NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2108

ANALYTICAL METHOD FOR DETERMINING TRANSMISSION AND ABSORPTION OF TIME-DEPENDENT RADIATION THROUGH THICK ABSORBERS

III - ABSORBER WITH RADIOACTIVE DAUGHTER PRODUCTS

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SUMMARY

A theoretical treatment of absorption problems is presented in which the following cases are simultaneously considered:

(a) Radiation is normal to an absorber of which the stations are plane parallel surfaces.

(b) Radiations are of several polyenergetic types.

(c) Induced radioactive isotopes decay to stable atoms in multistep decay processes.

(d) Radiations from the absorber affect the time-dependency of the source activity.

INTRODUCTION

Matrix methods have been applied to the problem of calculating transmission, reflection, and conversion to heat of time-dependent nuclear radiations (references 1 and 2). The analyses that were made, however, assumed single-step decay of induced radioactive atoms to stable ones. Many isotopes decay in several steps to stable materials, emitting a variety of types of radiation during the process. The transuranic radioactive isotopes are examples of multistep decay elements, as are certain isotopes of elements commonly used as structural materials such as iron and copper.

The matrix methods of references 1 and 2 developed at the NACA Lewis laboratory were therefore extended to include the multistep decay processes. The addition of these processes to the problems considered in references 1 and 2 makes the defining equations quite lengthy even though matrix notation is used. In order to facilitate presentation, tensor notation and the summation convention, when feasible, are used.

The assumption is made in references 1 and 2 that the source intensity is unaffected by back-reflected radiation from the absorber; that is, the time-dependency of the source is assumed to be known explicitly for all times of interest. In practice, however, the radiation back-reflected to the source includes radiation from the radioactivities within the absorber. Thus, although it is unlikely that the source will be noticeably affected by radiation reflected in the literal sense, enough of the aforementioned radiation originating within the absorber may quite possibly reach the source to induce a new radioactivity therein. If the contribution of the new activity to the total source intensity is not negligible, the time-dependecy of the radiation from the source will be significantly altered.

The change in source intensity as a function of time described in the preceding paragraph can be quantitatively calculated by methods shown herein.

SYMBOLS AND NOTATION

The following symbols are used in this report:

$[A]$, $[B]$, $[C]$, $[D]$	matrix coefficients
E	energy
H	thermal power generated
h	rate of conversion of energy of nuclear radiation absorbed to thermal energy
I	power of radiation, going to right, incident upon or emerging from station of absorber
N	number of radioactive atoms
n	number of stations in absorber
P	one-half of power of radiation from radioactivity emerging from station of absorber

q	number of generations of daughters
R	power of radiation, going to left, incident upon or emerging from station of absorber
r	power back-scattering coefficient
t	power-transmission coefficient
κ	rate of conversion of radioactive energy of parents to radioactive energy of daughters
λ	decay constant
μ	rate of conversion of radiation absorbed to energy of radioactivity
τ	time

Superscripts and subscripts:

$(\ )^{iesd}_{\epsilon\sigma}$	( ) of energy ϵ and type σ in i^{th} station that becomes d^{th} generation radiation of energy e and type s
δ	δ^{th} generation of daughters
0	initial condition

The following conventions have been adopted in the notation used:

(1) If an index appears both as a subscript and as a superscript in the same term, a summation over the range of the index is indicated. As used herein, "term" means any algebraic quantity or quantities separated from other algebraic quantities by +, -, or = signs.

(2) In order to avoid confusion with the summation convention in tensor analysis, the further provision is made that only repeated Greek symbols will indicate summation. This provision is made because English symbols may be repeated in the same term but never both as a subscript and a superscript and will therefore not indicate summation.

(3) A symbol like $\mu_{\epsilon\sigma}^{ies0}$ denotes the rate of conversion of radiation of energy ϵ and type σ absorbed in the i^{th} station to radioactivity of a parent atom emitting radiation of energy e and type s. Transformations from energy ϵ and type σ to energy e and type s will be symbolically indicated as in the preceding example.

