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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

FOR REFERENCE

No. 1070

AN EMPIRICAL EQUATION FOR THE COEFFICIENT OF HEAT TRANSFER
TO A FLAT SURFACE FROM A PLANE HEATED—AIR JET
DIRECTED TANGENTIALLY TO THE SURFACE

By John Zerbe and James Selna

Ames Aeronautical Laboratory
Moffett Field, Calif.

Washington
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SUMMARY

An investigation of the heat transfer to a surface from plane heated-air jets discharged tangentially to the surface was conducted to provide heat-transfer relationships required in the design of heated-air jet installations for aircraft windshield fog prevention.

Experimental temperature, velocity, and heat-transfer data were obtained by tests in which the initial jet temperature and velocity were varied from 101° to 156° F and from 52 to 218 feet per second, respectively. The jets were produced by three nozzles of different depths: namely, 0.102, 0.313, and 0.547 inch.

The resulting data were correlated to yield relation-ships for the maximum profile jet velocities and temperatures, and the coefficients of heat transfer from the jet to the surface, in terms of the nozzle-exit jet velocity and temperature and of the distance from the apparent jet origin. The test results are presented in tabular form and the correlations of the data are illustrated graphically.

INTRODUCTION

During an analytical investigation of the use of heated-air jets to prevent fog formations on the inside surface of bullet-resisting windshields, it was found that the required heat-transfer data were not available; consequently, the present investigation on the heat transfer to a surface from

a plane heated-air jet directed tangentially to the surface
(hereinafter designated as a surface jet) was undertaken.

The rate of transfer of heat from a surface jet to the
surface at a point any distance from the nozzle exit may be
defined as equivalent to the product of a coefficient of heat
transfer and the difference between the maximum profile tem-
perature in the jet and the temperature of the surface at
that point. The coefficient of heat transfer is dependent
principally upon the jet velocity and temperature. Equations
establishing the relationship between the jet velocity and
the major jet parameters of unheated surface jets are avail-
able in reference 1 and theoretical temperature relationships
for freely expanding heated jets are available in reference
2. The application of this information to heated-surface
jets has not been previously accomplished and there are no
available expressions for the coefficient of heat transfer
for surface jets.

The purpose of the present investigation was to establish
relationships generalizing the variation of velocity, tem-
perature, and coefficients of heat transfer in heated-surface
jets in order that a rational approach to the use of heated-
air jets for fog prevention could be made.

The investigation included the experimental evaluation
of the velocity, temperature, and heat-transfer character-
istics of heated-surface jets emerging from nozzles of three
different configurations at several initial velocity and
temperature conditions. The resulting data are correlated
in terms of the jet properties at the jet nozzle exit and the
relationships developed are generally applicable to the heat
transfer from surface jets to smooth flat surfaces when the
heat flow to the surface is comparable to that which prevails
in a plane heated-air jet installation for aircraft wind-
shield fog prevention.

This investigation was conducted as part of a general
study of aircraft windshield fog prevention which was under-
taken by the Ames Aeronautical Laboratory at the request of
the Bureau of Aeronautics, Navy Department.

SYMBOLS

The symbols used in this report are defined as follows:

c   constant

d   surface jet-nozzle depth, feet

d_r reference surface jet-nozzle depth (1/12 ft)

e distance from apparent jet origin to nozzle exit ($d/\tan \alpha$), feet

h coefficient of heat transfer for surface jet, Btu per hour, square foot, $^\circ F$

k_l thermal conductivity of Lucite plate, Btu per hour, square foot, $^\circ F$ per foot

k thermal conductivity of air, Btu per hour, square foot, $^\circ F$ per foot

L distance from nozzle exit to point under consideration, feet

l thickness of Lucite plate, feet

p barometric pressure, inches of mercury

q unit heat transfer through the Lucite plate, Btu per hour, square foot

T_m maximum profile jet temperature at any distance x, $^\circ F$

T_o jet temperature at nozzle exit, $^\circ F$

T_a ambient-air temperature, $^\circ F$

T_t temperature of top surface of Lucite plate, $^\circ F$

T_b temperature of bottom surface of Lucite plate, $^\circ F$

θ_m maximum profile jet temperature rise above ambient-air temperature, at any distance x, $^\circ F$

θ_o jet temperature at nozzle exit above ambient-air temperature, $^\circ F$

U velocity at any point in the jet, feet per second

U_m maximum profile jet velocity at any distance x, feet per second

U_o jet velocity at nozzle exit, feet per second

x distance from apparent jet origin, feet

y distance perpendicular to surface, feet

y_r distance perpendicular to surface where jet velocity
           is one—half U_m, feet

$\dfrac{hx}{k}$ Nusselt number

$\dfrac{U_o x \rho}{\mu}$ Reynolds number

ρ density of air, slugs per cubic foot

α angle of expansion of surface jet, degrees

μ absolute viscosity of air, pound—seconds per square
           foot

EQUIPMENT AND TEST PROCEDURE

The test equipment employed (fig. 1) consisted of a
surface jet nozzle discharging heated air over a Lucite
plate (12 in. wide by 24 in. long by 1/4 in. thick), which
was mounted above an ice bath. Surface—type thermocouples
were installed on the upper and lower surfaces of the Lucite
plate (fig. 2) at four stations 5, 10, 15, and 20 inches
from the nozzle exit, to measure the temperature drop through
the plate. The thermal conductivity of the plate (k_l =
0.125 Btu/hr, sq ft, $^\circ$F/ft) was determined experimentally
and the plate was employed as a heat meter.

