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RESEARCH MEMORANDUM

A STUDY OF SEVERAL PARAMETERS CONTROLLING THE TRAJECTORIES OF A SUPERSONIC ANTIAIRCRAFT MISSILE POWERED WITH SOLID- OR LIQUID-FUEL ROCKETS

By Ralph F. Huntsberger

Ames Aeronautical Laboratory
Moffett Field, Calif.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

The trajectories for a supersonic antiaircraft missile were calculated by a step-by-step integration method for a number of different conditions. The effects of changing drag, initial thrust ratio (ratio of initial thrust to initial gross weight), and weight ratio (ratio of initial gross weight to weight after all fuel has burned), which are the principal variables controlling the trajectory for a fixed launching angle, were investigated. The results of the analysis indicated that: (1) The rate of change of range and altitude of the missile would become increasingly favorable with reduction of drag; (2) in general, there would be an optimum initial thrust ratio giving maximum range or altitude; above this optimum value the range and altitude would decrease because of the large amount of energy expended in overcoming drag at low altitudes; and (3) increase of the weight ratio of the missile, within the limits investigated, would improve the range and altitude obtainable with fuel of a given specific impulse.

INTRODUCTION

The design of a supersonic self-propelled missile presents many problems in the fields of aerodynamics and thermodynamics upon which very little work has been done. Among these problems is that of calculating the performance of such a missile and how it will be affected by changes in the aerodynamic and power characteristics. This problem has many ramifications, but its simplest form is that of determining the zero-lift trajectory of the missile when launched from the ground under a given set of initial conditions. Since, for a missile of this type the designer

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is chiefly interested in the rate of climb, maximum altitude obtainable, and range, the determination of the trajectory can serve as a guide to the performance of the missile in these respects. While it is recognized that lift forces will alter the performance of the missile, especially the range at low altitudes, the complexity of any analysis which includes lift forces becomes so great that for the preliminary study presented in this paper these forces have been neglected.

The zero-lift trajectory of a missile is dependent upon the drag of the missile, the launching angle, the weight and the type of fuel carried, and the initial acceleration. The weight of the fuel can be conveniently expressed by the ratio of initial gross weight to the weight after all the fuel has burned (weight ratio). The initial acceleration is given by the ratio of thrust to initial gross weight (initial thrust ratio) where the thrust is determined by the thermodynamic properties of the fuel and the rate at which the fuel burns.

Since the drag of a missile will be a function of both its size and shape, no general analysis of missile trajectories is possible. However, if the drag characteristics of a particular missile are specified, it is then possible to study the effects of changing the other parameters which affect the trajectories.

For the purpose of such a study a design was chosen which was amenable to analysis within the present limited scope of knowledge concerning supersonic aerodynamics. It was assumed that, except for lifting wings to give higher normal accelerations and shorter turning radii, the design selected would be typical of a supersonic antiaircraft missile for operation at altitudes below 50,000 feet. From the available types of power plants (rocket, ram-jet, and turbo-jet) rocket power was selected because its operation could be most completely divorced from the aerodynamics of the missile. Also, the rocket was the only type of power plant capable of delivering sufficient thrust to give a very high rate of climb.

The drag and power characteristics for the missile were determined primarily from theoretical considerations except in the case of solid-fuel rockets for which some data on thrust were available.

The trajectories of the missile with both solid- and liquid-fuel rocket power and fixed values of weight and initial thrust ratios were calculated for several launching angles and for drag values ranging from zero to twice the values estimated from supersonic aerodynamic theory. The effects on the trajectories of varying initial thrust ratio were determined for each type of rocket power considered, and in the case of the missile with solid-fuel [redacted] was made of the effect of varying the weight ratio.

A theoretical analysis was carried out for the case of a missile with zero drag fired vertically, and the results were compared with those obtained from a step-by-step solution of the equations of motion for the missile with normal drag.

COEFFICIENTS AND SYMBOLS

The following coefficients and symbols have been used in the presentation of the analysis and results:

I specific impulse of fuel, pounds per pound per second $\left(I = \frac{T}{K}\right)$

λ weight ratio of missile $\left(\frac{W_o}{W_1}\right)$

β initial thrust ratio $\left(\frac{T}{W_o}\right)$

C_D drag coefficient $\left(\frac{D}{\rho\frac{V^2}{2}S_F}\right)$

where

D drag, pounds

S_F frontal area, square feet

T thrust, pounds

V velocity, feet per second

W_o initial gross weight, pounds

W_1 weight after fuel has burned, pounds

K rate of fuel consumption, pounds per second

ρ air density, slugs per cubic foot

In addition, the following symbols have been used:

H altitude, feet

H_1 altitude at end of power flight, feet

X horizontal distance, feet

[redacted]

M Mach number

V_1 velocity at end of power flight, feet per second

W weight at any time, pounds

W_F fuel weight, pounds

g gravitational acceleration, feet per second per second

t time, seconds

t_1 time at which all fuel has been burned, seconds

θ angle between tangent to flight path and horizontal, degrees

θ_o launching angle, degrees

θ_1 angle at end of power flight, degrees

γ_j ratio of specific heat at constant pressure to that at constant volume (1.22 for products of combustion)

R gas constant, feet per degree Rankine

T_j jet temperature, degrees Rankine

p_o fuel-tank pressure, pounds per square inch

p_j jet exhaust pressure, pounds per square inch

METHODS OF ANALYSIS

The flight of rocket-propelled missiles consists of motion in two regimes. In the first regime, thrust is created by burning the fuel and thus the weight continuously decreases with time. After the fuel has been exhausted the weight remains constant and the missile enters the second regime (coasting flight) in which the thrust is zero. If the curvature of the earth is neglected the force system controlling the complete trajectory of such a missile may be represented as shown in figure 1. The equations of motion which determine this trajectory are:

$$T \sin \theta + D \sin \theta + W + \frac{W}{g} \frac{d^2H}{dt^2} = 0 \tag{1a}$$

$$T \cos \theta + D \cos \theta + \frac{W}{g} \frac{d^2X}{dt^2} = 0 \tag{1b}$$

where the thrust, drag, weight, and flight-path direction will, in general, be functions of the time. In the second regime of flight, these equations reduce to:

$$D \sin \theta + W_1 + \frac{W_1}{g} \frac{d^2H}{dt^2} = 0 \tag{2a}$$

$$D \cos \theta + \frac{W_1}{g} \frac{d^2X}{dt^2} = 0 \tag{2b}$$

No direct analytical solution of these equations is possible; however, a step-by-step solution can be used to yield approximate trajectories. The method is quite laborious but no other has been found which allows consideration of all variables.

To simplify the analysis the assumption can be made that, during the powered flight, the thrust has a constant value which is given by the equation

$$T = IK \tag{3}$$

The assumption does not give a completely rigorous result because the specific impulse increases with a reduction of the pressure at which the rocket gases exhaust and thus will be higher at high altitudes. However, for the cases which are considered, the change in altitude during the power flight is generally less than 20,000 feet which would result in less than a 10-percent increase in specific impulse.

Assumption of a constant thrust permits expression of the weight during the power flight by the equation

$$W = W_o - Kt \tag{4}$$

The drag is given by the equation

$$D = C_D \rho \frac{V^2}{2} S_F \tag{5}$$

where

$$V^2 = \left(\frac{dH}{dt}\right)^2 + \left(\frac{dX}{dt}\right)^2$$

and the instantaneous flight-path direction is given by the equation

$$\tan \theta = \frac{dH}{dt} \div \frac{dX}{dt} \tag{6}$$

The relationships given in equations (3) to (6) permit solution of equations (1) and (2).

Trajectories for launching angles from 30° to vertical were calculated from the foregoing equations by assuming the values of drag coefficient, weight, flight-path direction, and Mach number to remain constant over short time increments. Initially, calculations were carried out to determine the trajectories of the missile powered with either solid or liquid fuels and an initial thrust ratio of 11.12. These calculations were based on the normal drag of the missile, shown in figure 2, as estimated from references 1, 2, and 3, and on the characteristics of the atmosphere taken from NACA tables (reference 4) to an altitude of 80,000 feet and from reference 5 above this height.

In order to study the effects of increasing or decreasing the estimated normal drag, trajectories were calculated for drag values of 0, 50, 150, and 200 percent of the normal value as shown in figure 2. Again, these calculations were made for an initial thrust ratio of 11.12 and both types of rocket fuel.

The effect of varying the thrust ratio was studied by calculating the trajectories of the missile with constant weight, normal drag, and both types of rocket fuel for values of the initial thrust ratio of 2.78 and 5.56. The results of these calculations were compared with those obtained for an initial thrust ratio of 11.12.

The variation of weight ratio was studied for the missile powered with solid rocket fuel by assuming the cannister weight could be reduced to allow sufficient fuel to be carried to give weight ratios of 2.0 and 2.5 for the same total missile weight. Trajectories, calculated for these weight ratios, were compared with those obtained from calculations for the actual estimated weight ratio of 1.53. These calculations were made for initial thrust ratios of 2.78, 5.56, and 11.12. All calculations for the missile powered with liquid rocket fuel were made for the estimated weight ratio of 1.86.

The trajectory of a missile without drag has been considered in the appendix, and a solution giving maximum altitude has been obtained for a missile fired vertically.

DESCRIPTION OF MISSILE

General Description

The missile which was assumed to be typical of a supersonic aircraft interceptor is shown in figure 3. Control in pitch and yaw could be obtained by rotating the appropriate tail surfaces, or a hinged portion thereof, about a spanwise axis. Roll stabilization would be afforded by gyro operation of the control surfaces. The turning radius of this missile would be limited by the lift which could be developed by the body.

The missile design chosen included a 200-pound war head and control equipment weighing 150 pounds. The remainder of the internal volume was consumed by structural members, fuel cannisters or tanks, fuel, and exhaust nozzle. The internal volume fixed the amount of fuel, either solid or liquid, which could be carried. Thus, a comparison was afforded between solid- and liquid-fuel rocket power for a missile of fixed external size and configuration.

Missile with Solid-Fuel Rocket Power

The initial gross weight of the missile with solid-fuel rocket power was 2500 pounds of which 500 pounds were structure, controls, and war head. The ratio of fuel weight to cannister weight was assumed to be 0.77 which was the average for three Monsanto rockets now in production. Upon the basis of this assumption 870 pounds were fuel and 1130 pounds were cannister and nozzle. The specific impulse of the solid fuel was assumed to be 160 pounds per pound per second, which was the average value for the three rockets previously mentioned.

Missile with Liquid-Fuel Rocket Power

The initial gross weight of the missile with gasoline fuel and liquid oxygen was 1100 pounds. Of this weight 475 pounds were structure, controls, and war head; 25 pounds were a tank of compressed nitrogen for supplying fuel system pressure; 113 pounds were gasoline; 397 pounds were oxygen; and 90 pounds were tanks and nozzle.

The weights of gasoline and oxygen were based on a chemically correct mixture. The available internal volume fixed the size of fuel and oxygen tanks. The gasoline tanks were wire-wrapped spheres. The oxygen tanks were of standard Army Air Force design constructed of stainless steel to withstand the low temperature of liquid oxygen. (See reference 6.)

The specific impulse for the liquid fuel was taken from the curves of figures 4, 5, and 6. Figure 4 presents the variation of gross weight of the missile and specific impulse of the fuel with the pressure within fuel and oxygen containers. This pressure was assumed equal to the combustion chamber pressure. The values of specific impulse were calculated from the following equation derived from information given in reference 7:

$$I = \frac{1}{g}\sqrt{\frac{2\gamma_j gRT_j}{\gamma_j - 1}\left[\left(\frac{p_o}{p_j}\right)^{\frac{\gamma_j-1}{\gamma_j}} - 1\right]}$$

For the calculations the jet-exhaust stagnation temperature was assumed to be 7000° Rankine. The values obtained from these calculations were arbitrarily reduced by 10 percent for practical application.

Figure 5 gives the variation of tank pressure and specific impulse with weight ratio. Figure 6 presents the results of calculations of the altitude the missile with zero drag would reach if fired vertically for the relation between specific impulse and weight ratio given in figure 5. These calculations, based on equation (21) in the appendix, indicated that a specific impulse of 220 pounds per pound per second would give nearly the maximum altitude for all values of the initial thrust ratio (ratio of thrust to initial gross weight) considered and this value was selected. It was assumed that the optimum conditions obtained for zero drag would also apply to a missile fired in air. The corresponding tank pressure was 310 psi.

RESULTS AND DISCUSSION

Effects of Drag

<u>Trajectories</u>.- The calculated trajectories for the missile with normal drag and both solid-fuel and liquid-fuel rocket power are given in figure 7 for a ratio of initial thrust to initial gross weight (initial thrust ratio) of 11.12 which was used throughout the study of the effects of drag on performance. Curves giving typical variation of Mach number along the trajectories are shown in figure 8. The trajectories for each

type of power plant had approximately the same shape at the same launching angle; however, the missile with liquid-fuel rocket power traversed a much greater distance in each case. Similar trajectories were obtained from calculations for the missile with the drag assumed at 0, 50, 150, and 200 percent of the normal value for each Mach number. These trajectories are given for each type of power plant and a launching angle of 60° in figure 9. For launching angles other than 60° the trajectories maintained the same relationships with respect to both drag and type of power plant.

The trajectories for launching angles near the vertical closely approximated parabolic shapes because in traversing the upper atmosphere the drag of the missile was small as a result of the low air density. For launching angles below 60° the effect of drag was to decrease rapidly the horizontal velocity component in gliding flight and the resulting trajectories departed from parabolic shapes by having steeper slopes over their descending portions. This result is apparent in figure 7 for the missile with normal drag. Inspection of figure 9 reveals that, for a given launching angle, increases in the drag caused progressively larger departures of the trajectories from the parabolic form.

Maximum range.- The calculated trajectories for the missile with each assumed variation of drag coefficient with Mach number yielded the results which are summarized in figure 10 for solid-fuel power and in figure 11 for liquid-fuel power. The maximum range with normal drag and solid-fuel power was found to be 14.3 miles as compared to 24.0 miles with liquid-fuel power. The increase of range in the latter instance was attributed to the increase of the ratio of initial gross weight to weight after all fuel has burned (weight ratio) and to the higher specific impulse of the liquid fuel. The effect of increasing the drag from zero to twice its normal value was to reduce considerably the range. Also, in the case of the missile with solid-fuel power, increase of drag caused a small increase in the launching angle required to give a maximum range. In the case of the missile with liquid-fuel power the launching angle for maximum range first increased and then decreased as the drag was progressively increased. The apparent discrepancy in the variation of launching angle with increase of drag for the same missile but with different types of power was attributed to the fact that the missile with liquid-fuel power attained approximately twice the Mach number attained with solid-fuel power; this would place the major portion of its flight at Mach numbers where the drag characteristics would be different from those for the missile with solid-fuel power, since the drag coefficient varied with Mach number.

The maximum range of the missile as a function of the percentage of normal drag is given in figure 12 for each type of power plant. In each case increase of drag reduced the maximum range; however, the greatest reduction occurred as the drag was increased from zero to its normal value. Obviously the range would decrease asymptotically to zero as the drag was infinitely increased. The percentage reduction in range from the value for zero drag was 45 percent for the missile with solid-fuel power and normal drag characteristics, while it was 76 percent for the case of liquid-fuel power. The larger reduction in the latter case can be explained by consideration of the energy components which determine the trajectory of a given missile.

Since the missiles under comparison had the same external dimensions, their drags at a given Mach number would be the same, but the kinetic energy of the heavier missile (solid fuel) would be greater than that of the lighter missile (liquid fuel). Thus the energy expended in overcoming drag would be a smaller percentage of the total available energy in the case of the heavier missile, and increase or decrease of drag by a given amount would have less effect on the heavier missile at a given Mach number. Although the lighter missile attained higher velocities, thereby increasing its kinetic energy, the drag was increased in approximately the same proportion so that in all cases considered for the lighter missile, the energy required to overcome drag was a larger fraction of the total energy available.

