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CRITICAL AXIAL-COMPRESSIVE STRESS OF A CURVED RECTANGULAR PANEL WITH A CENTRAL CHORDWISE STIFFENER

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SUMMARY

The theoretical critical stress is derived for a simply supported curved rectangular panel in axial compression having a central chordwise stiffener offering no torsional restraint. The results are presented in the form of computed curves and a table.

Because a panel of moderate or large curvature buckles in compression at a stress considerably below the theoretical value, a method is suggested to aid in determining the critical stress for use in design.

INTRODUCTION

A simplified method was recently developed for determining the theoretical buckling stresses of unstiffened cylindrical shells under various loading conditions (references 1 and 2). In the present paper the theory is extended to include stiffened shells (appendix B), and the particular case of curved rectangular panels in axial compression, reinforced by a centrally located chordwise stiffener of zero torsional stiffness, is treated in detail (appendix C). Numerical results for this case are given in figures and a table. Because tests show that unstiffened curved panels in compression buckle at a load considerably below the theoretical (see reference 3), a procedure is suggested to permit the estimation of the actual critical stress of a rectangular panel with a central chordwise stiffener.

RESULTS AND DISCUSSION

<u>Theoretical critical stress</u>.– The effect of a central chordwise stiffener on the critical axial stress of a curved rectangular panel

is to increase the buckling stress. The increase is very small or zero for $\beta \leqq 0.7$ (β is the ratio of the circumferential dimension to the axial dimension) or when Z (curvature parameter, see appendix A) is greater than the values given in the following table:

β	Z
0.83	20.0
1.0	14.0
1.2	11.2
1.5	9.3
2.0	7.6
3.0	6.3

Only when $\beta > 0.7$ and Z is less than the corresponding value given in this table, is the buckling stress for the panel with a central chordwise stiffener appreciably greater than the buckling stress for the panel without a stiffener. The percentage increase that can be expected is shown for a number of cases in table 1.

The critical stress for a curved plate with a central chordwise stiffener is conveniently computed by the use of the standard buckling equation for a curved plate (see equation in fig. 1) with the buckling coefficient k_x for the unstiffened plate increased by an amount Δk_x due to the presence of the stiffener. The critical-stress coefficient k_x for an unstiffened curved plate of various aspect ratios may be obtained from figure 1. Figure 1(a) gives the variation of k_x with aspect ratio for a flat plate ($Z = 0$); whereas figure 1(b) gives k_x for curved plates. The increase in critical-axial-stress coefficient Δk_x due to the presence of the stiffener is given in figure 2. The curves in figure 2(a) give the maximum possible increase in critical stress and apply when a stiffener is used for which the stiffness is equal to or greater than the critical stiffness, that is, the value of stiffness beyond which a further increase does not cause any increase in the buckling stress. Figure 2(b) can be used to find the increase in critical stress when the stiffness is below the critical value.

In order to find the theoretical critical-stress coefficient for a given curved plate with a central chordwise stiffener $(k_x + \Delta k_x)$, first calculate the values of Z and γ for this plate-stiffener combination (where γ is twice the ratio of stiffener stiffness to plate stiffness $2\frac{EI}{Db}$). Then find the critical-stress coefficient for the unstiffened plate k_x from figure 1. Next find the maximum possible increase of critical-stress coefficient Δk_x due to the presence of a stiffener from figure 2(a) and the critical stiffness corresponding to this maximum possible increase from the curve in figure 2(b). If the given stiffener has a stiffness equal to or greater than the critical stiffness, add together the stress coefficient k_x for the unstiffened plate and the increase Δk_x just found from figure 2(a); the sum is the stress coefficient for the plate-stiffener combination. If the given stiffener has a stiffness less than the critical stiffness, add together the stress coefficient k_x for the unstiffened plate and the increase Δk_x found from figure 2(b).

The curves of figure 2 apply to plate-stiffener combinations buckling in one wave in either direction. As the curvature of a panel increases, the axial wave length tends to diminish. In order to show why the use of a central stiffener does not increase appreciably the critical compressive stress of the unstiffened plate when the values of Z are greater than those given in the preceding table, the curves of figure 3 have been drawn for $\beta = 1$ to include consideration of cases of more than one half wave in the axial direction. These curves show that for two half waves (node at the stiffener, $14 < Z < 23$) no increase in stress due to a stiffener occurs and that for three half waves $(23 < Z < 38)$, only a slight increase in stress is possible and even this slight increase requires a stiffener of high stiffness. Only if the unstiffened panel buckles into one half wave in the axial direction, does it appear worthwhile to use a single chordwise stiffener. Computations carried out for curved plates with the circumferential dimension 1.5 and 2 times the axial dimension showed the same results. In all of these cases the minimum load was found when there was one wave in the circumferential direction.

Estimation of design critical stresses.- Panels having moderate and large values of Z buckle at compressive stresses considerably lower than the stresses predicted on the basis of the small-deflection theory. In the absence of experimental data on the compressive buckling stress of panels with a stiffener having a flexural stiffness less than the critical stiffness, the fraction of the total possible increase in experimental critical stress, actually achieved by use of the stiffener, may be assumed to be equal to the fraction predicted by the theoretical solution (obtainable from figs. 1 and 2). This assumption permits an estimate to be made for the compressive stress of such a panel when the stresses for the limiting cases of an unstiffened panel and for a panel with a stiffener having a flexural stiffness equal to or greater than the critical stiffness are known. According to the aforementioned assumption the critical stress $\sigma_{exp}(\gamma)$ for the stiffened panel is given by

$$\sigma_{exp}(\gamma) = \sigma_{exp}(0) + R\left[\sigma_{exp}(\gamma_{cr}) - \sigma_{exp}(0)\right] \tag{1}$$

where

$$R = \frac{\Delta k_x(\gamma)}{\Delta k_x(\gamma_{cr})}$$

The increase in the theoretical critical-stress coefficients Δk_x needed to calculate R for panels having a wide range of ratios of circumferential dimension to axial dimension is obtained from figure 2.

