NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1716

TABLES OF HYPERGEOMETRIC FUNCTIONS FOR USE IN COMPRESSIBLE-FLOW THEORY

By Vera Huckel

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ERRATA AND ADDENDA

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Errata

Pages 10 and 11: The captions for table 3 and table 3 concluded should read:

THE FUNCTIONS $-\frac{2}{\beta k}\frac{dY_k}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k

Pages 12 and 13: The captions for table 4 and table 4 concluded should read:

THE FUNCTIONS $-\frac{2}{\beta k}\frac{dY_{-k}}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k

Addenda

For completeness the additional formulas of reference 1 used in the computation of the functions are as follows:

For arbitrary positive indices

$$Y_k(\tau) = F\left(a_k,\ b_k,\ k+1;\ \tau\right)$$

For negative nonintegral indices

$$\overline{Y}_k(\tau) = \tau^{-k} F\left(a_k-k,\ b_k-k,\ 1-k;\ \tau\right)$$

where

$$a_k + b_k = k - \beta$$

$$a_k b_k = -\frac{k}{2}(k+1)\beta$$

and

$$F(a,\ b,\ c;\ \tau) = 1 + \frac{ab}{c}\tau + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}\tau^2 + \ldots$$

For negative integral indices

$$Y_{-k}(\tau) = 1 - \frac{(a_k - k)(b_k - k)}{1!\,(k-1)}\tau + \frac{(a_k - k)(a_k - k + 1)(b_k - k)(b_k - k + 1)}{2!\,(k-1)(k-2)}\tau^2$$

$$- \frac{(a_k - k)(a_k - k + 1)(a_k - k + 2)(b_k - k)(b_k - k + 1)(b_k - k + 2)}{3!\,(k-1)(k-2)(k-3)}\tau^3 + \ldots$$

$$+ (-1)^{k-1}\frac{(a_k - k)(a_k - k + 1)\ldots(a_k - 2)(b_k - k)(b_k - k + 1)\ldots(b_k - 2)}{(k-1)!\,(k-1)!}\tau^{k-1}$$

$$+ c\left[\tau^k F(a_k, b_k, k+1; \tau)\log\tau + \frac{a_k b_k}{1!\,(k+1)}\left(\frac{1}{a_k} + \frac{1}{b_k} - \frac{1}{1} - \frac{1}{k+1}\right)\tau^{k+1}\right.$$

$$\left. + \frac{a_k(a_k+1)b_k(b_k+1)}{2!\,(k+1)(k+2)}\left(\frac{1}{a_k} + \frac{1}{a_k+1} + \frac{1}{b_k} + \frac{1}{b_k+1} - \frac{1}{1} - \frac{1}{2} - \frac{1}{k+1} - \frac{1}{k+2}\right)\tau^{k+2} + \ldots\right]$$

and

$$c = (-1)^{k+1}\frac{(a_k - 1)(a_k - 2)\ldots(a_k - k)(b_k - 1)(b_k - 2)\ldots(b_k - k)}{k!\,(k-1)!}$$

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1716

TABLES OF HYPERGEOMETRIC FUNCTIONS FOR USE IN COMPRESSIBLE-FLOW THEORY

By Vera Huckel

SUMMARY

In the hodograph method of treating plane potential compressible flows the differential equation, originally obtained by Chaplygin in his study on gas jets, plays a significant role. This paper tabulates various hypergeometric functions which arise as particular solutions of Chaplygin's differential equation. The tables should prove useful in the tabulation of other auxiliary functions which may arise in various compressible-flow problems. The adiabatic index for air has been taken as 1.4.

INTRODUCTION

Any general theory of compressible potential flow will probably involve the hodograph variables. A reason for this statement is that in the hodograph plane, in which the independent variables are the magnitude and the direction of the fluid velocity, the equations of motion are linear; whereas, in the physical plane they are in general an intractable set of nonlinear partial differential equations.

The simplification due to the use of hodograph variables, however, presents certain difficulties which do not appear in the physical plane. For example, application of the necessary boundary conditions for uniform compressible flow past an arbitrary body is almost impossible, at least up to the present time. Certain singularities in the flow also appear, notably, near the sonic speed and in the undisturbed flow at infinity. Nevertheless, the possibility of getting around these and other difficulties in the near future justifies the publication of tables for the fundamental set of functions which represent the particular solutions of the flow equations in the hodograph plane.

The following section contains equations and definitions necessary for the understanding of the several functions listed in the tables. The reader is referred to the original paper (reference 1) in which the particular flow solutions are derived in detail.

EQUATIONS AND DEFINITIONS

The linear equations in the hodograph variables θ and q, which relate the velocity potential ϕ and the stream function ψ for the steady two-dimensional flow of a nonviscous compressible fluid, are

$$\left.\begin{aligned} \frac{\partial \phi}{\partial \theta} &= \lambda_1(q)\,\frac{\partial \psi}{\partial q} \\ \frac{\partial \phi}{\partial q} &= -\lambda_2(q)\,\frac{\partial \psi}{\partial \theta} \end{aligned}\right\} \tag{1}$$

in which, for the adiabatic equation of state between the pressure and density,

$$\lambda_1(q) = \frac{\rho_o}{\rho} q$$

$$= \frac{q}{(1 - \tau)^{\beta}}$$

and

$$\lambda_2(q) = -q \frac{d}{dq}\left(\frac{\rho_o}{\rho q}\right)$$

$$= \frac{\rho_o}{\rho q}(1 - M^2)$$

$$= \frac{1 - (2\beta + 1)\tau}{q(1 - \tau)^{\beta+1}}$$

where

q magnitude of fluid velocity

θ angle included by velocity vector and positive direction of x-axis

ρ density of fluid

a velocity of sound in fluid

M Mach number (q/a)

$$\beta = \frac{1}{\gamma - 1}$$

γ ratio of specific heats at constant pressure and at constant volume, taken as 1.4 for air

τ dimensionless speed variable $\left(\tau = \frac{q^2}{2\beta a_o^2} = \frac{M^2}{2\beta + M^2}\right)$

The index o refers to stagnation point $q = 0$.

Observe that the Mach number is given in terms of τ by the relation

$$M^2 = \frac{2\beta\tau}{1 - \tau}$$

For the tables the numerical value $\beta = 2.5$, corresponding to $\gamma = 1.4$, is used. Hence

$$M^2 = \frac{5\tau}{1 - \tau}$$

and $M = 1$ corresponds to $\tau = \frac{1}{6}$.

By substituting in equations (1) the product-type solutions

$$\left.\begin{aligned} \phi_k &= P_k(q) \begin{matrix}\cos\\ \sin\end{matrix} (k\theta) \\ \psi_k &= Q_k(q) \begin{matrix}\sin\\ \cos\end{matrix} (-k\theta) \end{aligned}\right\} \qquad (2)$$

and by observing that from equations (1)

$$\left.\begin{aligned} kP_k(q) &= \frac{\rho_o}{\rho} q \frac{dQ_k(q)}{dq} \\ \frac{dP_k(q)}{dq} &= -kq \frac{d}{dq}\left(\frac{\rho_o}{\rho q}\right) Q_k(q) \end{aligned}\right\} \qquad (3)$$

the functions $Q_k(q)$ can be shown to satisfy the following second-order differential equation:

$$q^2 \frac{d^2Q_k}{dq^2} + (1 + M^2)q \frac{dQ_k}{dq} - k^2(1 - M^2)Q_k = 0 \qquad (4)$$

The functions $P_k(q)$ can be obtained from $Q_k(q)$ by means of the first of equations (3). Equation (4) may be reduced to a standard type by the introduction of the dimensionless speed variable τ as the independent variable. Thus, let

$$Q_k(q) = q^k Y_k(\tau) \qquad (5)$$

where clearly $Y_k(\tau) \rightarrow 1$ as $\tau \rightarrow 0$ (incompressible case). After some elementary operations the desired differential equation is

$$\tau(1 - \tau) \frac{d^2Y_k}{d\tau^2} + \left[(k + 1) - (k + 1 - \beta)\tau\right] \frac{dY_k}{d\tau} + \frac{1}{2}\beta k(k + 1)Y_k = 0 \qquad (6)$$

Equation (6), which is of the hypergeometric type, was first introduced by Chaplygin in his memoir on gas jets (reference 2).

In the present paper, tables of numerical values have been prepared for a selected number of the complete set of particular solutions of equation (6). These solutions extend the results of Chaplygin into the supersonic range and to negative values of the index k.

DESCRIPTION OF TABLES

Tables 1 and 2 have been prepared for the functions Y_k and tables 3 and 4 for the functions $\frac{dY_k}{d\tau}$ for both positive and negative values of the index k ranging from 0.5 to 15 in increments of 0.5 and for the speed variable τ ranging from 0.01 to 0.50 in increments of 0.01. The critical value of τ is 1/6 for air; hence the present tables extend considerably into the supersonic range. Thus, corresponding to the value $\tau = 0.50$, the Mach number is $\sqrt{5}$.

For large values of the index k (for example, greater than 15), it is possible to develop and utilize asymptotic expressions which involve the function $h(\tau)$ for $M < 1$ (see discussion following equation (42) of reference 1) and involve the function $\theta(M)$ for $M > 1$ (see equation (57) of reference 1).

The numerical evaluation of the functions listed in the tables was performed with both manual computing and with the aid of an IBM computing machine. The tables may be considered accurate as listed although the actual computations made full use of the capacity of the machine and involved many more places.

It is hoped that the tables presented in this paper will be found adequate and useful for the numerical evaluation of auxiliary functions which may arise in the solution of problems of compressible flow.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., May 20, 1948

REFERENCES

1. Garrick, I. E., and Kaplan, Carl: On the Flow of a Compressible Fluid by the Hodograph Method. II – Fundamental Set of Particular Flow Solutions of the Chaplygin Differential Equation. NACA Rep. No. 790, 1944.

2. Chaplygin, S. A.: On Gas Jets. (Text in Russian.) Sci. Ann., Moscow Imperial Univ., Math.-Phys. Sec., vol. 21, 1904, pp. 1–121. (Available as NACA TM No. 1063, 1944.)

TABLE 1.- THE FUNCTIONS Y_k FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k

