REPORT No. 917

THE EFFECTS OF AERODYNAMIC HEATING AND HEAT TRANSFER ON THE SURFACE TEMPERATURE OF A BODY OF REVOLUTION IN STEADY SUPERSONIC FLIGHT

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SUMMARY

An approximate method for determining the convective cooling requirement in the laminar boundary-layer region of a body of revolution in high-speed flight was developed and applied to an example body. The cooling requirement for the example body was determined as a function of Mach number, altitude, size, and a surface-temperature parameter. The maximum value of Mach number considered was 3.0 and the altitudes considered were those within the lower constant-temperature region of the atmosphere (40,000 to 120,000 ft). The extent of the laminar boundary layer was determined approximately at each condition as a function of the variables considered.

The convective cooling requirements were found to be small for the range of Mach numbers considered, but increased rapidly with increasing Mach number. For thin, fair bodies the body length for a completely laminar boundary layer is of the order of magnitude of 50 feet for moderate supersonic Mach numbers (3.0) and medium altitudes (100,000 ft).

INTRODUCTION

One of the many problems encountered at high speeds is that of aerodynamic heating due to the compression and friction of the ambient air moving over the surfaces of the aircraft. Flight tests of German V–2 missiles have shown the effects of aerodynamic heating to be appreciable and several theoretical studies have been made of the heating of the warheads on the V–2 and Wasserfall missiles. A study of the heating of the V–2 missile, considering the transient conditions of temperature and velocity along the missile trajectory, is reported in reference 1.

The problem of determining the theoretical heat-transfer characteristics of boundary layers dates back to the works of Pohlhausen (reference 2) and L. Crocco (reference 3). In 1935 and again in 1938 von Kármán treated the subject of heat transfer through laminar boundary layers. (See references 4 and 5.) A study of the heat transfer through a laminar boundary layer to a flat plate in a compressible fluid is presented in reference 5. More recently a study was made of the temperature attained by a flat plate in a high-speed air stream at the condition of equilibrium between the convective heat transfer from the boundary layer to the plate, and the thermal radiation from the plate to the atmosphere. (See reference 6.)

Most of the studies, up to the present time, have been concerned with the determination of the temperature of a flat plate or cone without internal cooling. The results of these studies have served to emphasize the necessity for internal cooling to maintain the surfaces of supersonic aircraft at temperatures which will not cause damage to the aircraft structure and pay load, or discomfort to the occupants.

It is the purpose of this report to present a method for determining the convective cooling requirements in the laminer boundary-layer region for any body of revolution in steady supersonic flight, and to present the results of the application of this method to a representative body. An estimate of the extent of the laminar boundary layer on the example body is also determined.

SYMBOLS

The following symbols have been used in the presentation of the method and its application:

c_p	specific heat at constant pressure, Btu per pound, °F absolute
c_v	specific heat at constant volume, Btu per pound, °F absolute
d	maximum body diameter, feet
g	gravitational acceleration, feet per second squared
H	altitude, feet
J	mechanical equivalent of heat, 778 foot-pounds per Btu
k	coefficient of thermal conductivity, Btu per second, °F absolute, square foot per foot
L	length ratio (l/l_a), dimensionless
l	length of body, feet
M	Mach number, dimensionless
m	Mach number parameter $\left(\frac{\gamma-1}{2}M_v^2\right)$, dimensionless
P	pressure coefficient $\left[(p_v-p_1)/\frac{1}{2}\rho_1V_1^2\right]$, dimensionless
Pr	Prandtl number $(c_p\mu/k)$, dimensionless
p	pressure, pounds per square foot
Q	total rate of heat transfer, Btu per second
$\frac{Q}{(\sigma^*)^{1/2}L^{3/2}}$	total heat-transfer parameter, Btu per second
q	local rate of heat transfer, Btu per second, square foot
$q\left(\frac{L}{\sigma^*}\right)^{1/2}$	local heat-transfer parameter, Btu per second, square foot
R	gas constant, feet per °F absolute

Re_δ laminar boundary-layer Reynolds number $(\rho_v V\delta/\mu_s)$, dimensionless
r radius of body at any point, feet
S frontal area of body, square feet
s distance from nose along the axis of the body, feet
T temperature, ° F absolute
u tangential velocity at any point within the boundary layer, feet per second
V velocity just outside the boundary layer, feet per second
y distance normal to the surface, feet
β surface temperature parameter $[(T_s-T_v)/(T_0-T_v)]$, dimensionless
γ ratio of specific heats (c_p/c_v), dimensionless
δ boundary-layer thickness, feet
θ angle of the nose shock wave with the horizontal, degrees
μ absolute viscosity, pound-second per foot squared
ρ air density, slugs per cubic foot
σ^* air density ratio (ρ/ρ_a), dimensionless
τ surface unit shear, pounds per square foot

In addition the following subscripts have been used.

a reference length or air density
s body surface
v any point along the body just outside the boundary layer
x location of a particular limit of integration along the length of the body
0 stagnation condition
1 ambient condition
2 condition at the rear of the nose wave
3 condition at the nose of the body just outside the boundary layer

THEORY

METHOD

The following analysis is based on the fundamental concept that the rate of heat transfer, by conduction, either into or out of any unit of surface area, by a surrounding fluid, is a function of the temperature gradient in the fluid, and the thermal conductivity of the fluid, at the surface. In order to obtain the temperature gradient through a boundary layer, the boundary-layer thickness must be determined. Therefore, to determine the boundary-layer thickness, in the procedure of the present report, a knowledge of the temperature, velocity, Mach number, and air density of the fluid along the surface is necessary. The required information can be obtained from the pressure distribution over the surface, which in turn is derived from the shape and speed of the body, and the flight altitude.

Pressure distribution.—For the case of a given body of revolution in steady supersonic flight, at high altitudes, the conditions of flight speed and ambient temperature and pressure will be fixed. If the body has a small nose angle in relation to the shock wave angle at the nose, its pressure distribution can be determined by the approximate method of von Kármán and Moore. (See reference 7.) For more blunt bodies, or at higher Mach numbers, it is necessary to resort to the more complex, but exact, three-dimensional method of characteristics. (See reference 8.)

The air stream approaching the body will undergo an increase in temperature and pressure upon passing through the nose wave. The static temperature just aft of the nose wave is given by the relation

$$T_2=T_1\frac{[2\gamma M_1^2\sin^2\theta-(\gamma-1)][(\gamma-1)M_1^2\sin^2\theta+2]}{(\gamma+1)^2M_1^2\sin^2\theta} \tag{1}$$

and the temperature at the nose, just outside the boundary layer T_3 is given by the isentropic relation

$$T_3=T_2\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{2}$$

where

$$p_2=p_1\left(\frac{7M_1^2\sin^2\theta-1}{6}\right) \tag{3}$$

The pressure p_3 is obtained directly from the pressure distribution for the particular body. The value of the nose wave angle θ can be obtained from reference 9.

Most bodies for supersonic aircraft can be expected to have smooth surfaces and fair contours in order to have the minimum possible drag. The air flow about a well-designed body of revolution will therefore be shock free from aft of the nose wave to the rear wave, and as a result, outside the boundary layer, the air flow will be isentropic.

