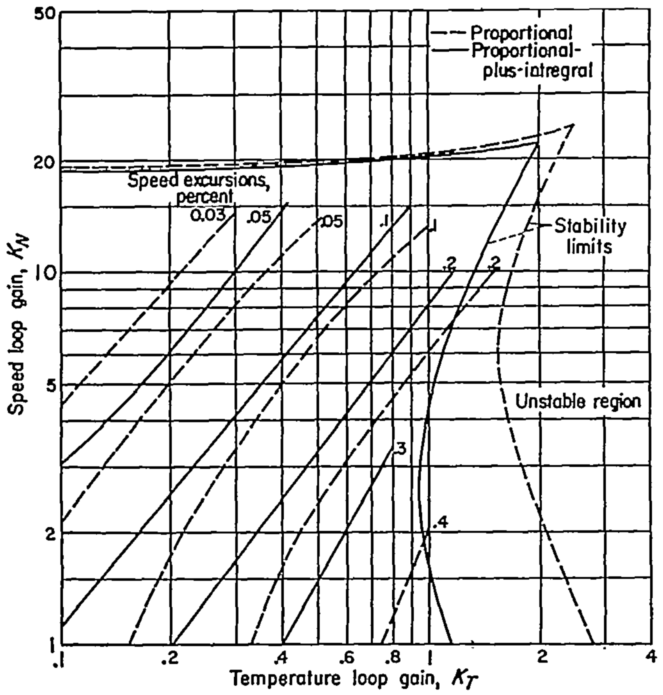


FIGURE 5.—Maximum speed excursion characteristics of proportional control compared with proportional-plus-integral control.

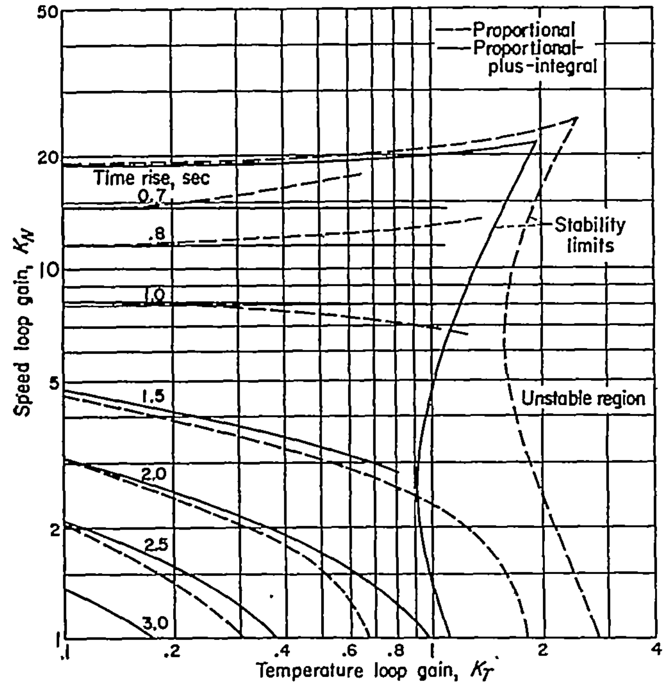


FIGURE 7.—Time to reach maximum speed excursion with proportional control compared with time when using proportional-plus-integral control.

Figure 7 presents a comparison of the time required to reach the point of maximum speed excursion for both systems. Contour lines of constant time on the stability limit map indicate that at low values of temperature loop gain the times are very nearly equal. As this loop gain is increased, the divergence also increases with the proportional control being a little faster for the greater part of the range of speed loop gain. At high-speed loop gains, the system with added integral term has a slight advantage. This, however, is in an

