

REPORT 1260

STUDIES OF THE SPEED STABILITY OF A TANDEM HELICOPTER IN FORWARD FLIGHT¹

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SUMMARY

Flight-test measurements, related analytical studies, and corresponding pilots' opinions of the speed stability of a tandem-rotor helicopter are presented. An undesirable instability, evidenced by rearward stick motion with increasing forward speed at constant power, is indicated to be caused by variations with speed of the front-rotor downwash at the rear rotor. An analytical expression for predicting changes in speed stability caused by changes in rotor geometry is derived and constants for use with the analytical expression are presented in chart form. Means for improving stability with speed are studied both analytically and experimentally.

The test results also give some information as to the flow conditions at the rear rotor.

INTRODUCTION

For the past several years the National Advisory Committee for Aeronautics has been studying the flying qualities of helicopters in order to set up flying-qualities criterions and to provide a basis for improvement. Information obtained during flying-qualities studies of a tandem helicopter in reference 1 indicated the tandem-rotor configuration to be susceptible to instability with speed in forward flight. That this instability with speed was a basic problem resulting from effects of front-rotor downwash on the rear rotor appeared likely. Hence, this aspect of the tandem configuration seemed worthy of study in order to provide a basis for improvement.

Basically, speed stability may be defined as the variation of pitching moment with speed. If an increase in forward speed of the helicopter, with control stick fixed, produces a nose-down moment, the speed will increase further due to the resulting nose-down attitude. Such an aircraft is unstable with speed. If a nose-up moment is associated with an increase in speed from trim with stick fixed, the resulting nose-up attitude tends to reduce the speed to the trim value. An aircraft exhibiting the latter characteristics is stable with speed. A more complete discussion of helicopter stability may be found in chapter 11 of reference 2.

Stability with speed is important primarily when a helicopter is being operated at or near the placard speed. At this condition, instability with speed increases the likelihood of inadvertently exceeding the placard speed with possible damage to the aircraft. At lower speeds, stability with speed is desirable as it simplifies maintaining desired speeds

and provides a logical variation of control position with speed. The military and civilian regulatory agencies are now generally requiring helicopters to exhibit speed stability. (See refs. 3 and 4.)

The investigation herein was undertaken to determine the minimum satisfactory speed stability for a tandem-rotor helicopter and to determine the factors that affect speed stability in order to provide a basis for improvement.

SYMBOLS

b	number of blades per rotor
r	radial distance to blade element, ft
R	blade radius, ft
c	blade-section chord, ft
c_e	equivalent blade chord (on thrust basis), $\frac{\int_0^R cr^2 dr}{\int_0^R r^2 dr}$, ft
σ	rotor solidity, $bc_e/\pi R$
Θ	instantaneous blade-section pitch angle; angle between line of zero lift of blade section and plane perpendicular to rotor shaft, radians
θ	collective pitch, average value around azimuth of Θ , radians
ρ	mass density of air, slugs/cu ft
ρ_0	mass density of air at sea level, slugs/cu ft
V	true airspeed of helicopter along flight path, fps
Ω	rotor angular velocity, radians/sec
α	rotor angle of attack; angle between flight path and plane perpendicular to axis of no feathering, positive when axis is inclined rearward, radians
μ	tip-speed ratio, $V \cos \alpha / \Omega R$, assumed equal to $V / \Omega R$
T	rotor thrust, component of rotor resultant force parallel to axis of no feathering, lb
C_T	rotor-thrust coefficient, $\frac{T}{\pi R^2 \rho (\Omega R)^2}$
L	rotor lift, lb
C_L	rotor-lift coefficient, $\frac{L}{\frac{1}{2} \rho \pi R^2 V^2}$
ϵ	angle of downwash at rear rotor due to front rotor (assumed equal to C_T/μ^2), radians
W	helicopter gross weight, lb
ΔL	difference in lift of front and rear rotors, positive when lift of rear rotor is greater, lb

¹ Supersedes recently declassified NACA Research Memorandum L53F15a by Robert J. Tapscott and Kenneth B. Amer, 1953.

- ΔT difference in thrust of front and rear rotors, positive when thrust of rear rotor is greater, lb
 - $\Delta \theta$ difference in collective-pitch angle of front and rear rotors, positive when pitch of rear rotor is greater, radians
 - ΔR difference in blade radius of front and rear rotors, positive when blade radius of rear rotor is greater, ft
 - $\Delta(\Omega R)$ difference in blade tip speed of front and rear rotors, positive when tip speed of rear rotor is greater, fps
 - $\Delta \sigma$ difference in rotor solidity of front and rear rotors, positive when rotor solidity of rear rotor is greater
 - $\Delta \alpha$ total difference in angle of attack of front and rear rotors, positive when rear rotor is greater, radians
 - $\Delta \alpha_d$ difference in angle of attack of front and rear rotors due to swashplate dihedral, positive when angle of attack of rear rotor is greater, radians
 - δ longitudinal position of control stick, positive when forward, in. from neutral
 - α' longitudinal angle between rotor force vector and axis of no feathering, deg
 - x distance from center of gravity to midpoint between line of action of thrust vectors or effective midpoint, positive when center of gravity is forward of effective midpoint, in.
 - x' distance from effective midpoint to the midpoint between rotor shafts, positive when effective midpoint is forward, in.
 - x_0 distance from midpoint between rotor shafts to center of gravity, positive when center of gravity is forward, in.
 - h vertical distance from center of gravity to the plane of the rotors, in.
 - B_1 longitudinal cyclic pitch with respect to shaft, positive when axis of no feathering is tilted forward with respect to the shaft, deg
- Subscripts:
- fr front rotor
 - av average value

DESCRIPTION OF TEST HELICOPTER

The tandem helicopter used in the tests is shown in figure 1. It has a normal gross weight of approximately 7,000 pounds and has two rotors of equal size, each having a diameter of 41 feet. The rotors have equal rotational speed and solidity and are of equal distance above the center of gravity. There is no overlap of the swept areas of the rotors and the swashplates are parallel longitudinally to one another. The center-of-gravity range when measured along a line perpendicular to the shafts, which are parallel, is from 1 inch rearward to 18 inches forward of the midpoint between shafts. For the tests the center of gravity was 13 inches forward of this midpoint. The horizontal and twin vertical stabilizers have total areas of approximately 40 and 50 square

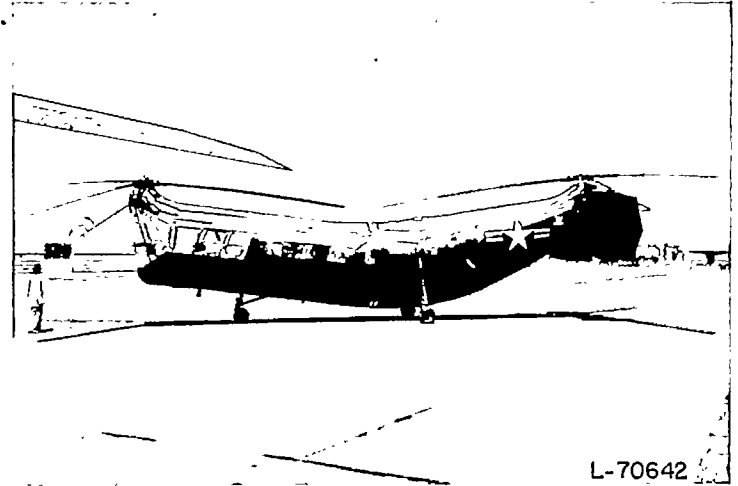


FIGURE 1.—Test helicopter.

feet, respectively. The helicopter has conventional pilot controls: stick, pedals, collective-pitch lever, and throttle. Longitudinal control is achieved by a longitudinal motion of the stick, which produces a combination of longitudinal cyclic pitch and differential collective pitch, the latter providing by far the larger magnitude of pitching moment. Lateral control is achieved by lateral motion of the stick which causes lateral cyclic pitch at both rotors; directional control is achieved by use of the pedals which causes differential lateral cyclic pitch. Longitudinal trim control is obtained through use of a control wheel which varies the differential collective pitch between rotors. For all of the tests the trim control was at an indicator setting of approximately 0.8° nose up.