The stresses $\sigma_{exp}(0)$ and $\sigma_{exp}(\gamma_{cr})$ may be regarded as the critical stresses of unstiffened panels: $\sigma_{exp}(0)$ is the critical stress of the original panel with no stiffener and $\sigma_{exp}(\gamma_{cr})$ is the critical stress of a panel having the radius, thickness, and circumferential dimension but half the axial dimension of the original panel. Because adequate design data on the buckling stresses of curved rectangular panels in axial compression are not available, some method of approximating these stresses must be used. Three lower limits may be given for the buckling of a curved panel; they are the buckling stress of the corresponding flat panel which may be obtained from figure 1(a), the buckling stress of the complete cylinder which may be obtained from figure 4 adapted from reference 4, and the buckling stress of the long curved strip, of which the curved panel may be considered a part, which may be obtained from figure 5 adapted from reference 3. The highest of these three lower limits represents a conservative approximation to the actual buckling stress of the panel. By use of values for $\sigma_{exp}(0)$, $\sigma_{exp}(\gamma_{cr})$, and R, determined in the manners just described, the design buckling stress $\sigma_{exp}(\gamma)$ may be determined from equation (1).

CONCLUSIONS

The theoretical analysis shows that for certain curvatures and aspect ratios designated in this paper, a curved rectangular panel can be appreciably strengthened to resist additional axial compression without buckling by the use of a central chordwise stiffener. The strengthening effect decreases as the curvature increases and as the ratio of the circumferential dimension to the axial dimension decreases.

With the aid of the semiempirical results contained herein, the axial-compressive buckling strength of a curved rectangular panel with a central chordwise stiffener can be estimated.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., March 11, 1948

APPENDIX A

SYMBOLS

a	axial or circumferential dimension of panel, whichever is larger
b	axial or circumferential dimension of panel, whichever is smaller
m, n, p, q	integers
r	radius of curvature of panel
t	thickness of panel
L	length of cylinder
u	displacement in axial (x–) direction of point in median surface of panel
v	displacement in circumferential (y–) direction of point in median surface of panel
w	displacement in radial direction of point in median surface of panel; positive outward
x	axial coordinate of panel
y	circumferential coordinate of panel
D	flexural stiffness of panel per unit length $\left(\frac{Et^3}{12(1 - \mu^2)}\right)$
E	Young's modulus of elasticity
F	Airy's stress function for median-surface stresses produced by buckle deformation $\left(\frac{\partial^2 F}{\partial y^2}\right.$, stress in axial direction; $\frac{\partial^2 F}{\partial x^2}$, stress in circumferential direction$\left.\right)$
I	moment of inertia of stiffener
Z	curvature parameter $\left(\frac{b^2}{rt}\sqrt{1 - \mu^2}\right.$ for panels or $\frac{L^2}{rt}\sqrt{1 - \mu^2}$ for cylinders$\left.\right)$

a_{kq}, a_{mn}, a_{pq} — deflection coefficients in trigonometric series

k_x — critical-axial-compressive-stress coefficient appearing in the formula $\sigma_x = \frac{k_x \pi^2 D}{b^2 t}$ for panels or $\sigma_x = \frac{k_x \pi^2 D}{L^2 t}$ for cylinders

p — lateral pressure

$\beta = \frac{a}{b}$

γ — ratio of flexural stiffness of stiffener to half flexural stiffness of plate in same direction $\left(2 \frac{EI}{Db}\right)$

γ_{cr} — lowest value of γ for which a buckle node occurs at stiffener location

μ — Poisson's ratio

τ — shear stress in shell

σ_x — axial-compressive stress

σ_y — circumferential-compressive stress

σ_{exp} — experimental axial buckling stress

Q — operator defined in appendix C

$\delta\left(x - \frac{b}{2}\right)$ — Dirac δ function defined in appendix B

$$M_{pq} = \left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4} \frac{Z^2 p^4 \beta^4}{\left(p^2 + q^2\beta^2\right)^2} - k_x \beta^2 p^2$$

$$N_{pq} = \left(p^2\beta^2 + q^2\right)^2 + \frac{12}{\pi^4} \frac{Z^2 p^4 \beta^8}{\left(p^2\beta^2 + q^2\right)^2} - k_x \beta^4 p^2$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \, \partial y^2} + \frac{\partial^4}{\partial y^4}$$

∇^{-4} — inverse of ∇^4 defined by $\nabla^{-4}\left(\nabla^4 w\right) = w$

APPENDIX B

EQUATION OF EQUILIBRIUM OF CYLINDRICAL SHELL WITH STIFFENERS

The equation of equilibrium of a cylindrical shell under the action of shear, both direct stresses, and lateral pressure is (reference 2):

$$D\nabla^4 w + \frac{Et}{r^2}\nabla^{-4}\frac{\partial^4 w}{\partial x^4} + t\left(\sigma_x\frac{\partial^2 w}{\partial x^2} + 2\tau\frac{\partial^2 w}{\partial x\,\partial y} + \sigma_y\frac{\partial^2 w}{\partial y^2}\right) + p = 0 \qquad \text{(B1)}$$

In this equation each term is a force per unit area or pressure. The first term is the restoring pressure due to the bending stiffness of the shell, the second is the restoring pressure due to the stretching stiffness of the shell, the third is the (negative) restoring pressure due to compressive stress in the axial direction, and so forth.

The equation describing all the effects of a stiffener riveted to the shell is quite complex, since even when all cross-sectional deformations are assumed zero any real stiffener has two principal bending stiffnesses, a torsional stiffness, a bending-torsion stiffness, and a stretching stiffness. In many problems it is sufficient to idealize the stiffener by considering it to be located along a line on the shell and to have only the bending stiffness that restrains radial deformations of the shell. This idealization is equivalent to considering the stiffener to be without torsional stiffness and attached to the plate by a frictionless bond which maintains contact but allows the plate to slide freely under the stiffener.