ANALYSIS

The analysis is based on the following assumptions:

(1) Stations of the absorber are plane parallel surfaces.

(2) Radiation is normal to absorber.

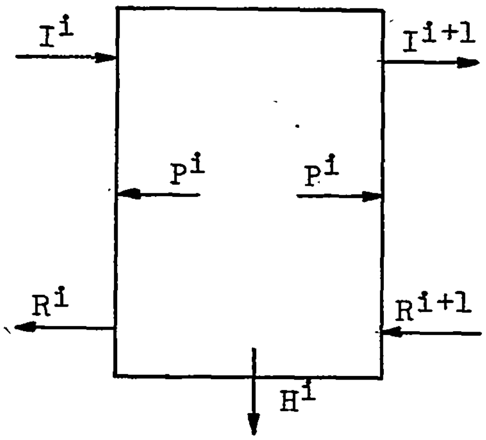
(3) Part of the radiation absorbed at each station of the absorber is transformed to thermal energy and part to induced radioactivity in the absorber.

(4) Radioactive atoms in the absorber decay in a finite number of steps to stable atoms.

(5) One-half of the radioactivity produced in each station of the absorber is emitted from each side of the absorber.

(6) At some instant of time, the time-dependency of the incident radiation is known explicitly.

Method. - Diagram (a) is included to clarify the discussion in the following paragraph.



As in references 1 and 2, the absorber is considered as being arbitrarily divided into n convenient stations normal to the path of the radiation. Radiation of power I^i from the $(i-1)^{th}$ station is incident upon the i^{th} station of the absorber; whereas radiation R^i from the i^{th} station is incident upon the $(i-1)^{th}$ station.

In the treatment presented herein, the values of the following physical constants must be known:

h rates of conversion of energy of nuclear radiation absorbed to thermal energy

r power back-scattering coefficients

t power-transmission coefficients

κ rates of conversion of radioactive energy of parents to radioactive energy of daughters

λ radioactive decay constants

μ rates of conversion of radiation absorbed to energy of radioactivity

In addition, the following boundary conditions must be known for a complete solution to the problem:

$\left[H^{i}\right]_0$ initial thermal power being emitted from i^{th} station of absorber

$\left[I^{1}(\tau)\right]_0$ initial radiation power incident upon absorber from source

$\left[P^{i}\right]_0$ one-half of initial power of radioactivity in i^{th} station of absorber for all i

$\left[R^{n+1}(\tau)\right]_0$ radiation power incident upon absorber from side of absorber opposite source

The problem is to find I^{i}, R^{i}, and H^{i} in terms of known quantities. The defining equations incorporating the preceding assumptions are:

$$I^{i+1,es} = t_{\epsilon}^{ies}\, I^{i\epsilon s} + r_{\epsilon}^{ies}\, R^{i+1,\epsilon s} + P^{ies} \tag{1}$$

$$R^{ies} = r_{\epsilon}^{ies}\, I^{i\epsilon s} + t_{\epsilon}^{ies}\, R^{i+1,\epsilon s} + P^{ies} \tag{2}$$

$$H^{i} = h^{i\epsilon\sigma}\left(I^{i}{}_{\epsilon\sigma} + R^{i+1}{}_{\epsilon\sigma}\right) \tag{3}$$

$$P^{ies} = P^{ies\delta}_{\delta} \tag{4a}$$

where the range of δ is from 0 to q and q is the number of generations of daughters.

$$\frac{dP^{ies0}}{d\tau} = \mu^{ies0}_{\epsilon\sigma}\left(I^{i\epsilon\sigma} + R^{i+1,\epsilon\sigma}\right) - \lambda^{ies0} P^{ies0} \tag{4b}$$

and for $d \geqq 1$,

$$\frac{dP^{iesd}}{d\tau} = \kappa^{ies,d-1}_{E\Sigma} P^{iE\Sigma,d-1} - \lambda^{iesd} P^{iesd} \tag{4c}$$

The derivation of equations (4b) and (4c) is presented in the appendix.