For the latter part of the tests, in order to change the speed stability of the test helicopter, the rigging of the rotors was modified to incorporate what will henceforth be referred to as swashplate "dihedral." This consisted of adjusting the longitudinal control cables to give rearward cyclic and forward cyclic pitch on the front and rear rotors, respectively, while the control stick was locked in longitudinal neutral position, thus producing a fixed difference in the longitudinal cyclic pitch of the front and rear rotors at any stick position. Aerodynamically, this is equivalent to physically inclining the shafts toward one another. The total longitudinal swashplate travel was reduced to prevent exceeding a cyclic pitch of 6° , a limit set by linkage and clearance between the blade and droopstop, by use of a reducing bar on the longitudinal cyclic control cables. A calibration of the longitudinal cyclic pitch and differential collective pitch for the configuration with approximately 4.5° of longitudinal swashplate dihedral and trim setting of 0.8° nose up is shown in figure 2.

The test helicopter was equipped with synchronized standard NACA recording instruments that measured control position, airspeed along the flight path, and angle of attack at the nose of the helicopter of the plane perpendicular to the rotor shafts. The angle-of-attack and airspeed pick-up installation is shown in figure 3.

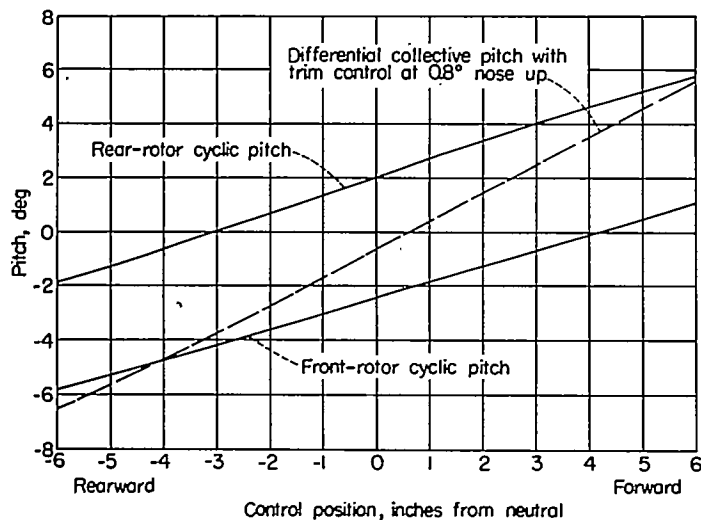


FIGURE 2.—Longitudinal control calibration of test helicopter with approximately 4.5° swashplate dihedral.

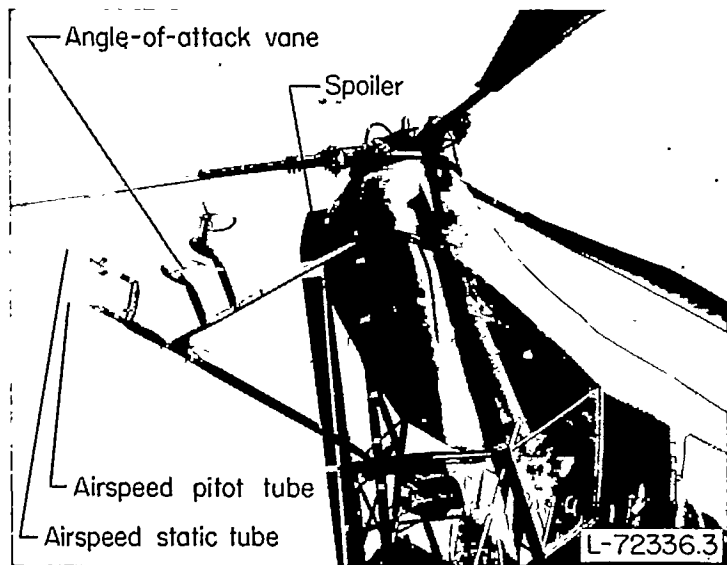


FIGURE 3.—Airspeed, angle-of-attack, and spoiler installation on test helicopter.

TESTING TECHNIQUE

In order to keep the pitching moments on the helicopter in flight trimmed during speed variation, any pitching moments resulting from speed changes must be counteracted by longitudinal motion of the control stick. Since speed stability is defined by the variation of pitching moment with speed, the variation of stick position in counteracting moments due to speed change is a measure of speed stability. Rearward stick motion would be needed to neutralize a nose-down moment while forward motion of the stick cancels a nose-up moment. Inasmuch as a nose-up moment associated with increased speed is stabilizing, forward stick motion with increasing speed signifies stability with speed.

Measurements were made in flight of the speed stability of the test helicopter in several configurations, the procedure being to trim the helicopter at a given speed and record stick

position and forward speed while varying the speed at constant power and collective pitch. It will be noted that the helicopter was not in level flight but descending or ascending as caused by increasing or decreasing speed at constant power.

The significance of this technique is that under the given conditions the stick motion is a measure of the speed stability exhibited by the helicopter in small disturbances from steady trimmed flight where power and collective pitch are constant. It is under these conditions that speed stability affects the pilots' opinions of flying qualities of the helicopter.

THEORETICAL ANALYSIS OF SPEED STABILITY

For purposes of subsequent comparison with experimental results and to form a basis for improvement of speed stability, the following theoretical analysis of speed stability is performed.

ASSUMPTIONS

The analysis in this report is based on the stability derivatives of reference 5 and therefore the assumptions of that reference are carried over. In addition, for the purposes of this analysis, the following simplifying assumptions are made:

(1) The pitching moments of the fuselage—horizontal-tail combination are zero. This assumption appears justified in view of the large magnitudes of the pitching moments caused by the rotors compared with the pitching moments caused by the fuselage-tail combination.

(2) The lift of the fuselage—horizontal-tail combination is zero.

(3) The front rotor is not affected by the rear rotor.

(4) The downwash angle at the rear rotor due to the lift of the front rotor is given by $C_{T_{fr}}/\mu^2 \approx C_{L_{fr}}/2$ where $C_{T_{fr}}$ and $C_{L_{fr}}$ are the thrust and lift coefficients, respectively, of the front rotor. Theoretically, this magnitude of downwash is not fully reached until at an infinite distance behind the front rotor; however, calculations of stability using the assumed value of downwash and theoretical calculations of downwash behind a rotor presented in reference 6 indicate it to be a reasonable assumption. The theory developed herein will be restricted to $\mu \geq 0.15$. Below this value of μ , the downwash formula becomes inaccurate.

(5) The stability with speed of the individual rotors and pitching moments due to changes in longitudinal cyclic pitch are neglected. Preliminary calculations show these quantities to be of only secondary importance.