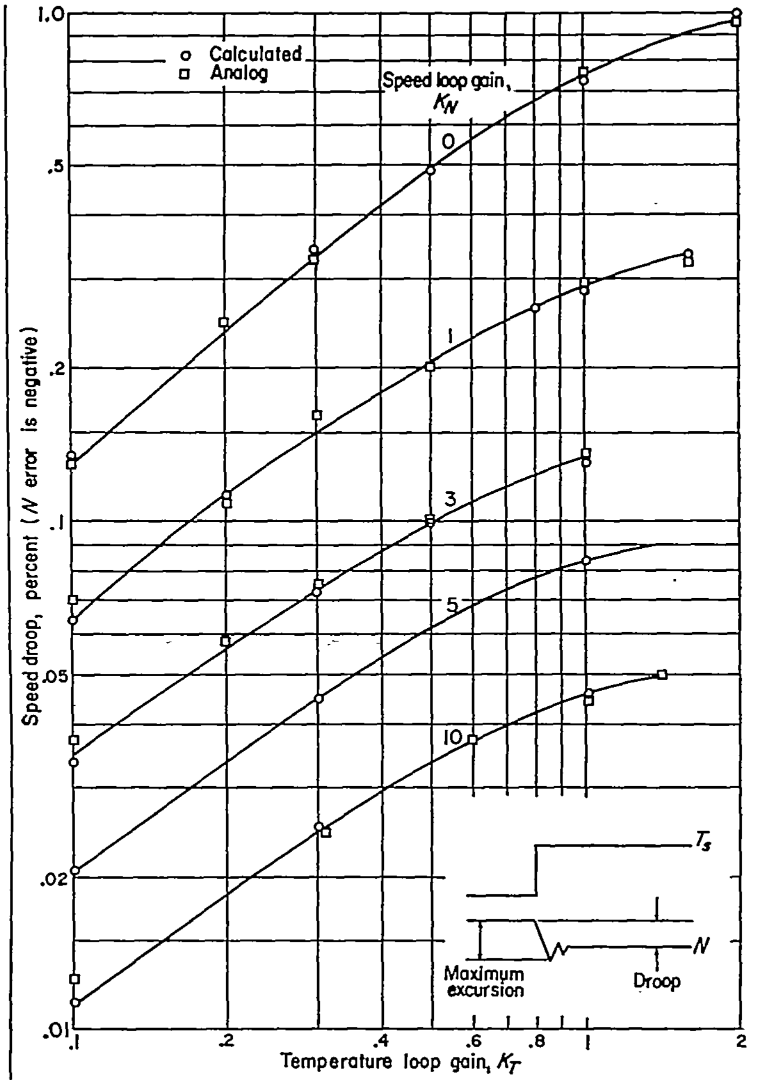
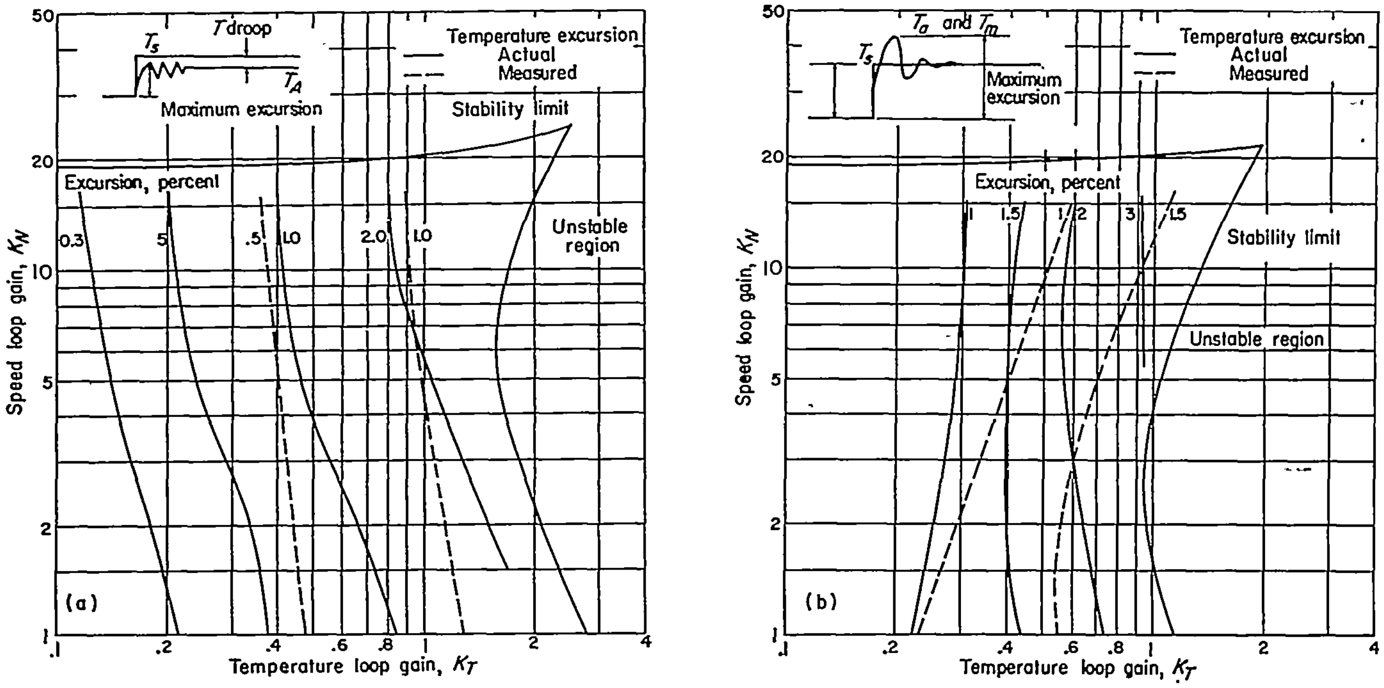


FIGURE 6.—Speed droop in proportional control with disturbance in set temperature.

undesirable region of control operation because the operating point is too close to the stability limit and the system is very oscillatory.

