NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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NUMERICAL DETERMINATION OF INDICIAL LIFT OF A TWO-DIMENSIONAL SINKING AIRFOIL AT SUBSONIC MACH NUMBERS FROM OSCILLATORY LIFT COEFFICIENTS WITH CALCULATIONS FOR MACH NUMBER 0.7

By Bernard Mazelsky

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

The reciprocal equations for relating the incompressible circulatory indicial lift to the lift due to harmonic oscillations have been modified to include the noncirculatory lift associated with apparent-mass effects. Although the apparent-mass effects are impulsive in nature in incompressible flow, the lift due to apparent-mass effects in compressible flow is a time-dependent function. The corresponding reciprocal equations for the total compressible lift are given. By use of the reciprocal equations for compressible flow, the indicial lift and moment functions due to an airfoil's experiencing a sudden acquisition of vertical velocity are determined numerically for Mach number 0.7. Lack of sufficient flutter coefficients prevents the calculation of these functions at other Mach numbers.

Although the indicial lift and moment functions due to penetration of a sharp-edge gust may be obtained from the oscillatory tab or aileron coefficients by a similar analysis, sufficient coefficients are not available at the present. However, an approximate method is shown for determining a portion of this unsteady-lift function.

When a comparison is made of the indicial lift functions at Mach numbers 0.0 and 0.7, it is noted that the growth of lift to the steady state appears to be less rapid for the compressible case than for the incompressible case. Consequently, the calculation of the gust load factor at high subsonic Mach numbers utilizing the two-dimensional incompressible indicial lift functions and an over-all correction for compressibility such as the Prandtl-Glauert factor might be conservative.

INTRODUCTION

A knowledge of transient flows is important in many aeronautical problems. In the study of transient flows, two types of airfoil motions have had special significance - a harmonically oscillating airfoil and an airfoil experiencing a sudden change in angle of attack. The lift function for an airfoil experiencing a sudden change in angle of attack and the lift function associated with the growth of lift on an airfoil due to penetration of a sharp-edge gust are commonly referred to as indicial lift functions. The present paper is concerned with the use of reciprocal relations for determining for compressible flow the indicial lift functions directly from the lift data that are available for the airfoil oscillating harmonically.

The indicial lift functions have been derived for two-dimensional incompressible flow; the function for a sudden change in angle of attack was derived by Wagner, see reference 1, while the penetration function was derived by Küssner, reference 2. An account of the relations that exist between these indicial lift functions and the lift coefficients for a two-dimensional oscillating airfoil is given by Garrick in reference 3.

In a recent paper by Lomax, Heaslet, and Sluder (reference 4), a method for determining the indicial lift and moment function is given. While the beginning portion of the indicial functions can be calculated readily and the final value is considered to be the steady-state lift value given by the Prandtl-Glauert factor, the intermediate or transition values of the indicial lift functions are difficult to obtain by this method and consequently numerical results are given only for a Mach number of 0.8 in reference 4.

The situation in regard to the subsonic compressible-flow coefficients for an oscillating airfoil is much better. Possio (reference 5) has formulated the problem in a linearized form for determining the lift and moment for the oscillatory case. Dietze, Schade, and Frazer and Skan (references 6, 7, and 8) present the lift coefficients of a harmonically oscillating airfoil at various Mach numbers up to 0.8 and for various values of reduced frequency up to about 2.5. It was felt desirable to see whether these available flutter data could be used in conjunction with the reciprocal relations to obtain the complete indicial lift functions for compressible flow since, for dynamic-load studies, knowledge of the indicial lift functions is needed over a larger range of chord lengths than that given in reference 4 for Mach numbers other than 0.8. This paper discusses the use of the reciprocal relations for the case of compressible flow and presents an evaluation of the indicial functions for $M = 0.7$ for an airfoil suddenly acquiring a vertical velocity, for which case it was found that sufficient oscillatory lift data were available.

The indicial lift function due to penetration of a sharp-edge gust could also be determined by superposition provided the oscillatory lift coefficients were known for a wide range of flap to chord ratios as well as for a range of reduced frequencies. Unfortunately, sufficient data are not available at the present time; however, in order to provide some insight as to the growth of lift for the gust case, an alternate method is shown for approximating most of this function.

SYMBOLS

s, s_1, σ	distance traveled, half-chords
ω	angular frequency
V	forward velocity of airfoil
c	chord
k	reduced-frequency parameter $\left(\frac{\omega c}{2V}\right)$
$L(s)$	lift per unit length of span
$M(s)$	moment per unit length of span about quarter-chord point
ρ	density
h	amplitude of vertical displacement, half-chords
$\dot{h}$, $\ddot{h}$	first and second derivatives of h with respect to s
M	Mach number
$\delta(s)$	impulse function, $\delta(s) = \infty$ for $s = 0$, $\delta(s) = 0$ for $s \neq 0$, and $\int_{-\infty}^{\infty} \delta(s)\, ds = 1$
$k_1(s)$	indicial lift function for an airfoil experiencing a sudden acquisition in vertical velocity as used in equation $L = -\pi\rho c V^2 \dot{h} k_1(s)$
U	velocity of sharp-edge gust
$k_2(s)$	indicial lift function due to penetration of a sharp-edge gust as used in equation $L = -\pi\rho c V U k_2(s)$

$m_1(s)$ indicial moment function for an airfoil experiencing a sudden acquisition of vertical velocity where moment is taken about quarter-chord position