DERIVATIONS OF EQUATIONS

In order that no pitching moments be produced as the speed of the helicopter is varied, the difference in the thrust of the front and rear rotors ΔT must remain at the trim value. The average thrust of rotors during steady flight is given by definition of the thrust coefficient as

$$T_{av} = \frac{W}{2} = \left(\frac{C_T}{\sigma}\right)_{av} \sigma_{av} \pi (R_{av})^2 \rho [(\Omega R)_{av}]^2 \quad (1)$$

Also, the thrust differential between rotors is

$$\Delta T = \Delta \left[\frac{C_T}{\sigma} \sigma \pi R^2 \rho (\Omega R)^2 \right]$$

Taking differentials,

$$\begin{aligned} \Delta T = \Delta \left(\frac{C_T}{\sigma} \right) [\sigma \pi R^2 \rho (\Omega R)^2]_{av} + \Delta \sigma \left[\frac{C_T}{\sigma} \pi R^2 \rho (\Omega R)^2 \right]_{av} \\ + 2R_{av} \Delta R \left[\frac{C_T}{\sigma} \sigma \pi \rho (\Omega R)^2 \right]_{av} + 2(\Omega R)_{av} \Delta(\Omega R) \left[\frac{C_T}{\sigma} \sigma \pi R^2 \rho \right]_{av} \end{aligned} \quad (2)$$

Dividing equation (2) by equation (1) gives

$$\frac{\Delta T}{W/2} = \frac{1}{\left(\frac{C_T}{\sigma} \right)_{av}} \Delta \left[\frac{C_T}{\sigma} + \frac{\Delta \sigma}{\sigma_{av}} + \frac{2 \Delta R}{R_{av}} + \frac{2 \Delta(\Omega R)}{(\Omega R)_{av}} \right] \quad (3)$$

However, $\Delta(C_T/\sigma)$ may be expressed as

$$\Delta \left(\frac{C_T}{\sigma} \right) = \left[\left(\frac{\partial C_T}{\partial \alpha} \right)_{av} \Delta \alpha + \left(\frac{\partial C_T}{\partial \theta} \right)_{av} \Delta \theta \right] \quad (4)$$

where $\left(\frac{\partial C_T}{\partial \alpha} \right)_{av}$ and $\left(\frac{\partial C_T}{\partial \theta} \right)_{av}$ are averages of the front and rear rotor derivatives.

Substituting equation (4) for $\Delta \left(\frac{C_T}{\sigma} \right)$ into equation (3), gives

$$\begin{aligned} \frac{\Delta T}{W/2} = \frac{1}{\left(\frac{C_T}{\sigma} \right)_{av}} \left[\left(\frac{\partial C_T}{\partial \alpha} \right)_{av} \Delta \alpha + \left(\frac{\partial C_T}{\partial \theta} \right)_{av} \Delta \theta \right] + \\ \frac{\Delta \sigma}{\sigma_{av}} + \frac{2 \Delta R}{R_{av}} + \frac{2 \Delta(\Omega R)}{(\Omega R)_{av}} \end{aligned} \quad (5)$$

When the above equation is solved for $\Delta \theta$ and differentiated with respect to μ , setting $\frac{\partial(\Delta T)}{\partial \mu} = 0$, the following expression is obtained:

$$\begin{aligned} \frac{d(\Delta \theta)}{d\mu} = - \frac{\left(\frac{C_T}{\sigma} \right)_{av}}{\left[\left(\frac{\partial C_T}{\partial \theta} \right)_{av} \right]^2} \frac{d \left(\frac{\partial C_T}{\partial \theta} \right)_{av}}{d\mu} \left\{ \frac{\Delta T}{W/2} \left[\frac{\Delta \sigma}{\sigma_{av}} + \frac{2 \Delta R}{R_{av}} + \frac{2 \Delta(\Omega R)}{(\Omega R)_{av}} \right] \right\} - \frac{\left(\frac{\partial C_T}{\partial \alpha} \right)_{av}}{\left(\frac{\partial C_T}{\partial \theta} \right)_{av}} \frac{d(\Delta \alpha)}{d\mu} - \\ \left\{ \frac{d \left(\frac{\partial C_T}{\partial \alpha} \right)_{av}}{d\mu} - \frac{\left(\frac{\partial C_T}{\partial \alpha} \right)_{av}}{\left[\left(\frac{\partial C_T}{\partial \theta} \right)_{av} \right]^2} \frac{d \left(\frac{\partial C_T}{\partial \theta} \right)_{av}}{d\mu} \right\} \Delta \alpha \end{aligned} \quad (6)$$

Under the assumption that the downwash angle at the rear rotor is

$$\epsilon = \left(\frac{C_T}{\mu^2} \right)_{fr} \quad (7)$$

the total difference in angle of attack between the front and rear rotors due to downwash and swashplate dihedral becomes

$$\Delta \alpha = - \left(\frac{C_T}{\mu^2} \right)_{fr} + \Delta \alpha_d \quad (8)$$

Equation (6) was determined in terms of average values of C_T and μ . Therefore, $C_{T_{fr}}$ and μ_{fr} in equation (8) will be replaced in terms of average values in order that substitution into equation (6) can be made. The mathematics for determining $\Delta \alpha$ in terms of $C_{T_{av}}$ and μ_{av} are presented in appendix A and the resulting expression is

$$\Delta \alpha = - \left(\frac{C_T}{\mu^2} \right)_{av} \left(1 - \frac{\Delta T}{W} + \frac{\Delta R}{R_{av}} \right) + \Delta \alpha_d \quad (9)$$

which when differentiated with respect to μ gives

$$\frac{d(\Delta \alpha)}{d\mu} = 2 \left(\frac{C_T}{\mu^3} \right)_{av} \left(1 - \frac{\Delta T}{W} + \frac{\Delta R}{R_{av}} \right) \quad (10)$$

Substituting equations (9) and (10) into equation (6) and simplifying results in the following expression:

$$\begin{aligned} \frac{d(\Delta \theta)}{d\mu} = K_1 \left(\frac{C_T}{\sigma} \right)_{av} \left(\frac{\Delta T}{W} - \frac{\Delta R}{R_{av}} \right) + K_2 \left(\frac{C_T}{\sigma} \right)_{av} \left[\frac{\Delta \sigma}{\sigma_{av}} + 2 \frac{\Delta(\Omega R)}{(\Omega R)_{av}} \right] + \\ K_3 \Delta \alpha_d + K_4 C_{T_{av}} \end{aligned} \quad (11)$$

where

$$\begin{aligned} K_1 = -2 \frac{\frac{d \left(\frac{\partial C_T}{\partial \theta} \right)_{av}}{d\mu}}{\left[\left(\frac{\partial C_T}{\partial \theta} \right)_{av} \right]^2} + 2 \frac{\left(\frac{\partial C_T}{\partial \alpha} \right)_{av} \sigma_{ar}}{\left(\frac{\partial C_T}{\partial \theta} \right)_{av} (\mu_{av})^3} - \\ \left\{ \frac{d \left(\frac{\partial C_T}{\partial \alpha} \right)_{av}}{d\mu} - \frac{d \left(\frac{\partial C_T}{\partial \theta} \right)_{av}}{d\mu} \frac{\left(\frac{\partial C_T}{\partial \alpha} \right)_{av}}{\left[\left(\frac{\partial C_T}{\partial \theta} \right)_{av} \right]^2} \right\} \frac{\sigma_{av}}{(\mu_{av})^2} \end{aligned}$$

$$K_2 = \frac{d\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}}{\left[\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}\right]^2}$$

$$K_3 = -\frac{\frac{d\left(\frac{\partial C_T/\sigma}{\partial \alpha}\right)_{av}}{d\mu} + \left(\frac{\partial C_T/\sigma}{\partial \alpha}\right)_{av}}{\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}} + \frac{d\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}}{\left[\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}\right]^2} \frac{d\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}}{d\mu}$$

$$K_4 = -\frac{\left(\frac{\partial C_T/\sigma}{\partial \alpha}\right)_{av}}{\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}} \frac{2}{(\mu_{av})^3} +$$

$$\left\{ \frac{d\left(\frac{\partial C_T/\sigma}{\partial \alpha}\right)_{av}}{d\mu} - \frac{\left(\frac{\partial C_T/\sigma}{\partial \alpha}\right)_{av}}{\left[\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}\right]^2} \frac{d\left(\frac{\partial C_T/\sigma}{\partial \theta}\right)_{av}}{d\mu} \right\} \frac{1}{(\mu_{av})^2}$$

Note that in equation (11) changes in R are assumed to take place at constant σ and ΩR and that the derivatives are the average of front- and rear-rotor values.