The results presented in figure 12 also indicate that decreasing the drag would increase the range in such a manner that the rate of change of range would become increasingly favorable with reduction of drag. This result was found to be true for the missile with either type of power plant.

Maximum altitude.- The variation of maximum altitude attained with launching angle is presented in figure 13 for solid-fuel power and in figure 14 for liquid-fuel power. As with range, the effect of drag was to reduce the altitude attained as the drag was increased. The effect was greatest for vertical launching and decreased with launching angle. Increasing the drag from zero to its normal value had more effect on reducing the altitude than did increases above the normal value. These results were consistent with the effects of drag on range, since the primary effect of drag was to alter the trajectory.

Effects of Initial Thrust Ratio

Trajectories.- The trajectories for the missile with each type of power plant and normal drag were calculated for two additional ratios of thrust to initial gross weight (initial thrust ratio). (This parameter gives the initial acceleration which the missile would have in horizontal flight in units of g.) These trajectories had the same general shape characteristics as those for the thrust ratio of 11.12 previously mentioned.

Maximum range.- The trajectory calculations for different initial thrust ratios yielded the range as a function of launching angle according to the curves of figure 15. These results were obtained for the missile with each type of power. For the heavier missile with solid-fuel power the range increased slightly with increase of the initial thrust ratio, while the reverse was true for the lighter missile with liquid-fuel power. This paradoxical result can be explained by consideration of energy components as before.

The largest missile velocity and therefore greatest kinetic energy was attained with the highest value of the initial acceleration (or initial thrust ratio). If no drag were present it could be shown that this would lead to the greatest range for the missile. The same result would be true if the energy expended in overcoming the drag were a small percentage of the total energy. However, a point would be reached, as was the case for the missile with liquid-fuel power, where the energy expanded in overcoming drag at the higher velocities would become so large a fraction of the total available energy that the range of the missile would be decreased by increasing the value of the initial thrust ratio.

The launching angle for maximum range was found to increase as the value of the thrust ratio decreased. This result was true for each type of power plant. From the study of projectiles it is well known that the maximum range is attained in a vacuum if the projectile is fired from the ground at an angle of 45°. For the missile carrying its own propelling charge this is no longer true. It can be shown that the angle to give a maximum range for the coasting flight of such a missile in a vacuum is given within 1 percent by the formula

$$\theta_1 = \frac{1}{2} \cos^{-1}\left[\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4gH_1}{V_1^2}}\right] \tag{7}$$

provided that

$$\frac{4gH_1}{V_1^2} \leqq 0.30$$

Since the angle for maximum range is less than 45°, this would indicate that the launching angle would decrease as the altitude at the end of the power flight increased. However, the launching angle does not decrease because the flight path under power is always concave downward due to the gravitational attraction. Also, as the time of the power flight to a given altitude increases, the launching angle must increase to give maximum range, since the flight-path angle decreases continuously under constant gravitational acceleration.

The results shown in figure 15 indicate that the launching angle for maximum range increased with the time of power flight, since for a fixed quantity of fuel with a given specific impulse the time of power flight will be inversely proportional to the initial thrust ratio. For the case of the missile with liquid fuel the time of power flight was further increased because of the greater specific impulse of the fuel which decreased the rate of fuel consumption. Thus, the launching angle for maximum range in the case of the missile with liquid fuel was greater than that for the missile with solid fuel at corresponding initial thrust ratios.

Maximum altitude.- The maximum altitude which would be reached by the missile with each type of power plant and three initial thrust ratios is shown as a function of launching angle in figure 16. The maximum altitude attained decreased rapidly as the launching angle was reduced from the vertical. This phenomenon was particularly marked for the missile powered with liquid fuel and can be explained as an effect of the longer time of power flight with the liquid fuel.

For the missile powered with solid fuel the maximum altitude increased with increase of the initial thrust ratio. This same result was generally true for the missile with liquid-fuel power except at launching angles near the vertical and an initial thrust ratio of 11.12. Here the maximum altitude was attained with a lower value of the initial thrust ratio because of the large drag associated with the high initial accelerations.

The variation of the maximum altitude attained (vertical firing) by the missile with initial thrust ratio for each type of power plant is shown in figure 17. These results indicate that an initial thrust ratio of 6.0 would give a maximum altitude for the missile with liquid-fuel power; whereas an initial thrust ratio of more than 11.0 would be required for the missile with solid-fuel. The maximum altitude obtainable with the missile powered with liquid fuel was 20.8 miles as compared to slightly more than 8.7 miles for the missile with solid-fuel power.

It should be noted here that the results obtained for the missile with liquid-fuel power would have been modified if fuel pumps instead of pressure tanks had been used. The weight of fuel pumps would increase with initial thrust ratio because of the greater quantity of fuel handled per unit time. This would decrease the weight ratio for a fixed initial gross weight.

Effects of Weight Ratio

Trajectories.- In order to study the effects of changing weight ratio on the performance of the missile with normal drag the trajectories were calculated for solid-fuel power and the same initial thrust ratios previously considered. However, the calculations were based on assumed weight ratios of 2.0 and 2.5 with the missile assumed to have the same external dimensions. The use of the same fuel (Monsanto) was assumed throughout the calculations.

The results of these calculations are given in figure 18 for a launching angle of 60°. The trajectories for the different weight ratios were similar for each value of the initial thrust ratio and departed progressively from a parabolic form as the launching angle was reduced.

Maximum range.- The results of the calculations of range as a function of launching angle for the different weight ratios are summarized in figure 19. (Note that the scale of range has been changed for each weight ratio.) The maximum range was increased approximately fourfold for a two-thirds increase in weight ratio from 14.2 miles for a weight ratio of 1.53 to 57.0 miles for a weight ratio of 2.50. The effect of launching angle and initial thrust ratio on range at the higher weight ratios was substantially the same as that found for the missile with a weight ratio of 1.53. It should be pointed out, however, in figure 19(c) that the maximum range did not occur at the highest value of the initial thrust ratio (11.12) with a weight ratio of 2.50 but rather at a value near 5.56. This fact indicates that the missile attained such a high velocity in the dense lower atmosphere that a considerable portion of its total energy was expended in overcoming drag with a resulting decrease in range.

Maximum altitude.- The results of altitude calculations for the missile with different weight ratios are summarized in figure 20. The maximum altitude attained by the missile at different launching angles likewise demonstrated a marked increase with increase of weight ratio for launching angles greater than 60°. For launching angles less than this value, the increase in altitude was small because of the longer burning time required for the higher weight ratios. The altitude attained was generally greater with the high values of initial thrust ratio than that for the low values except at the nearly vertical launching angles for the missile with a weight ratio of 2.50. As has been pointed out, this was due to the large drag encountered by the missile in the lower atmosphere.

The altitudes attained by the missiles of different weight ratios launched vertically have been compared with the theoretical values which missiles of the same specific impulse would attain with zero drag, as determined from equation (21) of the appendix. These results, shown in figure 21, indicate that there will be an increase in the altitude attained with increase of initial thrust ratio above the value of 1.0, whether drag is considered or not. However, continued increase of initial thrust ratio when drag is considered may result in a decrease of altitude beyond some maximum value as was the case for the missile with a weight ratio of 2.50. Also, the increase of maximum altitude with increase of weight ratio does not proceed as rapidly for the missile with finite drag as would occur for the same missile with zero drag. The increase in altitude for a 63-percent increase in weight ratio was 324 percent when drag was considered; whereas the theory predicted a 372-percent increase with zero drag for an initial thrust ratio of 6.0.

CONCLUSIONS

Consideration of the missile drag, initial thrust ratio (ratio of initial thrust to initial gross weight), and weight ratio (ratio of initial gross weight to weight after all fuel has burned) in the analysis of the trajectories of a rocket-powered supersonic missile of the aircraft-interceptor type as determined for different launching angles indicated the following:

1. The rate of change of range and altitude would become increasingly favorable with reduction of drag.

2. In general, there would be an optimum initial thrust ratio giving maximum range or altitude; above this optimum value the range and altitude would decrease because of the large amount of energy expended in overcoming drag at low altitudes.

3. Increase of the weight ratio of the missile, within the limits investigated, would improve the range and altitude obtainable with fuel of a given specific impulse.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif.

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APPENDIX

MISSILES WITHOUT DRAG

If the drag is neglected, equations (1a) and (1b), when combined with equation (4), become

$$T \sin \theta - (W_o - Kt) - \left(\frac{W_o - Kt}{g}\right) \frac{d^2H}{dt^2} = 0 \tag{8a}$$

$$T \cos \theta - \left(\frac{W_o - Kt}{g}\right) \frac{d^2X}{dt^2} = 0 \tag{8b}$$

A solution has not been found for these equations except for the case of a rocket fired vertically where equation (8b) vanishes and (8a) becomes

$$T - (W_o - Kt) - \left(\frac{W_o - Kt}{g}\right) \frac{d^2H}{dt^2} = 0 \tag{9}$$

Integration of equation (9) for the missile starting from rest at sea level with a constant thrust instantaneously applied yields, for the vertical velocity,

$$\frac{dH}{dt} = V_H = \frac{gT}{K} \ln \frac{W_o}{W_o - Kt} - gt \tag{10}$$

Integration of equation (10) to determine the altitude gives

$$H = \frac{gT}{K}\left(\ln \frac{W_o}{W_o - Kt} - \frac{W_o}{Kt} \ln \frac{W_o}{W_o - Kt} + 1\right) t - \frac{1}{2} gt^2 \tag{11}$$

At the end of the power flight

$$t = t_1 = \frac{W_F I}{T} \tag{12}$$

Introducing the weight ratio,

$$\lambda = \frac{W_o}{W_o - W_F} \tag{13}$$

and the initial thrust-weight ratio,

$$\beta = \frac{T}{W_o} \tag{14}$$

gives

$$t_1 = \frac{I}{\beta} \frac{\lambda - 1}{\lambda}$$

Equations (10) and (11) become, respectively,

$$V_1 = gI \left[\ln \lambda - \frac{1}{\beta}\left(\frac{\lambda - 1}{\lambda}\right)\right] \tag{15}$$

and

$$H_1 = gI^2\left[\frac{1}{\beta}\left(1 - \frac{\ln\lambda}{\lambda} - \frac{1}{\lambda}\right) - \frac{1}{2\beta^2}\left(\frac{\lambda - 1}{\lambda}\right)^2\right] \tag{16}$$

Equation (2a) for the missile without drag becomes

$$g + \frac{d^2H}{dt^2} = 0 \tag{17}$$

Integrating equation (17) twice with t_1 as the lower limit and t as the upper limit yields

$$H = H_1 + V_1 (t - t_1) - \frac{1}{2} g (t - t_1)^2 \tag{18}$$

which is the total altitude the missile would attain.

Differentiating equation (18) with respect to time to obtain a maximum indicates that such will occur when

$$t - t_1 = \frac{V_1}{g} \tag{19}$$

or

$$H_{max} = H_1 + \frac{V_1^2}{2g} \tag{20}$$

Substituting the values of V_1 and H_1 from equations (15) and (16) gives for the maximum altitude

$$H_{max} = g \frac{I^2}{2} \left[(\ln\lambda)^2 + \frac{2}{\beta} \left(1 - \ln\lambda - \frac{1}{\lambda}\right) \right] \tag{21}$$

which may be written

$$H_{max} = \frac{gI^2}{2} (\zeta) \tag{22}$$

where

$$\zeta = (\ln\lambda)^2 + \frac{2}{\beta} \left(1 - \ln\lambda - \frac{1}{\lambda}\right) \tag{23}$$

Values of ζ for weight ratios of 1.5 to 10.0 and initial thrust ratios of 1.0 to 10.0 are given in figure 22.

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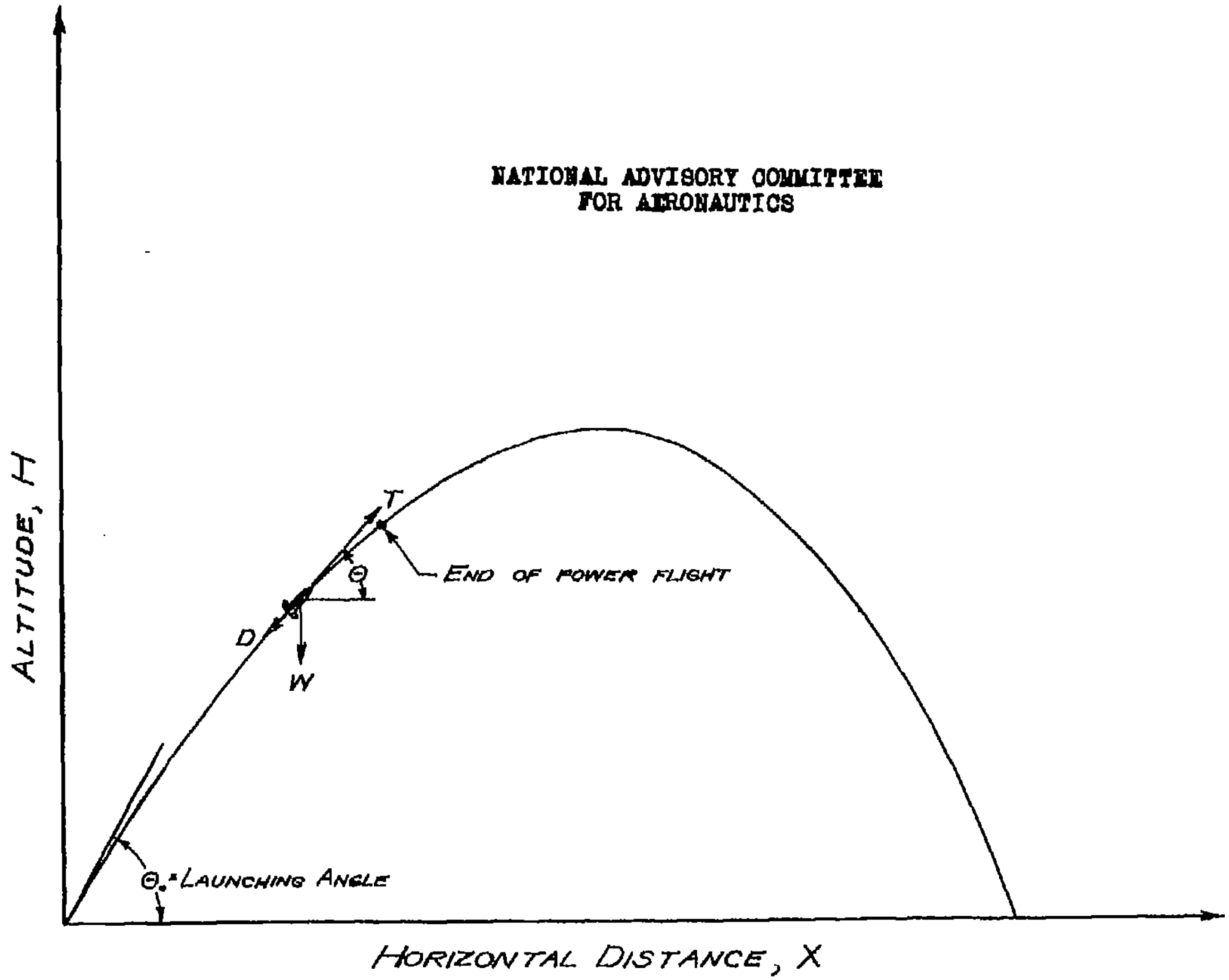

FIGURE 1.- COORDINATE SYSTEM AND FORCE DIAGRAM USED IN ANALYSIS OF SUPERSONIC MISSILES.