Maximum excursion of turbine-discharge temperature is presented in figure 8(a) for the basic configuration and in figure 8(b) for the system with integral added. Temperature data were recorded at two locations in the temperature loop. One signal represents actual gas temperature T_a, while the other is the thermocouple output or measured temperature T_m. Under practical conditions the thermocouple indication is the more realistic one to use because it is the actual control parameter and also because it offers a better indication of turbine blade temperature. When operating a control system with low loop gains, the entire system response is slow and a condition of no overshoot or at least of very small overshoot beyond final value can be established. Under these conditions a thermocouple can follow actual gas temperature with reasonable accuracy. However, at higher loop gains this is not true, and a greater divergence between actual and measured maximum temperature excursion can be expected. These conditions are shown in figures 8(a) and (b).



(a) Proportional control.

(b) Proportional-plus-integral control.

FIGURE 8.—Contour lines of actual and measured temperature excursions on stability limit map.

Without the integral term it is possible at low values of temperature loop gain to have maximum excursions of temperature that are less than the required change. In addition, the final value is always less than the required change because of the characteristic droop associated with proportional control systems. The addition of the integral term results in zero steady-state error, and the maximum excursion will be at least equal to the required change in set temperature, even at low values of temperature loop gain. This basic difference of the two systems is shown in figures 8(a) and (b). These figures also show that at comparable operating points of speed and temperature loop gains the maximum excursion is greater for the system having the integral terms included in the configuration.

Temperature droop for the proportional system is shown in figure 9. The data indicate that droop decreases as temperature loop gain increases. Increases in speed loop gain also tend to decrease droop, but to a lesser degree.