Values for K_1 , K_2 , K_3 , and K_4 are plotted against μ in figure 4 for values of μ from 0.15 to 0.50 and for $\sigma=0.03$, 0.06, and 0.09. A direct calculation of the speed stability of a given configuration may be accomplished by using the K values of figure 4 and equation (11) when C_T , μ , and σ are known.

RESULTS AND DISCUSSION

SPEED STABILITY OF ORIGINAL CONFIGURATION

Measurements of speed stability.—Figure 5 (a) shows a plot of stick position against forward speed for the original configuration trimmed at approximately 70 knots in level flight which is approximately the cruising speed. The curve shows that rearward stick motion was necessary to maintain trim longitudinally as the speed increased throughout the speed range from 50 knots to 105 knots. The nose-up control moment was applied to counteract a nose-down moment due to the increased speed, since to maintain zero pitching acceleration the sum of the moments must be zero. Thus, figure 5 (a) indicates the test helicopter in its original configuration to be unstable with speed from 50 knots to 105 knots. The variation of slope with speed indicates some tendency for the instability to become smaller with increased speed particularly at the lower speeds. At 70 knots the stick slope is approximately -0.01 inch per knot.

Pilots' opinions.—The instability of the test helicopter was considered by the pilots to be undesirable in that it increased the likelihood of the placard speed being exceeded inadvertently. However, they considered this instability to be

less serious than the maneuver instability and lateral-directional instabilities reported in reference 1.

Source of instability.—The unstable variation of pitching moments with speed may be caused by the rotors or the fuselage. Chapter 11 of reference 2 indicates that the individual rotors are stable with speed and calculations indicate the contribution of the fuselage-tail combination to the moment variation with speed to be small with respect to that contributed by the rotors for the test helicopter. It is probable, therefore, that the greater part of speed instability is contributed by the rotor configuration and is caused by the variations of front-rotor downwash acting on the rear rotor.

In forward flight the rear rotor is operating in the downwash of the front rotor and is trimmed accordingly. As forward speed increases, the downwash angle is reduced because of the larger mass of air handled per second by the front rotor. The reduction of downwash angle with increased speed causes an increase in the rear-rotor angle of attack so that at constant control position a thrust increase is produced resulting in a nose-down or unstable pitching moment. The K_4 term in equation (11) accounts for this effect. Equation (10) shows that the rate of change of downwash with speed is reduced as the speed is increased. This reduction occurs more rapidly at the lower speeds. Experimental verification of this trend is noted in figure 5(a) wherein a reduction in the instability with speed of the test helicopter as the speed increased is noticeable particularly at the lower speeds.

Computation of speed stability at 70 knots.—The basic tandem configuration used in the tests had equal radius, equal tip speed, and equal solidity of the front and rear rotors and no swashplate dihedral. Under these conditions, equation (11) reduces to:

$$\frac{d(\Delta\theta)}{d\mu} = K_1 \left(\frac{C_T}{\sigma}\right)_{av} \frac{\Delta T}{W} + K_4 C_{T_{av}} \quad (12)$$

The second term in equation (12) is the important term and is the one that accounts for the effects of downwash variation. The first expression in equation (12) is retained because the center of gravity was approximately 13 inches forward of the midpoint; a difference therefore results in front and rear rotor thrusts at the trim condition. In addition to a physical shift of the center of gravity there is an effective shift introduced by the tilt of the rotor thrust vectors from the shaft axis. In the case under consideration, the increment ΔT due to the vector tilt was examined and found to have a negligible effect on speed stability. However, in some high-speed cases where the longitudinal tilt of the rotor force vector from the shaft may be large and where the effect of a thrust difference is more significant, a significant difference in the speed stability might result. A derivation of the method of accounting for the difference in thrust ΔT due to tilt of the thrust vectors is presented in appendix B.

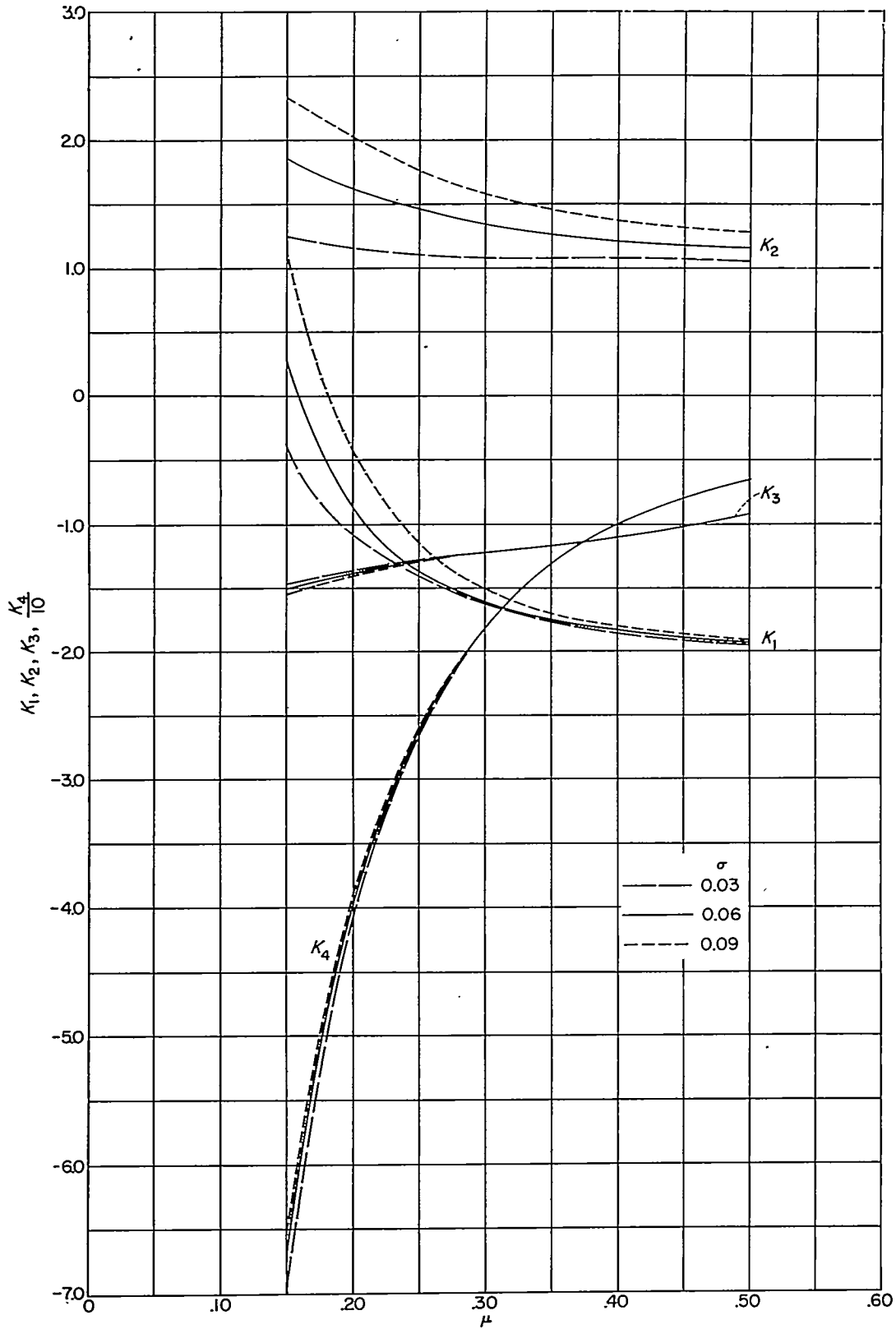


FIGURE 4.—Constants for use in speed-stability equation:

$$\frac{d(\Delta\theta)}{d\mu} = K_1 \left(\frac{C_T}{\sigma}\right)_{\sigma\sigma} \left(\frac{\Delta T}{W} - \frac{\Delta R}{R_{\sigma\sigma}}\right) + K_2 \left(\frac{C_T}{\sigma}\right)_{\sigma\sigma} \left(\frac{\Delta\sigma}{\sigma_{\sigma\sigma}} + 2 \frac{\Delta(\Omega R)}{(\Omega R)_{\sigma\sigma}}\right) + K_3 \Delta\alpha_d + K_4 C_{T_{\sigma\sigma}}$$

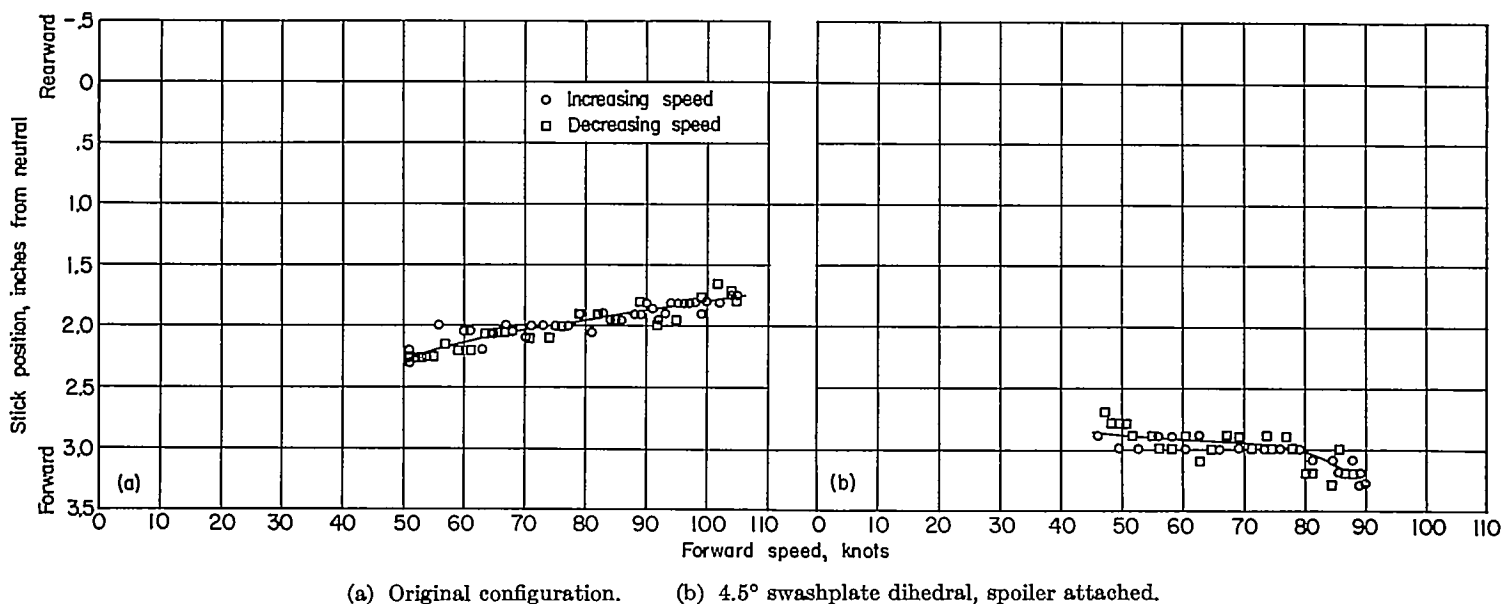


FIGURE 5.—Variation of longitudinal stick position with speed for test helicopter, trimmed at approximately 70 knots, in level flight.

For the test helicopter at 70 knots

$$\begin{aligned} \Delta T &= -320 \text{ lb} \\ W &= 6,750 \text{ lb} \\ \Delta T/W &= -0.0474 \\ \Omega R &= 537 \text{ ft/sec} \\ \sigma_{av} &= 0.052 \\ C_{T_{av}} &= 0.00424 \\ \left(\frac{C_T}{\sigma}\right)_{av} &= 0.0815 \\ \rho/\rho_0 &= 0.89 \\ \mu &= 0.22 \end{aligned}$$

From figure 4, $K_1 = -1.15$ and $K_4 = -33.5$. Substituting into equation (12)

$$\begin{aligned} \frac{d(\Delta\theta)}{d\mu} &= -1.15(0.0815)(-0.0474) - 33.5(0.00424) \\ &= 0.004 - 0.142 \\ \frac{d(\Delta\theta)}{d\mu} &= -0.138 \end{aligned}$$

Converting $\frac{d(\Delta\theta)}{d\mu}$ from radians per μ to degrees per knot gives

$$\frac{d(\Delta\theta)}{dV} = -0.025 \text{ degree per knot}$$

Knowing the ratio between differential collective pitch and stick motion, which as shown in figure 2 for the test helicopter is $1^\circ \Delta\theta$ per inch of stick travel, the stick travel per knot speed change can be computed. In this case,

$$\frac{d\delta}{dV} = -0.025 \text{ inch per knot}$$

Comparison of calculated and experimental values of speed stability.—The calculated value of speed stability for the test helicopter in its original configuration is -0.025 inch per knot whereas the measured value is -0.01 inch per knot. The orders of magnitude are in agreement and the difference, while large percentagewise, is probably within the accuracy of the data and the nature of the assumptions used in the theoretical analysis. Of the assumptions, the one neglecting the contribution of the fuselage-tail combination is considered most likely to be in error.

EFFECT OF SWASHPLATE DIHEDRAL ON SPEED STABILITY

Although, as indicated in the previous section, the assumptions used in the theory may cause some error in the estimation of the absolute value of speed stability, such errors should be due primarily to fuselage moments which remain constant with changes in rotor geometry. Hence the theory should be adequate to predict changes in speed stability brought about by changes in the rotor thrust contributions.

For the purposes of checking the theory and obtaining a condition of positive speed stability for pilots' opinions of flying qualities, swashplate dihedral was rigged into the control system of the test helicopter. It is understood that at least one manufacturer has experimented with swashplate dihedral with some success in improving the speed stability of the tandem configuration.