[redacted]

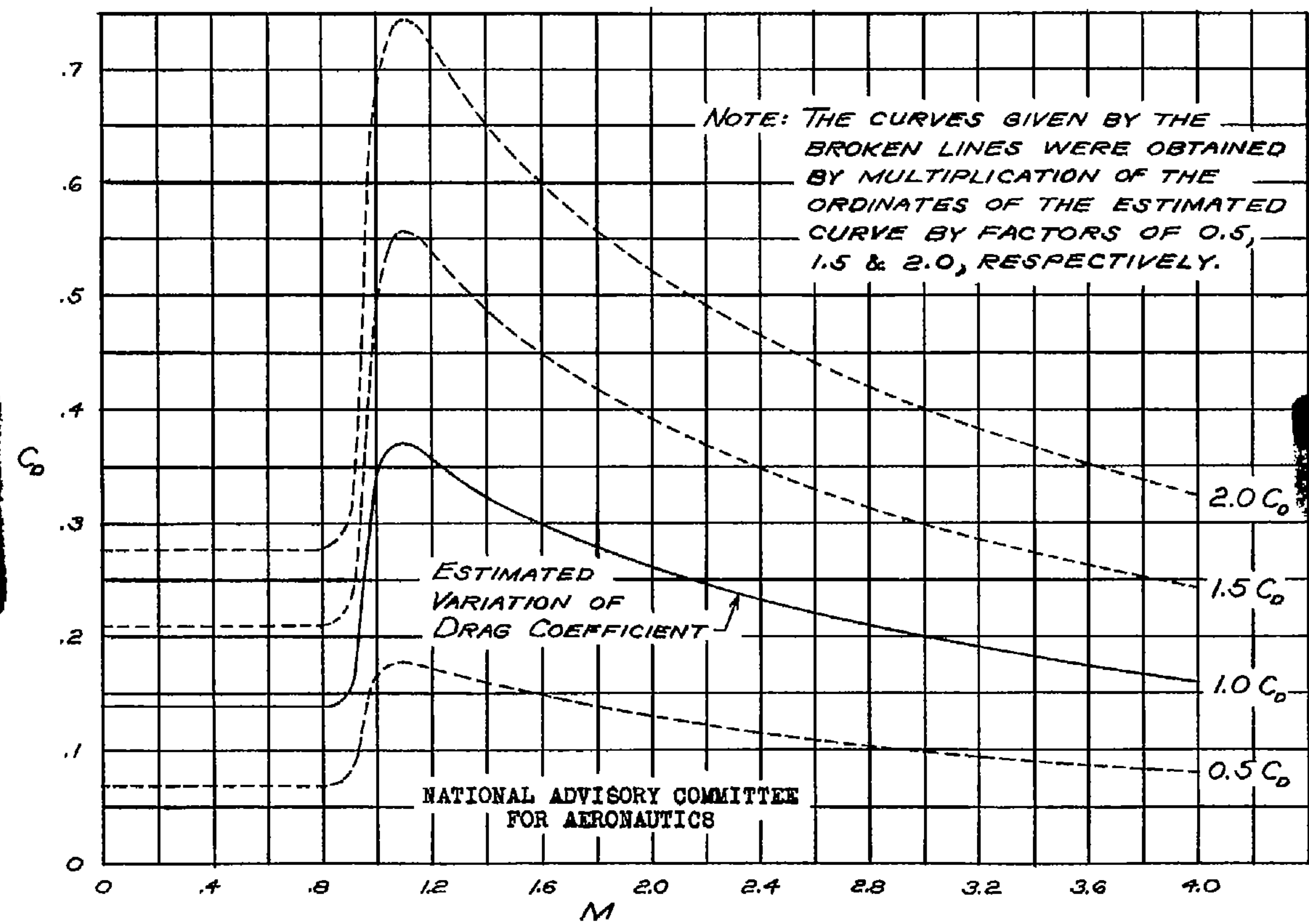

FIGURE 2. ESTIMATED VARIATION OF DRAG COEFFICIENT WITH MACH NUMBER FOR THE ASSUMED SUPERSONIC MISSILE AT ZERO LIFT.

[redacted]

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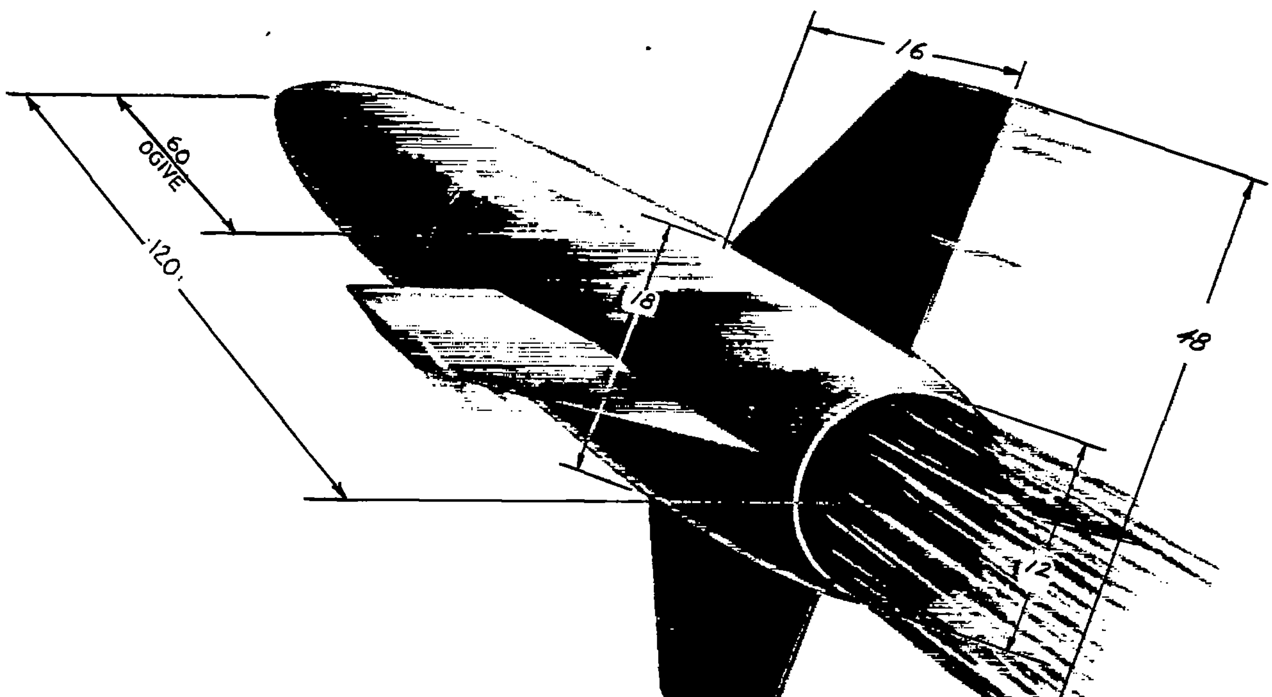

NOTE: ALL DIMENSIONS IN INCHES

GROSS WEIGHTS:

1. MONSANTO SOLID FUEL..............2500 LB.
2. GASOLINE - LIQUID OXYGEN FUEL.....1100 LB.

FIGURE 3. EXTERNAL CONFIGURATION OF ASSUMED GROUND-TO-AIR SUPERSONIC MISSILE USED TO COMPARE LIQUID AND SOLID FUEL ROCKET POWER PLANTS.

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12-17-45

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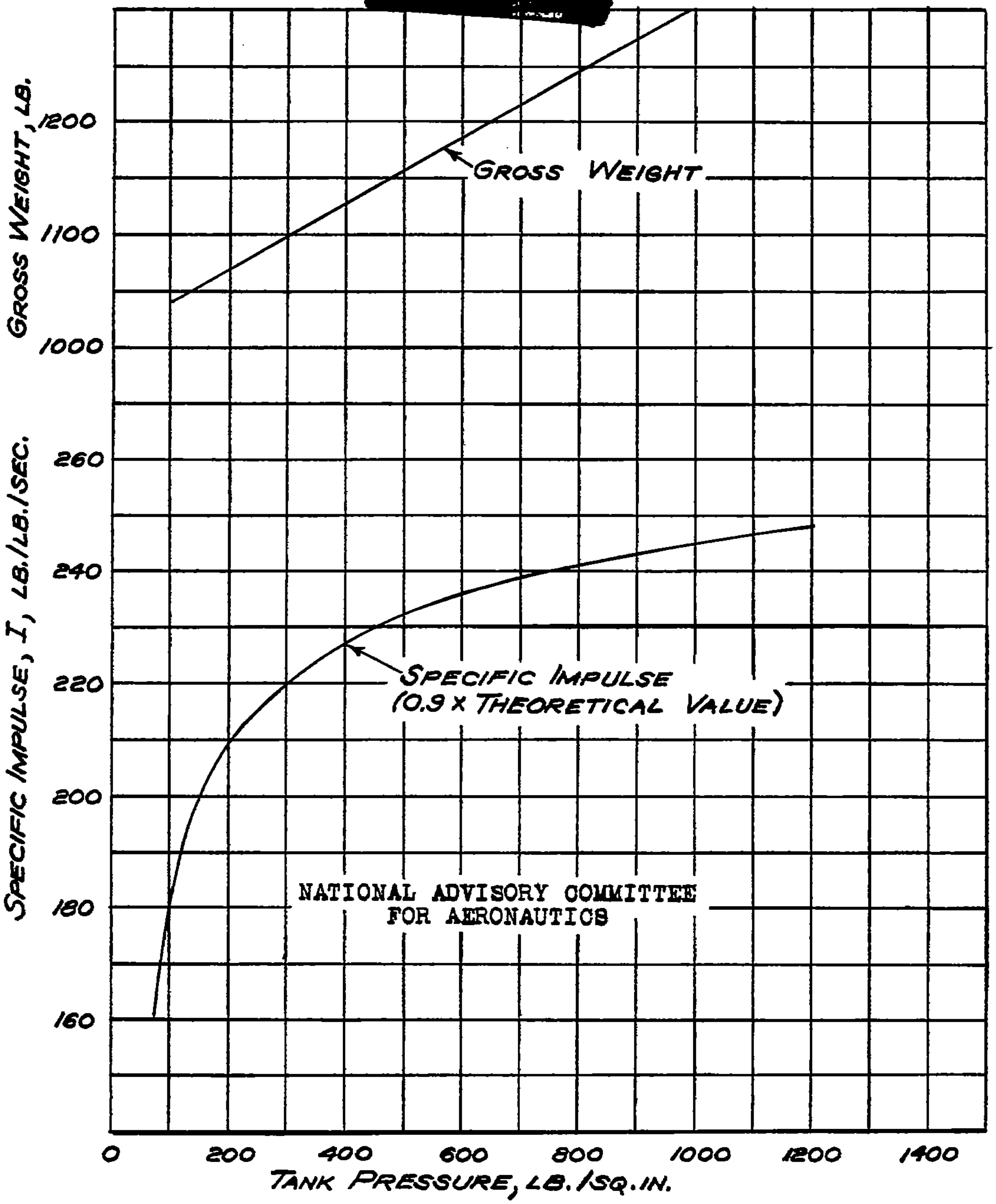

FIGURE 4.– VARIATION OF SPECIFIC IMPULSE AND INITIAL GROSS WEIGHT WITH FUEL-TANK PRESSURE FOR THE ASSUMED SUPERSONIC MISSILE WITH GASOLINE AND LIQUID-OXYGEN FUEL.

[redacted]

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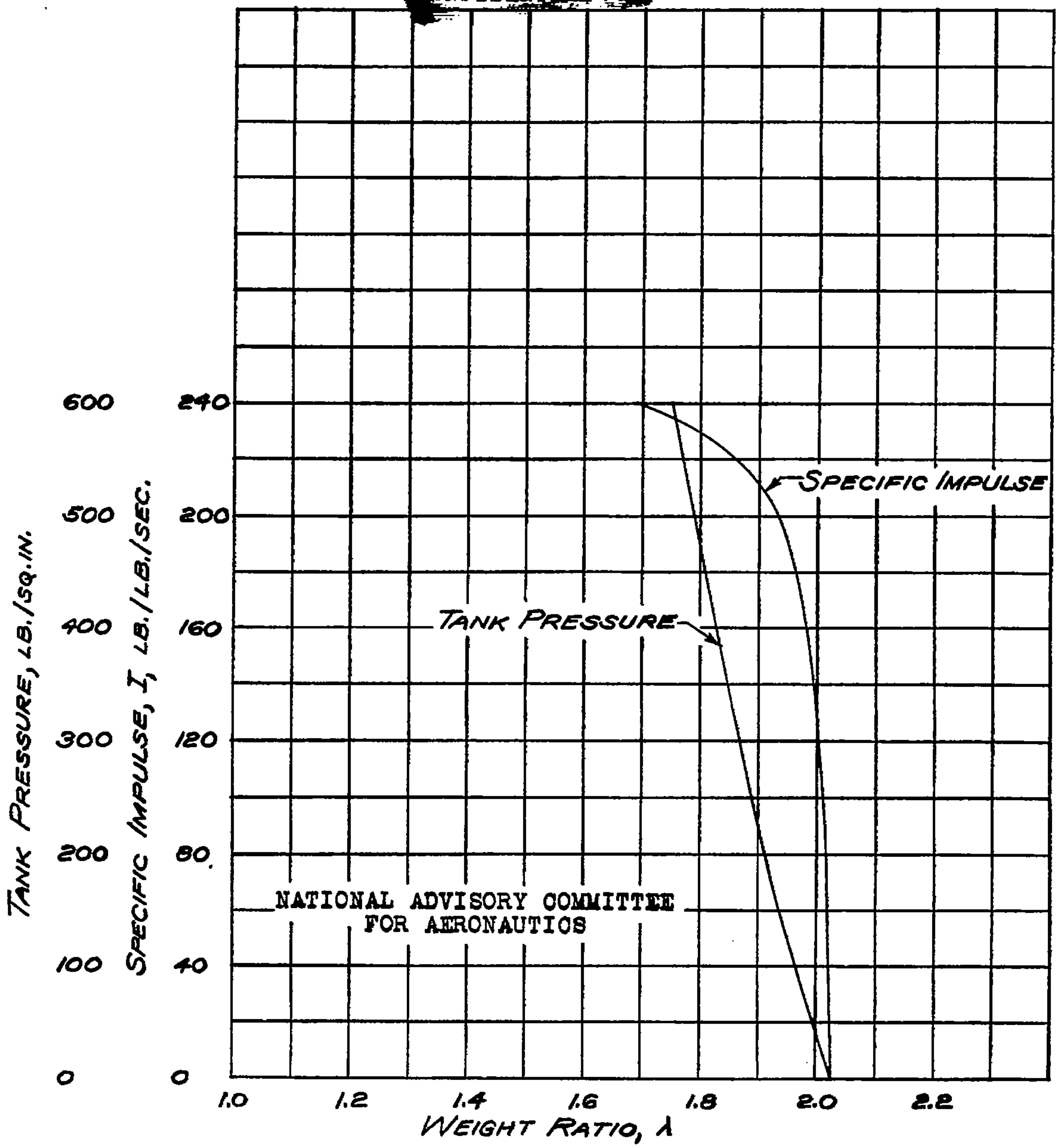


FIGURE 5.- VARIATION OF FUEL-TANK PRESSURE AND SPECIFIC IMPULSE WITH WEIGHT RATIO FOR THE ASSUMED SUPERSONIC MISSILE WITH GASOLINE AND LIQUID-OXYGEN FUEL.