In the preceding equations: t^{ies}_{ϵ} is the power-transmission coefficient for radiation of type s and energy ϵ incident upon the i^{th} station, which is transmitted with energy e; r^{ies}_{ϵ} is the power back-scattering coefficient for radiation of type s and energy ϵ incident upon the i^{th} station that is reflected as radiation of energy e; $h^{i\epsilon\sigma}$ is the rate of conversion of energy of nuclear radiation of type σ and energy ϵ in the i^{th} station of the absorber; $\mu^{ies0}_{\epsilon\sigma}$ is the rate of conversion of radiation of type σ and energy ϵ absorbed in the i^{th} station to radioactivity of a parent atom emitting radiation of type s and energy e; λ^{ies0} is the decay constant for the radioactive parent isotopes in the i^{th} station that emits radiation of type s and energy e; $\kappa^{ies,d-1}_{E\Sigma}$ is the rate of conversion of radioactive energy in the i^{th} station of the $(d-1)^{th}$ generation of the isotope emitting radiation of type Σ and energy E to radioactive energy of the d^{th} generation of this isotope that emits radiation of type s and energy e. Capital Greek subscripts are used here to emphasize that only a single type and energy is indicated because the increase in P^{iesd} is due entirely to the decay of one particular isotope.

The interactions of radiations with the absorber that result in change of type are all contained in the radioactivity term P. The inclusion of all the interaction terms into P is physically plausible because a change of radiation type can occur only by means of a nuclear reaction.

In order to solve the problem, the set of equations (1) and (2) are used to express all the I's and R's in terms of the P's, and the boundary conditions $\left[I^1(\tau)\right]_0$ and $\left[R^{n+1}(\tau)\right]_0$. The resulting expressions are then substituted in equation (4b) and the set of equations (4b) and (4c) becomes expressible in the following form:

$$\frac{dP^{ies}}{d\tau} = A^{ies}\,P^{ies} + B^{ies}\left[I^1(\tau)\right]_0 + C^{ies}\left[R^{n+1}(\tau)\right]_0 + D^{ies} \qquad (5)$$

If the indices are omitted, the solution to equation (5) may be written in the matrix form

$$[P] = e^{[A]\tau}\left\{[P]_0 + \int e^{-[A]\tau}\,[F(\tau)]\,d\tau\right\} \qquad (6)$$

where

$$[F(\tau)] = [B]\left[I^1(\tau)\right]_0 + [C]\left[R^{n+1}(\tau)\right]_0 + [D]$$

and

$[A],[B],[C],[D]$ matrices all the elements of which are constants

After all the P's have been found, I^i, R^i, and H^i may be found as in reference 1.

<u>Example</u>. - Assume that:

(1) The absorber is three stations in thickness.

(2) The initial incident radiation is

$$I^{1\;1\;\gamma}\Big|_{\tau=0} = (10 - 5\cos\pi\tau) \text{ roentgen per hour}$$

$$I^{1\;2\;\alpha}\Big|_{\tau=0} = (100 - 50\cos\pi\tau) \text{ roentgen per hour}$$

(3) The alpha-radiation induces a 1 Mev gamma-emitting isotope in the second station; this isotope decays to a 3 Mev neutron emitter which, in turn, decays to a stable isotope.

(4) The 3 Mev neutrons induce 2 Mev gamma-rays in the source of incident radiation.