Jet velocities and temperatures were measured in a
vertical plane extending through the lengthwise center line
of the plate. These measurements were made with a velocity—
temperature probe (fig. 1) containing a total and a static
pressure tube and a thermocouple probe. A micromanometer
was employed to determine the jet velocities and the jet
temperatures and Lucite—plate surface temperatures were
indicated by a Brown, direct—reading, self—balancing
potentiometer.

Tests were conducted with three surface jet nozzles
(fig. 3) 0.547, 0.313, and 0.102 inch deep at the nozzle
exits herein designated as nozzles A, B, and C, respectively.
Each nozzle was 12 inches wide. In order to insure uniform
jet air flow, the nozzles were all designed to have the
exit depth and width prevail for a length of at least 10

nozzle depths before the nozzle exit. Nozzle-exit jet tem-
peratures of approximately 100° and 150° F together with
nozzle-exit velocities ranging from 50 to 220 feet per second
were employed during the tests. Five to nine tests were con-
ducted with each nozzle, and during each test, measurements
of the maximum profile velocity and temperature in the jet
and of the heat transferred through the Lucite plate were
obtained at each of the four instrumented stations. Velocity
profiles were measured at each station during several of the
tests conducted with nozzle B.

RESULTS AND DISCUSSION

The results of the tests are presented in table I.
The jet velocities given were calculated from the difference
between the total pressure in the jet and the ambient static
pressure. The static-pressure measurements in the jet were
not used in these calculations because the standard static
tube employed erroneously recorded large negative pressures.
Other investigators (references 1 and 3) using similar
equipment also found this to be true. However, when more
refined equipment was employed, they found the correct static
pressure in the jet to be approximately one-half percent of
the dynamic pressure above ambient static pressure. There-
fore, the error involved in using the ambient static pres-
sure is negligible. The rate of heat transfer through the
Lucite plate and the coefficients of heat transfer were
evaluated by the relationships

$$q = \frac{k_{7}t}{l}\left(T_t - T_b\right)$$

and

$$h = \frac{q}{T_m - T_t}$$

A typical set of the velocity profile data obtained
during the tests of nozzle B has been plotted nondimen-
sionally, U/U_m as a function of y/y_r, in figure 4. The
curve drawn on this plot is from a similar plot given in
reference 1 for velocity profiles in an unheated surface
jet. Since the experimental points conform to the curve,
it is evident that the jet temperatures and the heat trans-
fer from the jet had no measurable effect on the velocity

profiles. Thus, it was concluded that relationships defin-
ing jet velocities in unheated surface jets could be applied
equally well to heated surface jets. Concerning the maximum
profile velocities, reference 1 states that the maximum pro-
file velocity $(U_m)_1$ at any point x_1 is related to the
maximum profile velocity $(U_m)_2$ at any other point x_2 in
the same jet by the relationship

$$\frac{(U_m)_1}{(U_m)_2} = \left(\frac{x_1}{x_2}\right)^{-\frac{1}{2}} \tag{1}$$

where x (fig. 5) is the distance from the apparent jet
origin to the point under consideration and may be defined
as

$$x = L + e \tag{2}$$

where

$$e = \frac{d}{\tan \alpha} \tag{3}$$

The angle α is indicated from the data of reference 1 to
be approximately $8\frac{1}{2}°$ and this value is in agreement with
that observed during the present tests. Measurements of
the maximum profile velocities for each nozzle showed that
the initial velocity U_o was equal to U_m for a length
of approximately $4d$ from each nozzle exit. Thus, equation
(1) may be written as follows:

$$\frac{U_m}{U_o} = \left(\frac{e + 4d}{x}\right)^{\frac{1}{2}} \tag{4}$$

This relationship is compared to the test data in figure 6.
The experimental points conform to the curve (equation (4))
to a better degree at the higher values of $\frac{e + 4d}{x}$, and it
is probable that the relationship is not strictly valid at
low values of $\frac{e + 4d}{x}$. Low values of $\frac{e + 4d}{x}$ are encountered
at distances x which are large with respect to the quantity
$e + 4d$, a constant for any given nozzle depth. The maxi-
mum deviation of any of the points from the curve is less
than 30 percent.

Theoretical temperature data for freely expanding plane
jets (reference 2) show that the maximum profile jet tem-
peratures above ambient-air temperature are proportional to
the maximum profile jet velocities. Thus the maximum pro-
file jet temperature data given in table I were plotted
nondimensionally (θ_m/θ_o) as a function of velocity (U_m/U_o)
in figure 7. The plot provides the relationship

$$\frac{\theta_m}{\theta_o} = \frac{U_m}{U_o} \qquad (5)$$

The maximum deviation of any of the test points from equa-
tion (5) is less than 25 percent. Equation (5) assumes
that the heat transfer to the surface has a negligible
effect on the temperature in the jet. This assumption is
based on the fact that the heat transferred to the surface
is very small in comparison with the heat content of the
jet and its validity is amply verified by figure 7. Further-
more, the heat flows obtained during the tests are com-
parable with those which would prevail in a plane heated-
air jet installation for aircraft windshield fog prevention.