[redacted]

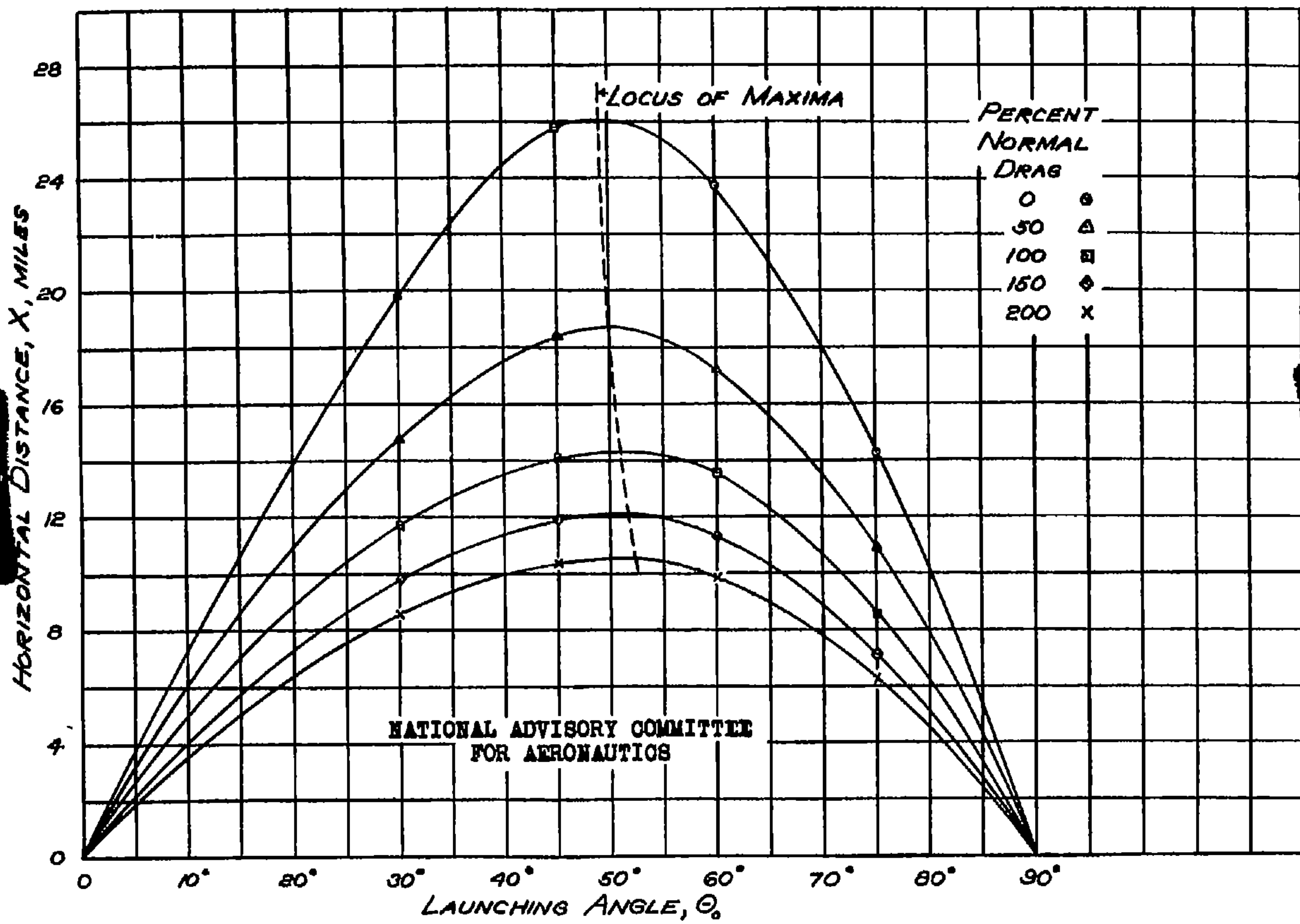
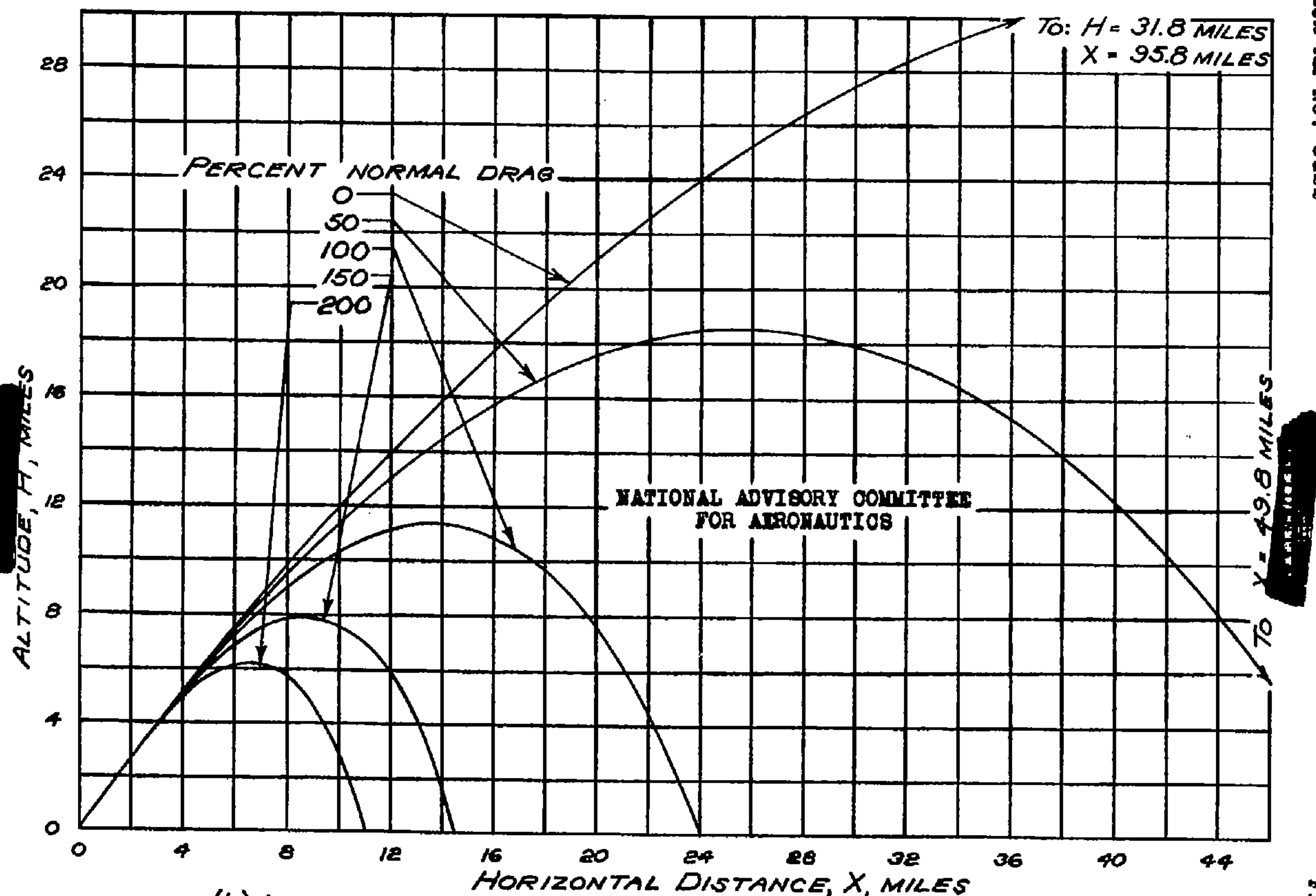
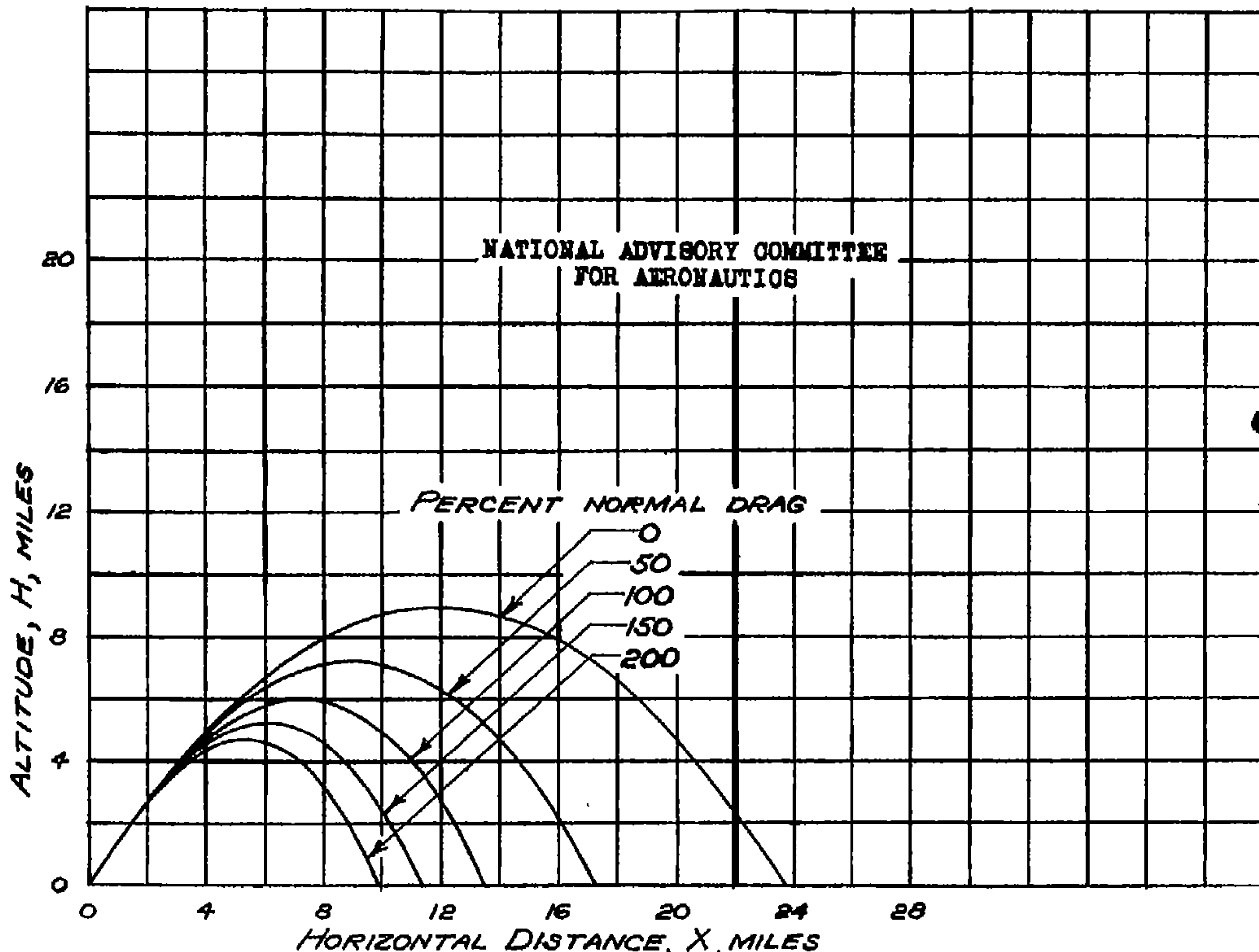


FIGURE 10.– VARIATION OF RANGE WITH LAUNCHING ANGLE FOR THE MISSILE WITH A SOLID-FUEL ROCKET POWER PLANT AND SEVERAL ASSUMED VARIATIONS OF THE DRAG COEFFICIENT WITH MACH NUMBER; $\beta = 11.12$.



(b) LIQUID FUEL POWER, $\lambda = 1.86$

FIGURE 9.- CONCLUDED.



(a) SOLID FUEL POWER, $\lambda = 1.53$.

FIGURE 9.- CALCULATED TRAJECTORIES FOR THE MISSILE WITH SEVERAL ASSUMED VARIATIONS OF THE DRAG COEFFICIENT WITH MACH NUMBER. $\beta = 11.12$, $\Theta_0 = 60°$

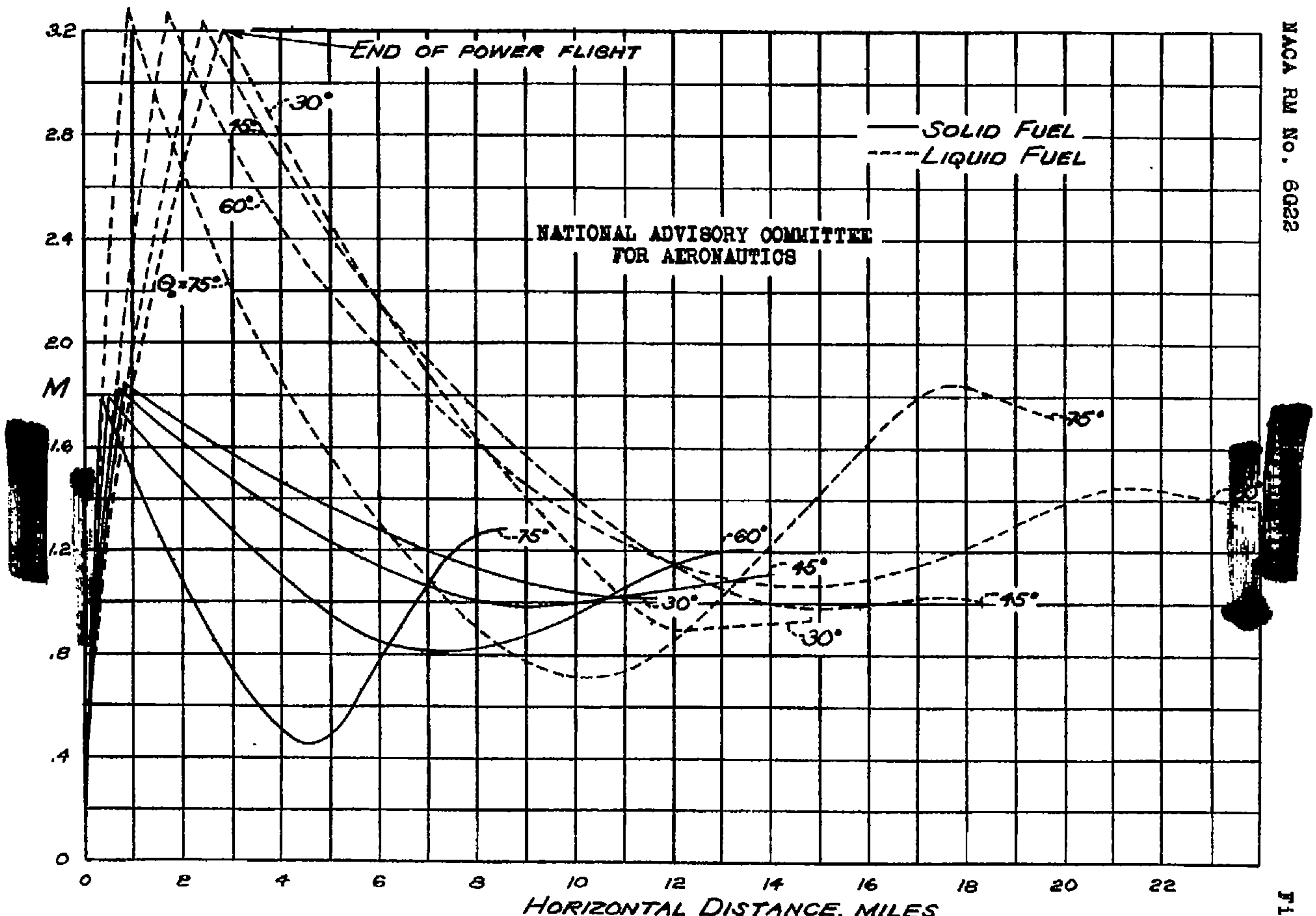
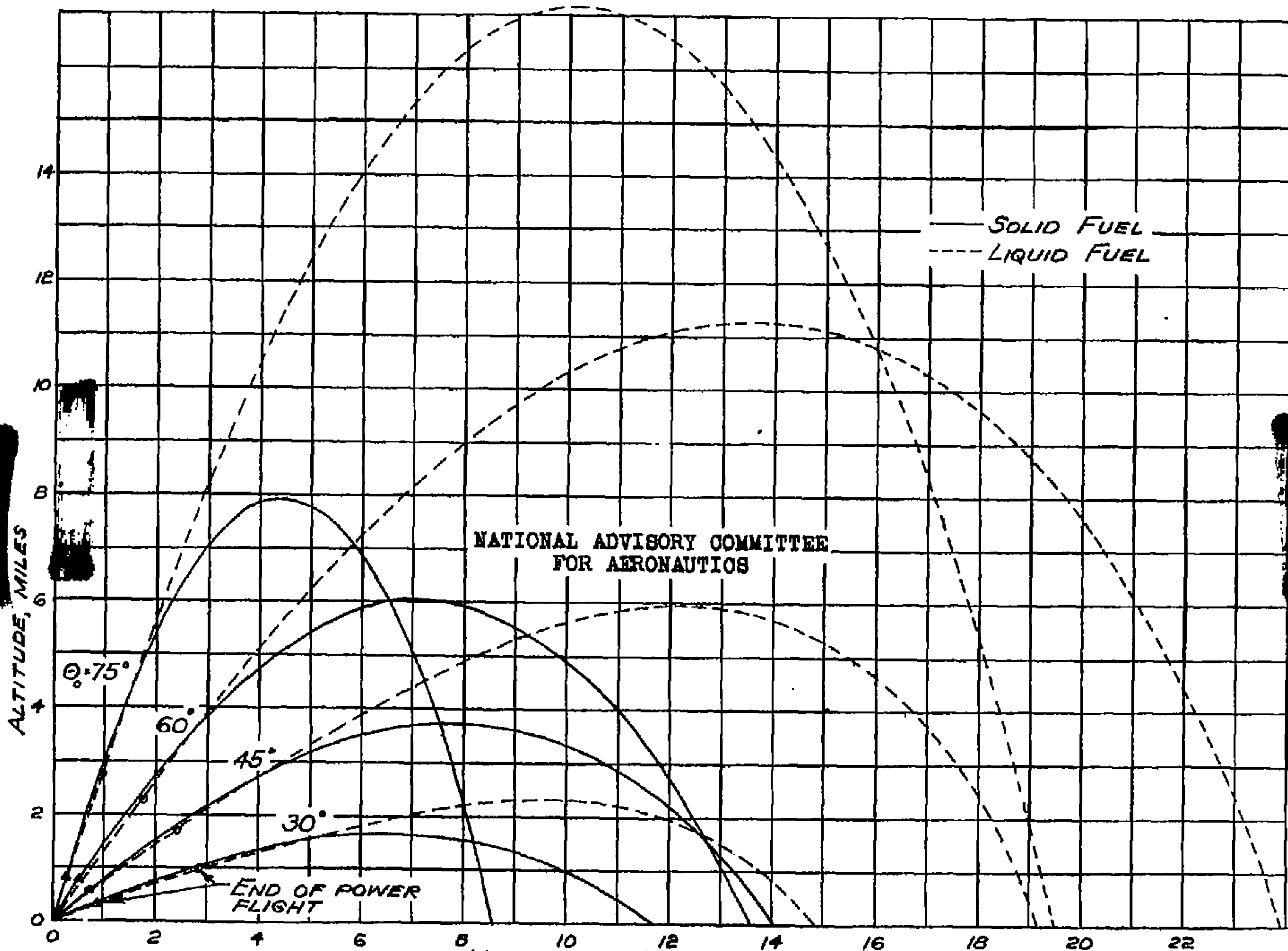


