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TECHNICAL NOTE 2527

A VELOCITY-CORRECTION FORMULA FOR THE CALCULATION OF TRANSONIC MACH NUMBER DISTRIBUTIONS OVER DIAMOND-SHAPED AIRFOILS

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A VELOCITY-CORRECTION FORMULA FOR THE CALCULATION OF TRANSONIC MACH NUMBER DISTRIBUTIONS OVER DIAMOND-SHAPED AIRFOILS

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SUMMARY

A velocity-correction formula is proposed for the purpose of calculating, from the known Mach number distribution for a diamond-shaped airfoil at a stream Mach number of 1.0, Mach number distributions on the same airfoil at speeds from a Mach number of about 0.8 to shock-attachment Mach number. The time required to calculate these additional Mach number distributions is small in comparison with the time required by rigorous methods. The accuracy of the results for stream Mach numbers near 1.0 is of the same order as the accuracy of the known Mach number distribution. Moreover, the results tend to become exact as the stream Mach number is increased toward that for shock attachment. An expression for the rate of change of local Mach number with stream Mach number is derived and an explicit equation for the drag coefficient as a function of stream Mach number and thickness ratio is given.

INTRODUCTION

The pressure distribution for a diamond-shaped airfoil at a stream Mach number of 1.0 has been calculated by Guderley and Yoshihara (reference 1). Calculations for a similar airfoil at four speeds between Mach number 1.0 and the shock-attachment Mach number have been performed by Vincenti and Wagoner (reference 2). According to reference 2, similar calculations have been made by Cole at slightly subsonic speeds. These rigorous results combined with reliable experimental results provide sufficient information for checking the accuracy of an approximate velocity-correction formula proposed for calculating Mach number distributions on a diamond-shaped airfoil. The concepts involved should facilitate the calculations for other shapes.

The justification for the proposed velocity-correction formula is based upon its good agreement with existing rigorous calculations. Moreover, it is in accord with the general transonic similarity rule,

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and with the special form of that rule for a stream Mach number of 1.0. Also, it agrees with Guderley's result that the local Mach numbers are constant for small departures of the stream Mach number from 1.0.

The present paper gives the derivation of the method of calculation and its application for the determination of the Mach number distributions over a 10-percent-thick diamond-shaped airfoil in the Mach number range $0.80 \leq M_\infty \leq 1.30$. The results are compared with the earlier results of Vincenti and Wagoner. A brief discussion is given of the movement of the shock wave along the airfoil surface as the stream Mach number is decreased below 1.0.

ANALYSIS AND DISCUSSION

The velocity-correction formula to be presented for diamond-shaped airfoils is based upon the combination of two heretofore unrelated concepts. One concept is obtained from the special form of the transonic similarity rule (reference 3) when the free stream is sonic and the other from an examination of the transonic approximation for Prandtl-Meyer flow.

Transonic similarity rule for sonic stream.- The form of the transonic similarity rule for a sonic stream is derived in the appendix and the result obtained is

$$M^2 - 1 = \delta^{2/3} C(x) \tag{1}$$

where M is the local Mach number, $C(x)$ is a function depending upon the body shape, and δ is a parameter (such as the airfoil thickness ratio or the angle of flow deflection for Prandtl-Meyer flow) used to differentiate bodies of the same family. The concept to be obtained from the similarity rule is the role played by $C(x)$. For the purpose of this paper, it is sufficient to consider the flow past the nose of a wedge and the Prandtl-Meyer expansion around a convex corner. The flow deflection δ is taken as positive for the wedge (compression) and negative for the Prandtl-Meyer corner (expansion). The stream is sonic. Equation (1) may be applied to both of these flows but $C(x)$ will be different for the two cases. For Prandtl-Meyer flow $C(x)$ is a constant and for the flow past the wedge $C(x)$ corresponds to the Mach number distribution given in reference 1. Thus, $C(x)$ determines the shape of the Mach number distribution along the surface. An illustration of equation (1) is presented in figure 1 to show the role of $C(x)$. It should be noted (fig. 1) that all curves originate at the point $M = 1$, $\delta = 0$. As δ is varied from zero, $M^2 - 1 \propto \delta^{2/3}$ for all points on either surface.

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Transonic approximation for Prandtl-Meyer flow.- The concept of the role played by δ^*, the amount the free stream must be deflected to reach sonic velocity, may be illustrated by considering Prandtl-Meyer flow. For Prandtl-Meyer flow, in order to reach sonic velocity the stream must be deflected through an angle δ^*, given approximately by (from reference 3 with $2(M_\infty - 1)$ replaced by $(M_\infty^2 - 1)$):

$$\delta^* \approx \left(\frac{M_\infty^2 - 1}{C}\right)^{3/2} \tag{2}$$

where M_∞ is the stream Mach number and $C = \left[\frac{3}{2}(\gamma + 1)\right]^{2/3}$, γ being the ratio of specific heats. The value of δ^* is taken to be positive.

Inasmuch as, for Prandtl-Meyer flow, the flow deflection may be considered to consist of more than one deflection (fig. 2), the expression for the local Mach number may be written:

$$M^2 - 1 = C\left|\delta - \delta^*\right|^{2/3} \tag{3}$$

where δ is taken as the flow deflection from the free-stream direction. From equation (2), $\delta^* = 0$ when $M_\infty = 1$, and, for this case, equation (3) reduces to the transonic similarity rule. A plot of equation (3) is shown in figure 3. It is important to note that the curve for $M_\infty > 1$ is the same as the curve for $M_\infty = 1$ except that the origin of the $M_\infty > 1$ curve has been shifted to the point δ^*. This result shows that, for Prandtl-Meyer flow, the expression for $M^2 - 1$ is of the same form as the transonic similarity rule for a sonic stream, providing δ is measured from the δ^* direction.

Velocity-correction formula for diamond-shaped airfoil.- The fundamental assumption made to develop a velocity-correction formula for the diamond-shaped airfoil is that the concept of $C(x)$ obtained from the similarity rule for a sonic stream and the concept of δ^* illustrated by Prandtl-Meyer flow may be combined in the form

$$M^2 - 1 = C(x)\left|\delta - \delta^*\right|^{2/3} \tag{4}$$

It should be emphasized that equation (4) is based upon an assumption and is not claimed to be rigorous. Typical plots of equation (4) are given in figure 4.

The $C(x)$ used for the diamond-shaped airfoil is obtained from the solution given in reference 1 for this airfoil for $M_\infty = 1$. Thus,

$$C(x) = \left(\frac{M^2 - 1}{\delta^{2/3}}\right)_{M_\infty = 1} \tag{5}$$

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If the stream Mach number is greater than 1.0, the flow must be decelerated (compressed through an angle δ^*) to reach sonic velocity. In the approximation of the transonic small-disturbance theory, the compression of the flow caused by a shock wave is isentropic. Thus, to the same order of approximation, δ^* may be determined from either Prandtl-Meyer flow or shock-wave relations. It can be shown that δ^* computed from shock-wave relations obeys the transonic similarity rule; that is,

$$\frac{M_\infty{}^2 - 1}{\delta^{*2/3}} = K^* \tag{6}$$

where K^* is a constant. Note that equation (6) is very similar to equation (2) which is valid for Prandtl-Meyer flow. The values of δ^* used in the computations were taken from shock-wave tables to insure that the results obtained by the use of equation (4) would fair smoothly into those given by shock-expansion theory. The angle δ^*, based on shock-wave relations, is shown as a function of M_∞ in figure 5.

The concept of δ^* previously presented appears to have physical significance only for stream Mach numbers greater than 1.0. In order to calculate flows with a stream Mach number less than 1.0, the curve for δ^* was extrapolated by assuming that δ^* was an odd function of $M_\infty{}^2 - 1$.

Conditions satisfied by proposed velocity-correction formula.- Equation (4) contains the transonic similarity rule as a special case. Eliminating δ^* from equations (4) and (6) leads to the following equation:

$$M^2 - 1 = C(x)\delta^{2/3}\left[1 - \frac{\left(M_\infty{}^2 - 1\right)^{3/2}}{\delta K^{*3/2}}\right]^{2/3} \tag{7}$$

When $M_\infty = 1$, the local $M^2 - 1$ distribution is proportional to $\delta^{2/3}$ as required by the transonic similarity rule. Moreover, when M_∞ and δ are varied so that $\frac{\left(M_\infty{}^2 - 1\right)^{3/2}}{\delta}$ remains constant, the expression for the local Mach number at the surface becomes

$$M^2 - 1 = C(x)\delta^{2/3}F\left[\frac{\left(M_\infty{}^2 - 1\right)^{3/2}}{\delta}\right]$$

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This expression is in agreement with the corresponding result of the transonic similarity rule.

Equation (4) is also in accord with the result of Guderley (reference 4) that $\left(\frac{dM}{dM_\infty}\right)_{M_\infty=1} = 0$ for all shapes having finite thickness. Thus, from equation (7), at a fixed position x,

$$\frac{dM}{dM_\infty} = \left(\frac{C(x)}{K^*}\right)^{3/2}\frac{M_\infty}{M}\left|\frac{M_\infty^2 - 1}{M^2 - 1}\right|^{1/2} \tag{8}$$

and hence

$$\left(\frac{dM}{dM_\infty}\right)_{M_\infty=1} = 0$$

The result of Guderley that $\left(\frac{dM}{dM_\infty}\right)_{M_\infty=1} = 0$ has been used by Vincenti and Wagoner (reference 2) to obtain the result that $\left(\frac{dc_d}{dM_\infty}\right)_{M_\infty=1} = -\frac{2}{\gamma + 1}\left(c_d\right)_{M_\infty=1}$ where c_d is the drag coefficient. The method of the present paper gives this same slope of the drag curve at $M_\infty = 1$.