$x_{cp}(s)$ center-of-pressure location in percent chord from leading edge for an airfoil experiencing a sudden acquisition of vertical velocity

$C(k)$ Theodorsen's circulatory lift function (reference 9), $F(k) + iG(k)$, as used in equation $L = -\pi\rho cV^2he^{iks}\left[-ikC(k)\right]$

$C_c(k)$ complex compressible-flow oscillatory lift coefficient which includes both circulatory- and noncirculatory-lift components, $C_c(k) = F_c(k) + iG_c(k)$, as used in equation $L = -\pi\rho cV^2e^{iks}h\left[-ikC_c(k)\right]$

$M(k)$ in-phase component of the complex moment corresponding to the $F_c(k)$ lift coefficient

$f(k) = F_c(k) - F_c(\infty)$

$m(k) = M(k) - M(\infty)$

Z_1, Z_2 in-phase and out-of-phase lift coefficients associated with translation of the airfoil as used in equation $L = \pi\rho cV^2e^{iks}\frac{h}{2}\left(Z_1 + iZ_2\right)$

M_1, M_2 in-phase and out-of-phase moment coefficient about quarter-chord position associated with translation of the airfoil as used in equation $M = \pi\rho c^2V^2e^{iks}\frac{h}{2}\left(M_1 + iM_2\right)$

A, B, C constants

Matrix notation:

$\lfloor\ \ \rceil$ rectangular matrix

$\{\ \ \}$ column matrix

ANALYSIS AND RESULTS

Superposition Integrals for the Incompressible and Compressible Cases

The reciprocal equations which are essentially a special form of the superposition integral have been derived for the incompressible circulatory lift by Garrick (reference 3). This section first discusses an extension of Garrick's work by incorporating the potential lift due to apparent-mass effects into the reciprocal equations for the incompressible case. The reciprocal equations for the compressible case are then indicated and the evaluation of the indicial lift and moment for an airfoil experiencing a sudden acquisition of vertical velocity is determined for the case of $M = 0.7$.

Incompressible case.- In the incompressible case, the lift in unsteady motion has been separated into two parts - a noncirculatory lift which is commonly referred to as an apparent-mass effect and a lift due to the circulation about the airfoil. The reciprocal equations for incompressible flow are expressed only in terms of Theodorsen's circulatory-lift function $C(k)$ and Wagner's indicial-lift function $k_1(s)$ and can be expressed in two separate forms as follows:

$$k_1(s) = \frac{2}{\pi}\int_0^{\infty} \frac{F(k)\ \sin ks}{k}\,dk \qquad (s > 0) \qquad (1a)$$

and

$$k_1(s) = 1 + \frac{2}{\pi}\int_0^{\infty} \frac{G(k)\ \cos ks}{k}\,dk \qquad (s > 0) \qquad (1b)$$

where $F(k)$ and $G(k)$ are, respectively, the in-phase and out-of-phase components of circulatory lift on a harmonically oscillating airfoil defined by the equation

$$L = -\pi\rho cV^2 he^{iks}\left[-ikC(k)\right]$$

where

$$C(k) = F(k) + iG(k)$$

The $k_1(s)$ function is defined such that the lift on an airfoil experiencing a sudden acquisition of vertical velocity may be expressed by the following equation:

$$L = -\pi \rho c V^2 \dot{h} k_1(s) \tag{2}$$

Two relations that can be obtained from equation (1a) are (1) that the value of $F(k)$ at $k = 0$ is equal to the asymptotic value of $k_1(s)$ as s tends to infinity, these values corresponding to the steady-state lift, and (2) that the initial value of $k_1(s)$ is equal in magnitude to the asymptotic value of $F(k)$ as k tends to infinity. This second relation is not readily evident from equation (1a); consequently, a mathematical proof of this second relation is shown in appendix A. As a result of these relationships, the initial value and the asymptote of the $k_1(s)$ function can be determined from the given $F(k)$ function. A knowledge of these end points is valuable in a numerical solution of the reciprocal equations.

As noted previously, equations (1a) and (1b) express only the circulatory part of the lift and, therefore, do not account for the apparent-mass effects. An expression for the apparent-mass effects for the indicial case, however, may be obtained as follows.

The lift due to apparent-mass effects alone for any arbitrary vertical motion of the airfoil when written in terms of the nondimensional displacement h is given in reference 9 as

$$L = -\pi \rho \frac{c^2}{4} \frac{c}{2} \frac{4V^2}{c^2} \ddot{h}(s) \tag{3}$$

When this equation is applied to the case of an airfoil experiencing a sudden vertical velocity, $\ddot{h}(s)$ becomes impulsive in character. The magnitude of this impulse may be defined by

$$\ddot{h}(s) = \dot{h}\delta(s)$$

where $\dot{h}$ is the instantaneous vertical velocity acquired by the airfoil and the function $\delta(s)$ is defined such that $\delta(s) = \infty$ for $s = 0$, $\delta(s) = 0$ for $s \neq 0$, and

$$\int_{-\infty}^{\infty} \delta(s)ds = 1$$

When $\dot{h}\delta(s)$ is substituted for the impulsive sinking acceleration $\ddot{h}(s)$ in equation (3), the resulting expression for the lift due to apparent mass alone becomes