FIGURE 8. VARIATION OF MACH NUMBER WITH HORIZONTAL DISTANCE FOR MISSILE POWERED WITH SOLID- OR LIQUID-FUEL ROCKETS; $\beta = 11.12$

[redacted]



[redacted]

FIGURE 7. CALCULATED TRAJECTORIES FOR MISSILE POWERED WITH SOLID- OR LIQUID-FUEL ROCKETS β = 11.12.

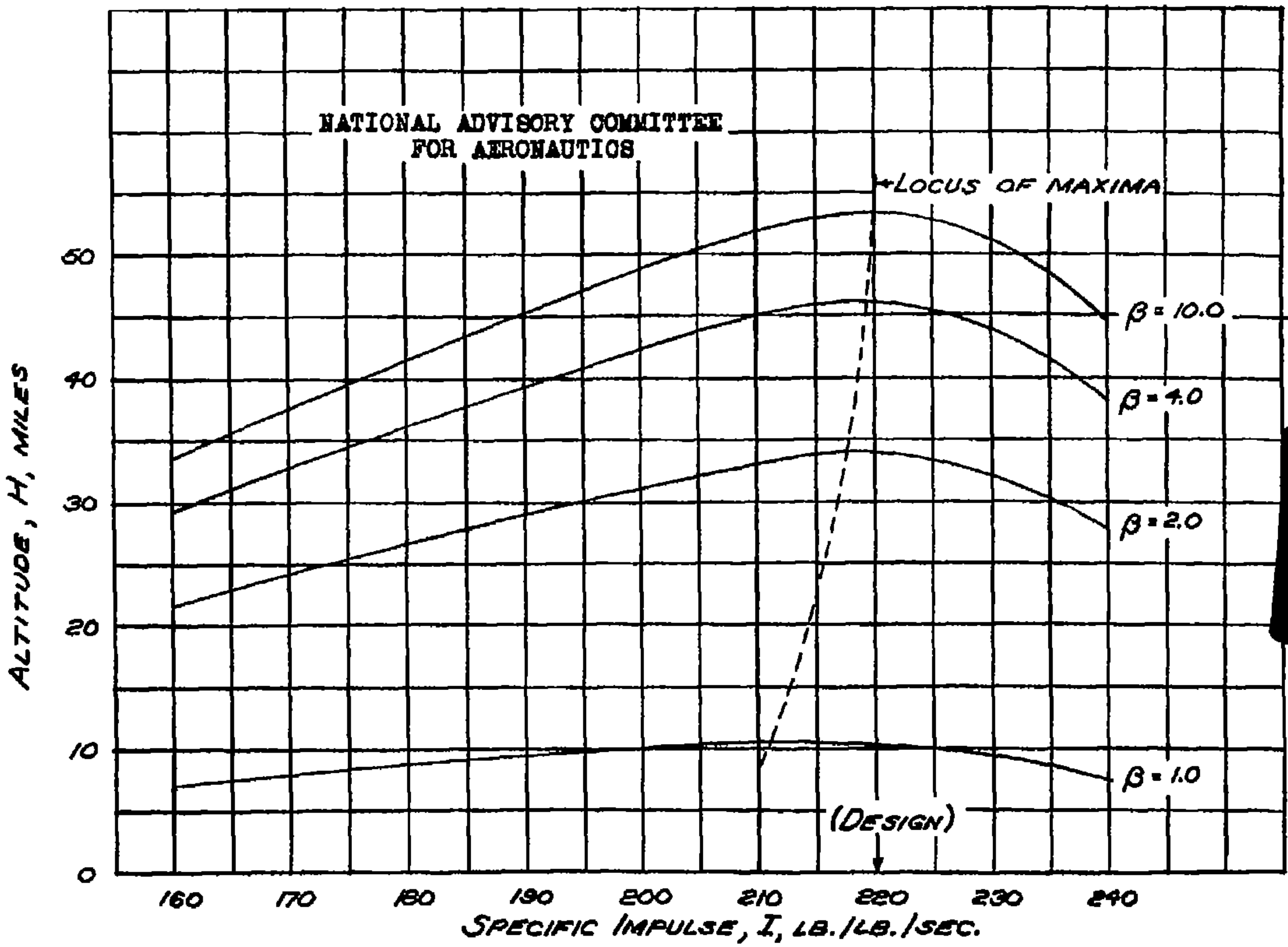


FIGURE 6. VARIATION OF ALTITUDE WITH SPECIFIC IMPULSE FOR THE ASSUMED SUPERSONIC MISSILE WITH ZERO DRAG USING GASOLINE AND LIQUID-OXYGEN FUEL AND HAVING SEVERAL VALUES OF THE RATIO OF THRUST TO INITIAL WEIGHT, β.

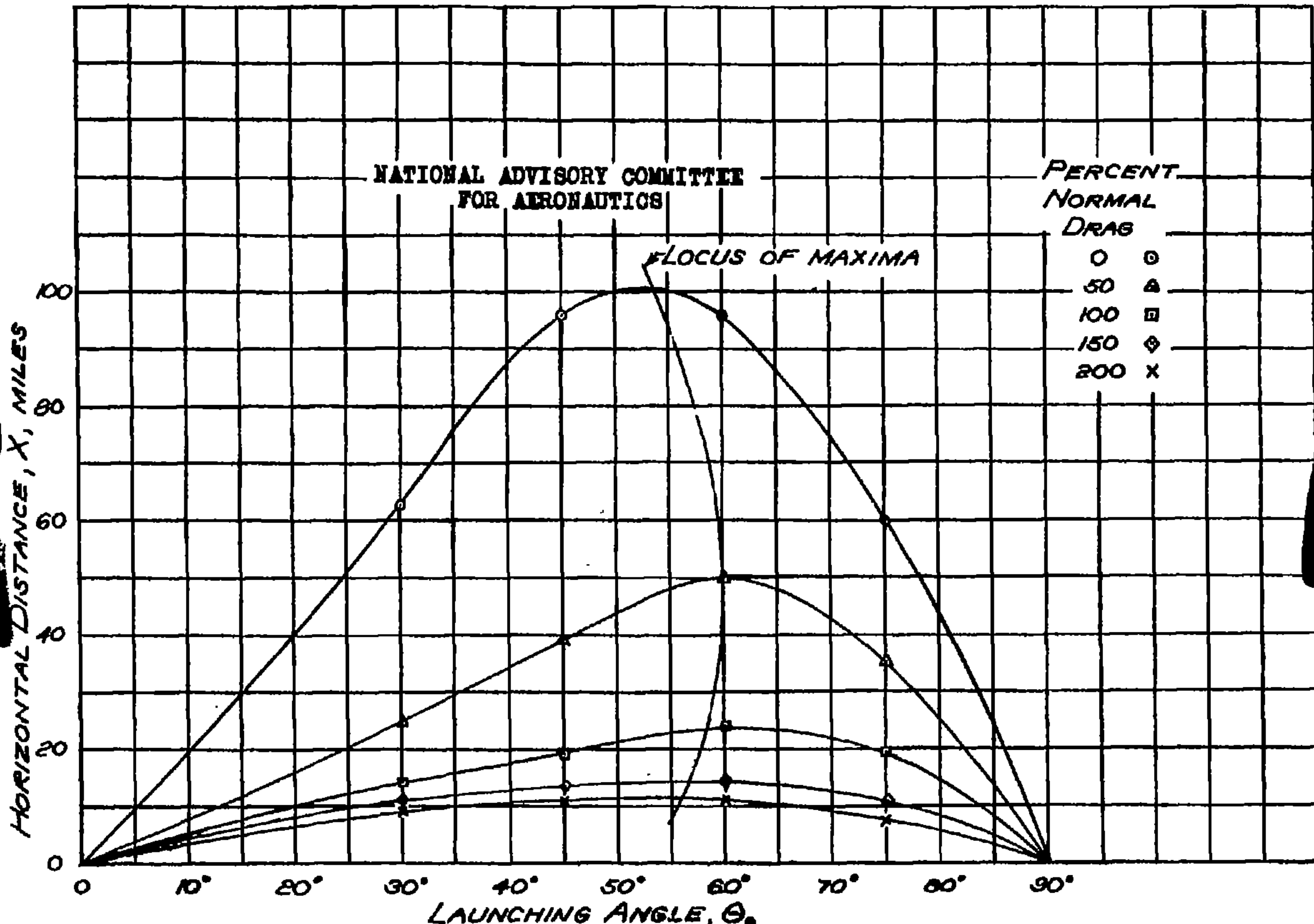


FIGURE 11.—VARIATION OF RANGE WITH LAUNCHING ANGLE FOR THE MISSILE WITH A LIQUID-FUEL ROCKET POWER PLANT AND SEVERAL ASSUMED VARIATIONS OF THE DRAG COEFFICIENT WITH MACH NUMBER, $\beta = 11.12$.

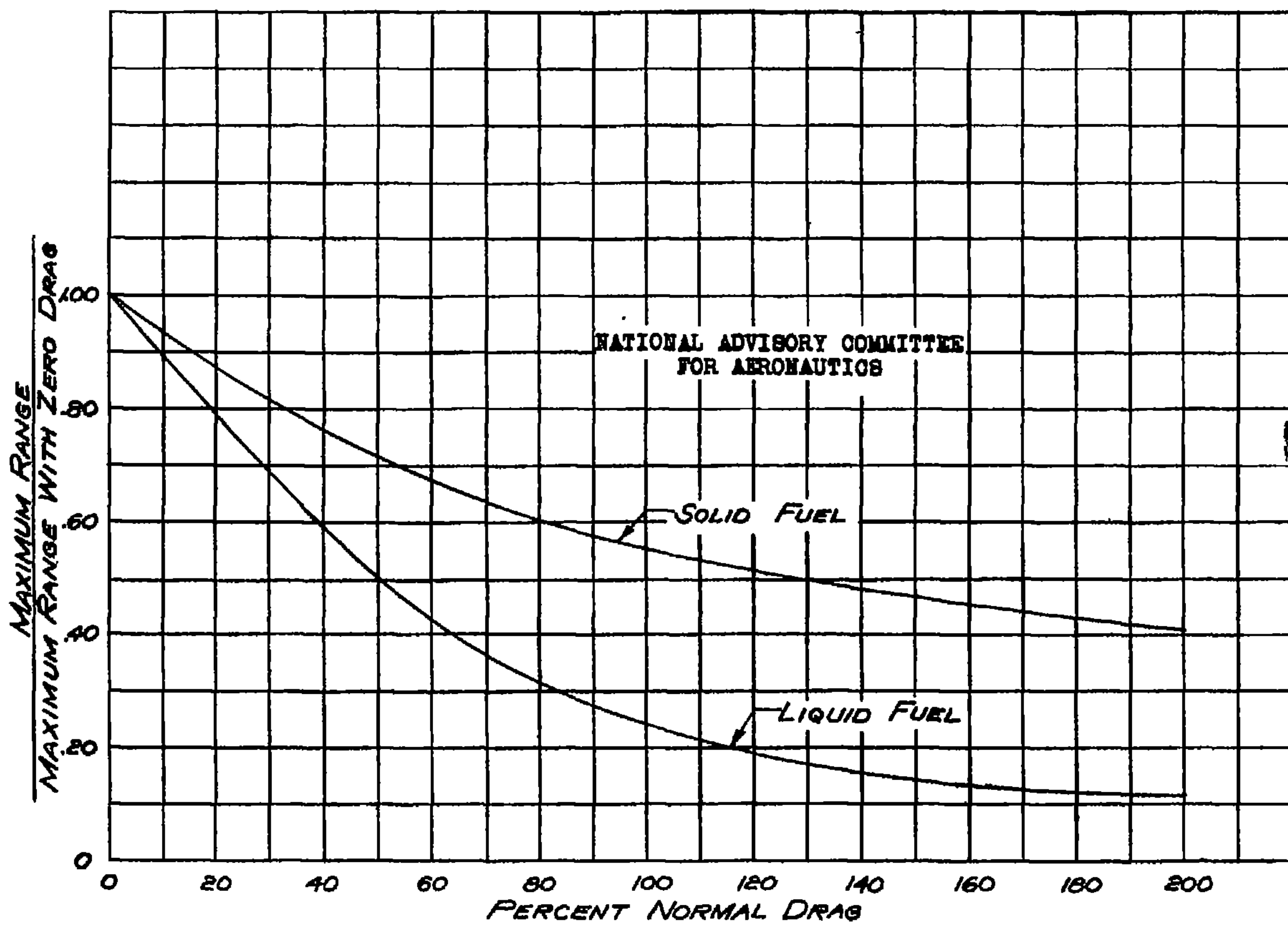


Figure 12.- Variation of maximum range with percent normal drag for the missile with solid- or liquid-fuel power: $\beta = 11.12$

[redacted]