(5) The other boundary conditions are as follows. At $\tau = 0$:

$$R^{4\,e\,t} = H^{1} = I^{1\,2\,\gamma} = P^{1} = 0$$

(6) The pertinent physical constants are:

$$t^{1\,1\,\gamma} = t^{1\,2\,\gamma} = t^{2\,1\,\gamma} = 0.9$$

$$t^{2\,2\,\gamma} = t^{3\,1\,\gamma} = 0.8$$

$$t^{3\,2\,\gamma} = 0.9$$

$$t^{1\,2\,\alpha} = 0.8$$

$$t^{2\,2\,\alpha} = 0.7$$

$$t^{3\,2\,\alpha} = 0.5$$

$$t^{1\,3\,n} = 0.9$$

$$t^{2\,3\,n} = 0.8$$

$$t^{3\,3\,n} = 0.9$$

$$r^{1\,e\,\gamma} = 0$$

$$r^{1\,2\,\alpha} = 0.1$$

$$r^{2\,2\,\alpha} = 0.2$$

$$r^{3\,2\,\alpha} = 0.3$$

$$r^{1\,3\,n} = 0.1$$

$$r^{2\,3\,n} = 0.2$$

$$r^{3\,3\,n} = 0.1$$

$$\kappa^{2\ 3\ n\ 1} = 0.5 \text{ second}^{-1}$$

$$\lambda^{0\ 2\ \alpha\ 0} = \lambda^{0\ 2\ \gamma\ 0} = 1 \text{ second}^{-1}$$

$$\lambda^{2\ 1\ \gamma\ 0} = 10^{-1} \text{ second}^{-1}$$

$$\lambda^{2\ 3\ n\ 1} = 10 \text{ second}^{-1}$$

$$\mu^{1\ e\ t} = \mu^{3\ e\ t} = \lambda^{1\ e\ t\ \delta} = 0$$

$$\mu^{2}\,{}^{1}_{2}\,{}^{\gamma}_{\alpha} = 10^{-3} \text{ second}^{-1}$$

$$\mu^{0}\,{}^{2}_{3}\,{}^{\gamma}_{n} = 10^{-2} \text{ second}^{-1}$$

All $h = 0$

Solution. - The preceding data and conditions assume no induced alpha-activity. The alpha-radiations may therefore be found simply by solving the simultaneous algebraic equations involved. The methods outlined in reference 1 are suitable and when applied yield:

$$\begin{bmatrix} I^{1\ 2\ \alpha} \\ R^{1\ 2\ \alpha} \end{bmatrix} = \begin{bmatrix} \frac{10}{8} & -\frac{1}{8} \\ \frac{1}{8} & \frac{8}{10}-\frac{1}{80} \end{bmatrix} \begin{bmatrix} \frac{10}{7} & -\frac{2}{7} \\ \frac{2}{7} & \frac{7}{10}-\frac{4}{70} \end{bmatrix} \begin{bmatrix} \frac{10}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{5}{10}-\frac{9}{50} \end{bmatrix} \begin{bmatrix} I^{4\ 2\ \alpha} \\ 0 \end{bmatrix}$$

The final results are:

$$I^{2\ 2\ \alpha} = 0.8296\ I^{1\ 2\ \alpha}$$

$$I^{3\ 2\ \alpha} = 0.6178\ I^{1\ 2\ \alpha}$$

$$I^{4\ 2\ \alpha} = 0.3089\ I^{1\ 2\ \alpha}$$

$$R^{1\ 2\ \alpha} = 0.3364\ I^{1\ 2\ \alpha}$$

$$R^{2\ 2\ \alpha} = 0.2956\ I^{1\ 2\ \alpha}$$

$$R^{3\ 2\ \alpha} = 0.1853\ I^{1\ 2\ \alpha}$$

The neutron intensities may all be expressed in terms of the 3 Mev activity of the second-generation daughter product in the second station. Thus,

$$I^{4\ 3\ n} = P^{2\ 3\ n\ 1}$$

$$I^{3\ 3\ n} = \frac{10}{9} P^{2\ 3\ n\ 1}$$

$$I^{2\ 3\ n} = \frac{1}{9} P^{2\ 3\ n\ 1}$$

$$R^{1\ 3\ n} = P^{2\ 3\ n\ 1}$$

$$R^{2\ 3\ n} = \frac{10}{9} P^{2\ 3\ n\ 1}$$

$$R^{3\ 3\ n} = \frac{1}{9} P^{2\ 3\ n\ 1}$$

The calculations of the intensities of the gamma-rays are more complicated; the methods of Case II (reference 1) are applicable to these computations and yield the following results:

$$I^{2\ 1\ \gamma} = 0.9000\ I^{1\ 1\ \gamma}$$

$$I^{2\ 2\ \gamma} = 0.9000\ I^{1\ 2\ \gamma}$$

$$I^{3\ 1\ \gamma} = P^{2\ 1\ \gamma} + 0.81\ I^{1\ 1\ \gamma}$$

$$I^{3\ 2\ \gamma} = 0.7200\ I^{1\ 2\ \gamma}$$

$$I^{4\ 1\ \gamma} = 0.8000\ P^{2\ 1\ \gamma} + 0.6480\ I^{1\ 1\ \gamma}$$

$$I^{4\ 2\ \gamma} = 0.6480\ I^{1\ 2\ \gamma}$$

$$R^{1\ 2\ \gamma} = R^{3\ 1\ \gamma} = 0$$

$$R^{1\ 1\ \gamma} = 0.9000\ P^{2\ 1\ \gamma}$$

$$R^{2\ 1\ \gamma} = P^{2\ 1\ \gamma}$$

If P's are now determined, the problem will be solved.

$$P^{2\,1\,\gamma} = P^{2\,1\,\gamma\,0}$$

The differential equation satisfied by $P^{2\,1\,\gamma\,0}$ is

$$\frac{dP^{2\,1\,\gamma\,0}}{d\tau} = 10^{-3}\left(I^{2\,2\,\alpha} - R^{3\,2\,\alpha}\right) - \frac{1}{10} P^{2\,1\,\gamma\,0}$$

$$= 10^{-3} \times 1.0149\, I^{1\,2\,\alpha} - \frac{1}{10} P^{2\,1\,\gamma\,0}$$

This equation is integrated and the given boundary conditions are substituted to result in

$$P^{2\,1\,\gamma\,0} = 1.0149 - 0.005136\left(\frac{\cos \pi\tau}{10} + \pi \sin \pi\tau\right)$$

Similarly,

$$\frac{dP^{2\,3\,n\,1}}{d\tau} = \frac{1}{2} P^{2\,1\,\gamma\,0} - 10\, P^{2\,3\,n\,1}$$

If the integrated expression for $P^{2\,1\,\gamma\,0}$ is substituted into this equation, it is found that

$$P^{2\,3\,n\,1} = 0.05075 + 0.0002073 \cos \pi\tau - 0.0007269 \sin \pi\tau$$

Finally,

$$\frac{dP^{0\,2\,\gamma\,0}}{d\tau} = 10^{-2} R^{1\,3\,n} - P^{0\,2\,\gamma\,0}$$

$$= 10^{-2} P^{2\,3\,n\,1} - P^{0\,2\,\gamma\,0}$$

Therefore,

$$P^{0\,2\,\gamma\,0} = 0.0005075 + 0.000002292 \cos \pi\tau - 0.0000000696 \sin \pi\tau$$

All the P's are to be evaluated at $\tau = 1$ day = 86,400 seconds. The values resulting therefrom are substituted into the preceding expressions for the I's and R's. Inasmuch as $I^{1\ 2\ \gamma} = P^{0\ 2\ \gamma} = P^{0\ 2\ \gamma\ 0}$, the following results are readily obtained:

$I^{2\ 2\ \alpha} = 41.48$ roentgens per hour

$I^{3\ 2\ \alpha} = 30.89$ roentgens per hour

$I^{4\ 2\ \alpha} = 15.45$ roentgens per hour

$R^{1\ 2\ \alpha} = 16.82$ roentgens per hour

$R^{2\ 2\ \alpha} = 14.78$ roentgens per hour

$R^{3\ 2\ \alpha} = 9.265$ roentgens per hour

$I^{2\ 3\ n} = 5.662 \times 10^{-3}$ roentgen per hour

$I^{3\ 3\ n} = 5.662 \times 10^{-2}$ roentgen per hour

$I^{4\ 3\ n} = 5.0957 \times 10^{-2}$ roentgen per hour

$R^{1\ 3\ n} = 5.0957 \times 10^{-2}$ roentgen per hour

$R^{2\ 3\ n} = 5.672 \times 10^{-2}$ roentgen per hour

$R^{3\ 3\ n} = 5.672 \times 10^{-3}$ roentgen per hour

$I^{2\ 1\ \gamma} = 4.500$ roentgens per hour

$I^{2\ 2\ \gamma} = 4.588 \times 10^{-4}$ roentgen per hour

$I^{3\ 1\ \gamma} = 5.064$ roentgens per hour

$I^{3\ 2\ \gamma} = 3.670 \times 10^{-4}$ roentgen per hour

$I^{4\ 1\ \gamma} = 4.052$ roentgens per hour

$I^{4\ 2\ \gamma} = 3.303 \times 10^{-4}$ roentgen per hour

$R^{1\ 1\ \gamma} = 9.130 \times 10^{-1}$ roentgen per hour

$R^{1\ 2\ \gamma} = 0$ roentgen per hour

$R^{2\ 1\ \gamma} = 1.015$ roentgens per hour

$$R^{2\ 2\ \gamma} = 0 \text{ roentgen per hour}$$

$$R^{3\ 1\ \gamma} = 0 \text{ roentgen per hour}$$

$$R^{3\ 2\ \gamma} = 0 \text{ roentgen per hour}$$

DISCUSSION

Many induced radioactive isotopes decay to one or more intermediate radioactivities before becoming stable atoms. Because, in many cases, the radiations from these intermediate isotopes are more harmful than the original activity, the entire gamut of radiations must be accounted for.

In addition, the possibility exists that radioactivity from the absorber will induce a radioactivity in the source itself that differs from the original radiation. Theoretically, this consideration may be as important as the one in the preceding paragraph because the decay of almost all the induced radioactivities can be predicted from the available data, whereas the effect of the back-reflected radiation on the source is not as readily obtained. Furthermore, any change in the source results in possible subsequent changes in the radiation at the various stations of the absorber.

The methods of references 1 and 2 have been generalized to include the preceding two considerations. The main limitations of the method are:

(1) The assumption of normal incidence at all stations of the absorber

(2) The assumption of stations of the absorber as plane parallel surfaces

(3) For any practical problem, a large number of physical constants are required for solution. A great many of the required constants are not yet available in the literature. Nevertheless, the method should currently prove useful for obtaining approximate results in absorption problems in which reasonable estimates of the constants may be made.

SUMMARY OF RESULTS

The problem of determining transmission and absorption of time-dependent radiation through thick absorbers has now been generalized to include the following time-dependent cases:

(a) Radiation is normal to an absorber of which the stations are plane parallel surfaces.

(b) An arbitrary number of energies and types of radiation are involved in the source and in the radioactivities of the absorber.

(c) Radioactive isotopes decay to stable atoms in a finite number of steps.

(d) The source is an arbitrary function of time and its activity may be affected by the back-reflected radiation from the absorber.

Thus, the amount of heat and the activities in various portions of an absorber can be calculated for problems of complexity embodying cases (a), (b), (c), and (d).

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, December 9, 1949.

APPENDIX - DERIVATION OF EQUATIONS 4(b) AND 4(c)

Let N_a^{iesd} equal the number of atoms in the i^{th} station of the absorber in the d^{th} generation that are capable of emitting radiation of type s and energy e and let the number of particles of radiation be denoted by N^{iesd}. The following equation may then be written:

$$\frac{d}{d\tau} N^{iesd} = C^{iesd} \frac{d}{d\tau} N_a^{iesd} \tag{A1}$$

where C^{iesd} is a constant that accounts for the possibility that in the decay process an atom may emit more than one radioactive particle of type s and energy e.