$$L_{impulse} = -\pi\rho cV^2\dot{h}\,\frac{\delta(s)}{2} \tag{4}$$

If the impulsive part of the unsteady-lift functions is designated by $k_1(s)_{impulse}$ and if the form of equation (2) is retained, then the expression for the impulsive lift may be written

$$L_{impulse} = -\pi\rho cV^2\dot{h}k_1(s)_{impulse} \tag{5}$$

The value of $k_1(s)$ due to apparent-mass effects in the incompressible case may then be determined by a comparison of equations (4) and (5) to give

$$k_1(s)_{impulse} = \frac{1}{2}\,\delta(s) \tag{6}$$

Addition of the $k_1(s)$ function due to circulation, equations (1), and the $k_1(s)_{impulse}$ function, equation (6), then gives the expressions for the lift for the indicial case as follows:

$$k_1(s)_{total} = \frac{2}{\pi}\int_0^{\infty} \frac{F(k)\ \sin ks}{k}\,dk + \frac{1}{2}\,\delta(s) \qquad (s \geqq 0) \tag{7a}$$

and

$$k_1(s)_{total} = 1 + \frac{2}{\pi}\int_0^\infty \frac{G(k)\ \cos ks}{k}\,dk + \frac{1}{2}\,\delta(s) \qquad (s \geqq 0) \qquad (7b)$$

Compressible case.- While in the incompressible case it has been found convenient and natural to consider the circulatory and noncirculatory lifts separately (this separation is especially desirable because of the impulsive nature of the noncirculatory part), these circumstances do not exist in the compressible case. The perturbation velocities are finite in the compressible case (since speed of sound is considered finite) and, consequently, the effects of the pressure disturbances in the whole flow field are not felt instantaneously as for incompressible flow. Consequently, the lift due to apparent-mass effects would not be impulsive but would remain finite and time dependent. Separation of the lifts is not necessary in the compressible case and, moreover, is not convenient since the available results for the compressible case are presented numerically only for the total lift. The reciprocal equations for the total compressible lift are, therefore, much simpler than the equations for the incompressible case since the impulse function may be disregarded.

For compressible flow, the case is exactly analogous to the treatment of indicial admittance given by Bush in reference 10. In this reference, equations are given which relate the indicial admittance to the in-phase and out-of-phase responses of a system subjected to an oscillatory forcing function. In notation similar to that used for incompressible flow, these expressions may be written as follows

$$k_1(s)_{total} = \frac{2}{\pi}\int_0^\infty \frac{F_c(k)\ \sin ks}{k}\,dk \qquad (s > 0) \qquad (8a)$$

and

$$k_1(s)_{total} = F_c(0) + \frac{2}{\pi}\int_0^\infty \frac{G_c(k)\ \cos ks}{k}\,dk \qquad (s > 0) \qquad (8b)$$

where $F_c(k)$ and $G_c(k)$ are, respectively, the in-phase and out-of-phase lift components on an oscillating airfoil and are defined by the equation

$$L = -\pi\rho c V^2 h e^{iks}\left[-ikC_c(k)\right] \tag{9}$$

where

$$C_c(k) = F_c(k) + iG_c(k)$$

Equations (8a) and (8b) are applicable for the compressible case providing the function $C_c(k)$ is continuous and finite in the interval 0 to ∞. From the physical conditions for the compressible case, it can reasonably be assumed that these functions adhere to those conditions.

It is of interest to note that, when these equations are applied to incompressible flow, they apply only to the circulatory part of the lift as given by equations (1a) and (1b). In this case, the factor $F_c(0)$ becomes, by definition, simply unity. For compressible flow, the factor $F_c(0)$ acquires a value given by the Prandtl-Glauert factor $\frac{1}{\sqrt{1 - M^2}}$.

The existing coefficients for the translatory case which are to be used in the reciprocal relations have been given in various forms beginning with Possio's work. The form in which all the results are commonly put is

$$L = \pi\rho c V^2 e^{iks} \frac{h}{2}\left(Z_1 + iZ_2\right) \tag{10a}$$

and

$$M = \pi\rho c^2 V^2 e^{iks} \frac{h}{2}\left(M_1 + iM_2\right) \tag{10b}$$

where M is the moment acting about the quarter-chord point. In order to make convenient use of the reciprocal relations given by equations (8a) and (8b), it is first necessary to convert the expression for lift given by equation (10a) to a form similar to the one used in equation (9). When the expressions (9) and (10a) are compared, the following correspondence is seen to exist for compressible flow:

$$F_c(k) = \frac{Z_2(k)}{2k} \tag{11a}$$

and

$$G_c(k) = \frac{-Z_1(k)}{2k} \tag{11b}$$

Summary of Available Flutter Coefficients and Related Data for Subsonic Compressible Flow

