REPORT 971

THEORETICAL STABILITY DERIVATIVES OF THIN SWEPTBACK WINGS TAPERED TO A POINT WITH SWEPTBACK OR SWEPTFORWARD TRAILING EDGES FOR A LIMITED RANGE OF SUPERSONIC SPEEDS

By FRANK S. MALVESTUTO, Jr. and KENNETH MARGOLIS

SUMMARY

The stability derivatives valid for a limited range of supersonic speeds are presented for a series of sweptback wings tapered to a point with sweptback or sweptforward trailing edges. These wings were derived by modifying the trailing edge of a basic triangular wing so that it coincided with lines drawn from the wing tips to the wing axis of symmetry. The stability derivatives were formulated by using the pressure distributions previously obtained for the basic triangular wing for angle of attack, constant vertical acceleration, sideslip, pitching, rolling, and yawing. Explicit expressions are given for the stability derivatives with respect to principal body axes, and conversion formulas are provided for the transformation to stability axes. The results are limited to Mach numbers for which the wing is contained within the Mach cones springing from the vertex and from the trailing edge of the center section of the wing.

INTRODUCTION

Methods based upon linearized potential flow have been developed in references 1 to 5 for determining the pressure distributions for angle of attack and sideslipping, pitching, and rolling motions of a triangular wing of small thickness traveling at supersonic speeds. The results of these investigations are valid for a range of Mach number for which the Mach cone springing from the apex of the wing may be behind or ahead of the leading edge of the wing. In reference 6 attention is given only to triangular wings contained within the Mach cone springing from the wing apex. Methods are obtained therein for determining the rolling moment due to yawing and the several side-force and yawing-moment derivatives, together with a collection of all the known stability derivatives for triangular wings at supersonic speeds. As pointed out in these previous investigations, if the trailing edge of the triangular wing is modified so as to coincide with any line which is inclined at an angle always greater than the Mach angle (fig. 1), a series of sweptback wings with sweptback or sweptforward trailing edges will be developed which will have the same pressure distribution over their surfaces as that determined for the basic triangular wing. This phenomenon is based on the well-known fact that, in linearized supersonic flow, disturbances cannot propagate any farther forward than the Mach cone from the origin of disturbances.

The object of the present report is to determine the stability derivatives at supersonic speeds for this limited series of sweptback wings with pointed tips by using the pressure distributions previously determined for the basic triangular wing. Explicit expressions are presented for these stability derivatives with respect to the principal body axes, and conversion formulas are provided for the transformation to stability axes.

The results are restricted to wings that are contained within the Mach cones springing from the apex and the trailing edge of the center section of the wing.

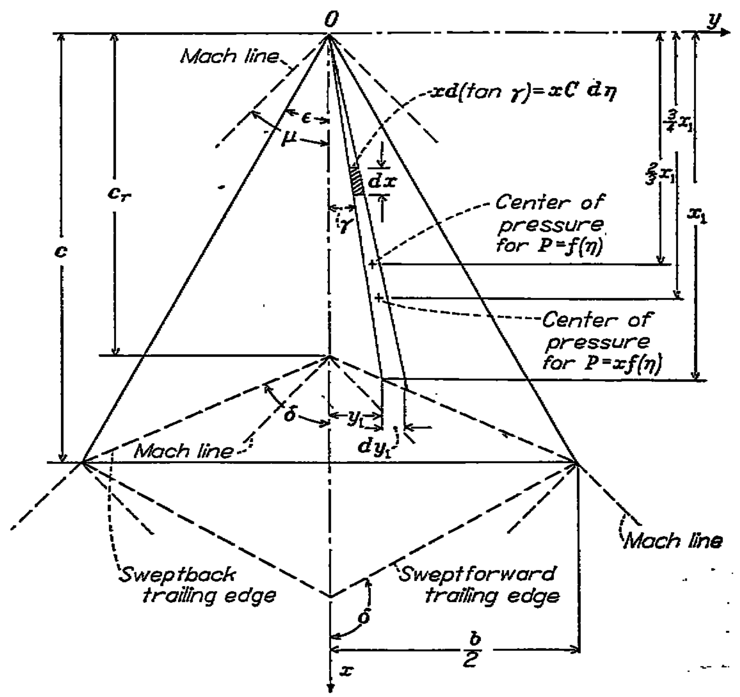


FIGURE 1.—Sweptback wing tapered to a point with sweptback or sweptforward trailing edges. The elemental triangle and associated data are shown with respect to wing with sweptback trailing edge. Note that trailing edge is alwaye inclined at an angle greater than the Mach angle ($|N|\leq BC$).

SYMBOLS

x, y, z rectangular coordinates (figs. 1 and 2)

u, v, w incremental flight velocities along x-, y-, and z-axes, respectively (fig. 3)

p, q, r angular velocities about x-, y-, and z-axes, respectively (fig. 3)

V flight speed

M stream Mach number (V/Speed of sound)

μ Mach angle $\left(\sin^{-1}\frac{1}{M}\right)$

B cotangent of Mach angle $(\sqrt{M^2-1})$

α angle of attack (w/V)

β angle of sideslip (v/V)

ϵ semivertex angle of triangle (fig. 1)

P local lifting pressure (pressure difference between upper and lower surface of airfoil)

ρ density of fluid

b wing span

c root chord of basic triangular wing (fig. 1)

c_r root chord of sweptback wing (fig. 1)

$\bar{c}$ mean aerodynamic chord

$$\left(\bar{c}=\frac{4}{bc_r}\int_0^{b/2}(\text{Local chord})^2\,dy=\frac{2}{3}c(1-N)\right)$$

A aspect ratio $\left(A=\frac{b^2}{S}=\frac{2b}{c(1-N)}\right)$

N ratio of slope of leading edge to slope of trailing edge $\left(N=\frac{c-c_r}{c}=\frac{\tan\epsilon}{\tan\delta}=1-\frac{4\cot\Lambda}{A}\right)$

S wing area $\left(S=\frac{1}{2}bc_r=\frac{1}{2}bc(1-N)=\frac{3}{4}b\bar{c}\right)$

C leading-edge slope $\left(C=\tan\epsilon=\frac{b}{2c}\right)$

Λ sweepback angle of leading edge $(90°-\epsilon)$

δ angle of trailing-edge slope (fig. 1)

$\gamma=\tan^{-1}\frac{y}{x}$ (fig. 1)

$\eta=\frac{y}{x\tan\epsilon}=\frac{y}{xC}$ (fig. 1)

x_{cg} distance of center of gravity forward of $\left(\frac{2}{3}c,0,0\right)$ position

$\frac{x_s}{\bar{c}}$ static margin $\left(\frac{x_s}{\bar{c}}=\frac{C_{m_\alpha}}{C_{L_\alpha}}\right)$

$k=\sqrt{1-B^2C^2}$

$E'(BC)$ complete elliptic integral of the second kind with modulus k $\left(\int_0^{\pi/2}\sqrt{1-k^2\sin^2 z}\,dz\right)$

$F'(BC)$ complete elliptic integral of the first kind with modulus k $\left(\int_0^{\pi/2}\frac{dz}{\sqrt{1-k^2\sin^2 z}}\right)$

$E''(BC)=\frac{1}{E'(BC)}$

$Q(BC)=\frac{[E''(BC)]^2}{\sqrt{1-B^2C^2}}$

$G(BC)=\frac{1-B^2C^2}{(1-2B^2C^2)E'(BC)+B^2C^2F'(BC)}$

$I(BC)=\frac{2(1-B^2C^2)}{(2-B^2C^2)E'(BC)-B^2C^2F'(BC)}$

$J(BC)=E''(BC)\,I(BC)\sqrt{1-B^2C^2}$

L' rolling moment

L normal force (approx. lift)

M' pitching moment

N' yawing moment

Y lateral force

C_L lift coefficient $\left(\frac{L}{\frac{1}{2}\rho V^2S}\right)$

C_l rolling-moment coefficient $\left(\frac{L'}{\frac{1}{2}\rho V^2Sb}\right)$

C_m pitching-moment coefficient $\left(\frac{M'}{\frac{1}{2}\rho V^2S\bar{c}}\right)$

C_n yawing-moment coefficient $\left(\frac{N'}{\frac{1}{2}\rho V^2Sb}\right)$

C_Y lateral-force coefficient $\left(\frac{Y}{\frac{1}{2}\rho V^2S}\right)$

C_{D_0} profile-drag coefficient $\left(\frac{\text{Profile drag}}{\frac{1}{2}\rho V^2S}\right)$

When α, $\dot{\alpha}$, q, p, β, and r are used as subscripts, a nondimensional derivative is indicated, and this derivative is the slope of the variation through zero. For example,

$$C_{m_{\dot{\alpha}}}=\left[\frac{\partial C_m}{\partial\left(\frac{\dot{\alpha}\bar{c}}{2V}\right)}\right]_{\dot{\alpha}\to 0}$$

$$C_{m_q}=\left[\frac{\partial C_m}{\partial\left(\frac{q\bar{c}}{2V}\right)}\right]_{q\to 0}$$

$$C_{l_p}=\left[\frac{\partial C_l}{\partial\left(\frac{pb}{2V}\right)}\right]_{p\to 0}$$

$$C_{l_\beta}=\left[\frac{\partial C_l}{\partial\beta}\right]_{\beta\to 0}$$

and

$$C_{l_r}=\left[\frac{\partial C_l}{\partial\left(\frac{rb}{2V}\right)}\right]_{r\to 0}$$

A dot above a symbol denotes differentiation with respect to time.

