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TECHNICAL NOTE

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THEORETICAL BASIC SPAN LOADING CHARACTERISTICS OF WINGS
WITH ARBITRARY SWEEP, ASPECT RATIO, AND TAPER RATIO

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SUMMARY

A procedure based on the Weissinger method has been devised so that the basic span loading and associated aerodynamic characteristics can be rapidly predicted for wings having arbitrary values of sweep, aspect ratio, taper ratio, and twist. A method for correcting for the effects of compressibility is given. A comparison of the results of this method with that of lifting-line and lifting-surface methods indicates that the accuracy is much better than that obtained with lifting-line methods and is comparable to that obtained with lifting-surface methods.

This report, together with NACA TN No. 1491, allows a simple and rapid prediction of both the basic and additional loading characteristics for wings of arbitrary plan form. The characteristics which can be found for a given wing are as follows:

1. Span load distribution due to twist (this report)
2. Span load distribution due to angle of attack (TN No. 1491)
3. Induced drag (this report and TN No. 1491)
4. Angle of zero lift (this report)
5. Lift-curve slope (TN No. 1491)
6. Pitching moment at zero lift (this report)
7. Location of aerodynamic center (TN No. 1491)

It is believed these predicted values are valid at all subcritical Mach numbers and for all lift coefficients where viscous and stall effects are negligible.

To establish the effects of sweep, aspect ratio, and taper ratio on the basic loading characteristics produced by uniform twist, the method presented in this report was applied to a few representative wing configurations and the results discussed.

INTRODUCTION

The need for information on the subsonic characteristics of swept wings to supplement the limited amount of existing experimental data has led to a theoretical study of their characteristics. As in most theoretical studies, the wing characteristics have been determined using the span loading which in this case has been predicted by the method of Weissinger. Since these characteristics are dependent upon the span loading, it has been found convenient to study the characteristics as those associated with additional-type loading (i.e., the loading due to wing angle of attack) and those associated with basic-type loading (i.e., the loading due to wing twist or effective twist).

In reference 1, the Weissinger method was applied to a series of wings encompassing the probable ranges of sweep, aspect ratio, and taper ratio to determine the wing characteristics associated with additional-type loading. The results (including span load distribution, spanwise center of pressure, lift-curve slope, and aerodynamic center) are presented in graphical form as a function of wing plan form.

The present report is an extension of reference 1 to facilitate determination of the wing characteristics associated with basic-type loading (span load distribution, angle of zero lift, and pitching moment) for a wide range of plan forms. Since the basic loading is a function of twist as well as the plan-form variables (sweep, aspect ratio, and taper ratio), it seemed impractical to present loading characteristics for all possible wing configurations. Therefore, it was intended that this report should present a simple procedure which would allow prediction of the basic loading for the wide range of plan forms investigated in reference 1, and should present the actual basic loading for a few representative configurations to establish the effects of the various geometric parameters. The results of this work then, together with reference 1, should enable a rapid evaluation of the wing characteristics associated with both the basic and additional types of loading for wings having sweep angles ranging from -45° to 75° , aspect ratios of 1.5 to 10, and taper ratios of 0 to 1.5.

SYMBOLS

Aerodynamic Parameters

| | |
|--------------------------------|---|
| $\frac{c_{l_a}^c}{C_{L_{av}}}$ | spanwise loading coefficient for unit wing lift coefficient (additional-type loading) |
| $\frac{c_{l_b}^c}{e_{cav}}$ | spanwise loading coefficient for unit twist (basic-type loading), per degree |
| $\frac{c_{l_a}^c}{c_{av}}$ | spanwise loading coefficient for additional-type loading |
| $\frac{c_{l_b}^c}{c_{av}}$ | spanwise loading coefficient for basic-type loading |
| $\frac{c_l^c}{c_{av}}, k$ | gross spanwise loading coefficient $\left(\frac{c_{l_a}^c}{c_{av}} + \frac{c_{l_b}^c}{c_{av}} \right)$ |
| c_{l_a} | section lift coefficient for additional-type loading $\left[\frac{(\text{local lift})_a}{qS} \right]$ |
| c_{l_b} | section lift coefficient for basic-type loading $\left[\frac{(\text{local lift})_b}{qS} \right]$ |
| c_l | gross section lift coefficient $(c_{l_a} + c_{l_b})$ |
| C_L | wing lift coefficient $\left(\frac{\text{total lift}}{qS} \right)$ |
| C_{L_α} | rate of change of lift coefficient with angle of attack measured at zero lift |
| G | dimensionless circulation $\left(\frac{\Gamma}{bV} \right)$ |
| Γ | circulation, feet squared per second |
| C_{D_i} | induced drag coefficient $\left(\frac{\text{induced drag}}{qS} \right)$ |

$C_{m_b}_0$ pitching-moment coefficient due to basic loading

$$\left(\frac{\text{pitching moment due to basic loading}}{qS\bar{c}} \right)$$

q free-stream dynamic pressure, pounds per square foot

M Mach number

V free-stream velocity, feet per second

Geometric Parameters

Λ angle of sweep of the quarter-chord line, positive for sweepback, degrees

A aspect ratio $\left(\frac{b^2}{S} \right)$

λ taper ratio $\left(\frac{\text{tip chord}}{\text{root chord}} \right)$

b wing span measured perpendicular to the plane of symmetry, feet

c wing chord measured parallel to the plane of symmetry, feet

c_{av} average wing chord $\left(\frac{S}{b} \right)$, feet

\bar{c} mean aerodynamic chord $\left(\frac{\int_0^{b/2} c^2 dy}{\int_0^{b/2} c dy} \right)$, feet

S wing area, square feet

$(\alpha_T)_0$ ¹angle of attack for zero lift of the three-quarter-chord point of the root section mean line, radians

α_v ¹angle of attack of the three-quarter-chord point of the spanwise station v section mean line, radians

$(\alpha_v)_0$ ¹the values of α_v for zero net lift on the wing, radians

¹All angles are measured in a plane parallel to the plane of symmetry.

- ϵ_v ¹twist of section mean line relative to the wing root measured at the three-quarter-chord point for the spanwise station v $[(\alpha_v)_0 - (\alpha_r)_0]$, radians unless noted otherwise
- ϵ ¹twist of the tip section mean line relative to the wing root measured at the three-quarter-chord point, degrees
- y lateral coordinate measured from wing root perpendicular to the plane of symmetry
- η dimensionless lateral coordinate $\left(\frac{y}{b/2}\right)$
- ϕ trigonometric spanwise coordinate $(\cos^{-1}\eta)$
- $a_{v,n}$ coefficient indicating the influence of circulation G at station n on the downwash angle at control point v where the location of n is defined by $\eta = \cos \frac{n\pi}{8}$ and the location of v is defined by $\eta = \cos \frac{v\pi}{8}$

Subscripts

- a parameters associated with additional loading
- b parameters associated with basic loading
- e equivalent geometric parameters
- o value of parameter at zero lift
- v,n integers defining specific span locations

PROCEDURE

Development of Method

In the Weissinger lifting-line method, which has been previously discussed in references 1 and 2, and is used herein, the gross

¹See footnote 1, p. 4.

circulation (representing additional and/or basic loading) is considered concentrated into a lifting line lying along the quarter-chord line. The boundary condition, fixing the spanwise strength distribution of the circulation, requires that the downwash of this lifting line and its system of trailing vortices produce at points along the three-quarter-chord line a downwash angle equal to the slope of the wing section mean line at these points. For the general case, the boundary condition is usually applied at seven spanwise stations distributed across the total span so that seven simultaneous equations can be formed involving unknown gross loadings at the seven spanwise stations. However, for the symmetric loading case, which is considered herein (fig. 1) and in reference 1, the loading and control points on only half of the wing need be considered so that determination of the span loading requires the solution of only four simultaneous equations of the form

$$\alpha_v = \sum_{n=1}^4 a_{v,n} G_n \quad (1)$$

where

α_v the section mean line angle of attack at the three-quarter-chord point for the spanwise stations v , where the locations of v are defined by $\eta = \frac{y}{b/2} = \cos \frac{v\pi}{8}$

$a_{v,n}$ influence coefficients involving the purely geometric wing parameters Λ , A , and λ

G_n the unknown dimensionless circulation $\frac{\Gamma}{bV}$ at the spanwise stations n where the locations of n are defined by $\eta = \frac{y}{b/2} = \cos \frac{n\pi}{8}$

In this form the set of equations may be used to obtain the gross loading on any wing for which the aspect ratio, sweep, taper ratio, twist, and angle of attack are specified.

Past experience has indicated that the gross loading can be better studied if broken down into the basic and the additional type of loading. The basic loading is that existing with zero net lift on the wing and is due to twist or effective twist (e.g., partial-span flap deflection or spanwise change in camber) of the wing chord

plane. In contrast, the additional loading is that producing net lift on the wing and is, in effect, the loading existing on an untwisted and uncambered wing. From this, it follows that the basic loading is a function of the variation in section angle of attack across the wing and is independent of the wing angle of attack; whereas the additional loading is a function of wing angle of attack and is independent of the variation in section angle of attack across the wing.

In determining the additional loading characteristics presented in reference 1 then, it was not necessary to consider variation of the angle of attack across the span (i.e., it was assumed $\alpha_{v=1} = \alpha_{v=2} = \alpha_{v=3} = \alpha_{v=4}$) and it was possible to use equation (1) in the following form:

$$l = \sum_{n=1}^4 a_{v,n} \frac{G_n}{\alpha}$$

However, in determining the basic loading, the angle of attack varies across the span and equation (1) must be used as given. Although the variation of α across the span is known from wing geometry for any wing angle of attack, its value for the unique condition of zero net lift on the wing is required and this, of course, is unknown. Thus, there results a set of four simultaneous equations which involve eight unknowns - the values of $(\alpha_v)_0$ and G_n . Since the twist distribution is known, three of these unknowns can be eliminated by the following expression:

$$(\alpha_v)_0 = (\alpha_r)_0 + \epsilon_v$$

where $(\alpha_r)_0$ is the angle of attack of the root mean line at the three-quarter-chord point for $C_L = 0$, and ϵ_v is the wing twist relative to the root. This, then, reduces the number of unknowns in the set of equations to five and, hence, a fifth equation is required to permit a solution.

The fifth equation is obtained from the expression for the total lift on the wing. This expression is given as equation (C51) of reference 2 and where the seven-point (symmetric loading) solution is used reduces to

$$C_L = \frac{\pi A}{8} \left(G_4 + 2 \sum_{n=1}^3 G_n \sin \varphi_n \right) \quad (2)$$

Since $\varphi_n = \frac{n\pi}{8}$ and since for basic loading $C_L = 0$, the only

unknowns in equation (2) are the loading factor G_n which also appear in equation (1). Thus, there are now five equations that contain the unknowns $(\alpha_r)_0$, G_1 , G_2 , G_3 , and G_4 . In the expanded form the equations are:

$$\begin{aligned} \epsilon_1 &= -(\alpha_r)_0 + a_{1,1}G_1 + a_{1,2}G_2 + a_{1,3}G_3 + a_{1,4}G_4 \\ \epsilon_2 &= -(\alpha_r)_0 + a_{2,1}G_1 + a_{2,2}G_2 + a_{2,3}G_3 + a_{2,4}G_4 \\ \epsilon_3 &= -(\alpha_r)_0 + a_{3,1}G_1 + a_{3,2}G_2 + a_{3,3}G_3 + a_{3,4}G_4 \\ 0 &= -(\alpha_r)_0 + a_{4,1}G_1 + a_{4,2}G_2 + a_{4,3}G_3 + a_{4,4}G_4 \\ C_L=0 &= 0.765G_1 + 1.414G_2 + 1.848G_3 + G_4 \end{aligned}$$

Solution of this set of equations will give the angle of attack of the mean line at the three-quarter-chord point of the root chord, and the loads G_1 , G_2 , G_3 , and G_4 at the span stations $\eta = 0.924$, 0.707 , 0.383 , and 0 .

Application of Method

Basic loading and angle of zero lift.— Figure 1 has been prepared to show the physical significance of the various loading and geometric parameters. Equation (3), which is included in figure 1, can be used to determine the loading on a wing having a

²The reader should note that strictly speaking ϵ and α represent the slope and not the angle of the mean line. That is, the equations should be written $\tan[\epsilon_1 + (\alpha_r)_0] = a_{1,1}G_1 + a_{1,2}G_2 + \dots$. Early in the derivation of equation (1), it was assumed that all slopes were small and, therefore, it was permissible to substitute the angle in radians for the slope. To avoid serious error in those cases where the twist is large, the equations should be written using the slope rather than the angle in radians. If it is desired to keep the error under 1 percent, the true slope should be used for angles of over 10° , and under 5 percent, true slopes should be used for angles over 20° .

given twist distribution or, conversely, to determine the twist distribution to provide a given basic loading. In either case the solution of the five simultaneous equations is a relatively simple matter. The most time-consuming and laborious portion of the process is that of computing the values of the $a_{v,n}$ coefficients from the geometry of the wing. This process is fully outlined in reference 2. Since these coefficients are a function of wing geometry alone, however, it is only necessary to compute them once for each plan form to study any variation of twist or camber on the plan form. These computations have been made for the range of plan forms shown in figure 2 and are presented in table I. (It should be noted that figure 2 shows only the range and not the total number of plan forms; approximately 200 wings were considered altogether.)

Thus, with the aid of table I, the problem of determining the basic loading and the angle of zero lift for any plan form given in table I is reduced to the following simple steps:

1. Insertion (in equation (3)) of the given values of twist for the four spanwise stations
2. Insertion of the values of $a_{v,n}$ obtained from table I
3. Simultaneous solution of the five equations

The resulting loading coefficients may be put in the more convenient coefficient form $\frac{c_{lb}^c}{c_{av}}$ by the following conversion:

$$\left(\frac{c_{lb}^c}{c_{av}} \right)_n = 2AG_n \quad (4)$$

To aid in fairing the loading curve, values of $\frac{c_{lb}^c}{c_{av}}$ at intermediate span stations may be obtained through use of the interpolation function (equation (A6)) given in reference 1. Determination of the twist for a given loading is, of course, a simple inversion of this process.

As will be evident later, it is not necessary to use the exact plan form to obtain a good approximation of the basic loading on a given wing. Consequently, if the basic loading is desired on a plan form between those given in table I, it is generally acceptable to use the coefficients $a_{v,n}$ given in table I for a plan form which

most closely approximates the desired plan form. If a more refined estimate of loading is required, experience has indicated that it is better to determine the loading for the bracketing plan forms and interpolate the loading rather than to determine the loading for the interpolated values of the coefficients $a_{v,n}$

Consideration must be given to the number of significant figures retained in the solution of the five simultaneous equations. Actually the number of significant figures required depends to a large extent on whether or not the equations are ill-conditioned; however, it is noteworthy that in solving for the basic loadings presented in this report, none of the sets of equations appeared to be ill-conditioned. Beyond possible effects of ill-conditioning, however, to be strictly correct, the number of significant figures to be retained should be examined at each step of the computations to maintain a given accuracy. When computations are made either longhand or with a slide rule, this procedure can be followed, but when computing machines are used it becomes more practical, even though not rigorously correct, to maintain a given number of decimal places throughout the computations. In an effort to establish the number of decimal places required, a number of computations were made using five places and then four, three, etc., for each of several plan forms. The results obtained were compared and it was concluded that satisfactory accuracy could be had if the value of ϵ_v in radians were given to four decimal places, if the coefficients $a_{v,n}$ were tabulated and used to two decimal places, and, if in the solution of simultaneous equations, five decimal places were retained.