The two-dimensional compressible flutter coefficients for the real and imaginary parts of the lift and moment oscillatory coefficients for sinking motion are given in table I for three Mach numbers, $M = 0.5$, $M = 0.6$, and $M = 0.7$. This table represents a summary of the results of three authors where the flutter coefficients by each have been converted to the Z_1, Z_2, M_1, and M_2 form for use in equations (10a) and (10b). The values of Z_1 and Z_2 correspond to the lift coefficients indicated in equation (10a), and the values of M_1 and M_2 indicated in equation (10b) correspond to the moment taken about the quarter-chord position. The values of these coefficients were obtained from three sources, and the range of reduced frequencies taken from each source is indicated in table I. Data for large values of reduced frequency do not appear to exist. The accuracy of these data for reduced frequencies lower than approximately 1 is better than the accuracy of the data at higher values of reduced frequency (see reference 8), but a reasonably good solution for the indicial lift and moment functions is still possible since this decrease in accuracy at the higher reduced frequencies will be shown to have little effect on the determination of either the unsteady lift or unsteady moment functions provided the value of the flutter coefficients at the infinite reduced frequency is known. Fortunately these values can be determined from the initial value of the indicial functions (see appendix A). If the flutter coefficients are known up to a reduced frequency where they have practically the same value as the flutter coefficient at the infinite reduced frequency, then they are known for a sufficient range of reduced frequencies for determining a reliable solution of the indicial functions. This condition exists for $M = 0.7$ since values of the flutter coefficients at a reduced frequency of 2.5 have been found to agree quite well with the values at the infinite reduced frequency. However, values of the flutter coefficients at $M = 0.5$ and $M = 0.6$ are known only for reduced frequencies up to 1. Also, the flutter coefficients at these Mach numbers must be known for higher values of reduced frequency than for $M = 0.7$ since the flutter coefficients approach the values at the infinite frequency at a much slower rate. Consequently, the only Mach number for which sufficient data were found available for obtaining the indicial functions is $M = 0.7$.

In order to aid in the numerical solution of the reciprocal equations, the end points of the $F_c(k)$ function should be determined. These end points may be determined independently of the flutter coefficients. The value of $F_c(0)$ which represents the steady-state lift can be given by the Prandtl-Glauert factor $\frac{1}{\sqrt{1 - M^2}}$. When an attempt is made to calculate this end point from the Z_1 and Z_2 coefficients given in table I, an indeterminate form is found; however, the magnitude of this indeterminate quantity when properly evaluated agrees very well with the factor $\frac{1}{\sqrt{1 - M^2}}$. The value of $F_c(\infty)$ which corresponds to the value of $k_1(s)$ at $s = 0$ (see appendix A) can be determined by the following equation

$$F_c(\infty) = \frac{2}{\pi M} \tag{12}$$

Numerical Solution of the Reciprocal Equation

The indicial lift function $k_1(s)$ can be determined from either of equations (8a) and (8b) where the coefficients $F_c(k)$ and $G_c(k)$ are determined from the oscillatory coefficients by equations (11a) and (11b). It has been noted that, of the data presented in table I, the only Mach number for which sufficient data are available for providing a reasonable estimate of the indicial lift and moment on the airfoil is $M = 0.7$ since values of the flutter coefficients are not given for values of $k > 1$ at the other Mach numbers. A plot of the $F_c(k)$ and $G_c(k)$ functions for this Mach number is shown in figure 1. Although a smooth curve can be drawn for the $F_c(k)$ function at the higher reduced frequencies, the data for the $G_c(k)$ function appear to be erratic in this region. Consequently, the latter function is represented by a broken line at the higher reduced frequencies. Also, the numerical evaluation of the $k_1(s)$ function by equation (8b) is more difficult than by equation (8a) because the integrand in equation (8b) is indeterminate at $k = 0$. In order to evaluate this indeterminate form, the value of the derivative of $G_c(k)$ at $k = 0$ must be known. Since the function $G_c(k)$ is not given in a closed form, this derivative would be difficult to determine accurately in the compressible case. Because of the erratic nature of the $G_c(k)$ function at the higher values of reduced frequency and because of the presence of this indeterminate form of $G_c(k)$ at $k = 0$, equation (8a) is used throughout the ensuing analysis for determining the indicial functions.

The graphical solution of equation (8a) may be simplified considerably by incorporating in the solution of this equation the known value of the flutter coefficient at the infinite reduced frequency by making the following substitution for the flutter coefficient $F_c(k)$:

$$f(k) = F_c(k) - F_c(\infty) \tag{13a}$$

or

$$F_c(k) = f(k) + F_c(\infty) \tag{13b}$$

The substitution of equation (13b) into equation (11a) leads to an alternate equation for $k_1(s)$:

$$k_1(s) = F_c(\infty) + \frac{2}{\pi}\int_0^{\infty} \frac{f(k)\ \sin ks}{k}\,dk \tag{14}$$

The graphical solution of $k_1(s)$ is thus simplified since the integral in this expression can be evaluated more readily than the integral in equation (8a) because the integrand approaches zero much more rapidly.

The $F_c(k)$ function for $M = 0.7$ is calculated from the flutter coefficient Z_2 by equation (11a) and is shown in figure 1 together with the value of the asymptote calculated by equation (12). The transformed function $f(k)$ as obtained by equation (13a) is shown in figure 2. A plot of the integrand in equation (14) for several values of s is shown in figure 3. These integrands were integrated by means of a planimeter. The $k_1(s)$ function thus found by means of equation (14) is shown in figure 4 along with the $k_1(s)$ function for $M = 0.0$. In this same figure a part of an independent solution for $M = 0.7$ given in reference 4 is also shown.