Because of the isentropic flow along the body surface, the static temperature distribution just outside the boundary layer can be calculated, knowing the static temperature and pressure at the nose and the pressure distribution, by the use of the isentropic relation

$$T_v=T_2\left(\frac{p_v}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4}$$

The velocity at each point on the body can be determined, knowing the static temperature distribution, because the energy in the air stream outside the boundary layer will be constant. Therefore,

$$T_0=T_v+\frac{V^2}{2Jgc_p} \tag{5}$$

or

$$V=[2Jgc_p(T_0-T_v)]^{\frac{1}{2}} \tag{6}$$

The Mach number of the air stream just outside the boundary layer M_v can be obtained from the results of equations (4) and (6).

$$M_v^2=\frac{V^2}{\gamma gRT_v} \tag{7}$$

Also, with the temperature distribution known, the air density along the body can be determined by the use of the relation

$$\rho_v=\frac{p_v}{gRT_v} \tag{8}$$

With the pressure, static temperature, velocity, Mach number and air-density distributions known, just outside the boundary layer, it is possible to determine the boundary-layer thickness.

Boundary-layer thickness.—The general momentum equation for a laminar boundary layer on a body of revolution is shown, in reference 10, to be, in the nomenclature of the present report:

$$\tau_s r=\frac{d}{ds}\left[\int_0^{\delta}(\rho_s V^2-\rho u^2)rdy\right]-V\frac{d}{ds}\left[\int_0^{\delta}(\rho_s V-\rho u)rdy\right] \tag{9}$$

Because most bodies of revolution for supersonic flight can be expected to be sharp nosed and have large fineness ratios (of the order of 10), the dimension s will be measured along the axis rather than along the surface of the body. Equation (9) expressed nondimensionally, becomes

$$\tau_s\left(\frac{r}{l}\right)l=\frac{d}{d\left(\frac{s}{l}\right)}\left\{\delta\rho_s V^2\left(\frac{r}{l}\right)\int_0^1\left[1-\frac{\rho}{\rho_s}\left(\frac{u}{V}\right)^2\right]d\left(\frac{y}{\delta}\right)\right\}$$
$$-V\frac{d}{d\left(\frac{s}{l}\right)}\left\{\delta\rho_s V\left(\frac{r}{l}\right)\int_0^1\left[1-\frac{\rho}{\rho_s}\left(\frac{u}{V}\right)\right]d\left(\frac{y}{\delta}\right)\right\} \tag{10}$$

where

$$\tau_s=\mu_s\left(\frac{\partial u}{\partial y}\right)_s$$

In order to solve the foregoing equation for the boundary-layer thickness, the change in velocity and air density with distance normal to the surface must be known. Since at any point on the body the static pressure will be constant through the boundary layer, the variation in air density is related to the temperature profile by the relation

$$\frac{T}{T_v}=\frac{\rho_v}{\rho} \tag{11}$$

The temperature profile within a laminar boundary layer, with heat transfer and with Prandtl's number assumed to be unity, is demonstrated in reference 4, to be given by the relation

$$\frac{T}{T_v}=\frac{T_s}{T_v}+\frac{u}{V}\left[\left(1+\frac{\gamma-1}{2}M_v^2\right)-\frac{T_s}{T_v}\right]-\left(\frac{u}{V}\right)^2\frac{\gamma-1}{2}M_v^2 \tag{12}$$

To simplify this expression let

$$\frac{\gamma-1}{2}M_v^2=m \tag{13}$$

and

$$\frac{T_s}{T_v}=1+\beta m \tag{14}$$

or

$$\frac{T_s-T_v}{T_0-T_v}=\beta \tag{15}$$

with these simplifications, equation (12) becomes

$$\frac{T}{T_v}=1+\beta m+\frac{u}{V}[m(1-\beta)]-\left(\frac{u}{V}\right)^2 m \tag{16}$$

and the air-density relation, equation (11), becomes

$$\frac{\rho_v}{\rho}=1+\beta m+\frac{u}{V}[m(1-\beta)]-\left(\frac{u}{V}\right)^2 m \tag{17}$$

The velocity profile within the boundary layer is assumed to be linear;

$$\frac{u}{V}=\frac{y}{\delta} \tag{18}$$

for all values of Mach number and surface-temperature parameter β, because the linear profile is a simple, yet reasonable, approximation of the actual laminar velocity profile at high Mach numbers. Examples of approximate laminar velocity profiles for various Mach numbers and for one very low value of surface temperature are shown in figures 3 and 4 of reference 5.

The momentum equation (10), upon substitution of the linear velocity profile relation, becomes

$$\mu_s l\left(\frac{V}{\delta}\right)\left(\frac{r}{l}\right)=\frac{d}{d\left(\frac{s}{l}\right)}\left\{\delta\rho_v V^2\left(\frac{r}{l}\right)\int_0^1\left[1-\frac{\rho}{\rho_v}\left(\frac{y}{\delta}\right)^2\right]d\left(\frac{y}{\delta}\right)\right\}-$$
$$V\frac{d}{d\left(\frac{s}{l}\right)}\left\{\delta\rho_v V\left(\frac{r}{l}\right)\int_0^1\left[1-\frac{\rho}{\rho_v}\left(\frac{y}{\delta}\right)\right]d\left(\frac{y}{\delta}\right)\right\} \tag{19}$$

Substitution of the density relation, equation (17), into equation (19), and performing the necessary integrations and algebraic manipulations, results in the following expression for the boundary-layer thickness:

$$\delta^2=\frac{2Cl}{\left[\left(\frac{r}{l}\right)B\,\rho_v\right]_x^2}\int_0^{\frac{s_x}{l}}\left(\frac{r}{l}\right)^2\frac{\rho_v\mu_s B}{CV}\left(\frac{V}{V_x}\right)^{(2A/B)}d\left(\frac{s}{l}\right) \tag{20}$$

in which

$$A=1+\frac{2}{m}+\frac{\beta-0.5}{m}\log_e(1+\beta m)+\left[\frac{\beta(1-\beta)-Y^2}{2mY}\right]\log_e Z$$

$$B=\frac{1}{m}+\frac{\beta}{2m}\log_e(1+\beta m)+\frac{1-\beta^2-Y^2}{4mY}\log_e Z$$

$$C=e^{2\int_{(A/B)_0}^{(A/B)_x}\log_e V\,d\left(\frac{A}{B}\right)}$$

$$Y=\left[\frac{4}{m}(1+\beta m)+(1-\beta)^2\right]^{\frac{1}{2}}$$

and

$$Z=\left[1-\frac{2}{(1-\beta)-Y}\right]\left[\frac{(1-\beta)+Y}{(1-\beta)-2+Y}\right]$$

The boundary-layer thickness for any length of body at any altitude can be determined from the calculated values for a fixed length and altitude when the ambient temperatures are identical by the use of the relation

$$\delta=\delta_a\left(\frac{L}{\sigma^*}\right)^{\frac{1}{2}} \tag{21}$$

where the subscript a denotes the calculation for a reference altitude and length.