All angles are measured in radians.

Unprimed stability derivatives refer to principal body axes; primed stability derivatives refer to stability axes.

ANALYSIS

The stability derivatives of a triangular wing of zero thickness at small angles of attack in a supersonic air stream have been determined theoretically in the investigations of references 1 to 6. These derivatives, with the exception of those which depend on skin friction, may be separated into two classes—the derivatives which depend upon the distribution of pressure over the wing and the derivatives which depend upon the suction force along the leading edge of the wing. Although the edge-suction derivatives have been summarized in this report, the pressure coefficients needed to determine these derivatives are not presented. The local lifting-pressure coefficients used to obtain the derivatives which are dependent on the pressure distribution over the triangular wing contained within the Mach cone springing from the apex are presented in table I. These lifting-pressure coefficients and obviously the lifting pressures $\left(\text{local pressure coefficient times } \frac{1}{2}\rho V^2\right)$ are of the general form $x^n f(\eta)$ where x is the x-component of the distance from the origin of the axes to a particular point on the wing and η is the ratio of the slope of a ray from the vertex of the wing through the point to the slope of the leading edge of the wing. (See fig. 1.) For the local lifting-pressure coefficients of the stability derivatives listed in table I, the exponent n of the distance x is either equal to 0 or to 1. For $n=0$, the pressure is constant along any ray η=Constant from the vertex; this case is termed "conical flow." For $n=1$, the pressure increases linearly along each such ray, and the flow may be termed "quasi-conical."

The particular form $x^n f(\eta)$ noted for the distribution of the lifting pressures suggests the "triangular" integration procedure for determining the forces and moments. Thus, the wing is considered as composed of an infinite number of elemental triangular areas (fig. 1). The lift and first moment of the lift are then determined for each elemental triangular area and the results summed up by integration to give the force and moment derivatives for the complete wing. Figures 2, 3, and 4 indicate the position and positive direction of the axes used in the analysis together with the positive direction of the velocities, forces, and moments relative to these axes.

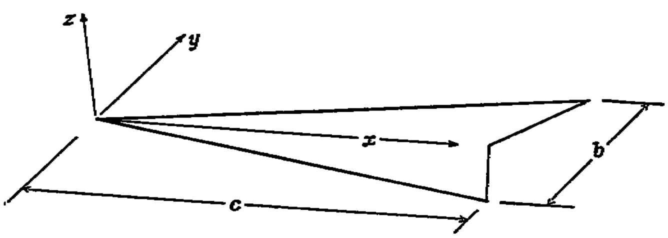


FIGURE 2.—Axes and notation used in analysis.

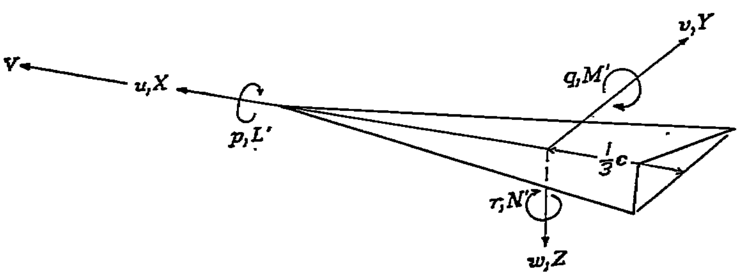


FIGURE 3.—Velocities, forces, and moments relative to principal axes with origin at $\frac{2}{3}$ c.

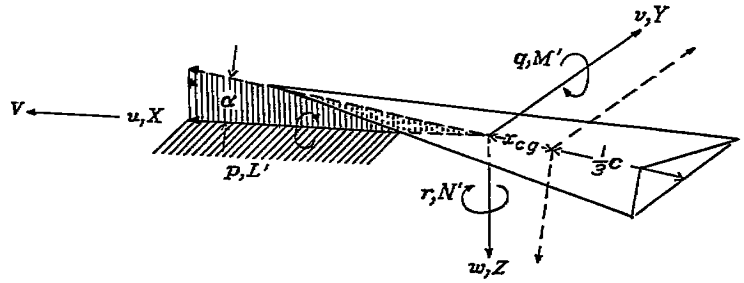


FIGURE 4.—Velocities, forces, and moments relative to stability axes with origin at $\frac{2}{3}c-x_{c_g}$. Principal axes of figure 3 dotted in for comparison.

Conical flows: Derivatives C_{L_α}, C_{m_α}, and C_{l_β}.—Table I shows that the local lifting-pressure coefficients of the derivatives C_{L_α}, C_{m_α}, and C_{l_β} depend solely upon η and therefore represent conical flows. The lift of an elemental triangle (see fig. 1) is

$$dL=\frac{1}{2}\,x_1\,dy_1\,P(\eta) \tag{1}$$

where $P(\eta)$ is the local lifting pressure for any of these three stability derivatives. Since x_1 and y_1 can be written as functions of η, that is,

$$x_1=\frac{3}{2}\,\bar{c}\,\frac{1}{1-N\eta}$$

$$dy_1=\frac{3}{2}\,\bar{c}C\,\frac{d\eta}{1-N\eta}$$

equation (1) becomes

$$dL=\frac{9}{8}\,\bar{c}^2C\,\frac{P(\eta)\;d\eta}{(1-N\eta)^2} \tag{2}$$

For the moment of lift of an elemental area (reference 1), consider the fact that for a conical-flow condition the resultant lift of a triangle acts at a point ⅔ the chord of the triangle from the vertex, or for this case $\frac{2}{3}x_1$ of the elemental triangle. Hence, the moment of the elemental lift about the y-axis (origin at the vertex of the triangle) is

$$dM'=-\frac{2}{3}x_1\,dL$$

and by the use of equation (2) this moment becomes

$$dM' = -\frac{9}{8}\bar{c}^3 C \frac{P(\eta)\, d\eta}{(1-N\eta)^3} \tag{3}$$

For moments about the x principal axis (rolling moments)

$$dL' = -y\, dL$$

where y is the y-coordinate of the position of the center of pressure for $P=f(\eta)$. Figure 1 indicates that for this condition

$$y = \frac{2}{3} C\eta x_1 = \bar{c} C \frac{\eta}{1-N\eta}$$

hence

$$dL' = -\frac{9}{8}\bar{c}^3 C^2 \frac{P(\eta)\eta\, d\eta}{(1-N\eta)^3} \tag{4}$$

Equations (2), (3), and (4) are the differential forms of the lift and the pitching and rolling moments of the wing when the pressure distribution is a function only of η. In order to obtain the lift and the pitching and rolling moments, the $P(\eta)$ functions $\left(\text{pressure coefficients multiplied by } \frac{1}{2}\rho V^2\right)$ for these motions obtained from table I are substituted into equations (2), (3), and (4) and these equations are integrated with respect to η over the entire wing. Because the wing is symmetrical with respect to the x principal axis the integration can be performed between the limits 0 to 1 and the results multiplied by 2. The nondimensional integral forms of the stability derivatives C_{L_α}, C_{m_α}, and C_{l_β} have been derived, converted to a different center-of-gravity position, and listed in table I. The new center of gravity is located a distance $\frac{2}{3}c$ from the vertex, and the shift affects only the derivative C_{m_α}. Integration of the integrals involved in these derivatives will produce functions of N which give the variation of the stability derivatives with N, the ratio of the slope of the leading edge to the slope of the trailing edge of the wing. The derivative C_{L_α} has previously been determined in reference 7 for the type of wing considered herein.