It will be useful to fit the $k_1(s)$ function for $M = 0.7$ given in figure 4 by some analytical function. Since the exponential function has a simple operational equivalent and has been found convenient in approximating the $k_1(s)$ function at $M = 0.0$ (see reference 11), a limited series of such functions was chosen to approximate the $k_1(s)$ function at $M = 0.7$. The function found to fit this curve quite well is

$$k_1(s) = 1.4\left(1 - 0.364e^{-0.0536s} - 0.405e^{-0.357s} + 0.419e^{-0.902s}\right) \qquad (15)$$

and is also shown in figure 4. The corresponding approximate expressions for the harmonically oscillating airfoil can be found from equation (15) in a similar manner shown in reference 11. The expressions for $F_c(k)$ and $G_c(k)$ are

$$F_c(k) = 1.4\left[1 - \frac{0.364k^2}{(0.0536)^2 + k^2} - \frac{0.405k^2}{(0.357)^2 + k^2} + \frac{0.419k^2}{(0.902)^2 + k^2}\right] \qquad (16a)$$

and

$$G_c(k) = 1.4\left[- \frac{0.01951k}{(0.0536)^2 + k^2} - \frac{0.1446k}{(0.357)^2 + k^2} + \frac{0.3779k}{(0.902)^2 + k^2}\right] \qquad (16b)$$

Both of these expressions are plotted in figure 1 for comparison with the data obtained from the flutter coefficients. Note that the approximate expression for the $F_c(k)$ function is in fairly good agreement for all of the k values considered. However, relatively poor agreement exists for the $G_c(k)$ function at the higher values of k. This comparison is to be expected since, as was previously cited, the $G_c(k)$ function obtained from the flutter coefficients appears to be quite erratic at the higher values of k.

The solution of the reciprocal equation has thus far been shown for the calculation of the $k_1(s)$ function. Since the reciprocal equations are also applicable to the determination of the unsteady moment $m_1(s)$ due to a sudden change in airfoil vertical velocity, the $m_1(s)$ function taken about the quarter-chord point can be shown to be given by

$$m_1(s) = \frac{2}{\pi}\int_0^\infty \frac{M(k)\ \sin ks}{k}\, dk \tag{17}$$

where $M(k)$ is considered herein to be determined from the flutter coefficient M_2 by the following equation:

$$M(k) = -\frac{M_2}{2k} \tag{18}$$

A plot of the function $M(k)$ for $M = 0.7$ is shown in figure 5 together with the value of $M(\infty)$ determined by

$$M(\infty) = -\frac{1}{2\pi M} \tag{19}$$

In a manner similar to that shown for the unsteady-lift case, the function $M(k)$ is then transformed in order that equation (17) can be evaluated graphically by the following substitution:

$$m(k) = M(k) - M(\infty) \tag{20a}$$

or

$$M(k) = m(k) + M(\infty) \tag{20b}$$

A plot of the function $m(k)$ for $M = 0.7$ is shown in figure 6. The substitution of equation (20b) into equation (17) leads to the expression

$$m_1(s) = M(\infty) + \frac{2}{\pi}\int_0^\infty \frac{m(k)\ \sin ks}{k}\, dk \tag{21}$$

Plots of the integrand in equation (21) are shown in figure 7 for several values of s, and the $m_1(s)$ function evaluated by this equation is shown in figure 8.

If the indicial lift function $k_1(s)$ and the indicial moment function $m_1(s)$ are known, the variation of indicial center-of-pressure

Location in percent chord can be calculated by means of the following equation:

$$x_{cp}(s) = 25 - \frac{m_1(s)100}{k_1(s)} \tag{22}$$

A plot of the center-of-pressure location is given in figure 9 and, for comparison, the case for $M = 0.0$ is also shown.

Approximate Considerations for the Indicial Lift Due to Penetration of a Sharp-Edge Gust

Because of the limited number of flutter coefficients available for flap motions, as noted in the introduction, the determination of the indicial lift function for penetration $k_2(s)$ by use of the reciprocal equations is not possible at the present time. An alternate and perhaps semirational method for determining a part of the penetration function $k_2(s)$ from the $k_1(s)$ function on the basis of the relations between these two functions for the incompressible case is suggested as follows. The relations between the $k_1(s)$ and $k_2(s)$ functions for the incompressible case are given in reference 3 and are as follows:

$$k_2(s) = \frac{1}{\pi}\int_0^s k_1(s - s_1)\sqrt{\frac{s_1}{2 - s_1}}\,ds_1 + \frac{1}{\pi}\sqrt{s(2 - s)} \qquad (0 < s < 2) \tag{23a}$$

$$k_2(s) = \frac{1}{\pi}\int_0^2 k_1(s - s_1)\sqrt{\frac{s_1}{2 - s_1}}\,ds_1 \qquad (s > 2) \tag{23b}$$