M	r	$Y_{0.5}$	$Y_{1.0}$	$Y_{1.5}$	$Y_{2.0}$	$Y_{2.5}$	$Y_{3.0}$	$Y_{3.5}$	$Y_{4.0}$	$Y_{4.5}$	$Y_{5.0}$	$Y_{5.5}$	$Y_{6.0}$	$Y_{6.5}$	$Y_{7.0}$	$Y_{7.5}$
0.22473	0.01	0.99377	0.98756	0.98138	0.97522	0.96909	0.96300	0.95695	0.95093	0.94495	0.93900	0.93309	0.92722	0.92138	0.91558	0.90981
.31944	.02	.98760	.97525	.96300	.95087	.93887	.92700	.91527	.90369	.89224	.88093	.86976	.85873	.84783	.83708	.82645
.39324	.03	.98147	.96306	.94487	.92695	.90932	.89198	.87493	.85822	.84180	.82568	.80986	.79434	.77911	.76416	.74950
.45644	.04	.97539	.95099	.92699	.90346	.88043	.85792	.83594	.81449	.79357	.77316	.75326	.73386	.71495	.69651	.67855
.51299	.05	.96935	.93905	.90936	.88039	.85220	.82481	.79823	.77245	.74747	.72326	.69981	.67710	.65511	.63383	.61322
.56493	.06	.96337	.92723	.89197	.85773	.82461	.79262	.76178	.73205	.70343	.67588	.64938	.62389	.59938	.57581	.55316
.61347	.07	.95743	.91554	.87482	.83549	.79766	.76135	.72656	.69325	.66139	.63094	.60184	.57403	.54752	.52218	.49800
.65938	.08	.95154	.90396	.85791	.81366	.77135	.73098	.69255	.65600	.62129	.58834	.55707	.52748	.49932	.47268	.44743
.70321	.09	.94570	.89251	.84124	.79224	.74565	.70149	.65972	.62027	.58306	.54798	.51495	.48384	.45457	.42704	.40114
.74536	.10	.93990	.88117	.82480	.77122	.72057	.67286	.62804	.58600	.54663	.50979	.47534	.44315	.41309	.38503	.35883
.78612	.11	.93415	.86996	.80861	.75060	.69609	.64508	.59748	.55316	.51195	.47367	.43815	.40521	.37469	.34641	.32022
.82572	.12	.92845	.85886	.79265	.73037	.67220	.61813	.56803	.52171	.47895	.43955	.40326	.36987	.33917	.31097	.28506
.86436	.13	.92279	.84789	.77692	.71053	.64891	.59201	.53965	.49160	.44759	.40733	.37056	.33699	.30659	.27849	.25308
.90219	.14	.91719	.83708	.76142	.69107	.62619	.56668	.51332	.46580	.41779	.37693	.33994	.30645	.27616	.24879	.22407
.93934	.15	.91162	.82639	.74615	.67200	.60405	.54215	.48801	.43526	.38931	.34838	.31138	.27811	.24833	.22166	.19779
.97590	.16	.90611	.81567	.73111	.65331	.58246	.51838	.46070	.40896	.36268	.32138	.28458	.25185	.22276	.19694	.17404
1.0120	.17	.90064	.80517	.71630	.63499	.56143	.49537	.43635	.38384	.33726	.29604	.25965	.22795	.19930	.17445	.15263
1.0476	.18	.89522	.79479	.70171	.61705	.54095	.47310	.41896	.35988	.31319	.27225	.23641	.20511	.17781	.15404	.13336
1.0830	.19	.88984	.78452	.68734	.59947	.52101	.45156	.40249	.33704	.29043	.24992	.21480	.18441	.15817	.13573	.11607
1.1180	.20	.88451	.77436	.67320	.58225	.50159	.43073	.38692	.31527	.26892	.22900	.19472	.16535	.14025	.11884	.10059
1.1529	.21	.87923	.76432	.65928	.56539	.48270	.41060	.34822	.29455	.24860	.20941	.17609	.14784	.12394	.10377	.08678
1.1875	.22	.87399	.75440	.64557	.54889	.46432	.39115	.32837	.27485	.22945	.19110	.15883	.13176	.10918	.09022	.07448
1.2221	.23	.86879	.74459	.63208	.53273	.44644	.37237	.30936	.25612	.21140	.17401	.14288	.11705	.09568	.07807	.06357
1.2566	.24	.86364	.73489	.61881	.51693	.42907	.35425	.29115	.23834	.19441	.15807	.12815	.10360	.08354	.06720	.05393
1.2910	.25	.85854	.72531	.60576	.50146	.41218	.33676	.27372	.22147	.17845	.14324	.11457	.09134	.07259	.05751	.04543
1.3254	.26	.85348	.71584	.59291	.48634	.39577	.31990	.25706	.20547	.16345	.12945	.10208	.08018	.06274	.04890	.03798
1.3599	.27	.84847	.70648	.58028	.47155	.37983	.30365	.24113	.19033	.14939	.11665	.09062	.07006	.05391	.04128	.03146
1.3944	.28	.84350	.69723	.56785	.45709	.36436	.28800	.22593	.17601	.13623	.10479	.08012	.06090	.04602	.03456	.02580
1.4291	.29	.83857	.68810	.55564	.44296	.34935	.27293	.21142	.16247	.12392	.09382	.07053	.05263	.03899	.02866	.02090
1.4638	.30	.83369	.67907	.54362	.42916	.33479	.25844	.19758	.14969	.11246	.08370	.06178	.04519	.03275	.02350	.01668
1.4988	.31	.82886	.67016	.53181	.41567	.32067	.24450	.18440	.13764	.10169	.07437	.05382	.03852	.02724	.01901	.01307
1.5339	.32	.82406	.66135	.52021	.40250	.30698	.23110	.17185	.12629	.09171	.06579	.04660	.03293	.02239	.01513	.01001
1.5693	.33	.81931	.65265	.50881	.38964	.29372	.21824	.15993	.11562	.08243	.05793	.04007	.02784	.01815	.01180	.00744
1.6049	.34	.81461	.64406	.49760	.37708	.28088	.20589	.14860	.10559	.07382	.05078	.03418	.02353	.01445	.00895	.00529
1.6408	.35	.80995	.63558	.48660	.36483	.26844	.19405	.13784	.09618	.06585	.04415	.02890	.01837	.01125	.00654	.00351
1.6771	.36	.80533	.62721	.47579	.35288	.25641	.18270	.12764	.08737	.05848	.03817	.02416	.01472	.00850	.00492	.00206
1.7136	.37	.80076	.61894	.46517	.34123	.24478	.17182	.11798	.07912	.05169	.03273	.01994	.01153	.00615	.00284	.00090
1.7506	.38	.79623	.61078	.45475	.32987	.23353	.16142	.10884	.07142	.04543	.02782	.01680	.00876	.00417	.00147	-.00001
1.7879	.39	.79174	.60273	.44452	.31879	.22266	.15147	.10020	.06424	.03969	.02339	.01289	.00637	.00251	.00036	-.00071
1.8257	.40	.78729	.59478	.43447	.30800	.21217	.14196	.09204	.05756	.03443	.01940	.00998	.00433	.00114	-.00051	-.00123
1.8640	.41	.78289	.58693	.42462	.29749	.20204	.13288	.08435	.05135	.02963	.01584	.00744	.00260	.00002	-.00119	-.00160
1.9028	.42	.77853	.57919	.41495	.28725	.19227	.12421	.07711	.04560	.02526	.01267	.00524	.00115	-.00087	-.00169	-.00184
1.9421	.43	.77422	.57155	.40547	.27729	.18284	.11595	.07030	.04027	.02130	.00985	.00335	-.00004	-.00157	-.00204	-.00197
1.9821	.44	.76994	.56401	.39616	.26760	.17377	.10809	.06390	.03535	.01771	.00737	.00174	-.00101	-.00209	-.00227	-.00202
2.0226	.45	.76571	.55658	.38704	.25817	.16508	.10061	.05791	.03082	.01448	.00520	.00038	-.00179	-.00247	-.00239	-.00200
2.0638	.46	.76152	.54925	.37810	.24900	.15660	.09349	.05229	.02665	.01158	.00331	-.00073	-.00238	-.00272	-.00243	-.00193
2.1057	.47	.75737	.54202	.36934	.24008	.14851	.08674	.04705	.02285	.00899	.00169	-.00167	-.00283	-.00286	-.00240	-.00182
2.1483	.48	.75326	.53488	.36075	.23142	.14073	.08033	.04216	.01937	.00669	.00030	-.00242	-.00314	-.00291	-.00232	-.00167
2.1918	.49	.74919	.52785	.35233	.22301	.13325	.07427	.03760	.01620	.00466	-.00088	-.00299	-.00334	-.00289	-.00219	-.00151
2.2361	.50	.74516	.52092	.34409	.21484	.12607	.06852	.03337	.01332	.00287	-.00185	-.00342	-.00343	-.00281	-.00203	-.00134

TABLE 1.- THE FUNCTIONS Y_k FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k - Concluded

M	τ	$Y_{8.0}$	$Y_{8.5}$	$Y_{9.0}$	$Y_{9.5}$	$Y_{10.0}$	$Y_{10.5}$	$Y_{11.0}$	$Y_{11.5}$	$Y_{12.0}$	$Y_{12.5}$	$Y_{13.0}$	$Y_{13.5}$	$Y_{14.0}$	$Y_{14.5}$	$Y_{15.0}$
0.22473	0.01	0.90408	0.89889	0.89273	0.88711	0.88152	0.87597	0.87045	0.86496	0.85952	0.85410	0.84872	0.84337	0.83806	0.83278	0.82753
.31944	.02	.81596	.80560	.79537	.78527	.77530	.76546	.75573	.74613	.73666	.72730	.71806	.70894	.69993	.69104	.68226
.39284	.03	.73511	.72100	.70716	.69358	.68026	.66719	.65438	.64180	.62947	.61738	.60551	.59388	.58246	.57127	.56029
.45644	.04	.66104	.64398	.62736	.61116	.59538	.58000	.56502	.55042	.53619	.52234	.50884	.49568	.48287	.47039	.45823
.51299	.05	.59328	.57398	.55530	.53722	.51972	.50280	.48642	.47057	.45523	.44039	.42604	.41215	.39871	.38571	.37313
.56493	.06	.53138	.51045	.49034	.47101	.45244	.43459	.41745	.40097	.38515	.36994	.35534	.34130	.32782	.31487	.30244
.61347	.07	.47692	.45390	.43189	.41085	.39072	.37148	.35308	.34049	.32466	.30956	.29516	.28143	.26834	.25588	.24394
.65998	.08	.42532	.40087	.37941	.35909	.33985	.32164	.30439	.28806	.27260	.25797	.24412	.23101	.21860	.20686	.19574
.70521	.09	.37679	.35390	.33238	.31216	.29315	.27530	.25852	.24276	.22795	.21404	.20098	.18871	.17719	.16636	.15620
.74996	.10	.33439	.31159	.29032	.27050	.25201	.23477	.21871	.20373	.18978	.17677	.16465	.15336	.14284	.13304	.12391
.78612	.11	.29399	.27355	.25280	.23361	.21585	.19944	.18426	.17023	.15725	.14527	.13419	.12395	.11449	.10574	.09767
.82772	.12	.26127	.23944	.21940	.20103	.18417	.16872	.15459	.14156	.12965	.11874	.10874	.09958	.09119	.08350	.07646
.86436	.13	.22995	.20890	.18975	.17233	.15649	.14209	.12901	.11712	.10631	.09650	.08759	.07949	.07214	.06547	.05941
.90019	.14	.20176	.18164	.16349	.14713	.13238	.11910	.10714	.09637	.08667	.07794	.07008	.06301	.05665	.05093	.04578
.93934	.15	.17644	.15735	.14029	.12506	.11146	.09932	.08849	.07882	.07021	.06252	.05567	.04957	.04413	.03929	.03497
.97390	.16	.15375	.13578	.11987	.10580	.09335	.08236	.07264	.06405	.05647	.04978	.04388	.03867	.03407	.03000	.02645
1.0320	.17	.13347	.11667	.10194	.08904	.07775	.06788	.05924	.05168	.04508	.03938	.03428	.02988	.02605	.02270	.01978
1.0476	.18	.11539	.09979	.08686	.07453	.06436	.05557	.04796	.04137	.03568	.03077	.02652	.02286	.01969	.01696	.01461
1.0830	.19	.09932	.08493	.07258	.05199	.05392	.04515	.03851	.03283	.02798	.02383	.02029	.01727	.01470	.01250	.01063
1.1180	.20	.08507	.07189	.06070	.05122	.04318	.03639	.03065	.02579	.02170	.01824	.01533	.01288	.01081	.00907	.00761
1.1529	.21	.07249	.06049	.05042	.04199	.03494	.02905	.02414	.02004	.01662	.01378	.01141	.00945	.00782	.00646	.00534
1.1875	.22	.06140	.05055	.04157	.03414	.02801	.02295	.01879	.01536	.01255	.01024	.00835	.00680	.00554	.00450	.00366
1.2221	.23	.05168	.04194	.03398	.02749	.02220	.01791	.01442	.01160	.00931	.00747	.00598	.00478	.00382	.00305	.00243
1.2566	.24	.04318	.03450	.02751	.02189	.01738	.01377	.01089	.00859	.00677	.00532	.00417	.00327	.00255	.00199	.00154
1.2910	.25	.03579	.02811	.02202	.01720	.01340	.01041	.00806	.00622	.00479	.00368	.00281	.00214	.00163	.00123	.00093
1.3254	.26	.02938	.02265	.01739	.01331	.01014	.00769	.00581	.00437	.00327	.00244	.00180	.00133	.00097	.00070	.00050
1.3599	.27	.02386	.01801	.01352	.01010	.00750	.00563	.00406	.00295	.00213	.00152	.00108	.00075	.00052	.00035	.00023
1.3944	.28	.01913	.01410	.01031	.00748	.00538	.00383	.00270	.00187	.00128	.00086	.00056	.00035	.00021	.00012	.00005
1.4291	.29	.01510	.01081	.00766	.00536	.00370	.00251	.00167	.00108	.00067	.00040	.00021	.00009	.00002	-.00002	-.00005
1.4638	.30	.01170	.00809	.00551	.00367	.00239	.00151	.00090	.00050	.00024	.00008	-.00001	-.00007	-.00009	-.00010	-.00010
1.4988	.31	.00884	.00585	.00377	.00234	.00139	.00076	.00035	.00010	-.00004	-.00012	-.00015	-.00016	-.00015	-.00013	-.00012
1.5339	.32	.00646	.00402	.00239	.00132	.00068	.00021	-.00003	-.00016	-.00022	-.00023	-.00022	-.00020	-.00017	-.00014	-.00011
1.5693	.33	.00450	.00256	.00131	.00054	.00009	-.00016	-.00028	-.00032	-.00032	-.00029	-.00025	-.00020	-.00017	-.00013	-.00010
1.6049	.34	.00290	.00140	.00049	-.00003	-.00030	-.00041	-.00043	-.00041	-.00036	-.00030	-.00024	-.00019	-.00015	-.00011	-.00008
1.6408	.35	.00163	.00050	-.00012	-.00043	-.00052	-.00056	-.00051	-.00044	-.00036	-.00029	-.00022	-.00017	-.00012	-.00009	-.00006
1.6771	.36	.00062	-.00017	-.00056	-.00069	-.00070	-.00063	-.00053	-.00043	-.00034	-.00025	-.00019	-.00014	-.00010	-.00007	-.00004
1.7136	.37	-.00016	-.00067	-.00085	-.00085	-.00077	-.00064	-.00051	-.00040	-.00030	-.00022	-.00015	-.00011	-.00007	-.00005	-.00003
1.7506	.38	-.00073	-.00101	-.00103	-.00093	-.00078	-.00062	-.00047	-.00035	-.00025	-.00017	-.00012	-.00008	-.00005	-.00003	-.00002
1.7879	.39	-.00113	-.00123	-.00112	-.00094	-.00075	-.00057	-.00041	-.00029	-.00020	-.00013	-.00008	-.00005	-.00003	-.00001	-.00001
1.8257	.40	-.00142	-.00134	-.00114	-.00091	-.00069	-.00050	-.00035	-.00024	-.00015	-.00009	-.00006	-.00003	-.00001	0	0
1.8640	.41	-.00159	-.00139	-.00111	-.00085	-.00061	-.00043	-.00028	-.00018	-.00011	-.00006	-.00003	-.00001	0	0	0
1.9028	.42	-.00167	-.00137	-.00105	-.00076	-.00053	-.00035	-.00022	-.00013	-.00007	-.00004	-.00001	0	0	.00001	.00001
1.9421	.43	-.00167	-.00130	-.00095	-.00066	-.00044	-.00028	-.00016	-.00009	-.00004	-.00001	0	.00001	.00001	.00001	.00001
1.9821	.44	-.00162	-.00121	-.00085	-.00056	-.00035	-.00021	-.00011	-.00005	-.00002	0	.00001	.00001	.00001	.00001	.00001
2.0226	.45	-.00153	-.00109	-.00073	-.00046	-.00027	-.00014	-.00007	-.00002	0	.00001	.00002	.00002	.00001	.00001	.00001
2.0638	.46	-.00141	-.00096	-.00061	-.00036	-.00020	-.00009	-.00003	0	.00002	.00002	.00002	.00002	.00001	.00001	.00001
2.1057	.47	-.00127	-.00082	-.00050	-.00027	-.00013	-.00005	0	.00002	.00002	.00002	.00002	.00001	.00001	.00001	0
2.1483	.48	-.00112	-.00069	-.00039	-.00019	-.00008	-.00001	.00002	.00003	.00003	.00002	.00002	.00001	.00001	0	0
2.1918	.49	-.00096	-.00056	-.00029	-.00012	-.00003	.00002	.00003	.00004	.00003	.00002	.00002	.00001	.00001	0	0
2.2361	.50	-.00081	-.00044	-.00020	-.00006	.00001	.00004	.00004	.00004	.00003	.00002	.00001	.00001	0	0	0

