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NOTE ON THE RELATIVE EFFECT OF THE DIHEDRAL AND THE SWEEP BACK OF AIRPLANE WINGS.

By Max M. Munk.

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Summary.

Dihedral and sweep back can properly be compared only to the extent to which their action is similar in a side slip. In general the rolling moments due to side slip are about three to six times greater for a given dihedral angle than for an equal angle of sweep back.

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1. Sweep back was used in some of the early airplanes in order to obtain lateral stability. It is now rarely used except in very moderate values, usually as a modification to correct a "tail-heavy" condition in an existing design. Dihedral, on the other hand, may be found in practically all airplanes. Since the balancing effects of these two angles are somewhat similar their ratio for the same angle is here investigated.

Only small dihedrals and sweep backs are considered in this note. Both have a small influence on the wing air forces, but it is not great and is not the subject of this note. The important effect is the unsymmetry of the air force at unsymmetrical flying maneuvers, caused by the dihedral and the sweep back respectively. True, even a wing without any sweep back or dihedral gives an unsymmetric air force, when moved unsymmetrically through the air. The subject of this note is the additional unsymmetry of the air force, i.e. in a rolling moment, a yawing moment and lateral force, caused by the sweep back or the dihedral in rolling, yawing or side slipping of the airplane.

Now, it cannot be seen how a wing without any dihedral can experience any considerable lateral force. On the other hand, the lateral force plays no very important part with respect to the lateral stability. Hence, this difference in the action of dihedral and sweep back, though of a fundamental nature, does not decide the question of equivalence or non-equivalence of the two angles.

The yawing moment is chiefly determined by the fuselage and the tail unit. The wings contribute to it no forces except such as are nearly parallel to the chord and these forces are always small. There remains then chiefly the rolling moment to be considered.

Now the rolling moment due to roll is large in itself but it is so even without dihedral or sweep back. It is only slightly modified by these two angles. The same holds true with the rolling moment due to yaw. There remains then at last only one combination, the rolling moment due to side slipping, which is chiefly determined by the magnitude of the dihedral or sweep back. The ratio of the effect of equal dihedral and sweep back on this rolling moment is therefore a sufficient approximation of the desired ratio of their balancing power.

2. I begin with the effect of the sweep back. The angle of sweep back may be σ, the velocity of flight V. Then in straight flight, only the component of the velocity $V \cos\sigma$ is effective for the creation of the lift. Since the lift coefficient may be written $2\pi \sin\alpha'$, the lift per unit area is

$$L/S = 2\pi V^2 \frac{\rho}{2} \cos^2\sigma \sin\alpha'$$

where α' is the effective angle of attack ($L = 0$ for $\alpha' = 0$). Now let v/V be the small angle of side slipping. Then the lift per unit area and per unit dynamic pressure $\frac{L}{S} V^2 \frac{\rho}{2}$ is increased or decreased by

$$2\pi \sin\alpha' \frac{d}{d\sigma} \cos^2\sigma \; d\sigma = -2\pi \sin\alpha' \sin 2\sigma \frac{v}{V} \tag{1}$$

since

$$\frac{v}{V} = d\sigma .$$

I turn now to the rolling moment, caused by the dihedral γ and side slipping. The effective angle of attack is increased on one side and decreased on the other side by $\frac{v(\sin\gamma)}{V}$ which is the component of the side velocity at right angles to the wing and divided by V. Hence the expression corresponding to (1) is now

$$2\pi \left(\sin\gamma \frac{v}{V}\right) \tag{2}$$

The ratio of (1) to (2) is approximately $2 \sin\alpha \frac{\sigma}{\gamma}$.

It appears from this rough investigation that the effect of the sweep back is always smaller than that of the dihedral. The ratio depends on the angle of attack, so that at very large angles of attack it may be as much as one-third, whereas at very low angles of attack it is less than one-tenth. The rolling moment at high angles of attack and with correspondingly low velocity of flight is more important than at small angles of attack, as in the latter case the controllability is very good in consequence of the high speed and the degree of the stability is then less important. It can therefore be said that for average conditions the effect of sweep back is 1/3 to 1/6 of that of the dihedral of equal magnitude.