The first term in equation (23a) is associated with only the circulatory part of the $k_1(s)$ function while the second term of the equation arises from the impulsive term in the $k_1(s)$ function, and hence is associated with the apparent-mass effects. This second term is known not to apply for compressible flow, since for the $k_1(s)$ function the apparent-mass effect is no longer impulsive in character but rather is time dependent. Therefore this equation cannot be applied to the compressible case. With regard to the applicability of equation (23b), it is assumed that this equation can evaluate the circulatory part of the lift for the $k_2(s)$ function provided that only the circulatory part of the $k_1(s)$ function is used. Although the $k_1(s)$ function calculated herein for compressible flow contains both the circulatory and noncirculatory lifts, the following argument may be used as a basis for the assumption that for values of s greater than approximately 4, the $k_1(s)$ function can be considered, for practical purposes, to be due to circulation only. If the assumption is made that the lift due to apparent-mass effects and the lift due to circulation act always at $\frac{1}{2}$-chord position and $\frac{1}{4}$-chord position, respectively, the moment function $m_1(s)$ shown in figure 8 must arise from the apparent-mass effects only. This figure shows that for practical purposes the moment function, referred to the quarter-chord position, has decayed to a negligible value for s greater than approximately 4. Thus the portion of the $k_1(s)$ function in figure 4 for s greater than approximately 4 may be associated with the lift due to circulation only. Inspection of equation (23b) shows that, if the evaluation of the $k_2(s)$ function is confined to values of s greater than 6, only the portion of the $k_1(s)$ function for s greater than approximately 4 (the portion attributed to circulation) will be used. A numerical solution of equations (23a) and (23b) is given in appendix B. The $k_2(s)$ function for $M = 0.0$ was calculated by using the $k_1(s)$ function for $M = 0.0$ shown in figure 4 as a means for estimating the accuracy of a numerical solution given by equation (B9) in appendix B. The values obtained were within 1.5 percent of the known Küssner function. For the compressible case the $k_2(s)$ function for s greater than approximately 6 was evaluated by utilizing the $k_1(s)$ function for $M = 0.7$. The results of this calculation are shown in figure 10. Although the solution for the compressible case is approximate, it is interesting to note that for $s > 6$, where the solution was reasoned to be applicable, the growth of lift is less rapid for the compressible case than for the incompressible case. This phenomenon would be expected since the growth of lift due to the $k_1(s)$ function, which was used to determine the $k_2(s)$ function, shows this same effect as evidenced by figure 4.

POSSIBLE EFFECTS ON GUST LOAD FACTOR

The fact that the growth of lift for the $k_1(s)$ and $k_2(s)$ functions is less for the $M = 0.7$ case than for the $M = 0.0$ case may have some significance in the determination of the response of an airplane to gusts. The current design procedure for the loads due to gusts on an airplane utilizes the two-dimensional incompressible-flow $k_1(s)$ and $k_2(s)$ functions and an over-all approximate correction for compressibility. If in the calculation of the gust load factor at about $M = 0.7$ the corresponding $k_1(s)$ and $k_2(s)$ functions for $M = 0.7$ were used instead of the incompressible $k_1(s)$ and $k_2(s)$ functions with the over-all approximate correction for compressibility, then the resulting load factor would probably be less than that found by using the incompressible $k_1(s)$ functions. This further suggests the desirability of obtaining the $k_2(s)$ function for the complete range of s and of obtaining the $k_2(s)$ functions for other values of Mach number so that a greater insight may be had as to the effect of compressibility on the gust load factor.

CONCLUDING REMARKS

The reciprocal equations which relate the incompressible circulatory lift to the lift due to harmonic oscillations have been modified to include the noncirculatory lift due to apparent-mass effects. While the lift due to apparent-mass effects for the indicial case is known to be impulsive for the incompressible case, the lift due to apparent-mass effects for the corresponding compressible case was found to be finite and a time-dependent function.

The reciprocal equations are evaluated numerically for determining the indicial lift and moment on an airfoil experiencing a sudden acquisition of vertical velocity at $M = 0.7$. Lack of sufficient flutter coefficients prevents the calculation of these functions at other Mach numbers.

The growth of lift to the steady-state value on an airfoil experiencing a sudden acquisition of vertical velocity was found to be less rapid for the compressible case than for the incompressible case.

An approximate calculation is made for a portion of the indicial-lift function due to penetration of a sharp-edge gust at $M = 0.7$. It is noted that the growth of lift to the steady-state value for this indicial lift function was also less rapid than for the incompressible case. As a consequence of these phenomena, the calculation of the gust load factor at high-subsonic Mach numbers utilizing the two-dimensional incompressible indicial lift functions and an over-all correction for compressibility such as the Prandtl-Glauert factor might be conservative.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., July 30, 1951

APPENDIX A

PROOF FOR $\lim_{s \to +0} k_1(s) = F_c(\infty)$

The function $k_1(s)$ can be expressed as a function of $F_c(k)$ by the following equation:

$$k_1(s) = \frac{2}{\pi}\int_0^{\infty} \frac{F_c(k) \sin ks}{k}\, dk \tag{A1}$$

provided $F_c(k)$ is a bounded and well-behaved function. The value of $k_1(s)$ as s tends to zero positively may by evaluated by taking the limit of both sides of equation (A1) as $s \to +0$:

$$\lim_{s \to +0} k_1(s) = \lim_{s \to +0} \frac{2}{\pi}\int_0^{\infty} \frac{F_c(k) \sin ks}{k}\, dk \tag{A2}$$

Before the limits in equation (A2) are evaluated, it is convenient to make the following substitution for $F_c(k)$:

$$F_c(k) = f(k) + F_c(\infty) \tag{A3}$$

Substitution of this expression for $F_c(k)$ into equation (A2) leads to the following expression for $k_1(s)$:

$$\lim_{s \to 0} k_1(s) = \lim_{s \to 0} \frac{2}{\pi}\left(\int_0^{\infty} \frac{f(k) \sin ks}{k}\, dk + F_c(\infty)\int_0^{\infty} \frac{\sin ks}{k}\, dk\right) \tag{A4}$$