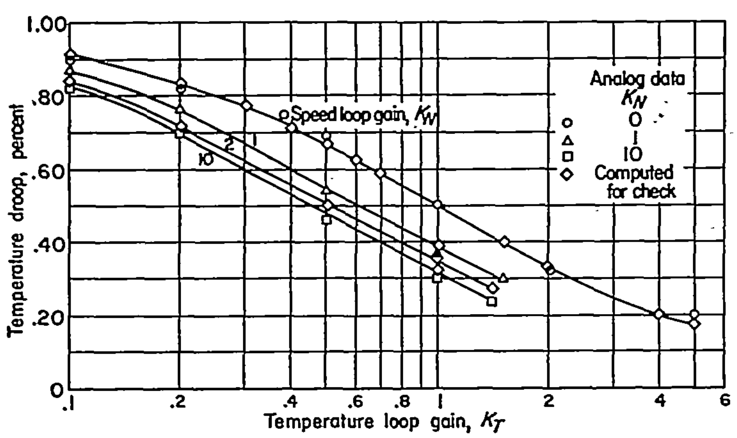
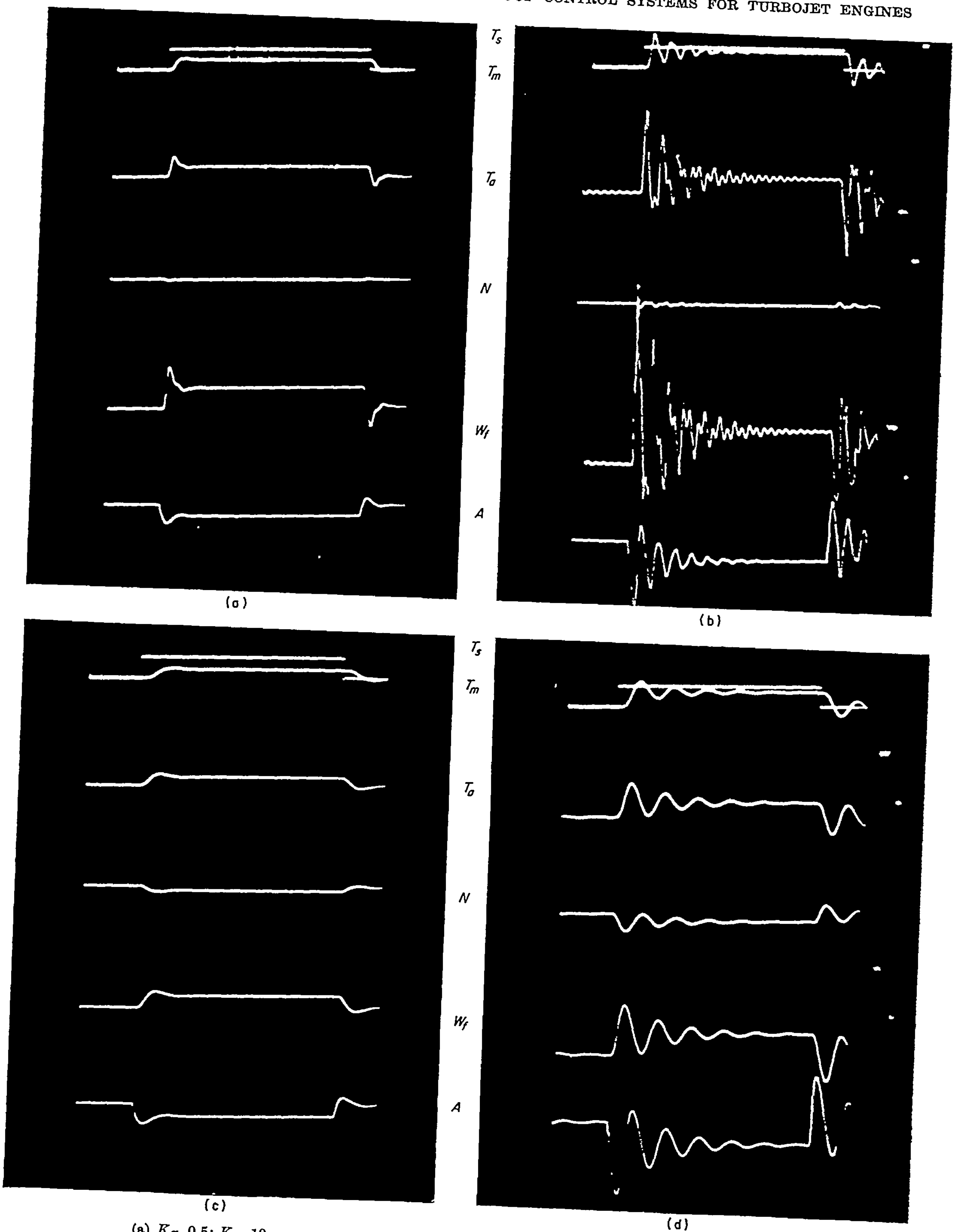


FIGURE 9.—Proportional-control temperature droop with disturbance in set temperature.

