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PLASTIC BUCKLING OF A LONG FLAT PLATE UNDER COMBINED SHEAR AND LONGITUDINAL COMPRESSION

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SUMMARY

The condition for plastic buckling of a long flat plate under combined shear and longitudinal compression is obtained by using the theory of plastic buckling developed for simple loading, provided the ratio of shear to longitudinal compression is assumed constant during the loading process. The plate may be elastically restrained along the edges. Combinations of shear and longitudinal compression were computed for three simply supported plates of 24S-T4 aluminum alloy, reduced to stress-ratio form, and compared with the known interaction curve in the elastic region. Departures from the elastic interaction curve were found to be slight, provided the coordinates of the interaction curve were modified to allow for variations in moduli.

INTRODUCTION

The combinations of shear and direct stress in the elastic range that will cause buckling in an infinitely long flat plate were presented in reference 1. The combinations proved to be given with high accuracy by the formula

$$R_c + {R_s}^2 = 1 \tag{1}$$

where R_c is the ratio of compressive stress when buckling occurs under the combined loading to the compressive stress when buckling occurs under compression alone and R_s is the ratio of shear stress when buckling occurs under the combined loading to the shear stress when buckling occurs under shear alone. The interaction curve (a parabola) represented by equation (1) was shown to be largely independent of any rotational elastic restraint along the edges of the plate.

In the plastic region, solutions have already been obtained for the buckling of an infinitely long flat plate in compression alone (reference 2) and in shear alone (reference 3). The purpose of the present paper is to obtain the corresponding solution for any combination of compression and shear when applied together. Since the differential equation for plastic buckling in this case has nearly the same form as the corresponding equation for the elastic buckling of an orthotropic plate, it might be expected that the solutions would be similar in the two cases. Balabukh, whose work is discussed in reference 4, concluded that for elastic buckling of an orthotropic plate equation (1) still applies with good accuracy. The solution to the present problem was sought, therefore, in the same general form as equation (1).

Plasticity theory in its present state is unable to deal with problems in combined stress in a general way. However, Ilyushin has shown (reference 5) that if combined stresses are applied in such a way that their ratio is a constant during the loading process, the combination of stresses may be treated as though it were a single stress. Thus, the theory of plastic buckling already derived is applicable to the present problem provided the ratio of shear to compression is constant during loading. This condition is often met or approximated in practice. The use of the deformation theory of plasticity in buckling problems has been justified by Batdorf (reference 6).

SYMBOLS

σ_x applied direct stress, positive in compression

τ applied shear stress

σ_i stress intensity $\left(\sqrt{\sigma_x^2 + 3\tau^2}\right)$

E elastic modulus

E_s secant modulus at stress intensity σ_i

E_t tangent modulus at stress intensity σ_i

k coefficient in buckling formula, a function only of length-width ratio of plate and elastic restraint in the elastic range and also of stress in the plastic range b plate width

h plate thickness

D bending stiffness of plate in elastic range $\left(\frac{Eh^3}{9}\right)$

$D' = \frac{E_s h^3}{9}$

S stiffness of restraining medium

ϵ magnitude of elastic restraint along parallel edges $\left(\frac{4Sb}{D'}\right)$

λ half wave length of buckles

ϕ angle of nodes with normal to plate edges (see fig. 1)

Subscripts:

c compression

s shear

pc pure compression

ps pure shear

σ_i under combined stress

INTERACTION CURVE

In the plastic range, the critical stress for pure compression is shown to be of the form given in the equation preceding equation (28) of reference 2:

$$(\sigma_x)_{pc} = k_{pc} \frac{\pi^2 D'}{b^2 h} = k_{pc} \frac{(E_s)_{pc}}{E} \frac{\pi^2 D}{b^2 h}$$

Similarly, the critical stress for pure shear is shown to be of the form given in equation (12) of reference 3:

$$\tau_{ps} = k_{ps} \frac{\pi^2 D'}{b^2 h} = k_{ps} \frac{(E_s)_{ps}}{E} \frac{\pi^2 D}{b^2 h}$$

In these expressions the subscripts pc and ps refer to pure compression and pure shear, respectively. The values of k depend upon the aspect ratio of the plate, the conditions of edge restraint, and the stress when the plate is stressed into the plastic region.