Rate of heat transfer.—The fundamental expression for the local rate of convective heat transfer from a unit surface area to or from a surrounding fluid is

$$q=-k_s\left(\frac{\partial T}{\partial y}\right)_s \tag{22}$$

where k_s is the thermal conductivity of the fluid adjacent to the surface and $(\partial T/\partial y)_s$ is the temperature gradient within the fluid at the surface. The temperature gradient, in terms of boundary-layer thickness, is obtained by the differentiation of the temperature profile relation (equation (16)) and, when y is equal to zero and the linear profile relation employed, the expression for the temperature gradient becomes

$$\left(\frac{\partial T}{\partial y}\right)_s=\frac{T_v}{\delta}m(1-\beta) \tag{23}$$

and therefore

$$q=-\frac{k_sT_vm}{\delta}(1-\beta) \tag{24}$$

By the use of equation (21), the local rate of heat transfer can be determined for many altitude and length conditions when the ambient-air temperature is constant, with but a single calculation of the boundary-layer-thickness distribution.

$$q_a=q\left(\frac{L}{\sigma^*}\right)^{\frac{1}{2}}=-\frac{k_sT_vm}{\delta_a}(1-\beta) \tag{25}$$

With the local rate of heat transfer known for any point on a body, the total rate of heat transfer or cooling requirement can be obtained by integrating over the surface of the body,

$$Q=2\pi l^2\int_0^{1.0}q\left(\frac{r}{l}\right)d\left(\frac{s}{l}\right) \tag{26}$$

or, for the more general case,

$$\frac{Q}{L^{3/2}(\sigma^*)^{1/2}}=2\pi l_a^2\int_0^{1.0}q_a\left(\frac{r}{l}\right)d\left(\frac{s}{l}\right) \tag{27}$$

The foregoing equation presumes that the laminar boundary layer will extend to the stern of the body and that no heat will be transferred from the surface by radiation or along the surface by conduction.

Boundary-layer transition.—In general, the boundary layer on a small, fine body flying at high altitudes and at moderate supersonic speeds should be laminar. However, the limiting length, altitude, and speed for a completely laminar boundary layer will have to be determined for each example considered.

The location of the transition point on a surface can be determined if the boundary-layer Reynolds number for transition is known. The effect of compressibility on the growth of the laminar boundary layer is presented in reference 10. As long as the flow is isentropic, increases in Mach number tend to stabilize the laminar boundary layer, causing transition to occur at higher Reynolds numbers. At slow speeds the minimum value of boundary-layer Reynolds number for transition to occur, using as the characteristic length the value of y at u equal to $0.707V$, is about 8,000; for the assumed linear velocity profile, using as the characteristic length the value of y at u equal to V, the value of the boundary-layer Reynolds number for transition is 11,300. Because no accurate method of determining the boundary layer Reynolds number for transition as a function of Mach number in the supersonic region has been developed, the best estimate of the value of laminar boundary-layer Reynolds number at transition appears to be the value, 11,300.

APPLICATION

In applying the foregoing method the cooling requirements for a body of revolution were calculated for a range of Mach numbers (M=1.2 to 3.0) and for a range of surface temperature parameters (β=0 to 1.0) for altitudes within the lower constant temperature region of the atmosphere (H= 40,000 to 120,000 ft).

A body with a fineness ratio of 10 was selected from reference 11 as being typical of the present design trend for rocket-powered missiles. The radius of the body selected, at any longitudinal station, is given by the equation

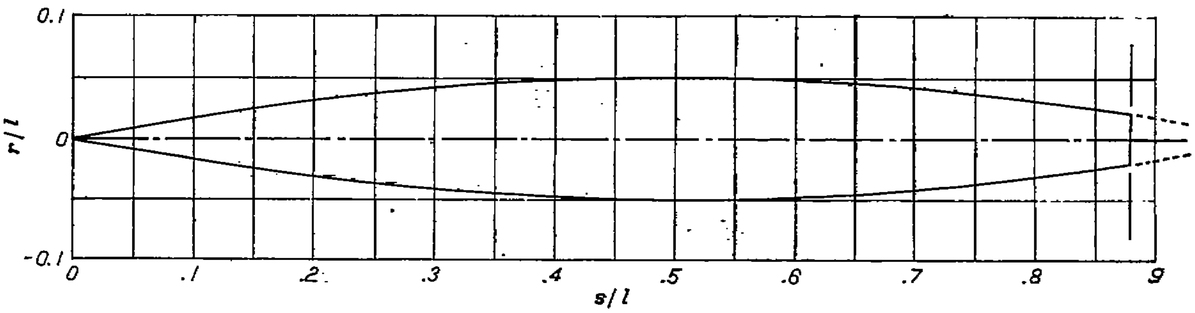


FIGURE 1.—Contour of the body of revolution,

$\frac{r}{l}=2\frac{d}{l}[(s/l)-(s/l)^2],\ \frac{d}{l}=\frac{1}{10}.$

$$\frac{r}{l}=2\frac{d}{l}[(s/l)-(s/l)^2] \tag{28}$$

where d/l=0.1. The contour of the body is shown in figure 1. Equation (28) defines a body which is pointed at both ends but, for the purpose of this example, the fineness ratio was reduced to 8.8 by cutting off the rear point at 88 percent of the length to allow a flat base for the power-plant nozzle outlet. The figure of 88 percent was selected arbitrarily to obtain a reasonable pressure recovery over the after portion of the body.

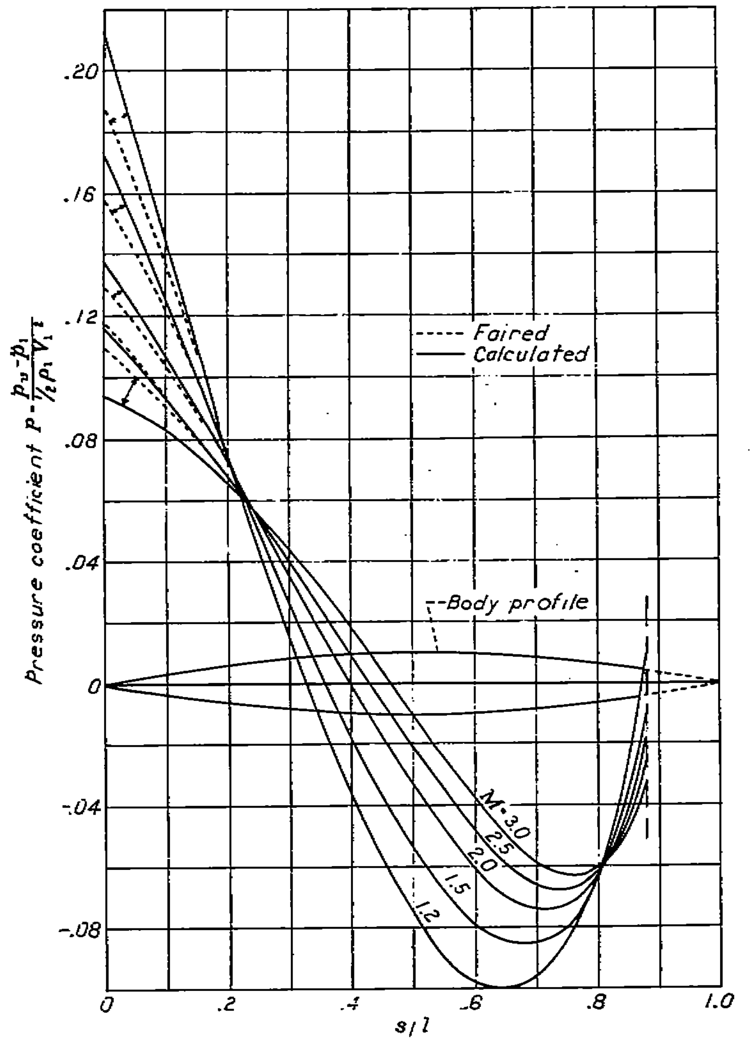


FIGURE 2.—Pressure distributions over the body of revolution for various Mach numbers.