Local lift, induced drag, and pitching moments.— With the basic

loading coefficient $\frac{c_{l_b}^c}{c_{av}}$ as determined above and the additional

loading coefficient $\frac{c_{l_a}^c}{c_{av}}$ as determined by method given in reference 1,

other wing characteristics, such as section gross local lift coefficient c_l , induced drag coefficient C_{D1} , and wing pitching-moment coefficients C_m are easily obtained. The gross value of c_l at any angle of attack is determined as follows:

$$c_l = c_{l_a} + c_{l_b} \quad (5)$$

where c_{l_a} is the lift coefficient due to additional-type loading from reference 1 and c_{l_b} is the lift coefficient due to the basic-type loading as determined by the relation

$$\begin{aligned}
 c_{l_b} &= \left(\frac{c_{l_b}^c}{c_{av}} \right) \left(\frac{c_{av}}{c} \right) \\
 &= \left(\frac{c_{l_b}^c}{c_{av}} \right) \left\{ \frac{(1+\lambda)}{2 [1-\eta(1-\lambda)]} \right\} \quad (6)
 \end{aligned}$$

In contrast to lift coefficient and loading, the induced drag cannot be determined for additional and basic loading separately and then be summed to get the total induced drag, rather the induced drag must be determined from the total loading distribution. Equation (3) of reference 1 has been modified to give the induced drag for the general case. The induced drag coefficient is then given by

$$\begin{aligned}
 C_{D_i} &= \frac{\pi}{8A} \left[k_1^2 + k_2^2 + k_3^2 + \frac{k_4^2}{2} - k_4 (0.0561 k_1 \right. \\
 &\quad \left. + 0.7887 k_3) - k_2 (0.7352 k_1 + 0.8445 k_3) \right] \quad (7)
 \end{aligned}$$

where

$$k_n = \left(\frac{c_{l_a}^c}{c_{av}} + \frac{c_{l_b}^c}{c_{av}} \right)_n$$

The ability of the Weissinger method to enable good predictions of the effect of load on wing pitching moment has been shown (reference 1) to be the result of:

1. The accurate prediction of spanwise distribution of load
2. The predominant effect that spanwise distribution of load has on pitching moments of swept wings as compared to the effect of chordwise distribution of load

Thus, even though in the Weissinger method the basic load distribution is concentrated along the quarter-chord line, the method should allow good predictions of the effect of basic loading on the pitching moment of swept wings.

The expression for C_m due to basic loading has been derived from equation (A4) of reference 1. Thus,

$$C_{m_b} = \frac{-b/2}{c} \tan \Lambda (0.1384 k_{1_b} + 0.1975 k_{2_b} + 0.1351 k_{3_b} + 0.0159 k_{4_b}) \quad (8)$$

It should be noted that, since the pitching moment due to basic loading is the result of a loading couple, the value of the pitching moment due to basic loading is independent of the location of the moment reference center. To obtain the gross pitching-moment coefficient for a wing, the pitching moment due to additional-type loading (reference 1) may be added directly to the pitching moment due to twist given by equation (8).

Effects of compressibility.— A means of correcting the wing characteristics associated with additional-type loading for compressibility was given in reference 1. This essentially consists of translating the effect of compressibility into an effective change in plan form in addition to the well-known increase in section pressures. These principles should apply equally well to basic loading characteristics. However, it should be noted that in the

case of additional loading the loading coefficient $\frac{c_l c}{C_L c_{av}}$ was not a function of angle of attack and consequently the effects of Mach number on loading were shown only as changes in load distribution

and not as changes in the average value of $\frac{c_l c}{C_L c_{av}}$. The change in

average value of loading was in effect absorbed in changes in $C_L c_{av}$. In contrast, the basic loading coefficient $\frac{c_{l_b} c}{c_{av}}$ is definitely a

function of the local angle of attack (twist) and is, therefore, a function of Mach number just as is lift-curve slope. Therefore, to obtain the value of $\frac{c_{l_b} c}{c_{av}}$ in compressible flow, it is necessary to:

1. Determine the value of $\frac{c_{l_b} c}{c_{av}}$ for the given twist and the equivalent plan form given by $\lambda_e = \lambda$, $A_e = A \sqrt{1-M^2}$

$$\text{and } \tan \Lambda_e = \frac{\tan \Lambda}{\sqrt{1-M^2}}$$

2. Multiply value of $\frac{c_{l_b} c}{c_{av}}$ obtained by $\frac{1}{\sqrt{1-M^2}}$

RESULTS AND DISCUSSION

Evaluation of Method

There is a scarcity of experimental basic loading data and consequently any evaluation of the accuracy of the Weissinger method in predicting basic loading must be indirect. Both references 1 and 2 proved the Weissinger method to be very accurate in predicting the additional-type loading, and similar accuracy should, therefore, be expected with regard to the basic loading.

To allow further evaluation of the method, a comparison is given in figure 3 between the basic loadings obtained by the Weissinger method, the method of reference 3 and the Falkner method for an unswept wing having an aspect ratio of 6.0 and a taper ratio of 0.5. The data from reference 3 were used for comparison since they are well known and have been widely used. The Falkner method was used because it is a lifting-surface method and should give better accuracy than either the method of reference 3 (a lifting-line method) or the Weissinger method (a modified lifting-line method). As presented in figure 3, the loading obtained by reference 3 is in serious disagreement; whereas the Weissinger loading shows relatively good agreement with that obtained by the Falkner method. These results are explainable on the basis of the following facts:

1. It can be readily shown that, even on high-aspect-ratio wings, the introduction of twist results in large induction effects.
2. Where induction effects are large, as for example the effects of induction on the lift-curve slope of low-aspect-ratio wings, it has been often demonstrated that unmodified lifting-line theory will not yield accurate results.
3. In reference 1 it was shown that the Weissinger method, which is a modified lifting-line method, overcomes the weakness of the unmodified theory and yields results on low-aspect-ratio wings comparable in accuracy to that obtained with lifting-surface theory.

In view of the foregoing comparisons, it is believed that (1) the basic loading characteristics of unswept wings can be predicted with much better accuracy by using the Weissinger method than by using the results of reference 3, and (2) that the Weissinger method is capable of predicting the basic loading characteristics on any wing with sufficient accuracy for preliminary design analysis.

**Effect of Plan-Form Variation on the
 Basic Loading Characteristics**

To study the effects of plan-form variation on the basic loading characteristics of uniformly twisted wings, the characteristics of a representative group of wings (see shaded wings, fig. 2) having unit³ washout have been computed and are presented in figures 4 to 9. The basic loading characteristics considered are the loading $\frac{c_{l_b}^c}{c_{av}}$ (figs. 4 to 7), the pitching moment due to twist C_{m_b} (fig. 8), and the angle of attack at the root for zero lift $(\alpha_r)_0$ (fig. 9).

Magnitude and spanwise distribution of load.— Examination of figures 4 to 7 reveals that the aspect ratio influences only the magnitude and is in fact the predominate influence on the magnitude. Reductions in aspect ratio from 6.0 to 3.5 and 1.5 result in approximately 35-percent and 70-percent reductions, respectively, in load due to twist for either the unswept or 45° swept-back wings (fig. 5).

Sweep, either forward or back, tends to reduce the magnitude of loading, although appreciable reductions are produced only by sweep angles greater than 45° (fig. 4). Sweep also affects the load distribution such that the load on the outer section of the wing is shifted inboard by sweepforward and toward the tip by sweepback; this is similar to the effect of sweep on the additional-type loading. Since increase in aspect ratio magnifies the loading, it also magnifies the effects of sweep on the loading as is shown in figure 4.

As shown in figures 6 and 7, taper ratio has little effect on the magnitude of basic loading; and variations in taper ratio, for taper ratios larger than 0.5, have little effect on the load distribution. However, for taper ratios less than 0.5, the loading on the outer section of the wing shifts inboard. These effects of taper ratio on loading are magnified by increases in aspect ratio.

³In this case, 1° was chosen, and for any larger amount of twist the effects are proportional within the limits of footnote 2, page 8.

Pitching moment.— That the pitching moment due to twist is primarily a function of sweep and aspect ratio is shown in figure 8. The magnitude of the pitching moment increases as either aspect ratio or sweep is increased so that pitching-moment coefficients as large as 0.008 for 1° of twist exist on wings having large aspect ratios and sweep angles. The effect of taper ratio is relatively small, the greatest being evidenced at the small values of taper ratio. For example, reducing the taper ratio from 0.5 to 0 reduces the pitching moment due to twist about 30 percent.

Angle of zero lift.— Although the effects of plan form on the angle of zero lift $(\alpha_r)_0$ may not be very important, some of the trends indicated in figure 9 are of interest. For the range of plan forms represented in figure 9, the angle of zero lift did not vary more than 27 percent. This is small compared to the effects of plan form on the magnitude and distribution of loading and on the pitching moment. In contrast to the small effect of taper noted previously, taper ratio appears to be the predominant influence on $(\alpha_r)_0$, particularly at large aspect ratios and large sweepback. The effect of aspect ratio and sweep are secondary but not negligible.

Consideration of Twist in Swept-Wing Design

The methods of this report enable a detailed study of two serious problems associated with the use of swept wings: First, the use of twist to control section lift distribution and hence stall; and, second, the degree to which bending, since it introduces twist, affects the aerodynamic characteristics. These two problems and the application of the methods of this report to this study are discussed in the sections immediately following.

Twist for separation control.— The induction effects on swept wings are such that large angles of attack are induced near the root of swept-forward wings and near the tip of swept-back wings. Consequently, for untwisted wings the sections at the root and tip, respectively, reach their $c_{l_{max}}$ before the rest of the wing and at a relatively low angle of attack of the wing. The resulting local separation produces the poor characteristics (large drag rise and large fore-and-aft movement of the aerodynamic center which occur at relatively low lift coefficients) which are typical of highly swept wings. To remedy these poor characteristics, some means must be provided which will cause the flow over the wing to separate more uniformly. This should be achieved if the wing is twisted and/or cambered so that all sections reach their $c_{l_{max}}$ at nearly the same

angle of attack of the wing. One possible approach would be to twist the wing to provide nearly uniform spanwise distribution of c_l and, hence, uniform separation, and to camber all sections to increase $C_{L_{max}}$.

An indication of the amount of twist required to produce uniform distribution of c_l for a given C_L can be had by (1) using reference 1, determine the c_l distribution for the given C_L (additional-type loading), (2) using this c_l distribution, determine the basic loading required to give uniform c_l distribution, and (3) using the method of this report, compute the twist distribution required to produce this basic loading.

For purposes of illustration, the foregoing procedure has been applied to a wing having 45° of sweepback, an aspect ratio of 6.0, and a taper ratio of 0.5. (See fig. 10.) It can be seen that the additional-type loading produces relatively large values of c_l over the outboard sections. The basic loading required to give uniform c_l distribution of 1.0 is shown shaded, and the basic loading ordinates to be used in equation (3) in order to determine the required twist are labeled $(c_{lb})^4$, $(c_{lb})^3$, etc. The twist, determined from solution of equation (3) to satisfy the condition of uniform load, is shown on the lower half of the figure. Such a variation would be difficult to build, and hence in the practical application some compromise twist variation would probably be chosen. The effect of compromising to the extent of using a linear variation in twist is also shown in figure 10. Although a uniform c_l distribution is not provided, the distribution is such that the characteristics of the wing at higher lift coefficients should be significantly improved. Undoubtedly the amount of twist and camber would also be compromised to some extent to provide good characteristics at high Mach numbers; however, it is likely that relatively large amounts of camber and twist could be tolerated on highly swept wings before the characteristics at high Mach numbers were jeopardized.

⁴It is recognized that the optimum spanwise distribution of c_l may not be a uniform distribution and that in the practical application better stalling characteristics will probably be exhibited if the c_l at the midsemispan is somewhat greater than at either the root or tip sections. It is apparent that further experimental data are needed to establish the optimum c_l distribution for swept wings.

Twist due to wing deflection.— Deflection of wings, whether unswept or swept, can be considered as composed of bending and torsional components. Torsional deflection of either unswept or swept wings obviously produces twist and, hence, affects the basic loading. On unswept wings, bending produces only an increment in the dihedral angle; whereas on swept wings, bending produces changes in section angle of attack (effective twist) as well as dihedral. Whether or not the twists due to bending and torsional deflections are additive or canceling depends upon the direction of sweep and the characteristics of the wing structure. For a given wing, the magnitude of the twist due to wing deflection is a function of the gross loading as well as the structural stiffness and, therefore, varies with acceleration in gusts or maneuvering flight. Thus, the aerodynamic characteristics of the wing in maneuvering flight are likely to differ greatly from those of the wing in steady flight.

For swept wings, the pitching moment resulting from this twist directly affects the trim and/or stability of the airplane and should, therefore, be given serious consideration. A qualitative analysis has shown that for a flexible wing in steady flight, the pitching-moment increments due to flexure result primarily in changes in trim but may also cause decrements in stability which increase in magnitude as lift coefficient is decreased. In accelerated flight, the flexible wing will probably experience serious decrements in stability at all lift coefficients.

These changes in trim and stability can be evaluated through use of equations (3) and (8) once the structural stiffness of the wing in both bending and torsion (hence, the twist components) has been determined. As can be seen in figure 8, the effects of twist on trim and stability are very dependent on the sweep and aspect ratio of the wing, and to some extent, dependent on the taper ratio. Reducing either aspect ratio or sweep will, therefore, minimize the influence of twist and wing bending on the trim and stability of the airplane.

CONCLUDING REMARKS

A method for predicting effects of twist on the span loading and associated characteristics for a wide range of plan forms has been presented. Comparison of the loadings obtained by this method and those obtained by lifting-surface (Falkner) and lifting-line (NACA TR No. 572) methods indicates that use of this method results in accuracy much better than that obtained with lifting-line methods and accuracy comparable to that obtained with lifting-surface methods.

It is believed, therefore, that (1) this method is capable of predicting the basic loading characteristics on any wing with sufficient accuracy for preliminary design analysis, and (2) the method should be particularly useful in determining the twist required for separation control and in evaluating effects of aeroelastic deformation.

To establish the effects of sweep, aspect ratio, and taper ratio on the basic loading characteristics produced by uniform twist, the method has been applied to a few representative wing plan forms. Some of the trends noted were:

1. The magnitude of basic loading due to uniform twist is primarily a function of aspect ratio; however, the influence of sweep also becomes important for sweep angles beyond 45° . Taper ratio has little effect.
2. The pitching moment due to uniform twist is a function of both sweep and aspect ratio and is likewise little affected by taper ratio.
3. The angle of zero lift of a uniformly twisted wing is a function of taper ratio as well as sweep and aspect ratio.

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Moffett Field, Calif.

REFERENCES

1. DeYoung, John: Theoretical Additional Span Loading Characteristics of Wings with Arbitrary Sweep, Aspect Ratio, and Taper Ratio. NACA TN No. 1491, 1947.
2. Van Dorn, Nicholas H., and DeYoung, John: A Comparison of Three Theoretical Methods of Calculating Span Load Distribution on Swept Wings. NACA TN No. 1476, 1947.
3. Anderson, Raymond F.: Determination of the Characteristics of Tapered Wings. NACA Rep. No. 572, 1936.