Improvement predicted by theory.—Inspection of equation (11) and figure 4 shows, inasmuch as K_3 is negative for $\mu = 0.15$ to 0.50 , that there will be a positive increment added to the speed stability when $\Delta\alpha_d$ is negative. Therefore, equation (11) suggests that a negative difference in angle of attack of rotors, that is, the swashplates tilted toward one another, improves the speed stability of the tandem configuration. The magnitude of the predicted improvement

is determined as follows:

For the test helicopter at cruise:

$$\begin{aligned} \mu & \text{-----} 0.22 \\ \sigma & \text{-----} 0.052 \end{aligned}$$

From figure 4, $K_3 = -1.33$ and, assuming $\Delta\alpha_a = -1^\circ$ or -0.0175 radian, equation (11) gives

$$\begin{aligned} \Delta \frac{d(\Delta\theta)}{d\mu} &= -1.33(-0.0175) \\ &= 0.023 \text{ radian per } \mu \text{ unit per degree dihedral} \end{aligned}$$

$$\Delta \frac{d(\Delta\theta)}{dV} = 0.004 \text{ degree per knot per degree dihedral}$$

Measured improvement.—Measurements in flight were made to confirm the effect of swashplate dihedral on the speed stability of the test helicopter. Data were obtained for the test helicopter with 2.7° and 4.5° of swashplate dihedral. Figure 5(b) is a plot of stick position against speed for the test helicopter with 4.5° of swashplate dihedral and shows the test helicopter now to have slightly positive speed stability. Comparison of figure 5(b) with figure 5(a) indicates a definite improvement in the speed stability with swashplate dihedral throughout the speed range from 50 knots to the maximum reached.

Comparison of experimental results with theory.—Spoilers were added to the fuselage for another investigation between the flights for obtaining the original data and the flights for obtaining the dihedral data. Intermediate flight tests indicated these spoilers to affect the speed stability adversely; thus, the incremental improvement in speed stability due to dihedral alone is best obtained by determining the improvement in going from 2.7° to 4.5° swashplate dihedral. The slope of the curve in figure 5 (b) at 70 knots and equivalent data for the 2.7° dihedral case are plotted in figure 6 along with the theoretical values. The experimental increment is computed to be 0.007 degree per knot per degree dihedral. Even though this value is somewhat higher than the value of 0.004 predicted by theory, the comparison is believed to be good enough to indicate the theory to be a useful tool for predicting changes in speed stability.

Pilots' opinions.—The pilots making the test flights considered the handling qualities of the test helicopter improved by the removal of the instability with speed.

CRITERIONS FOR SATISFACTORY SPEED-DISTURBANCE CHARACTERISTICS

While the pilots were certain that any instability with speed would be undesirable, they were not sure whether the speed-disturbance characteristics of the helicopter as modified with 4.5° of swashplate dihedral were satisfactory. As previously mentioned, the test helicopter with 4.5° swashplate dihedral was slightly stable with speed. When the controls of the helicopter were held fixed during flight in rough air, large disturbances in pitch attitude and hence in forward speed were produced from which the helicopter recovered slowly. Under contact conditions these large disturbances in speed were not bothersome in that they were easily prevented by control motion. Thus, for contact flight, slightly positive speed stability seems to be sufficient. However, the pilots felt that under blind-flying conditions,

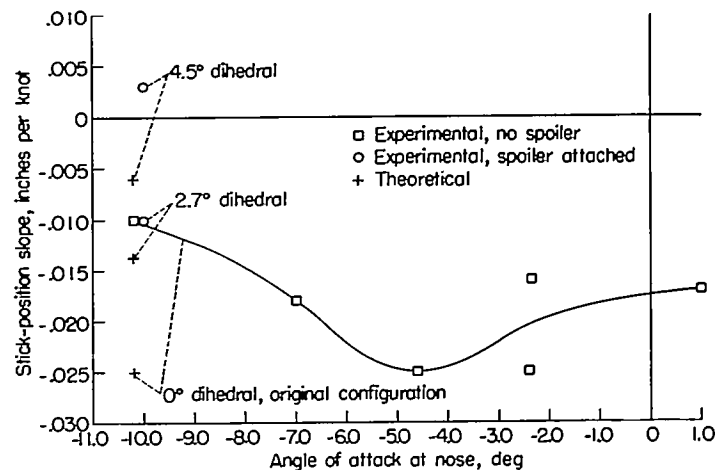


FIGURE 6.—Effect of swashplate dihedral and angle of attack at fuselage nose on speed stability at 70 knots.

these speed-disturbance characteristics might increase their difficulties excessively.

If, during blind flight, speed-disturbance characteristics such as those of the modified helicopter are actually found to be objectionable, it would appear desirable to modify such characteristics to reduce the amount of speed disturbance. From the pilots' point of view it might be desirable to limit the amount or percentage of speed disturbance after some period of time following a fixed longitudinal disturbance of the control stick. Modifications such as increases in stability with speed or in maneuver stability should tend to improve the helicopter's speed-disturbance characteristics.

In addition, it should be pointed out that an increase in speed stability will reduce the amount of forward longitudinal control available at the higher speeds for overcoming a nose-up divergence in pitch. Thus, it appears that an effort to remove any maneuver instability of a tandem helicopter should precede any attempt to increase the speed stability.

EFFECT OF ANGLE OF ATTACK ON SPEED STABILITY

Figure 6 also shows how the slope of stick motion with speed at 70 knots for the original configuration varies with angle of attack at the nose. The angle of attack was varied by changing the rate of descent and was measured by the vane shown in figure 3. Figure 6 is obtained from data such as in figure 5 (a). For example, the slope of the curve in figure 5 (a) at 70 knots is found to be -0.01 inch per knot. The angle of attack at the nose at this condition was measured to be -10.2° . These values determine one point of the curve of figure 6. The additional points were obtained similarly from other runs at several power conditions.

Figure 6 shows that a variation of speed stability with rate of descent exists, indicating that as the rear rotor changes position with respect to the line of flight through the front rotor, a different trim value and hence a different rate of change with speed of front-rotor downwash is apparently encountered. The maximum value of downwash appears to occur when the rear rotor is on the line of flight of the front rotor. This tends to be in agreement with the vertical traverse measurements of downwash angle behind a rotor in a wind tunnel, presented in reference 7, which also indicate

such changes in downwash angle with perpendicular distance from the line of flight of a rotor to exist. The significance of this downwash variation with respect to angle-of-attack stability was discussed in reference 1.

EXPLANATION OF EFFECTS OF CONFIGURATION CHANGES ON SPEED STABILITY

In addition to the effects of downwash (K_4) and swashplate dihedral (K_3) equation (11) shows the manner in which differences in front- and rear-rotor solidity, tip speed, radius, and trim thrust affect the speed stability. (The difference in trim thrust is affected primarily by the center-of-gravity location with respect to the midpoint between rotors.) Inasmuch as the theory was found to give good results in predicting the effects of swashplate dihedral, the theory should also be adequate in general for predicting the effects of other configuration changes.

SWASHPLATE DIHEDRAL

The stabilizing effect of swashplate dihedral is caused by the rear rotor operating at a more negative angle of attack than the front rotor. Under such conditions, an increase in forward speed causes a greater increase in the downflow through the rear rotor than through the front rotor because of the greater axial component of forward velocity. The greater increase in downflow through the rear rotor causes a larger reduction in rear-rotor thrust than that experienced by the front rotor, hence contributing a nose-up or stabilizing moment.

In addition to the stabilizing effect, there is a smaller destabilizing effect caused by swashplate dihedral. Since the difference in rotor angles of attack is more negative than without swashplate dihedral, the trim value of the difference in collective pitch of the rotors must be more positive than without swashplate dihedral in order to maintain the trim values of thrust. This difference in trim values of collective pitch causes a larger increase of thrust with speed for the rear rotor and a smaller increase of thrust with speed for the front rotor. The destabilizing effect due to differences in the pitch angles of the front and rear rotors increases with speed, thus accounting for the overall reduction in effectiveness of swashplate dihedral at higher speeds as shown in figure 4 by the reduction in absolute magnitude of K_3 at high values of μ .