Quasi-conical flows: Derivatives $C_{L_{\dot\alpha}}$, C_{L_q}, $C_{m_{\dot\alpha}}$, C_{m_q}, C_{l_p}, and C_{l_r}.—Table I indicates that the pressure coefficients for the derivatives $C_{L_{\dot\alpha}}$, C_{L_q}, $C_{m_{\dot\alpha}}$, C_{m_q}, C_{l_p}, and C_{l_r} are of the form $xf(\eta)$ where x is the x-component of the distance from the vertex of the wing to the point in question. For this case the lift of an elemental triangle is given by

$$dL = \int_{x=0}^{x=x_1} xf(\eta) xC\, d\eta\, dx$$

which can be rewritten as

$$dL = f(\eta) C\, d\eta \int_{x=0}^{x=x_1} x^2\, dx = f(\eta) C \frac{x_1^3}{3} d\eta$$

hence

$$dL = \frac{9}{8}\bar{c}^3 C \frac{f(\eta)\, d\eta}{(1-N\eta)^3} \tag{5}$$

Reference 4 indicates that when the pressure is of the form $xf(\eta)$ the resultant lift acts at the $\frac{3}{4}$-chord point of the triangle which for this case is equal to $\frac{3}{4}x_1$. The moment about the y-axis (origin at the vertex of triangle) is (see equation (5))

$$dM' = -\frac{3}{4} x_1\, dL$$

$$= -\frac{81}{64}\bar{c}^4 C \frac{f(\eta)\, d\eta}{(1-N\eta)^4} \tag{6}$$

In a manner similar to the development of equation (4), the following rolling-moment equation results when $P=xf(\eta)$:

$$dL' = -y\, dL$$

$$= -\frac{81}{64}\bar{c}^4 C^2 \frac{f(\eta)\eta\, d\eta}{(1-N\eta)^4} \tag{7}$$

Equations (5), (6), and (7) are the differential forms of the lift and pitching and rolling moments for cases where the pressure distributions are of the form $xf(\eta)$, that is, of a quasi-conical type. Substitution of the appropriate function $f(\eta)$ for $C_{L_{\dot\alpha}}$ and C_{L_q} in equation (5), for $C_{m_{\dot\alpha}}$ and C_{m_q} in equation (6), and for C_{l_p} and C_{l_r} in equation (7) will give these derivatives as a function of N after the necessary operations are performed and the resulting equations are reduced to coefficient form. Table I presents the nondimensional integral form of these derivatives with the origin shifted from the vertex to a point $\frac{2}{3}c$ from the vertex.

Edge-suction derivatives C_{n_β}, C_{n_p}, C_{n_r}, C_{Y_β}, C_{Y_p}, and C_{Y_r}.—The yawing and side-force derivatives depend upon the suction force along the leading edge of the wing (references 3 and 6). This suction force arises as a consequence of the subsonic nature of the external flow field in the vicinity of the leading edge of the wing when the leading edge is swept behind the Mach cone springing from the apex of the wing. Changes in the sweep of the trailing edge and area of the wing brought about by varying N have no effect on the leading-edge suction forces for the class of sweptback wings considered herein. These wings, as stated previously, are contained within the Mach cones springing from the vertex and from the trailing edge of the center section of the wing. The values of the coefficients are modified, however, because of the difference in the reference wing area; that is, the wing area of the sweptback wing is equal to $(1-N)$ times the wing area of the basic triangular wing. The derivatives obtained in reference 6 have been accordingly modified and are presented in table II of this report where the quantity $(1-N)$ has been denoted by $F_{11}(N)$. The degree of applicability of these suction-force derivatives to actual full-scale wings is somewhat uncertain for the reasons pointed out in reference 6 for triangular wings.

RESULTS AND DISCUSSION

The preceding section set forth a method for determining the stability derivatives for a limited series of sweptback wings with pointed tips and sweptback or sweptforward trailing edges as a function of the trailing-edge-sweep parameter

$$N=\frac{\tan \epsilon}{\tan \delta}=1-\frac{4 \cot \Lambda}{A}$$

The procedure employed pressure coefficients previously determined for the basic triangular wing. Table II gives the values of the stability derivatives in the principal-axes system with origin at $\left(\frac{2}{3}c,0,0\right)$ as shown in figure 3 and also the conversion formulas for determination of the derivatives in the stability system of axes with origin at a distance x_{cg} ahead of the $\left(\frac{2}{3}c,0,0\right)$ point as shown in figure 4. These formulas giving the conversion of the stability derivatives from the principal-axes system to the stability-axes system were obtained by an extension of the transformation equations of reference 8 to take into consideration the shift in the origin of the stability axes of distance x_{cg} ahead of the origin of the principal axes. In the conversion formulas for the stability-axes system, terms whose magnitudes are extremely small compared with unity have been omitted. The quantities $E''(BC)$, $Q(BC)$, $G(BC)$, $I(BC)$, and $J(BC)$ are the elliptic integral factors of the stability derivatives that determine their variation with Mach number. These factors are shown graphically in figure 5. The $F(N)$ factors of each of the derivatives are functions of N which give the effect of trailing-edge sweep on the derivatives. Figure 6 presents the variation of the $F(N)$ factors with N from $N=-1$, which corresponds to the case where the Mach lines coincide with the leading and trailing edges of the wing (symmetrical diamond plan form), to $N=1$, which corresponds to the limiting idealized case for which the trailing edge coincides with the leading edge of the wing. For $N=0$, of course, the plan form of the wing corresponds to that of the basic triangular wing. Because of the extremely rapid variation of some of the $F(N)$ factors with N, the product $(1-N)^n F(N)$ was plotted in figure 6 for these cases instead of merely the functions. The formulas for the $F(N)$ factors are listed in the appendix together with the solution of the definite forms of the integrals that appear in the evaluation of each of the $F(N)$ factors. For very accurate evaluations of the stability derivatives it is suggested that the necessary $F(N)$ factors be calculated using the formulas for these factors listed in the appendix instead of using the curves of the $F(N)$ factors presented in figure 6.

Typical variations of the stability derivatives with trailing-edge-sweep parameter N and with Mach number M are presented in figures 7 and 8, respectively. Because of the localized infinities at $BC=1$ for the suction derivatives C_{Y_β}, C_{n_β}, and C_{n_r} in the principal-axes system, all the lateral derivatives in the stability-axes system (determined by a transformation from the principal axes to the stability axes) which contain these suction derivatives will also become locally infinite at $BC=1$. For this reason the variations of the illustrative lateral derivatives in figure 8 are given for a range of Mach number with an upper limit slightly less than the Mach number corresponding to $BC=1$. It may be noted that such a localized infinity is defined with reference to an infinitesimal angle of sideslip, and the average derivative for a small but finite sideslip is not extremely large.