The coefficient of heat transfer h is presumed to be
defined from the theory of similarity of heat transfer (refer-
ence 4) as

$$\frac{hx}{k} = c\left(\frac{U_o x \rho}{\mu}\right)^n \qquad (6)$$

where c is a constant for any given surface jet. By use
of equations (3), (4), and (6) a constant c_1 may be defined
for any surface jet as follows: If two surface jets with a
common jet origin but of nozzle depths d and d_1, re-
spectively, are operated so that at a distance x_1 from the
apparent jet origin each has the same maximum profile tem-
perature and maximum profile velocity, the velocity profiles
at the point x_1 will be the same for each jet and there-
fore the coefficients of heat transfer for each jet at that
point will be identical. Thus from equation (6)

$$c \, (U_o)^n = c_1 \, (U_o)_1^n \qquad (7)$$

The combination of equation (7) with equations (3) and (4)
yields

$$\frac{c}{d^{n/2}} = \frac{c_1}{d_1^{\,n/2}}$$

and equation (6) may be written in terms of c_1 and d_1 as

$$\frac{hx}{k} = c_1 \left[\frac{U_0 x \rho}{\mu} \left(\frac{d}{d_1} \right)^{\frac{1}{2}} \right]^n \tag{8}$$

Values of hx/k and $\left(\dfrac{U_0 x \rho}{\mu} \right) \left(\dfrac{d}{d_1} \right)^{\frac{1}{2}}$ have been evaluated

in table I using a value of 1/12 foot for d_1, this value being defined as d_r. Logarithmic plots of hx/k as a

function of $\left(\dfrac{U_0 x \rho}{\mu} \right) \left(\dfrac{d}{d_r} \right)^{\frac{1}{2}}$ for each nozzle tested as well

as a comparison of these plots are presented in figure 8. The maximum deviation of any of the test points from the mean curve for any of the plots is less than 20 percent.

The curves of figure 8 all have a slope of 0.65; thus $n = 0.65$ for all the jet nozzles tested. The plots for nozzles A and B show c_1 to be 0.16, while that for nozzle C yields a value of c_1 of 0.21. It is probable that the value of 0.16 is more reliable, for as pointed out with regard to the velocities, the expressions developed herein may

not be strictly valid at low values of $\dfrac{e + 4d}{x}$ and all the

data for nozzle C were taken at low values of $\dfrac{e + 4d}{x}$. (See

fig. 6.) Also, the values of d may change slightly during actual test operation, and any change in d would affect the value of $(d/d_r)^{\frac{1}{2}}$ for nozzle C by a greater amount than it would for the other nozzles. Furthermore, little consideration need be given to the data of nozzle C since nozzles of such a small depth $(d = 0.102$ in.$)$ would have little practical use. Thus, the equation recommended for the evaluation of the coefficient of heat transfer for surface jets is

$$\frac{hx}{k} = 0.16 \left[\left(\frac{U_o x \rho}{\mu} \right) \left(\frac{d}{d_r} \right)^{\frac{1}{2}} \right]^{0.65} \tag{9}$$

where d_r is equal to 1/12 foot.

Ames Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Moffett Field, Calif., February 14, 1946.

REFERENCES

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2. Howarth, L.: Concerning the Velocity and Temperature
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    Proc. of the Cambridge Phil. Soc., vol. 34, pt. II,
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3. Prandtl, L.: The Mechanics of Viscous Fluids, Vol. III,
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4. Schack, Alfred: Industrial Heat Transfer. Translated
    from the German by Hans Goldschmidt and Everett P.
    Partridge. John Wiley and Sons, Inc., 1933.

TABLE I.- HEAT-TRANSFER DATA FOR SURFACE JETS DISCHARGED FROM NOZZLES A, B, AND C.