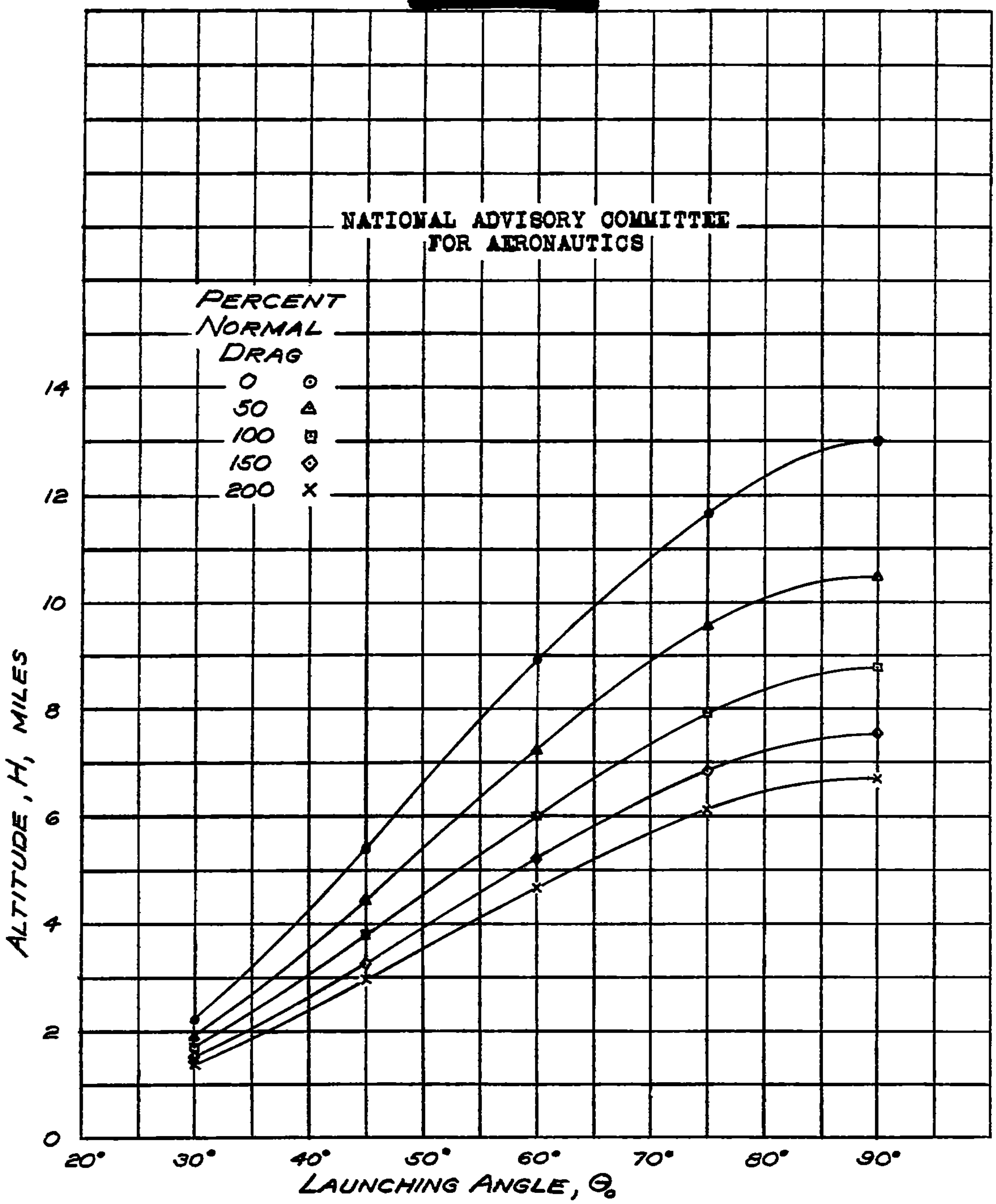


FIGURE 13.- VARIATION OF MAXIMUM ALTITUDE OF THE TRAJECTORY WITH LAUNCHING ANGLE FOR THE MISSILE WITH SOLID-FUEL ROCKET POWER AND SEVERAL ASSUMED VARIATIONS OF THE DRAG COEFFICIENT WITH MACH NUMBER. $\beta = 11.12$

[redacted]

M-2

[redacted]

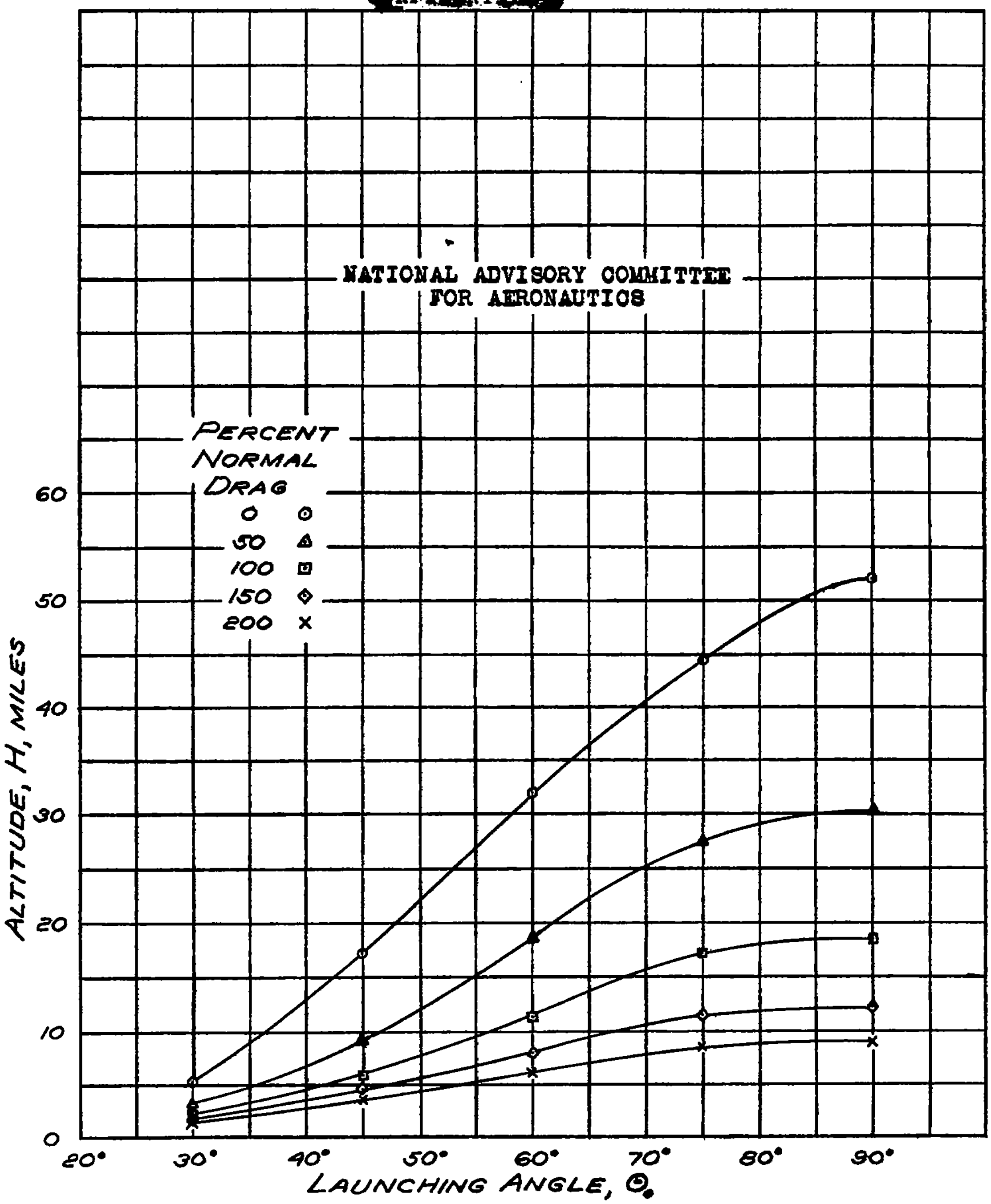


FIGURE 14.- VARIATION OF MAXIMUM ALTITUDE OF THE TRAJECTORY WITH LAUNCHING ANGLE FOR THE MISSILE WITH LIQUID-FUEL ROCKET POWER AND SEVERAL ASSUMED VARIATIONS OF THE DRAG COEFFICIENT WITH MACH NUMBER. β=11.12

[redacted]

M-2

[redacted]

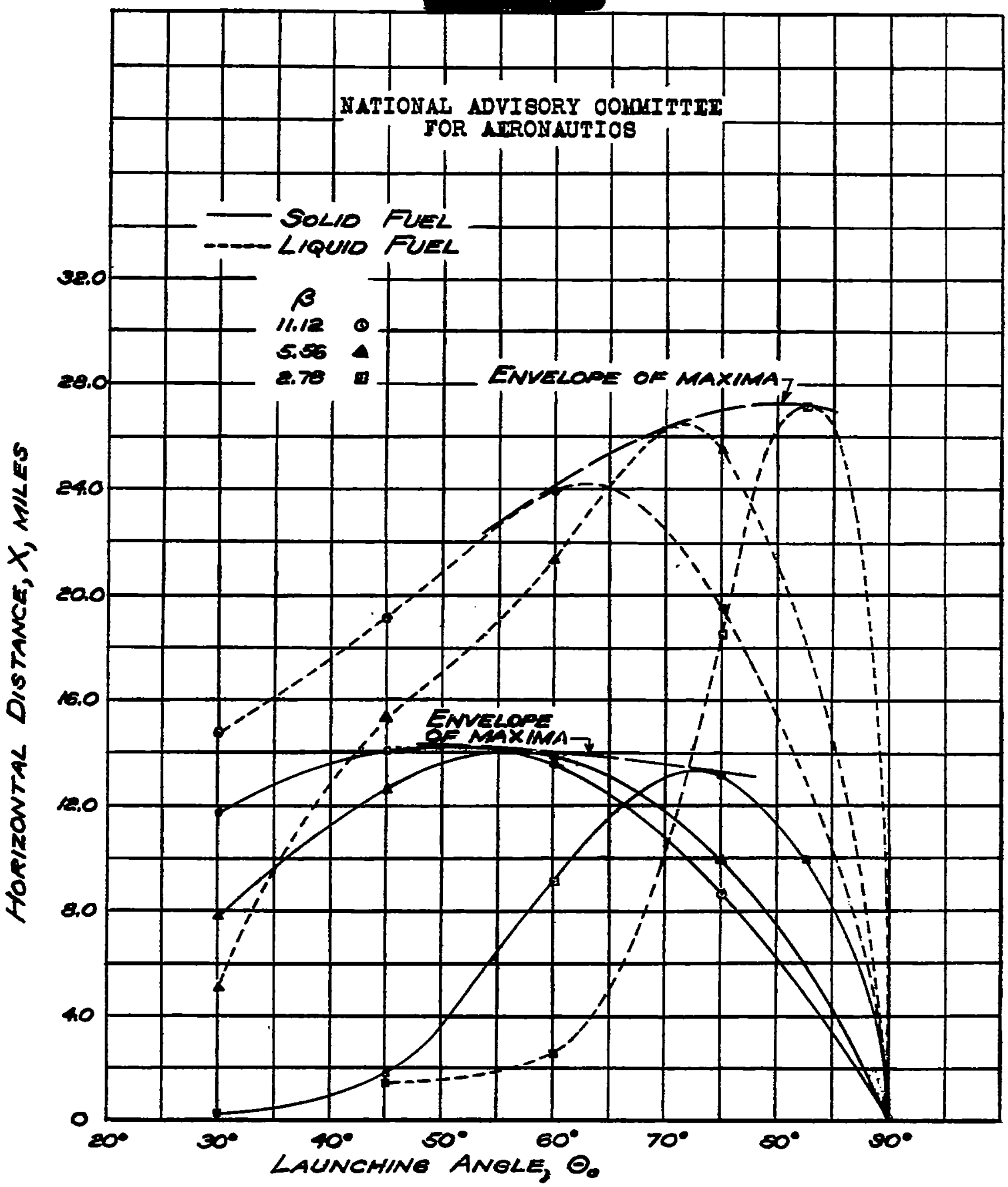


FIGURE 15.– VARIATION OF RANGE WITH LAUNCHING ANGLE FOR MISSILE WITH SOLID- OR LIQUID-FUEL ROCKET POWER FOR THREE VALUES OF INITIAL THRUST RATIO.

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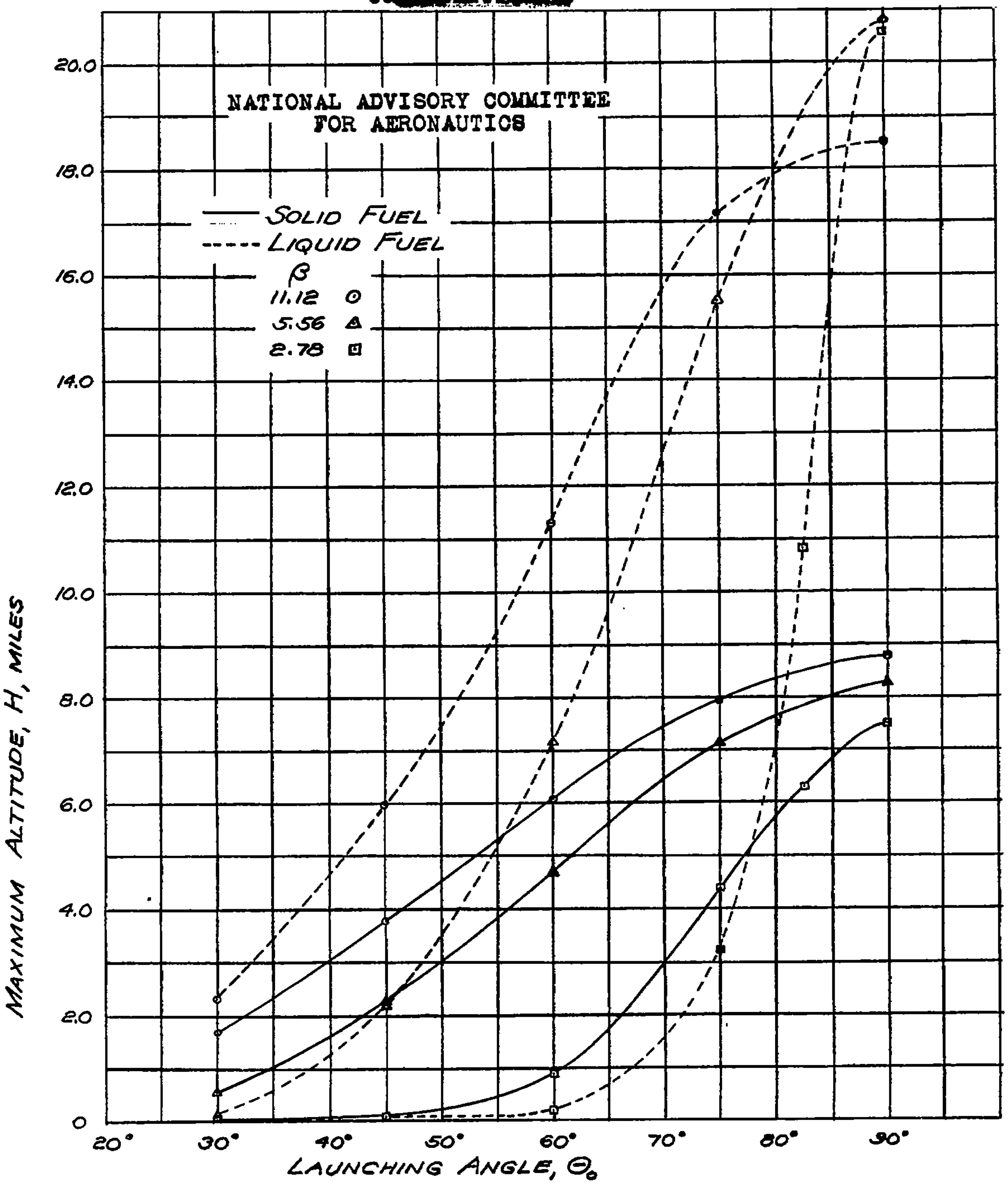


FIGURE 16.– VARIATION OF THE MAXIMUM ALTITUDE ATTAINED WITH LAUNCHING ANGLE FOR SOLID- OR LIQUID-FUEL ROCKET-POWERED MISSILES WITH THREE VALUES OF INITIAL THRUST RATIO.