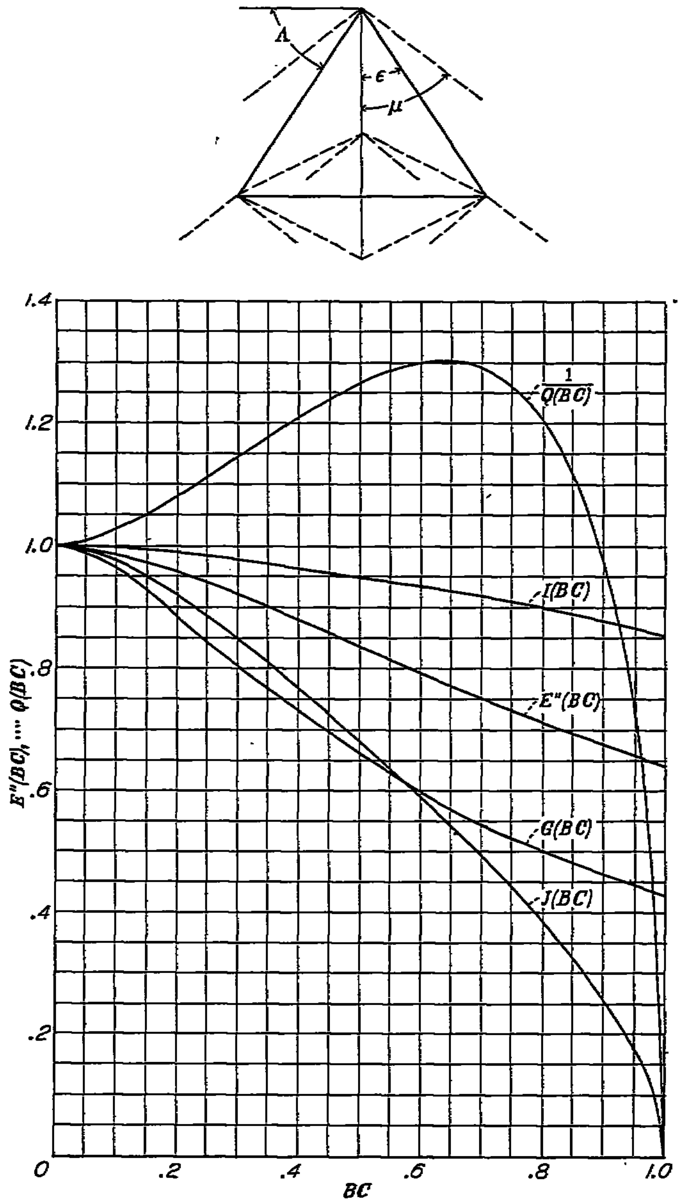


FIGURE 5.—Variation of the elliptic integral factors with BC.
$BC=\frac{\tan \epsilon}{\tan \mu}=\sqrt{M^2-1}\cot \Lambda$.

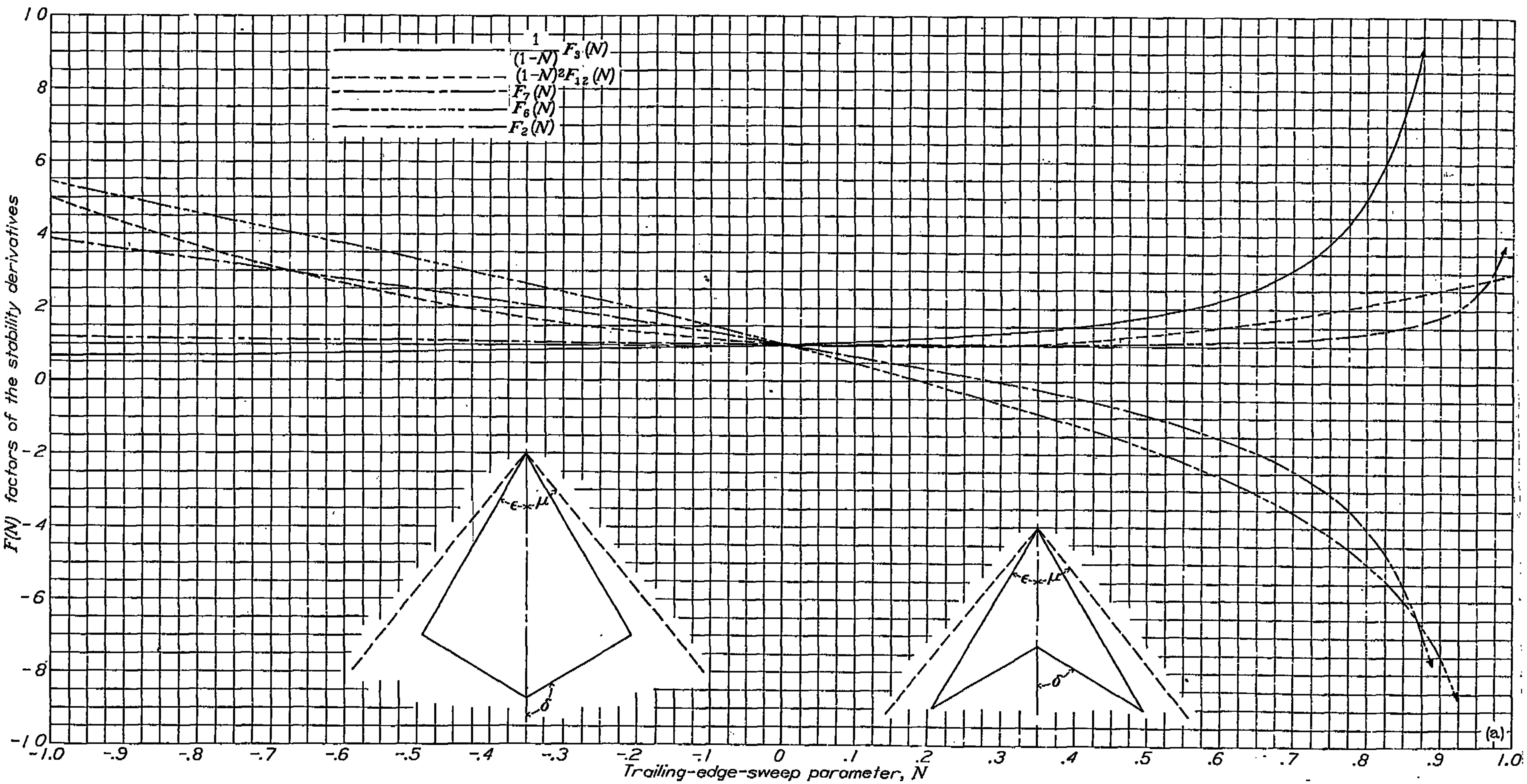


FIGURE 6.—The $F(N)$ factors of the stability derivatives that determine their variation with N. Valid only for Mach numbers for which $BC \geq |N| = \left|1 - \frac{4 \cot \Lambda}{A}\right|$.