Pressure distributions.—Pressure distributions over the body, obtained by the method of reference 7, for five Mach numbers, are shown in figure 2. Exact values of the pressure at the nose were obtained by using Taylor and Maccoll's values for cones. (See reference 9.) The curves, used for all subsequent calculations, were faired from the exact values at the nose to the curves given by the approximate method and are shown as dashed lines in figure 2.

The static temperature, velocity, Mach number, and air density distributions, just outside the boundary layer were calculated by the use of equations (1) to (8), assuming the ratio of specific heats γ to be constant. The angle of the nose wave θ in equations (1) and (3) was obtained from reference 9, and used to obtain the value of static temperature at the nose of the body for the range of Mach numbers being considered. With the data from equations (2), (4), (6), (7), and (8) known, the boundary-layer thickness along the body was calculated.

Boundary-layer thickness.—The boundary-layer thickness for a given body shape is determined by the following variables:

1. Size of body
2. Flight Mach number
3. Altitude
4. Rate of heat transfer or surface temperature

For the purpose of the calculations, the body was assumed to be of unit length and assumed to be flying at an altitude of 40,000 feet over the range of Mach numbers (1.2 to 3.0). In order to simplify the calculations, values of the rate of heat transfer were calculated for fixed values of the surface-temperature parameter β rather than for fixed surface temperatures. The result is that the surface temperature varies to some extent along the body because of the variation of local pressure. The exact variation in surface temperature with Mach number and surface-temperature parameter for a flat plate, or the approximate variation for the body, calculated from equation (16), considering T_v to be the ambient temperature, is shown in figure 3. The relation between Mach number and the surface-temperature parameter for a surface temperature of 520° Fahrenheit absolute is shown in figure 4.

The boundary-layer thickness was calculated by equation (20) using the values of absolute viscosity μ taken at the surface temperature. The calculations were made in successive steps by calculating each variable (A, B, C, Y, and Z) separately. The integrations to determine the variable C and the boundary-layer thickness δ were performed graphically.

Rate of heat transfer.—The values of boundary-layer thickness were used in equation (25) to obtain values of the

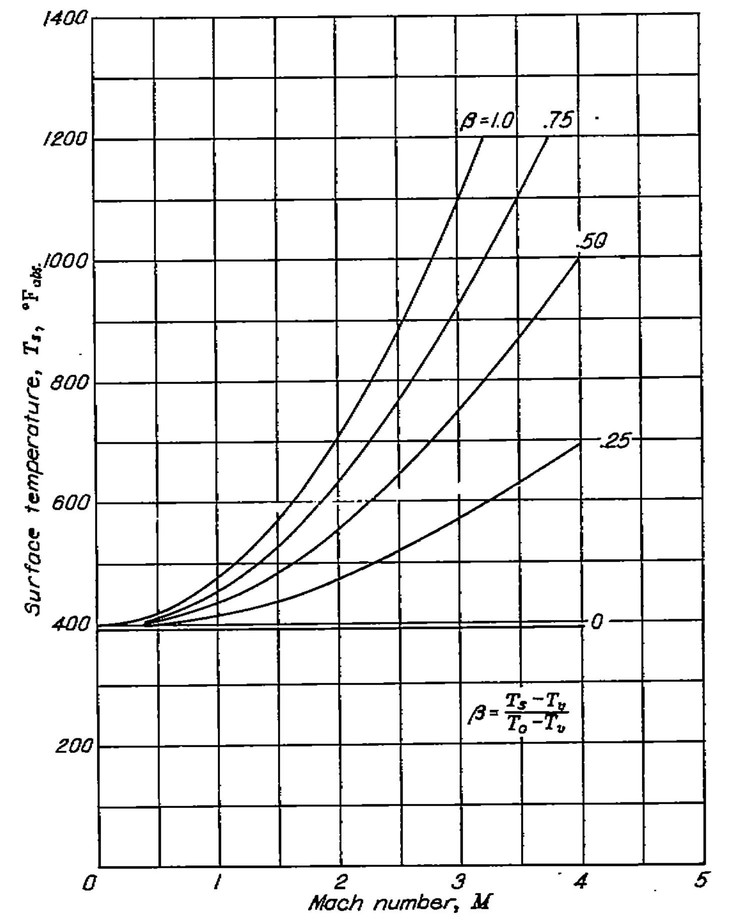


FIGURE 3.—The variation of surface temperature with Mach number for various values of surface-temperature parameter.

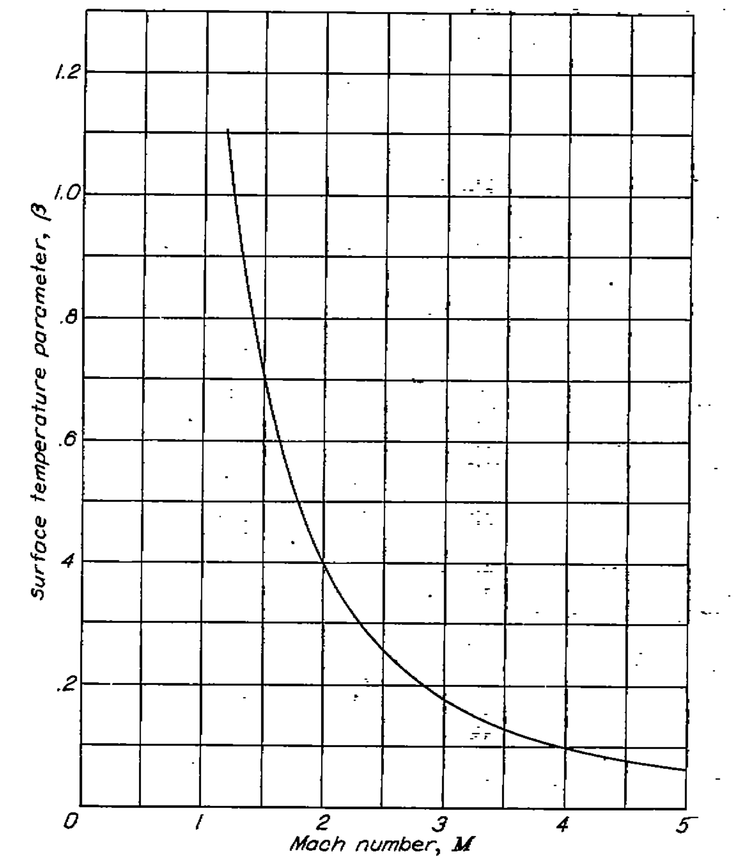
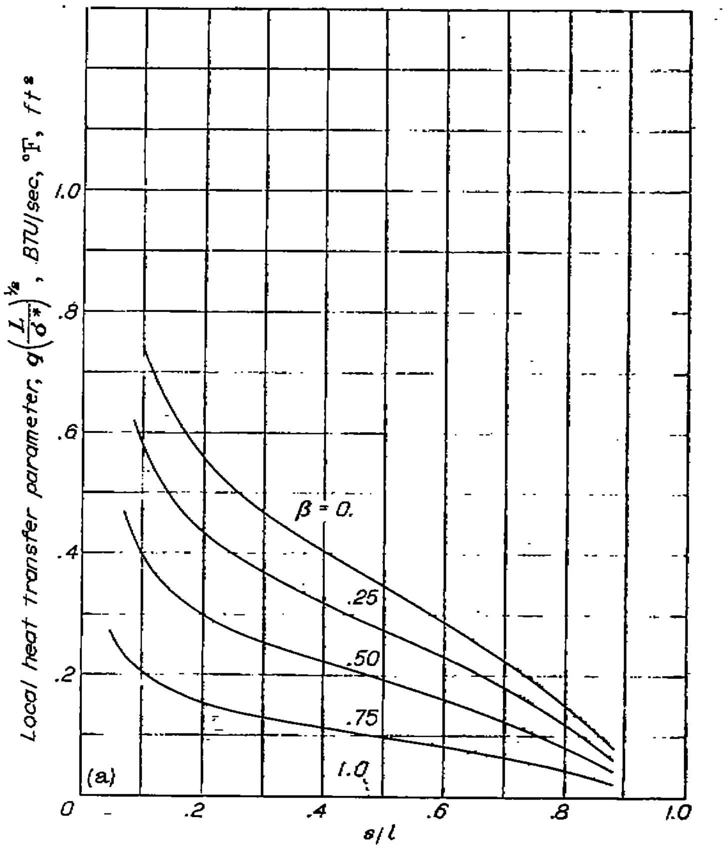