At a speed slightly above that for shock-wave attachment, the local Mach number over the front half of the diamond-shaped airfoil becomes sonic. Equation (8) gives $\left(\frac{dM}{dM_\infty}\right)_{M=1} = \infty$ for this condition, whereas shock-wave tables indicate that $\left(\frac{dM}{dM_\infty}\right)_{M=1}$ is extremely large but finite. This difference between infinity and a very large quantity is of no practical importance in the present considerations.

If the proposed velocity-correction formula is to yield good results throughout the speed range from $M_\infty = 1$ to the speed for shock attachment, the results should fair smoothly into those for purely supersonic flow with attached shock waves and also into those for subsonic stream Mach numbers. For the case of flow with attached shock waves, δ^* is greater than δ and the supersonic local Mach numbers may be estimated by expanding around a Prandtl-Meyer corner from the δ^* direction to the δ direction by use of equation (3) or, preferably, the exact

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expression for Prandtl-Meyer flow for the deflection $\delta^* - \delta$. Local Mach number distributions were calculated by both the present method and by the exact shock-expansion method (reference 5) for a 10-percent-thick diamond-shaped airfoil in the Mach number range $1.28 \leqq M_\infty \leqq 1.55$. The maximum difference in the local Mach numbers predicted by the two methods in this speed range was less than 0.01. This agreement indicates that the proposed velocity-correction formula not only fairs smoothly into the attached-shock calculations but may also be used as a means for calculating such flows with good accuracy.

The calculation for the rear half of the diamond-shaped airfoil at subsonic speeds is beyond the scope of this paper because of the presence of a shock wave on the rear surface. The location and strength of this shock wave seem to be strongly influenced by viscous effects.

The general shape of the curve of drag coefficient against Mach number calculated by means of equation (4) is very similar to the "possible interpolated" drag-coefficient curve given in reference 2 except near attachment Mach number.

The application of the method to the front and rear surfaces of the airfoil are treated separately.

Application to front surface of diamond-shaped airfoil.- In the sign convention adopted, δ^* is always positive $(M_\infty \geq 1)$. For a wedge, the flow is subsonic if $\delta - \delta^* > 0$ and supersonic if $\delta - \delta^* < 0$. When the local Mach numbers are subsonic, the $C(x)$ given by equation (5) is used. When the local Mach numbers are supersonic, Prandtl-Meyer concepts are employed as previously discussed.

Figure 5 shows the angle δ^* for sonic flow for a wedge as a function of stream Mach number as calculated from shock-wave relations. This curve is more accurate than that given by equation (6) since K^* is not quite constant. The difference between δ^* and the local slope δ' is a measure of the deviation of $M^2 - 1$ from zero. Figure 6 shows the variation of the $M^2 - 1$ distributions for the front half of the airfoil with the stream Mach number. These results were obtained by multiplying the $M^2 - 1$ distribution for $M_\infty = 1$ (reference 1) by the factor $\left(\frac{\delta - \delta^*}{\delta}\right)^{2/3}$ where δ is now considered the wedge thickness ratio. Figure 7 presents the corresponding local Mach number distributions.

Reexamination of equation (4) indicates that the proposed velocity-correction formula is equivalent to the assumption that the local Mach number distribution over the front surface of a diamond-shaped airfoil

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is the same as that for a thinner airfoil in a sonic stream; that is, $\delta - \delta^*$ may be considered as the effective airfoil thickness δ_e in a sonic stream. Figure 8 shows, for example, that a 10-percent-thick wedge at a stream Mach number of 1.167 has the same effective thickness as a 5-percent-thick wedge in a sonic stream. A 10-percent-thick wedge at $M_\infty = 1.278$ will have the same Mach number distribution as a flat plate in a sonic stream; namely, $M = 1$.

Application to rear surface of diamond-shaped airfoil.- For all Mach numbers below shock attachment, the local Mach number remains 1.0 just ahead of the corner, and therefore the Prandtl-Meyer expansion at the corner remains constant. The Mach number distribution on the rear is considered to result from a Prandtl-Meyer expansion and the reflected compression waves from the sonic line. A sketch illustrating this influence on the rear surface is given as figure 9. The subsonic influence (due to reflected compression waves from the sonic line) must decrease to zero when the local Mach number on the front becomes sonic. The subsonic influence on the rear is assumed to vary with stream Mach number in the same manner as the subsonic flow over the front and $C(x)$ for the rear is proportional to

$$(M^2 - 1)_{PM} - (M^2 - 1)_{M_\infty = 1}$$

where $(M^2 - 1)_{PM}$ corresponds to the Mach number obtained by a Prandtl-Meyer expansion through an angle 2δ. In other words, the subsonic influence on the rear for $M_\infty = 1$ given in reference 1 is multiplied by the factor $\left(\frac{\delta - \delta^*}{\delta}\right)^{2/3}$ to obtain the subsonic influence at other Mach numbers. The corresponding $M^2 - 1$ and M distributions are shown in figures 6 and 7, respectively.

Figure 10 gives the pressure distributions, based upon the exact formula for the pressure coefficient, corresponding to the Mach number distributions $(0.8 \leqq M_\infty \leqq 1.3)$ of figure 7. Note that the pressure-distribution curves form a confused pattern, whereas the Mach number distribution curves are more uniform. This behavior indicates that studies of transonic flow phenomena should be interpreted in terms of Mach number rather than pressure coefficient.

Figure 11 presents a comparison of the Mach number distributions calculated by the present method with those calculated in reference 2 for a 10-percent-thick diamond-shaped airfoil. The Mach number distributions

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obtained in reference 2 are presented as two curves. This choice arose in reference 2 because there was some question whether $M^2 - 1$ should be replaced by $2(M - 1)$. For the 10-percent-thick diamond-shaped airfoil, the Mach number is known to be exactly 1.0 just ahead of the corner and 1.485 just after the Prandtl-Meyer expansion at the corner for inviscid flow.

Drag coefficient.- Figure 12 shows the variation of drag coefficient with stream Mach number for the 10-percent-thick diamond-shaped airfoil. Separate curves are presented for the front and rear surfaces in order to emphasize the change in relative importance of the drag contributed by the two parts as the Mach number varies. An attempt has been made to reproduce faithfully the peculiarities and breaks in the curve, especially as the local Mach number on the front surface becomes sonic. The present results fair smoothly into the exact supersonic results; in fact, the supersonic results can be obtained to a high degree of accuracy by the present method.

Figure 13 presents a comparison of the drag curve given by the proposed method with the "possible interpolated curve" of reference 2 for a 7.87-percent-thick diamond-shaped airfoil. The "boxes" shown in figure 13 represent the choice of drag coefficients which are due to the choice of pressure coefficients and stream Mach numbers presented in reference 2. The two curves are very similar except near attachment Mach number. Reference 2 shows a fairly small slope for the drag curve in this region, whereas shock-expansion theory and the present method indicate that this slope becomes extremely large.

By use of the present concepts an explicit expression may be derived for the drag coefficient of a diamond-shaped airfoil for $M_\infty \geqq 1$. The drag coefficient may be expressed in terms of the airfoil thickness ratio δ and average pressure coefficient $\overline{P}$ as follows:

$$c_d = \delta\left(\overline{P}_{front} - \overline{P}_{rear}\right)$$

Except in the equations to follow, the exact pressure relations have been used throughout this paper. If the exact expression for the pressure coefficient were used in the analysis to follow, the formula obtained for the drag coefficient would yield the drag curve shown in figure 12. However, for the sake of simplicity, the following approximate relation obtained from Bernoulli's equation and limited to Mach numbers in the neighborhood of unity is used:

$$P \approx -\frac{2}{\gamma + 1}\left[\overline{(M^2 - 1)} - \overline{(M_\infty^2 - 1)}\right]$$

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Then

$$c_d \approx \frac{2\delta}{\gamma + 1}\left[\overline{(M^2 - 1)_{PM}} - \overline{C}(x)_R(\delta - \delta^*)^{2/3} + \overline{C}(x)_F(\delta - \delta^*)^{2/3}\right]$$

where $\overline{C}(x)_F = \frac{\overline{M^2 - 1}}{\delta^{2/3}}$ for the front surface at $M_\infty = 1$ and

$$\overline{C}(x)_R = \frac{(M^2 - 1)_{PM} - \overline{(M^2 - 1)_R}}{\delta^{2/3}}$$ for the rear surface at $M_\infty = 1$.

For Prandtl-Meyer flow, $M^2 - 1 = \left(3\,\frac{\gamma + 1}{2}\,\delta\right)^{2/3}$, and replacing $\overline{C}(x)_F$ and $\overline{C}(x)_R$ in terms of the drag coefficient for $M_\infty = 1$ leads to the following approximate expression for the drag coefficient:

$$c_d = \frac{2\delta^{5/3}}{(\gamma + 1)^{1/3}}3^{2/3} + \left[c_{d_0} - \frac{2\delta^{5/3}3^{2/3}}{(\gamma + 1)^{1/3}}\right]\left[1 - \frac{1}{\delta}\left(\frac{M_\infty{}^2 - 1}{K^*}\right)^{3/2}\right]^{2/3} \tag{9}$$

where c_{d_0} is the drag coefficient for $M_\infty = 1$.