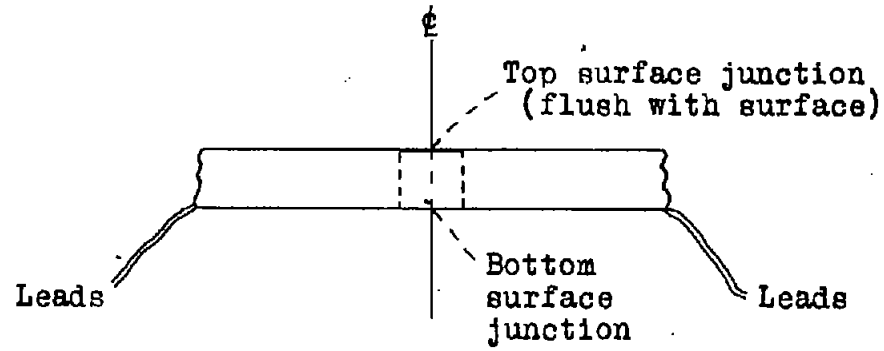| Surface jet nozzle		Nozzle A, d=0.547 in.					Nozzle B, d=0.313 in.									Nozzle C, d=0.102 in.							
Run number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Barometric pressure, p (in. Hg)		29.9	29.9	29.9	29.9	29.9	29.9	30.0	29.9	30.0	29.9	29.8	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Ambient air temp., T_a (°F)		78	77	77	81	82	78	76	78	77	79	81	81	78	76	79	77	77	79	78	80	82	77
Jet temp. at nozzle exit, T_0 (°F)		111	101	101	155	150	115	117	109	115	105	146	150	141	140	109	105	105	110	107	156	145	147
Jet velocity at nozzle exit, U_0 (ft/sec)		52	107	156	56	112	54	72	108	126	154	56	76	104	130	113	120	148	163	210	119	169	218
Maximum jet temp., T_m (°F)	1	104	97	97	139	137	102	105	100	105	99	123	127.5	125	123	91.5	91	90	93	91.5	115.5	111	112
	2	97	90	90	125	120	94.5	97	93	99	94.5	110	116	112	110	----	----	----	----	----	----	----	----
	3	93.5	89	89	118.5	117	91.5	92.5	90	97	93	105	111	108	106	85	83	83	85	84	99	96	97.5
	4	91	87.5	88	115	115	91	94	89	97	92	105	109	107	105	----	81.5	81	----	----	----	94.5	----
Maximum jet velocity, U_m (ft/sec)	1	43	89	134	44	91	37	49	76	89	116	38	49	72	92	49	51	64	74	99	48	68	96
	2	33	65	95	32	67	26	36	54	65	89	27	36	51	67	31	34	42	48	65	31	47	63
	3	26	56	82	28	59	22	30	45	53	76	23	29	42	53	23	26	33	38	50	24	35	48
	4	24	55	78	24	57	21	26	41	48	69	21	26	38	47	20	22	28	31	42	21	29	41
Lucite plate top surface temp., T_t (°F)	1	76	77.5	80	95	105	75	76	78	78	79	88	90	91	93	72	73	75	77	77	89	89	91
	2	70	73	75	85	89	67	69	73	74	77	74.5	78	80	81	----	----	----	----	----	----	----	----
	3	68	70	73	80	87	64.5	66	70	71	74.5	72.	74	76	77	63	63.5	66	66	70	73	77	78
	4	65	70	70	74	82	60.5	63	65	67.5	72	67.5	68	71	72.5	----	60	62	----	----	----	71	----
Lucite plate bottom surface temp., T_b (°F)	1	35	37	35	41.5	36	39	34.5	34	32	34	37	38	33.5	34	34	40	39.5	37	38	35	33	35
	2	33	40	40	36	34	36	35	36.5	33	37	35	34	36.5	38	----	----	----	----	----	----	----	----
	3	37	33	37	38	37	36	35	36	35	33	38	33	34	35	34	34	34	36.5	37	37	40	38
	4	37	40	39	36	34	34	35	36.5	34	40	36	34	34	36	----	34	36	----	----	----	37	----