When compressive stress σ_x and shear stress τ are applied simultaneously, as they are considered to be in this paper, they may also be expressed in the same form

$$\sigma_x = k_c \frac{(E_s)_{\sigma_i}}{E} \frac{\pi^2 D}{b^2 h}$$

$$\tau = k_s \frac{(E_s)_{\sigma_i}}{E} \frac{\pi^2 D}{b^2 h}$$

where the subcript σ_i signifies that the secant modulus is to be taken at the stress intensity $\sigma_i = \sqrt{\sigma_x^2 + 3\tau^2}$. The values of k_c and k_s depend on the ratio of σ_x to τ in a manner shown in the appendix.

The stress ratios are

$$R_c = \frac{\sigma_x}{(\sigma_x)_{pc}} = \frac{k_c}{k_{pc}} \frac{(E_s)_{\sigma_i}}{(E_s)_{pc}}$$

$$R_s = \frac{\tau}{\tau_{ps}} = \frac{k_s}{k_{ps}} \frac{(E_s)_{\sigma_i}}{(E_s)_{ps}}$$

For numerical calculation, a series of plates was selected, the first of which would buckle in the elastic range, with successive plates buckling at higher and higher stresses up to a stress somewhat beyond the yield stress. The plate material was taken as 24S-T4 aluminum alloy with a yield stress of 46 ksi. The edges of the plates were assumed simply supported. Calculations for critical combinations of applied shear and compression were made by an energy method, details of which are given in the appendix.

Results of the calculations for three plates are shown in figure 2. When R_c and R_s are used as coordinates, the curves of figure 2(a) are obtained. When R_c and R_s are modified to allow for variations in moduli, the curves of figure 2(b) are obtained. When plotted in this way the separation of the curves is appreciably reduced.

The plate with the lowest value of $\frac{\pi^2 D}{b^2 h}$ buckles elastically; the modulus ratios are therefore all unity and the corresponding interaction curve is the parabola of equation (1). The plate with the intermediate value of $\frac{\pi^2 D}{b^2 h}$ buckles elastically in pure compression but plastically in pure shear; the corresponding interaction curve falls slightly below the parabola in figure 2(b). The plate with the highest value of $\frac{\pi^2 D}{b^2 h}$ buckles plastically for all combinations of shear and compression and its interaction curve falls still farther below the parabola in figure 2(b). The sum of the abscissa and the square of the ordinate (see equation (1)) is always unity at the ends of the interaction curves but may differ slightly in between. For the top (elastic) curve, the sum is always unity; whereas, for the bottom (plastic) curve, the sum may drop to 0.95. Since this difference is of the same order of magnitude as the inaccuracies in knowledge of material properties at very high stresses, the difference is not of too great consequence. As a practical matter it seems advantageous to set up the algebraic relation

$$R_c \frac{(E_s)_{pc}}{(E_s)_{\sigma_i}} + \left[R_s \frac{(E_s)_{ps}}{(E_s)_{\sigma_i}}\right]^2 = 1 \qquad (2)$$

which applies to the top curve of figure 2(b).

By use of this relation, it is possible to discover whether any given combination of stresses σ_x and τ will cause a plate to buckle. The stress intensity $\sigma_i = \sqrt{\sigma_x^2 + 3\tau^2}$ is known; therefore, $(E_s)_{\sigma_i}$ is known. The values of the stresses $(\sigma_x)_{pc}$ and τ_{ps} under the two simple loadings can be found from references 2 and 3; at the same time the moduli $(E_s)_{pc}$ and $(E_s)_{ps}$ are also determined. These quantities are substituted into equation (2); the plate will or will not buckle depending upon whether the value of the left-hand side of equation (2) is greater or less than unity.

ILLUSTRATIVE PROBLEM

A simply supported plate of 24S-T4 aluminum alloy with a value of $\frac{\pi^2 D}{b^2 h}$ equal to 9000 psi is considered. The stress-strain curve of the material gives values of $\frac{E_s}{E}$ as in curve A, figure 1, of reference 2. A longitudinal compressive stress of 20 ksi is applied to the plate and the shear stress necessary to buckle the plate is required.