EFFECT OF TIP SPEED OR SOLIDITY DIFFERENTIAL

Since K_2 is shown by figure 4 to be positive at all values of μ from 0.15 to 0.50, equation (11) shows that positive differences in tip speed or solidity (rear rotor greater) have a stabilizing effect. (It should be noted, as previously pointed out, that in equation (11) changes in one parameter are assumed to cause no change in other parameters.) Figure 4 also shows that K_2 decreases as μ increases, indicating that tip-speed and solidity differences have a maximum effect at the lower speeds and decrease in effectiveness as the speed increases.

EFFECT OF CENTER-OF-GRAVITY LOCATION OR RADIUS DIFFERENTIAL

The effect on speed stability of center-of-gravity location or radius differential may be understood by considering

each parameter in the expression $K_1 (C_T/\sigma)_{av} \left(\frac{\Delta T}{W} - \frac{\Delta R}{R_{av}} \right)$ of equation (11). Figure 4 shows K_1 to be negative over most of the range of μ values covered. Since $(C_T/\sigma)_{av}$ is always positive, the above expression will generally show a positive increment of speed stability when ΔT is negative or when ΔR is positive. Inasmuch as ΔT is negative by definition when the front-rotor thrust is greater, location of the center of gravity forward of the midpoint between rotors will generally improve the speed stability.

Inasmuch as figure 4 shows that at the lower tip-speed ratios and higher solidities K_1 becomes small and may even become positive, forward center-of-gravity location or positive radius differential in such cases become less effective and may even have an adverse effect on speed stability. It is believed that the loss in effectiveness of these two parameters at low speeds is due to the fact that the front-rotor lift coefficient is increased. An increase in the effect of destabilizing downwash is therefore obtained, which overshadows the stabilizing tendency at low speeds. At high speeds the destabilizing effect due to downwash decreases and the stabilizing effect predominates.

MEANS FOR IMPROVING SPEED STABILITY

MAGNITUDE OF CONFIGURATION CHANGES REQUIRED TO ACHIEVE NEUTRAL STABILITY FOR THE TEST HELICOPTER

In order to compare the effectiveness of the various methods for improving speed stability, the calculated magnitudes of the changes in each parameter needed to make the test helicopter neutrally stable with speed are shown in table I. At $\mu=0.17$ the amount of thrust or radius differential needed, as shown by -2.3 and 2.3 , respectively, is impossible. The values of 0.6 and 0.3 for $\Delta\sigma/\sigma_{av}$ and $\Delta(\Omega R)/(\Omega R)_{av}$, respectively, indicate that relatively large though not impossible differences would be required. However, in the event that moderate amounts of these latter differentials were used to improve other characteristics, such as angle-of-attack stability, the effect on speed stability would be in the proper direction. Table I shows that at $\mu=0.17$, -3° of swashplate dihedral, a reasonable value, will cause the test helicopter to be neutrally stable with speed. Swashplate dihedral therefore seems to be the most practical means of improving the speed stability of the tandem-rotor helicopter at low speeds. The higher value of swashplate dihedral actually used on the test helicopter was needed because of the adverse effect of the spoiler installation.

At high speeds, as represented by values for $\mu=0.30$ in table I, the test helicopter could be made neutrally stable

TABLE I
 MAGNITUDES OF CONFIGURATION CHANGES NEEDED TO GIVE NEUTRAL STABILITY ON TEST HELICOPTER

μ	Measured $\frac{d(\Delta\theta)}{d\mu}$ for test helicopter, radians/ μ	Difference in angle of attack due to swash- plate dihedral, $\Delta\alpha_p$, deg	$\frac{\Delta T}{W}$	$\frac{\Delta R}{R_{av}}$	$\frac{\Delta\sigma}{\sigma_{av}}$	$\frac{\Delta(\Omega R)}{(\Omega R)_{av}}$
0.17	-0.077	-3	-2.3	2.3	0.6	0.3
.30	-.027	-1.2	-.2	.2	.26	.13

with speed by using any of the methods individually. The values for $\Delta T/W$, $\Delta R/R_{av}$, $\Delta \sigma/\sigma_{av}$, and $\Delta(\Omega R)/(\Omega R)_{av}$ represent large although feasible differences in these parameters, while the value of -1.2° for swashplate dihedral is small. As at low speeds, swashplate dihedral is apparently the most effective single change. However, moderate amounts of other changes could be used simultaneously with good results. Although swashplate dihedral and solidity and tip-speed differential become less effective with increased speed, the lower amount of instability of the original configuration at the higher speed results in less configuration change needed for neutral stability than at the lower speed.

PRACTICAL CONSIDERATIONS REGARDING SWASHPLATE DIHEDRAL

The means for incorporating swashplate dihedral in the test helicopter, described in the section entitled "Description of Test Helicopter," was an expedient method and there are practical considerations to be given to its use. Because of the tilt of the swashplates at the neutral stick position, it was necessary to reduce the longitudinal cyclic-pitch range to avoid linkage interference. In addition, the droopstop clearance in flight of one or both rotors tends to be reduced. The pilots reported the reduction in longitudinal cyclic pitch produced no appreciable change in longitudinal control in flight. However, since cyclic pitch is the only longitudinal control available for taxiing, the reduction in longitudinal cyclic-pitch range might prove to be objectionable during attempts to taxi in high winds. For the test helicopter with the swashplate dihedral, no attempt was made to taxi in high winds.

For a helicopter in the design stages, a more suitable means of incorporating swashplate dihedral might be the inclination of the rotor shafts towards one another. By inclining the rotor shafts, the necessity for reducing the longitudinal cyclic-pitch range to avoid linkage interference and the possibility of blades hitting the droopstops are virtually eliminated. However, inclining the rotor shafts will not eliminate the problem of clearance between the rotors and fuselage.

Another practical consideration regarding swashplate dihedral—its effect on rotor stalling—is discussed in the next section.

In view of these adverse conditions which may arise from swashplate dihedral, some practical considerations must be given to its use.

EFFECT OF STALLING ON SPEED STABILITY

With the load equally distributed between the two rotors of the tandem-rotor configuration, the rear rotor, operating in the downwash of the front rotor, is in more of a climb condition and tends to stall first. When the rear rotor stalls, its lift decreases and with constant stick position a nose-up moment about the center of gravity is contributed. As the forward speed increases, the stalled area of the rotor disk becomes larger and with the stick position constant a nose-up moment is obtained due to the speed increase.

Thus, as rear-rotor stalling is encountered there is an increase in the speed stability. Although stalling of the rear rotor appears to be desirable for speed stability at high forward speeds, it is undesirable for angle-of-attack stability and performance. The effects of rear-rotor stalling on angle-of-attack stability and of stalling in general on performance are discussed, respectively, in reference 1 and chapter 10 of reference 2.

When swashplate dihedral is incorporated in the tandem-rotor configuration, the axis of no feathering of the rear rotor is inclined forward and the component of forward flight velocity along the axis of no feathering is increased. The increased downflow through the rear rotor causes it to be in more of a climb condition than normal thereby decreasing the forward speed at which it begins to stall. Calculations of angles of attack at the tip of the retreating blades for the configuration with 4.5° swashplate dihedral at a forward speed of 80 knots show that the rear rotor is beginning to stall while the front rotor is well below stalled conditions. These differences in stalling apparently account for the increase in speed stability of the modified configuration above approximately 80 knots as indicated by the change in the slope of the curve in figure 5 (b).

Other configuration changes that may be made for stability purposes, such as forward center of gravity and increased solidity or tip speed of the rear rotor, will tend to cause the front rotor to stall first.

It appears that the most desirable conditions regarding stalling from a performance standpoint would be the simultaneous stalling of both rotors. Under such conditions, with a fixed average value of C_T/σ , the forward speed at which stall begins would be a maximum. By considering, during the design stages, the amount of the various configuration changes needed for satisfactory stability and performance, a suitable combination of rotor geometry and center-of-gravity location might be attained whereby optimum stalling characteristics would result.