Although the second integral in equation (A4) can be evaluated readily to give

$$\lim_{s \to +0} \int_0^\infty \frac{\sin ks}{k} dk = \frac{\pi}{2} \tag{A5}$$

the evaluation of the first integral in equation (A4) is not readily evident; however it can be shown (see lemma) that

$$\lim_{s \to 0} \int_0^\infty \frac{f(k) \sin ks}{k} dk = 0 \tag{A6}$$

since

$$\lim_{k \to \infty} f(k) = 0$$

As a consequence of equations (A5) and (A6) the value of the limit in equation (A4) is

$$\lim_{s \to +0} k_1(s) = F_c(\infty) \tag{A7}$$

Lemma:

Prove that

$$\lim_{s \to +0} \int_0^\infty \frac{f(k) \sin ks}{k} dk = 0 \tag{A8}$$

provided $f(k)$ is a bounded and integrable function within the limits of integration and

$$\lim_{k \to \infty} f(k) = 0$$

If the following substitution is made for ks

$$ks = x$$

an alternate expression for equation (A8) is had

$$\lim_{s \to +0} \int_0^{\infty} f\left(\frac{x}{s}\right) \frac{\sin x}{x}\, dx = 0 \tag{A9}$$

Since $f\left(\frac{x}{s}\right)$ is bounded and integrable within the limits of integration and since the integral

$$\int_0^{\infty} \frac{\sin x}{x}\, dx$$

is absolutely convergent, then equation (A9) is uniformly convergent for $s \geqq 0$ within the limits of integration. Consequently, there is a positive number X such that

$$\left| \int_X^{\infty} f\left(\frac{x}{s}\right) \frac{\sin x}{x}\, dx \right| < \frac{1}{3}\epsilon \tag{A10}$$

where ϵ is an arbitrary number chosen as small as desired and the number X is independent of the value of s. Also a number x_o, independent of s, may be chosen so that

$$\left| \int_0^{x_o} f\left(\frac{x}{s}\right) \frac{\sin x}{x}\, dx \right| < \frac{1}{3}\epsilon \tag{A11}$$

However,

$$\int_{x_o}^{X} f\left(\frac{x}{s}\right) \frac{\sin x}{x}\, dx = f\left(\frac{x_o}{s}\right) \int_{x_o}^{\xi} \frac{\sin x}{x}\, dx + f\left(\frac{X}{s}\right) \int_{\xi}^{X} \frac{\sin x}{x}\, dx$$

where

$$x_o \leqq \xi \leqq X$$

It can be shown that

$$\left| \int_p^q \frac{\sin x}{x} \, dx \right| < \pi \qquad (q > p > 0)$$

therefore

$$\left| \int_{x_o}^{X} f\left(\frac{x}{s}\right) \frac{\sin x}{x} \, dx \right| < 2\pi f\left(\frac{x_o}{s}\right)$$

thus a value of S can be chosen as small as desired so that

$$\left| \int_{x_o}^{X} f\left(\frac{x}{s}\right) \frac{\sin x}{x} \, dx \right| < \frac{1}{3} \epsilon \qquad (s \leqq S) \qquad \text{(A12)}$$

Combining the results given by equations (A10), (A11), and (A12) yields the equation

$$\left| \int_0^{\infty} f\left(\frac{x}{s}\right) \frac{\sin x}{x} \, dx \right| < \frac{1}{3} \epsilon + \frac{1}{3} \epsilon + \frac{1}{3} \epsilon \qquad (s \leqq S) \qquad \text{(A13a)}$$

thus, in the limit as $S \longrightarrow 0$ positively

$$\lim_{s \longrightarrow +0} \int_0^{\infty} f\left(\frac{x}{s}\right) \frac{\sin x}{x} \, dx = 0 \qquad \text{(A13b)}$$

or

$$\lim_{s \longrightarrow +0} \int_0^{\infty} \frac{f(k) \sin ks \, dk}{k} = 0 \qquad \text{(A13c)}$$

APPENDIX B

NUMERICAL EVALUATION OF INCOMPRESSIBLE-FLOW INDICIAL LIFT FUNCTION

DUE TO PENETRATION OF A SHARP-EDGE GUST

The indicial lift function due to penetration of a sharp-edge gust $k_2(s)$ expressed in terms of the indicial lift function $k_1(s)$ is given by equations (23a) and (23b). Although a numerical evaluation of the first term in equation (23a) may be performed with the aid of Simpson's rule, the numerical evaluation of the integral in equation (23b) is difficult in that the integrand is infinite at $s_1 = 2$. In order to overcome this difficulty, a parabolic-arc segment can be fitted to the function $k_1(s - s_1)$, between the limits $s_1 = \frac{3}{2}$ and $s_1 = 2$. Because of the similarity between equations (23a) and (23b), the numerical solution of both equations can be written in one equation with the aid of matrix notation. For simplicity, the numerical solution is calculated at integral values of s. However, for increased accuracy the interval of integration is taken at quarter multiples of s. In view of these considerations, the numerical solution for the $k_2(s)$ function may be performed in three steps:

(1) Integration of integrand for the limits $s_1 = 0$ to $s_1 = 1$

With the aid of Simpson's integrating factors the following integral is evaluated, a quarter multiple of s being assumed for the interval of integration:

$$\frac{1}{\pi}\int_0^1 k_1(s - s_1)\sqrt{\frac{s_1}{2 - s_1}}\,ds_1 \approx \frac{1}{\pi}\,\frac{1}{4}\left[\frac{1}{3}(0)k_1(s - 0) + \frac{4}{3}\sqrt{\frac{1}{7}}\,k_1\left(s - \frac{1}{4}\right) + \frac{2}{3}\sqrt{\frac{1}{3}}\,k_1\left(s - \frac{1}{2}\right) + \frac{4}{3}\sqrt{\frac{3}{5}}\,k_1\left(s - \frac{3}{4}\right) + \frac{1}{3}(1)k_1(s - 1)\right] \tag{B1}$$