If the force per unit length exerted by the stiffener on the panel is q and this force is applied uniformly over the width of the stiffener ϵ, the pressure becomes q/ϵ. A term must therefore be added to equation (B1) which has the magnitude q/ϵ under the stiffener and is zero everywhere else. As ϵ approaches zero this term becomes simply $q\delta$ where δ is the Dirac delta function (reference 5), defined by the properties

$$\delta(t - t_o) = 0$$

when

$$t \neq t_o$$

and

$$\int_{t_1}^{t_2} \delta(t - t_o)\, dt = 1$$

so that

$$\int_{t_1}^{t_2} f(t)\, \delta(t - t_o)\, dt = f(t_o)$$

when t_o is within the interval (t_1, t_2) and $f(t)$ is a continuous function in the neighborhood of t_o. The effect of a stiffener located at $x = x_o$ would be represented by the term $q\, \delta(x - x_o)$ added to equation (B1) and of a stiffener at $y = y_o$ would be represented by the addition of this term $q\, \delta(y - y_o)$.

If the stiffener runs axially, the beam theory gives

$$q = EI \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} \tag{B2}$$

If the stiffener runs circumferentially, the beam theory gives

$$q = EI \frac{\partial^4 w}{\partial y^4} + P \frac{\partial^2 w}{\partial y^2} \tag{B3}$$

where EI is the stiffness of and P the compressive force in the stiffener and w is the deflection of the plate at the stiffener. In the problem of the present paper the stiffener is located at $x = \frac{b}{2}$, P is zero, and only axial stress is applied to the shell so that the equation of equilibrium becomes

$$D \nabla^4 w + \frac{Et}{r^2} \nabla^{-4} \frac{\partial^4 w}{\partial x^4} + EI \frac{\partial^4 w}{\partial y^4} \delta\left(x - \frac{b}{2}\right) + \sigma_x t \frac{\partial^2 w}{\partial x^2} = 0 \tag{B4}$$

$D \nabla^4 w + Et \quad + \frac{EI}{b} \frac{\partial^4 w}{\partial y^4} + \sigma_x t \frac{\partial^2 w}{\partial x^2} = 0$

APPENDIX C

THEORETICAL SOLUTION FOR CRITICAL AXIAL STRESS OF CURVED RECTANGULAR PANEL WITH CENTRAL CHORDWISE STIFFENER

Equation of equilibrium.- The critical axial-compressive stresses of curved rectangular panels having a central chordwise stiffener of zero torsional and stretching stiffness placed at $x = \frac{b}{2}$ may be found by solving the equation of equilibrium, equation (B4). Division of equation (B4) by D followed by appropriate substitutions gives

$$\nabla^4 w + \frac{12Z^2}{b^4}\nabla^{-4}\frac{\partial^4 w}{\partial x^4} + \frac{EI}{D}\delta\left(x - \frac{b}{2}\right)\frac{\partial^4 w}{\partial y^4} + k_x\frac{\pi^2}{b^2}\frac{\partial^2 w}{\partial x^2} = 0 \tag{C1}$$

The equation of equilibrium may be represented by

$$Q(w) = 0 \tag{C2}$$

where Q is the operator defined by

$$Q = \nabla^4 + \frac{12Z^2}{b^4}\nabla^{-4}\frac{\partial^4}{\partial x^4} + \frac{EI}{D}\delta\left(x - \frac{b}{2}\right)\frac{\partial^4}{\partial y^4} + k_x\frac{\pi^2}{b^2}\frac{\partial^2}{\partial x^2} \tag{C3}$$

Method of solution.- Equation (C2) may be solved by the Galerkin method as outlined in references 2 and 6. As suggested in reference 2 for simply supported rectangular panels, the following series expansion is used for w

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{a} \tag{C4}$$

(The coordinate system used is shown in fig. 6(a).) The coefficients a_{mn} are then chosen to satisfy the equations

$$\int_0^a\int_0^b \sin\frac{p\pi x}{b}\sin\frac{q\pi y}{a}\,Q(w)\,dx\,dy = 0 \tag{C5}$$

When the operations indicated in equation (C5) are performed, a set of homogeneous linear algebraic equations in a_{pq} is obtained with k_x appearing as a parameter. The solution for the critical-axial-stress coefficient k_x is then found to be the minimum value of k_x for which the algebraic equations have a nonvanishing solution for a_{pq}, that is, for which the plate is in equilibrium in a deflected state.

The boundary conditions implied by the method of solution are zero radial deflection and end moment at each edge, zero displacement along each edge, and free displacement normal to each edge in the median surface of the panel (see reference 1); that is,

$$\left.\begin{aligned}
&w(0,y) = w(b,y) = 0 && w(x,0) = w(x,a) = 0 \\
&\frac{\partial^2 w}{\partial x^2}(0,y) = \frac{\partial^2 w}{\partial x^2}(b,y) = 0 && \frac{\partial^2 w}{\partial y^2}(x,0) = \frac{\partial^2 w}{\partial y^2}(x,a) = 0 \\
&v(0,y) = v(b,y) = 0 && u(x,0) = u(x,a) = 0 \\
&\frac{\partial^2 F}{\partial y^2}(0,y) = \frac{\partial^2 F}{\partial y^2}(b,y) = 0 && \frac{\partial^2 F}{\partial x^2}(x,0) = \frac{\partial^2 F}{\partial x^2}(x,a) = 0
\end{aligned}\right\} \tag{C6}$$