$a_{1,1}$

(a) $a_{1,n}$

$a_{1,2}$

| A | λ | | A | | | | | | |
|-----|------|------|-------|-------|-------|-------|-------|-------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 11.92 | 13.45 | 15.07 | 16.65 | --- | --- | --- |
| | .25 | .25 | 10.60 | 10.92 | 11.33 | 11.78 | --- | --- | --- |
| | .50 | .50 | 10.31 | 10.65 | 10.89 | 11.16 | 11.61 | --- | --- |
| | 1.00 | 1.00 | 10.48 | 10.54 | 10.66 | --- | --- | --- | --- |
| | 1.50 | 1.50 | 10.48 | 10.51 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | 11.62 | 12.91 | 14.28 | 15.70 | 17.22 | --- | --- |
| | .25 | .25 | 10.55 | 10.81 | 11.13 | 11.48 | 12.06 | --- | --- |
| | .50 | .50 | 10.31 | 10.61 | 10.79 | 10.98 | 11.34 | --- | --- |
| | 1.00 | 1.00 | 10.48 | 10.54 | 10.62 | 10.74 | 10.94 | --- | --- |
| | 1.50 | 1.50 | 10.48 | 10.51 | 10.58 | 10.66 | 10.82 | --- | --- |
| 0 | 0 | 0 | 11.43 | 12.64 | 14.00 | 15.45 | 17.65 | 20.72 | 23.80 |
| | .25 | .25 | 10.57 | 10.75 | 11.00 | 11.32 | 11.84 | 12.61 | 13.44 |
| | .50 | .50 | 10.31 | 10.60 | 10.73 | 10.90 | 11.19 | 11.69 | 12.16 |
| | 1.00 | 1.00 | 10.48 | 10.53 | 10.60 | 10.69 | 10.86 | 11.14 | 11.45 |
| | 1.50 | 1.50 | 10.48 | 10.51 | 10.56 | 10.63 | 10.76 | 10.97 | 11.22 |
| 30 | 0 | 0 | 11.56 | 13.16 | 15.07 | 17.03 | 20.16 | 24.38 | 28.63 |
| | .25 | .25 | 10.56 | 10.75 | 11.04 | 11.41 | 12.07 | 13.09 | 14.27 |
| | .50 | .50 | 10.31 | 10.60 | 10.73 | 10.91 | 11.26 | 11.82 | 12.49 |
| | 1.00 | 1.00 | 10.48 | 10.53 | 10.60 | 10.69 | 10.87 | 11.19 | 11.77 |
| | 1.50 | 1.50 | 10.48 | 10.51 | 10.56 | 10.62 | 10.76 | 10.99 | 11.28 |
| 45 | 0 | 0 | 11.86 | 14.13 | 16.76 | 19.48 | 23.59 | 29.14 | 34.71 |
| | .25 | .25 | 10.57 | 10.78 | 11.15 | 11.64 | 12.59 | 14.08 | 15.69 |
| | .50 | .50 | 10.31 | 10.59 | 10.77 | 10.97 | 11.43 | 12.24 | 13.21 |
| | 1.00 | 1.00 | 10.48 | 10.53 | 10.60 | 10.71 | 10.92 | 11.34 | 11.88 |
| | 1.50 | 1.50 | 10.48 | 10.51 | 10.56 | 10.63 | 10.79 | 11.08 | 11.47 |
| 60 | 0 | 0 | 12.98 | 16.92 | 20.99 | 25.09 | 31.27 | --- | --- |
| | .25 | .25 | 10.58 | 10.91 | 11.61 | 12.60 | 14.35 | --- | --- |
| | .50 | .50 | 10.31 | 10.63 | 10.87 | 11.27 | 12.19 | --- | --- |
| | 1.00 | 1.00 | 10.48 | 10.54 | 10.63 | 10.74 | 11.17 | --- | --- |
| | 1.50 | 1.50 | 10.48 | 10.51 | 10.58 | 10.67 | 10.92 | --- | --- |
| 75 | 0 | 0 | 18.59 | 26.78 | 34.99 | 43.20 | --- | --- | --- |
| | .25 | .25 | 10.77 | 12.72 | 15.19 | 17.72 | --- | --- | --- |
| | .50 | .50 | 10.54 | 11.03 | 12.34 | 14.15 | --- | --- | --- |
| | 1.00 | 1.00 | 10.49 | 10.59 | 11.04 | 12.04 | --- | --- | --- |
| | 1.50 | 1.50 | 10.48 | --- | --- | --- | --- | --- | --- |

| A | λ | | A | | | | | | |
|-----|------|------|-------|--------|--------|--------|--------|--------|--------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -1.32 | 0.72 | 2.73 | 4.79 | --- | --- | --- |
| | .25 | .25 | -3.38 | -2.81 | -2.16 | -1.54 | --- | --- | --- |
| | .50 | .50 | -3.62 | -3.27 | -2.87 | -2.44 | -1.79 | --- | --- |
| | 1.00 | 1.00 | -3.72 | -3.22 | -3.26 | --- | --- | --- | --- |
| | 1.50 | 1.50 | -3.73 | -3.60 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -2.20 | -1.86 | .48 | 1.79 | 3.83 | --- | --- |
| | .25 | .25 | -3.53 | -3.15 | -2.77 | -2.35 | -1.72 | --- | --- |
| | .50 | .50 | -3.68 | -3.46 | -3.21 | -2.93 | -2.51 | --- | --- |
| | 1.00 | 1.00 | -3.75 | -3.62 | -3.46 | -3.27 | -2.98 | --- | --- |
| | 1.50 | 1.50 | -3.77 | -3.67 | -3.54 | -3.40 | -3.16 | --- | --- |
| 0 | 0 | 0 | -3.06 | -2.57 | -2.12 | -1.71 | -1.12 | -.37 | 0.36 |
| | .25 | .25 | -3.66 | -3.48 | -3.29 | -3.12 | -2.88 | -2.58 | -2.30 |
| | .50 | .50 | -3.73 | -3.62 | -3.50 | -3.37 | -3.19 | -2.96 | -2.75 |
| | 1.00 | 1.00 | -3.77 | -3.68 | -3.62 | -3.53 | -3.40 | -3.22 | -3.05 |
| | 1.50 | 1.50 | -3.78 | -3.73 | -3.66 | -3.59 | -3.48 | -3.32 | -3.17 |
| 30 | 0 | 0 | -3.60 | -3.83 | -4.17 | -4.55 | -5.22 | -6.11 | -7.02 |
| | .25 | .25 | -3.72 | -3.69 | -3.61 | -3.62 | -3.66 | -3.81 | -4.02 |
| | .50 | .50 | -3.77 | -3.70 | -3.66 | -3.62 | -3.60 | -3.64 | -3.72 |
| | 1.00 | 1.00 | -3.79 | -3.74 | -3.71 | -3.66 | -3.62 | -3.61 | -3.62 |
| | 1.50 | 1.50 | -3.79 | -3.77 | -3.73 | -3.69 | -3.63 | -3.62 | -3.61 |
| 45 | 0 | 0 | -3.97 | -4.70 | -5.65 | -6.62 | -8.10 | -10.13 | -12.14 |
| | .25 | .25 | -3.73 | -3.73 | -3.79 | -3.91 | -4.21 | -4.72 | -5.28 |
| | .50 | .50 | -3.78 | -3.73 | -3.74 | -3.76 | -3.86 | -4.09 | -4.41 |
| | 1.00 | 1.00 | -3.79 | -3.77 | -3.74 | -3.76 | -3.76 | -3.83 | -3.98 |
| | 1.50 | 1.50 | -3.80 | -3.77 | -3.76 | -3.74 | -3.74 | -3.78 | -3.87 |
| 60 | 0 | 0 | -4.78 | -6.60 | -8.46 | -10.37 | -13.23 | --- | --- |
| | .25 | .25 | -3.79 | -3.88 | -4.16 | -4.61 | -5.40 | --- | --- |
| | .50 | .50 | -3.79 | -3.80 | -3.87 | -4.01 | -4.41 | --- | --- |
| | 1.00 | 1.00 | -3.80 | -3.79 | -3.80 | -3.84 | -3.98 | --- | --- |
| | 1.50 | 1.50 | -3.80 | -3.80 | -3.79 | -3.81 | -3.88 | --- | --- |
| 75 | 0 | 0 | -8.00 | -12.28 | -16.56 | -20.65 | --- | --- | --- |
| | .25 | .25 | -3.93 | -4.94 | -6.23 | -7.55 | --- | --- | --- |
| | .50 | .50 | -3.83 | -4.02 | -4.81 | -5.69 | --- | --- | --- |
| | 1.00 | 1.00 | -3.82 | -3.85 | -4.07 | -4.58 | --- | --- | --- |
| | 1.50 | 1.50 | -3.82 | --- | --- | --- | --- | --- | --- |

$a_{1,3}$

$a_{1,4}$

| A | λ | | A | | | | | | |
|-----|------|------|-------|-------|-------|-------|-------|------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -0.47 | -1.59 | -2.80 | -3.86 | --- | --- | --- |
| | .25 | .25 | -.36 | -.24 | -.02 | -.35 | --- | --- | --- |
| | .50 | .50 | -.28 | -.36 | -.27 | -.10 | -0.22 | --- | --- |
| | 1.00 | 1.00 | -.18 | -.33 | -.37 | --- | --- | --- | --- |
| | 1.50 | 1.50 | -.15 | -.29 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -.15 | -.80 | -1.45 | -2.18 | -3.22 | --- | --- |
| | .25 | .25 | -.24 | -.23 | -.09 | -.07 | -.37 | --- | --- |
| | .50 | .50 | -.18 | -.26 | -.23 | -.16 | 0 | --- | --- |
| | 1.00 | 1.00 | -.12 | -.21 | -.26 | -.25 | -.18 | --- | --- |
| | 1.50 | 1.50 | -.09 | -.18 | -.23 | -.23 | -.23 | --- | --- |
| 0 | 0 | 0 | .05 | -.10 | -.24 | -.37 | -.54 | -.77 | -0.98 |
| | .25 | .25 | -.12 | -.13 | -.11 | -.07 | -.01 | -.08 | -.18 |
| | .50 | .50 | .09 | -.13 | -.13 | -.12 | .09 | .03 | -.04 |
| | 1.00 | 1.00 | .06 | .09 | .13 | .14 | .13 | .09 | .05 |
| | 1.50 | 1.50 | .05 | .11 | .12 | .13 | .13 | .12 | .08 |
| 30 | 0 | 0 | .19 | .47 | .84 | 1.24 | 1.91 | 2.80 | 3.69 |
| | .25 | .25 | .07 | .08 | .11 | .15 | .25 | .45 | .69 |
| | .50 | .50 | .05 | .08 | .08 | .10 | .13 | .21 | .34 |
| | 1.00 | 1.00 | .04 | .06 | .07 | .08 | .10 | .13 | .19 |
| | 1.50 | 1.50 | .03 | .06 | .07 | .08 | .09 | .11 | .14 |
| 45 | 0 | 0 | .37 | 1.01 | 1.61 | 2.62 | 3.87 | 5.55 | 7.23 |
| | .25 | .25 | .05 | .09 | .18 | .30 | .56 | 1.01 | 1.48 |
| | .50 | .50 | .04 | .05 | .07 | .13 | .23 | .46 | .73 |
| | 1.00 | 1.00 | .03 | .03 | .05 | .07 | .12 | .21 | .35 |
| | 1.50 | 1.50 | .02 | .04 | .05 | .06 | .08 | .16 | .24 |
| 60 | 0 | 0 | .86 | 2.30 | 3.79 | 5.29 | 7.55 | --- | --- |
| | .25 | .25 | .04 | .13 | .37 | .73 | 1.36 | --- | --- |
| | .50 | .50 | .02 | .05 | .11 | .25 | .58 | --- | --- |
| | 1.00 | 1.00 | .01 | .03 | .05 | .10 | .22 | --- | --- |
| | 1.50 | 1.50 | .02 | .03 | .04 | .07 | .13 | --- | --- |
| 75 | 0 | 0 | 3.25 | 6.55 | 9.85 | 13.15 | --- | --- | --- |
| | .25 | .25 | .11 | .09 | 1.09 | 2.90 | --- | --- | --- |
| | .50 | .50 | .03 | .03 | .79 | 1.47 | --- | --- | --- |
| | 1.00 | 1.00 | .01 | -.05 | -.22 | -.62 | --- | --- | --- |
| | 1.50 | 1.50 | .01 | --- | --- | --- | --- | --- | --- |

| A | λ | | A | | | | | | |
|-----|------|------|------|------|--------|-------|-------|-------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 0.20 | 0.70 | 1.20 | 1.69 | --- | --- | --- |
| | .25 | .25 | -.21 | -.13 | .01 | .15 | --- | --- | --- |
| | .50 | .50 | -.21 | -.20 | -.14 | -.05 | 0.10 | --- | --- |
| | 1.00 | 1.00 | -.21 | -.21 | -.20 | --- | --- | --- | --- |
| | 1.50 | 1.50 | -.22 | -.21 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | .01 | .30 | .61 | .92 | 1.36 | --- | --- |
| | .25 | .25 | -.21 | -.17 | -.11 | -.02 | .12 | --- | --- |
| | .50 | .50 | -.22 | -.21 | -.18 | -.12 | -.05 | --- | --- |
| | 1.00 | 1.00 | -.23 | -.22 | -.21 | -.19 | -.14 | --- | --- |
| | 1.50 | 1.50 | -.24 | -.22 | -.21 | -.20 | -.17 | --- | --- |
| 0 | 0 | 0 | -.17 | -.10 | -.04 | .01 | .07 | .15 | 0.23 |
| | .25 | .25 | -.24 | -.22 | -.20 | -.17 | -.14 | -.10 | -.07 |
| | .50 | .50 | -.26 | -.24 | -.22 | -.21 | -.18 | -.16 | -.12 |
| | 1.00 | 1.00 | -.26 | -.25 | -.25 | -.23 | -.21 | -.19 | -.16 |
| | 1.50 | 1.50 | -.27 | -.26 | -.25 | -.23 | -.22 | -.20 | -.18 |
| 30 | 0 | 0 | -.31 | -.43 | -.62 | -.81 | -1.13 | -1.56 | -1.97 |
| | .25 | .25 | -.26 | -.26 | -.27 | -.29 | -.34 | -.43 | -.54 |
| | .50 | .50 | -.27 | -.27 | -.26 | -.27 | -.28 | -.32 | -.37 |
| | 1.00 | 1.00 | -.27 | -.27 | -.27 | -.27 | -.27 | -.28 | -.30 |
| | 1.50 | 1.50 | -.28 | -.27 | -.26 | -.27 | -.27 | -.28 | -.28 |
| 45 | 0 | 0 | -.43 | -.73 | -1.08 | -1.46 | -2.05 | -2.84 | -3.63 |
| | .25 | .25 | -.28 | -.29 | -.32</ | | | | |