Equation (9) is not sufficiently accurate for general use because of the rather severe limitations of the approximate pressure formula but is useful for illustrating the variation of drag coefficient with stream Mach number.

Equation (9) separates the drag coefficient for a diamond-shaped airfoil into two terms: (1) a supersonic term contributed by the Prandtl-Meyer corner which remains constant and (2) a subsonic term which decreases to zero in the vicinity of shock-attachment Mach number.

At stream Mach numbers less than 1.0, the drag contributed by the supersonic term is no longer constant but is decreased because a shock wave moves forward along the rear surface. This forward movement decreases the extent of the supersonic region which is gradually replaced by the symmetrical subsonic pressure distribution. The existence of a shock wave on the rear surface at these speeds makes the calculation of the pressure distribution very difficult since the location of the shock wave is strongly affected by boundary-layer separation. For this reason, the total drag coefficients in the subsonic range are not presented.

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CONCLUDING REMARKS

A velocity-correction formula has been proposed for the purpose of calculating, from the known Mach number distribution for a diamond-shaped airfoil at a stream Mach number of 1.0, Mach number distributions on the same airfoil at speeds from a Mach number of about 0.8 to shock-attachment Mach number. The formula exhibits the following properties:

(1) The formula contains the general transonic similarity rule as well as the special form for a sonic stream.

(2) The local Mach number over the front surface of the airfoil is 1.0 at the correct stream Mach number.

(3) The drag coefficient is that given by Guderley and Yoshihara when $M_\infty = 1$.

(4) The rate of change of drag coefficient with stream Mach number at $M_\infty = 1$ is the same as that given by Vincenti and Wagoner.

(5) Calculated Mach number distributions are in agreement with calculations based on shock-expansion theory above shock-wave attachment Mach numbers.

(6) The formula reduces to the proper form for Prandtl-Meyer flow.

Pressure and Mach number distributions for a 10-percent-thick diamond-shaped airfoil are presented for the Mach number range $0.8 \leqq M_\infty \leqq 1.3$. The variation of local Mach number distribution with stream Mach number is regular, whereas the pressure-distribution curves form a confused pattern. This behavior indicates that studies of transonic flow phenomena should be interpreted in terms of Mach number rather than pressure coefficient.

By the use of the velocity-correction formula proposed in the present paper, an approximate expression is derived for the drag coefficient of a diamond-shaped airfoil as a function of stream Mach number and thickness ratio. The drag coefficient is separated into two terms: (1) a supersonic term contributed by the Prandtl-Meyer corner and (2) a subsonic term which decreases to zero in the vicinity of attachment Mach number.

In its present form, the proposed velocity-correction formula appears to be applicable only to diamond-shaped airfoils at transonic

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speeds and will probably require modification if it is to be applied to curved airfoils. Presumably, the present concepts may be used to obtain corresponding relations for a cone.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX

DERIVATION OF TRANSONIC SIMILARITY RULE FOR A SONIC STREAM

The results presented in reference 3 have shown that, when a perturbation velocity potential ϕ is defined by

$$\left.\begin{aligned} u &= a^* + \phi_x \\ v &= \phi_y \end{aligned}\right\} \tag{A1}$$

where a^* is the velocity of sound for $M = 1$, and x and y are Cartesian coordinates the transonic approximation to the differential equation for the flow of a compressible fluid may be written as

$$\frac{\gamma + 1}{a^*} \phi_x \phi_{xx} - \phi_{yy} = 0 \tag{A2}$$

In addition, when $2(M - 1)$ is replaced by $M^2 - 1$,

$$\frac{\gamma + 1}{a^*} \phi_x = M^2 - 1 \tag{A3}$$

Also, from reference 3, the appropriate boundary conditions for equation (2) when the stream is sonic are

$$\frac{\phi_y(x,0)}{a^*} = \delta h\left(\frac{x}{c}\right) \tag{A4}$$

and at infinity

$$\phi_x = \phi_y = 0 \tag{A5}$$

where δ denotes the airfoil thickness ratio, c is the chord, and h is a function describing the body shape.

A solution $\phi_1(x_1, y_1)$ of equations (A2), (A4), and (A5) is assumed to be known. Two flows are considered to be similar if a solution $\phi_2(x_2, y_2)$ satisfying equations similar to (A2), (A4), and (A5) can be

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related to ϕ_1. In particular, similarity will exist providing A, B, C can be so determined that

$$\left.\begin{aligned} \phi_2 &= A\phi_1 \\ x_2 &= Bx_1 \\ y_2 &= Cy_1 \end{aligned}\right\} \qquad \text{(A6)}$$

if, in addition, ϕ_1 and ϕ_2 both satisfy equation (A2), and

$$\left.\begin{aligned} \phi_{1_{y_1}}(x_1, 0) &= \delta_1 h\left(\frac{x_1}{c_1}\right) \\ \phi_{2_{y_2}}(x_2, 0) &= \delta_2 h\left(\frac{x_2}{c_2}\right) \end{aligned}\right\} \qquad \text{(A7)}$$

and at infinity

$$\phi_{1_{y_1}} = \phi_{1_{x_1}} = \phi_{2_{y_2}} = \phi_{2_{x_2}} = 0$$

With full generality, c_1 may be taken equal to c_2 since both bodies are in the flow field which extends infinitely far in every direction. If $c_1 = c_2$, then $B = 1$. Flows involving different values of γ need not be considered in this analysis, which is primarily concerned with the result for a particular gas - namely, air.

If both ϕ_1 and ϕ_2 are to satisfy equation (A2), it is easily found that

$$AC^2 = 1 \qquad \text{(A8)}$$

From equations (A6) and (A7)

$$\phi_{1_{y_1}}(x,0) = \delta_1 h\left(\frac{x}{c}\right) = \frac{C}{A}\,\phi_{2_{y_2}}(x,0) = \frac{C}{A}\,\delta_2 h\left(\frac{x}{c}\right)$$

from which

$$\frac{C}{A} = \frac{\delta_1}{\delta_2} \qquad \text{(A9)}$$

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From equations (A8) and (A9),

$$C = \left(\frac{\delta_1}{\delta_2}\right)^{1/3} \tag{A10}$$

$$A = \left(\frac{\delta_2}{\delta_1}\right)^{2/3} \tag{A11}$$

From equation (A3)

$$\phi_{2_x}(x,0) = \frac{a^*}{\gamma + 1}\left(M_2{}^2 - 1\right) = A\phi_{1_x}(x,0) = A\,\frac{a^*}{\gamma + 1}\left(M_1{}^2 - 1\right) \tag{A12}$$

and, from equations (A11) and (A12),

$$M_2{}^2 - 1 = \delta_2{}^{2/3}\,\frac{M_1{}^2 - 1}{\delta_1{}^{2/3}}$$

Inasmuch as ϕ_1 was assumed to be the known solution, the value of $\frac{M_1{}^2 - 1}{\delta_1{}^{2/3}}$ at the surface may be replaced by $C(x)$, a known function, and the transonic similarity rule for a sonic stream yields the result that, on the surface,