Solution for circumferential dimension greater than axial dimension.— Substitution of the expressions for Q and w given by equations (C3) and (C4) into equation (C5) leads to the following set of algebraic equations:

$$a_{pq}\left[\left(p^2\beta^2 + q^2\right)^2 + \frac{12}{\pi^4}\frac{Z^2p^4\beta^8}{\left(p^2\beta^2 + q^2\right)^2} - k_x\beta^4p^2\right]$$

$$+ 2\frac{EI}{Db}q^4 \sin\frac{p\pi}{2}\sum_k a_{kq}\sin\frac{k\pi}{2} = 0 \tag{C7}$$

where $p = 1,2, \ldots$ and $q = 1,2, \ldots$

which may be written in two sets of equations as follows:

$$a_{pq}N_{pq} + \gamma q^4(-1)^{\frac{p-1}{2}} \sum_{k=1,3}^{\infty} a_{kq}(-1)^{\frac{k-1}{2}} = 0 \qquad \text{(C8)}$$

where $p = 1,3, \ldots$ and $q = 1,2, \ldots$

and

$$a_{pq}N_{pq} = 0 \qquad \text{(C9)}$$

where $p = 2,4, \ldots$ and $q = 1,2, \ldots$

For buckling across the stiffeners there must be an odd number of half waves in the axial direction; hence, the set of equations (C8) applies. Dividing through the set of equations by N_{pq} gives

$$a_{pq} + \frac{\gamma q^4}{N_{pq}}(-1)^{\frac{p-1}{2}} \sum_{k=1,3}^{\infty} a_{kq}(-1)^{\frac{k-1}{2}} = 0$$

where $p = 1,3, \ldots$ and $q = 1,2, \ldots$ Multiplication of each equation by $(-1)^{\frac{p-1}{2}}$ and summing with respect to p gives

$$\sum_{p=1,3}^{\infty} a_{pq}(-1)^{\frac{p-1}{2}} + \gamma q^4 \sum_{p=1,3}^{\infty} \frac{1}{N_{pq}} \sum_{k=1,3}^{\infty} a_{kq}(-1)^{\frac{k-1}{2}} = 0 \qquad \text{(C10)}$$

where $q = 1,2, \ldots$ Equation (C10) reduces to

$$1 + \gamma q^4 \sum_{p=1,3}^{\infty} \frac{1}{N_{pq}} = 0 \tag{C11}$$

or

$$\gamma = -\frac{1}{q^4 \sum_{p=1,3}^{\infty} \frac{1}{N_{pq}}}$$

where $q = 1,2, \ldots$

It is known that at low values of the curvature parameter Z one half wave $(q = 1)$ gives the minimum buckling load

$$\gamma = -\frac{1}{\sum_{p=1,3}^{\infty} \frac{1}{N_{p1}}} \tag{C12}$$

Equation (C12) gives the minimum value of γ for which the plate-stiffener combination is in equilibrium in a deflected state at a given value of k_x. Figures 2 and 3 present results obtained from equation (C12).

The solution to equations (C9) corresponds to no buckling across stiffeners and is given by

$$N_{pq} = 0$$

where $p = 2,4, \ldots$ and $q = 1,2, \ldots$ The lowest roots of these equations are the values of k_x given in column (b) of table 1 for various panel shapes and stiffener stiffness.

<u>Solution for axial dimension greater than circumferential dimension</u>.— When the axial dimension is greater than the circumferential dimension, interchanging a and b in equation (C4) is convenient in order to retain b as the shorter dimension

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{C13}$$

The coordinate system used is shown in figure 6(b). The use of these coordinates involves a slight modification of the equation of equilibrium as the stiffener is now placed at $x = \frac{a}{2}$. The modified equation is now

$$D \nabla^4 w + \frac{Et}{r^2} \nabla^{-4} \frac{\partial^4 w}{\partial x^4} + EI\, \delta\left(x - \frac{a}{2}\right) \frac{\partial^4 w}{\partial y^4} + \sigma_x t \frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{(C14)}$$

and

$$Q = \nabla^4 + \frac{12Z^2}{b^4} \nabla^{-4} \frac{\partial^4}{\partial x^4} + \frac{EI}{D} \delta\left(x - \frac{a}{2}\right) \frac{\partial^4}{\partial y^4} + k_x \frac{\pi^2}{b^2} \frac{\partial^2}{\partial x^2} \qquad \text{(C15)}$$

This problem is solved in a manner similar to that in the previous problem by a substitution of the expressions for Q and w given by equations (C13) and (C15) into equation (C5) (also modified by interchanging a and b). The following set of algebraic equations results

$$a_{pq}\left[\left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4} \frac{Z^2 p^4 \beta^4}{\left(p^2 + q^2\beta^2\right)^2} - k_x \beta^2 p^2\right] + 2\frac{EI}{Db} \beta^3 q^4 \sin\frac{p\pi}{2} \sum_k a_{kq} \sin\frac{k\pi}{2} = 0 \qquad \text{(C16)}$$

where $p = 1,2, \ldots$ and $q = 1,2, \ldots$

As in the previous problem for buckling across the stiffener, an odd number of half waves in the axial direction is necessary and for no buckling across the stiffener an even number is necessary. For this case again at low values of Z one half wave in the circumferential direction gives the minimum critical stress to cause the buckling of the stiffener. An equation similar to equation (C11) results, in which N_{pq} is replaced by

$$M_{pq} = \left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4} \frac{Z^2 p^4 \beta^4}{\left(p^2 + q^2\beta^2\right)^2} - k_x \beta^2 p^2$$

and the γ is replaced by $\gamma\beta^3$. For low values of Z, $q = 1$ and

$$\gamma = -\frac{1}{\beta^3 \sum_{p=1,3}^{\infty} \frac{1}{M_{p1}}} \tag{C17}$$

Equation (C17) gives the minimum value of γ for which the plate-stiffener combination is in equilibrium in a deflected state at a given value of k_x. Few results are shown for this case as a panel having the axial length equal to or greater than about 1.5 times the circumferential length would not be significantly strengthened by addition of a central chordwise stiffener of zero torsional and stretching stiffness. (See table 1.)

The solution which corresponds to no buckling across the stiffener is given by

$$M_{pq} = 0$$

where $p = 2,4,6 \ldots$ and $q = 1,2,3 \ldots$ The lowest roots of these equations are the values of k_x given in column (b) of table 1.