From reference 2, the value of $(\sigma_x)_{pc}$ is found to be 32 ksi with $\left(\frac{E_s}{E}\right)_{pc} = 0.94$. From reference 3, the value of τ_{ps} is found to be 27.6 ksi with $\left(\frac{E_s}{E}\right)_{ps} = 0.62$. The value of R_c can now be found to be $\frac{20}{32} = 0.625$. Equation (2) therefore gives

$$\left[R_s \frac{0.62}{\left(\frac{E_s}{E}\right)_{\sigma_i}}\right]^2 = 1 - 0.625 \frac{0.94}{\left(\frac{E_s}{E}\right)_{\sigma_i}}$$

In order to fix the value of $\left(\frac{E_s}{E}\right)_{\sigma_i}$, a value of σ_i must be assumed. The value of σ_i will fall between the values of σ_i for pure compression and pure shear; that is, $32 < \sigma_i < 48$ ksi. As a first approximation, σ_i was taken as 40 ksi; therefore, $\left(\frac{E_s}{E}\right)_{\sigma_i}$ was found equal to 0.84. Upon substitution in the formula, R_s is found to be equal to 0.74. The shear stress is then

$$\tau = R_s \tau_{ps}$$

$$= 0.74 \times 27.6$$

$$= 20.4 \text{ ksi}$$

The combination of σ_x and τ results in a stress intensity

$$\sigma_i = \sqrt{{\sigma_x}^2 + 3\tau^2}$$

$$= \sqrt{(20)^2 + 3(20.4)^2}$$

$$= 40.6 \text{ ksi}$$

No significant change will result if the problem is reworked with the new value of σ_i.

CONCLUDING REMARKS

A solution has been obtained for the buckling stress of an infinitely long flat plate under combined shear and longitudinal compression in the plastic region. A detailed calculation for a number of simply supported plates made of a typical aluminum alloy shows that an approximate algebraic relationship, equation (2), will suffice for determination of buckling stress.

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National Advisory Committee for Aeronautics
Langley Air Force Base, Va., September 26, 1949

APPENDIX

ANALYSIS

Energy expressions in terms of Cartesian coordinates (x',y').- The general expression for the net strain energy of a supported plate in the plastic region is given in reference 2 as formula (18). If only longitudinal compression σ_x and shear τ are applied, the separate expressions for the strain energy V_1 and the work T during buckling become, in terms of the deflection w at the point (x',y') in a Cartesian system,

$$V_1 = \frac{D'}{2}\iint\left\{C_1\left(\frac{\partial^2 w}{\partial x'^2}\right)^2 - C_2\frac{\partial^2 w}{\partial x'^2}\frac{\partial^2 w}{\partial x'\partial y'} + C_3\left[\left(\frac{\partial^2 w}{\partial x'\partial y'}\right)^2 + \frac{\partial^2 w}{\partial x'^2}\frac{\partial^2 w}{\partial y'^2}\right] + \left(\frac{\partial^2 w}{\partial y'^2}\right)^2\right\}dx'dy' \qquad \text{(A1)}$$

$$T = \frac{h}{2}\iint\left[\sigma_x\left(\frac{\partial w}{\partial x'}\right)^2 + 2\tau\frac{\partial w}{\partial x'}\frac{\partial w}{\partial y'}\right]dx'dy' \qquad \text{(A2)}$$

where the values of the plasticity coefficients are as follows:

Plasticity coefficient	Pure compression	Pure shear	Combined compression and shear
C_1	$\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}$	1	$1 - \dfrac{\frac{3}{4}}{1 + 3\left(\frac{\tau}{\sigma_x}\right)^2}\left(1 - \frac{E_t}{E_s}\right)$
C_2	0	0	$\dfrac{3\frac{\tau}{\sigma_x}}{1 + 3\left(\frac{\tau}{\sigma_x}\right)^2}\left(1 - \frac{E_t}{E_s}\right)$
C_3	1	$\frac{1}{2} + \frac{1}{2}\frac{E_t}{E_s}$	$1 - \dfrac{\frac{3}{2}\left(\frac{\tau}{\sigma_x}\right)^2}{1 + 3\left(\frac{\tau}{\sigma_x}\right)^2}\left(1 - \frac{E_t}{E_s}\right)$

If, in addition, a restraining medium of rotational stiffness S is along one longitudinal edge of the plate, the strain energy V_2 in the restraint itself is taken to be, from expression (19) of reference 2,

$$V_2 = \frac{D'\epsilon}{2b}\int\left[\left(\frac{\partial w}{\partial y'}\right)_{y'=y'_o}\right]^2 dx' \tag{A3}$$

where

$$\epsilon = \frac{4Sb}{D'}$$

if y'_o is the edge coordinate.