$$M^2 - 1 = \delta^{2/3}\,C(x) \tag{A13}$$

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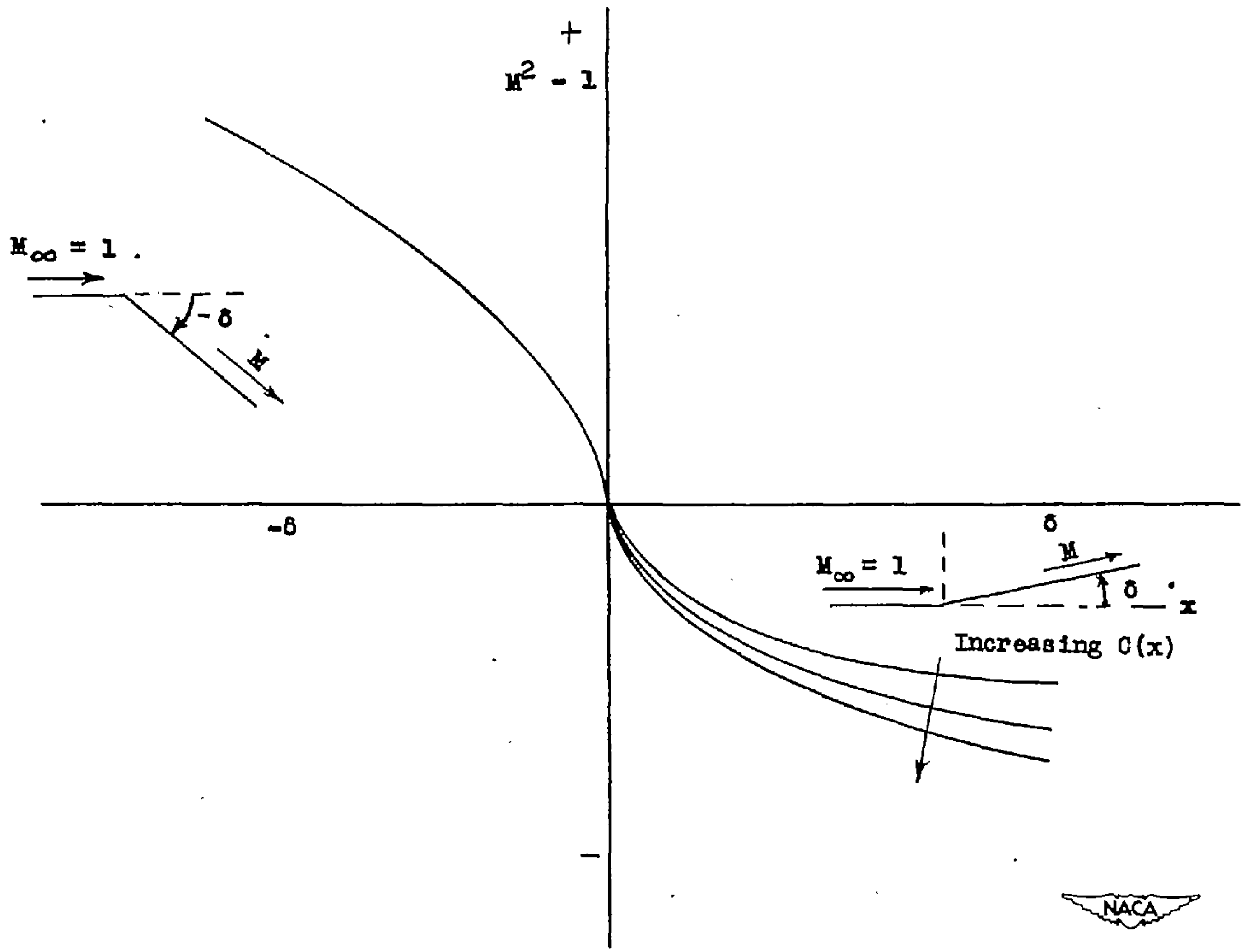

Figure 1.- Transonic similarity rule plotted for $M_\infty = 1$ to illustrate role of $C(x)$.

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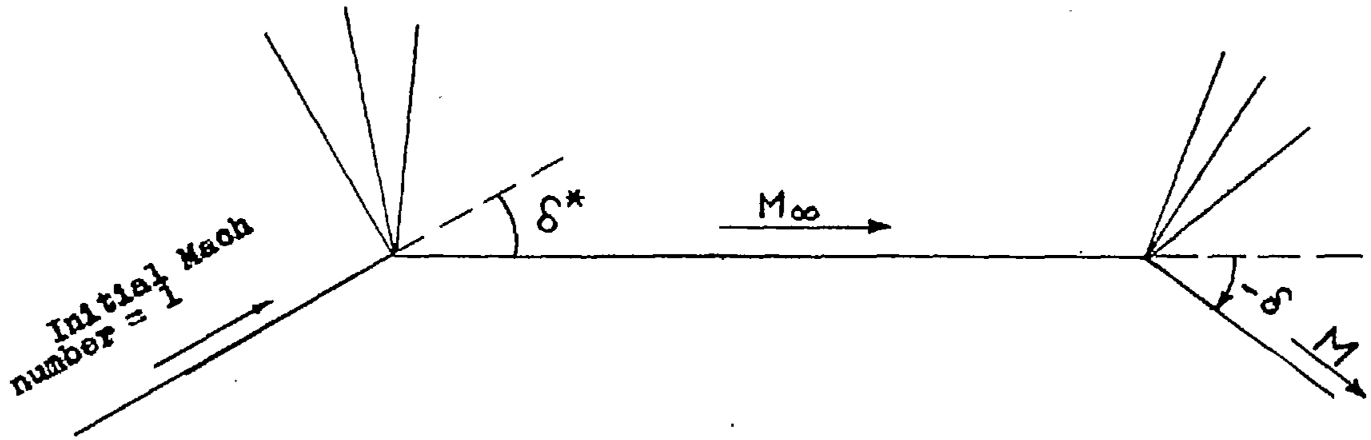

Figure 2.- Introduction of concept of free stream into Prandtl-Meyer flow.

$$M^2 - 1 = C|\delta - \delta^*|^{2/3}; \quad \delta^* = \left(\frac{M_\infty^2 - 1}{C}\right)^{3/2}.$$

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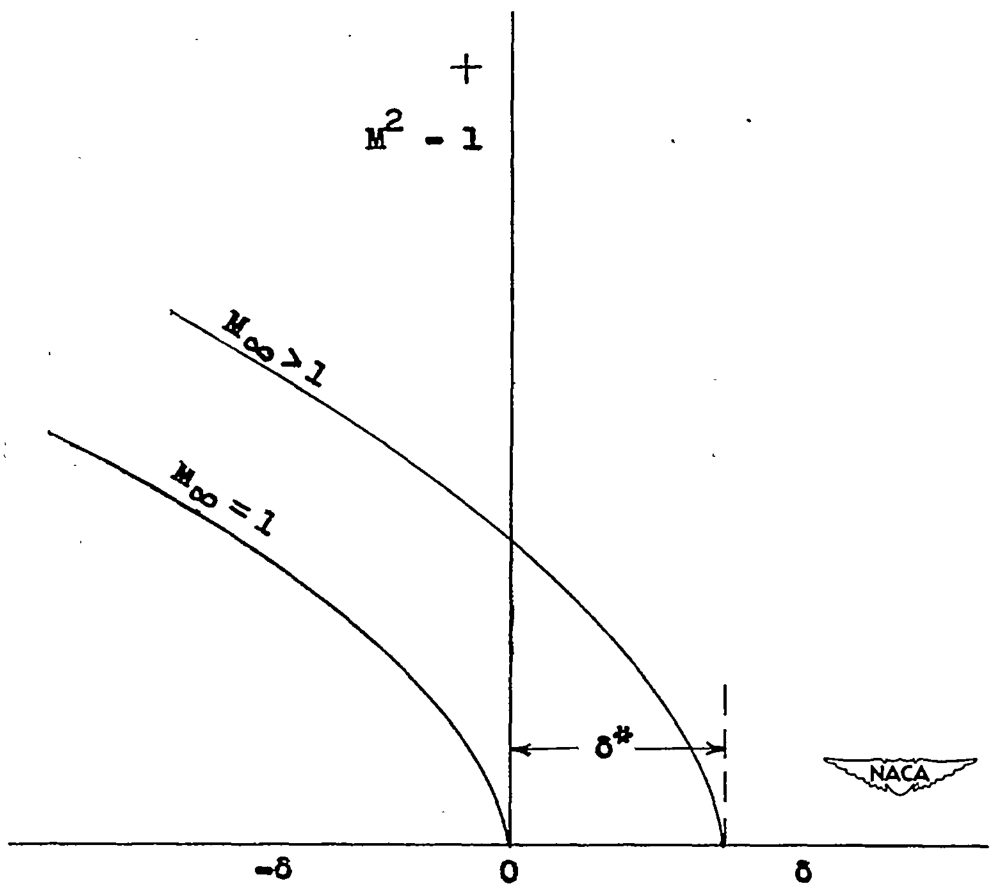

Figure 3.- Prandtl-Meyer flow plotted to illustrate role of δ*.

$$M^2 - 1 = C|\delta - \delta^*|^{2/3}; \quad \delta^* = \left(\frac{M_\infty^2 - 1}{C}\right)^{3/2}.$$

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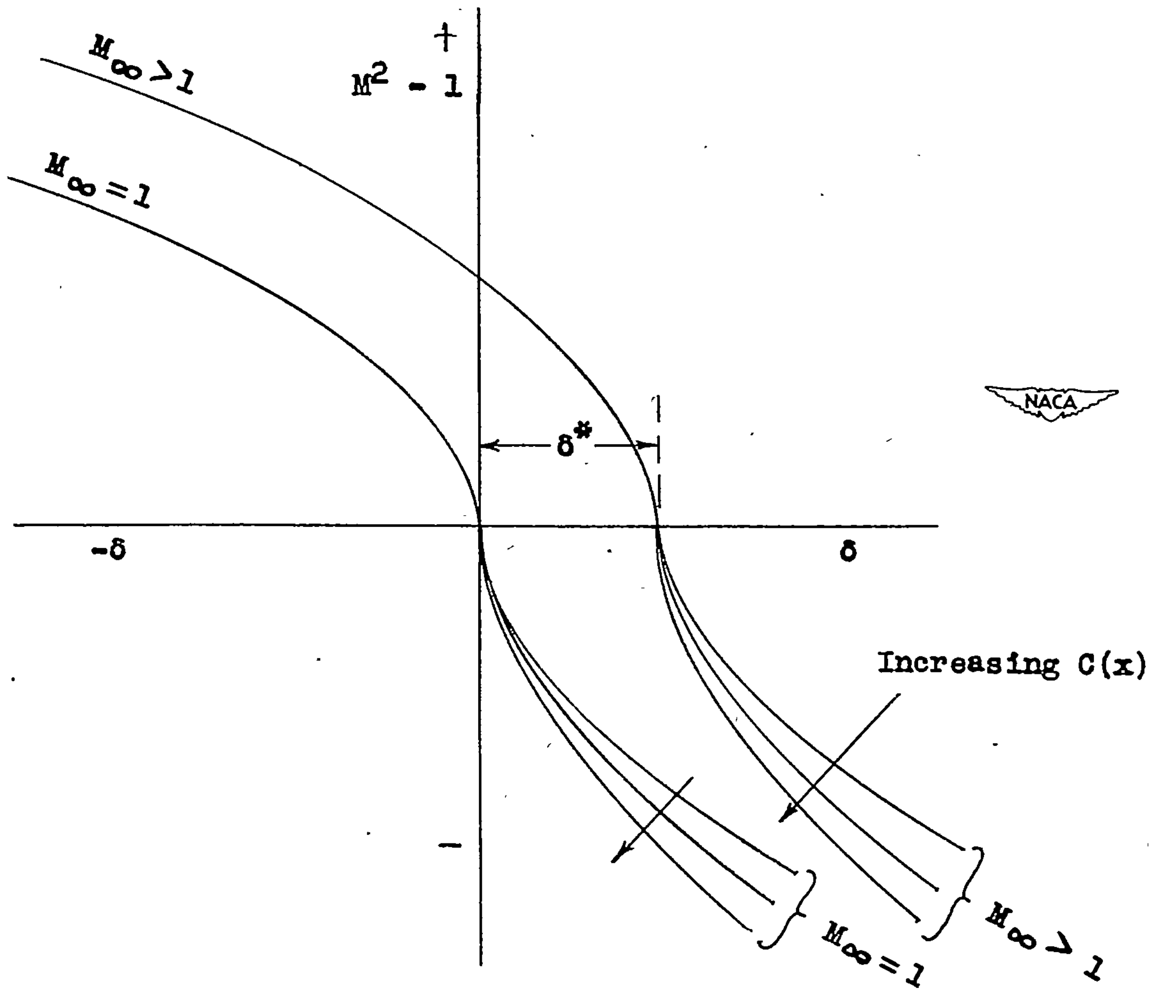