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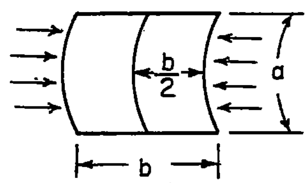
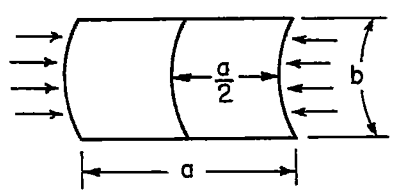
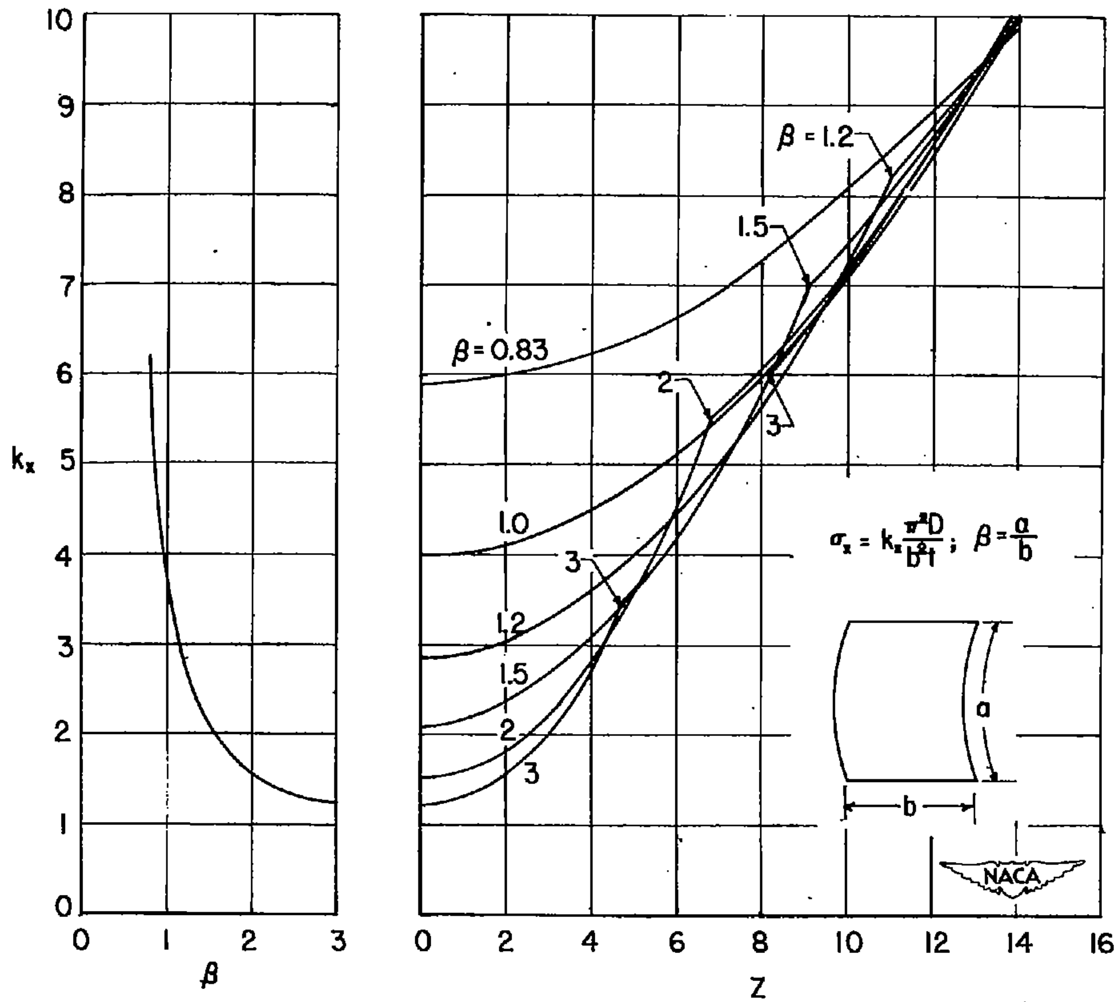
TABLE 1

CRITICAL-AXIAL-COMPRESSIVE-STRESS COEFFICIENTS FOR BUCKLING WITHOUT STIFFENER AND FOR NO BUCKLING ACROSS STIFFENER OFFERING ZERO TORSIONAL RESTRAINT