NACA

TABLE 2.- THE FUNCTIONS Y_{-k} FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k

M	τ	$Y_{-0.5}$	$Y_{-1.0}$	$Y_{-1.5}$	$Y_{-2.0}$	$Y_{-2.5}$	$Y_{-3.0}$	$Y_{-3.5}$	$Y_{-4.0}$	$Y_{-4.5}$	$Y_{-5.0}$	$Y_{-5.5}$	$Y_{-6.0}$	$Y_{-6.5}$	$Y_{-7.0}$	$Y_{-7.5}$	$Y_{-8.0}$	$Y_{-8.5}$
0.22473	0.01	1.00621	1.01234	1.01802	1.02834	1.03377	1.04023	1.04634	1.05282	1.05936	1.06600	1.07271	1.07947	1.08629	1.09316	1.10007	1.10703	1.11404
.31944	.02	1.01236	1.02438	1.03461	1.06098	1.07187	1.08708	1.09870	1.11260	1.12595	1.14016	1.15444	1.16911	1.18398	1.19910	1.21443	1.22999	1.24575
.39324	.03	1.01844	1.03611	1.04982	1.09633	1.11323	1.14154	1.15790	1.18160	1.20151	1.22485	1.24731	1.27139	1.29756	1.32604	1.34613	1.37829	1.39896
.45644	.04	1.02444	1.04735	1.06370	1.13347	1.15692	1.20400	1.22480	1.26215	1.28774	1.32332	1.35396	1.38993	1.42438	1.46184	1.49329	1.53869	1.57875
.51299	.05	1.03038	1.05859	1.07689	1.17171	1.20202	1.27443	1.29744	1.35639	1.38508	1.43954	1.47738	1.52984	1.57475	1.62862	1.67945	1.73604	1.79207
.56493	.06	1.03625	1.06954	1.08763	1.21049	1.24773	1.35248	1.37703	1.46612	1.49749	1.57789	1.62061	1.69775	1.75182	1.82943	1.89386	1.97447	2.04836
.61347	.07	1.04205	1.08011	1.09778	1.24935	1.29331	1.43760	1.46207	1.59865	1.62233	1.74890	1.78635	1.90171	1.96125	2.07570	2.16143	2.26864	2.36012
.65938	.08	1.04779	1.09040	1.10677	1.28793	1.33805	1.52903	1.53139	1.73673	1.76026	1.93888	1.97659	2.15078	2.20837	2.38198	2.46235	2.63886	2.74303
.70321	.09	1.05346	1.10041	1.11465	1.32588	1.38136	1.62588	1.64361	1.89864	1.91021	2.16941	2.19223	2.45456	2.49865	2.76563	2.85727	3.11187	3.21542
.74536	.10	1.05906	1.11015	1.12146	1.36292	1.42267	1.72716	1.73720	2.07790	2.07035	2.43746	2.43282	2.82240	2.83471	3.24609	3.28593	3.78071	3.79685
.78612	.11	1.06460	1.11962	1.12724	1.39882	1.46147	1.83181	1.83049	2.27359	2.23813	2.74469	2.69685	3.26869	3.21749	3.84963	3.81553	4.50378	4.50562
.82572	.12	1.07007	1.12883	1.13203	1.43336	1.49733	1.93871	1.92177	2.48415	2.41034	3.09145	2.97898	3.78203	3.64449	4.42869	4.42859	5.30226	5.39516
.86436	.13	1.07548	1.13778	1.13587	1.46637	1.52986	2.04673	2.00989	2.70756	2.58316	3.47697	3.27401	4.35437	4.10919	5.46489	5.12135	6.15730	6.34967
.90219	.14	1.08083	1.14648	1.13881	1.49769	1.55872	2.15471	2.09188	2.94130	2.75229	3.89726	3.57481	5.07033	4.60050	6.51684	5.88073	8.30480	7.47903
.93934	.15	1.08611	1.15493	1.14087	1.52718	1.58861	2.26151	2.16605	3.18243	2.91300	4.34908	3.87146	5.89648	5.10882	7.73785	6.68353	10.17028	8.71364
.97590	.16	1.09133	1.16313	1.14211	1.55475	1.60430	2.36601	2.23193	3.42769	3.06029	4.82594	4.15879	6.67493	5.59394	9.12281	7.49455	12.36275	9.99967
1.0120	.17	1.09648	1.17110	1.14295	1.58029	1.62058	2.46712	2.28735	3.67350	3.18900	5.38014	4.40623	7.57292	6.04921	10.65223	8.26600	14.85862	11.29518
1.0476	.18	1.10157	1.17883	1.14222	1.60374	1.63229	2.56378	2.33087	3.91607	3.29390	5.82253	4.61813	8.51870	6.43800	12.30573	8.93751	17.64587	12.36800
1.0830	.19	1.10660	1.18632	1.14118	1.62503	1.63932	2.65502	2.36116	4.15148	3.36985	6.32471	4.77412	9.47162	6.78640	14.03106	9.43705	20.61987	13.19549
1.1180	.20	1.11157	1.19359	1.13945	1.64411	1.64158	2.73988	2.37702	4.37570	3.41188	6.80907	4.85951	10.42232	6.87787	15.77370	9.68277	23.67688	13.56709
1.1589	.21	1.11648	1.20063	1.13706	1.66096	1.63903	2.81750	2.37742	4.58471	3.41533	7.26922	4.89975	11.33331	6.85438	17.46225	9.58373	26.66857	13.28949
1.1875	.22	1.12133	1.20746	1.13405	1.67555	1.63166	2.88708	2.36149	4.77458	3.37997	7.69010	4.76089	12.16949	6.61791	19.01855	9.05393	29.40679	12.15492
1.2221	.23	1.12612	1.21407	1.13045	1.68787	1.61947	2.94789	2.32854	4.94128	3.29004	8.05838	4.54987	12.89311	6.13190	20.32951	7.99478	31.66990	9.92808
1.2566	.24	1.13085	1.22047	1.12689	1.69793	1.60252	2.99328	2.27805	5.08129	3.13438	8.36067	4.21534	13.46466	5.36098	21.30981	6.32378	33.20880	6.47978
1.2910	.25	1.13552	1.22666	1.12161	1.70572	1.58089	3.04069	2.20968	5.19109	2.96651	8.58385	3.74772	13.84401	4.48895	21.84515	3.96624	33.75675	1.56235
1.3254	.26	1.14013	1.23265	1.11643	1.71127	1.55467	3.07164	2.12388	5.26750	2.74467	8.71936	3.19976	13.99164	2.86848	21.82680	.86484	33.02663	-4.93353
1.3599	.27	1.14469	1.23814	1.11079	1.71460	1.52398	3.09173	2.01888	5.30767	2.48786	8.74354	2.38684	13.86982	1.10658	21.14697	-3.01566	30.75892	-13.08186
1.3944	.28	1.14919	1.24403	1.10471	1.71575	1.48897	3.10064	1.89667	5.30911	2.07593	8.65785	1.48733	13.44398	-1.00799	19.70948	-7.68806	26.69798	-22.88090
1.4291	.29	1.15363	1.24944	1.09823	1.71474	1.44979	3.09814	1.75703	5.26973	1.66935	8.44917	.44273	12.68389	-3.46955	17.42869	-13.11396	20.63157	-34.23864
1.4638	.30	1.15801	1.25465	1.09136	1.71163	1.40664	3.08408	1.60093	5.18788	1.21025	8.11002	-.74209	11.56494	-6.26106	14.23785	-19.24725	12.40018	-46.96123
1.4988	.31	1.16214	1.25969	1.08414	1.70645	1.35970	3.05840	1.42785	5.06235	.70041	7.63483	-2.05882	10.06924	-9.35346	10.09018	-25.98772	1.91481	-60.74320
1.5339	.32	1.16661	1.26454	1.07660	1.69928	1.30990	3.02111	1.23987	4.89243	.14324	7.02006	-3.49571	8.18560	-12.70549	4.96906	-33.19905	-10.82812	-75.17496
1.5693	.33	1.17048	1.26922	1.06875	1.69015	1.25334	2.97230	1.03739	4.67787	-.43734	6.26436	-5.03761	5.91543	-16.26379	-1.11433	-40.70736	-25.73396	-89.72579
1.6049	.34	1.17499	1.27373	1.06063	1.67913	1.19838	2.91815	.82214	4.41892	-1.09626	5.36873	-6.66611	3.26340	-19.96341	-8.11521	-48.30878	-42.60005	-103.77319
1.6408	.35	1.17910	1.27807	1.05225	1.66630	1.13855	2.84090	.59480	4.11633	-1.76832	4.33655	-8.35977	.24786	-23.78877	-15.93379	-55.74331	-61.10970	-116.60884
1.6771	.36	1.18316	1.28224	1.04365	1.65170	1.07610	2.75889	.35694	3.77132	-2.46727	3.17359	-10.09438	-3.10385	-27.47487	-24.51447	-62.76042	-80.89069	-127.46301
1.7136	.37	1.18716	1.28626	1.03484	1.63541	1.01130	2.66639	.11001	3.38561	-3.18642	1.88608	-11.84357	-6.75444	-31.10905	-33.64609	-69.06620	-101.21884	-135.53324
1.7506	.38	1.19111	1.29012	1.02585	1.61751	.94440	2.56355	-.14442	2.96136	-3.91892	.49059	-13.57818	-10.62690	-34.53893	-43.16348	-74.36207	-121.62692	-140.01821
1.7879	.39	1.19501	1.29383	1.01670	1.59806	.87569	2.45203	-.40475	2.50120	-4.65598	-1.00606	-15.26876	-14.75800	-37.64472	-52.80021	-78.34851	-141.31905	-140.15613
1.8257	.40	1.19886	1.29739	1.00740	1.57714	.80543	2.33119	-.66928	2.00816	-5.39085	-2.58589	-16.88412	-18.98405	-40.34182	-62.46269	-80.73694	-159.49064	-135.26589
1.8640	.41	1.20266	1.30080	.99799	1.55482	.73389	2.20202	-.93630	1.48864	-6.11498	-4.23498	-18.39287	-23.87186	-42.58351	-71.73541	-81.85819	-175.29327	-124.78942
1.9028	.42	1.20640	1.30407	.98847	1.53118	.66134	2.06518	-1.20406	.93744	-6.82013	-5.93169	-19.76388	-27.54002	-44.69388	-80.38731	-79.67936	-187.86430	-108.33338
1.9421	.43	1.21010	1.30721	.97887	1.50629	.58806	1.92135	-1.47082	.36763	-7.49805	-7.65672	-20.96662	-31.70326	-44.96469	-88.12924	-75.81093	-196.35998	-89.70796
1.9821	.44	1.21375	1.31021	.96921	1.48023	.51432	1.77185	-1.73480	-.21942	-8.14058	-9.38878	-21.97235	-35.68114	-45.05835	-94.67165	-69.51994	-199.99135	-56.96069
2.0226	.45	1.21735	1.31308	.95950	1.45306	.44038	1.61563	-1.99428	-.81910	-8.73982	-11.10548	-22.75418	-39.37978	-44.31063	-99.73579	-60.73962	-198.06151	-22.40319
2.0638	.46	1.22090	1.31582	.94976	1.42487	.36630	1.45329	-2.24754	-1.42663	-9.28816	-12.78388	-23.28794	-42.71378	-42.67336	-103.05831	-49.47784	-190.00888	17.37133
2.1057	.47	1.22440	1.31843	.94001	1.39573	.29293	1.29100	-2.49292	-2.03707	-9.77843	-14.40078	-23.55271	-45.99820	-40.11668	-104.40522	-35.82375	-175.41289	61.48150
2.1483	.48	1.22785	1.32093	.93026	1.36570	.21993	1.12350	-2.72881	-2.64341	-10.20400	-15.93307	-23.53140	-47.95258	-36.63100	-103.57881	-19.95208	-154.08696	108.76362
2.1918	.49	1.23126	1.32331	.92053	1.33485	.14779	.97391	-2.92367	-3.24658	-10.55886	-17.35811	-23.21096	-49.70301	-32.22923	-100.46684	-2.12405	-126.04419	157.79103
2.2361	.50	1.23462	1.32558	.91088	1.30327	.07660	.78274	-3.16605	-3.83955	-10.83773	-18.65408	-22.58346	-50.78406	-26.94606	-94.89051	17.31941	-91.55277	206.90752