<u>Energy expressions in terms of oblique coordinates (x,y)</u>.- In order to obtain corresponding energy expressions in oblique coordinates, apply the transformation

$$\left.\begin{aligned} x' &= x - y \sin \phi \\ y' &= y \cos \phi \end{aligned}\right\} \tag{A4}$$

where the two coordinate systems are related as in figure 1. In the oblique coordinates, V_1, T, and V_2 become

$$\begin{aligned} V_1 = \frac{D'}{2}\iint\Bigg[&\left(C_1 - C_2 \tan\phi + 2C_3 \tan^2\phi + \tan^4\phi\right)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{\cos^4\phi}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 \\ &+ \left(\frac{C_3}{\cos^2\phi} + \frac{4\tan^2\phi}{\cos^2\phi}\right)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{C_3}{\cos^2\phi} + \frac{2\tan^2\phi}{\cos^2\phi}\right)\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \\ &+ \left(\frac{4\tan^3\phi}{\cos\phi} + 4C_3\frac{\tan\phi}{\cos\phi} - \frac{C_2}{\cos\phi}\right)\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x \partial y} + 4\frac{\tan\phi}{\cos^3\phi}\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial x \partial y}\Bigg]dx\,dy \cos\phi \end{aligned} \tag{A5}$$

$$T = \frac{h}{2}\iint\left[\left(\sigma_x + 2\tau \tan \phi\right)\left(\frac{\partial w}{\partial x}\right)^2 + \frac{2\tau}{\cos \phi}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]dxdy \cos \phi \tag{A6}$$

$$V_2 = \frac{D'\epsilon}{2b_1 \cos \phi}\int\left[\frac{1}{\cos \phi}\left(\frac{\partial w}{dy}\right)_{y=y_o} + \tan \phi\left(\frac{\partial w}{dx}\right)_{y=y_o}\right]^2 dx \tag{A7}$$

Energy relation at buckling.- When buckling occurs, the energy relation is as follows:

$$T = V_1 + V_2 \tag{A8}$$

Deflection surface.- In order to obtain a practical solution the deflection function or a reasonable approximation of it is required. As in references 1 to 3, the deflection surface is taken to be

$$w = \left[\frac{\pi\epsilon}{2}\left(\frac{y^2}{{b_1}^2} - \frac{1}{4}\right) + \left(1 + \frac{\epsilon}{2}\right) \cos \frac{\pi y}{b_1}\right] \cos \frac{\pi x}{\lambda}$$

When this expression with its derivatives is substituted into the expressions for T, V_1, and V_2,

$$T = \left(\sigma_x + 2\tau \tan \phi\right)\frac{\pi^2 b_1 h}{4\lambda}\left[\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}\right] \cos \phi$$

$$V_1 = \frac{D'}{2}\left\{\frac{\pi^4 b_1}{2\lambda^3}\left(C_1 - C_2 \tan \phi + 2C_3 \tan^2\phi + \tan^4\phi\right)\left[\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2\right.\right.$$

$$\left. + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}\right] + \frac{\pi^4\lambda}{2{b_1}^3}\frac{1}{\cos^4\phi}\left[\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}\right]$$

$$\left. + \frac{\pi^4}{2b_1\lambda}\frac{1}{\cos^2\phi}\left(2C_3 + 6 \tan^2\phi\right)\left[\left(\frac{5}{24} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}\right]\right\}$$

$$V_2 = \frac{\pi^2 D' \epsilon \lambda}{2{b_1}^3 \cos^3 \phi}$$

Critical combination of stresses.- When the foregoing values of T, V_1, and V_2 are substituted into equation (A8), the following equation is obtained:

$$\sigma_x + 2\tau \tan \phi = \left[\left(\frac{b_1}{\lambda}\right)^2 \left(C_1 - C_2 \tan \phi + 2C_3 \tan^2 \phi + \tan^4 \phi\right) + \frac{F_1(\epsilon)}{\left(\frac{b_1}{\lambda}\right)^2 \cos^4 \phi} + \frac{C_3 + 3 \tan^2 \phi}{\cos^2 \phi} F_2(\epsilon)\right] \frac{\pi^2 D'}{{b_1}^2 h} \tag{A9}$$

where

$$\left.\begin{aligned} F_1(\epsilon) &= \frac{\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{2}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}} \\ F_2(\epsilon) &= 2\frac{\left(\frac{5}{24} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}} \end{aligned}\right\} \tag{A10}$$