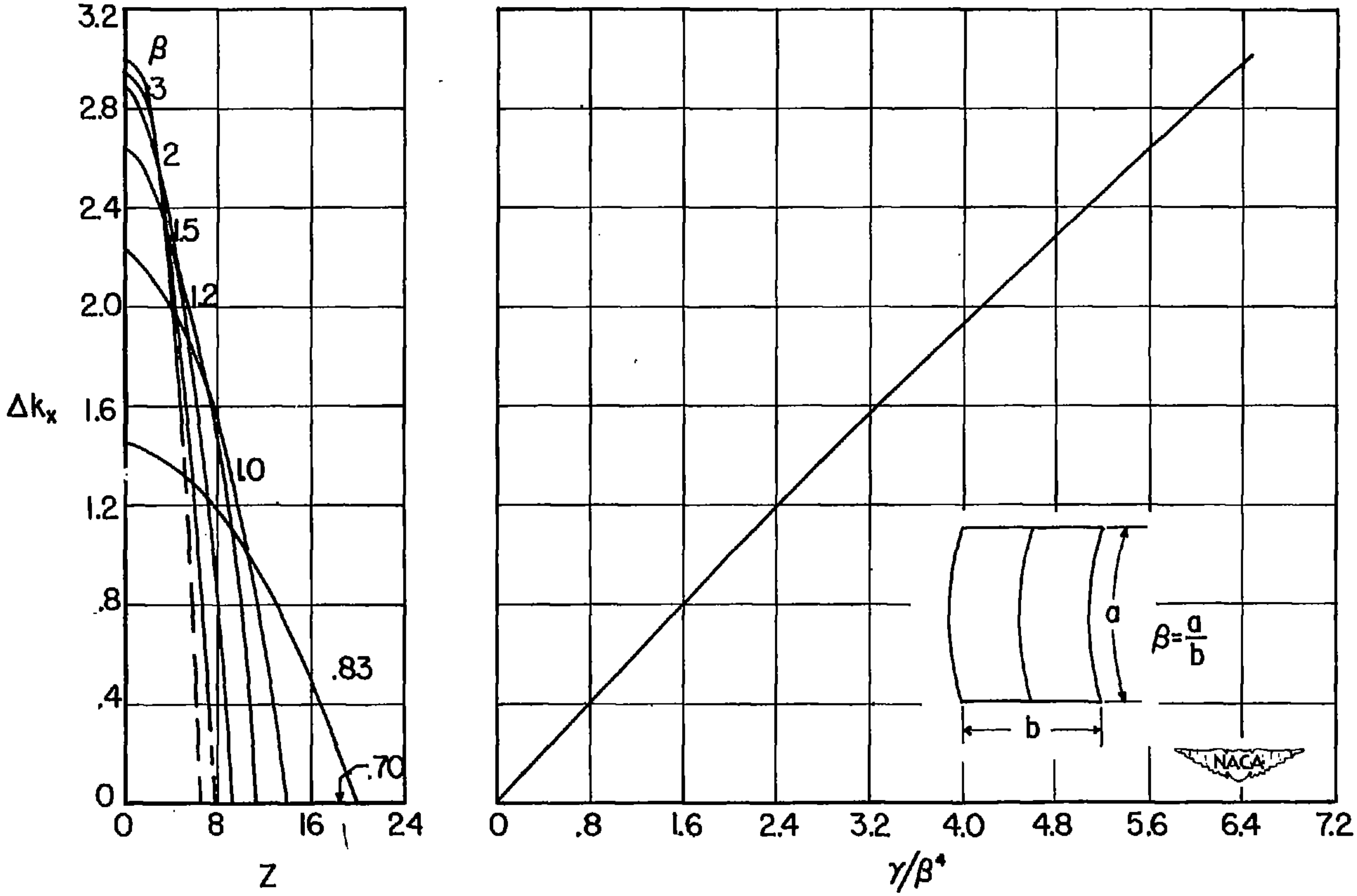
$\frac{a}{b}$	$Z = \frac{b^2}{rt}\sqrt{1 - \mu^2}$	$k_x = \frac{\sigma_x t b^2}{D\pi^2}$ (a) Buckling without stiffener	$k_x = \frac{\sigma_x t b^2}{D\pi^2}$ (b) No buckling across stiffener	Percentage increase $\frac{(b) - (a)}{(a)} \times 100$
				
1	0	4.00	6.25	56
	5	4.77	6.74	41
	10	7.08	8.22	16
	30	21.1	22.2	5
	100	70.3	70.3	0
	1000	703.	703.	0
1.5	0	2.09	4.94	136
	1	2.15	4.96	131
	5	3.56	5.54	56
	10	7.42	7.42	0
	30	21.1	21.6	2
	100	70.3	70.3	0
2	0	1.56	4.52	190
	1	1.64	4.52	176
	5	3.53	5.20	47
	10	7.08	7.22	2
	30	21.1	21.1	0
				
1.5	0	4.34	4.34	0
	5	5.04	5.04	0
	10	7.17	7.17	0
	30	21.2	21.2	0
	100	70.2	70.2	0
	1000	704.	704.	0
2	0	4.00	4.00	0
	10	7.08	7.08	0
	100	70.3	70.3	0
	1000	704.	704.	0



(a) Flat panels.

(b) Curved panels.

Figure 1.- Critical-axial-compressive-stress coefficient for rectangular panels.



(a) Maximum possible increase.

(b) Increase for given stiffness.

Figure 2.- Increase in critical-axial-compressive-stress coefficient due to the presence of a central chordwise stiffener over the value for the unstiffened panel.