TABLE 2.- THE FUNCTIONS Y_{-k} FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k - Concluded

M	τ	$Y_{-9.0}$	$Y_{-9.5}$	$Y_{-10.0}$	$Y_{-10.5}$	$Y_{-11.0}$
0.22473	0.01	1.12109	1.12820	1.13534	1.14253	1.14977
.31944	.02	1.26173	1.27792	1.29433	1.31096	1.32780
.39324	.03	1.42621	1.45401	1.48239	1.51134	1.54087
.45644	.04	1.62030	1.66283	1.70668	1.75166	1.79793
.51299	.05	1.85215	1.91308	1.97710	2.04273	2.11119
.56493	.06	2.13375	2.21656	2.30801	2.39958	2.49811
.61347	.07	2.48306	2.58995	2.72111	2.84336	2.98490
.65938	.08	2.92651	3.05526	3.24946	3.40361	3.61220
.70321	.09	3.50151	3.64048	3.94256	4.12018	4.44325
.74536	.10	4.25830	4.37867	4.87150	5.04379	5.57405
.78612	.11	5.26038	5.30511	6.13305	6.23400	7.14387
.82772	.12	6.58282	6.45212	7.85083	7.75292	9.34325
.86436	.13	8.30784	7.64141	10.17177	9.65375	12.41548
.90219	.14	10.51724	9.47404	13.25610	11.96329	16.64687
.93934	.15	13.28171	11.31865	17.25956	14.65895	22.34143
.97590	.16	16.64743	13.29916	22.30764	17.64160	29.77649
1.0120	.17	20.62066	15.28313	28.46234	20.70700	39.13830
1.0476	.18	25.15160	17.07262	35.68367	23.51973	50.43942
1.0830	.19	30.11885	18.39938	43.78883	25.59430	63.42357
1.1180	.20	35.31635	18.92588	52.41374	26.28858	77.46752
1.1529	.21	40.44430	18.25389	60.98180	24.81459	91.49304
1.1875	.22	45.10602	15.94132	68.68563	20.27077	103.90450
1.2221	.23	48.81201	11.52791	74.48708	11.69838	112.56889
1.2566	.24	50.99235	4.56929	77.14027	-1.88743	114.85431
1.2910	.25	51.01808	-5.32177	75.24094	-21.14405	107.73999
1.3254	.26	48.23157	-18.42898	67.30369	-46.78018	88.00604
1.3599	.27	41.98512	-34.88336	51.86633	-78.92492	52.50363
1.3944	.28	31.68655	-54.61604	27.61793	-117.24766	-1.50183
1.4291	.29	16.84971	-77.31472	-6.45545	-160.79144	-75.93201
1.4638	.30	-2.85265	-102.39013	-50.91592	-207.88361	-171.46880
1.4988	.31	-27.53686	-128.95522	-105.72084	-256.08792	-287.13105
1.5339	.32	-57.06047	-155.82128	-170.08412	-302.21110	-419.90984
1.5693	.33	-90.98237	-181.51408	-242.36552	-342.37479	-564.48809
1.6049	.34	-128.53485	-204.31206	-320.00136	-372.15965	-713.11234
1.6408	.35	-168.60700	-222.30763	-399.48924	-386.82428	-855.64701
1.6771	.36	-209.75465	-233.49106	-476.43733	-381.59525	-979.84982
1.7136	.37	-250.21825	-235.85499	-545.68568	-352.01841	-1071.88873
1.7506	.38	-287.97084	-227.51592	-601.50286	-294.35538	-1117.10429
1.7879	.39	-320.78370	-206.84752	-637.85655	-206.00201	-1101.00073
1.8257	.40	-346.31315	-172.61913	-648.75111	-85.90121	-1010.42629
1.8640	.41	-362.20543	-124.13168	-628.61974	65.08846	-834.88056
1.9028	.42	-366.21647	-61.34228	-578.75302	243.86079	-567.86488
1.9421	.43	-356.34176	15.03145	-477.74080	444.92726	-208.17371
1.9821	.44	-330.95064	103.43658	-341.89994	660.30164	238.98784
2.0226	.45	-288.91846	201.44780	-165.65783	879.64071	761.18047
2.0638	.46	-229.74923	305.77552	48.14025	1090.53396	1338.31908
2.1057	.47	-153.68135	412.32463	294.03074	1278.99233	1942.76944
2.1483	.48	-61.76869	516.30700	563.86702	1430.12568	2540.05406
2.1918	.49	44.06996	612.40869	846.91373	1528.99037	3090.17801
2.2361	.50	161.03945	695.00836	1111.91889	1561.57261	3549.61274