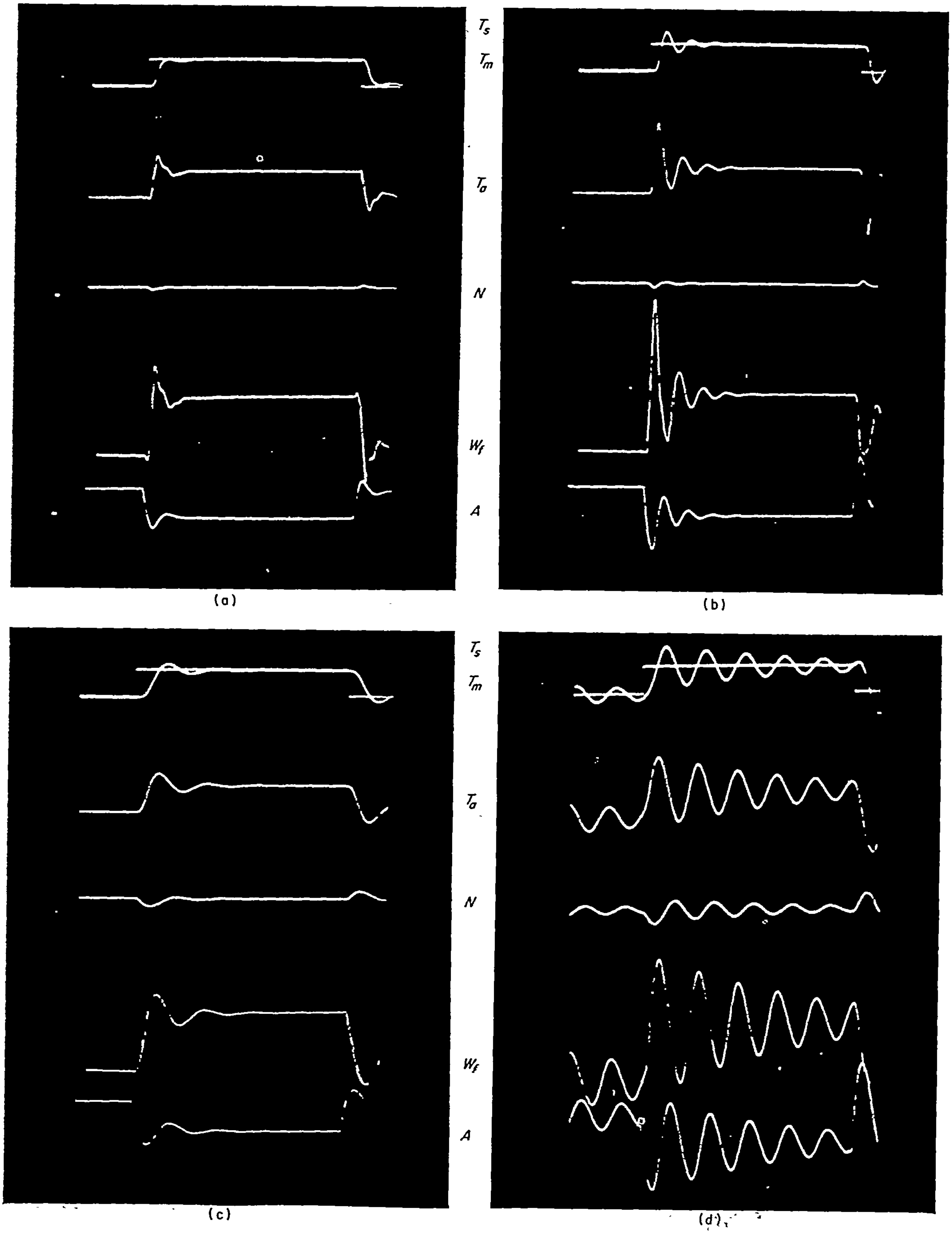
Photographs of significant traces are shown in figure 10 for the proportional control system and in figure 11 for the proportional-plus-integral system. Examination of these photographs in conjunction with data already presented indicates the magnitude and nature of transient responses at various operating points of the systems: Figure 10(c), taken with a speed loop gain K_N of 1.0 and a temperature loop gain K_T of 0.5, shows that the proportional system is very stable with small overshoots, but that it is inherently slow in response and has a droop in both speed and temperature. Figure 10(d), taken with K_T increased to 2.0, shows that the system now becomes more oscillatory with a relatively low frequency of superimposed oscillation. Temperature droop is noticeably reduced. Figure 10(a) presents the conditions when K_T is again set at 0.5, but K_N is increased to 10. These responses indicate a much faster system than present in figure 10(c), but fuel flow and actual temperature excursions are greater. The increased actual temperature overshoot, however, is of such short duration that it does not contribute significantly to the maximum excursion of measured temperature, which is more nearly representative of the manner in which turbine blades respond.

Figure 10(b) presents responses taken with K_N set at 18 and K_T at 2.0. These responses indicate two modes of oscillation before stable operation is achieved. Investigation of this action shows that the lower frequency is due primarily to the temperature loop which contains the slower servos, while the higher frequency is due to action of the speed loop which includes faster servos. Actual values of superimposed frequencies are not directly determinable from consideration of each loop independently because of the effect of the interaction loop.



(a) K_T, 0.5; K_N, 10.
(b) K_T, 2.0; K_N, 18.
(c) K_T, 0.5; K_N, 1.0.
(d) K_T, 2.0; K_N, 1.0.

FIGURE 10.—Transient response to disturbance in set temperature. Proportional control.



(a) K_T, 0.5; K_N, 10.
(b) K_T, 1.0; K_N, 10.
(c) K_T, 0.5; K_N, 1.0.
(d) K_T, 1.0; K_N, 1.0.

FIGURE 11.—Transient response to disturbance in set temperature. Proportional-plus-integral control.

A comparison of figure 11 with figure 10 shows that the addition of the integral term does not alter the trends indicated by the proportional system. The two points of difference are (1) with the integral, the droop in speed and temperature is eliminated, and (2) the over-all system responses are slower and more oscillatory.