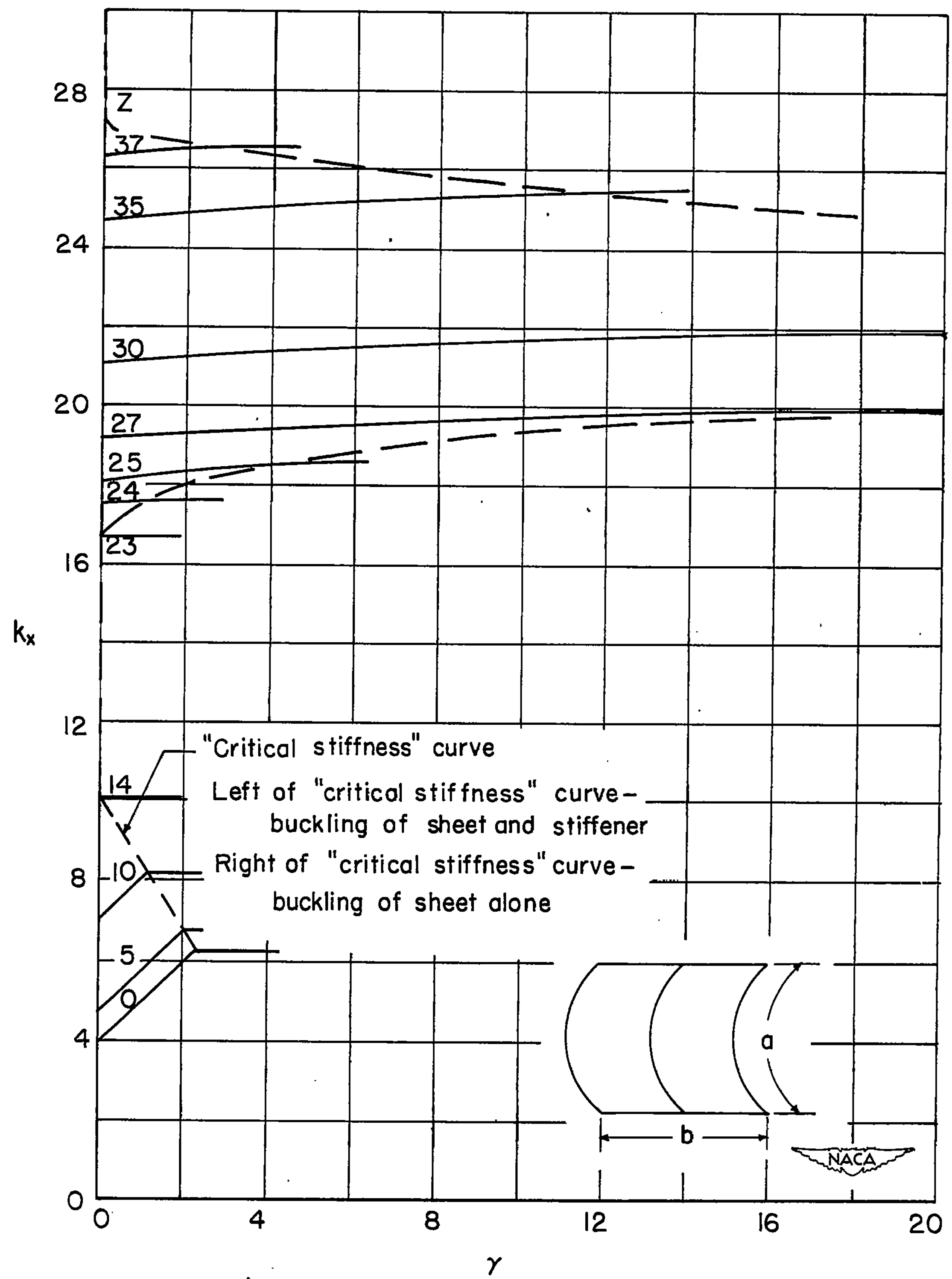


Figure 3.- Effect of central chordwise stiffener upon critical-axial-compressive-stress coefficients for simply supported curved rectangular panels of $\frac{a}{b} = 1$.

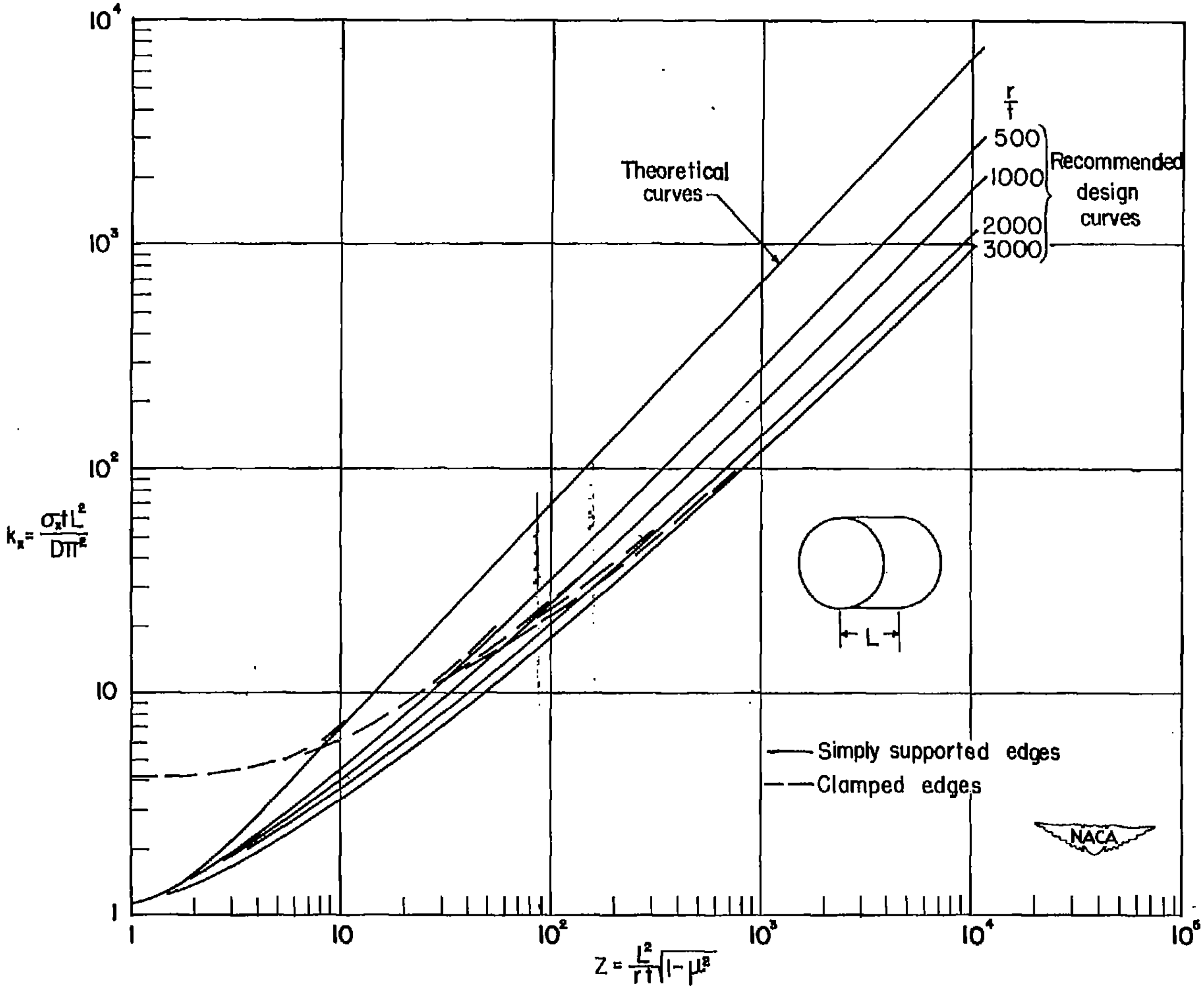


Figure 4.- Critical-axial-compressive-stress coefficients for simply supported cylinders. (Figure adapted from reference 4.)