M	τ	$Y_{-11.5}$	$Y_{-12.0}$	$Y_{-12.5}$	$Y_{-13.0}$	$Y_{-13.5}$	$Y_{-14.0}$	$Y_{-14.5}$	$Y_{-15.0}$
0.22473	0.01	1.15706	1.16439	1.17177	1.17920	1.18667	1.19419	1.20176	1.20938
.31944	.02	1.34487	1.36215	1.37966	1.39740	1.41537	1.43358	1.45201	1.47069
.39324	.03	1.57099	1.60171	1.63304	1.66500	1.69758	1.73081	1.76470	1.79925
.45644	.04	1.84542	1.89423	1.94434	1.99580	2.04865	2.10292	2.15863	2.21584
.51299	.05	2.18167	2.25487	2.33040	2.40866	2.48951	2.57318	2.65967	2.74913
.56493	.06	2.59854	2.70506	2.81470	2.93004	3.04939	3.17438	3.30409	3.43959
.61347	.07	3.12279	3.27669	3.43066	3.59897	3.77035	3.95453	4.14456	4.34650
.65938	.08	3.79283	4.01938	4.22797	4.47610	4.71454	4.98795	5.25861	5.56117
.70321	.09	4.66286	5.01237	5.27768	5.65958	5.97484	6.39561	6.76576	7.23250
.74536	.10	5.80618	6.38120	6.68167	7.31014	7.68834	8.38040	8.84686	9.61427
.78612	.11	7.31552	8.31853	8.57671	9.68601	10.04918	11.28133	11.76987	13.14450
.82772	.12	9.29718	11.10378	11.13201	13.18426	13.31348	15.64628	15.90843	18.56319
.86436	.13	11.85596	15.11898	14.93208	18.37894	17.78406	22.31221	21.73571	27.06023
.90219	.14	15.06779	20.84427	18.93771	26.03917	23.75939	32.46717	29.76409	40.41914
.93934	.15	18.93890	28.82866	24.41880	37.10348	31.43017	47.65058	40.39510	61.08451
.97590	.16	23.35228	39.62098	30.85682	52.58311	40.71195	69.63353	53.64562	92.04144
1.0120	.17	28.00565	53.65449	37.82088	73.36916	51.01265	100.11537	68.73300	136.36578
1.0476	.18	32.34833	71.08334	44.42986	99.92783	60.95240	140.18349	83.53502	196.30550
1.0830	.19	35.52780	91.57802	49.22253	131.88685	68.07542	189.51619	93.99146	271.80292
1.1180	.20	36.35926	114.09781	50.06599	167.54191	68.61889	245.36520	93.57512	358.47985
1.1529	.21	33.33250	136.66964	44.13928	203.34340	57.42443	301.43124	73.02111	445.28124
1.1875	.22	24.66942	156.81272	28.02983	233.45519	28.08943	346.83287	20.54614	512.19221
1.2221	.23	8.44296	168.45541	-2.02392	249.50334	-26.54923	365.45022	-77.20349	528.66893
1.2566	.24	-17.83790	167.99211	-49.72297	240.64604	-113.53151	335.97994	-233.28347	453.60113
1.2910	.25	-55.97907	148.52329	-118.12998	194.09094	-238.47108	233.04436	-457.58292	237.68678
1.3254	.26	-102.79983	103.30904	-208.94751	96.15751	-403.88284	29.63809	-759.05476	-171.01744
1.3599	.27	-163.79987	25.84653	-321.70655	-66.07198	-607.27622	-298.94314	-1111.37809	-818.13317
1.3944	.28	-235.82528	-89.24659	-452.94976	-302.54760	-839.24025	-769.76637	-1508.62341	-1729.27264
1.4291	.29	-316.17195	-245.18664	-595.52039	-617.62386	-1081.82940	-1385.86504	-1901.81292	-2897.74078
1.4638	.30	-400.37001	-441.90413	-738.08664	-1007.24507	-1307.61763	-2129.46855	-2227.25310	-4265.16683
1.4988	.31	-482.09434	-674.95476	-865.03689	-1456.28066	-1479.80719	-2955.81759	-2401.84593	-5710.72290
1.5339	.32	-553.24315	-934.60176	-956.87099	-1936.42773	-1553.74259	-3788.80202	-2328.17966	-7042.04508
1.5693	.33	-604.21632	-1205.21914	-982.18404	-2405.14227	-1480.08167	-4519.75595	-1904.05671	-7996.74136
1.6049	.34	-624.42755	-1465.16693	-944.29072	-2806.05367	-1209.71071	-5010.69776	-1036.49434	-8256.42715
1.6408	.35	-603.01053	-1687.27261	-793.47330	-3071.24750	-700.26787	-5103.02542	340.47168	-7475.85696
1.6771	.36	-529.76900	-1840.01778	-519.75784	-3125.66477	76.12552	-4632.16784	2245.69421	-5327.73603
1.7136	.37	-396.25667	-1889.47599	-111.03756	-2893.66235	1124.60600	-3447.96086	4630.90202	-1561.30502
1.7506	.38	-196.96037	-1801.97836	434.72034	-2307.52142	2419.72124	-1439.61956	7363.87065	3930.17013
1.7879	.39	69.51839	-1547.40309	1106.52053	-1317.39725	3899.13591	1436.85665	10218.34547	11040.93968
1.8257	.40	399.39180	-1102.89857	1878.02303	98.09661	5460.13001	5131.79255	12874.33337	19393.97410
1.8640	.41	782.84017	-456.76598	2706.26679	1921.74948	6960.09190	9479.51060	14931.89167	28304.23061
1.9028	.42	1203.52139	387.84570	3531.81250	4089.34552	8222.00565	14184.63576	15940.44769	36776.67337
1.9421	.43	1638.54210	1409.83150	4280.61433	6484.37310	9045.55362	18821.39155	15443.98617	43550.09119
1.9821	.44	2058.98761	2568.03143	4867.81305	8937.19925	9223.90679	22849.44729	13040.23605	47193.89448
2.0226	.45	2431.06979	3800.54923	5208.48133	11229.85409	8565.58896	25648.64806	8449.46586	46258.99836
2.0638	.46	2717.91017	5025.84087	5200.15387	13107.19054	6920.03048	26973.04584	1585.94414	39475.95789
2.1057	.47	2881.92277	6145.82456	4781.75446	14294.63654	4204.64910	25022.22079	-7377.10357	25984.44439
2.1483	.48	2887.70234	7051.11980	3893.30725	14522.06569	430.60433	20525.12465	-17959.70301	5569.03124
2.1918	.49	2705.26342	7628.34244	2510.61420	13552.52655	-4276.12894	12828.88453	-29355.53393	-21131.52198
2.2361	.50	2313.41765	7769.17495	648.91701	11212.76015	-9661.92845	1982.53129	-40450.46470	-52479.19996

– see errata in front of report

TABLE 3.- THE FUNCTIONS $-\frac{2}{\rho K}\int \frac{dY_k}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR

SEVERAL VALUES OF THE INDEX k

M	τ	$\frac{dY_{0.5}}{d\tau}$	$\frac{dY_{1.0}}{d\tau}$	$\frac{dY_{1.5}}{d\tau}$	$\frac{dY_{2.0}}{d\tau}$	$\frac{dY_{2.5}}{d\tau}$	$\frac{dY_{3.0}}{d\tau}$	$\frac{dY_{3.5}}{d\tau}$	$\frac{dY_{4.0}}{d\tau}$	$\frac{dY_{4.5}}{d\tau}$	$\frac{dY_{5.0}}{d\tau}$	$\frac{dY_{5.5}}{d\tau}$	$\frac{dY_{6.0}}{d\tau}$	$\frac{dY_{6.5}}{d\tau}$	$\frac{dY_{7.0}}{d\tau}$	$\frac{dY_{7.5}}{d\tau}$
0.22473	0.01	0.9923	0.9900	0.9866	0.9826	0.9781	0.9732	0.9682	0.9630	0.9577	0.9523	0.9468	0.9414	0.9359	0.9303	0.9248
.31944	.02	.9845	.9801	.9734	.9653	.9564	.9469	.9370	.9269	.9166	.9063	.8958	.8854	.8750	.8646	.8543
.39324	.03	.9768	.9702	.9602	.9482	.9350	.9210	.9065	.8918	.8769	.8619	.8469	.8320	.8173	.8026	.7881
.45644	.04	.9692	.9603	.9471	.9313	.9139	.8956	.8767	.8576	.8383	.8191	.8001	.7812	.7625	.7442	.7261
.51299	.05	.9615	.9505	.9340	.9145	.8930	.8706	.8475	.8243	.8010	.7779	.7552	.7327	.7107	.6892	.6681
.56493	.06	.9539	.9407	.9211	.8978	.8725	.8460	.8190	.7919	.7649	.7383	.7122	.6866	.6617	.6375	.6139
.61347	.07	.9463	.9309	.9082	.8814	.8522	.8219	.7911	.7603	.7299	.7001	.6710	.6427	.6153	.5888	.5633
.65938	.08	.9387	.9212	.8954	.8650	.8322	.7982	.7638	.7297	.6961	.6634	.6317	.6011	.5719	.5432	.5161
.70321	.09	.9311	.9115	.8827	.8489	.8124	.7749	.7372	.6999	.6635	.6282	.5941	.5615	.5302	.5004	.4720
.74536	.10	.9236	.9019	.8701	.8329	.7930	.7520	.7111	.6709	.6319	.5943	.5582	.5239	.4912	.4603	.4311
.78612	.11	.9160	.8923	.8576	.8170	.7737	.7296	.6857	.6428	.6014	.5617	.5240	.4882	.4545	.4228	.3930
.82572	.12	.9085	.8827	.8451	.8013	.7548	.7076	.6608	.6155	.5719	.5305	.4913	.4545	.4200	.3877	.3576
.86436	.13	.9010	.8732	.8327	.7858	.7361	.6860	.6366	.5890	.5435	.5006	.4602	.4225	.3875	.3549	.3249
.90219	.14	.8936	.8637	.8204	.7704	.7177	.6648	.6130	.5632	.5161	.4719	.4306	.3923	.3569	.3244	.2945
.93934	.15	.8861	.8543	.8082	.7552	.6996	.6440	.5899	.5383	.4897	.4444	.4024	.3637	.3283	.2959	.2665
.97590	.16	.8787	.8449	.7961	.7402	.6817	.6236	.5674	.5141	.4642	.4180	.3756	.3368	.3015	.2695	.2405
1.0120	.17	.8713	.8355	.7840	.7253	.6641	.6036	.5455	.4907	.4397	.3929	.3501	.3113	.2763	.2449	.2167
1.0476	.18	.8640	.8262	.7721	.7105	.6468	.5840	.5241	.4679	.4161	.3688	.3260	.2874	.2528	.2220	.1947
1.0830	.19	.8566	.8170	.7602	.6959	.6297	.5649	.5033	.4460	.3934	.3458	.3030	.2648	.2309	.2009	.1745
1.1180	.20	.8493	.8077	.7484	.6815	.6129	.5461	.4830	.4247	.3716	.3239	.2813	.2436	.2104	.1813	.1560
1.1529	.21	.8420	.7985	.7367	.6672	.5963	.5277	.4632	.4041	.3507	.3030	.2608	.2237	.1914	.1633	.1390
1.1875	.22	.8347	.7894	.7251	.6531	.5800	.5097	.4440	.3842	.3306	.2831	.2414	.2051	.1737	.1466	.1235
1.2221	.23	.8274	.7802	.7135	.6392	.5640	.4920	.4254	.3650	.3113	.2641	.2230	.1876	.1572	.1313	.1094
1.2566	.24	.8202	.7712	.7021	.7254	.5482	.4748	.4072	.3464	.2928	.2460	.2057	.1712	.1419	.1173	.0966
1.2910	.25	.8130	.7621	.6907	.6117	.5327	.4579	.2895	.3285	.2751	.2289	.1894	.1559	.1278	.1044	.0849
1.3254	.26	.8058	.7532	.6794	.5982	.5174	.4414	.3724	.3112	.2581	.2126	.1740	.1417	.1148	.0926	.0743
1.3599	.27	.7987	.7442	.6682	.5849	.5024	.4253	.3557	.2946	.2419	.1971	.1596	.1284	.1027	.0818	.0648
1.3944	.28	.7915	.7353	.6571	.5717	.4877	.4095	.3395	.2785	.2264	.1825	.1460	.1161	.0917	.0720	.0562
1.4291	.29	.7844	.7264	.6461	.5587	.4732	.3941	.3239	.2631	.2116	.1686	.1333	.1046	.0815	.0631	.0485
1.4638	.30	.7773	.7176	.6351	.5459	.4589	.3791	.3086	.2482	.1974	.1555	.1214	.0940	.0722	.0550	.0416
1.4988	.31	.7703	.7088	.6243	.5332	.4449	.3644	.2939	.2339	.1839	.1431	.1103	.0841	.0636	.0477	.0354
1.5339	.32	.7632	.7001	.6135	.5206	.4311	.3501	.2796	.2201	.1711	.1314	.0999	.0751	.0558	.0411	.0300
1.5693	.33	.7562	.6914	.6029	.5083	.4176	.3361	.2658	.2069	.1589	.1204	.0902	.0667	.0488	.0352	.0251
1.6049	.34	.7492	.6828	.5923	.4960	.4044	.3225	.2524	.1943	.1473	.1101	.0812	.0590	.0423	.0299	.0209
1.6408	.35	.7422	.6742	.5818	.4840	.3914	.3092	.2394	.1821	.1363	.1004	.0728	.0520	.0365	.0252	.0171
1.6771	.36	.7353	.6656	.5713	.4721	.3786	.2963	.2269	.1705	.1258	.0912	.0650	.0455	.0313	.0210	.0138
1.7136	.37	.7284	.6571	.5610	.4603	.3661	.2837	.2148	.1594	.1159	.0827	.0578	.0396	.0266	.0174	.0110
1.7506	.38	.7215	.6486	.5508	.4487	.3538	.2714	.2032	.1487	.1065	.0747	.0512	.0343	.0223	.0141	.0086
1.7879	.39	.7146	.6402	.5406	.4373	.3418	.2594	.1919	.1385	.0977	.0672	.0451	.0294	.0186	.0113	.0065
1.8257	.40	.7078	.6318	.5306	.4260	.3300	.2478	.1811	.1288	.0893	.0603	.0395	.0251	.0152	.0088	.0047
1.8640	.41	.7010	.6235	.5206	.4149	.3184	.2365	.1706	.1196	.0815	.0538	.0344	.0211	.0123	.0067	.0032
1.9028	.42	.6942	.6152	.5107	.4039	.3071	.2256	.1605	.1108	.0740	.0478	.0297	.0176	.0097	.0048	.0020
1.9421	.43	.6874	.6070	.5010	.3931	.2960	.2149	.1508	.1024	.0671	.0423	.0254	.0144	.0074	.0033	.0009
1.9821	.44	.6807	.5988	.4913	.3825	.2851	.2046	.1415	.0944	.0605	.0371	.0215	.0116	.0055	.0020	.0001
2.0226	.45	.6740	.5906	.4817	.3720	.2745	.1946	.1326	.0868	.0544	.0324	.0180	.0091	.0038	.0009	-.0005
2.0638	.46	.6673	.5825	.4721	.3616	.2642	.1848	.1240	.0797	.0487	.0281	.0149	.0069	.0024	0	-.0010
2.1057	.47	.6606	.5745	.4627	.3515	.2540	.1754	.1158	.0729	.0434	.0241	.0121	.0050	.0012	-.0007	-.0014
2.1483	.48	.6540	.5665	.4534	.3414	.2441	.1663	.1079	.0665	.0384	.0204	.0095	.0033	.0002	-.0012	-.0016
2.1918	.49	.6474	.5585	.4442	.3316	.2344	.1574	.1004	.0604	.0338	.0171	.0073	.0019	-.0007	-.0016	-.0018
2.2361	.50	.6408	.5506	.4350	.3219	.2250	.1489	.0932	.0547	.0296	.0141	.0053	.0007	-.0013	-.0019	-.0019