TABLE I.- CONTINUED

(b) $a_{2,n}$

$a_{2,1}$

$a_{2,2}$

| A | λ | | A | | | | | | |
|-----|------|------|-------|-------|-------|-------|-------|-------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -2.03 | -2.03 | -2.11 | -2.13 | --- | --- | --- |
| | .25 | .25 | -1.93 | -2.06 | -2.08 | -2.10 | --- | --- | --- |
| | .50 | .50 | -1.83 | -2.06 | -2.07 | -2.09 | -2.11 | --- | --- |
| | 1.00 | 1.00 | -1.73 | -2.05 | -2.06 | --- | --- | --- | --- |
| -30 | 0 | 0 | -2.03 | -2.02 | -2.00 | -1.98 | -1.96 | --- | --- |
| | .25 | .25 | -1.93 | -2.03 | -2.01 | -1.99 | -1.98 | --- | --- |
| | .50 | .50 | -1.83 | -2.03 | -2.01 | -1.99 | -1.99 | --- | --- |
| | 1.00 | 1.00 | -1.73 | -2.03 | -2.02 | -2.01 | -1.99 | --- | --- |
| 0 | 0 | 0 | -2.00 | -1.98 | -1.83 | -1.74 | -1.60 | -1.40 | -1.22 |
| | .25 | .25 | -1.90 | -1.97 | -1.83 | -1.74 | -1.60 | -1.40 | -1.22 |
| | .50 | .50 | -1.80 | -1.98 | -1.83 | -1.74 | -1.60 | -1.40 | -1.22 |
| | 1.00 | 1.00 | -1.70 | -1.97 | -1.83 | -1.74 | -1.60 | -1.40 | -1.22 |
| 30 | 0 | 0 | -1.94 | -1.76 | -1.56 | -1.36 | -1.06 | -.69 | -.24 |
| | .25 | .25 | -1.84 | -1.87 | -1.72 | -1.56 | -1.33 | -1.00 | -.69 |
| | .50 | .50 | -1.74 | -1.97 | -1.79 | -1.66 | -1.45 | -1.17 | -.89 |
| | 1.00 | 1.00 | -1.64 | -1.97 | -1.83 | -1.74 | -1.56 | -1.32 | -1.08 |
| 45 | 0 | 0 | -1.87 | -1.59 | -1.26 | -.98 | -.74 | -.55 | -.64 |
| | .25 | .25 | -1.77 | -1.73 | -1.51 | -1.28 | -.93 | -.71 | -.61 |
| | .50 | .50 | -1.67 | -1.82 | -1.62 | -1.41 | -1.11 | -.77 | -.50 |
| | 1.00 | 1.00 | -1.57 | -1.82 | -1.72 | -1.54 | -1.28 | -.86 | -.58 |
| 60 | 0 | 0 | -1.66 | -1.17 | -.70 | -.23 | .47 | --- | --- |
| | .25 | .25 | -1.56 | -1.44 | -1.06 | -.70 | -.35 | --- | --- |
| | .50 | .50 | -1.46 | -1.56 | -1.22 | -.83 | -.43 | --- | --- |
| | 1.00 | 1.00 | -1.36 | -1.59 | -1.38 | -1.10 | -.69 | --- | --- |
| 75 | 0 | 0 | -1.50 | -.96 | -.43 | 1.00 | --- | --- | --- |
| | .25 | .25 | -1.40 | -1.23 | -.87 | -.29 | 1.04 | --- | --- |
| | .50 | .50 | -1.30 | -1.21 | -.95 | -.61 | --- | --- | --- |
| | 1.00 | 1.00 | -1.20 | -.94 | -.97 | -.80 | --- | --- | --- |

| A | λ | | A | | | | | | |
|-----|------|------|------|------|-------|-------|------|------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 5.86 | 6.22 | 6.61 | 7.01 | --- | --- | --- |
| | .25 | .25 | 5.77 | 6.01 | 6.30 | 6.60 | --- | --- | --- |
| | .50 | .50 | 5.75 | 5.82 | 6.16 | 6.43 | 6.84 | --- | --- |
| | 1.00 | 1.00 | 5.72 | 5.82 | 6.04 | --- | --- | --- | --- |
| -30 | 0 | 0 | 5.83 | 6.10 | 6.41 | 6.75 | 7.28 | --- | --- |
| | .25 | .25 | 5.76 | 5.93 | 6.15 | 6.41 | 6.81 | --- | --- |
| | .50 | .50 | 5.74 | 5.87 | 6.05 | 6.26 | 6.60 | --- | --- |
| | 1.00 | 1.00 | 5.72 | 5.82 | 5.96 | 6.13 | 6.41 | --- | --- |
| 0 | 0 | 0 | 5.81 | 6.04 | 6.32 | 6.63 | 7.15 | 7.85 | 8.58 |
| | .25 | .25 | 5.76 | 5.91 | 6.10 | 6.32 | 6.69 | 7.23 | 7.77 |
| | .50 | .50 | 5.73 | 5.85 | 6.01 | 6.19 | 6.50 | 6.95 | 7.42 |
| | 1.00 | 1.00 | 5.72 | 5.81 | 5.93 | 6.06 | 6.33 | 6.69 | 7.10 |
| 30 | 0 | 0 | 5.82 | 6.09 | 6.46 | 6.90 | 7.57 | 8.52 | 9.50 |
| | .25 | .25 | 5.76 | 5.93 | 6.17 | 6.46 | 6.97 | 7.68 | 8.43 |
| | .50 | .50 | 5.73 | 5.82 | 6.05 | 6.29 | 6.70 | 7.32 | 7.95 |
| | 1.00 | 1.00 | 5.72 | 5.81 | 5.96 | 6.13 | 6.47 | 6.97 | 7.51 |
| 45 | 0 | 0 | 5.84 | 6.24 | 6.74 | 7.33 | 8.26 | 9.52 | 10.79 |
| | .25 | .25 | 5.76 | 5.97 | 6.33 | 6.74 | 7.44 | 8.41 | 9.39 |
| | .50 | .50 | 5.74 | 5.89 | 6.15 | 6.50 | 7.08 | 7.93 | 8.76 |
| | 1.00 | 1.00 | 5.72 | 5.83 | 6.02 | 6.28 | 6.76 | 7.45 | 8.17 |
| 60 | 0 | 0 | 5.92 | 6.69 | 7.59 | 8.52 | 9.92 | --- | --- |
| | .25 | .25 | 5.80 | 6.23 | 6.90 | 7.60 | 8.68 | --- | --- |
| | .50 | .50 | 5.74 | 6.05 | 6.60 | 7.23 | 8.13 | --- | --- |
| | 1.00 | 1.00 | 5.72 | 5.90 | 6.32 | 6.81 | 7.61 | --- | --- |
| 75 | 0 | 0 | 6.99 | 8.24 | 10.71 | 12.57 | --- | --- | --- |
| | .25 | .25 | 6.30 | 7.22 | 9.25 | 10.72 | --- | --- | --- |
| | .50 | .50 | 6.13 | 7.35 | 8.62 | 9.99 | --- | --- | --- |
| | 1.00 | 1.00 | 5.89 | 6.93 | 8.01 | 9.10 | --- | --- | --- |

$a_{2,3}$

$a_{2,4}$

| A | λ | | A | | | | | | |
|-----|------|------|-------|-------|-------|-------|-------|-------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -1.94 | -1.42 | -.87 | -.33 | --- | --- | --- |
| | .25 | .25 | -1.84 | -1.72 | -1.32 | -.83 | --- | --- | --- |
| | .50 | .50 | -1.66 | -1.85 | -1.50 | -1.12 | -0.56 | --- | --- |
| | 1.00 | 1.00 | -1.56 | -1.66 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -2.09 | -1.77 | -1.43 | -1.04 | -.50 | --- | --- |
| | .25 | .25 | -1.97 | -1.93 | -1.63 | -1.17 | -1.00 | --- | --- |
| | .50 | .50 | -1.83 | -2.04 | -1.80 | -1.58 | -1.21 | --- | --- |
| | 1.00 | 1.00 | -1.73 | -2.11 | -1.92 | -1.72 | -1.41 | --- | --- |
| 0 | 0 | 0 | -2.22 | -2.09 | -1.93 | -1.82 | -1.64 | -1.40 | -1.17 |
| | .25 | .25 | -2.17 | -2.15 | -2.06 | -1.99 | -1.80 | -1.61 | -1.42 |
| | .50 | .50 | -2.09 | -2.19 | -2.10 | -2.01 | -1.87 | -1.71 | -1.54 |
| | 1.00 | 1.00 | -2.01 | -2.23 | -2.14 | -2.06 | -1.93 | -1.80 | -1.66 |
| 30 | 0 | 0 | -2.31 | -2.29 | -2.33 | -2.39 | -2.51 | -2.67 | -2.87 |
| | .25 | .25 | -2.31 | -2.29 | -2.33 | -2.39 | -2.51 | -2.67 | -2.87 |
| | .50 | .50 | -2.32 | -2.28 | -2.28 | -2.28 | -2.35 | -2.45 | -2.57 |
| | 1.00 | 1.00 | -2.34 | -2.30 | -2.28 | -2.29 | -2.32 | -2.39 | -2.49 |
| 45 | 0 | 0 | -2.34 | -2.44 | -2.60 | -2.80 | -3.14 | -3.60 | -4.07 |
| | .25 | .25 | -2.33 | -2.36 | -2.47 | -2.61 | -2.84 | -3.20 | -3.56 |
| | .50 | .50 | -2.34 | -2.34 | -2.41 | -2.52 | -2.71 | -2.99 | -3.32 |
| | 1.00 | 1.00 | -2.35 | -2.34 | -2.36 | -2.44 | -2.59 | -2.84 | -3.11 |
| 60 | 0 | 0 | -2.41 | -2.76 | -3.20 | -3.65 | -4.34 | --- | --- |
| | .25 | .25 | -2.37 | -2.75 | -3.20 | -3.65 | -4.34 | --- | --- |
| | .50 | .50 | -2.36 | -2.65 | -2.71 | -3.11 | -3.73 | --- | --- |
| | 1.00 | 1.00 | -2.36 | -2.60 | -2.61 | -2.82 | -3.20 | --- | --- |
| 75 | 0 | 0 | -3.06 | -4.10 | -5.14 | -6.18 | --- | --- | --- |
| | .25 | .25 | -2.73 | -3.52 | -4.30 | -5.15 | --- | --- | --- |
| | .50 | .50 | -2.58 | -3.28 | -3.97 | -4.68 | --- | --- | --- |
| | 1.00 | 1.00 | -2.45 | -3.02 | -3.63 | -4.24 | --- | --- | --- |

| A | λ | | A | | | | | | |
|-----|------|------|------|------|-------|-------|-------|-------|-------|
| | λ | A | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 0.17 | 0.07 | -0.09 | -0.27 | --- | --- | --- |
| | .25 | .25 | .16 | .13 | .04 | -.09 | --- | --- | --- |
| | .50 | .50 | .15 | .16 | .08 | -.02 | -0.19 | --- | --- |
| | 1.00 | 1.00 | .13 | .17 | .13 | --- | --- | --- | --- |
| -30 | 0 | 0 | .12 | .09 | .01 | -.09 | -.24 | --- | --- |
| | .25 | .25 | .10 | .12 | .07 | .01 | -.10 | --- | --- |
| | .50 | .50 | .09 | .12 | .10 | .05 | -.04 | --- | --- |
| | 1.00 | 1.00 | .08 | .12 | .11 | .08 | -.01 | --- | --- |
| 0 | 0 | 0 | .06 | .08 | .06 | .04 | .01 | -0.04 | -0.09 |
| | .25 | .25 | .05 | .07 | .07 | .05 | .04 | .01 | -.04 |
| | .50 | .50 | .05 | .07 | .07 | .07 | .05 | .02 | -.01 |
| | 1.00 | 1.00 | .04 | .06 | .07 | .07 | .06 | .04 | .02 |
| 30 | 0 | 0 | .03 | .05 | .10 | .16 | .23 | .34 | .45 |
| | .25 | .25 | .03 | .05 | .07 | .10 | .16 | .23 | .33 |
| | .50 | .50 | .03 | .04 | .06 | .07 | .13 | .20 | .28 |
| | 1.00 | 1.00 | .02 | .04 | .04 | .07 | .10 | .16 | .22 |
| 45 | 0 | 0 | .02 | .09 | .18 | .29 | .46 | .67 | .90 |
| | .25 | .25 | .02 | .05 | .12 | .19 | .31 | .48 | .65 |
| | .50 | .50 | .02 | .04 | .08 | .14 | .24 | .36 | .54 |
| | 1.00 | 1.00 | .02 | .03 | .06 | .10 | .17 | .31 | .44 |
| 60 | 0 | 0 | .04 | .21 | .41 | .62 | .92 | --- | --- |
| | .25 | .25 | .02 | .10 | .25 | .41 | .69 | --- | --- |
| | .50 | .50 | .01 | .07 | .18 | .37 | .72 | --- | --- |
| | 1.00 | 1.00 | .01 | .04 | .13 | .23 | .41 | --- | --- |
| 75 | 0 | 0 | .30 | .76 | 1.22 | 1.68 | --- | --- | --- |
| | .25 | .25 | .16 | .51 | .94 | 1.22 | --- | --- | --- |
| | .50 | .50 | .09 | .40 | .71 | 1.02 | --- | --- | --- |
| | 1.00 | 1.00 | .03 | .29 | .56 | .82 | --- | --- | --- |



(c) $a_{3,n}$

$a_{3,1}$

| A | λ | | λ | | | | | | |
|-----|------|-----|------|------|------|------|------|------|------|
| | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 | | |
| -45 | 0 | 0 | 0.01 | 0.02 | 0.03 | 0.04 | --- | --- | --- |
| | .25 | 0 | .02 | .03 | .04 | --- | --- | --- | --- |
| | .50 | .01 | .02 | .03 | .04 | 0.05 | --- | --- | --- |
| | 1.00 | .01 | .03 | .04 | --- | --- | --- | --- | --- |
| | 1.50 | .02 | .03 | --- | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | .02 | .02 | .03 | .04 | --- | --- | --- |
| | .25 | .01 | .03 | .03 | .04 | .04 | --- | --- | --- |
| | .50 | .02 | .02 | .03 | .04 | .04 | --- | --- | --- |
| | 1.00 | .02 | .03 | .03 | .04 | .04 | --- | --- | --- |
| | 1.50 | .02 | .03 | .04 | .04 | --- | --- | --- | --- |
| 0 | 0 | 0 | .02 | .04 | .04 | .05 | .05 | 0.04 | 0.02 |
| | .25 | .02 | .04 | .05 | .05 | .05 | .03 | .02 | --- |
| | .50 | .02 | .04 | .05 | .05 | .05 | .03 | .01 | --- |
| | 1.00 | .03 | .04 | .05 | .05 | .04 | .03 | 0 | --- |
| | 1.50 | .03 | .04 | .05 | .05 | .04 | .02 | 0 | --- |
| 30 | 0 | 0 | .04 | .07 | .09 | .09 | .07 | .04 | 0 |
| | .25 | .04 | .07 | .09 | .09 | .07 | .03 | -.02 | --- |
| | .50 | .04 | .06 | .09 | .08 | .06 | .01 | -.04 | --- |
| | 1.00 | .05 | .08 | .09 | .08 | .05 | 0 | -.06 | --- |
| | 1.50 | .05 | .08 | .08 | .07 | .05 | .01 | -.07 | --- |
| 45 | 0 | 0 | .06 | .12 | .11 | .11 | .07 | .01 | -.06 |
| | .25 | .10 | .12 | .11 | .11 | .06 | -.02 | -.09 | --- |
| | .50 | .07 | .12 | .12 | .09 | .03 | -.03 | -.12 | --- |
| | 1.00 | .08 | .12 | .11 | .08 | .03 | -.06 | -.15 | --- |
| | 1.50 | .08 | .12 | .11 | .07 | .01 | -.08 | -.17 | --- |
| 60 | 0 | 0 | .12 | .16 | .13 | .08 | 0 | --- | --- |
| | .25 | .17 | .16 | .12 | .06 | -.03 | --- | --- | --- |
| | .50 | .14 | .16 | .10 | .04 | -.06 | --- | --- | --- |
| | 1.00 | .15 | .15 | .09 | .02 | -.09 | --- | --- | --- |
| | 1.50 | .16 | .13 | .08 | 0 | -.11 | --- | --- | --- |
| 75 | 0 | 0 | .20 | .08 | -.04 | -.17 | --- | --- | --- |
| | .25 | .19 | .06 | -.09 | -.22 | --- | --- | --- | --- |
| | .50 | .17 | .04 | -.11 | -.23 | --- | --- | --- | --- |
| | 1.00 | .16 | .01 | -.15 | -.30 | --- | --- | --- | --- |
| | 1.50 | .15 | --- | --- | --- | --- | --- | --- | --- |