Replacement of b_1 by $\frac{b}{\cos \phi}$ in equation (A9) gives, for the condition of buckling,

$$\sigma_x + 2\tau \tan \phi = \left[\left(\frac{b}{\lambda}\right)^2 \left(C_1 - C_2 \tan \phi + 2C_3 \tan^2\phi + \tan^4\phi\right) + \frac{F_1(\epsilon)}{\left(\frac{b}{\lambda}\right)^2} + \left(C_3 + 3 \tan^2\phi\right) F_2(\epsilon)\right] \frac{\pi^2 D'}{b^2 h} \tag{A11}$$

When τ is considered as a given constant, the wave length λ may be adjusted to make σ_x a minimum from the relation $\frac{\partial \sigma_x}{\partial \left(\frac{b}{\lambda}\right)^2} = 0$, which gives

$$\left(\frac{b}{\lambda}\right)^2 = \sqrt{\frac{F_1(\epsilon)}{C_1 - C_2 \tan \phi + 2C_3 \tan^2\phi + \tan^4\phi}} \tag{A12}$$

for elastic restraint ϵ independent of buckle wave length λ. Substitution of this value of $\left(\frac{b}{\lambda}\right)^2$ in equation (A11) to obtain the expression for σ_x gives

$$\sigma_x = \left[2\sqrt{F_1(\epsilon)}\sqrt{C_1 - C_2 \tan \phi + 2C_3 \tan^2\phi + \tan^4\phi} + \left(C_3 + 3 \tan^2\phi\right) F_2(\epsilon)\right] \frac{\pi^2 D'}{b^2 h} - 2\tau \tan \phi \tag{A13}$$

The angle ϕ of the waves may likewise be adjusted to make σ_x a minimum from the relation $\frac{\partial \sigma_x}{\partial (\tan \phi)} = 0$, which gives

$$\frac{2\tan^3\phi + 2C_3\tan\phi - \frac{1}{2}C_2}{\sqrt{C_1 - C_2\tan\phi + 2C_3\tan^2\phi + \tan^4\phi}}\sqrt{F_1(\epsilon)} + 3F_2(\epsilon)\tan\phi = \frac{\tau}{\frac{\pi^2 D'}{b^2 h}} \qquad \text{(A14)}$$

In the elastic range, equations (A11), (A12), and (A14) reduce to

$$\sigma_x + 2\tau\tan\phi = \left[\frac{\left(\frac{b}{\lambda}\right)^2}{\cos^4\phi} + \frac{F_1(\epsilon)}{\left(\frac{b}{\lambda}\right)^2} + \frac{1 + 2\sin^2\phi}{\cos^2\phi}F_2(\epsilon)\right]\frac{\pi^2 D}{b^2 h}$$

$$\left(\frac{b}{\lambda}\right)^2 = \sqrt{F_1(\epsilon)}\cos^2\phi$$

$$\left[2\sqrt{F_1(\epsilon)} + 3F_2(\epsilon)\right]\tan\phi = \frac{\tau}{\frac{\pi^2 D}{b^2 h}}$$

which agree with equations (B7), (B10), and (B11), respectively, of reference 1.

Computation of interaction curves.— In order to find the shear stress τ that will cause buckling for a given compressive stress σ_x, the following procedure was used. Simply supported plates of 24S-T4 aluminum alloy were selected for the computation; hence, $\epsilon = 0$, $F_1(\epsilon) = 1$, and $F_2(\epsilon) = 2$. Three values of $\frac{\pi^2 D}{b^2 h}$ were selected; for each value, equation (A14) gives the relation between the angle ϕ and shear stress τ. Pairs of values of ϕ and τ were then inserted by trial and error into equation (A13) until it was satisfied. Both the values of σ_x and the corresponding τ were then divided by σ_{pc} and τ_{ps}, respectively, to obtain the stress ratios. A complete interaction curve was obtained for each of the assumed values of $\frac{\pi^2 D}{b^2 h}$.