NACA

$\tau = 2/\beta^{k}$

TABLE 3.- THE FUNCTIONS $\frac{dY_k}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k - Concluded

M	τ	$\frac{dY_{8.0}}{d\tau}$	$\frac{dY_{8.5}}{d\tau}$	$\frac{dY_{9.0}}{d\tau}$	$\frac{dY_{9.5}}{d\tau}$	$\frac{dY_{10.0}}{d\tau}$	$\frac{dY_{10.5}}{d\tau}$	$\frac{dY_{11.0}}{d\tau}$	$\frac{dY_{11.5}}{d\tau}$	$\frac{dY_{12.0}}{d\tau}$	$\frac{dY_{12.5}}{d\tau}$	$\frac{dY_{13.0}}{d\tau}$	$\frac{dY_{13.5}}{d\tau}$	$\frac{dY_{14.0}}{d\tau}$	$\frac{dY_{14.5}}{d\tau}$	$\frac{dY_{15.0}}{d\tau}$
0.22473	0.01	0.9193	0.9138	0.9083	0.9028	0.8973	0.8918	0.8864	0.8809	0.8755	0.8701	0.8648	0.8595	0.8541	0.8489	0.8436
.31944	.02	.8440	.8338	.8236	.8136	.8036	.7938	.7840	.7743	.7647	.7553	.7459	.7366	.7275	.7184	.7094
.39304	.03	.7738	.7596	.7457	.7319	.7184	.7051	.6920	.6790	.6664	.6539	.6416	.6295	.6177	.6061	.5946
.45644	.04	.7084	.6910	.6740	.6573	.6409	.6250	.6093	.5941	.5791	.5645	.5503	.5364	.5228	.5096	.4967
.51099	.05	.6476	.6276	.6081	.5891	.5707	.5527	.5353	.5184	.5020	.4860	.4706	.4556	.4411	.4270	.4133
.56493	.06	.5911	.5690	.5476	.5269	.5070	.4877	.4691	.4512	.4339	.4172	.4012	.3857	.3708	.3565	.3427
.61347	.07	.5387	.5150	.4922	.4704	.4494	.4293	.4100	.3916	.3739	.3570	.3409	.3254	.3106	.2965	.2829
.65938	.08	.4901	.4652	.4415	.4190	.3974	.3769	.3574	.3389	.3213	.3045	.2886	.2735	.2592	.2455	.2326
.70321	.09	.4451	.4195	.3953	.3723	.3506	.3301	.3107	.2924	.2751	.2588	.2435	.2290	.2154	.2025	.1904
.74536	.10	.4035	.3775	.3530	.3301	.3085	.2882	.2692	.2515	.2348	.2192	.2046	.1910	.1782	.1663	.1551
.78612	.11	.3651	.3390	.3146	.2919	.2707	.2510	.2326	.2155	.1997	.1849	.1712	.1585	.1468	.1358	.1257
.82572	.12	.3297	.3037	.2797	.2574	.2368	.2178	.2003	.1841	.1691	.1554	.1427	.1310	.1203	.1104	.1013
.86436	.13	.2971	.2715	.2480	.2264	.2066	.1884	.1718	.1566	.1427	.1300	.1183	.1077	.0981	.0892	.0812
.90219	.14	.2672	.2422	.2193	.1986	.1796	.1624	.1468	.1327	.1198	.1082	.0977	.0881	.0795	.0717	.0647
.93934	.15	.2397	.2154	.1934	.1736	.1557	.1395	.1250	.1119	.1002	.0896	.0802	.0717	.0641	.0573	.0511
.97590	.16	.2145	.1911	.1701	.1512	.1344	.1194	.1059	.0940	.0833	.0739	.0654	.0579	.0513	.0454	.0402
1.0120	.17	.1915	.1690	.1490	.1313	.1156	.1017	.0894	.0785	.0689	.0605	.0531	.0465	.0408	.0357	.0313
1.0476	.18	.1704	.1490	.1302	.1136	.0990	.0862	.0750	.0653	.0567	.0493	.0427	.0371	.0322	.0279	.0241
1.0830	.19	.1513	.1310	.1133	.0978	.0844	.0728	.0627	.0539	.0463	.0398	.0342	.0293	.0251	.0215	.0185
1.1180	.20	.1339	.1148	.0982	.0839	.0716	.0611	.0520	.0442	.0376	.0319	.0271	.0230	.0195	.0165	.0140
1.1529	.21	.1181	.1001	.0848	.0716	.0605	.0510	.0429	.0361	.0303	.0254	.0213	.0178	.0149	.0125	.0104
1.1875	.22	.1038	.0870	.0728	.0609	.0507	.0423	.0351	.0292	.0242	.0200	.0166	.0137	.0113	.0093	.0077
1.2221	.23	.0909	.0753	.0623	.0514	.0423	.0348	.0285	.0234	.0191	.0156	.0127	.0104	.0084	.0068	.0055
1.2566	.24	.0793	.0649	.0530	.0431	.0351	.0284	.0230	.0186	.0150	.0120	.0097	.0077	.0062	.0049	.0039
1.2910	.25	.0688	.0556	.0448	.0360	.0288	.0230	.0183	.0146	.0115	.0091	.0072	.0057	.0044	.0035	.0027
1.3254	.26	.0595	.0474	.0376	.0298	.0235	.0184	.0144	.0113	.0088	.0068	.0053	.0041	.0031	.0024	.0018
1.3599	.27	.0511	.0401	.0314	.0244	.0189	.0146	.0112	.0086	.0066	.0050	.0038	.0028	.0021	.0016	.0012
1.3944	.28	.0437	.0337	.0259	.0198	.0151	.0114	.0086	.0064	.0048	.0035	.0026	.0019	.0014	.0010	.0007
1.4291	.29	.0371	.0281	.0212	.0159	.0119	.0088	.0065	.0047	.0034	.0024	.0017	.0012	.0008	.0006	.0004
1.4638	.30	.0312	.0233	.0172	.0126	.0092	.0066	.0047	.0033	.0023	.0016	.0011	.0007	.0005	.0003	.0002
1.4988	.31	.0261	.0190	.0138	.0098	.0070	.0049	.0033	.0023	.0015	.0010	.0006	.0004	.0002	.0001	0
1.5339	.32	.0216	.0154	.0108	.0075	.0051	.0035	.0023	.0014	.0009	.0005	.0003	.0001	0	0	0
1.5693	.33	.0177	.0123	.0084	.0056	.0037	.0023	.0014	.0008	.0004	.0002	.0001	0	-.0001	-.0001	-.0001
1.6049	.34	.0143	.0096	.0063	.0040	.0025	.0015	.0008	.0004	.0001	0	-.0001	-.0001	-.0001	-.0001	-.0001
1.6408	.35	.0113	.0073	.0046	.0028	.0016	.0008	.0003	.0001	-.0001	-.0002	-.0002	-.0002	-.0002	-.0001	-.0001
1.6771	.36	.0088	.0055	.0032	.0018	.0008	.0003	0	-.0002	-.0002	-.0002	-.0002	-.0002	-.0002	-.0001	-.0001
1.7136	.37	.0067	.0039	.0021	.0010	.0003	-.0001	-.0002	-.0003	-.0003	-.0003	-.0002	-.0002	-.0001	-.0001	-.0001
1.7506	.38	.0050	.0026	.0012	.0004	-.0001	-.0003	-.0004	-.0004	-.0005	-.0003	-.0003	-.0002	-.0002	-.0001	-.0001
1.7879	.39	.0034	.0016	.0005	-.0001	-.0004	-.0005	-.0005	-.0004	-.0003	-.0003	-.0002	-.0001	-.0001	-.0001	0
1.8257	.40	.0022	.0007	-.0001	-.0004	-.0006	-.0005	-.0005	-.0004	-.0003	-.0002	-.0002	-.0001	-.0001	0	0
1.8640	.41	.0012	.0001	-.0005	-.0006	-.0007	-.0006	-.0005	-.0004	-.0003	-.0002	-.0001	-.0001	-.0001	0	0
1.9028	.42	.0004	-.0004	-.0007	-.0008	-.0007	-.0006	-.0004	-.0003	-.0002	-.0002	-.0001	-.0001	0	0	0
1.9421	.43	-.0003	-.0008	-.0009	-.0008	-.0007	-.0005	-.0004	-.0003	-.0002	-.0001	-.0001	0	0	0	0
1.9821	.44	-.0007	-.0010	-.0010	-.0009	-.0007	-.0005	-.0003	-.0002	-.0001	-.0001	0	0	0	0	0
2.0226	.45	-.0011	-.0012	-.0011	-.0008	-.0006	-.0004	-.0003	-.0002	-.0001	-.0001	0	0	0	0	0
2.0638	.46	-.0013	-.0013	-.0010	-.0008	-.0006	-.0004	-.0002	-.0001	-.0001	0	0	0	0	0	0
2.1057	.47	-.0015	-.0013	-.0010	-.0007	-.0005	-.0003	-.0002	-.0001	0	0	0	0	0	0	0
2.1483	.48	-.0015	-.0013	-.0009	-.0006	-.0004	-.0002	-.0001	-.0001	0	0	0	0	0	0	0
2.1918	.49	-.0016	-.0012	-.0008	-.0005	-.0003	-.0002	-.0001	0	0	0	0	0	0	0	0
2.2361	.50	-.0015	-.0011	-.0007	-.0005	-.0003	-.0001	0	0	0	0	0	0	0	0	0

TABLE 4.- THE FUNCTIONS $\frac{-2}{B^k}\frac{d\Upsilon_{-k}}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k