$a_{3,2}$

| A | λ | | λ | | | | | | |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 | | |
| -45 | 0 | 0 | -1.81 | -1.80 | -1.80 | -1.80 | --- | --- | --- |
| | .25 | -1.81 | -1.80 | -1.80 | -1.80 | --- | --- | --- | --- |
| | .50 | -1.80 | -1.80 | -1.80 | -1.80 | -1.81 | --- | --- | --- |
| | 1.00 | -1.80 | -1.80 | -1.80 | --- | --- | --- | --- | --- |
| | 1.50 | -1.80 | -1.80 | --- | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -1.80 | -1.78 | -1.75 | -1.74 | -1.72 | --- | --- |
| | .25 | -1.80 | -1.77 | -1.75 | -1.73 | -1.70 | --- | --- | --- |
| | .50 | -1.79 | -1.77 | -1.75 | -1.73 | -1.69 | --- | --- | --- |
| | 1.00 | -1.79 | -1.76 | -1.74 | -1.72 | -1.69 | --- | --- | --- |
| | 1.50 | -1.79 | -1.76 | -1.73 | -1.71 | -1.68 | --- | --- | --- |
| 0 | 0 | 0 | -1.78 | -1.74 | -1.69 | -1.63 | -1.55 | -1.44 | -1.34 |
| | .25 | -1.78 | -1.73 | -1.67 | -1.61 | -1.52 | -1.41 | -1.29 | --- |
| | .50 | -1.77 | -1.72 | -1.66 | -1.59 | -1.50 | -1.38 | -1.26 | --- |
| | 1.00 | -1.77 | -1.70 | -1.64 | -1.57 | -1.48 | -1.34 | -1.21 | --- |
| | 1.50 | -1.76 | -1.70 | -1.63 | -1.56 | -1.45 | -1.32 | -1.18 | --- |
| 30 | 0 | 0 | -1.76 | -1.67 | -1.57 | -1.45 | -1.29 | -1.06 | -.85 |
| | .25 | -1.76 | -1.65 | -1.53 | -1.42 | -1.24 | -1.01 | -.77 | --- |
| | .50 | -1.74 | -1.64 | -1.51 | -1.38 | -1.20 | -.95 | -.71 | --- |
| | 1.00 | -1.73 | -1.61 | -1.47 | -1.34 | -1.14 | -.88 | -.61 | --- |
| | 1.50 | -1.72 | -1.59 | -1.45 | -1.30 | -1.10 | -.83 | -.55 | --- |
| 45 | 0 | 0 | -1.74 | -1.58 | -1.42 | -1.26 | -1.03 | -.72 | -.42 |
| | .25 | -1.72 | -1.55 | -1.38 | -1.20 | -.95 | -.62 | -.29 | --- |
| | .50 | -1.71 | -1.53 | -1.34 | -1.16 | -.89 | -.55 | -.20 | --- |
| | 1.00 | -1.69 | -1.49 | -1.30 | -1.10 | -.81 | -.44 | -.06 | --- |
| | 1.50 | -1.68 | -1.47 | -1.26 | -1.05 | -.75 | -.36 | -.03 | --- |
| 60 | 0 | 0 | -1.69 | -1.37 | -1.12 | -.87 | -.51 | -.19 | --- |
| | .25 | -1.61 | -1.32 | -1.05 | -.79 | -.39 | -.02 | --- | --- |
| | .50 | -1.59 | -1.28 | -.99 | -.72 | -.30 | --- | --- | --- |
| | 1.00 | -1.58 | -1.22 | -.92 | -.62 | -.17 | --- | --- | --- |
| | 1.50 | -1.53 | -1.17 | -.85 | -.55 | -.09 | --- | --- | --- |
| 75 | 0 | 0 | -1.23 | -.72 | -.22 | .27 | --- | --- | --- |
| | .25 | -1.16 | -.62 | -.09 | .45 | --- | --- | --- | --- |
| | .50 | -1.09 | -.54 | .03 | .59 | --- | --- | --- | --- |
| | 1.00 | -1.04 | -.43 | .18 | .79 | --- | --- | --- | --- |
| | 1.50 | -1.00 | --- | --- | --- | --- | --- | --- | --- |

$a_{3,3}$

| A | λ | | λ | | | | | | |
|-----|------|------|------|------|------|------|------|------|------|
| | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 | | |
| -45 | 0 | 0 | 4.41 | 4.54 | 4.69 | 4.84 | --- | --- | --- |
| | .25 | 4.42 | 4.56 | 4.73 | 4.90 | --- | --- | --- | --- |
| | .50 | 4.43 | 4.59 | 4.76 | 4.95 | 5.24 | --- | --- | --- |
| | 1.00 | 4.43 | 4.62 | 4.81 | --- | --- | --- | --- | --- |
| | 1.50 | 4.46 | 4.64 | --- | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | 4.40 | 4.50 | 4.63 | 4.76 | 4.98 | --- | --- |
| | .25 | 4.41 | 4.53 | 4.66 | 4.81 | 5.04 | --- | --- | --- |
| | .50 | 4.42 | 4.55 | 4.70 | 4.85 | 5.10 | --- | --- | --- |
| | 1.00 | 4.44 | 4.57 | 4.73 | 4.90 | 5.17 | --- | --- | --- |
| | 1.50 | 4.44 | 4.59 | 4.76 | 4.94 | 5.22 | --- | --- | --- |
| 0 | 0 | 0 | 4.40 | 4.49 | 4.60 | 4.73 | 4.93 | 5.24 | 5.54 |
| | .25 | 4.41 | 4.51 | 4.64 | 4.78 | 5.01 | 5.34 | 5.67 | --- |
| | .50 | 4.42 | 4.53 | 4.67 | 4.81 | 5.06 | 5.41 | 5.77 | --- |
| | 1.00 | 4.43 | 4.55 | 4.70 | 4.87 | 5.15 | 5.52 | 5.91 | --- |
| | 1.50 | 4.43 | 4.57 | 4.73 | 4.91 | 5.20 | 5.60 | 6.01 | --- |
| 30 | 0 | 0 | 4.39 | 4.50 | 4.64 | 4.81 | 5.11 | 5.54 | 5.98 |
| | .25 | 4.41 | 4.52 | 4.68 | 4.86 | 5.21 | 5.68 | 6.16 | --- |
| | .50 | 4.41 | 4.54 | 4.72 | 4.93 | 5.29 | 5.79 | 6.31 | --- |
| | 1.00 | 4.43 | 4.57 | 4.77 | 5.01 | 5.40 | 5.95 | 6.51 | --- |
| | 1.50 | 4.43 | 4.59 | 4.81 | 5.07 | 5.47 | 6.05 | 6.65 | --- |
| 45 | 0 | 0 | 4.39 | 4.51 | 4.73 | 4.97 | 5.37 | 5.98 | 6.58 |
| | .25 | 4.42 | 4.56 | 4.80 | 5.01 | 5.53 | 6.18 | 6.83 | --- |
| | .50 | 4.42 | 4.59 | 4.84 | 5.14 | 5.67 | 6.33 | 7.03 | --- |
| | 1.00 | 4.43 | 4.62 | 4.92 | 5.26 | 5.80 | 6.55 | 7.30 | --- |
| | 1.50 | 4.44 | 4.66 | 4.97 | 5.34 | 5.91 | 6.70 | 7.49 | --- |
| 60 | 0 | 0 | 4.40 | 4.64 | 5.04 | 5.48 | 6.16 | --- | --- |
| | .25 | 4.40 | 4.71 | 5.17 | 5.65 | 6.38 | --- | --- | --- |
| | .50 | 4.40 | 4.78 | 5.26 | 5.78 | 6.55 | --- | --- | --- |
| | 1.00 | 4.45 | 4.85 | 5.35 | 5.96 | 6.80 | --- | --- | --- |
| | 1.50 | 4.47 | 4.92 | 5.46 | 6.08 | 6.97 | --- | --- | --- |
| 75 | 0 | 0 | 4.70 | 5.61 | 6.53 | 7.45 | --- | --- | --- |
| | .25 | 4.81 | 5.80 | 6.80 | 7.79 | --- | --- | --- | --- |
| | .50 | 4.93 | 5.96 | 7.00 | 8.05 | --- | --- | --- | --- |
| | 1.00 | 5.02 | 6.01 | 7.28 | 8.42 | --- | --- | --- | --- |
| | 1.50 | 5.10 | --- | --- | --- | --- | --- | --- | --- |

$a_{3,4}$

| A | λ | | λ | | | | | | |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 | | |
| -45 | 0 | 0 | -1.62 | -1.47 | -1.29 | -1.08 | --- | --- | --- |
| | .25 | -1.60 | -1.43 | -1.22 | -1.00 | --- | --- | --- | --- |
| | .50 | -1.60 | -1.40 | -1.18 | -.95 | -.68 | --- | --- | --- |
| | 1.00 | -1.58 | -1.36 | -1.11 | -.88 | --- | --- | --- | --- |
| | 1.50 | -1.57 | -1.34 | --- | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -1.65 | -1.55 | -1.43 | -1.30 | -1.13 | --- | --- |
| | .25 | -1.63 | -1.53 | -1.40 | -1.25 | -1.01 | --- | --- | --- |
| | .50 | -1.63 | -1.51 | -1.37 | -1.21 | -.97 | --- | --- | --- |
| | 1.00 | -1.61 | -1.48 | -1.32 | -1.17 | -.92 | --- | --- | --- |
| | 1.50 | -1.61 | -1.46 | -1.29 | -1.13 | -.87 | --- | --- | --- |
| 0 | 0 | 0 | -1.66 | -1.61 | -1.55 | -1.50 | -1.42 | -1.32 | -1.21 |
| | .25 | -1.65 | -1.60 | -1.54 | -1.48 | -1.39 | -1.28 | -1.17 | --- |
| | .50 | -1.65 | -1.59 | -1.53 | -1.46 | -1.37 | -1.25 | -1.14 | --- |
| | 1.00 | -1.65 | -1.58 | -1.51 | -1.44 | -1.35 | -1.22 | -1.09 | --- |
| | 1.50 | -1.64 | -1.57 | -1.50 | -1.43 | -1.33 | -1.19 | -1.07 | --- |
| 30 | 0 | 0 | -1.67 | -1.65 | -1.64 | -1.63 | -1.64 | -1.67 | -1.70 |
| | .25 | -1.67 | -1.64 | -1.63 | -1.63 | -1.64 | -1.68 | -1.71 | --- |
| | .50 | -1.67 | -1.64 | -1.63 | -1.63 | -1.63 | -1.69 | -1.73 | --- |
| | 1.00 | -1.66 | -1.64 | -1.64 | -1.64 | -1.66 | -1.70 | -1.74 | --- |
| | 1.50 | -1.66 | -1.64 | -1.63 | -1.64 | -1.67 | -1.71 | -1.75 | --- |
| 45 | 0 | 0 | -1.68 | -1.68 | -1.70 | -1.73 | -1.81 | -1.93 | -2.05 |
| | .25 | -1.69 | -1.68 | -1.70 | -1.75 | -1.84 | -1.97 | -2.10 | --- |
| | .50 | -1.68 | -1.68 | -1.71 | -1.77 | -1.87 | -2.00 | -2.14 | --- |
| | 1.00 | -1.67 | -1.68 | -1.73 | -1.79 | -1.89 | -2.04 | -2.19 | --- |
| | 1.50 | -1.67 | -1.68 | -1.74 | -1.80 | -1.91 | -2.08 | -2.23 | --- |
| 60 | 0 | 0 | -1.68 | -1.73 | -1.84 | -1.96 | -2.15 | --- | --- |
| | .25 | -1.72 | -1.75 | -1.87 | -2.00 | -2.21 | --- | --- | --- |
| | .50 | -1.68 | -1.77 | -1.90 | -2.07 | -2.26 | --- | --- | --- |

(d) $a_{4,n}$

$a_{4,1}$

$a_{4,2}$

| Δ | λ | | λ | | | | | | |
|----------|-----------|----------|-----------|-------|-------|-------|-------|-------|-------|
| | λ | Δ | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -0.22 | -0.22 | -0.21 | -0.21 | --- | --- | --- |
| | .25 | .25 | .22 | .22 | .21 | .21 | --- | --- | --- |
| | .50 | .50 | .22 | .22 | .21 | .21 | -0.20 | --- | --- |
| | 1.00 | 1.00 | .22 | .22 | .21 | .21 | --- | --- | --- |
| | 1.50 | 1.50 | .21 | .21 | .21 | .21 | --- | --- | --- |
| -30 | 0 | 0 | .22 | .22 | .21 | .21 | -.20 | --- | --- |
| | .25 | .25 | .22 | .22 | .21 | .21 | -.20 | --- | --- |
| | .50 | .50 | .22 | .22 | .21 | .21 | -.19 | --- | --- |
| | 1.00 | 1.00 | .21 | .21 | .21 | .21 | -.19 | --- | --- |
| | 1.50 | 1.50 | .21 | .21 | .21 | .21 | -.19 | --- | --- |
| 0 | 0 | 0 | .21 | .20 | .19 | .19 | -.18 | -0.17 | -0.17 |
| | .25 | .25 | .21 | .20 | .19 | .19 | -.18 | -.17 | -.17 |
| | .50 | .50 | .21 | .20 | .19 | .18 | -.17 | -.15 | -.14 |
| | 1.00 | 1.00 | .20 | .19 | .18 | .17 | -.15 | -.14 | -.12 |
| | 1.50 | 1.50 | .20 | .18 | .17 | .16 | -.14 | -.12 | -.09 |
| 30 | 0 | 0 | .20 | .19 | .17 | .17 | -.16 | -.14 | -.13 |
| | .25 | .25 | .19 | .17 | .17 | .16 | -.14 | -.12 | -.11 |
| | .50 | .50 | .19 | .17 | .15 | .15 | -.13 | -.11 | -.09 |
| | 1.00 | 1.00 | .18 | .16 | .15 | .13 | -.11 | -.07 | -.04 |
| | 1.50 | 1.50 | .17 | .16 | .14 | .11 | -.09 | -.05 | 0 |
| 45 | 0 | 0 | .19 | .17 | .15 | .15 | -.14 | -.12 | -.09 |
| | .25 | .25 | .18 | .15 | .15 | .14 | -.12 | -.09 | -.07 |
| | .50 | .50 | .17 | .15 | .14 | .13 | -.10 | -.05 | -.04 |
| | 1.00 | 1.00 | .16 | .15 | .12 | .10 | -.07 | -.01 | .03 |
| | 1.50 | 1.50 | .16 | .14 | .11 | .08 | -.03 | -.01 | .09 |
| 60 | 0 | 0 | .15 | .14 | .14 | .13 | -.10 | --- | --- |
| | .25 | .25 | .14 | .15 | .13 | .11 | -.08 | --- | --- |
| | .50 | .50 | .15 | .14 | .12 | .09 | -.05 | --- | --- |
| | 1.00 | 1.00 | .15 | .12 | .08 | .05 | .01 | --- | --- |
| | 1.50 | 1.50 | .15 | .10 | 0 | .06 | --- | --- | --- |
| 75 | 0 | 0 | .15 | .12 | .09 | .05 | --- | --- | --- |
| | .25 | .25 | .14 | .10 | .05 | 0 | --- | --- | --- |
| | .50 | .50 | .14 | .08 | .02 | .04 | --- | --- | --- |
| | 1.00 | 1.00 | .11 | .03 | .05 | .13 | --- | --- | --- |
| | 1.50 | 1.50 | .08 | --- | --- | --- | --- | --- | --- |

| Δ | λ | | λ | | | | | | |
|----------|-----------|----------|-----------|------|------|------|------|------|------|
| | λ | Δ | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 0.01 | 0.01 | 0.02 | 0.02 | --- | --- | --- |
| | .25 | .25 | .01 | .02 | .02 | .02 | --- | --- | --- |
| | .50 | .50 | .01 | .02 | .02 | .02 | 0.01 | --- | --- |
| | 1.00 | 1.00 | .02 | .04 | .02 | --- | --- | --- | --- |
| | 1.50 | 1.50 | .02 | .02 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | .01 | .02 | .03 | .03 | .04 | --- | --- |
| | .25 | .25 | .02 | .03 | .03 | .04 | .03 | --- | --- |
| | .50 | .50 | .02 | .03 | .04 | .04 | .03 | --- | --- |
| | 1.00 | 1.00 | .03 | .04 | .04 | .03 | .02 | --- | --- |
| | 1.50 | 1.50 | .04 | .04 | .03 | .02 | --- | --- | --- |
| 0 | 0 | 0 | .02 | .04 | .07 | .08 | .09 | 0.10 | 0.09 |
| | .25 | .25 | .03 | .06 | .08 | .09 | .10 | .09 | .07 |
| | .50 | .50 | .05 | .07 | .08 | .10 | .10 | .07 | .05 |
| | 1.00 | 1.00 | .06 | .08 | .09 | .09 | .07 | .03 | .01 |
| | 1.50 | 1.50 | .07 | .09 | .09 | .08 | .05 | -.01 | -.08 |
| 30 | 0 | 0 | .04 | .09 | .14 | .17 | .20 | .19 | .17 |
| | .25 | .25 | .06 | .12 | .17 | .20 | .20 | .17 | .12 |
| | .50 | .50 | .08 | .14 | .19 | .20 | .18 | .13 | .07 |
| | 1.00 | 1.00 | .12 | .18 | .20 | .18 | .13 | .05 | -.05 |
| | 1.50 | 1.50 | .15 | .20 | .18 | .13 | .07 | -.05 | -.18 |
| 45 | 0 | 0 | .06 | .16 | .23 | .26 | .26 | .23 | .17 |
| | .25 | .25 | .10 | .19 | .26 | .26 | .24 | .17 | .09 |
| | .50 | .50 | .14 | .25 | .27 | .25 | .20 | .11 | .01 |
| | 1.00 | 1.00 | .20 | .27 | .24 | .20 | .11 | -.02 | -.17 |
| | 1.50 | 1.50 | .24 | .26 | .21 | .13 | .01 | -.13 | -.35 |
| 60 | 0 | 0 | .15 | .31 | .35 | .33 | .27 | --- | --- |
| | .25 | .25 | .21 | .35 | .33 | .29 | .20 | --- | --- |
| | .50 | .50 | .28 | .35 | .30 | .23 | .11 | --- | --- |
| | 1.00 | 1.00 | .35 | .31 | .21 | .11 | -.05 | --- | --- |
| | 1.50 | 1.50 | .35 | .26 | .14 | -.02 | --- | --- | --- |
| 75 | 0 | 0 | .44 | .39 | .27 | .16 | --- | --- | --- |
| | .25 | .25 | .44 | .32 | .17 | .03 | --- | --- | --- |
| | .50 | .50 | .43 | .24 | .07 | -.11 | --- | --- | --- |
| | 1.00 | 1.00 | .33 | .10 | -.14 | -.37 | --- | --- | --- |
| | 1.50 | 1.50 | .25 | --- | --- | --- | --- | --- | --- |