Heat transferred through lucite plate, q (Btu/hr, ft²)	1	246	243	269	321	413	216	249	264	276	270	306	312	345	354	228	197	213	240	234	324	336	336
	2	222	198	210	294	330	186	204	219	246	240	237	264	261	258	----	----	----	----	----	----	----	----
	3	186	222	216	252	300	171	186	204	216	249	204	246	252	252	177	177	192	177	198	216	222	240
	4	168	180	186	228	288	159	168	171	201	192	189	204	222	219	----	156	156	----	----	----	204	----
Coefficient of heat transfer from jet to plate, h (Btu/hr, ft², °F)	1	8.8	12.5	15.8	7.3	12.9	8.0	8.6	12.0	10.2	13.5	8.7	8.3	10.1	11.8	11.7	11.0	14.2	15.0	16.1	12.2	15.3	16.0
	2	8.2	11.6	14.0	7.3	10.6	6.8	7.3	11.0	9.8	13.7	6.7	7.0	8.2	8.9	----	----	----	----	----	----	----	----
	3	7.3	11.7	13.5	6.5	10.0	6.3	6.3	10.2	8.3	13.4	6.2	6.6	7.9	8.7	8.0	9.1	11.3	9.3	14.1	8.3	11.7	12.3
	4	6.5	10.3	10.3	5.6	8.7	5.2	5.4	7.1	6.8	9.6	5.0	5.0	6.2	6.7	----	7.2	8.2	----	----	----	8.7	----
Nusselts number $\frac{hr}{k}$	1	410	580	730	320	570	300	320	450	380	510	320	300	370	430	350	330	430	450	480	360	450	470
	2	600	840	1020	500	730	440	470	700	630	880	420	440	510	560	----	----	----	----	----	----	----	----
	3	730	1160	1340	620	950	570	570	920	750	1220	550	580	700	770	660	760	940	770	1170	680	950	1000
	4	830	1300	1310	680	1050	610	630	830	800	1130	570	570	710	770	----	790	900	----	----	----	970	----
Reynolds number $\frac{U_0 x \rho}{\mu} \times 10^{-4}$	1	21	44	66	21	43	18	24	36	42	52	16	22	30	36	30	32	40	44	56	31	43	55
	2	33	69	100	34	68	27	41	62	72	88	28	37	51	64	----	----	----	----	----	----	----	----
	3	46	94	137	47	93	44	58	87	102	124	39	53	72	90	83	89	109	121	155	85	117	151
	4	58	120	177	59	118	56	75	113	132	161	51	68	93	117	----	118	144	----	----	----	155	----
$\frac{U_0 x \rho}{\mu}\left(\frac{d}{d_1}\right)^{\frac{1}{2}} \times 10^{-4}$	1	16	32	47	16	32	10	13	20	24	29	9	12	17	20	10	10	12	14	18	10	14	18
	2	24	51	74	25	50	17	23	35	40	49	16	21	28	36	----	----	----	----	----	----	----	----
	3	34	70	101	35	69	25	32	49	57	69	22	30	40	50	26	28	35	30	49	26	37	48
	4	43	89	131	44	87	31	42	63	74	90	29	38	52	65	----	37	46	----	----	----	49	----
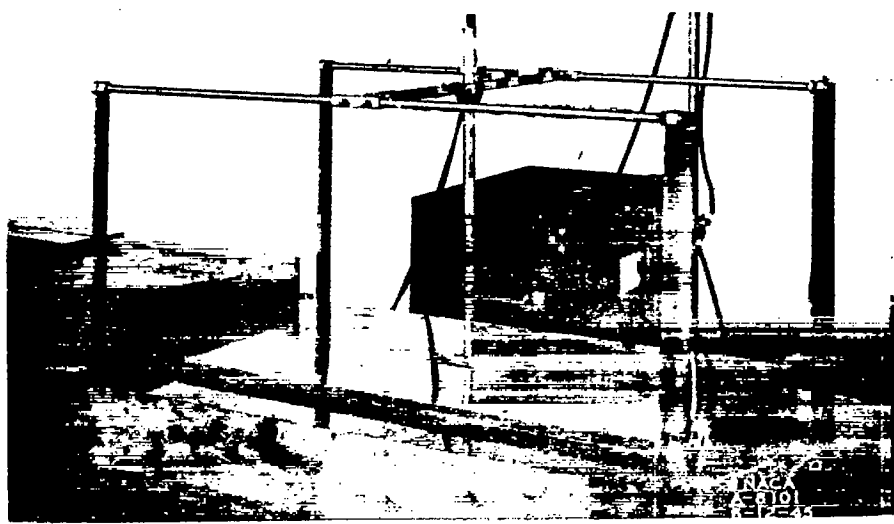
(a) Side view
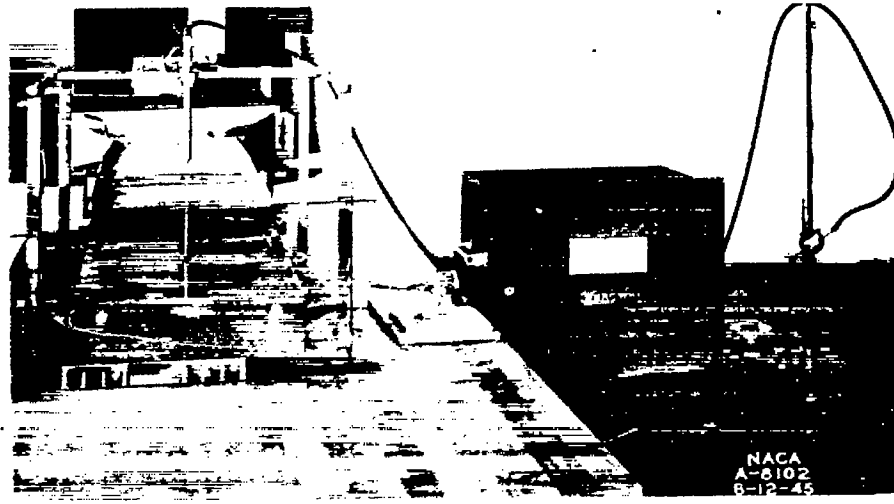
(b) Front view