CONTROLS WITH ADDED COMPENSATION FOR INTERACTION

The investigation was continued with an analysis of the transient response characteristic of proportional and proportional-plus-integral systems after a compensation term X_A was added, as shown in figure 2. Data indicated that these two compensated systems followed similar trends in regard to the characteristics of responses; therefore, subsequent discussion will be based on the compensated integral system. The only significant difference is that the compensated proportional system has a temperature droop which is predictable from consideration of equation (7). No speed droop is obtained when a disturbance is introduced in set temperature because the compensation term is so designed that no steady-state speed change will result from that disturbance. The system will have a speed droop if the disturbance is introduced elsewhere in the configuration.

The compensated proportional-plus-integral system is a little slower in response than the one without the integral term, but the advantages of the integral action in eliminating steady-state speed and temperature errors regardless of where disturbance occurs make the integral action more attractive.

Figure 12 shows the maximum speed excursion data and the time to reach this peak point for a disturbance in set temperature. Maximum speed excursion increases with increasing temperature loop gain and decreases with increasing values of speed loop gain. However, comparison with figure 5, a plot of the function for a noncompensated system, shows that the addition of the compensation term greatly reduces the speed-loop-gain effect on the system and, in addition, that the magnitude of peak error is greatly reduced at comparable loop-gain points. This indicates the effect of the compensation term in the system. The small speed-loop-gain effect would be eliminated completely if the compensation term had incorporated in it the necessary dynamic characteristics as required by equation (9).

Contour lines of time to reach maximum speed excursion point appear to follow the general shape of the stability limit. Comparison of these data with figure 7 shows that the compensated system is much faster than the noncompensated control.

Turbine-discharge temperature characteristics are presented in figure 13. With low temperature loop gains and over the full range of speed loop gains, no overshoot in actual temperature occurs, so that the maximum temperature excursion becomes equal to the required value. This temperature change to final value is primarily due to integral action in the system. The same effect can be observed in measured temperature data, but it continues to higher values of temperature loop gain because of the inability of the thermocouples to follow overshoots in temperature. At higher temperature-loop-gain values, the data show that temperature excursion is dependent on and increases with temperature loop gain.

These data also show that actual and measured temperatures are practically independent of speed loop gain up to their respective limiting lines, shown on the map and designated "limit T_a" and "limit T_m." At speed loop gains above these limits, a pronounced dependency does exist. Examination of figure 14, which consists of photographs of typical transient responses, will serve to define the nature of these limits. Maximum excursion of actual temperature

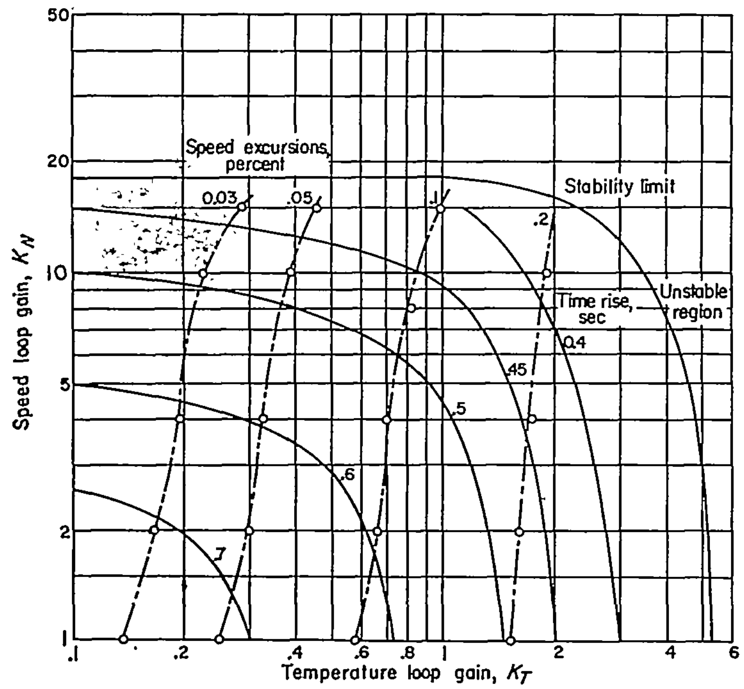


FIGURE 12.—Maximum speed excursion and time to maximum speed excursion for noninteracting proportional-plus-integral control superimposed upon stability map.