Figure 4.- Illustration of proposed velocity-correction formula showing combination of concepts of $C(x)$ and δ^*. $M^2 - 1 = C(x)\left|\delta - \delta^*\right|^{2/3}$; $\delta^* = \left(\frac{M_\infty^2 - 1}{C}\right)^{3/2}$.

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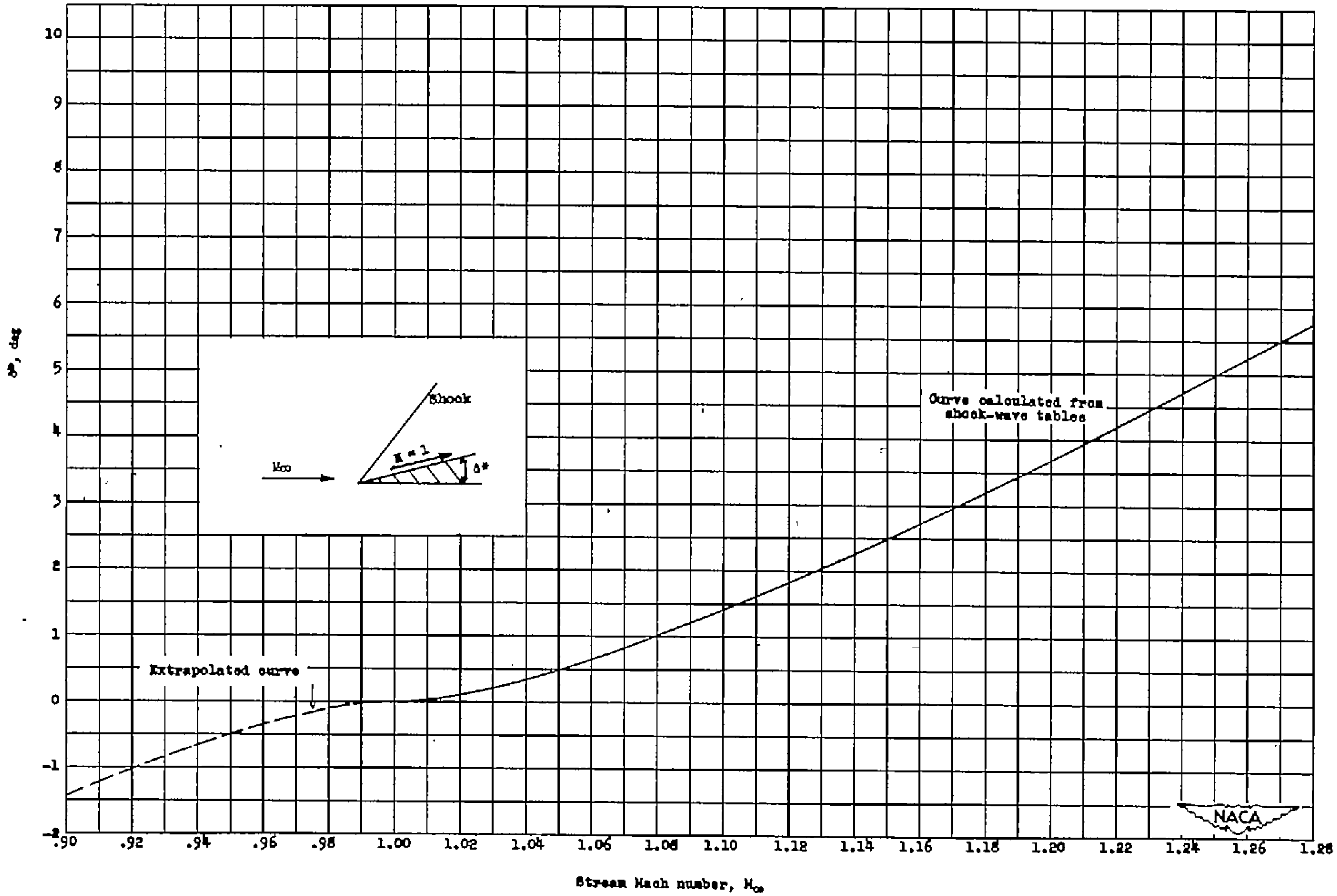

Figure 5.- Variation of δ* with stream Mach number.

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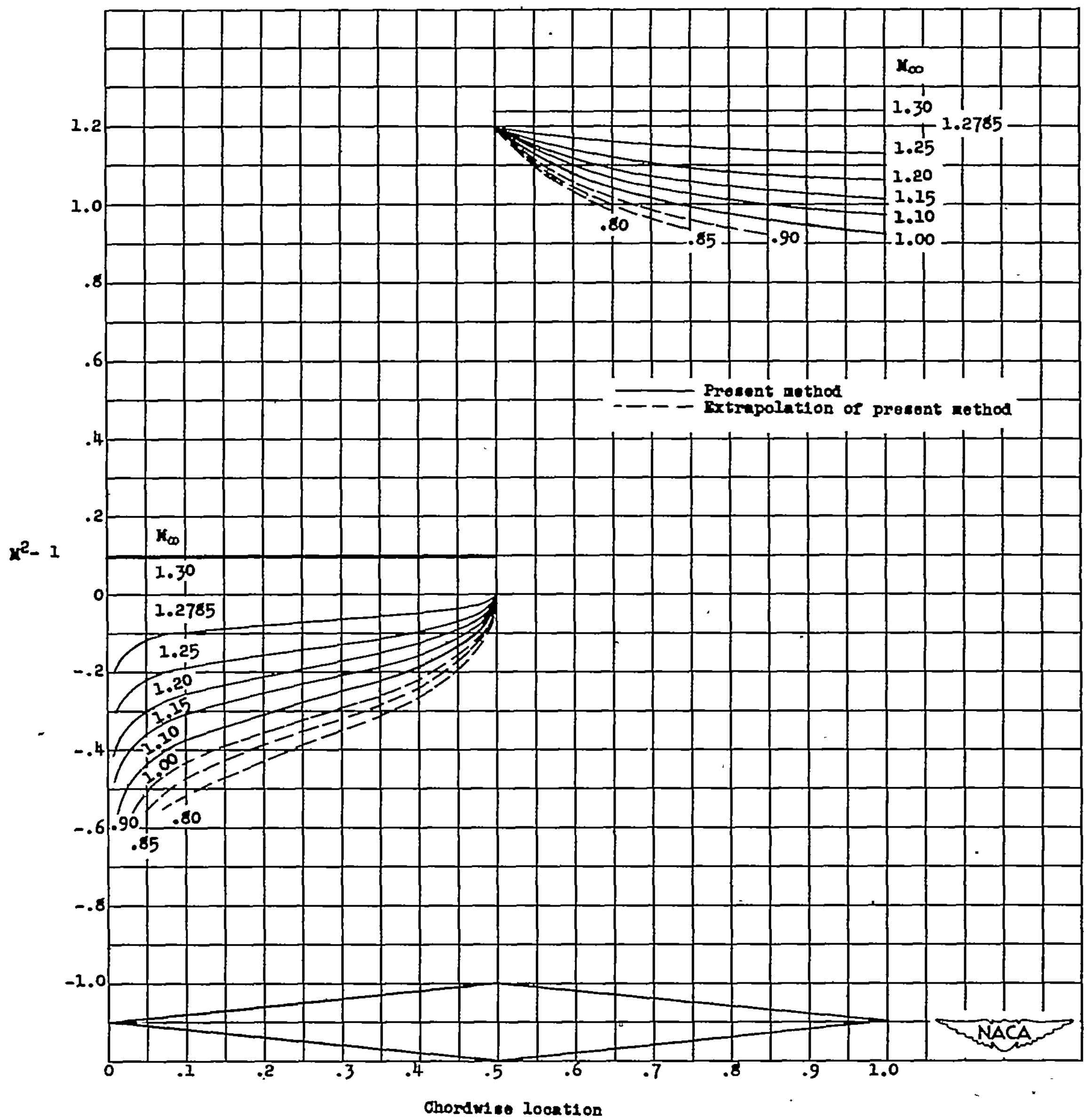

Figure 6.- Calculation of $M^2 - 1$ distribution for 10-percent-thick diamond-shaped airfoil.