Figure 1.- Side and front views of surface jet
test apparatus.

Figure 3.- Surface jet nozzles A, B, and C.
(Right to left.)

Top surface junction
(flush with surface)

Leads

Bottom
surface
junction

Leads

Section A-A

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Surface jet nozzle

A

d

5" 5" 5" 5"

1/4"

Station
1

Station
2

Station
3

Station
4

A

At least 10d
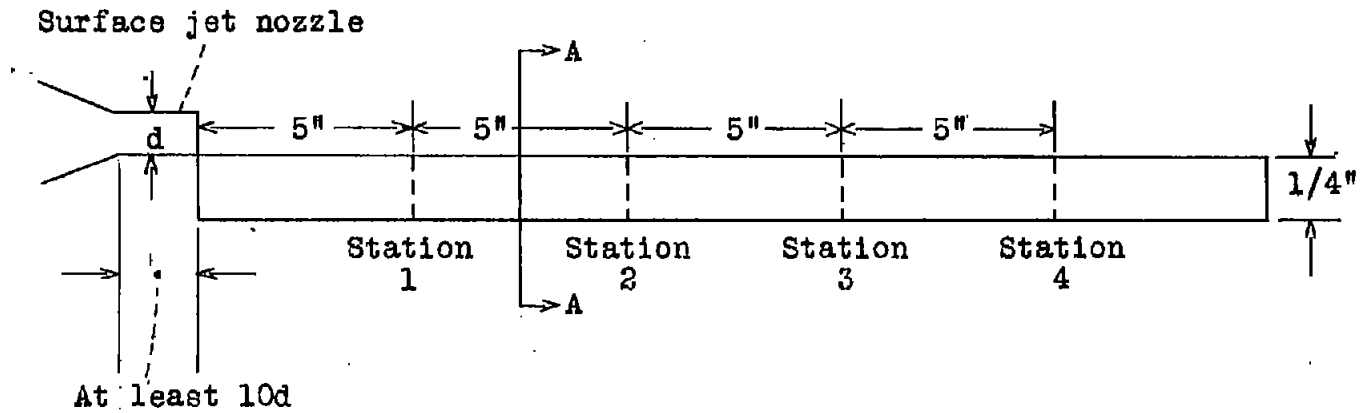
Figure 2.- Location of surface thermocouples on Lucite plate.

Fig. 2

	Station	1
o	"	1
+	"	2
×	"	3
□	"	4

- - - Reproduced from figure 14, reference 1

U/U_m (vertical axis)
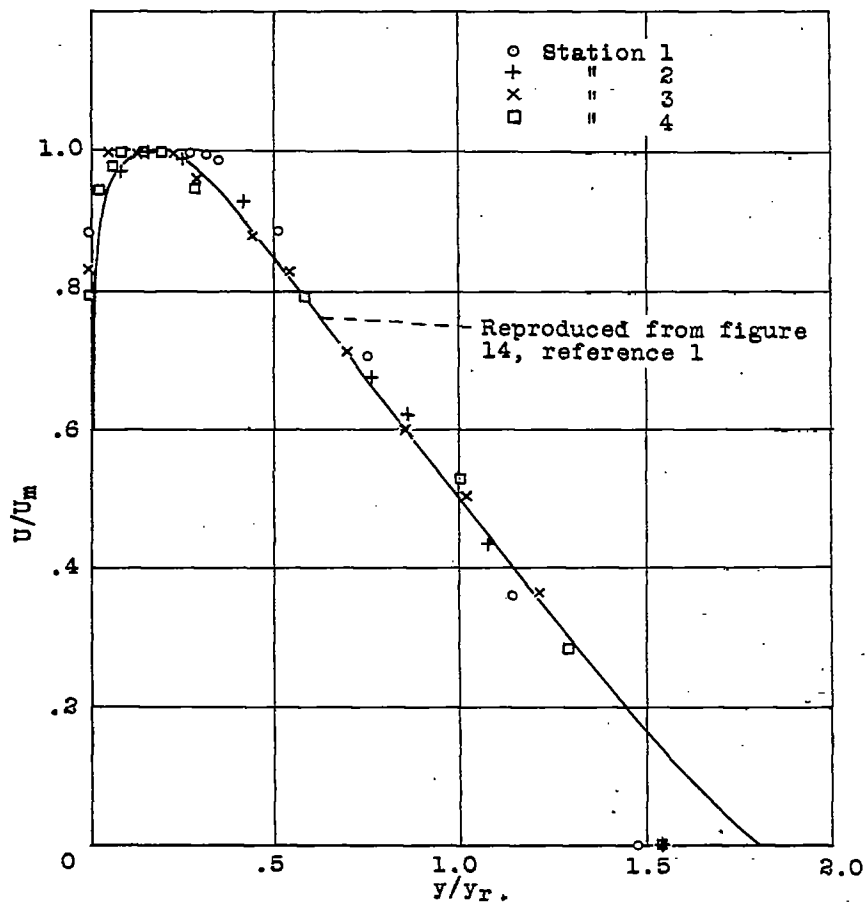
y/y_r (horizontal axis)

Figure 4.- Typical set of velocity profile data compared with nondimensional velocity profile curve of reference 1.
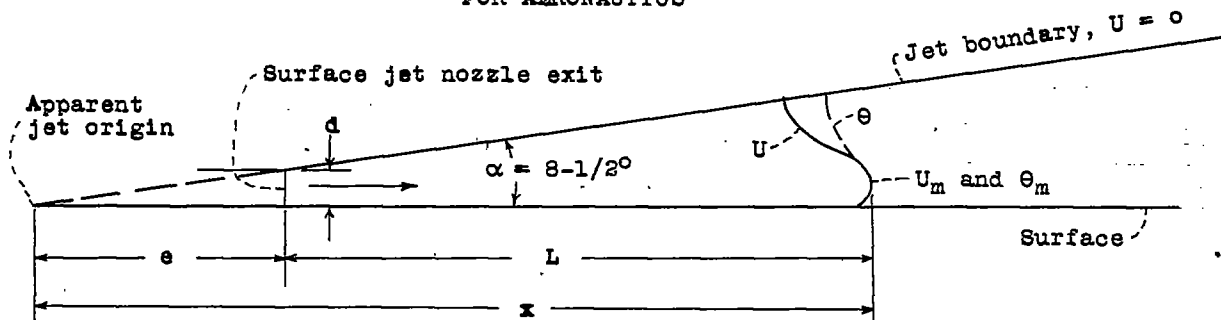
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Jet boundary, $U = 0$