M	τ	$\frac{d\Upsilon_{-0.5}}{d\tau}$	$\frac{d\Upsilon_{-1.0}}{d\tau}$	$\frac{d\Upsilon_{-1.5}}{d\tau}$	$\frac{d\Upsilon_{-2.0}}{d\tau}$	$\frac{d\Upsilon_{-2.5}}{d\tau}$	$\frac{d\Upsilon_{-3.0}}{d\tau}$	$\frac{d\Upsilon_{-3.5}}{d\tau}$	$\frac{d\Upsilon_{-4.0}}{d\tau}$	$\frac{d\Upsilon_{-4.5}}{d\tau}$	$\frac{d\Upsilon_{-5.0}}{d\tau}$	$\frac{d\Upsilon_{-5.5}}{d\tau}$	$\frac{d\Upsilon_{-6.0}}{d\tau}$	$\frac{d\Upsilon_{-6.5}}{d\tau}$	$\frac{d\Upsilon_{-7.0}}{d\tau}$	$\frac{d\Upsilon_{-7.5}}{d\tau}$	$\frac{d\Upsilon_{-8.0}}{d\tau}$	$\frac{d\Upsilon_{-8.5}}{d\tau}$
0.22473	0.01	-0.9888	-0.9752	-0.9225	-1.2340	-1.1556	-1.1548	-1.1238	-1.1190	-1.1147	-1.1161	-1.1188	-1.1230	-1.1278	-1.1332	-1.1390	-1.1450	-1.1513
.31944	.02	-.9777	-.9507	-.8476	-1.3671	-1.2767	-1.3480	-1.2729	-1.2799	-1.2583	-1.2632	-1.2640	-1.2728	-1.2819	-1.2936	-1.3061	-1.3197	-1.3339
.39324	.03	-.9666	-.9267	-.7752	-1.4551	-1.3657	-1.5583	-1.4346	-1.4878	-1.4333	-1.4555	-1.4440	-1.4625	-1.4715	-1.4921	-1.5108	-1.5342	-1.5578
.45644	.04	-.9556	-.9030	-.7053	-1.5116	-1.4251	-1.7727	-1.5963	-1.7413	-1.6369	-1.7062	-1.6658	-1.7100	-1.7083	-1.7464	-1.7664	-1.8050	-1.8373
.51299	.05	-.9447	-.8796	-.6378	-1.5437	-1.4573	-1.9819	-1.7495	-2.0345	-1.8824	-2.0245	-1.9322	-2.0357	-2.0038	-2.0817	-2.0899	-2.1587	-2.1929
.56493	.06	-.9339	-.8567	-.5727	-1.5597	-1.4644	-2.1787	-1.8855	-2.3590	-2.1000	-2.4150	-2.2412	-2.4599	-2.3665	-2.5300	-2.5000	-2.6342	-2.6511
.61347	.07	-.9231	-.8341	-.5099	-1.5512	-1.4486	-2.3576	-1.9976	-2.7047	-2.3379	-2.8767	-2.5852	-2.9992	-2.7998	-3.1268	-3.0131	-3.2829	-3.2420
.65938	.08	-.9124	-.8118	-.4495	-1.5325	-1.4120	-2.5146	-2.0804	-3.0602	-2.5633	-3.4032	-2.9512	-3.6643	-3.2986	-3.9070	-3.6392	-4.1662	-3.9946
.70321	.09	-.9018	-.7900	-.3913	-1.5016	-1.3566	-2.6463	-2.1296	-3.4138	-2.7628	-3.9848	-3.3211	-4.4573	-3.8487	-4.8994	-4.3767	-5.3492	-4.9286
.74536	.10	-.8913	-.7684	-.3354	-1.4604	-1.2843	-2.7506	-2.1422	-3.7537	-2.9234	-4.5994	-3.6728	-5.3705	-4.4251	-6.1215	-5.2080	-6.8919	-6.0454
.78612	.11	-.8809	-.7473	-.2816	-1.4101	-1.1969	-2.8257	-2.1161	-4.0685	-3.0326	-5.2329	-3.9809	-6.3855	-4.9923	-7.5740	-6.0952	-8.8378	-7.3179
.82572	.12	-.8705	-.7265	-.2300	-1.3522	-1.0962	-2.8707	-2.0501	-4.3474	-3.0790	-5.8603	-4.2183	-7.4725	-5.5052	-9.2366	-6.9786	-11.2021	-8.6809
.86436	.13	-.8602	-.7060	-.1805	-1.2876	-.9838	-2.8851	-1.9439	-4.5806	-3.0530	-6.4561	-4.3572	-8.5915	-5.9108	-11.0648	-7.7757	-13.9595	-10.0242
.90219	.14	-.8500	-.6859	-.1330	-1.2173	-.8614	-2.8688	-1.7980	-4.7591	-2.9463	-6.9946	-4.3703	-9.6931	-6.1502	-12.9883	-8.3833	-17.0340	-11.1886
.93934	.15	-.8398	-.6661	-.0876	-1.1422	-.7305	-2.8223	-1.6135	-4.8734	-2.7530	-7.4478	-4.2324	-10.7203	-6.1617	-14.9112	-8.6807	-20.8915	-11.9673
.97590	.16	-.8298	-.6467	-.0442	-1.0631	-.5924	-2.7462	-1.3921	-4.9229	-2.4688	-7.7910	-3.9212	-11.6101	-5.8842	-16.7139	-8.5361	-23.5362	-12.1120
1.0120	.17	-.8198	-.6276	-.0028	-.9806	-.4486	-2.6416	-1.1362	-4.8968	-2.0919	-7.9991	-3.4184	-12.2968	-5.2603	-18.2570	-7.8139	-26.5119	-11.3453
1.0476	.18	-.8099	-.6089	.0368	-.8956	-.3004	-2.5097	-.8483	-4.7931	-1.6225	-8.0500	-2.7110	-12.7137	-4.2400	-19.3864	-6.3832	-28.9084	-9.3777
1.0830	.19	-.8000	-.5905	.0744	-.8084	-.1490	-2.3520	-.5316	-4.6098	-1.0630	-7.9243	-1.7912	-12.7964	-2.7839	-19.9400	-4.1277	-30.3732	-5.9303
1.1180	.20	-.7903	-.5724	.1102	-.7197	.0045	-2.1701	-.1894	-4.3457	-.4176	-7.6061	-.6576	-12.4856	-.8662	-19.7556	-.9556	-30.5285	-.7600
1.1529	.21	-.7806	-.5547	.1443	-.6300	.1588	-1.9660	.1745	-4.0013	.3072	-7.0836	.6847	-11.7291	1.5226	-18.6793	3.1908	-28.9922	6.3129
1.1875	.22	-.7710	-.5373	.1765	-.5398	.3131	-1.7415	.5563	-3.5783	1.1035	-6.3492	2.2239	-10.4850	4.3736	-16.5744	8.3661	-25.4019	15.3774
1.2221	.23	-.7615	-.5203	.2071	-.4493	.4664	-1.4987	.9520	-3.0796	1.9613	-5.4001	3.9416	-8.7234	7.6584	-13.3302	14.4150	-19.4426	26.4059
1.2566	.24	-.7520	-.5035	.2359	-.3591	.6177	-1.2398	1.3774	-2.5092	2.8695	-4.2381	5.8132	-6.4311	11.3279	-8.8696	21.3662	-10.8733	39.2334
1.2910	.25	-.7427	-.4871	.2631	-.2695	.7663	-.9668	1.7685	-1.8722	3.8156	-2.8700	7.8079	-3.5996	15.3129	-3.1576	29.0298	.4461	53.5417
1.3254	.26	-.7334	-.4711	.2887	-.1809	.9113	-.6821	2.1811	-1.1747	4.7863	-1.3073	9.8896	-.2530	19.5844	3.7935	37.1969	14.5309	68.8924
1.3599	.27	-.7242	-.4553	.3128	-.0934	1.0521	-.3878	2.5910	-.4237	5.7673	.4338	12.0174	3.5785	23.8556	11.9194	45.6017	31.2515	84.5276
1.3944	.28	-.7150	-.4399	.3353	-.0074	1.1879	-.0853	2.9943	.3732	6.7438	2.3328	14.1467	7.8448	28.1838	21.1004	53.9274	51.3187	99.7825
1.4291	.29	-.7060	-.4248	.3563	.0768	1.3182	.2203	3.3899	1.2075	7.7007	4.3651	16.2299	12.4792	32.3737	31.1613	61.8141	71.2766	113.7075
1.4638	.30	-.6970	-.4100	.3758	.1590	1.4425	.5298	3.7651	2.0705	8.6228	6.5024	18.2174	17.3994	36.8807	41.8725	68.8700	93.5020	125.3006
1.4988	.31	-.6881	-.3955	.3939	.2390	1.5602	.8398	4.1254	2.9528	9.4958	8.7132	20.0590	22.3089	39.7552	52.9535	74.6852	116.2135	133.5100
1.5339	.32	-.6793	-.3813	.4107	.3167	1.6710	1.1489	4.4642	3.8449	10.3033	10.9632	21.7047	27.6985	42.6467	64.0787	78.8469	138.4889	137.2846
1.5693	.33	-.6705	-.3674	.4261	.3918	1.7744	1.4536	4.7785	4.7368	11.0330	13.2164	23.1059	32.8490	44.8087	74.8853	80.9566	159.2912	135.6313
1.6049	.34	-.6619	-.3539	.4401	.4642	1.8701	1.7533	5.0654	5.6187	11.6713	15.4348	24.2166	37.8335	46.1036	84.9828	80.6478	177.5031	127.6797
1.6408	.35	-.6533	-.3406	.4529	.5337	1.9519	2.0456	5.3221	6.4809	12.2079	17.5798	24.9946	42.5202	46.4073	93.9645	77.6049	191.9689	112.7245
1.6771	.36	-.6448	-.3277	.4645	.6003	2.0374	2.3286	5.5463	7.3137	12.6260	19.6188	25.4023	46.7758	45.6140	101.4200	71.5801	201.5425	90.3253
1.7136	.37	-.6364	-.3150	.4749	.6637	2.1085	2.6008	5.7359	8.1075	12.8220	21.4953	25.4080	50.4687	43.6409	106.9496	62.4107	205.1395	60.3218
1.7506	.38	-.6280	-.3027	.4841	.7240	2.1711	2.8604	5.8892	8.8535	13.0859	23.1903	24.9862	53.4718	40.4302	110.1777	50.0335	201.7923	22.8997
1.7879	.39	-.6197	-.2905	.4921	.7810	2.2251	3.1060	6.0048	9.5428	13.1112	24.6625	24.1191	55.6669	35.9567	110.7683	34.4964	190.7058	-21.3786
1.8257	.40	-.6116	-.2789	.4991	.8347	2.2703	3.3362	6.0815	10.1676	12.9933	25.8790	22.7969	56.9470	30.2262	108.4386	15.9681	171.3075	-71.5550
1.8640	.41	-.6034	-.2674	.5050	.8850	2.3068	3.5498	6.1185	10.7203	12.7291	26.8100	21.0182	57.2201	23.2797	102.9724	-5.2579	143.3019	-126.2723
1.9028	.42	-.5954	-.2562	.5099	.9319	2.3346	3.7455	6.1155	11.1943	12.3183	27.4295	18.7903	56.4120	15.1939	94.2330	-28.7620	106.7064	-183.7915
1.9421	.43	-.5875	-.2453	.5138	.9733	2.3538	3.9224	6.0783	11.5836	11.7612	27.7155	16.1299	54.4687	6.0818	82.1730	-54.0027	61.8860	-242.0285
1.9821	.44	-.5796	-.2347	.5167	1.0153	2.3644	4.0796	5.9890	11.8830	11.0611	27.6506	13.0626	51.3591	-3.9087	66.8423	-80.3252	9.5737	-298.6106
2.0226	.45	-.5718	-.2243	.5187	1.0517	2.3666	4.2163	5.8663	12.0883	10.2226	27.2226	9.6229	47.0767	-14.5959	48.3974	-106.9755	-49.1224	-350.9541
2.0638	.46	-.5641	-.2143	.5199	1.0847	2.3605	4.3320	5.7050	12.1963	9.2526	26.4644	5.8539	41.6408	-25.7678	27.0974	-133.1180	-112.7267	-396.3594
2.1057	.47	-.5565	-.2045	.5201	1.1141	2.3463	4.4261	5.5062	12.2046	8.1596	25.2547	-1.8069	35.0978	-37.1854	3.3108	-157.8592	-179.4194	-432.1212
2.1483	.48	-.5489	-.1950	.5196	1.1401	2.3248	4.4983	5.2714	12.1120	6.9540	23.7181	-2.4595	27.5211	-48.5877	-22.4916	-180.2737	-247.0770	-455.6580
2.1918	.49	-.5414	-.1857	.5183	1.1627	2.2946	4.5485	5.0024	11.9182	5.6476	21.8248	-6.8799	19.0112	-59.6971	-49.7429	-199.4350	-313.3276	-464.6116
2.2361	.50	-.5340	-.1768	.5162	1.1818	2.2576	4.5764	4.7012	11.6239	4.2542	19.5912	-11.3835	9.6942	-70.2260	-77.7905	-214.4478	-375.6230	-457.0407