The following two equations define the rate of change of N_a^{iesd}:

$$\frac{d}{d\tau} N_a^{ies0} = \frac{a^{ies}_{\epsilon\sigma}}{E^i_{SR_0}} \left(I^{i\epsilon\sigma} + R^{i+1,\epsilon\sigma}\right) - \lambda^{ies0} N_a^{ies0} \tag{A2}$$

where the 0 generation is defined as the original induced member of the radioactive decay series. The term $a^{ies}_{\epsilon\sigma}$ is the fraction of the energy of radiation of type σ and energy ϵ absorbed in forming radioactive atoms of N_a^{ies0} and $E^i_{SR_0}$ is the energy required to change a stable atom to the radioactive parent of the series.

For $d \geqq 1$

$$\frac{d}{d\tau} N_a^{iesd} = \lambda^{ies,d-1} N_a^{ies,d-1} - \lambda^{iesd} N_a^{iesd} \tag{A3}$$

Now

$$P^{ies} = P^{ies\delta}_{\delta} \tag{A4}$$

where the range of δ is 0 to q.

However,

$$P^{iesd} = E^i_{R_dR_{d+1}} C^{iesd} \lambda^{iesd} N_a^{iesd} \tag{A5}$$

where $E^{i}_{R_dR_{d+1}}$ is the energy emitted when a radioactive atom of the d^{th} generation changes to a radioactive atom of the $(d+1)^{th}$ generation. Therefore,

$$\frac{dP^{iesd}}{d\tau} = E^{i}_{R_dR_{d+1}} C^{iesd} \lambda^{iesd} \frac{d}{d\tau} N_a{}^{iesd} \tag{A6}$$

If equations (A2) and (A3) are substituted into equation (A6), there results:

$$\frac{dP^{ies0}}{d\tau} = E^{i}_{R_0R_1} C^{ies0} \lambda^{ies0} \left[\frac{a^{ies}_{\epsilon\sigma} \left(I^{i\epsilon\sigma} + R^{i+1,\epsilon\sigma}\right)}{E^{i}_{SR_0}} - \lambda^{ies0} N_a{}^{ies0}\right] \tag{A7}$$

and for $d \geqq 1$,

$$\frac{dP^{iesd}}{d\tau} = E^{i}_{R_dR_{d-1}} C^{iesd} \lambda^{iesd} \left[\lambda^{ies,\,d-1} N_a{}^{ies,\,d-1} - \lambda^{iesd} N_a{}^{iesd}\right] \tag{A8}$$

Finally, if equation (A5) is substituted into equations (A7) and (A8), they may be written

$$\frac{dP^{ies0}}{d\tau} = \mu^{ies0}_{\epsilon\sigma} \left(I^{i\epsilon\sigma} + R^{i+1,\epsilon\sigma}\right) - \lambda^{ies0} P^{ies0} \tag{A9}$$

and for $d \geqq 1$,

$$\frac{dP^{iesd}}{d\tau} = \kappa^{ies,\,d-1}_{E\Sigma} P^{iE\Sigma,\,d-1} - \lambda^{iesd} P^{iesd} \tag{A10}$$

where $\mu^{ies0}_{\epsilon\sigma}$ is

$$\frac{E^{i}_{R_0R_1} C^{ies0} \lambda^{ies0} a^{ies}_{\epsilon\sigma}}{E^{i}_{SR_0}}$$

and $\kappa_{E\Sigma}^{ies,\ d-1}$ is

$$\frac{E^{i}_{R_dR_{d+1}}\ C^{iesd}\ \lambda^{iesd}}{E^{i}_{R_{d-1}R_d}\ C^{ies,\ d-1}}$$

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