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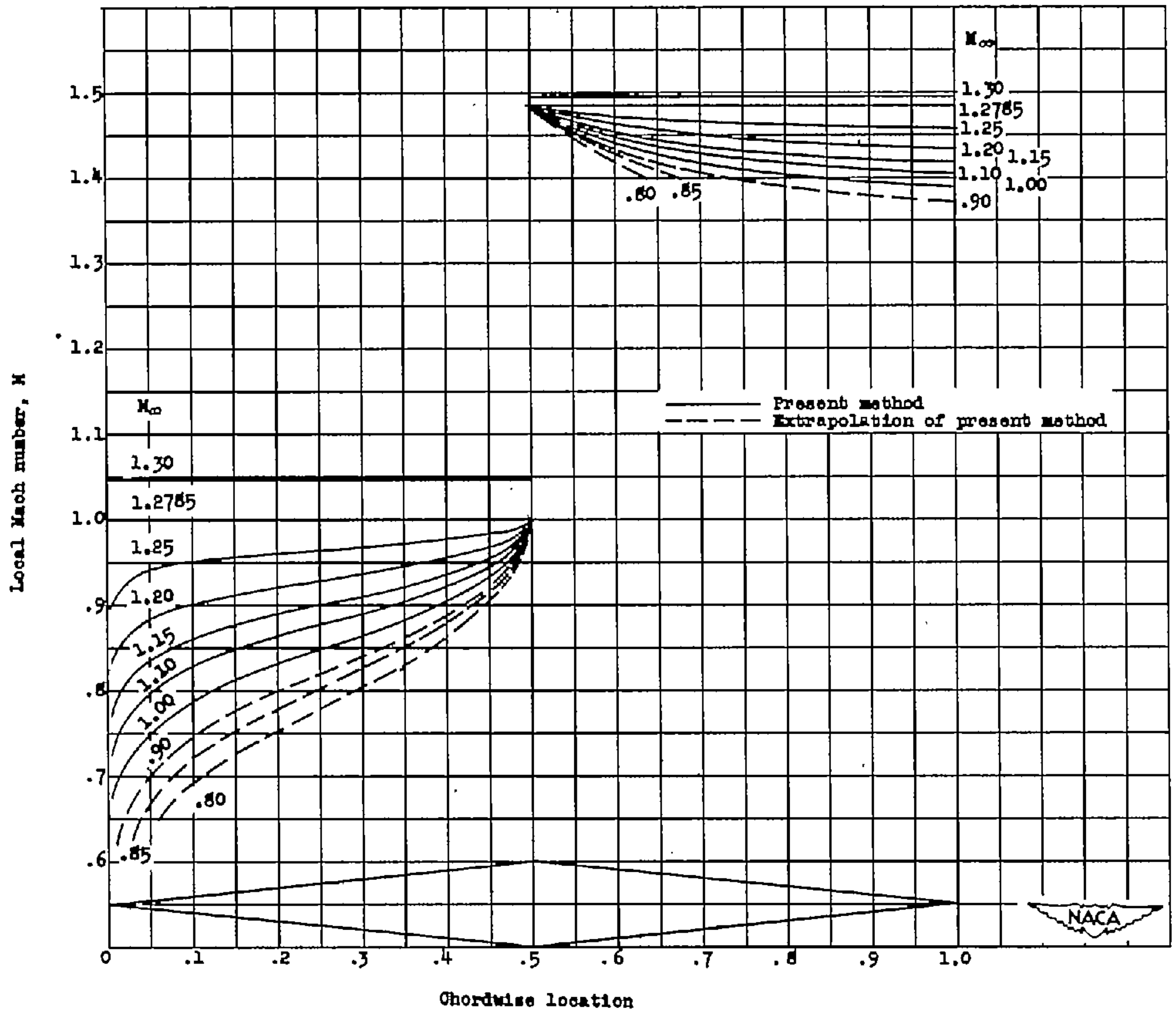

Figure 7.- Local Mach number distributions on 10-percent-thick diamond-shaped airfoil.

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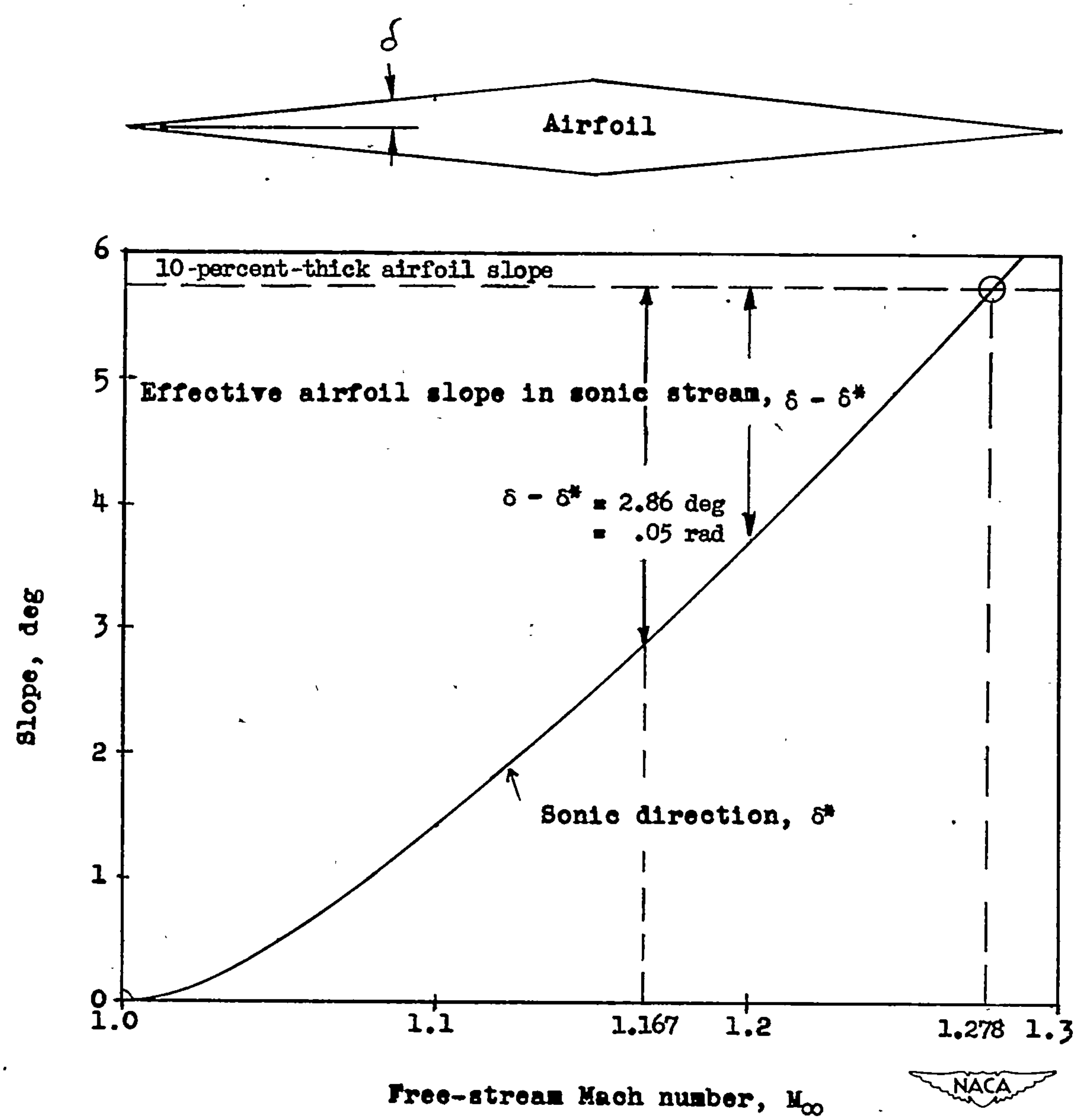

Figure 8.- Illustration of method of calculation.