FIGURE 4.—The variation of surface-temperature parameter with Mach number for a surface temperature of 520° F absolute on a flat plate.



(a) $M=1.2$.

FIGURE 6.—Variation of the local heat-transfer parameter with distance along the axis of a body of revolution for several values of the surface-temperature parameter.

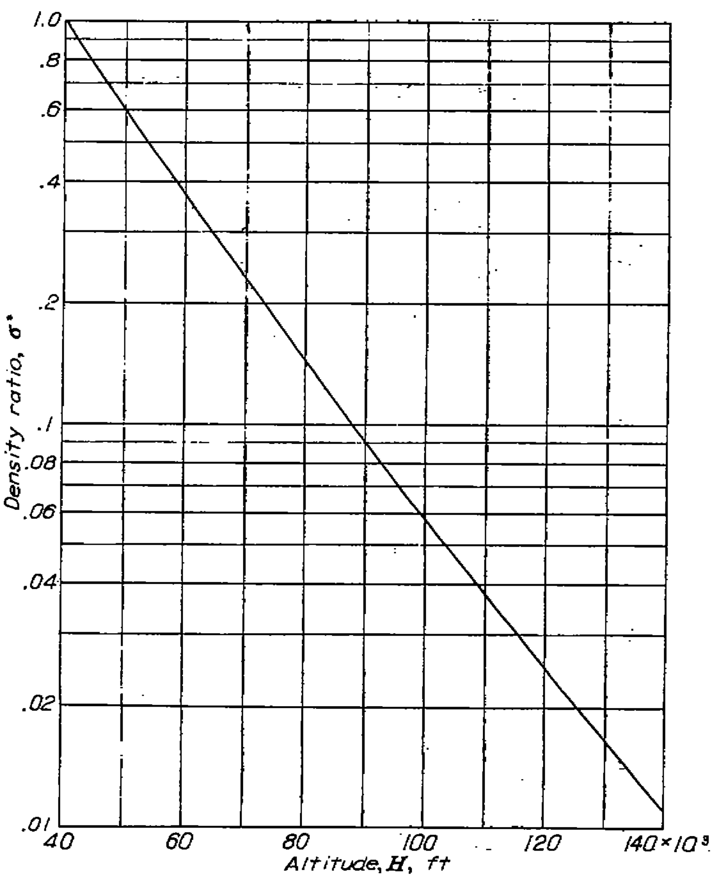
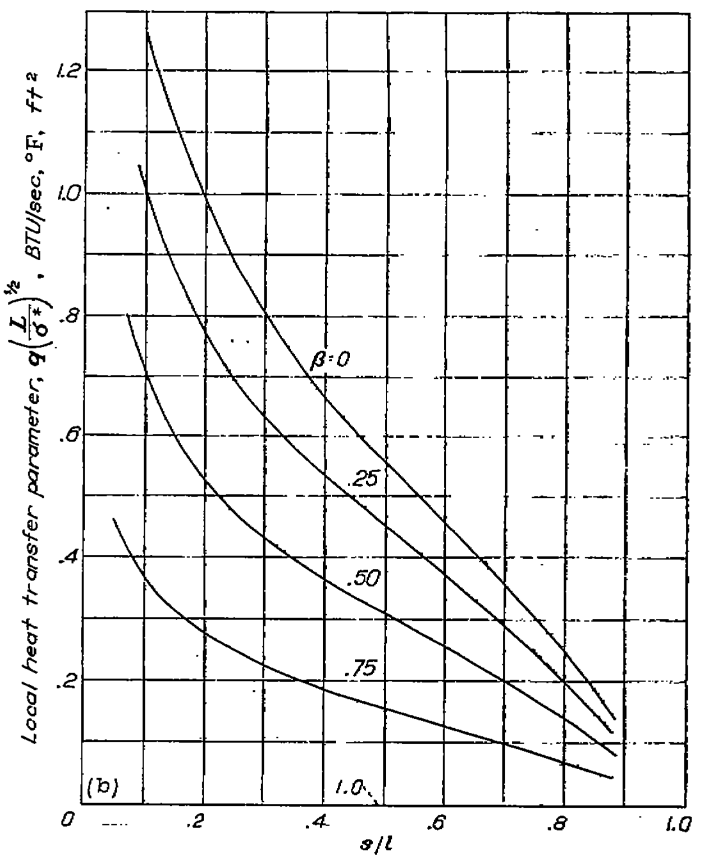
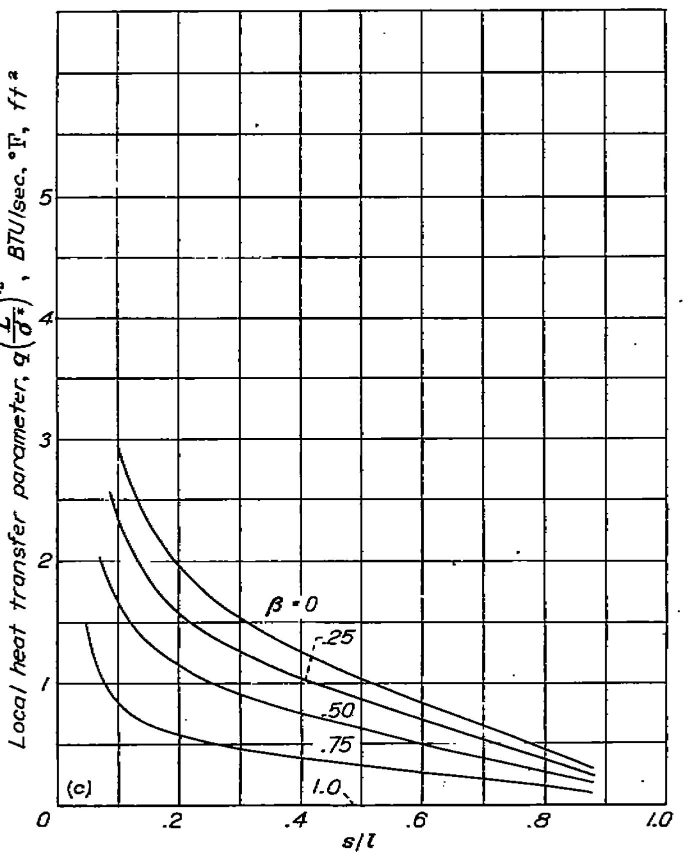


FIGURE 5.—The variation of air-density ratio with altitude for the lower constant temperature portion of the atmosphere.