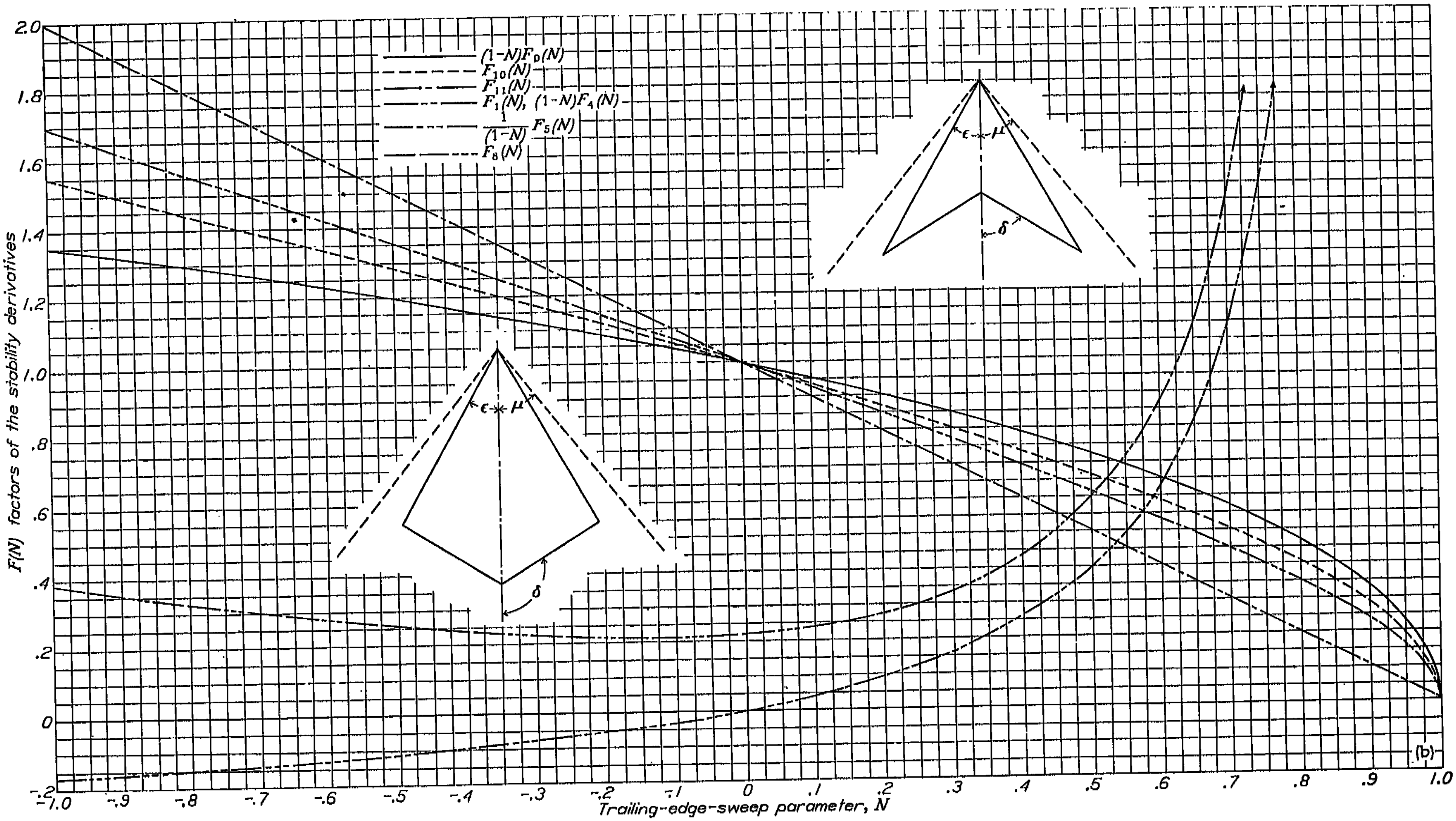


FIGURE 6.—Concluded.

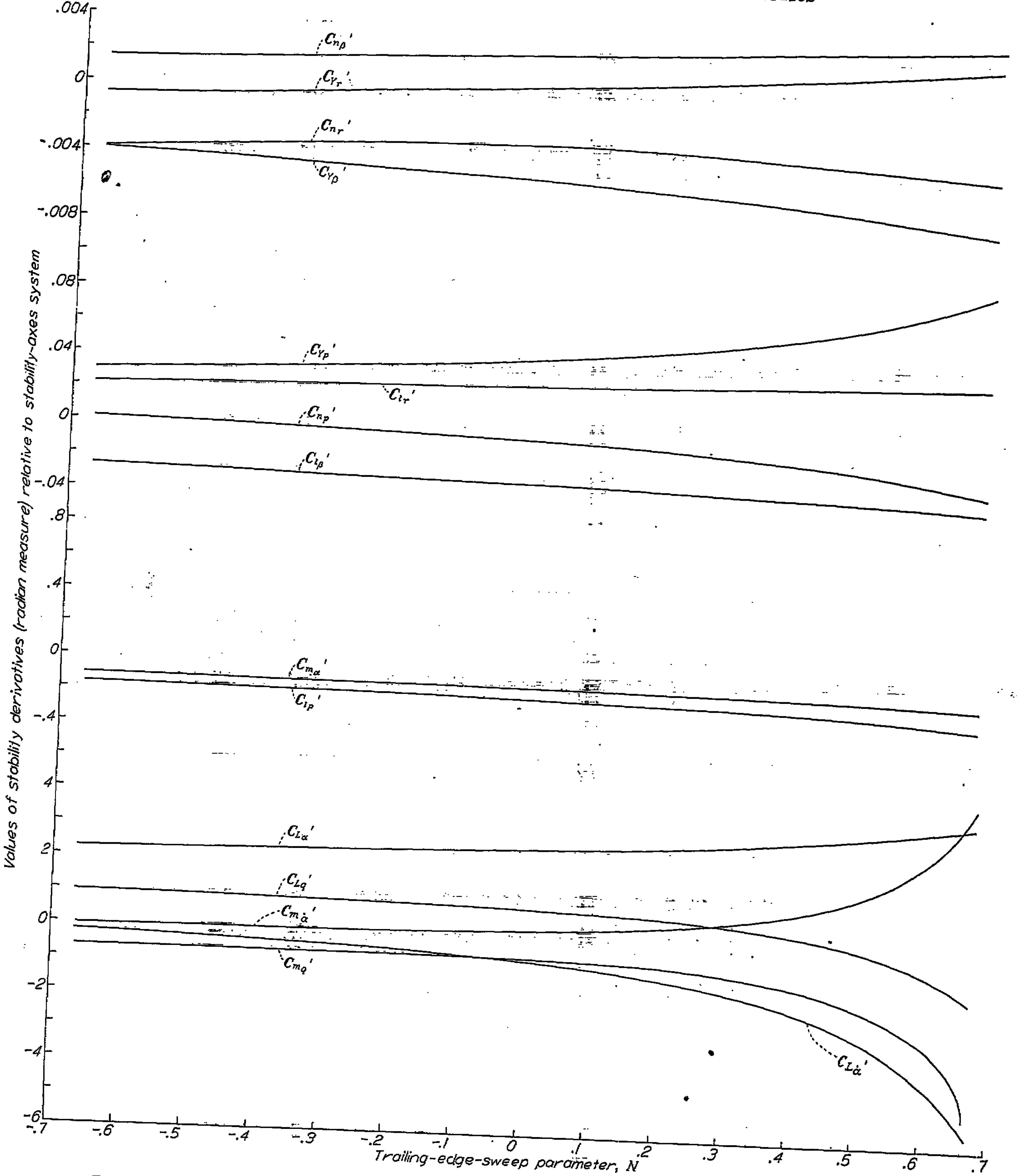
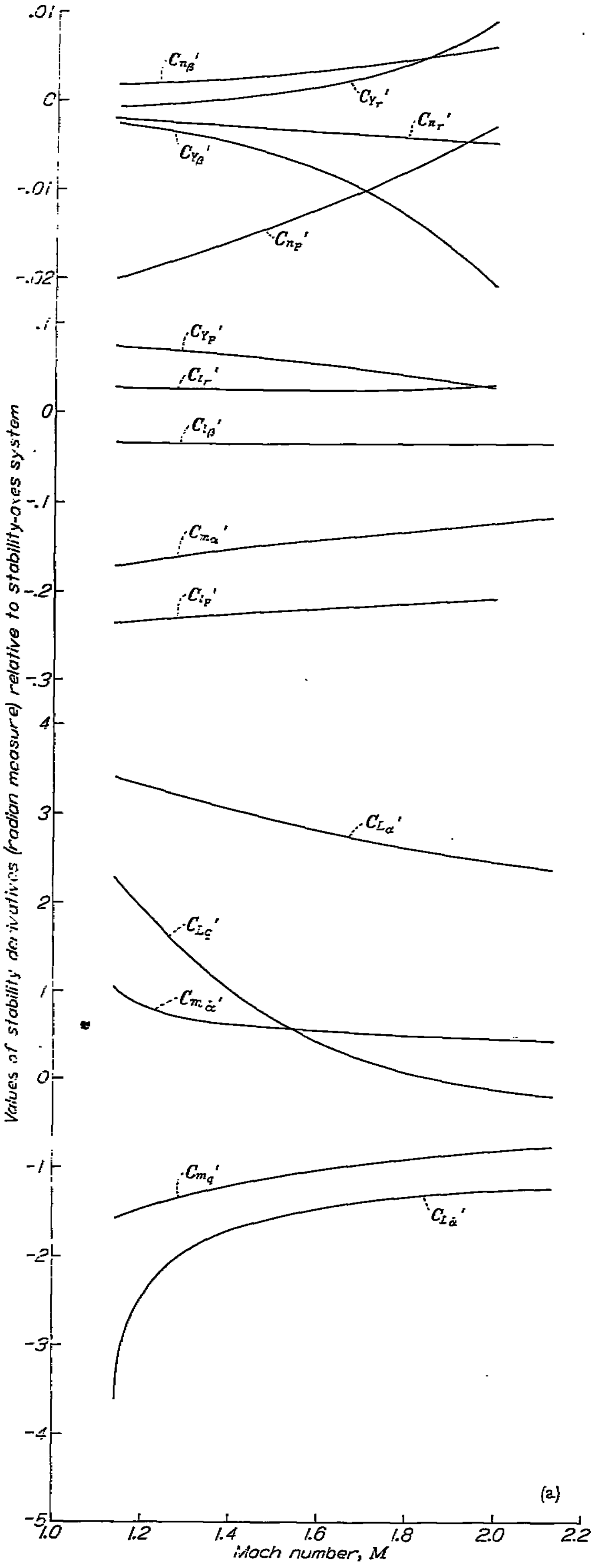
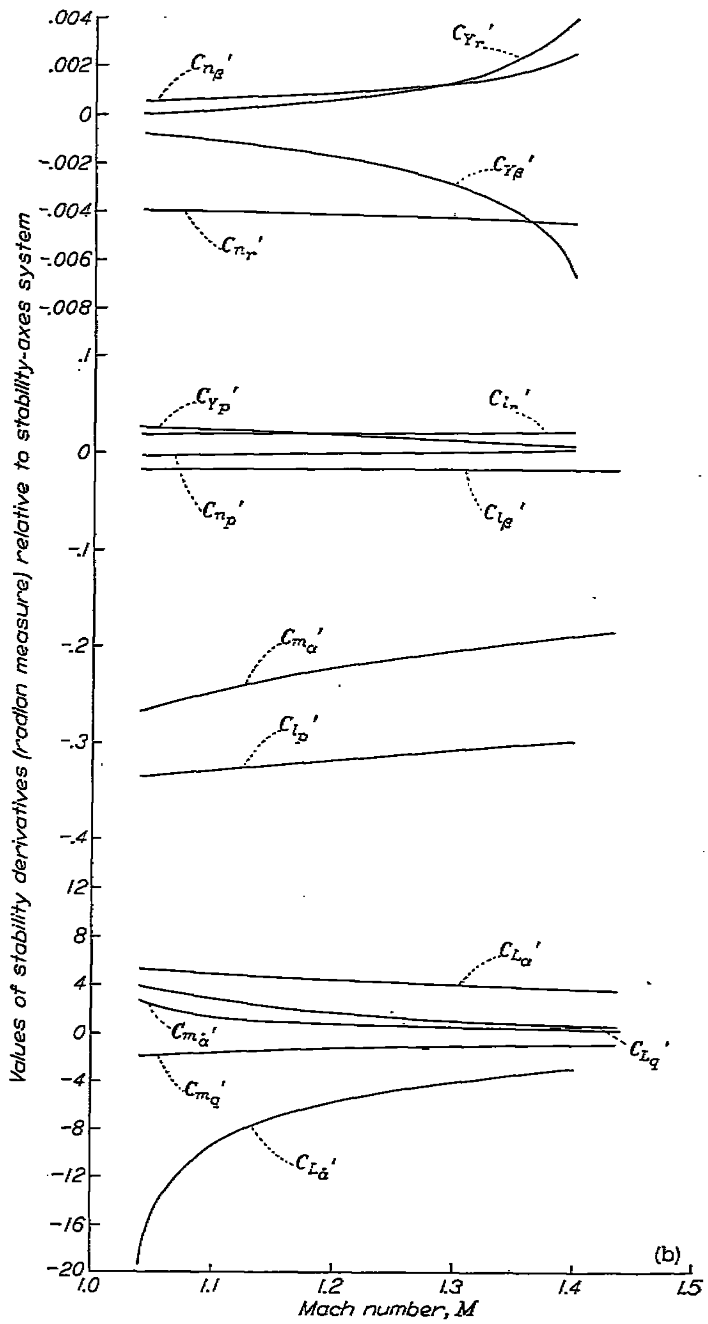