Surface jet nozzle exit

Apparent jet origin

d

$\alpha = 8\text{-}1/2^\circ$

θ

U

U_m and θ_m

Surface

e

L

x
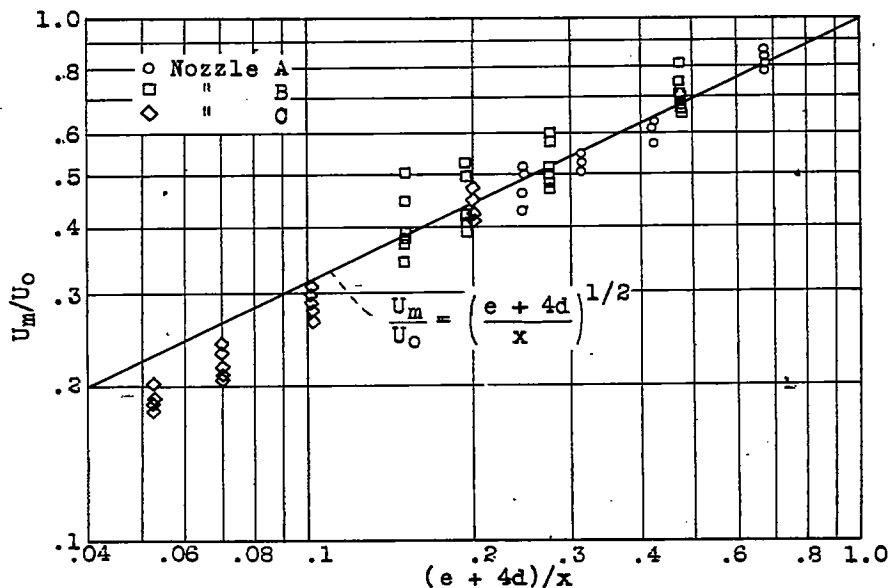
Figure 5.- Surface jet configuration.

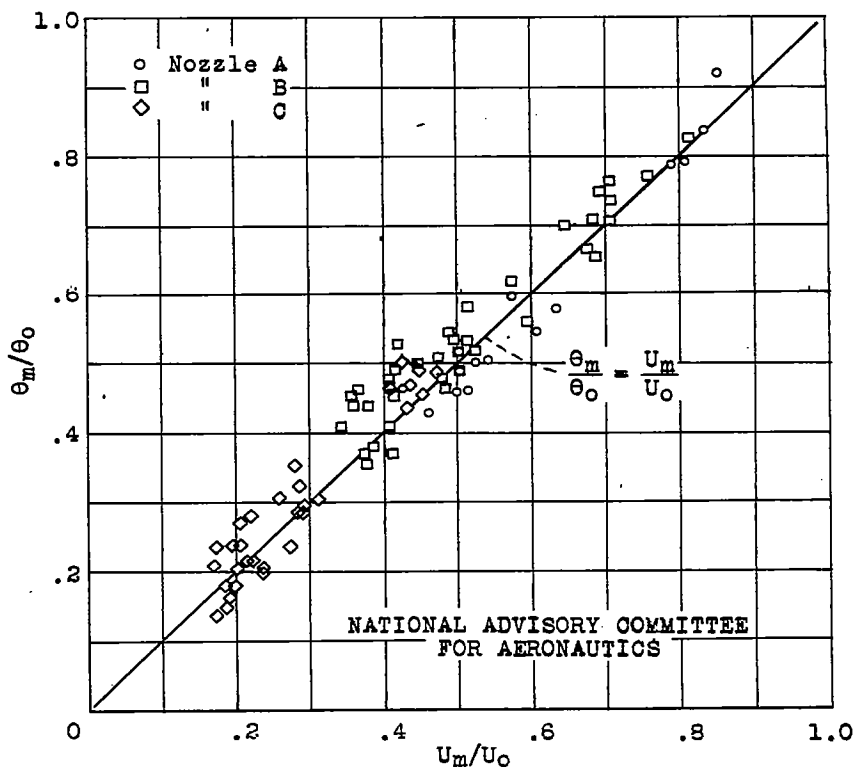Figure 6.- Velocity ratio U_m/U_O as a function of distance ratio $(e + 4d)/x$.

Figure 7.- Temperature ratio θ_m/θ_O as a function of velocity ratio U_m/U_O.