CON[redacted]

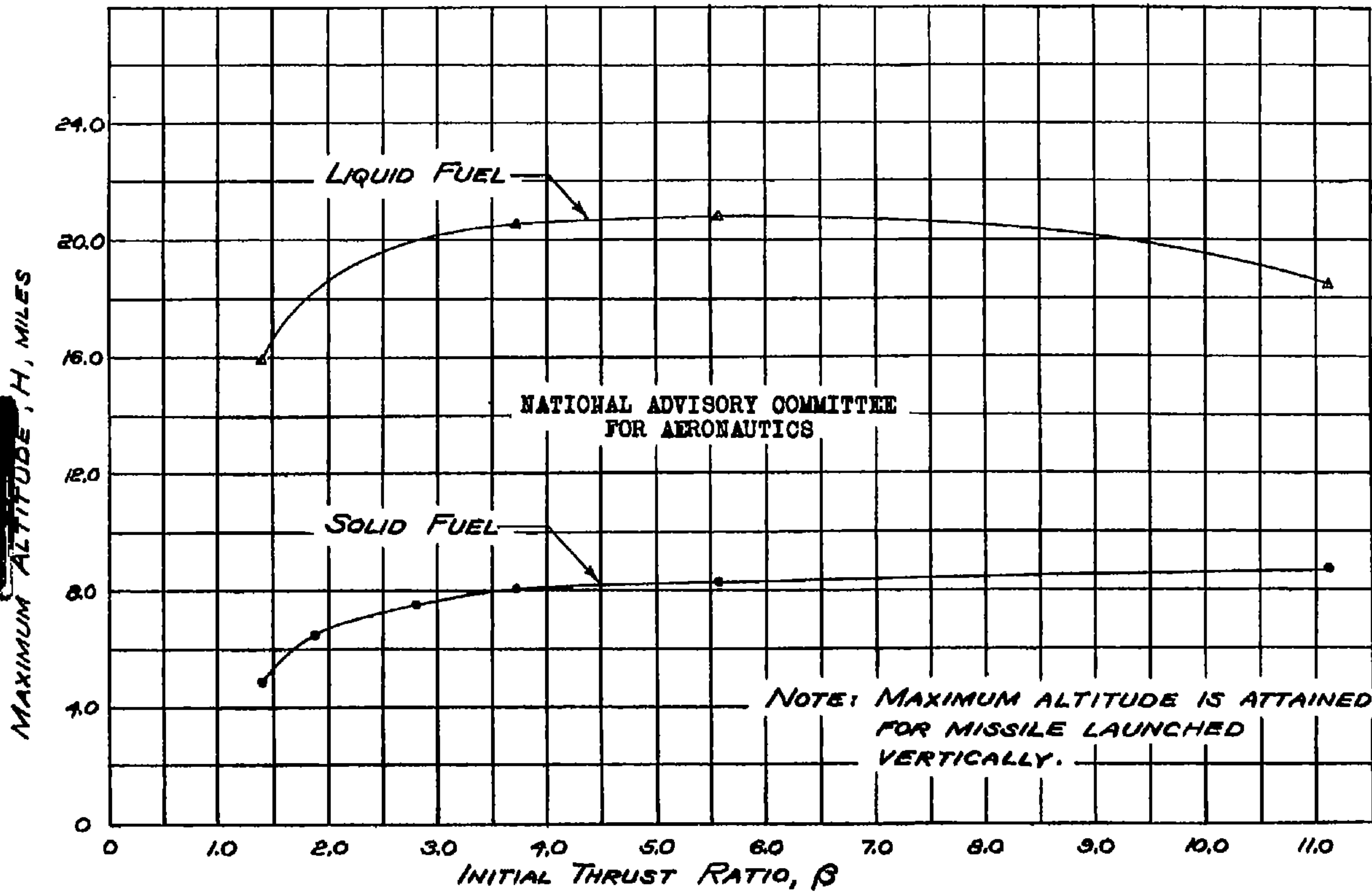
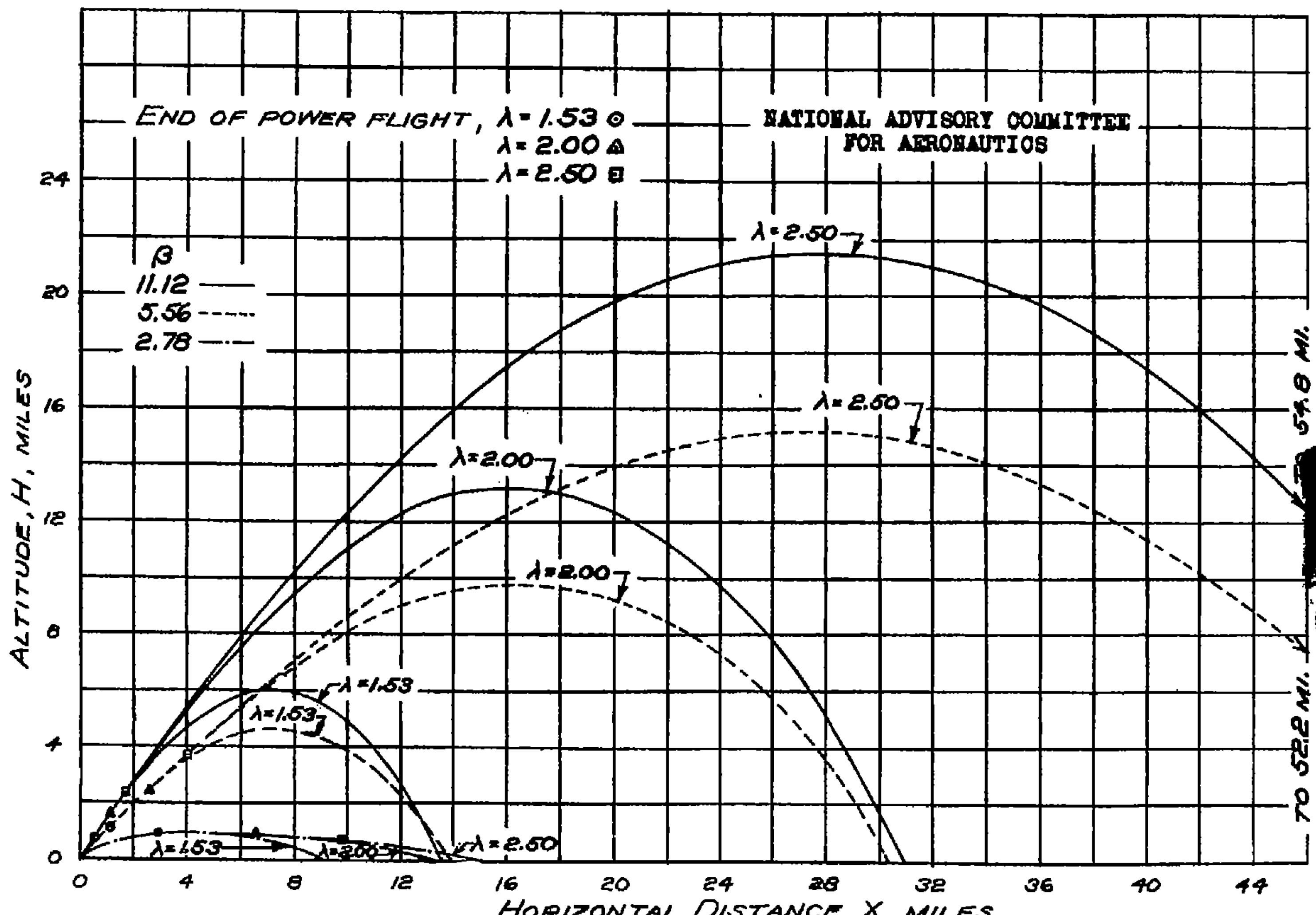


FIGURE 17.- VARIATION WITH INITIAL THRUST RATIO OF THE MAXIMUM ALTITUDE WHICH MISSILE WOULD ATTAIN IF POWERED WITH SOLID- OR LIQUID-FUEL ROCKETS.

Fig. 17

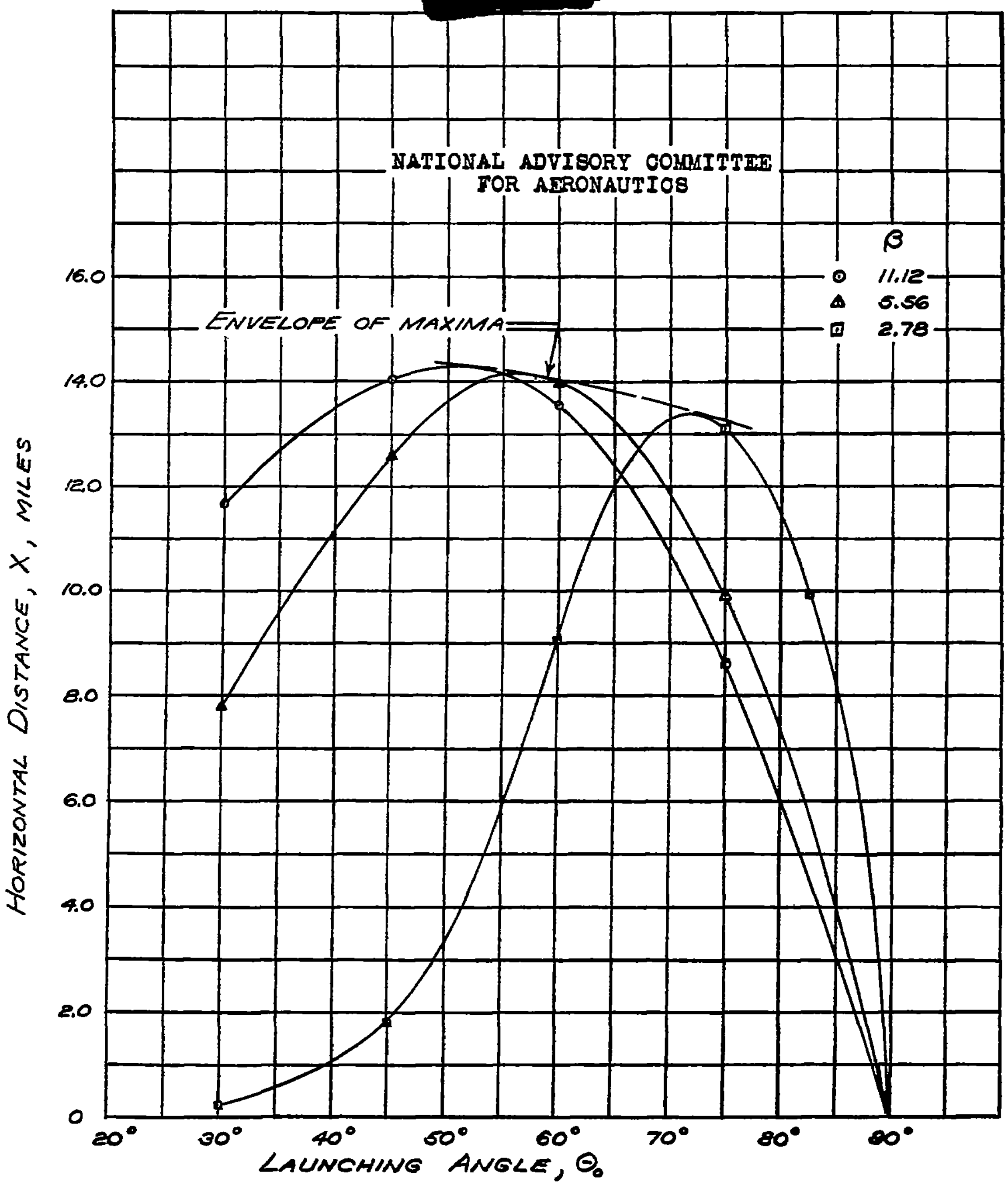
[redacted]



[redacted]

FIGURE 18.- COMPARISON OF TRAJECTORIES FOR SOLID-FUEL ROCKET POWERED MISSILE WITH THREE WEIGHT RATIOS AND THREE INITIAL THRUST RATIOS FOR A LAUNCHING ANGLE OF 60°.

M-2



(a) $\lambda = 1.53$.

FIGURE 19.- VARIATION OF RANGE WITH LAUNCHING ANGLE FOR SOLID-FUEL-POWERED MISSILE WITH THREE VALUES OF INITIAL THRUST RATIO.

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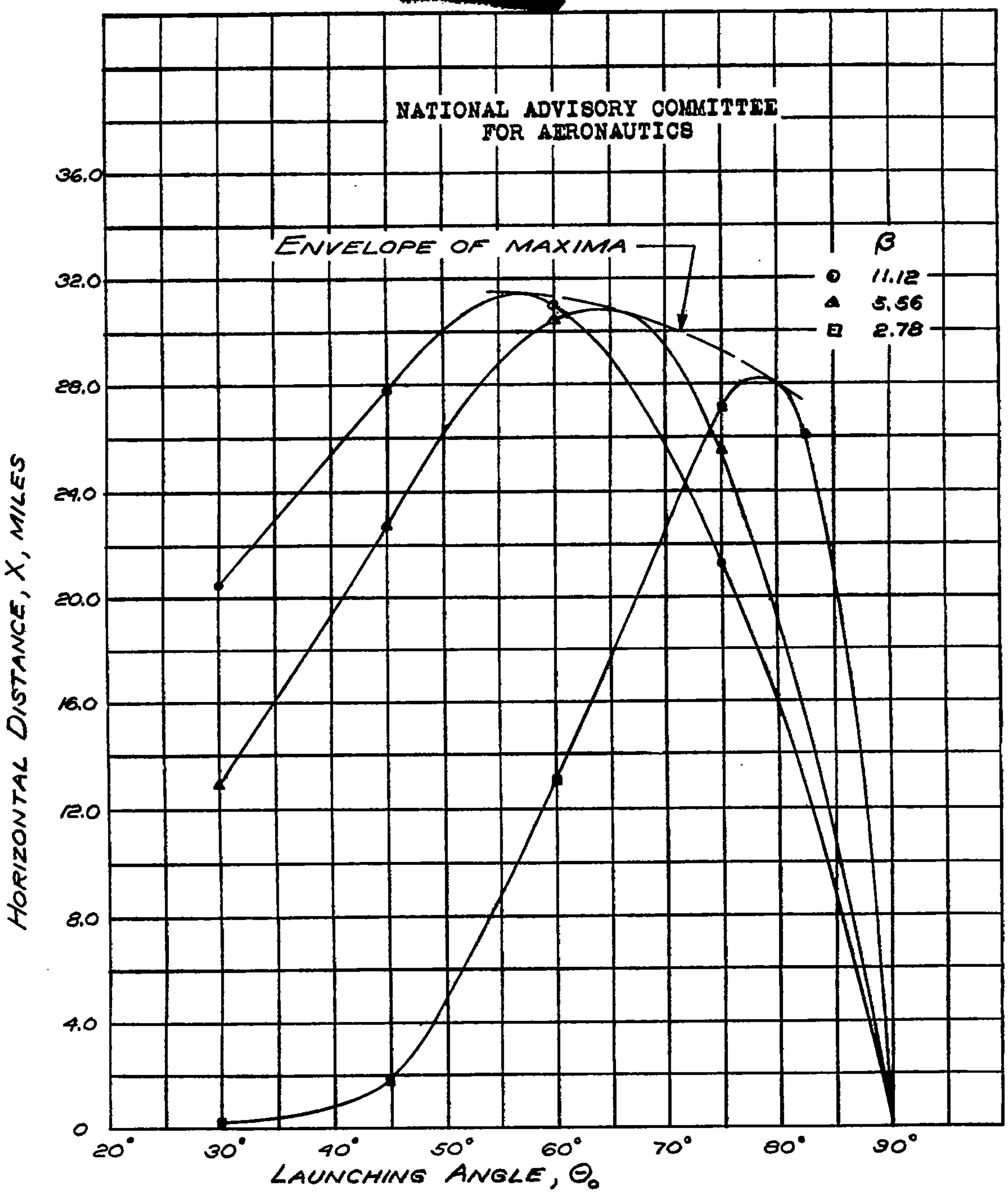

(b) $\lambda = 2.00$.

FIGURE 19.— CONTINUED.