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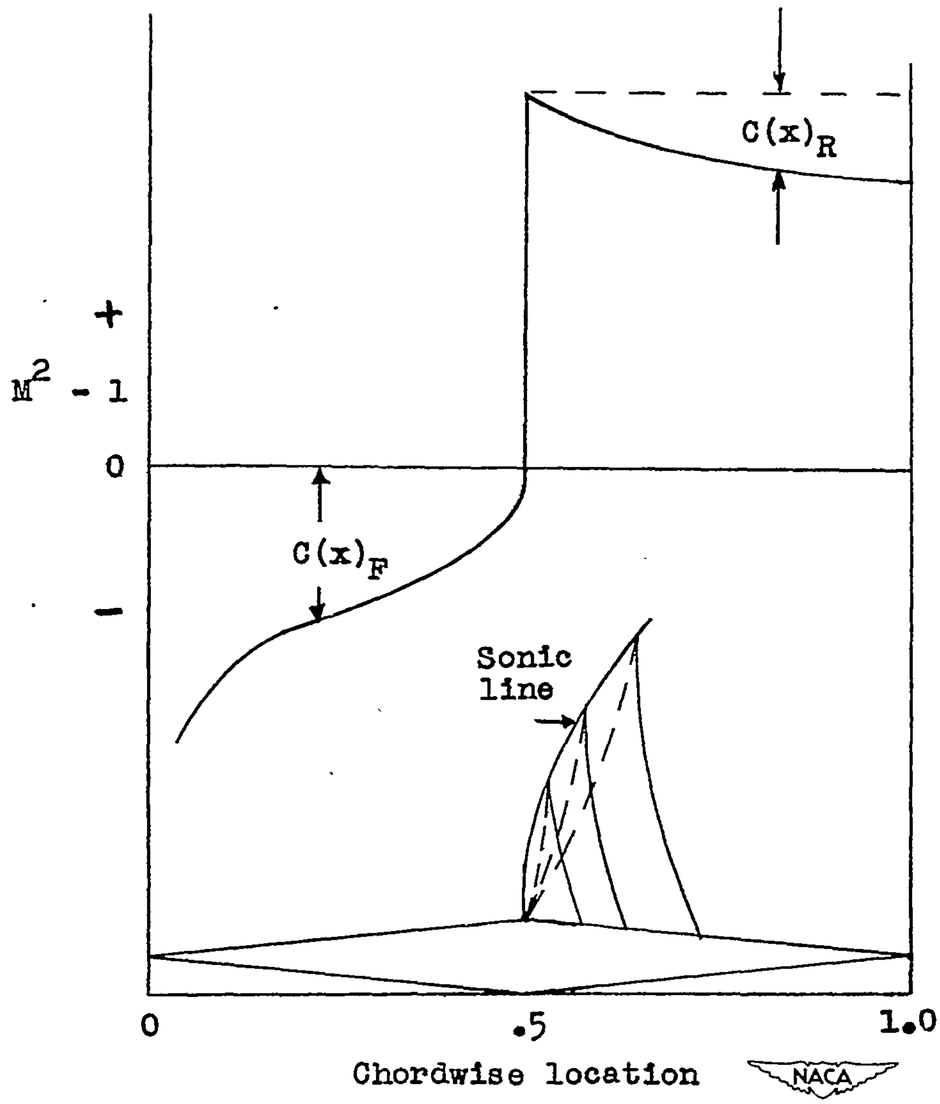

Figure 9.- Subsonic influence on rear half of diamond-shaped airfoil. $M_\infty = 1$.

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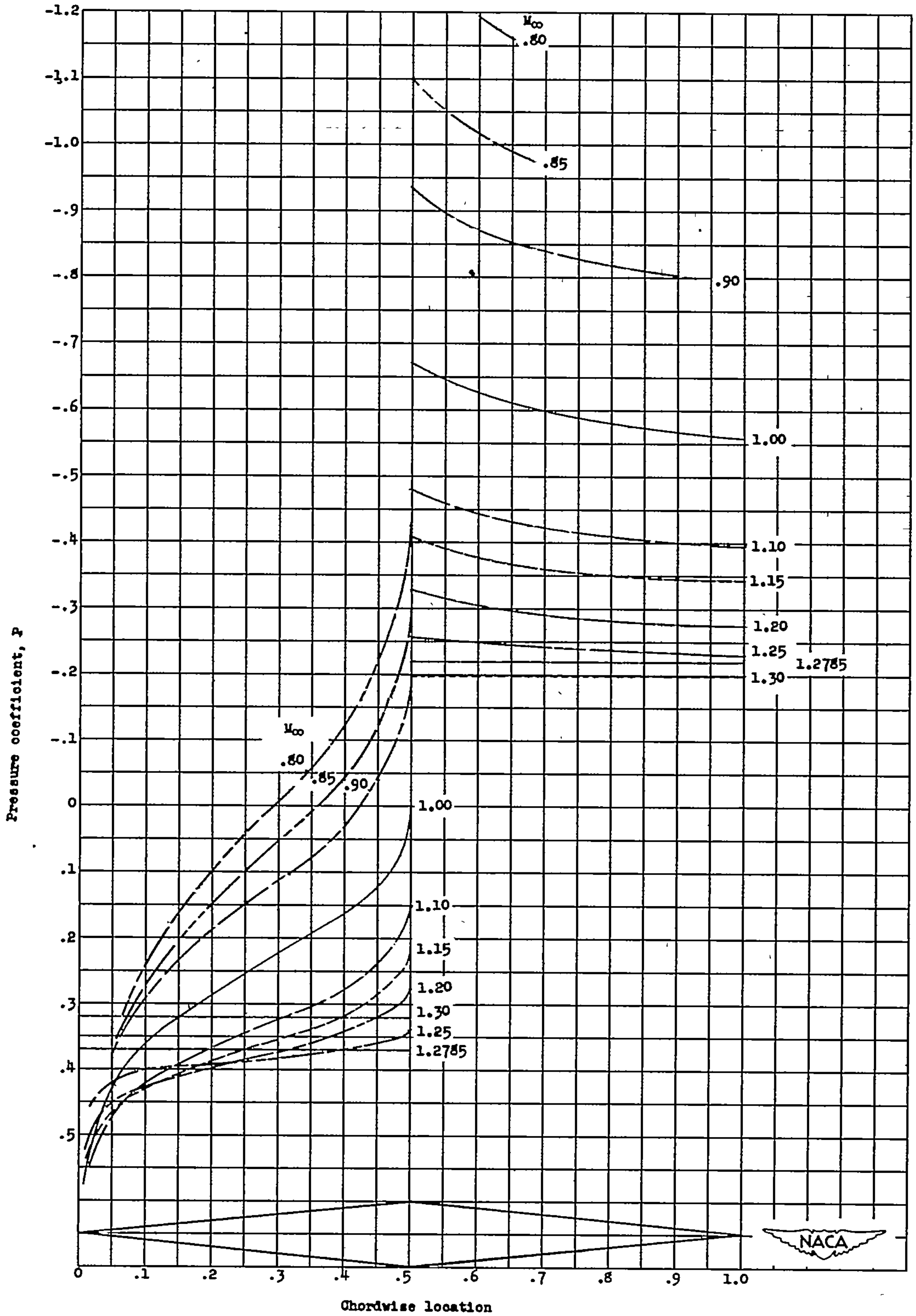

Figure 10.- Pressure distributions on a 10-percent-thick diamond-shaped airfoil.

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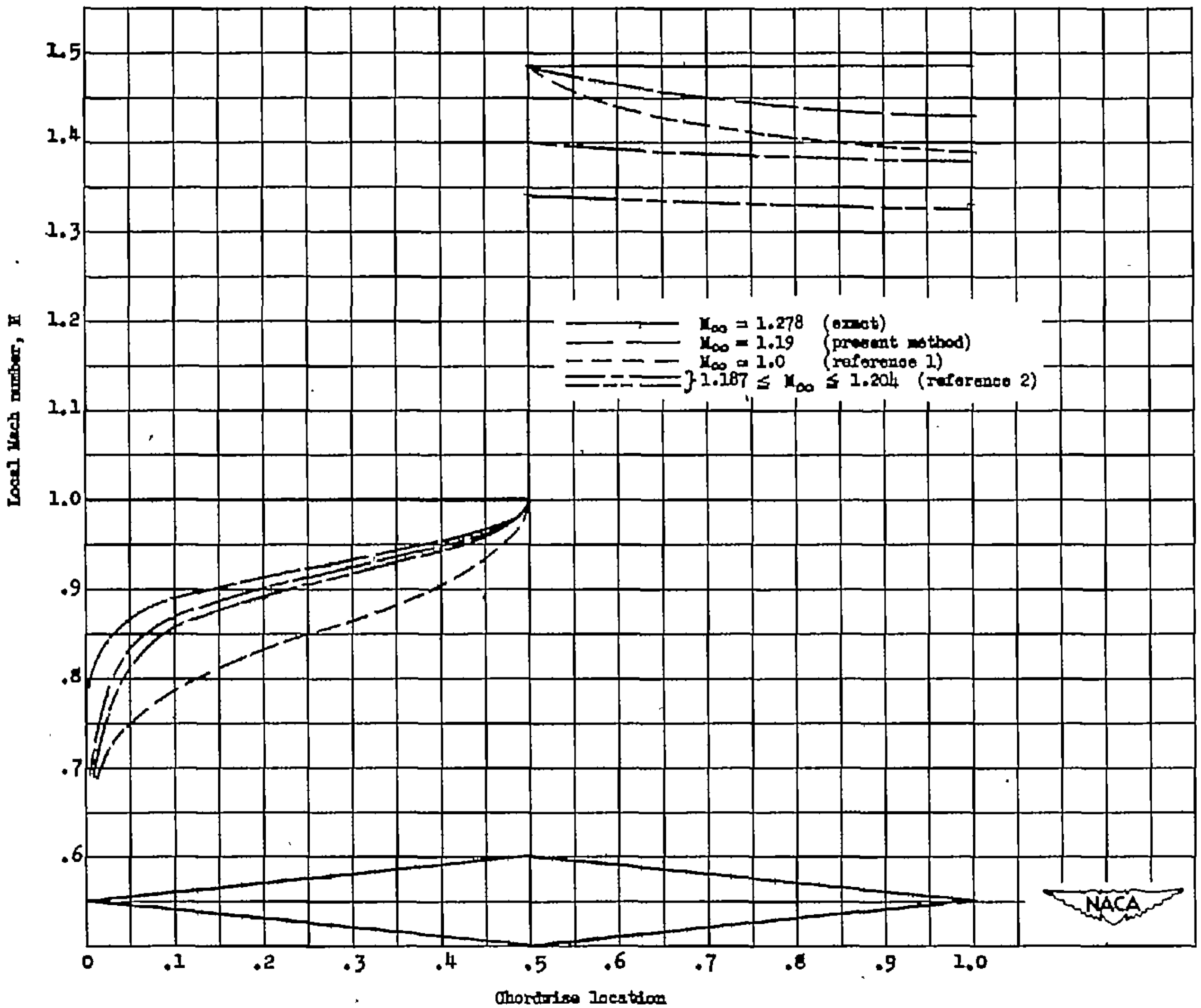

Figure 11.- Comparison of Mach number distributions on a 10-percent-thick diamond-shaped airfoil.