FIGURE 7.—The variation of the stability derivatives with N. $\Lambda=62°$; $M=1.6$; $\frac{x_a}{\bar{c}}=0.05$; $C_{D_0}=0.006$; $C_L=0.10$. $|N|=\left|1-\frac{4\cot\Lambda}{A}\right|\leq BC.$



(a) Λ=62°.

FIGURE 8.—The variation of the stability derivatives with Mach number; $A=3$; $\frac{x_s}{c}=0.05$; $C_{D_0}=0.006$; $C_L=0.10$. $BC \geq |N| = \left|1-\frac{4 \cot \Lambda}{A}\right|$.



(b) Λ=46°.

FIGURE 8.—Concluded.

In reference 9 the consideration of skin friction is shown to yield an appreciable damping moment. The skin-friction moment has been evaluated therein as

$$(N')_f = \int_0^c \int_{-xC}^{xC} C_{D_0} \frac{1}{2} \rho V_R^2 \left[\left(x-\frac{2}{3}c\right)\beta + y\right] dy\, dx \quad (8)$$

where V_R is the resultant velocity and

$$V_R^2 = (V-ry)^2 + r^2\left(x-\frac{2}{3}c\right)^2$$

and β is the local sideslip angle and equals $-\frac{r\left(x-\frac{2}{3}c\right)}{V}$ to the first order in r. Equation (8) also applies to the wings considered herein provided the necessary changes are made in the limits of integration. Substituting the proper limits for the sweptback wing in the integrals of equation (8) and performing the necessary operation yields the following nondimensional form of the skin-friction couple that is a part of the derivative C_{n_r}:

$$(C_{n_r})_f=2\frac{\partial}{\partial\left(\frac{rb}{2V}\right)}\left\{\frac{1}{\frac{1}{2}\rho V^2Sb}\int_0^{b/2}\int_{y/C}^{y\cot\delta+c_r}C_{D_0}\frac{1}{2}\rho V_R^2\right.$$

$$\left.\left[\left(x-\frac{2}{3}c\right)\beta+y\right]dx\,dy\right\} \tag{9}$$

where, in terms of N and A, the upper limit $y \cot \delta+c_r$ of the inner integral is equal to $\frac{2}{A}\frac{N(2y-b)+b}{1-N}$ and the lower limit y/C is equal to $\frac{4y}{A(1-N)}$. The evaluation of equation (9) gives, to the first order in r, in the body-axes system,

$$(C_{n_r})_f=-C_{D_0}\left[\frac{1}{6}+\frac{4}{9A^2}\frac{1-N+3N^2}{(1-N)^2}\right]$$

where the function $\frac{1-N+3N^2}{(1-N)^2}$ designated by $F_{12}(N)$ is plotted against N in figure 6.

In the formulation of the derivative C_{l_r} the associated local lifting-pressure coefficient listed in table I and originally determined in reference 6 does not include the effect of the spanwise variations in local Mach number caused by yawing (although the variation in forward speed is taken into account). In the text of reference 6 based on the results of calculations on an infinite-aspect-ratio rectangular wing using the Ackeret theory, it is indicated that the spanwise variation of the compressibility effect to the first order in r will produce first-order changes in the local lifting pressures and hence in the rolling moment due to yawing. The value of C_{l_r} presented herein and obtained under the approximation of zero spanwise variation of the local Mach number is therefore subject to doubt and should be considered only as a rough indication of the true value.

The stability derivatives of this report are valid only above a certain minimum Mach number given by $BC \geqq |N| = \left|\frac{\tan \epsilon}{\tan \delta}\right|$ which is the condition that the trailing edge be swept less than the Mach lines. An additional limitation is that the Mach number must be sufficiently above unity for the linearized theory to apply. In addition to these limitations on the range of validity of the derivatives, the limitations for the basic triangular wing discussed in reference 6 also apply to the sweptback wing of this report.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *September 23, 1948.*

APPENDIX

EVALUATION OF THE $F(N)$ FACTORS OF THE STABILITY DERIVATIVES

The determination of the $F(N)$ factors necessitates the evaluation of the definite integrals given in table I. The integrals of table I are or can be formed from the following basic integrals:

$$I_1=\int_0^1 \frac{d\eta}{(1-N\eta)^2\sqrt{1-\eta^2}}=\left[\frac{1}{(N^2-1)\sqrt{1-N^2}}\sin^{-1}\frac{N-\eta}{1-N\eta}+\frac{N\sqrt{1-\eta^2}}{(N^2-1)(1-N\eta)}\right]_0^1=\frac{\frac{\pi}{2}+\sin^{-1}N+N\sqrt{1-N^2}}{(1-N^2)^{3/2}}$$

$$I_2=\int_0^1 \frac{d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}=\left[\frac{-(N^2+2)}{2(N^2-1)^2\sqrt{1-N^2}}\sin^{-1}\frac{N-\eta}{1-N\eta}+\frac{(N^3+3N^2\eta-4N)\sqrt{1-\eta^2}}{2(N^2-1)^2(1-N\eta)^2}\right]_0^1$$

$$=\frac{(2+N^2)\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(4-N^2)\sqrt{1-N^2}}{2(1-N^2)^{5/2}}$$