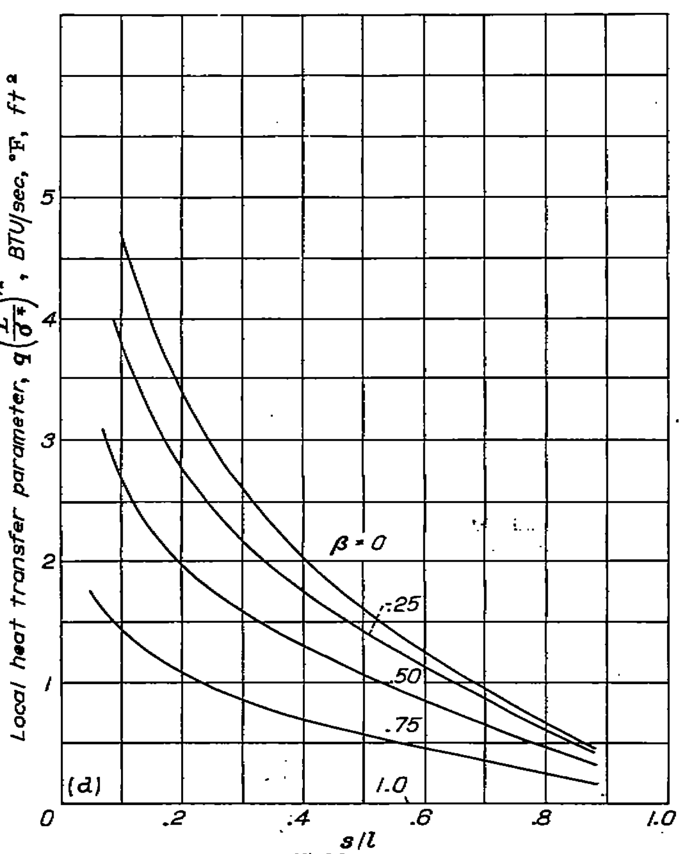
(b) $M=1.5$.

FIGURE 6.—Continued.

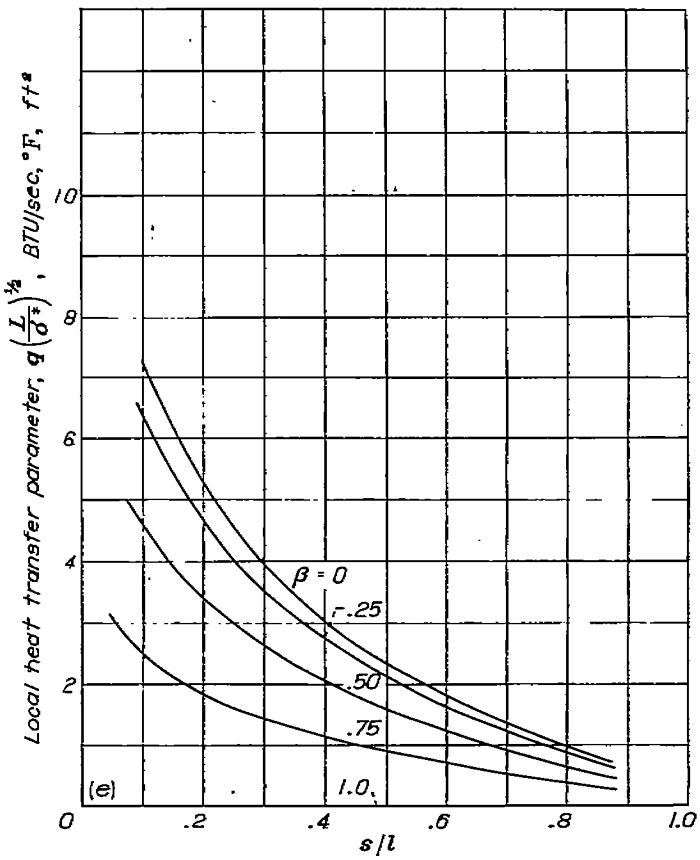


(c) $M=2.0$.
FIGURE 6.—Continued.

local heat-transfer parameter on the body. The value of the air-density ratio as a function of altitude is shown in figure 5, based on the air density at 40,000 feet for convenience. The results of the heat-transfer calculations are presented in figure 6 in such a manner as to include the effects of length, altitude, and surface-temperature parameter for each Mach number.



(d) $M=2.5$.
FIGURE 6.—Continued.



(e) $M=3.0$.
FIGURE 6.—Concluded.

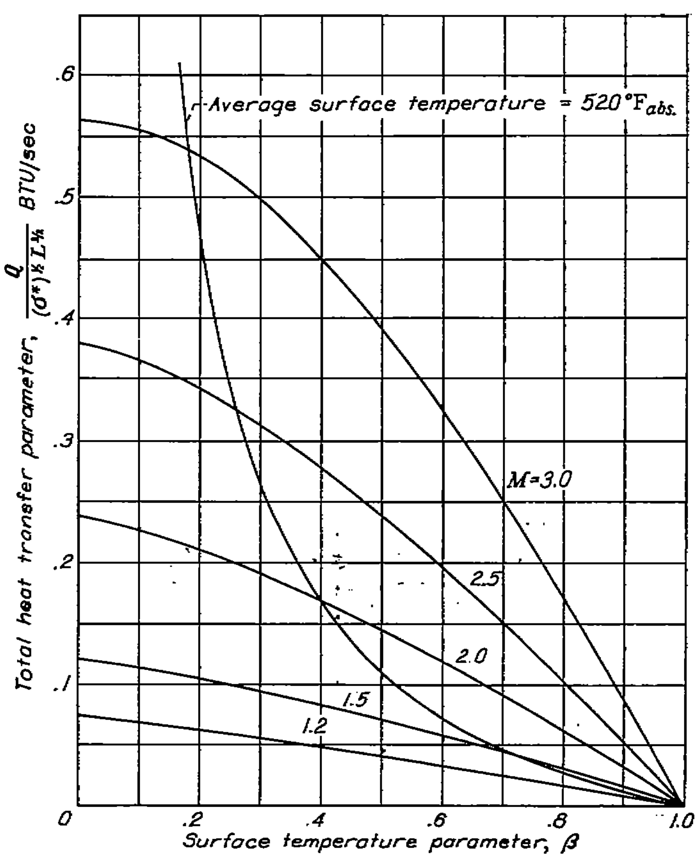


FIGURE 7.—The variation of total heat-transfer parameter with surface-temperature parameter and Mach number.