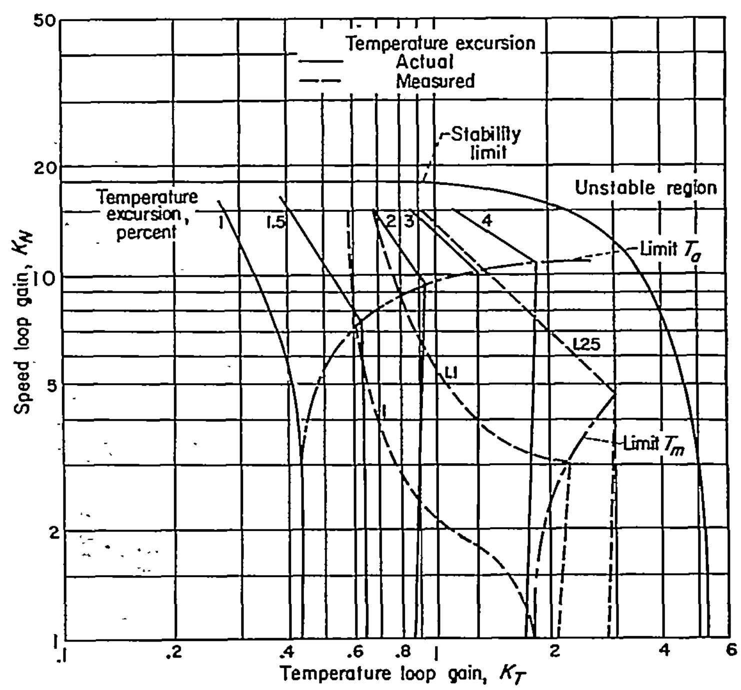
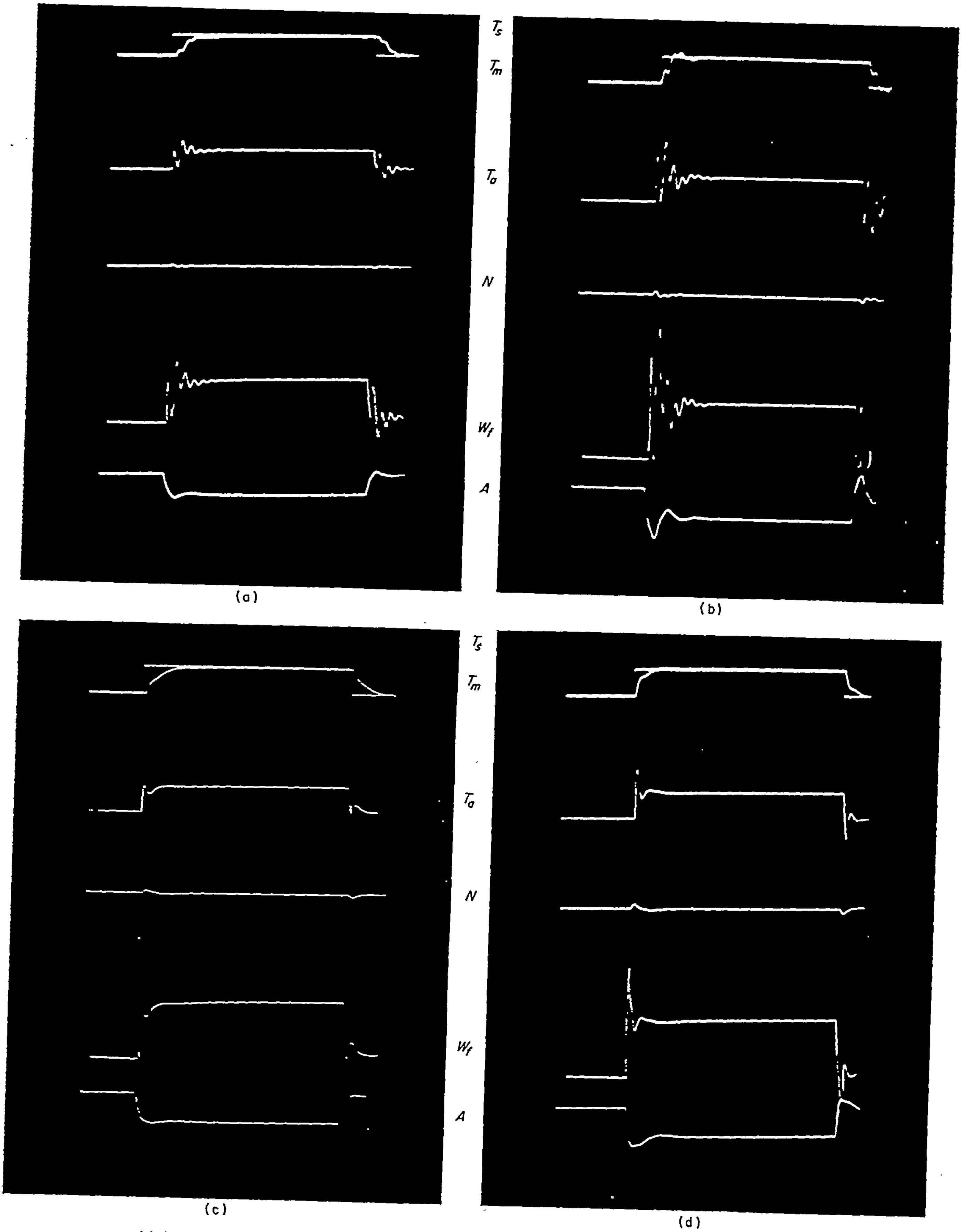


FIGURE 13.—Actual and measured maximum temperature excursions for noninteracting proportional-plus-integral control superimposed upon stability map.