$a_{4,3}$

$a_{4,4}$

| Δ | λ | | λ | | | | | | |
|----------|-----------|----------|-----------|-------|-------|-------|-------|-------|-------|
| | λ | Δ | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | -3.14 | -3.11 | -3.09 | -3.06 | --- | --- | --- |
| | .25 | .25 | -3.13 | -3.10 | -3.06 | -3.03 | --- | --- | --- |
| | .50 | .50 | -3.12 | -3.08 | -3.05 | -3.03 | -2.95 | --- | --- |
| | 1.00 | 1.00 | -3.10 | -3.03 | -2.99 | --- | --- | --- | --- |
| | 1.50 | 1.50 | -3.09 | -3.01 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | -3.13 | -3.10 | -3.07 | -3.03 | -2.98 | --- | --- |
| | .25 | .25 | -3.11 | -3.08 | -3.03 | -2.99 | -2.92 | --- | --- |
| | .50 | .50 | -3.11 | -3.06 | -3.01 | -2.95 | -2.87 | --- | --- |
| | 1.00 | 1.00 | -3.09 | -3.01 | -2.93 | -2.87 | -2.76 | --- | --- |
| | 1.50 | 1.50 | -3.06 | -2.97 | -2.88 | -2.79 | -2.66 | --- | --- |
| 0 | 0 | 0 | -3.13 | -3.08 | -3.01 | -2.96 | -2.86 | -2.73 | -2.61 |
| | .25 | .25 | -3.11 | -3.04 | -2.97 | -2.89 | -2.76 | -2.61 | -2.45 |
| | .50 | .50 | -3.09 | -3.01 | -2.91 | -2.81 | -2.67 | -2.48 | -2.30 |
| | 1.00 | 1.00 | -3.04 | -2.93 | -2.79 | -2.67 | -2.48 | -2.24 | -1.99 |
| | 1.50 | 1.50 | -3.00 | -2.85 | -2.69 | -2.52 | -2.30 | -1.99 | -1.69 |
| 30 | 0 | 0 | -3.11 | -3.03 | -2.92 | -2.80 | -2.62 | -2.38 | -2.17 |
| | .25 | .25 | -3.08 | -2.96 | -2.81 | -2.66 | -2.45 | -2.17 | -1.90 |
| | .50 | .50 | -3.05 | -2.90 | -2.71 | -2.53 | -2.28 | -1.96 | -1.64 |
| | 1.00 | 1.00 | -2.98 | -2.74 | -2.50 | -2.29 | -1.96 | -1.54 | -1.12 |
| | 1.50 | 1.50 | -2.89 | -2.59 | -2.31 | -2.04 | -1.69 | -1.11 | -.60 |
| 45 | 0 | 0 | -3.09 | -2.95 | -2.76 | -2.60 | -2.35 | -2.04 | -1.75 |
| | .25 | .25 | -3.04 | -2.83 | -2.62 | -2.41 | -2.12 | -1.75 | -1.38 |
| | .50 | .50 | -3.00 | -2.72 | -2.47 | -2.23 | -1.89 | -1.46 | -1.02 |
| | 1.00 | 1.00 | -2.87 | -2.51 | -2.20 | -1.90 | -1.45 | -.69 | -.30 |
| | 1.50 | 1.50 | -2.73 | -2.31 | -1.94 | -1.57 | -1.02 | -.25 | .42 |
| 60 | 0 | 0 | -3.04 | -2.75 | -2.44 | -2.19 | -1.89 | --- | --- |
| | .25 | .25 | -2.94 | -2.54 | -2.22 | -1.93 | -1.51 | --- | --- |
| | .50 | .50 | -2.81 | -2.38 | -2.02 | -1.68 | -1.17 | --- | --- |
| | 1.00 | 1.00 | -2.57 | -2.07 | -1.60 | -1.17 | -.50 | --- | --- |
| | 1.50 | 1.50 | -2.37 | -1.79 | -1.21 | -1.67 | .16 | --- | --- |
| 75 | 0 | 0 | -2.53 | -2.04 | -1.58 | -1.13 | --- | --- | --- |
| | .25 | .25 | -2.33 | -1.76 | -1.19 | -.62 | --- | --- | --- |
| | .50 | .50 | -2.13 | -1.47 | -.79 | -.11 | --- | --- | --- |
| | 1.00 | 1.00 | -1.81 | -.90 | 0 | .90 | --- | --- | --- |
| | 1.50 | 1.50 | -1.46 | --- | --- | --- | --- | --- | --- |

| Δ | λ | | λ | | | | | | |
|----------|-----------|----------|-----------|------|------|------|------|------|------|
| | λ | Δ | 1.5 | 2.5 | 3.5 | 4.5 | 6.0 | 8.0 | 10.0 |
| -45 | 0 | 0 | 4.02 | 4.04 | 4.06 | 4.10 | --- | --- | --- |
| | .25 | .25 | 4.02 | 4.05 | 4.09 | 4.14 | --- | --- | --- |
| | .50 | .50 | 4.03 | 4.07 | 4.12 | 4.18 | 4.27 | --- | --- |
| | 1.00 | 1.00 | 4.05 | 4.10 | 4.18 | --- | --- | --- | --- |
| | 1.50 | 1.50 | 4.07 | 4.15 | --- | --- | --- | --- | --- |
| -30 | 0 | 0 | 4.01 | 4.04 | 4.07 | 4.12 | 4.18 | --- | --- |
| | .25 | .25 | 4.02 | 4.06 | 4.11 | 4.17 | 4.26 | --- | --- |
| | .50 | .50 | 4.03 | 4.08 | 4.15 | 4.22 | 4.34 | --- | --- |
| | 1.00 | 1.00 | 4.06 | 4.14 | 4.23 | 4.34 | 4.51 | --- | --- |
| | 1.50 | 1.50 | 4.08 | 4.20 | 4.33 | 4.47 | 4.69 | --- | --- |
| 0 | 0 | 0 | 4.02 | 4.05 | 4.09 | 4.15 | 4.26 | 4.43 | 4.61 |
| | .25 | .25 | 4.03 | 4.07 | 4.14 | 4.23 | 4.38 | 4.61 | 4.86 |
| | .50 | .50 | 4.04 | 4.11 | 4.21 | 4.32 | 4.52 | 4.80 | 5.11 |
| | 1.00 | 1.00 | 4.07 | 4.19 | 4.34 | 4.52 | 4.81 | 5.21 | 5.63 |
| | 1.50 | 1.50 | 4.11 | 4.28 | 4.49 | 4.73 | 5.11 | 5.63 | 6.17 |
| 30 | 0 | 0 | 4.02 | 4.06 | 4.12 | 4.21 | 4.41 | 4.73 | 5.05 |
| | .25 | .25 | 4.03 | 4.09 | 4.21 | 4.36 | 4.64 | 5.06 | 5.50 |
| | .50 | .50 | 4.04 | 4.14 | 4.31 | 4.52 | 4.83 | 5.42 | 5.96 |
| | 1.00 | 1.00 | 4.09 | 4.28 | 4.56 | 4.89 | 5.41 | 6.15 | 6.90 |
| | 1.50 | 1.50 | 4.14 | 4.44 | 4.84 | 5.28 | 5.95 | 6.89 | 7.89 |
| 45 | 0 | 0 | 4.02 | 4.07 | 4.17 | 4.31 | 4.63 | 5.10 | 5.61 |
| | .25 | .25 | 4.03 | 4.12 | 4.30 | 4.55 | 4.98 | 5.61 | 6.26 |
| | .50 | .50 | 4.05 | 4.20 | 4.47 | 4.79 | 5.35 | 6.12 | 6.91 |
| | 1.00 | 1.00 | 4.10 | 4.41 | 4.86 | 5.35 | 6.12 | 7.18 | 8.24 |
| | 1.50 | 1.50 | 4.19 | 4.69 | 5.29 | 5.93 | 6.91 | 8.22 | 9.58 |
| 60 | 0 | 0 | 4.02 | 4.09 | 4.34 | 4.69 | 5.28 | | |

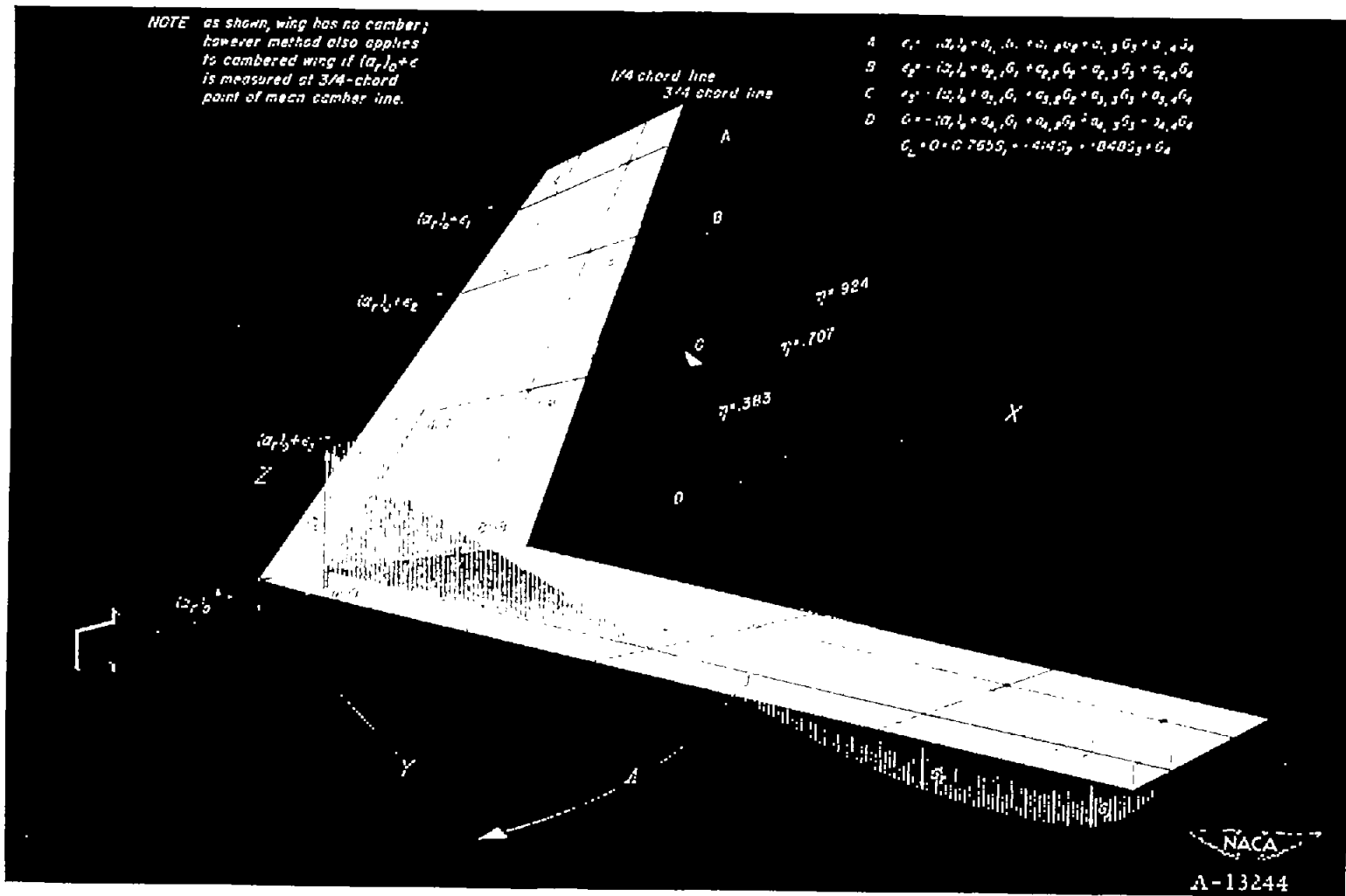


Figure 1.— Relationship between wing geometry and the equations used in determining the basic loading.

Note: Basic loading characteristics of shaded wings are discussed in text.

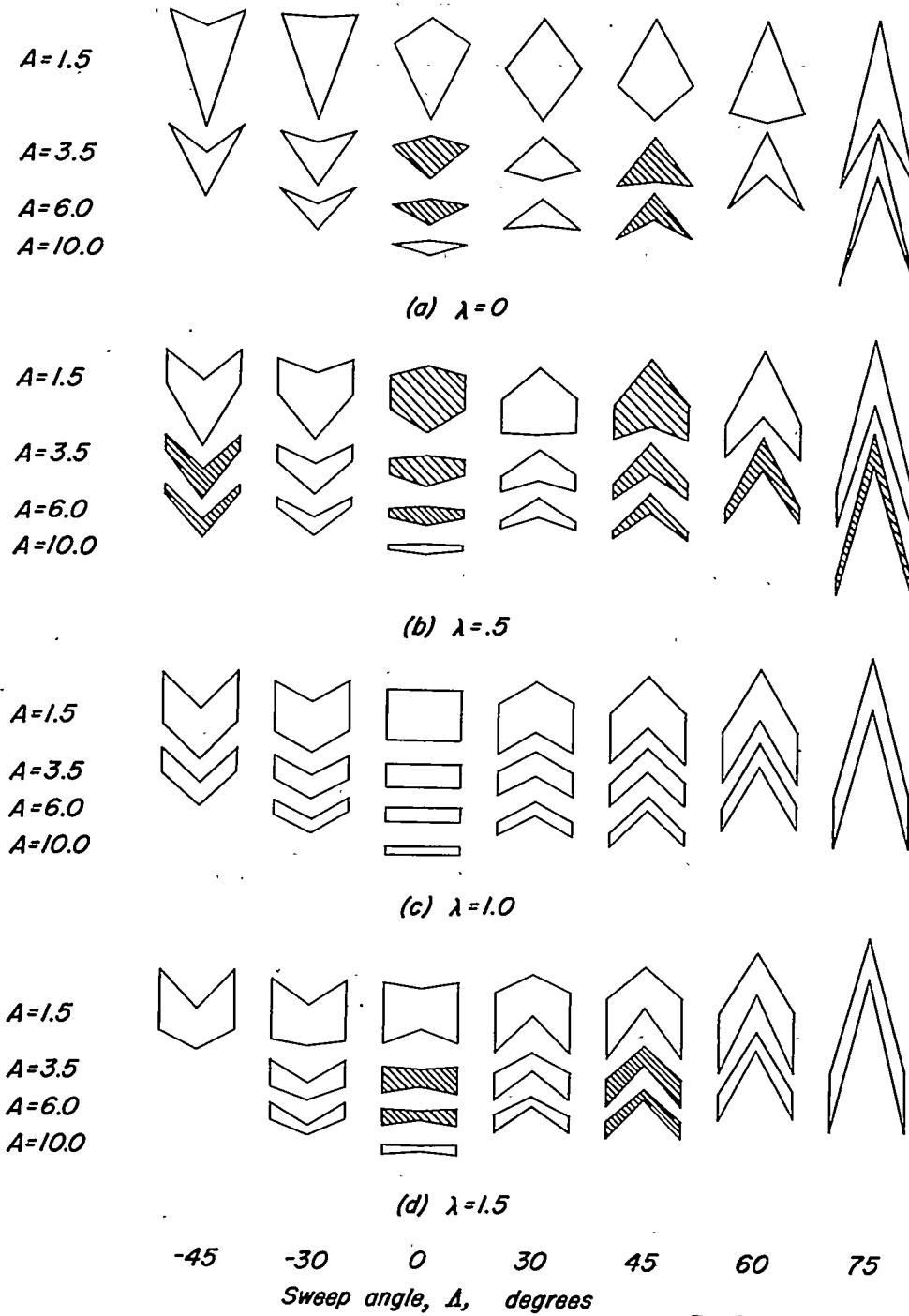


Figure 2.— Approximate range of planforms for which values of coefficients $a_{\nu n}$ are presented in table I.