$$I_3=\int_0^1\frac{d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}=\left[\frac{3N^2+2}{2(N^2-1)^3\sqrt{1-N^2}}\sin^{-1}\frac{N-\eta}{1-N\eta}+\frac{[\eta^2(4N^5+11N^3)-\eta(3N^4+27N^2)+2N^5-5N^3+18N]\sqrt{1-\eta^2}}{6(N^2-1)^3(1-N\eta)^3}\right]_0^1$$

$$=\frac{3(3N^2+2)\left(\frac{\pi}{2}+\sin^{-1}N\right)+(2N^5-5N^3+18N)\sqrt{1-N^2}}{6(1-N^2)^3\sqrt{1-N^2}}$$

$$I_4=\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}=\left[\frac{-(2N^2+1)}{2(N^2-1)^2\sqrt{1-N^2}}\sin^{-1}\frac{N-\eta}{1-N\eta}+\frac{[\eta(4N^4-N^2)-3N^3]\sqrt{1-\eta^2}}{2N^2(N^2-1)^2(1-N\eta)^2}\right]_0^1=\frac{(2N^2+1)\left(\frac{\pi}{2}+\sin^{-1}N\right)+3N\sqrt{1-N^2}}{2(1-N^2)^2\sqrt{1-N^2}}$$

$$I_5=\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}=\left[\frac{4N^2+1}{2(N^2-1)^3\sqrt{1-N^2}}\sin^{-1}\frac{N-\eta}{1-N\eta}+\frac{[\eta^2(6N^5+10N^3-N)-\eta(6N^4+27N^2-3)+2N^3+13N]\sqrt{1-\eta^2}}{6(N^2-1)^3(1-N\eta)^3}\right]_0^1$$

$$=\frac{3(4N^2+1)\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(2N^2+13)\sqrt{1-N^2}}{6(1-N^2)^3\sqrt{1-N^2}}$$

The $F(N)$ factors are formulated by referring to tables I and II and by using the evaluation of the five basic integrals and are as follows:

$$F_1(N)=\frac{2}{\pi}(1-N)^2I_1=\frac{2(1-N)^{1/2}}{\pi(1+N)^{3/2}}\left(\frac{\pi}{2}+\sin^{-1}N+N\sqrt{1-N^2}\right)$$

$$F_2(N)=\frac{2}{\pi}(1-N)^2I_2=\frac{(2+N^2)\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(4-N^2)\sqrt{1-N^2}}{\pi(1+N)^{5/2}(1-N)^{1/2}}$$

$$F_3(N)=\frac{4}{3\pi}(1-N)^2(2I_2-I_4)=\frac{2}{3\pi(1+N)^{5/2}(1-N)^{1/2}}\left[3\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(5-2N^2)\sqrt{1-N^2}\right]$$

$$F_4(N)=\frac{2}{\pi}(1-N)I_1=\frac{2}{\pi(1+N)^{3/2}(1-N)^{1/2}}\left(\frac{\pi}{2}+\sin^{-1}N+N\sqrt{1-N^2}\right)$$

$$F_5(N)=\frac{2}{\pi}(1-N)[I_1-(1-N)I_2]=\frac{N}{\pi(1+N)^{5/2}(1-N)^{1/2}}\left[(2-N)\left(\frac{\pi}{2}+\sin^{-1}N\right)+(N^2+2N-2)\sqrt{1-N^2}\right]$$

$$F_6(N)=\frac{16}{\pi}(1-N)^2\left[\frac{9}{4}(I_3-I_5)-\frac{2}{1-N}(I_2-I_4)\right]=\frac{2}{\pi(1+N)^{5/2}(1-N)^{1/2}}\left[(1-8N)\left(\frac{\pi}{2}+\sin^{-1}N\right)-N(6N^2+8N-7)\sqrt{1-N^2}\right]$$

$$F_7(N)=\frac{32}{3\pi}(1-N)^2\left[\frac{9}{8}(2I_3-I_5)-\frac{1}{1-N}(2I_2-I_4)\right]=\frac{2}{3\pi(1+N)^{7/2}(1-N)^{3/2}}\left[3(6N^2-8N+1)\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(12N^4+16N^3-20N^2-40N+29)\sqrt{1-N^2}\right]$$

$$F_8(N)=(1-N)^2\left[\frac{81}{64}I_3-\frac{9}{4(1-N)}I_2+\frac{9}{8}\left(\frac{1}{1-N}\right)^2I_1\right]$$

$$=\frac{9(2-16N^3+27N^2)\left(\frac{\pi}{2}+\sin^{-1}N\right)+9N\sqrt{1-N^2}(6N^4+16N^3+17N^2-32N+6)}{128(1-N^2)^{3/2}(1+N)^2}$$

$$F_9(N)=\frac{4}{\pi}(1-N)^2I_4=\frac{2}{\pi(1+N)^{5/2}(1-N)^{1/2}}\left[(2N^2+1)\left(\frac{\pi}{2}+\sin^{-1}N\right)+3N\sqrt{1-N^2}\right]$$

$$F_{10}(N)=\frac{4}{\pi}(1-N)^4I_5=\frac{2(1-N)^{1/2}}{3\pi(1+N)^{7/2}}\left[3(4N^2+1)\left(\frac{\pi}{2}+\sin^{-1}N\right)+N(2N^2+13)\sqrt{1-N^2}\right]$$

$$F_{11}(N)=1-N$$

$$F_{12}(N)=\frac{3N^2-N+1}{(1-N)^2}$$

The factor $F_{11}(N)$ is merely the ratio of area of the sweptback wing to basic triangular wing. The factor $F_{12}(N)$ is associated with the skin-friction contribution to the derivative C_{n_r}. (See section entitled "Results and Discussion.")

The variation of each of the $F(N)$ factors with N from $N=-1$ to $N=1$ is presented in figure 6.

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TABLE I.—PRESSURE COEFFICIENTS AND INTEGRAL FORMS OF THE STABILITY DERIVATIVES AT SUPERSONIC SPEEDS OF A SERIES OF SWEPTBACK WINGS TAPERED TO A POINT WITH SWEPTBACK OR SWEPTFORWARD TRAILING EDGES ($|N| \leq BC$)

Stability derivative	Reference for pressure coefficient	Pressure coefficient, $\frac{P}{\frac{1}{2}\rho V^2}$ (origin of axes at apex of wing)	Integral form of stability derivative $\left(\text{origin of principal axes at } \frac{2}{3}c \text{ from vertex}\right)$
C_{L_α}	2, 3	$\frac{4C\alpha}{E'(BC)\sqrt{1-\eta^2}}$	$AE''(BC)(1-N)^2\int_0^1\frac{d\eta}{(1-N\eta)^2\sqrt{1-\eta^2}}$
$C_{L_{\dot\alpha}}$	6	$\frac{4\dot\alpha CM^2x}{VB^2}\left[\left(\frac{2-\eta^2}{\sqrt{1-\eta^2}}\right)G(BC)-\left(\frac{1}{\sqrt{1-\eta^2}}+\frac{1-\eta^2}{M^2\sqrt{1-\eta^2}}\right)E''(BC)\right]$	$-2A\frac{M^2}{B^2}(1-N)^2\left[-G(BC)\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}+E''(BC)\left(\int_0^1\frac{d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}+\frac{1}{M^2}\int_0^1\frac{(1-\eta^2)d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}\right)\right]$
C_{L_q}	4	$\frac{4qCG(BC)}{V}\left[\frac{x(2-\eta^2)}{\sqrt{1-\eta^2}}\right]$	$2AG(BC)(1-N)^2\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}-2AE''(BC)(1-N)\int_0^1\frac{d\eta}{(1-N\eta)^2\sqrt{1-\eta^2}}$
C_{m_α}	2, 3	$\frac{4C\alpha}{E'(BC)\sqrt{1-\eta^2}}$	$AE''(BC)\left[(1-N)\int_0^1\frac{d\eta}{(1-N\eta)^2\sqrt{1-\eta^2}}-(1-N)^2\int_0^1\frac{d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}\right]$
$C_{m_{\dot\alpha}}$	6	$\frac{4\dot\alpha CM^2x}{VB^2}\left[\left(\frac{2-\eta^2}{\sqrt{1-\eta^2}}\right)G(BC)-\left(\frac{1}{\sqrt{1-\eta^2}}+\frac{1-\eta^2}{M^2\sqrt{1-\eta^2}}\right)E''(BC)\right]$	$\frac{AM^2G(BC)(1-N)}{B^2}\left[2\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}-\frac{9(1-N)}{4}\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}\right]-\frac{A(M^2+1)E''(BC)(1-N)}{B^2}\left[2\int_0^1\frac{d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}-\frac{9(1-N)}{4}\int_0^1\frac{d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}\right]+\frac{A(1-N)E''(BC)}{B^2}\left[2\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}-\frac{9(1-N)}{4}\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}\right]$
C_{m_q}	4	$\frac{4qCG(BC)}{V}\left[\frac{x(2-\eta^2)}{\sqrt{1-\eta^2}}\right]$	$-2AG(BC)\left[\frac{9}{8}(1-N)^2\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}-(1-N)\int_0^1\frac{(2-\eta^2)d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}\right]-2AE''(BC)\left[\int_0^1\frac{d\eta}{(1-N\eta)^2\sqrt{1-\eta^2}}-(1-N)\int_0^1\frac{d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}\right]$
C_{l_β}	5, 6	$\frac{4\alpha\beta\eta}{E'(BC)\sqrt{1-\eta^2}}$	$-\frac{4}{3}\alpha E''(BC)(1-N)^2\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}$
C_{l_p}	4	$\frac{2pC^2I(BC)}{V}\frac{x\eta}{\sqrt{1-\eta^2}}$	$-\frac{1}{8}AI(BC)(1-N)^4\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}$
C_{l_r}	6, 8	$-\frac{4r\alpha}{VE'(BC)}[x(1+C^2)]\frac{\eta}{\sqrt{1-\eta^2}}$	$\frac{4\alpha E''(BC)}{A(1-N)}\left\{\left[\frac{1}{1-N}+\frac{A^2(1-N)}{16}\right](1-N)^4\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^4\sqrt{1-\eta^2}}-\frac{8}{9}(1-N)^2\int_0^1\frac{\eta^2\,d\eta}{(1-N\eta)^3\sqrt{1-\eta^2}}\right\}$