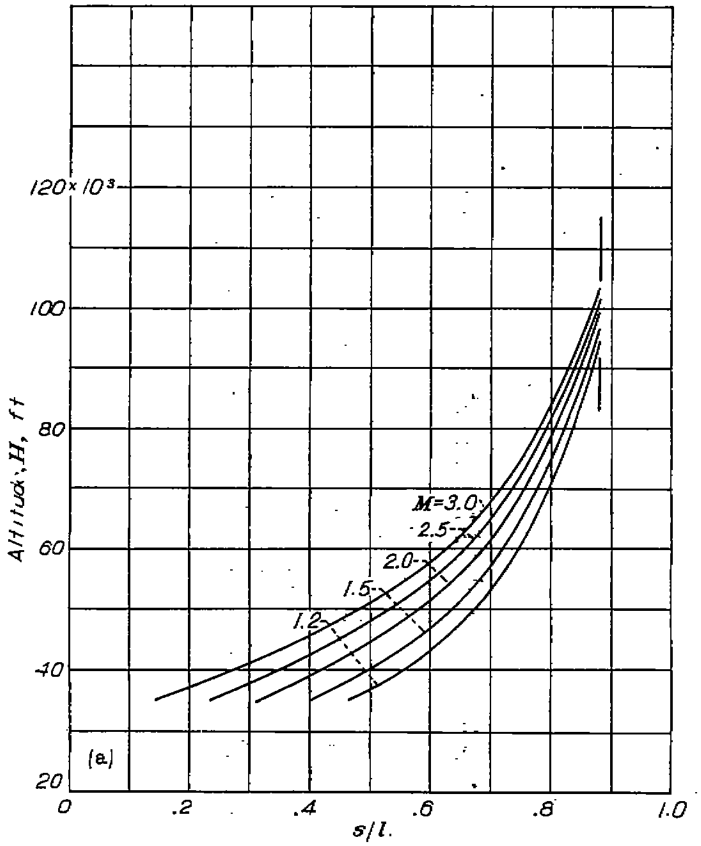


FIGURE 8.—The location of the transition point on a body of revolution as a function of altitude and Mach number for a body length of 100 feet and a boundary-layer Reynolds number for transition of 11,300. (a) β=1.0.

The total rate of heat transfer as a function of Mach number and surface-temperature parameter was obtained by the integration over the surface of the body. The total

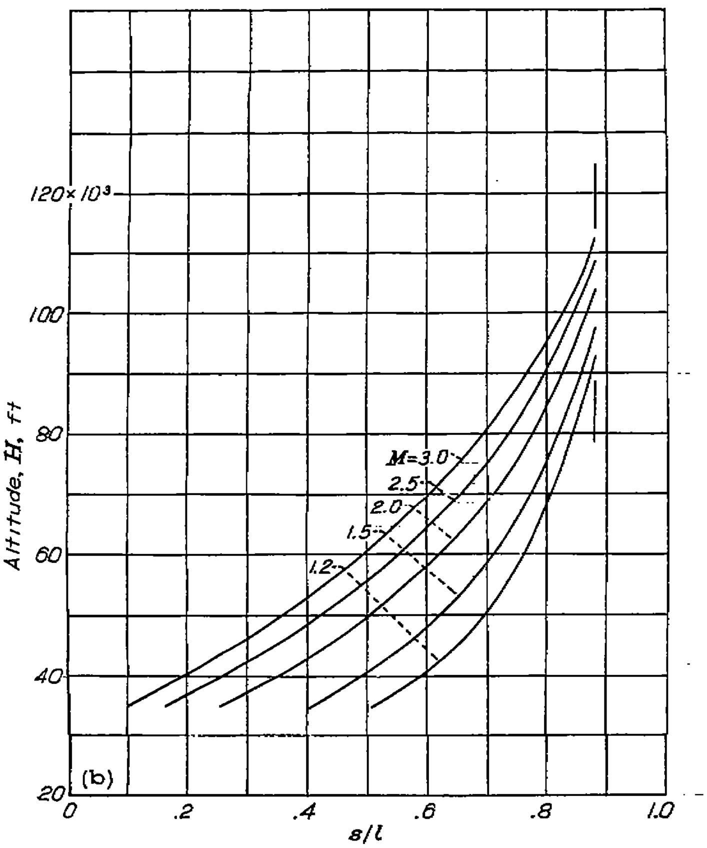


FIGURE 8.—Concluded. (b) β=0.

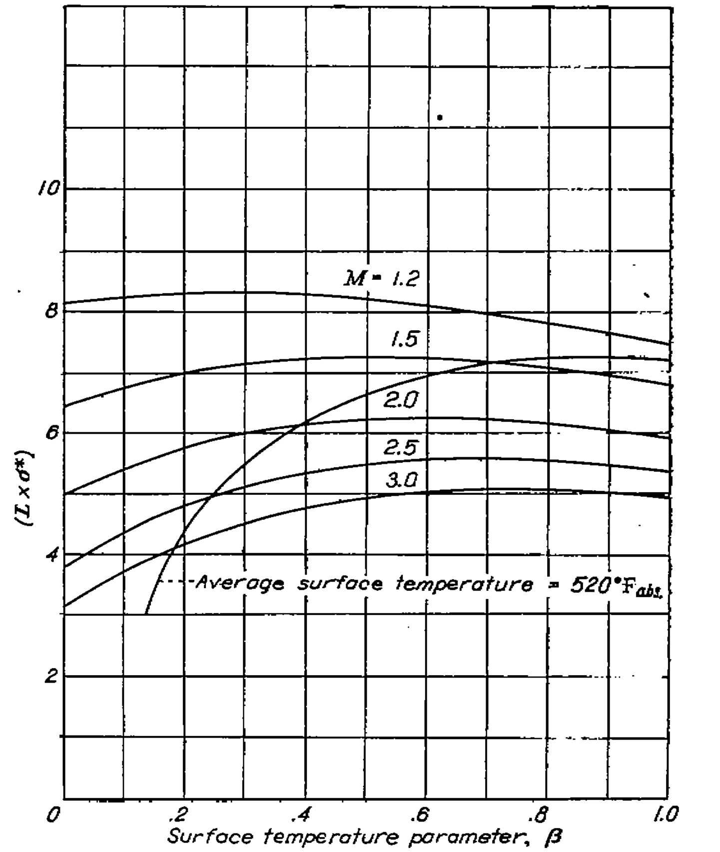


FIGURE 9.—The variation of body length for a completely laminar boundary layer with altitude, Mach number, and surface-temperature paramater for a boundary-layer Reynolds number at transition of 11,300.

heat-transfer parameter is presented in figure 7 with the curve of figure 4 cross-plotted to indicate the cooling required to maintain an average surface temperature of 520° Fahrenheit absolute.

Boundary-layer transition.—The location of the transition point for a 100-foot-long body was determined by calculating the boundary-layer Reynolds number using the local values of air density and viscosity, velocity, and boundary-layer thickness. The transition points were picked from curves of boundary-layer Reynolds number versus length and are shown as a function of Mach number and altitude for two values of the surface-temperature parameter in figure 8. The body length for a completely laminar boundary layer as a function of altitude, Mach number, and surface-temperature parameter is shown in figure 9. The curve of figure 4 is again cross-plotted to indicate the effect of a constant surface temperature.

Example.—As an example of the applicaion of the foregoing general curves to a specific body, approximately the size of the V–2 missile, assume:

l =50 feet
H =100,000 feet
$T_{s_{av}}$=520° Fahrenheit absolute
M =variable

The required cooling rate for this example from the data of figure 7 as a function of Mach number, is shown in figure 10. Also, from figure 9, the boundary layer will probably be completely laminar up to Mach numbers greater than those considered herein.