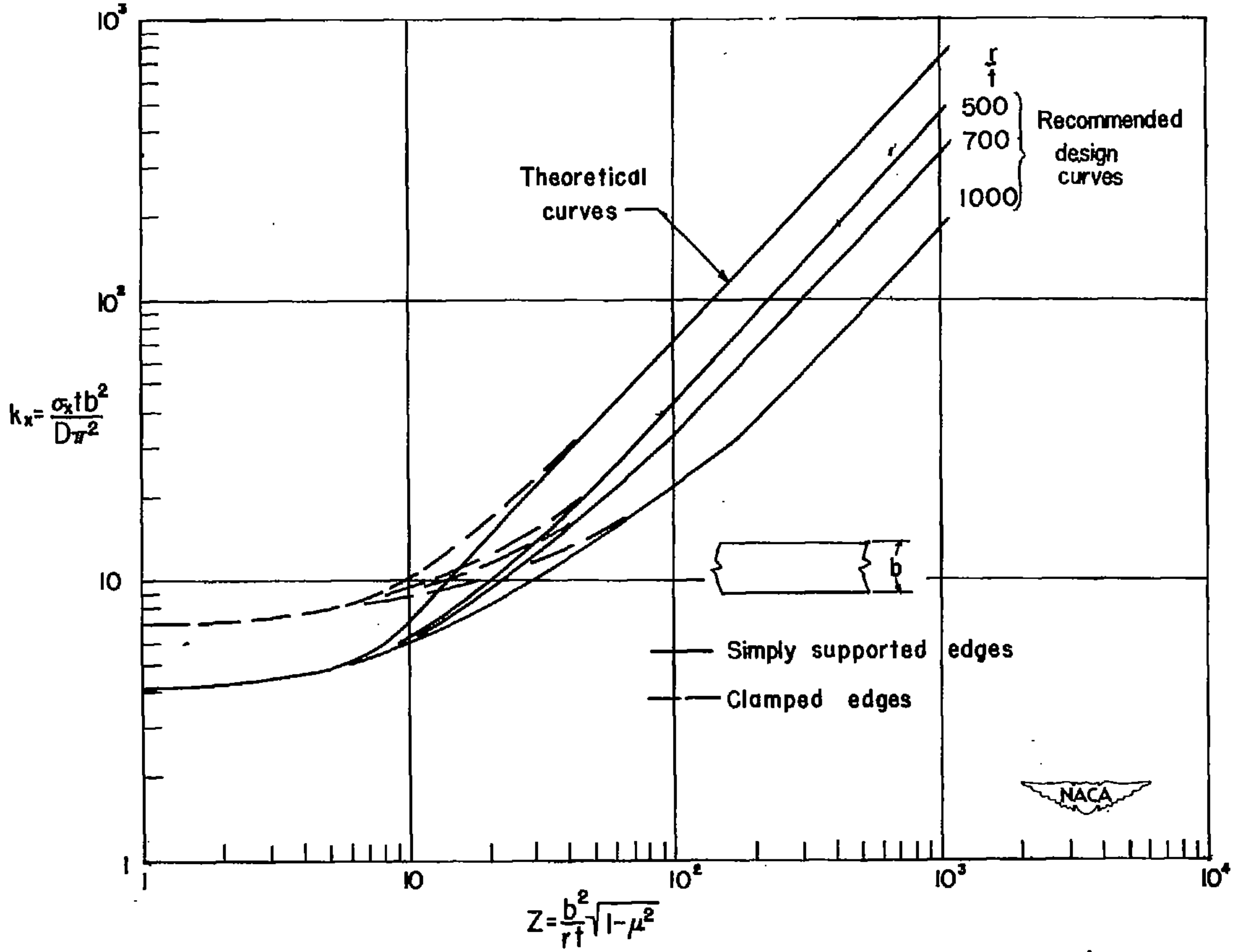
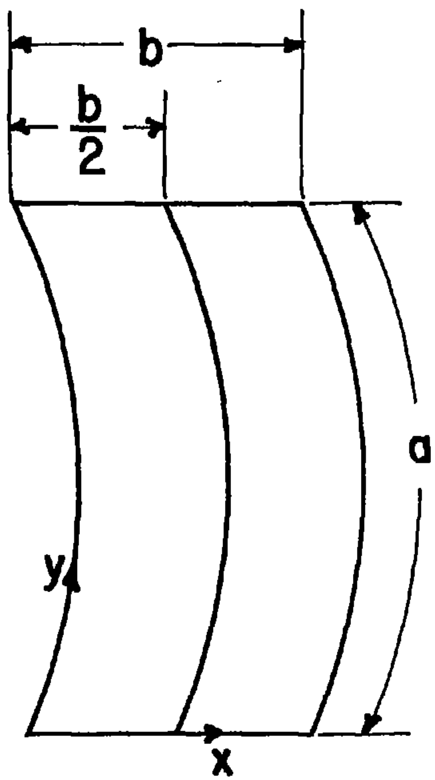
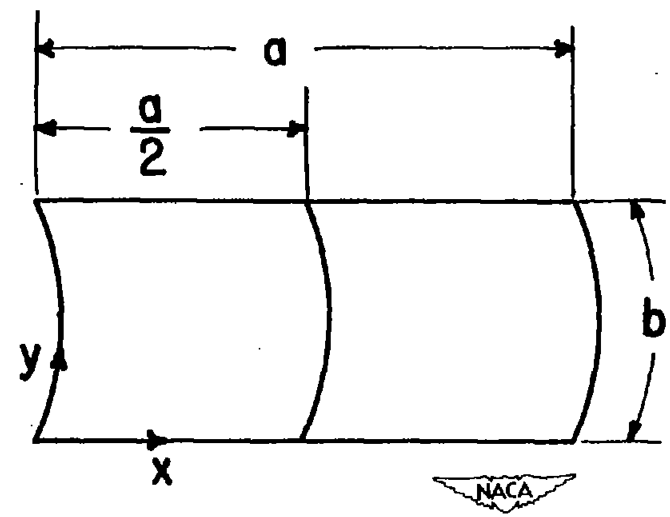


Figure 5.- Critical-axial-compressive-stress coefficients for a simply supported infinitely long strip with transverse curvature. (Figure adapted from reference 3.)



(a) Panels with the circumferential dimension larger than the axial dimension.



(b) Panels with the axial dimension larger than the circumferential dimension.

Figure 6.- Coordinate systems used in theoretical analysis.