Figure 2 shows the results of the calculation. When the stress ratios are used as coordinates without making allowance for changes in moduli, the interaction curves are as shown in figure 2(a). When the stress ratios are modified to allow for changes in moduli, the interaction curves are as shown in figure 2(b). Since the separation of the curves is appreciably reduced by use of the modified coordinates, curves of the type shown in figure 2(b) are recommended for use.

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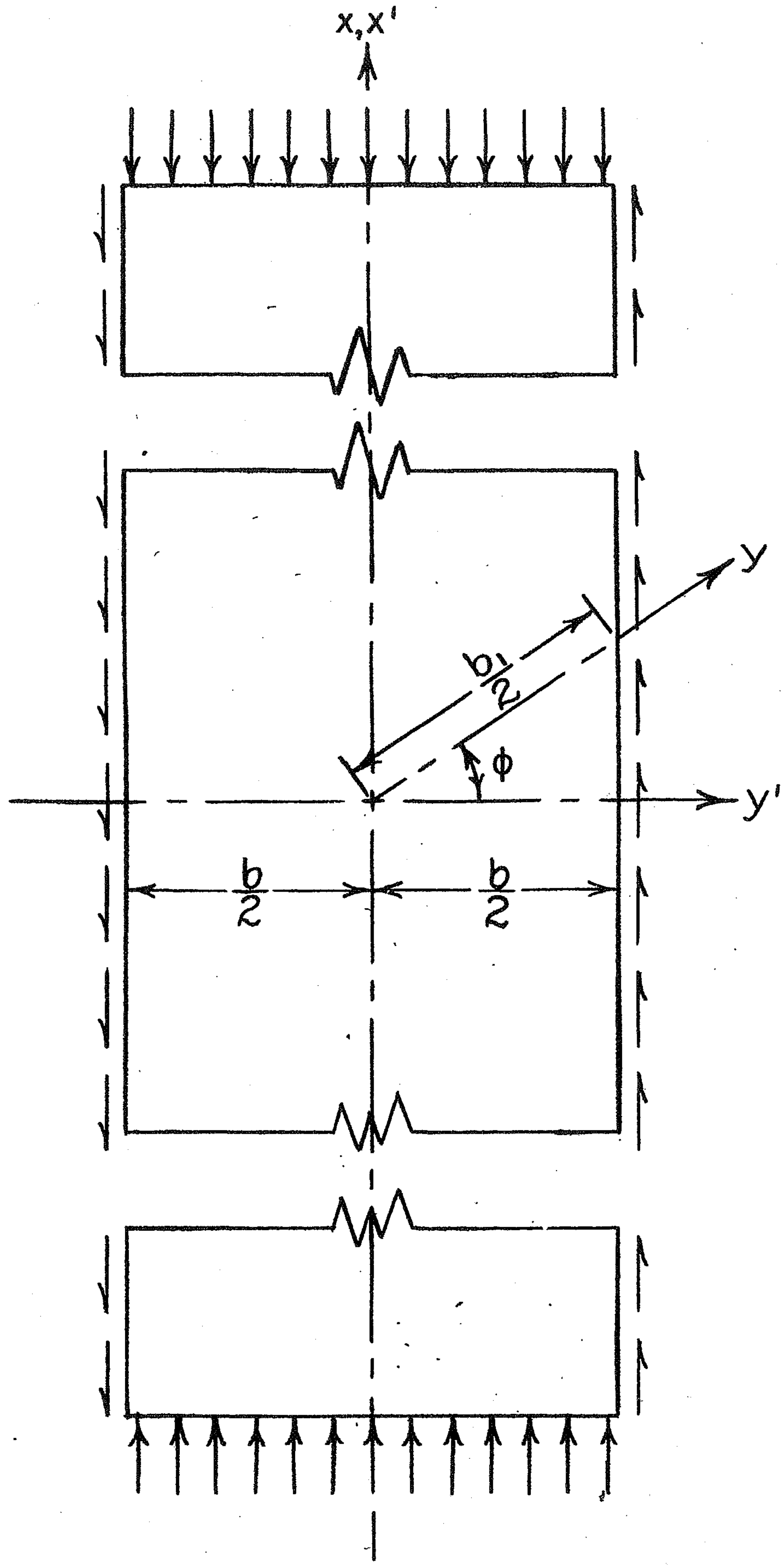
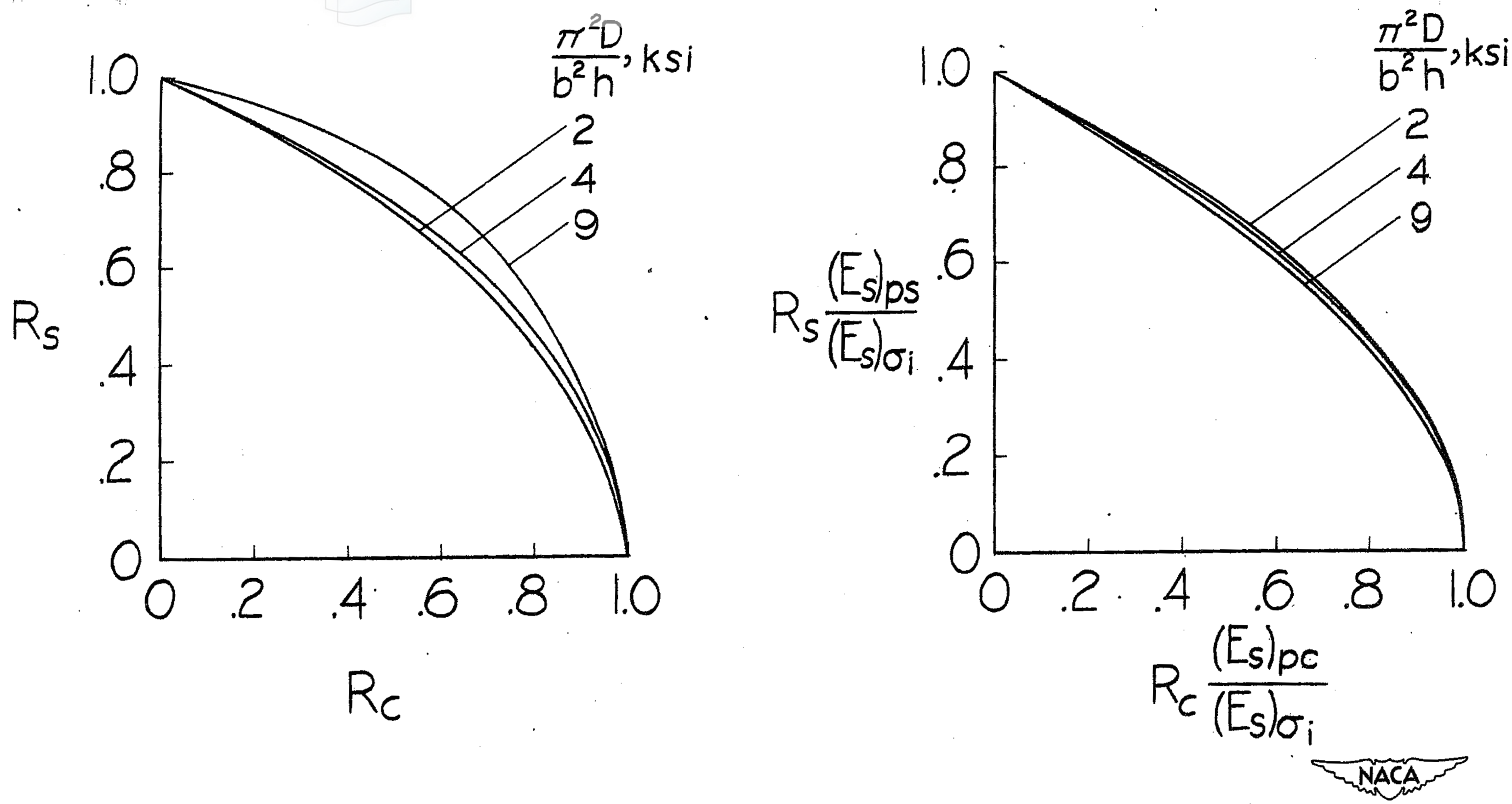


Figure 1.—Oblique coordinate system.



(a) With coordinates R_s and R_c.

(b) With coordinates R_s and R_c modified to allow for changes in moduli.

Figure 2.– Interaction curves for the buckling of a long flat plate of 24S-T4 aluminum alloy under combined longitudinal compression and shear.