TABLE 4.- THE FUNCTIONS $-\frac{2}{BK}\frac{d\Gamma_{-k}}{d\tau}$ FOR AIR ($\gamma = 1.4$) FOR SEVERAL VALUES OF THE INDEX k - Concluded

M	τ	$\frac{d\Gamma_{-9.0}}{d\tau}$	$\frac{d\Gamma_{-9.5}}{d\tau}$	$\frac{d\Gamma_{-10.0}}{d\tau}$	$\frac{d\Gamma_{-10.5}}{d\tau}$	$\frac{d\Gamma_{-11.0}}{d\tau}$	$\frac{d\Gamma_{-11.5}}{d\tau}$	$\frac{d\Gamma_{-12.0}}{d\tau}$	$\frac{d\Gamma_{-12.5}}{d\tau}$	$\frac{d\Gamma_{-13.0}}{d\tau}$	$\frac{d\Gamma_{-13.5}}{d\tau}$	$\frac{d\Gamma_{-14.0}}{d\tau}$	$\frac{d\Gamma_{-14.5}}{d\tau}$	$\frac{d\Gamma_{-15.0}}{d\tau}$
0.22473	0.01	-1.1377	-1.1643	-1.1710	-1.1779	-1.1848	-1.1919	-1.1990	-1.2063	-1.2136	-1.2209	-1.2284	-1.2359	-1.2435
.31944	.02	-1.3488	-1.3682	-1.3800	-1.3962	-1.4188	-1.4298	-1.4471	-1.4647	-1.4826	-1.5008	-1.5193	-1.5382	-1.5573
.39324	.03	-1.5837	-1.6103	-1.6388	-1.6672	-1.6971	-1.7277	-1.7593	-1.7917	-1.8249	-1.8589	-1.8937	-1.9292	-1.9656
.45644	.04	-1.8787	-1.9185	-1.9632	-2.0084	-2.0565	-2.1059	-2.1575	-2.2106	-2.2657	-2.3224	-2.3809	-2.4411	-2.5032
.51299	.05	-2.2605	-2.3119	-2.3682	-2.4458	-2.5208	-2.5939	-2.6744	-2.7556	-2.8423	-2.9311	-3.0243	-3.1204	-3.2206
.56493	.06	-2.7719	-2.8234	-2.9408	-3.0184	-3.1383	-3.2366	-3.3629	-3.4787	-3.6138	-3.7454	-3.8912	-4.0379	-4.1939
.61347	.07	-3.4768	-3.4962	-3.7122	-3.7812	-3.9903	-4.1013	-4.3127	-4.4997	-4.6798	-4.8597	-5.0937	-5.3045	-5.5568
.65938	.08	-4.4619	-4.3799	-4.8067	-4.8065	-5.2092	-5.2834	-5.6761	-5.8189	-6.2138	-6.4211	-6.8287	-7.0980	-7.3279
.70321	.09	-5.8347	-5.5232	-6.3770	-6.1773	-6.9938	-6.9062	-7.7009	-7.7858	-8.5134	-8.6498	-9.4469	-9.6967	-10.5183
.74536	.10	-7.7136	-6.9597	-8.6148	-7.9798	-9.6219	-9.1090	-10.7616	-10.3920	-12.0617	-11.8493	-13.5522	-13.5116	-15.2661
.78612	.11	-10.2189	-8.6899	-11.7352	-10.2436	-13.4485	-12.0154	-15.3763	-14.0467	-17.5829	-16.3892	-20.1132	-19.0861	-23.0339
.82578	.12	-13.4201	-10.6601	-15.9465	-12.9716	-18.8445	-15.6802	-22.1870	-18.8619	-26.0590	-22.6067	-30.5605	-27.0210	-35.8094
.86436	.13	-17.3698	-12.7417	-21.4046	-16.0303	-26.1925	-20.0130	-31.8899	-24.8376	-38.6665	-30.6829	-46.7321	-37.7645	-56.4036
.90219	.14	-22.0152	-14.7189	-28.1557	-19.1375	-35.7888	-24.6869	-45.0688	-31.6395	-56.9854	-40.3407	-70.7812	-51.2187	-88.2732
.93934	.15	-27.2017	-16.2491	-36.4714	-21.8171	-47.4451	-29.0437	-62.0123	-38.4055	-80.6474	-50.5121	-104.4588	-66.1429	-134.8450
.97590	.16	-32.6490	-16.9823	-44.7697	-23.3768	-60.9371	-32.0169	-82.3849	-43.5566	-110.8164	-58.9376	-148.4449	-79.4006	-198.1728
1.0120	.17	-37.9188	-16.2148	-53.6418	-22.9106	-75.2817	-32.0936	-104.9228	-44.6778	-145.5989	-61.8113	-201.0310	-85.1860	-276.7499
1.0476	.18	-42.4734	-13.5474	-61.6609	-19.3345	-88.8198	-27.3416	-127.1215	-38.3889	-181.0127	-53.5917	-256.6824	-74.4643	-362.7373
1.0830	.19	-45.5495	-8.3291	-67.9405	-11.4613	-99.8961	-15.5203	-145.0098	-20.6845	-210.6351	-27.1830	-304.6183	-34.9452	-438.8915
1.1180	.20	-46.4068	.0169	-69.6918	1.8869	-103.6809	5.7099	-153.0820	12.9070	-224.6029	25.7902	-327.7650	48.0736	-476.0468
1.1529	.21	-44.1685	11.9849	-66.3207	21.7264	-98.3977	38.4772	-144.4625	66.6206	-210.0728	113.8551	-302.6937	189.6490	-432.1938
1.1875	.22	-37.9796	27.6744	-55.5333	48.7406	-79.5447	84.3083	-111.3600	143.6364	-151.9399	241.5805	-200.8894	401.8951	-294.3881
1.2221	.23	-27.0972	47.2862	-35.6022	83.0866	-43.8794	143.6728	-45.8439	243.0613	-34.4045	413.0718	8.4501	689.0518	115.3798
1.2566	.24	-10.7681	70.4617	-4.9668	124.2142	13.7215	215.3834	59.0706	368.8312	156.6460	623.5413	352.7732	1042.6544	789.6949
1.2910	.25	11.8306	96.5296	37.1500	170.7083	93.6952	296.8789	208.0780	508.6824	430.5410	859.8999	849.7479	1435.4962	1680.4180
1.3254	.26	39.2137	124.4133	91.6128	220.1817	197.2027	382.3814	402.1033	653.3705	788.2705	1098.2701	1498.9964	1817.6378	2780.6185
1.3599	.27	72.7192	152.6234	157.1585	269.2381	328.5594	464.8721	696.8858	786.3353	1218.8784	1303.1880	2273.1519	2114.5871	4144.7935
1.3944	.28	111.0947	179.2831	238.1947	313.9239	465.3565	534.1111	901.4432	885.9968	1696.3506	1428.3904	3111.8288	2289.4900	5578.4952
1.4291	.29	153.1640	202.1854	313.7495	347.8827	618.1419	578.5843	1177.8051	926.8932	2177.6308	1419.6472	3913.5827	2050.4628	6889.8574
1.4638	.30	197.8798	218.8919	397.5801	366.6179	770.1941	585.3250	1440.2692	881.3222	2602.4106	1219.8722	4542.5497	1465.7261	7639.8312
1.4988	.31	241.3456	226.8495	478.2656	363.8599	908.4194	542.0836	1676.3287	722.7366	2893.8691	776.4769	4889.0863	372.3667	7608.3707
1.5339	.32	282.8695	223.5771	549.3694	334.0464	1016.4809	436.6263	1788.2851	428.6879	2970.5617	90.5743	4986.9502	-1280.5066	6397.9463
1.5693	.33	319.0303	206.7389	603.7170	272.3520	1077.0713	260.2278	1755.8503	-14.9664	2740.1863	-972.7498	4634.5771	-3486.6291	3539.8916
1.6049	.34	346.8950	174.5379	633.7801	173.4547	1072.4174	8.9439	1589.7974	-609.7993	2114.9212	-2274.8988	3823.7367	-6149.0087	-1080.8012
1.6408	.35	363.3697	125.7099	632.1540	41.8959	985.9125	-318.1714	1286.3755	-1341.2397	1062.8631	-3791.9459	-527.8536	-9064.3166	-7541.4786
1.6771	.36	365.5376	59.8081	598.1092	-127.8873	809.8151	-710.7724	712.2329	-2175.5794	-470.9748	-5415.6181	-4799.3940	-11916.0363	-15600.8091
1.7136	.37	350.8026	-22.6969	508.1863	-328.2080	517.0498	-1153.3213	-90.5685	-3058.3296	-2854.9872	-6986.3186	-9042.5703	-14282.8876	-24673.3518
1.7506	.38	317.0398	-120.0928	376.8003	-523.8865	122.9233	-1621.8449	-1109.9783	-3915.6965	-4811.3394	-8401.9928	-13954.4668	-15665.9731	-33811.5703
1.7879	.39	262.8123	-229.7330	196.8119	-693.9011	-373.2588	-2081.9999	-2310.2113	-4655.3980	-7397.7980	-9130.3847	-18874.3772	-15536.1936	-41704.9314
1.8257	.40	187.5397	-347.7839	-29.9790	-1035.1475	-957.0813	-2496.2190	-3629.9315	-5174.0044	-10006.8959	-9288.9624	-23197.0283	-13400.8052	-46785.4964
1.8640	.41	91.6458	-468.9819	-298.4618	-1861.6998	-1604.3468	-2819.7720	-4982.0832	-5363.9171	-12972.7629	-9378.4380	-26209.9863	-8884.7877	-47368.9648
1.9028	.42	-83.3281	-587.5878	-600.0105	-1855.5966	-2281.1164	-3006.5807	-6256.2597	-5183.6818	-15185.9450	-9384.0695	-27195.8949	-1819.2819	-41874.0954
1.9421	.43	-154.6578	-696.5930	-922.4815	-1598.1861	-2944.5478	-3018.2844	-7324.5696	-4969.8499	-15116.8958	-9170.0486	-25317.6985	7673.2003	-29090.5622
1.9821	.44	-296.1243	-788.9303	-1230.4448	-1670.5975	-3544.5941	-2798.0704	-8049.8037	-5049.5621	-14847.0739	1247.9001	-20121.4288	19114.8565	-8470.7809
2.0226	.45	-449.5651	-857.1556	-1565.6642	-1695.8906	-4026.5774	-2335.6897	-8393.4167	-5132.7338	-13106.6997	6703.8115	-11247.3919	31627.1672	19538.1259
2.0638	.46	-601.9913	-894.1756	-1847.8383	-1539.8706	-4334.5755	-1611.2176	-7934.2481	1277.5217	-9714.9006	12881.8582	1262.7629	43933.1999	53546.4356
2.1057	.47	-748.7728	-898.9520	-2075.5468	-1312.8597	-4415.4879	-629.8681	-6876.5248	4137.6789	-4619.7897	19898.1504	16918.1651	54509.9765	90273.6473
2.1483	.48	-882.3892	-850.2013	-2227.4933	-971.0830	-4223.5702	584.0184	-5066.8674	7261.1969	2066.9631	25314.1697	34786.1878	61520.4850	126636.4143
2.1918	.49	-995.0412	-760.1260	-2283.8051	-517.9399	-3785.1969	1979.2106	-2507.9730	10421.7347	10045.7820	30219.0078	53190.6864	63186.8958	156717.7921
2.2361	.50	-1079.0124	-622.7356	-2227.4964	36.4582	-2903.2300	3483.4884	734.3962	13343.0286	18819.9998	33233.6588	70373.4736	57971.3756	175267.5291