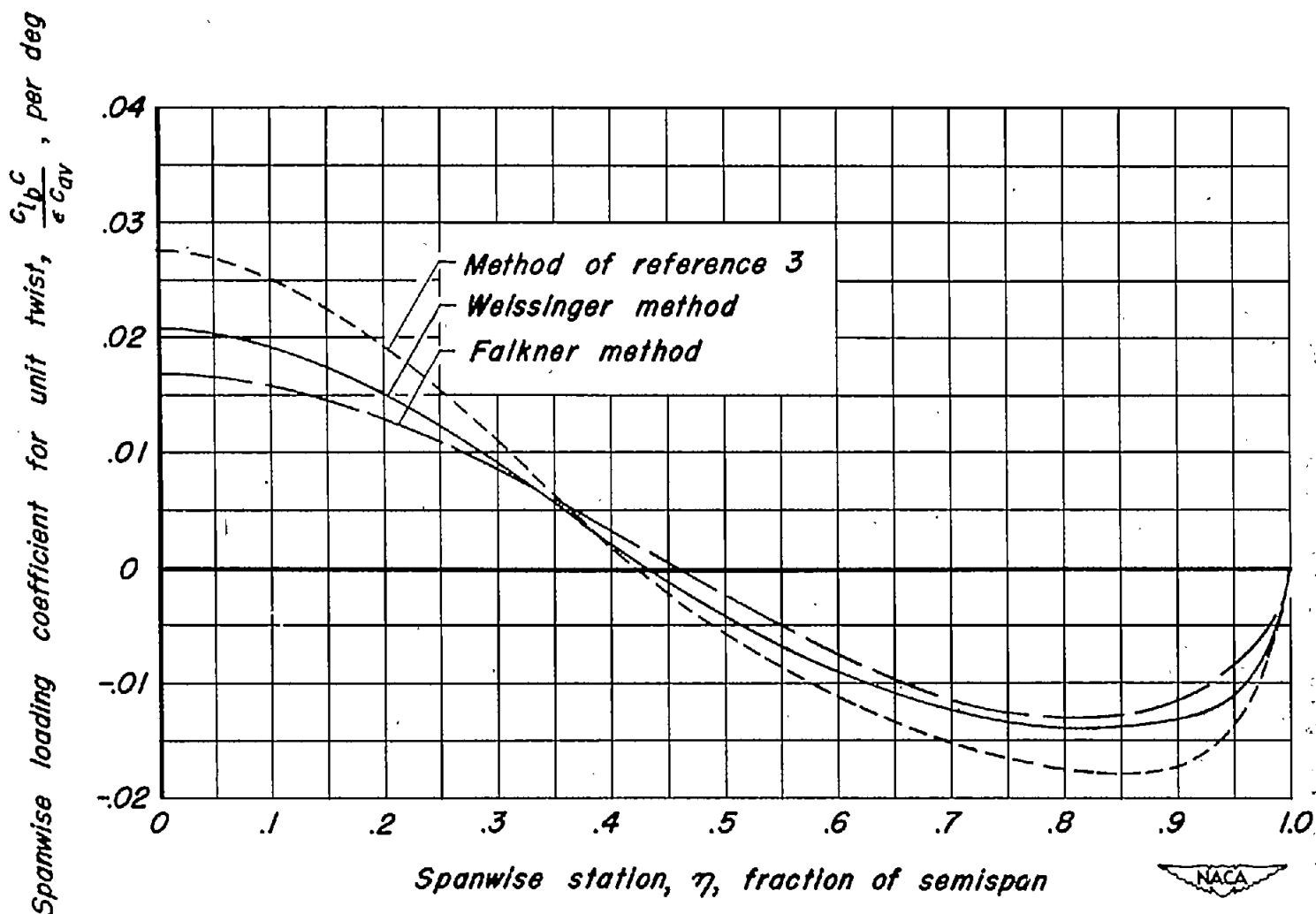


Figure 3.- A comparison of the basic loadings predicted by three theoretical methods for an unswept wing having an aspect ratio of 6.0 and taper ratio of 0.5.

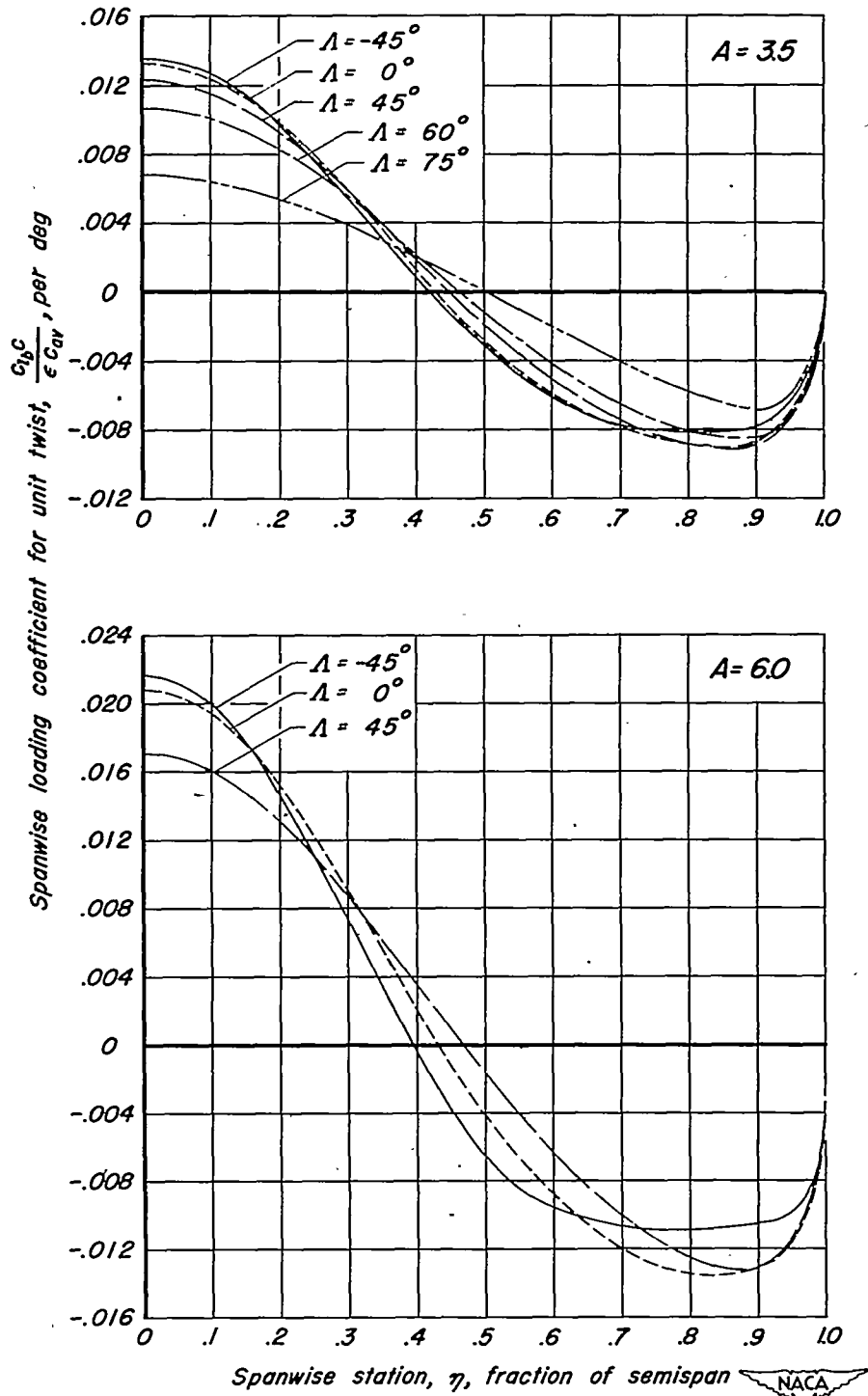


Figure 4.- The effect of sweep on the basic loading of wings having a taper ratio of 0.5.

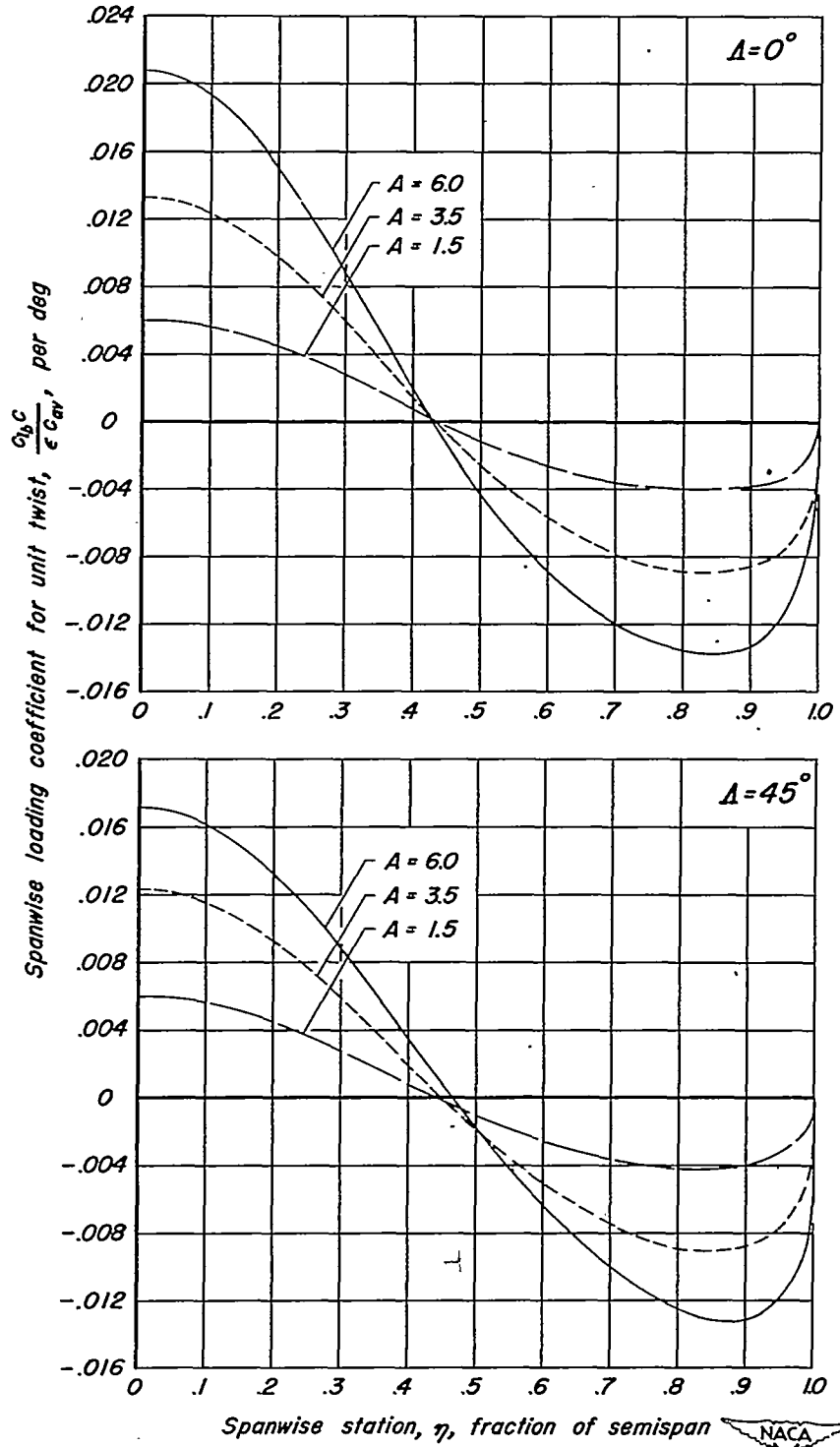


Figure 5.—The effect of aspect ratio on the basic loading of wings having a taper ratio of 0.5.

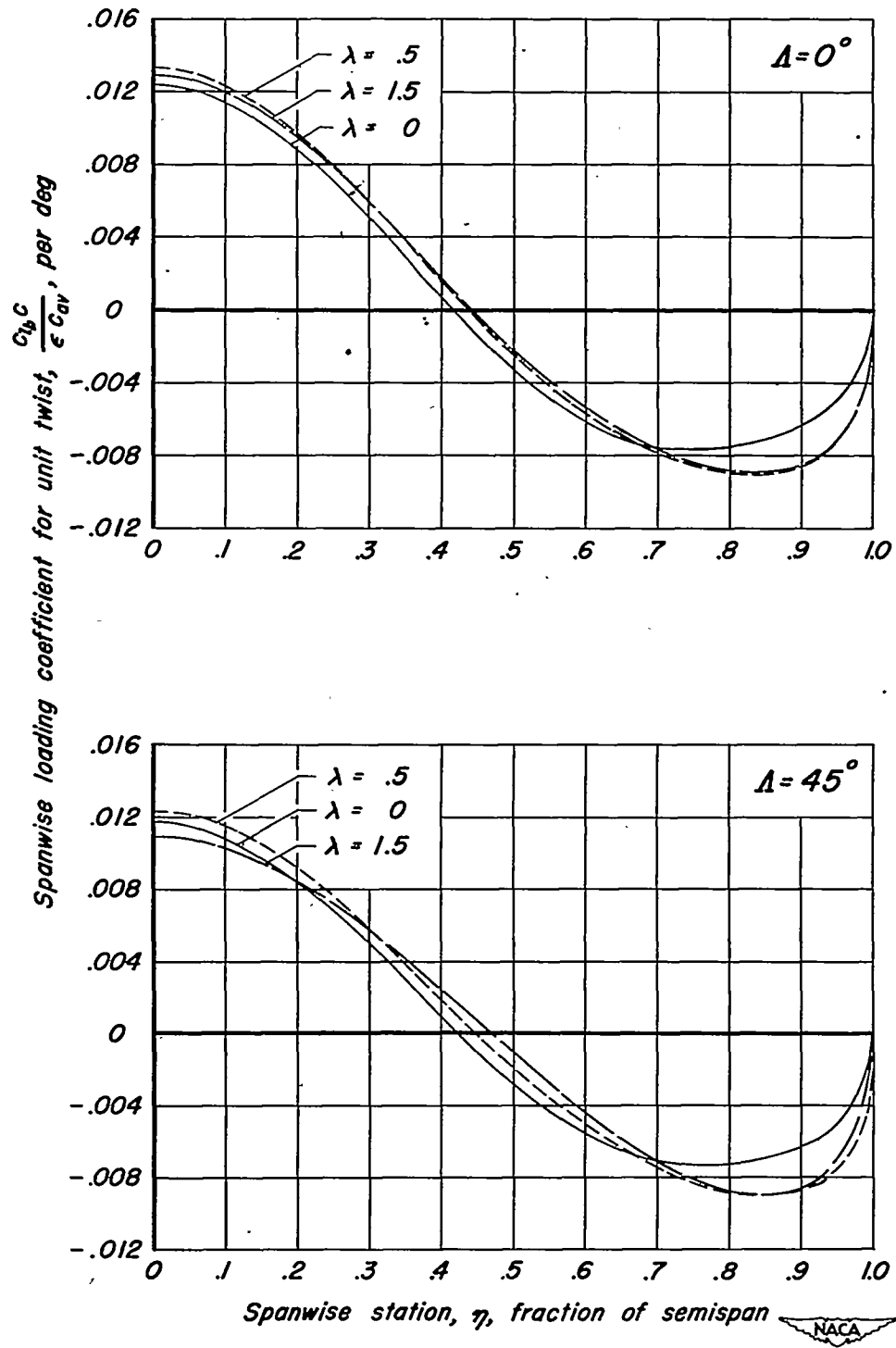


Figure 6.—The effect of taper ratio on the basic loading of wings having an aspect ratio of 3.5.