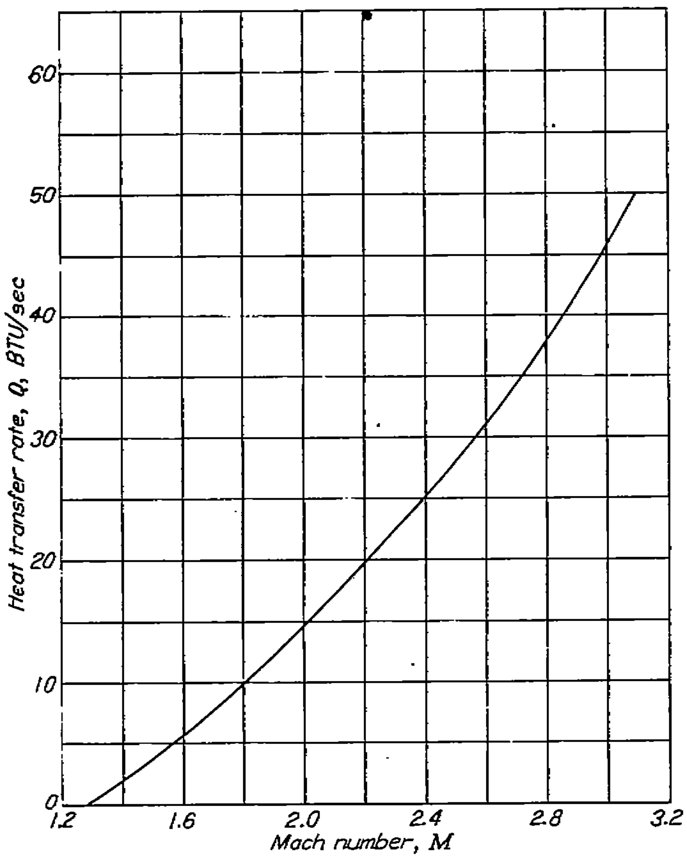


FIGURE 10.—The required cooling rate for the example body as a function of Mach number for a body length of 50 feet, an average surface temperature of 520° F absolute, and an altitude of 100,000 feet.

DISCUSSION

RESULTS

The results of the application of the foregoing analysis to a particular body are presented in figures 6 to 10. Although the results were obtained for a body of a specific shape, the general conclusions obtained are believed to be applicable to all fine, smooth, and fair bodies of revolution in steady supersonic flight.

The local rate of heat transfer is shown to approach infinity at the nose of the body (fig. 6), but at the stern, the local rate of heat transfer is small. The cooling requirement, for a fixed value of surface-temperature parameter, increases rapidly with increasing Mach number (fig. 7), until, at the highest Mach number ($M=3.0$) and at the maximum rate of heat transfer considered ($\beta=0$), the rate of increase becomes almost constant. However, for a fixed value of surface temperature, the cooling requirement increases even more rapidly with Mach number.

The location of the transition point on a body with a basic length of 100 feet for the conditions of zero and maximum rates of heat transfer, as a function of altitude and Mach number, is shown in figure 8. The rate of change of the position of transition with altitude becomes less with increasing altitude because of the increased rate of boundary-layer growth in the adverse pressure gradient over the after portion of the body. Therefore, if the after portion of the body is considered to be uncooled, the maximum body length for the condition of a laminar boundary layer up to the edge of the uncooled portion will be markedly greater than the maximum body length for the condition of a completely laminar boundary layer.

The assumption of a fixed value of boundary-layer Reynolds number for transition is identical to assuming a fixed degree of stability for the laminar boundary layer. As a result of this assumption the transition point, for a given Mach number, tends to move forward on the body with increasing rates of heat transfer because the boundary-layer Reynolds number at a given point on the body increases with increasing heat transfer due to the effect of surface temperature on the local kinematic viscosity.

When the length, speed, and altitude conditions are such that the boundary-layer Reynolds numbers become small, laminar separation is likely to occur over the rear portion of the body. However, because laminar separation is equivalent to an abrupt thickening of the boundary layer, the convective heat transfer from the boundary layer to the body will decrease in the separated region.

Because of the marked increase in heat transfer at transition or the marked reduction at laminar separation, the curves of figure 7 are only applicable when a completely laminar boundary layer exists on the body. However, the results presented in figure 6 are applicable over the forward portion of the body aft to the transition or separation point for all cases within the range or variables for which the calculations were made. The curves presented in figure 9 indicate the conditions of length, altitude, Mach number, and surface-temperature parameter which satisfy the condition of transition for a completely laminar boundary layer. The curve of figure 4 is cross-plotted to indicate the conditions for a given value of surface temperature.

The cooling requirement, or required rate of heat transfer, for an example body approximately the size of the German V–2 missile is presented in figure 10 as a function of Mach number. It should be noted that no cooling is required at speeds up to a Mach number of 1.25, but the required cooling increases rapidly with increasing Mach number.

For bodies less fine than the example considered and for those with less fair contours, the cooling requirements can be expected to be greater than those presented herein. For more blunt bodies the increased adverse velocity gradient at the stern will probably cause transition to occur at lower flight Reynolds numbers while the increased favorable velocity gradient over the nose should promote thinner laminar boundary layers and consequently greater cooling requirements in this region.

REVIEW OF ASSUMPTIONS

In order to make possible the development of the method presented herein, certain simplifying assumptions were necessary. It was assumed in the analysis that the boundary-layer velocity profile was linear for all Mach numbers and all rates of heat transfer. However, it has been shown in reference 5 that the laminar boundary layer does approach the assumed profile as the Mach number is increased. The effect of cooling at low Mach numbers (subsonic) is to decrease the linearity of the velocity profile, but as the Mach number is increased this latter effect becomes less marked.

It was assumed in the analysis that radiation could be neglected because in relation to the convective heat transfer, at least at the lower values of surface-temperature parameter, the radiant heat transfer would be small. However, if the aircraft, which is to be cooled, is designed with a thick outer surface of low thermal conductivity, the surface-temperature parameter will not be small, and, at higher Mach numbers, radiation must be considered.

It was assumed that the effects of heat conduction in the surface and its supporting structure could be neglected. In steady flight, the internal structure of a body would be at the same temperature as the surface; therefore, if a system were designed to give a constant skin temperature, there would be no internal heat conduction. It was assumed that Prandtl's number in unity in order to prevent the analysis from becoming excessively complicated. Although it is known that this assumption reduces the accuracy of the results, its effect is believed to be small. It was also assumed that the ratio of specific heats was constant with changes in temperature. The effect of this assumption on the results of the analysis over the range of Mach numbers considered is negligible.

CONCLUSIONS

The following conclusions can be drawn from the results obtained from the application of the method developed herein:

1. The convective cooling requirements for a body of revolution in steady supersonic flight at medium altitudes (40,000 to 120,000 ft.) with a completely laminar boundary layer are small for the range of Mach numbers considered (1.2 to 3.0) but increase rapidly with increasing Mach number.

2. For thin, fair bodies of revolution, the body length for a completely laminar boundary layer is of the order of 50 feet for moderate flight Mach number (3.0) and medium altitudes (100,000 ft.).

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National Advisory Committee for Aeronautics,
Moffett Field, Calif., *October 14, 1946.*

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