TABLE II.—STABILITY DERIVATIVES AT SUPERSONIC SPEEDS OF A SERIES OF SWEPTBACK WINGS TAPERED TO A POINT WITH SWEPTBACK OR SWEPTFORWARD TRAILING EDGES ($\lvert N\rvert \leqq BC$)

Principal axes (origin at $\left(\frac{2}{3}c, 0, 0\right)$)		Stability axes (origin at distance x_{cg} ahead of $\frac{2}{3}c$ point)	
Stability derivative	Formula	Stability derivative	Conversion formula
C_{L_α}	$\frac{\pi}{2} AE''(BC)F_1(N)$	$C_{L_\alpha}{}'$	C_{L_α}
$C_{L_{\dot{\alpha}}}$	$-\frac{\pi AM^2}{2B^2}\left[-3G(BC)F_3(N)+2E''(BC)F_2(N)+\frac{1}{M^2}E''(BC)F_1(N)\right]$	$C_{L_{\dot{\alpha}}}{}'$	$C_{L_{\dot{\alpha}}}$
C_{L_q}	$\frac{\pi}{2} A[3G(BC)F_3(N)-2E''(BC)F_4(N)]$	$C_{L_q}{}'$	$C_{L_q}+2\frac{x_{cg}}{\bar{c}}C_{L_\alpha}$
C_{m_α}	$\frac{\pi}{2} AE''(BC)F_5(N)$	$C_{m_\alpha}{}'$	$C_{m_\alpha}-\frac{x_{cg}}{\bar{c}}C_{L_\alpha}$
$C_{m_{\dot{\alpha}}}$	$-\frac{3\pi AM^2}{16B^2}\left[G(BC)F_7(N)+\frac{16}{3}E''(BC)\frac{F_5(N)}{F_{11}(N)}\right]+\frac{16AM^2}{9B^2}E''(BC)F_8(N)+\frac{\pi A}{16B^2}E''(BC)F_6(N)$	$C_{m_{\dot{\alpha}}}{}'$	$C_{m_{\dot{\alpha}}}-\frac{x_{cg}}{\bar{c}}C_{L_{\dot{\alpha}}}$
C_{m_q}	$-\frac{3}{16}\pi A\left\{G(BC)F_7(N)+\frac{16}{3}E''(BC)\left[\frac{F_5(N)}{F_{11}(N)}\right]\right\}$	$C_{m_q}{}'$	$C_{m_q}+\frac{x_{cg}}{\bar{c}}\left(2C_{m_\alpha}-C_{L_q}-2\frac{x_{cg}}{\bar{c}}C_{L_\alpha}\right)$
C_{l_β}	$-\frac{\pi}{3}\alpha E''(BC)F_9(N)$	$C_{l_\beta}{}'$	C_{l_β}
C_{l_p}	$-\frac{\pi A}{32}I(BC)F_{10}(N)$	$C_{l_p}{}'$	$C_{l_p}+\alpha\left(C_{l_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{l_\beta}+C_{n_p}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_p}\right)+\alpha^2\left[C_{n_r}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}\left(2C_{n_\beta}+C_{Y_r}\right)+\frac{32}{9}\left(\frac{x_{cg}}{\bar{c}}\right)^2\left(\frac{1}{A}\right)^2C_{Y_\beta}\right]$
C_{l_r}	$\frac{\pi\alpha E''(BC)}{AF_{11}(N)}\left\{\left[\frac{1}{F_{11}(N)}+\frac{A^2}{16}F_{11}(N)\right]F_{10}(N)-\frac{8}{9}F_9(N)\right\}$	$C_{l_r}{}'$	$C_{l_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{l_\beta}-\alpha\left[C_{l_p}-C_{n_r}+\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}\left(C_{Y_r}+2C_{n_\beta}\right)-\frac{32}{9}\left(\frac{x_{cg}}{\bar{c}}\right)^2\left(\frac{1}{A}\right)^2C_{Y_\beta}\right]-\alpha^2\left(C_{n_p}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_p}\right)$
C_{n_β}	$\frac{\pi}{48}\alpha^2A^2M^2Q(BC)F_{11}(N)$	$C_{n_\beta}{}'$	$C_{n_\beta}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_\beta}-\alpha C_{l_\beta}$
C_{n_p}	$-\pi\alpha\left\{\frac{1}{9A}\frac{1}{[F_{11}(N)]^2}+\frac{A}{16}\right\}J(BC)$	$C_{n_p}{}'$	$C_{n_p}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_p}-\alpha\left\{C_{l_p}-\left[C_{n_r}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}\left(2C_{n_\beta}+C_{Y_r}\right)+\frac{32}{9}\left(\frac{x_{cg}}{\bar{c}}\right)^2\left(\frac{1}{A}\right)^2C_{Y_\beta}\right]\right\}-\alpha^2\left[C_{l_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{l_\beta}+C_{n_p}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_\beta}\right]$
C_{n_r}	$-C_{D_0}\left[\frac{1}{6}+\frac{4}{9A^2}F_{12}(N)\right]-\frac{\pi\alpha^2}{36}\left\{\frac{4}{A[F_{11}(N)]^2}+\frac{A}{2}+\frac{9A^3[F_{11}(N)]^2}{64}\right\}M^2Q(BC)$	$C_{n_r}{}'$	$C_{n_r}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}\left(2C_{n_\beta}+C_{Y_r}\right)+\frac{32}{9}\left(\frac{x_{cg}}{\bar{c}}\right)^2\left(\frac{1}{A}\right)^2C_{Y_\beta}-\alpha\left(C_{l_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{l_\beta}+C_{n_p}-\frac{4}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_p}\right)+\alpha^2C_{l_p}$
C_{Y_β}	$-\frac{\pi}{4}\alpha^2AM^2Q(BC)$	$C_{Y_\beta}{}'$	C_{Y_β}
C_{Y_p}	$\frac{2\pi}{3}\alpha J(BC)\frac{1}{F_{11}(N)}$	$C_{Y_p}{}'$	$C_{Y_p}+\alpha\left(C_{Y_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_\beta}\right)$
C_{Y_r}	$\frac{\pi}{24}\alpha^2A^2M^2Q(BC)F_{11}(N)$	$C_{Y_r}{}'$	$C_{Y_r}-\frac{8}{3}\frac{x_{cg}}{\bar{c}}\frac{1}{A}C_{Y_\beta}-\alpha C_{Y_p}$