(a) Nozzle A (d = 0.547 in.)

$$\frac{hx}{k} = 0.16\left[\left(\frac{U_0 x\rho}{\mu}\right)\left(\frac{d}{d_r}\right)^{\frac{1}{2}}\right]^{0.65} \quad \left(\text{where } d_r = \frac{1}{12} \text{ foot}\right)$$

(c) Nozzle C (d = 0.108 in.)

$$\frac{hx}{k} = 0.21\left[\left(\frac{U_0 x\rho}{\mu}\right)\left(\frac{d}{d_r}\right)^{\frac{1}{2}}\right]^{0.65} \quad \left(\text{where } d_r = \frac{1}{12} \text{ foot}\right)$$

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(b) Nozzle B (d = 0.313 in.)

$$\frac{hx}{k} = 0.16\left[\left(\frac{U_0 x\rho}{\mu}\right)\left(\frac{d}{d_r}\right)^{\frac{1}{2}}\right]^{0.65} \quad \left(\text{where } d_r = \frac{1}{12} \text{ foot}\right)$$

—— Nozzles A and B
— — Nozzle C

$c_1 = 0.21$
$c_1 = 0.16$

$$\frac{hx}{k} = c_1\left[\left(\frac{U_0 x\rho}{\mu}\right)\left(\frac{d}{d_r}\right)^{\frac{1}{2}}\right]^{0.65} \quad \left(\text{where } d_r = \frac{1}{12} \text{ foot}\right)$$
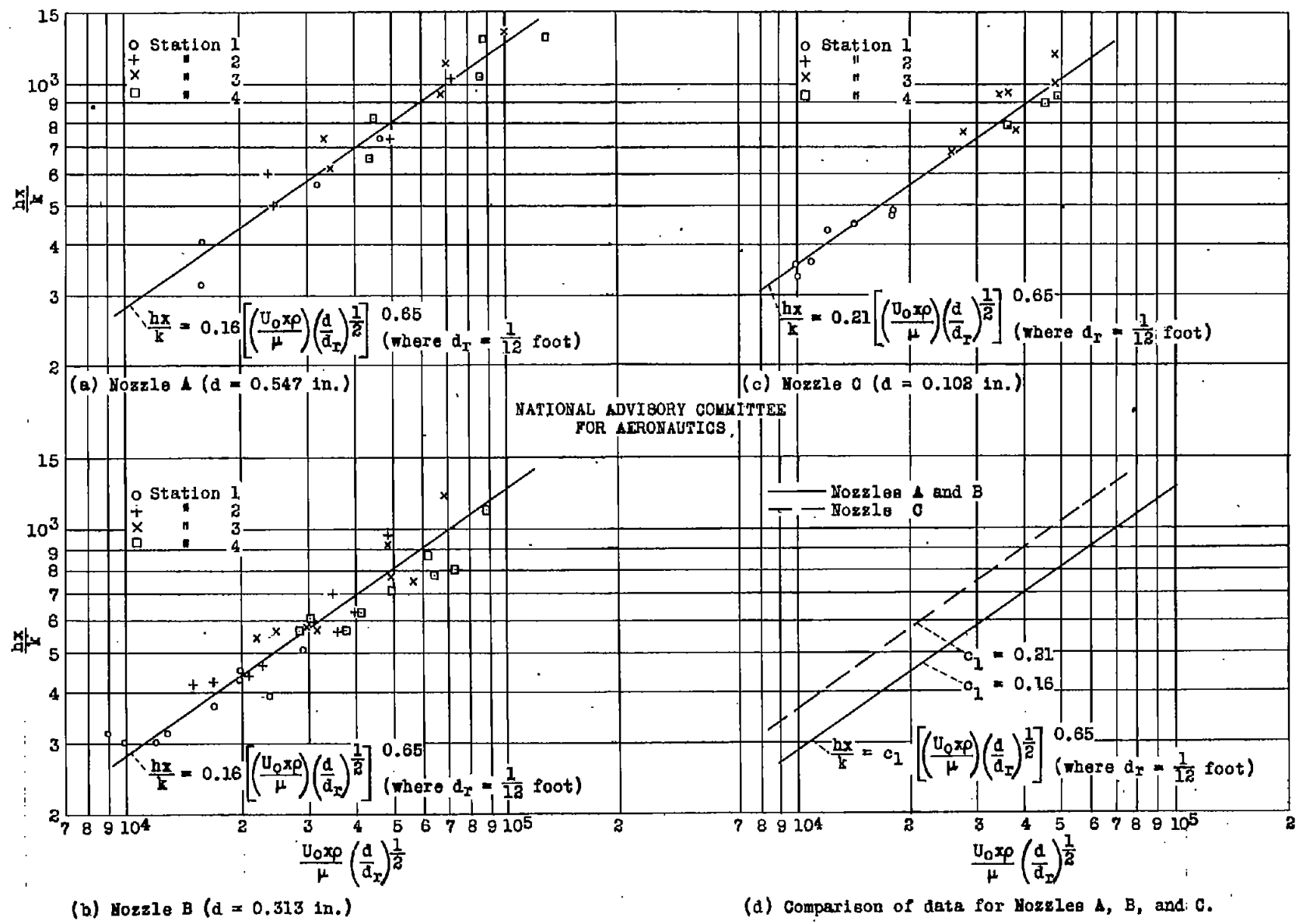
(d) Comparison of data for Nozzles A, B, and C.

Figure 8.- Nusselts number as a function of $\frac{U_0 x\rho}{\mu}\left(\frac{d}{d_r}\right)^{\frac{1}{2}}$ for surface jets tested.

Fig. 8