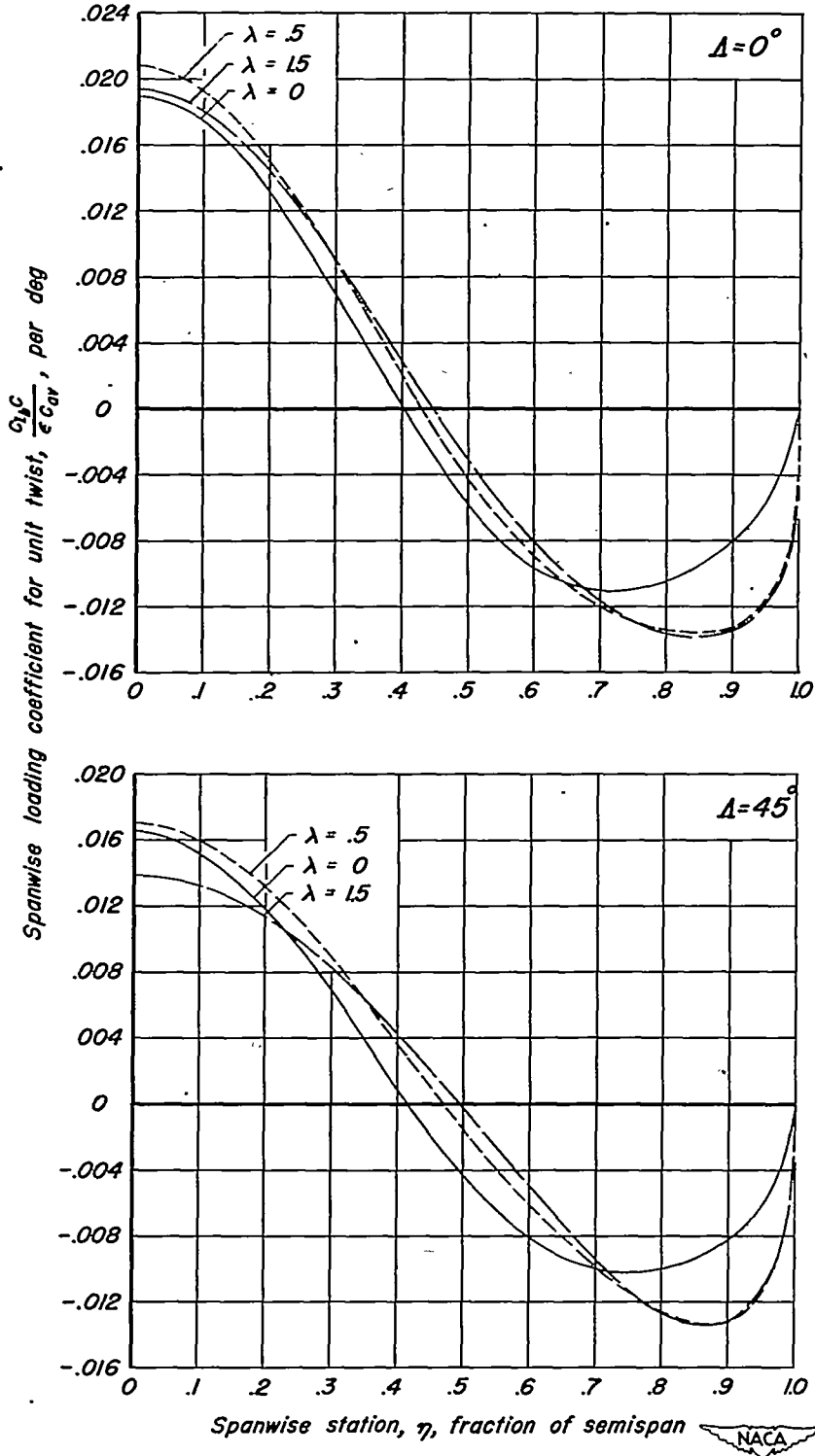


Figure 7.—The effect of taper ratio on the basic loading of wings having an aspect ratio of 6.0.

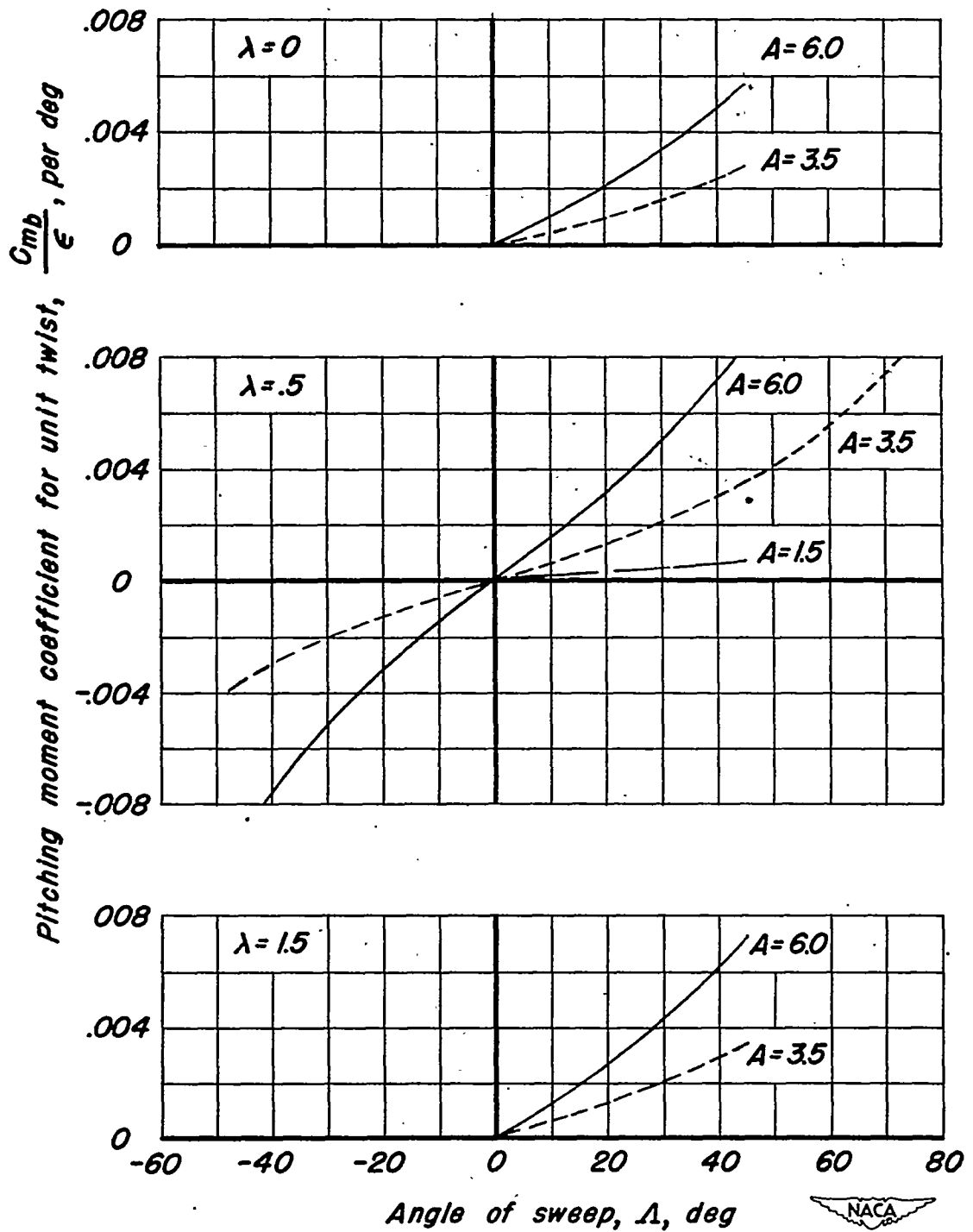


Figure 8.—The effect of planform on the pitching moment due to twist.

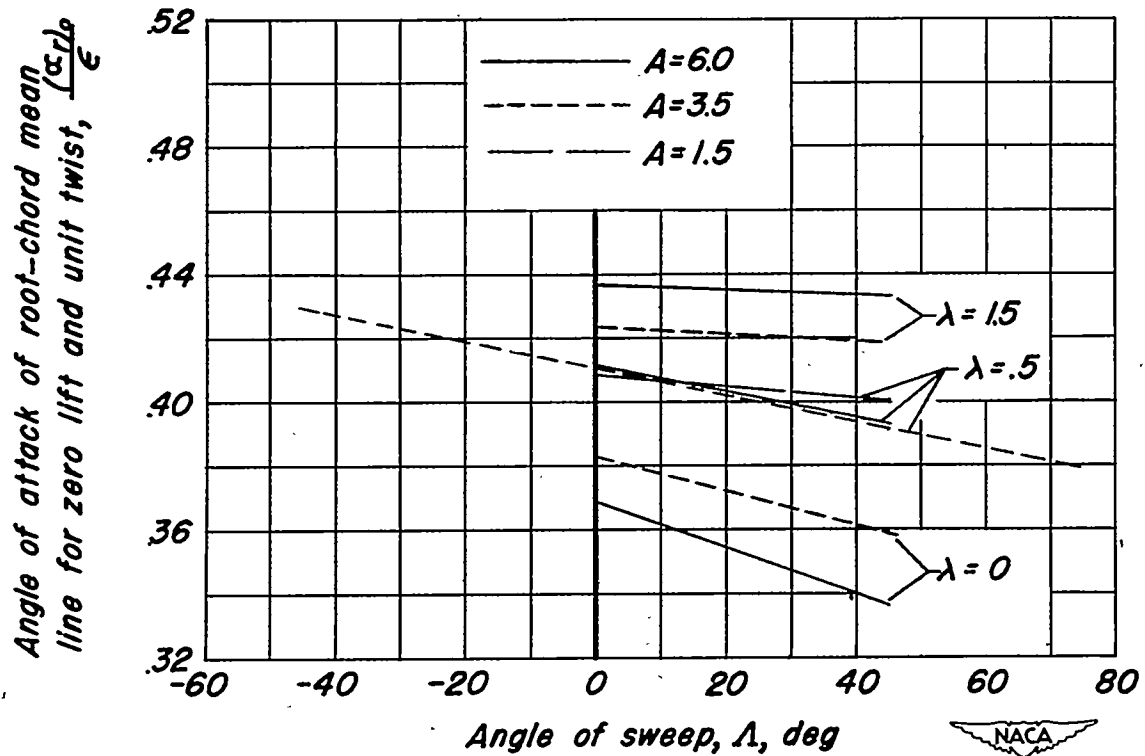


Figure 9.-The effect of planform on the angle of zero lift.

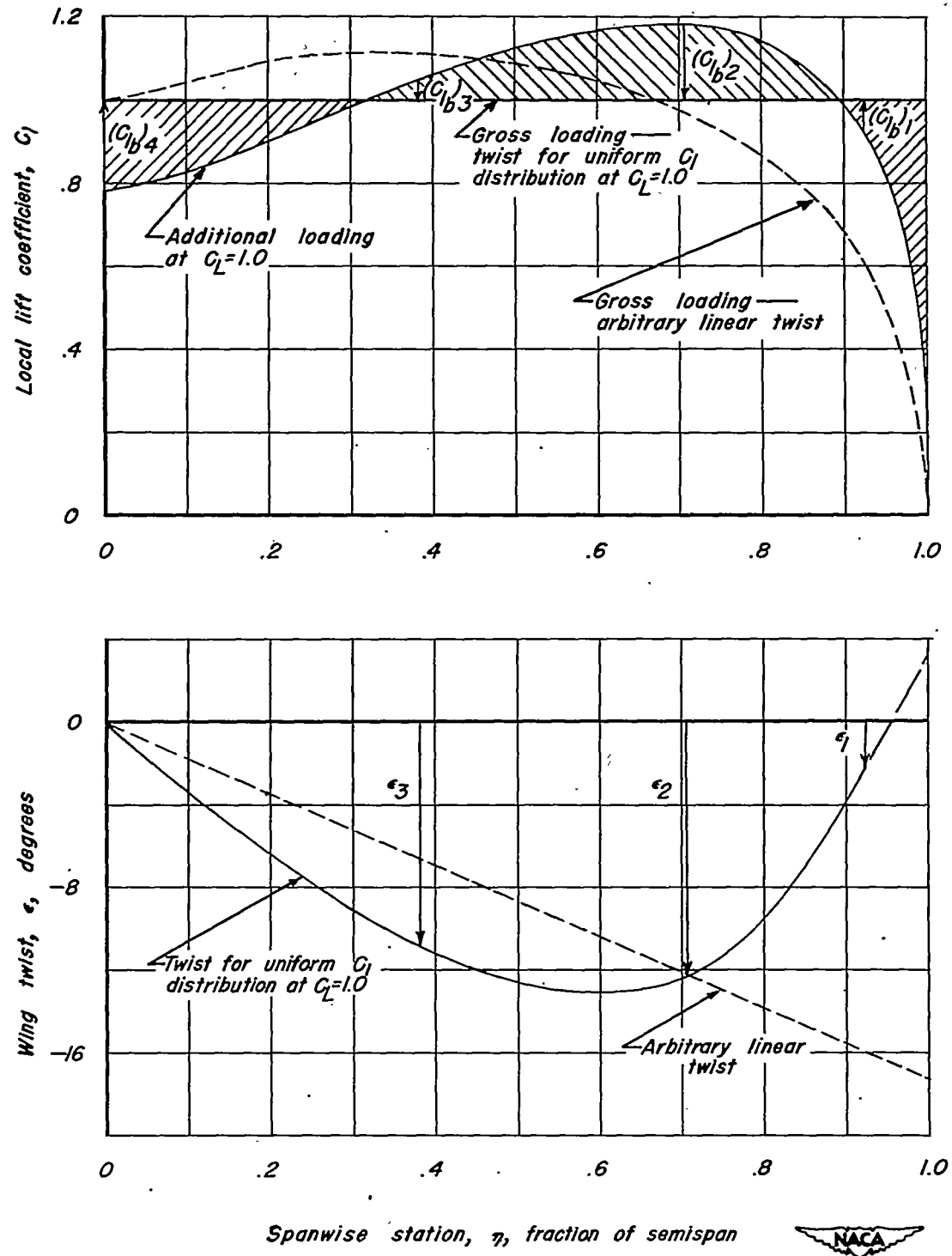


Figure 10.—Example illustrating use of twist to control spanwise distribution of local lift coefficient for a wing having 45° of sweepback, an aspect ratio of 6.0, and a taper ratio of 0.5.