(a) K_T, 0.5; K_N, 10.
(b) K_T, 1.0; K_N, 10.
(c) K_T, 0.5; K_N, 1.0.
(d) K_T, 1.0; K_N, 1.0.

FIGURE 14.—Transient response to disturbance in set temperature. Noninteracting proportional-plus-integral control.

in figure 14(d) occurs on the first peak of the oscillatory response; while in figure 14(b) it occurs on the second peak. The values of loop gains which result in equal amplitude of the first and second peaks define the limits shown.

Comparison of responses shown in figure 14 with those of figure 11 shows that compensation produces very desirable improvements in system performance in the region of operation defined by figures 14(c) and (d) because the system responds faster with less superimposed oscillation. Further examination of figure 14 shows that two modes of oscillation occur at the higher speed-loop-gain settings as shown in figures 14(a) and (b). The higher frequency is the result of interaction of the speed loop with the temperature loop. Therefore, when speed loop gain is low, the assumption can be made that the simple gain compensation for interaction is sufficient to allow analysis based on single-loop considerations. However, at high values of speed loop gain this assumption is no longer valid and additional compensation for dynamic terms is required if it is desired to make the two basic loops independent of each other.

CONCLUDING REMARKS

Addition of integral terms to the speed and temperature loops compresses the stability limits and makes the system slower and more oscillatory than would be the case with proportional control only. However, integral action, by eliminating droop, becomes desirable for control application because it makes final values of system parameters independent of the source of disturbance.

Addition of complete compensation (dynamic compensating terms) for engine interaction eliminates the effect of system interaction, which then permits each loop to be developed individually for a desired response. This compensation has the particular advantage that the complicated double-loop interacting system has been reduced to two noninteracting single loops, and the analysis and synthesis procedures of a single-loop servo theory can be applied.

Addition of proper partial compensation (gain compensating terms) results in considerable improvement in the characteristics of an interacting control system and, in a practical sense, is considerably easier to apply to a system than is complete compensation.

In particular, it was found that with the engine operating near maximum speed, where an increase in thrust is obtained by increasing the temperature, a single partial-compensation term from temperature error to speed error resulted in appreciable improvements in system characteristics. The system was more stable, and faster response times were observed. These improvements in characteristics can be considered advantageous in comparison with the noncompensated system. At comparable gain points, the compensated system is not only faster, but also has a definitely larger margin of gain to instability. It also follows that, for comparable responses of the two systems, the requirements on response of the control servos need not be so severe when compensation is used.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, *December 21, 1953*

APPENDIX

SYMBOLS

GENERAL SYMBOLS

A	area of variable-area exhaust nozzle
K_N	gain of fuel flow to speed control loop
K_{NA}	engine gain of speed to area
K_{NW}	engine gain of speed to fuel flow
K_T	gain of area to temperature control loop
K_{TA}	engine gain of turbine-discharge temperature to area
K_{TN}	engine gain of turbine-discharge temperature to speed
K_{TW}	engine gain of turbine-discharge temperature to fuel flow
K_X	gain of compensation term
N	actual engine speed
N_e	speed error, $N_s - H_1N$
N_m	measured engine speed, H_1N
N_s	desired engine speed
p	complex Laplacian operator
T or T_a	actual turbine-discharge temperature (T_a used when differentiating from T_m)
T_e	temperature error, $T_s - H_2T$
T_m	measured turbine-discharge temperature
T_s	desired turbine-discharge temperature
W_f	engine fuel flow

TRANSFER FUNCTIONS

E_1	speed to fuel flow
E_2	temperature to fuel flow
E_3	speed to area
E_4	temperature to area
G_1	fuel-flow controller
G_2	area controller
H_1	speed sensor
H_2	temperature sensor
X	complete-compensation term from temperature error to speed error
X_A	partial-compensation term from temperature error to speed error
X_B	partial-compensation term from area to speed error
X_C	partial-compensation term from area to fuel flow
Y	complete-compensation term from speed error to temperature error

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