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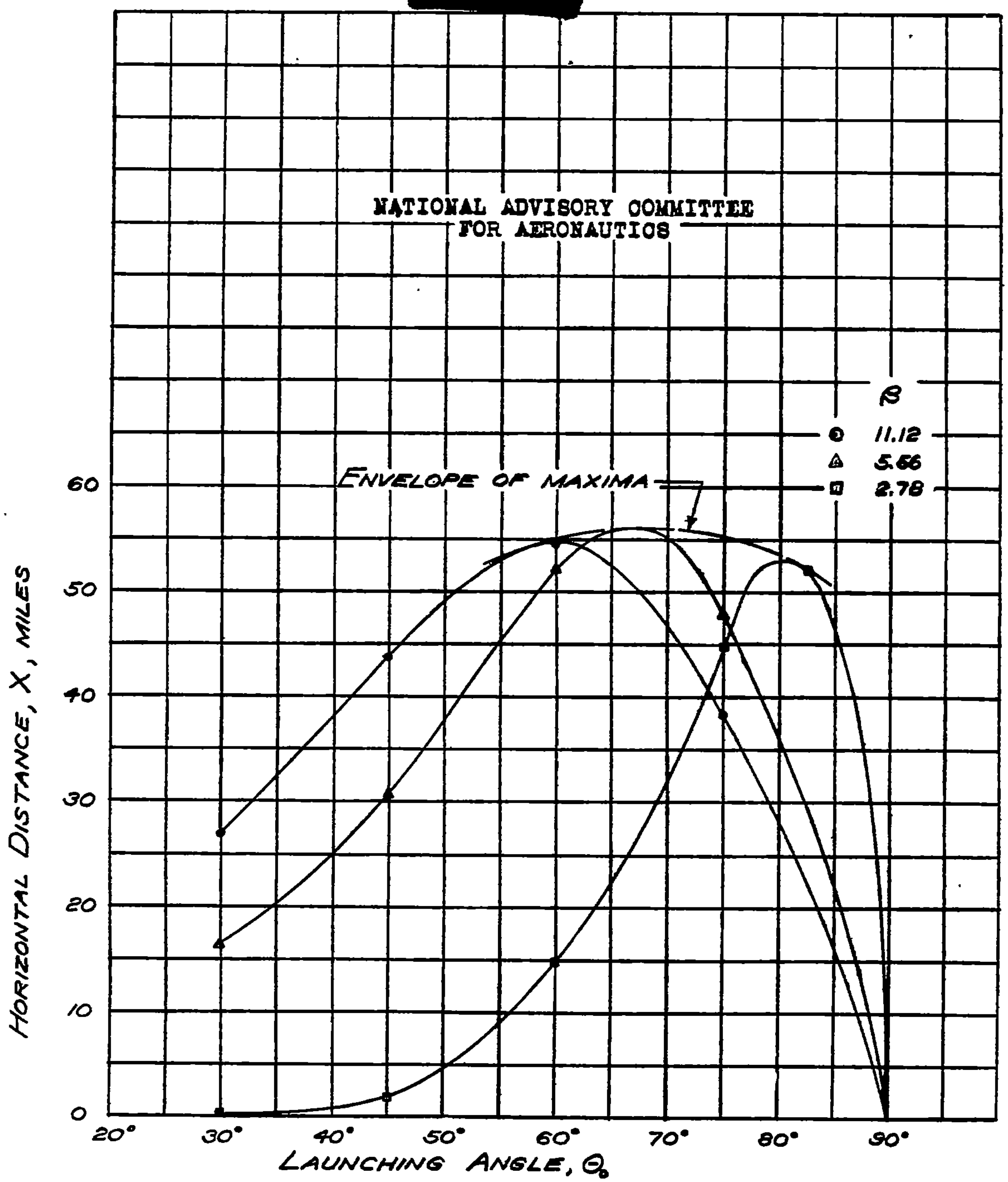
N-2

M-2



(c) $\lambda = 2.50$

FIGURE 19. CONCLUDED.

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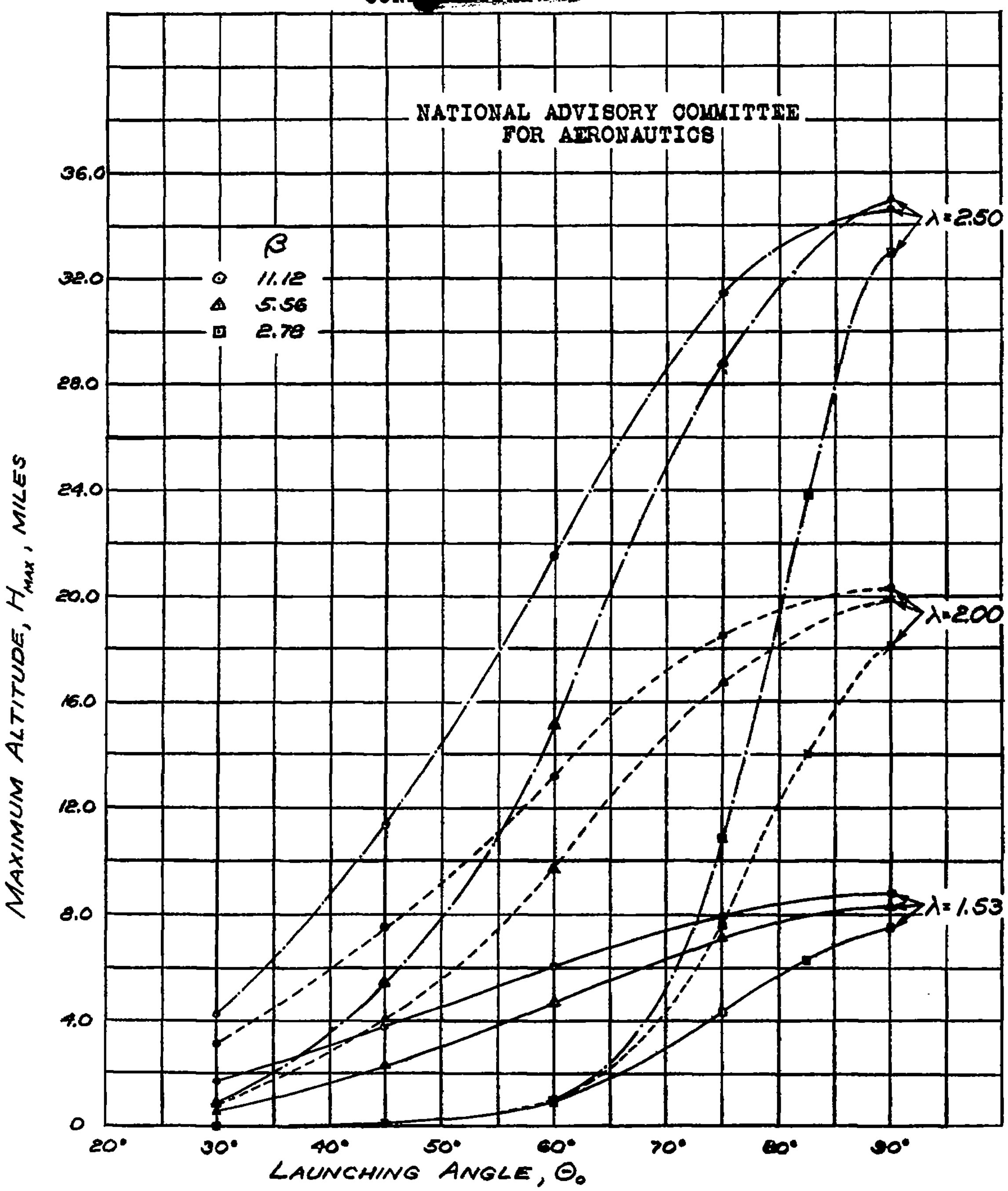

FIGURE 20.-VARIATION OF MAXIMUM ALTITUDE ATTAINED BY SOLID-FUEL-POWERED MISSILE WITH LAUNCHING ANGLE FOR THREE WEIGHT RATIOS AND THREE INITIAL THRUS[redacted]

[redacted]AL

M-2

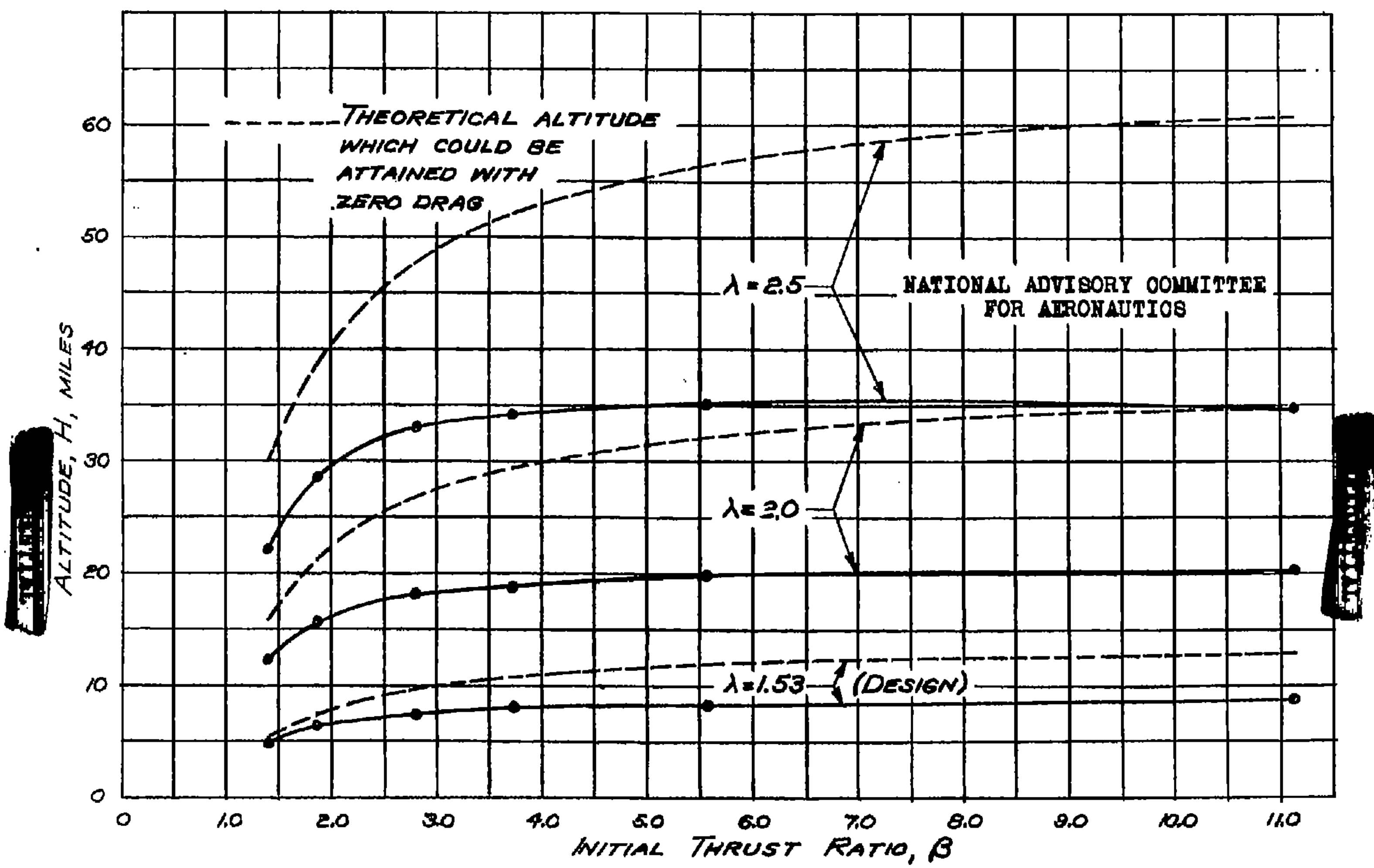

FIGURE 21.- VARIATION OF ALTITUDE ATTAINED IN THE ATMOSPHERE WITH THE RATIO OF THRUST TO INITIAL WEIGHT FOR THE ASSUMED SUPERSONIC MISSILE POWERED WITH SOLID FUEL AND LAUNCHED VERTICALLY.

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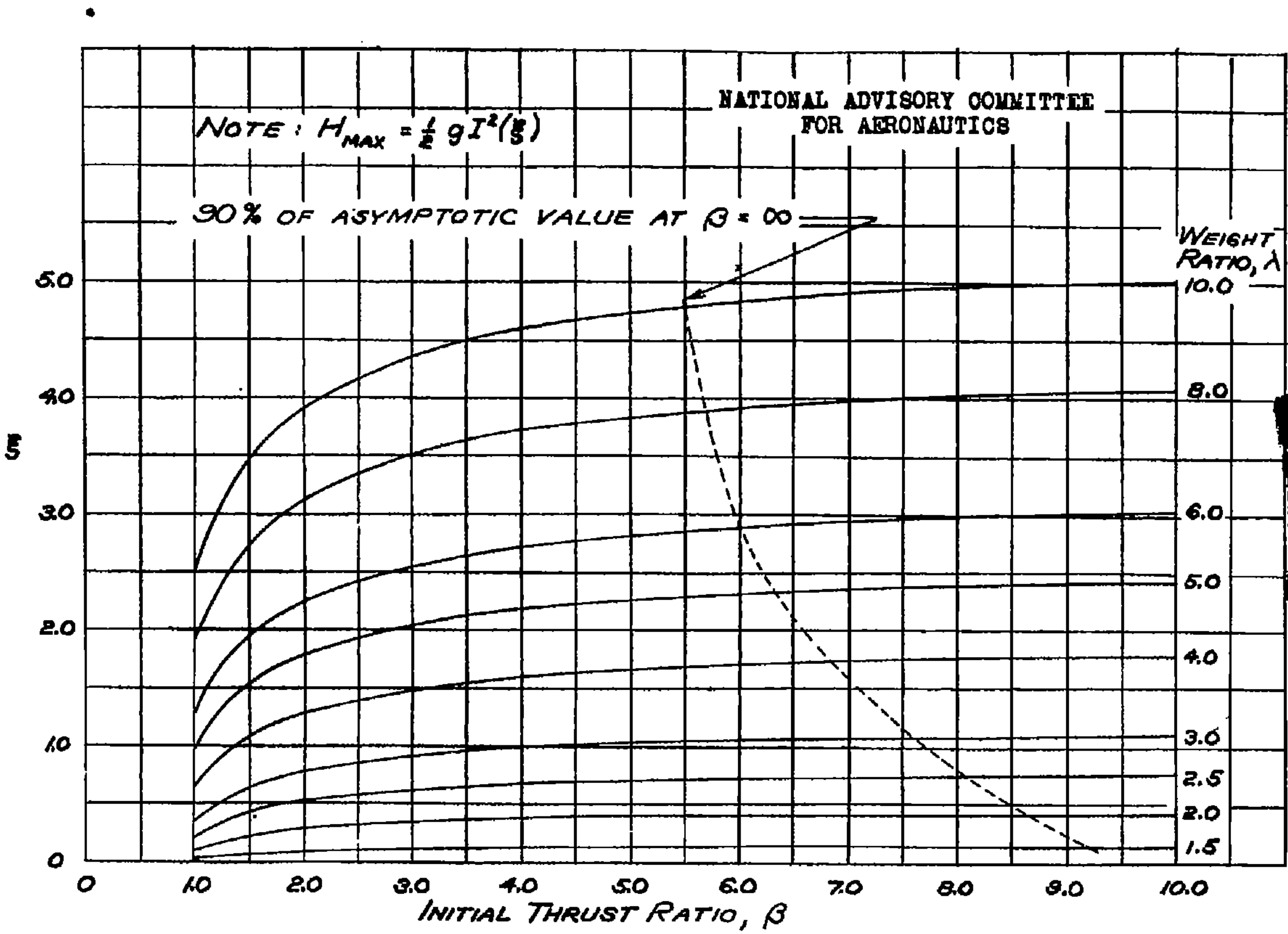

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FIGURE 22. – VARIATION OF THE FACTOR ξ WITH INITIAL THRUST RATIO FOR SEVERAL VALUES OF THE WEIGHT RATIO.