CONCLUSIONS

A study of the speed stability of a tandem-rotor helicopter in forward flight indicates the following conclusions:

1. The test helicopter is unstable with speed from 50 knots to 105 knots, which is the speed range covered in the tests, in that the stick position moved rearward with increasing forward speed at constant power. This result applies both with and without fuselage spoilers attached during the tests. The pilots consider this characteristic unsatisfactory.
2. An effort to remove any maneuver instability of the tandem helicopter should precede any attempt to improve the speed stability.
3. Instability with speed of the test helicopter is caused primarily by variations with speed of the front rotor downwash at the rear rotor and can be approximately predicted by theory.

4. Swashplate "longitudinal dihedral" (swashplates inclined towards each other) improves the stability with speed of the tandem-rotor helicopter. A value of 4.5° of swashplate dihedral made the test helicopter slightly stable (in spite of the adverse effect of fuselage spoilers) from 50 knots, the minimum speed tested, to the maximum speed tested. Some considerations must be given to the practical aspects of the use of swashplate dihedral.

5. The pilots considered the speed-disturbance characteristics of the test helicopter with only slightly positive speed stability to be satisfactory under contact conditions. The possible need for an additional criterion to limit the amount of speed disturbance during blind flight in rough air remains to be determined.

6. Improvement in speed stability due to swashplate dihedral can be predicted approximately by theory.

7. The speed stability of the tandem helicopter can be studied conveniently by a theoretical chart which is presented.

8. Instability with speed varies with rate of descent, probably as a result of the variation of downwash behind a rotor with perpendicular distance from the line of flight through the rotor.

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 NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
 LANGLEY FIELD, VA., June 4, 1953.

APPENDIX A

DETERMINATION OF $\Delta\alpha$ IN TERMS OF AVERAGE VALUES OF C_T AND μ

The difference in angle of attack of the front and rear rotors is the sum of the downwash angle and the difference in angle due to the geometric swashplate dihedral and is expressed as follows:

$$\begin{aligned} \Delta\alpha &= -\frac{C_{T_{fr}}}{(\mu_{fr})^2} + \Delta\alpha_d \\ &= -\frac{C_{L_{fr}}}{2} + \Delta\alpha_d \end{aligned} \quad (A1)$$

where $C_{L_{fr}}$ is a front-rotor term. In order to express $\Delta\alpha$ in terms of average quantities, it is necessary to determine an expression for $C_{L_{fr}}$ in terms of $C_{L_{av}}$. By definition,

$$C_{L_{fr}} = \frac{L_{fr}}{\frac{1}{2} \rho V^2 \pi (R_{fr})^2} \quad (A2)$$

and

$$C_{L_{av}} = \frac{L_{av}}{\frac{1}{2} \rho V^2 \pi (R_{av})^2} \quad (A3)$$

Dividing equation (A2) by equation (A3) and solving for $C_{L_{fr}}$ gives:

$$C_{L_{fr}} = C_{L_{av}} \frac{L_{fr}}{L_{av}} \left(\frac{R_{av}}{R_{fr}} \right)^2 \quad (A4)$$

Expressing L_{fr} and R_{fr} in terms of average values gives:

$$L_{fr} = L_{av} \left(1 - \frac{\Delta L}{W} \right) \quad (A5)$$

and

$$R_{fr} = R_{av} \left(1 - \frac{1}{2} \frac{\Delta R}{R_{av}} \right) \quad (A6)$$

Substituting equations (A5) and (A6) into equation (A4) and retaining only linear terms gives the following expression for $C_{L_{fr}}$ in terms of $C_{L_{av}}$:

$$C_{L_{fr}} = C_{L_{av}} \left(1 - \frac{\Delta L}{W} \right) \frac{1}{1 - \frac{\Delta R}{R_{av}}}$$

Expanding by the binomial theorem and once again retaining only linear terms

$$C_{L_{fr}} = C_{L_{av}} \left(1 - \frac{\Delta L}{W} + \frac{\Delta R}{R_{av}} \right) \quad (A7)$$

With the expression for $C_{L_{fr}}$ in terms of $C_{L_{av}}$ substituted into the original expression for $\Delta\alpha$, that expression becomes

$$\Delta\alpha = -\frac{C_{L_{av}}}{2} \left(1 - \frac{\Delta L}{W} + \frac{\Delta R}{R_{av}} \right) + \Delta\alpha_d$$

and assuming $L = T$

$$\Delta\alpha = -\left(\frac{C_T}{\mu^2} \right)_{av} \left(1 - \frac{\Delta T}{W} + \frac{\Delta R}{R_{av}} \right) + \Delta\alpha_d \quad (A8)$$

APPENDIX B

METHOD OF DETERMINING THE EFFECTIVE LOCATION OF THE CENTER OF GRAVITY

In order to determine accurately the load carried by each rotor, the center-of-gravity location with respect to the midpoint between the lines of action of the rotor resultant-force vectors, rather than the center-of-gravity location with respect to the midpoint between rotor shafts, must be considered. For the purposes of this analysis the rotor resultant-force vector is assumed to be equal in magnitude to the rotor thrust. A schematic diagram of the tandem-rotor system is shown in figure 7. From figure 7 the distance from the actual midpoint between rotors to the effective midpoint is

$$x' = h \tan(a' - B_1)_{ar} \quad (B1)$$

Then the location of the center of gravity with respect to the effective midpoint is

$$x = x_0 - x'$$

Using this location of the center of gravity, the thrust carried by each rotor in steady flight can be determined accurately for known conditions of flight.

Sample calculations of effective center-of-gravity location.—For a sample case, assume

- $(C_T/\sigma)_{ar} = 0.10$
- $\mu = 0.30$
- $h = 100$ inches
- $\theta_{ar} = 8^\circ$
- $B_{1,ar} = 2.0^\circ$
- $x_0 = 12$ inches

The preceding quantities pertaining to the rotors are average values and may be obtained from flight data or calculated. With the preceding quantities a'_{ar} can be determined from figure 3 of reference 5. For the sample case

$$a'_{ar} = 6.5^\circ$$

Substituting into equation (B1)

$$\begin{aligned} x' &= 100 \tan(6.5^\circ - 2.0^\circ) \\ &= 7.8 \text{ inches} \end{aligned}$$

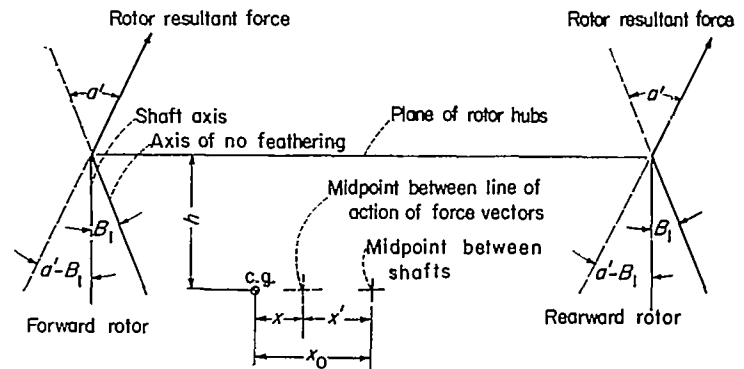


FIGURE 7.—Schematic side view of rotors of a tandem helicopter in forward flight.

The effective location of the center of gravity is $x = 12 - 7.8 = 4.2$ inches forward of the midpoint. In this sample case, note that the distance from the effective midpoint to the center of gravity is about one-third the distance from the geometric midpoint between shafts to the center of gravity. Failure to consider this difference might give misleading results.

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