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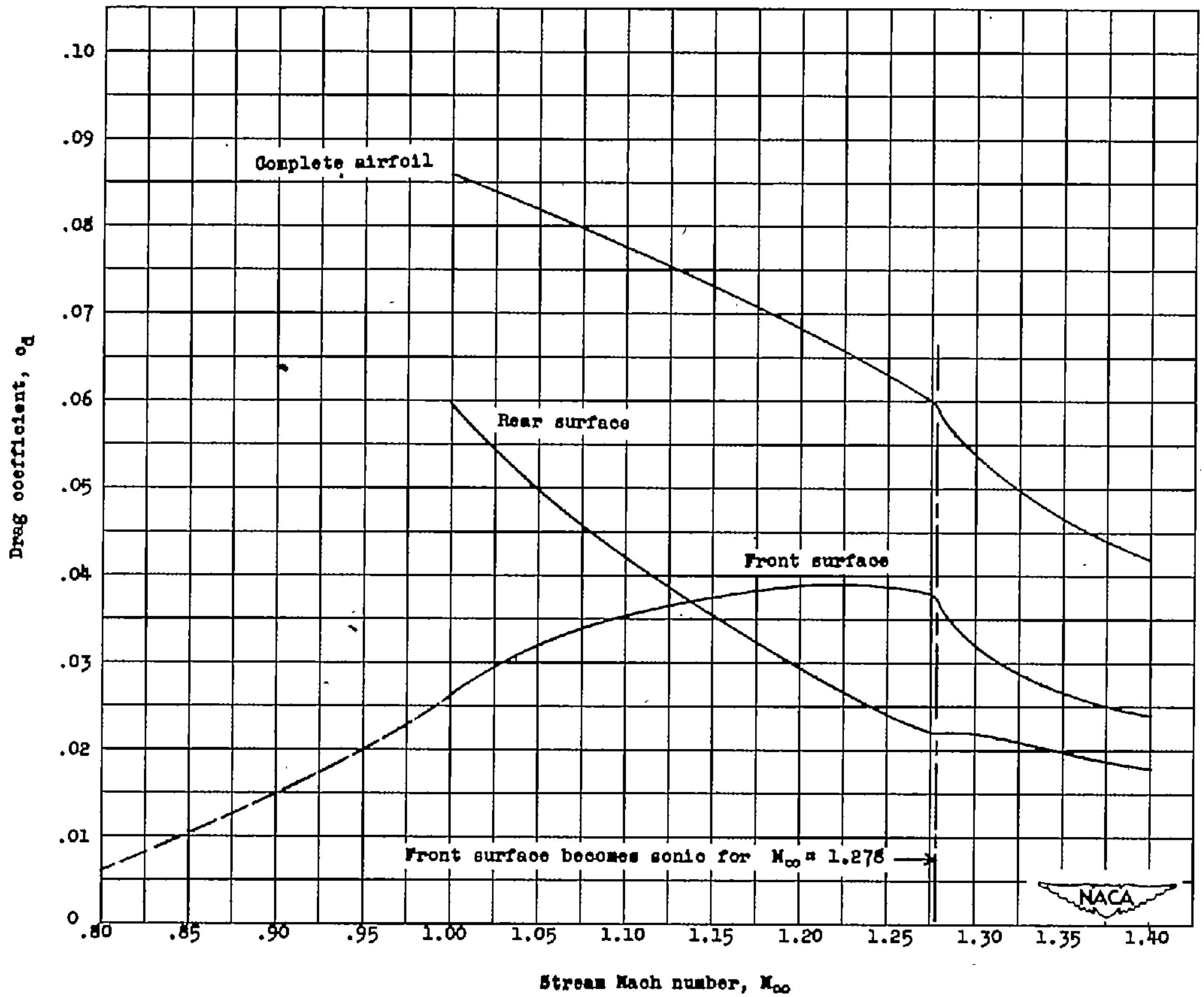

Figure 12.- Drag coefficient for a 10-percent-thick diamond-shaped airfoil as a function of Mach number.

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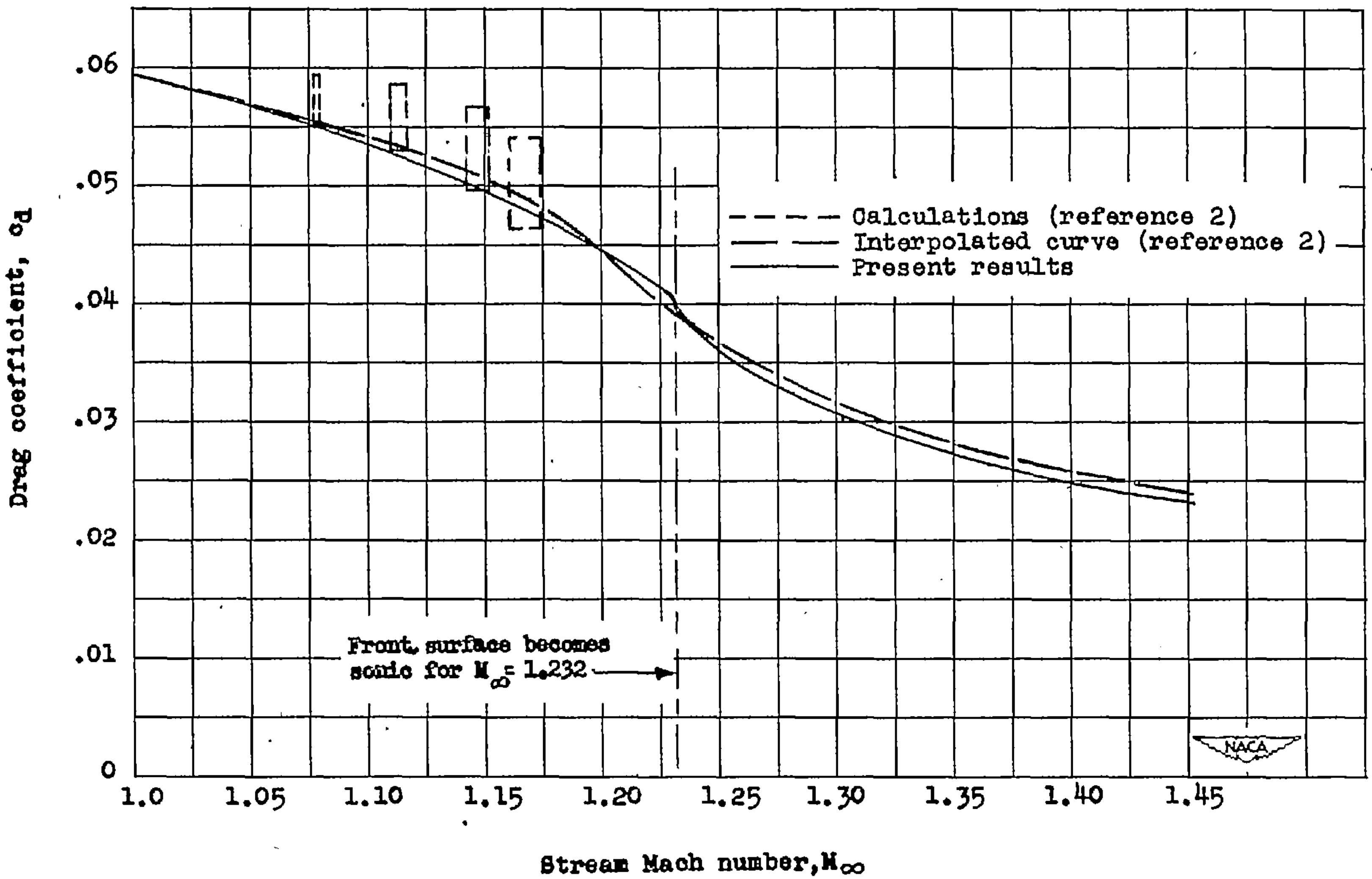

Figure 13.- Comparison of curves of drag coefficient for diamond-shaped airfoil. δ = 0.0787.

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