(2) Integration of integrand for the limits $s_1 = 1$ to $s_1 = \frac{3}{2}$

In a manner similar to that shown for the evaluation of the integral in step 1, Simpson's integrating factors can be utilized to calculate the same integral over the limits $s_1 = 1$ to $s_1 = \frac{3}{2}$ as follows:

$$\frac{1}{\pi}\int_{1}^{3/2} k_1(s - s_1)\sqrt{\frac{s_1}{2 - s_1}}\,ds_1 \approx \frac{1}{\pi}\,\frac{1}{4}\left[\frac{1}{3}(1)k_1(s - 1) + \frac{4}{3}\sqrt{\frac{5}{3}}\,k_1\left(s - \frac{5}{4}\right) + \frac{1}{3}\sqrt{3}k_1\left(s - \frac{3}{2}\right)\right] \tag{B2}$$

(3) Integration of integrand for the limits $s_1 = \frac{3}{2}$ to $s_1 = 2$

As noted previously the integration for these limits is not possible by means of the simple Simpson's factors since the integrand is infinite at $s_1 = 2$. With the change in variable $s_1 = 2 - \sigma$, a parabolic-arc segment is fitted to the function $k_1(s - s_1)$ between the limits $s_1 = \frac{3}{2}$ to $s_1 = 2$ by use of the following equation:

$$k_1(s - 2 + \sigma) \approx A + B\sigma + C\sigma^2 \tag{B3}$$

A set of simultaneous equations relating the function $k_1(s - 2 + \sigma)$ with the constants A, B, and C in the parabolic-arc segment may be obtained by evaluating equation (B3) at the points $\sigma = 0$, $\sigma = \frac{1}{4}$, and $\sigma = \frac{1}{2}$ as follows: at $\sigma = 0$,

$$k_1(s - 2) = A \tag{B4}$$

at $\sigma = \frac{1}{4}$,

$$k_1\left(s - \frac{7}{4}\right) = A + \frac{1}{4}B + \frac{1}{16}C \tag{B5}$$

at $\sigma = \frac{1}{2}$

$$k_1\left(s - \frac{3}{2}\right) = A + \frac{1}{2}B + \frac{1}{4}C \tag{B6}$$

When equations (B4), (B5), and (B6) are solved simultaneously for the constants A, B, and C, the resulting expressions are

$$\left.\begin{aligned} A &= k_1(s - 2) \\ B &= -6k_1(s - 2) + 8k_1\left(s - \frac{7}{4}\right) - 2k_1\left(s - \frac{3}{2}\right) \\ C &= 8k_1(s - 2) - 16k_1\left(s - \frac{7}{4}\right) + 8k_1\left(s - \frac{3}{2}\right) \end{aligned}\right\} \tag{B7}$$

If the expression for $k_1(s - 2 + \sigma)$ given by equation B(3) is substituted into the integrand under consideration together with the values of A, B, and C determined by equations (B7), the integral can be evaluated in closed form in terms of the unsteady-lift function $k_1(s - s_1)$ from $s_1 = \frac{3}{2}$ to $s_1 = 2$. The results of this closed-form integration are as follows:

$$\frac{1}{\pi}\int_{3/2}^{2} k_1(s - s_1)\sqrt{\frac{s_1}{2 - s_1}}\, ds_1 \approx \frac{14\pi - 4\sqrt{3}}{12\pi} k_1(s - 2) +$$

$$\frac{3\sqrt{3} - 2\pi}{\pi} k_1\left(s - \frac{7}{4}\right) + \frac{14\pi - 21\sqrt{3}}{12\pi} k_1\left(s - \frac{3}{2}\right) \tag{B8}$$

Combining equations (B1), (B2), and (B8) allows the solution of the two equations (23a) and (23b) for the $k_2(s)$ function to be written in a single matrix equation for integral multiples of s as follows:

$$
\begin{Bmatrix} k_2(1) \\ k_2(2) \\ k_2(3) \\ k_2(4) \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}
= \frac{1}{\pi}
\begin{bmatrix}
\frac{1}{12} & \frac{\sqrt{15}}{15} & \frac{\sqrt{3}}{18} & \frac{\sqrt{7}}{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\frac{14\pi-9\sqrt{3}}{12} & 3\sqrt{3}-2\pi & \frac{7\pi-10\sqrt{3}}{6} & \frac{\sqrt{15}}{9} & \frac{1}{6} & \frac{\sqrt{15}}{15} & \frac{\sqrt{3}}{18} & \frac{\sqrt{7}}{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & \frac{14\pi-9\sqrt{3}}{12} & 3\sqrt{3}-2\pi & \frac{7\pi-10\sqrt{3}}{6} & \frac{\sqrt{15}}{9} & \frac{1}{6} & \frac{\sqrt{15}}{15} & \frac{\sqrt{3}}{18} & \frac{\sqrt{7}}{21} & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{14\pi-9\sqrt{3}}{12} & 3\sqrt{3}-2\pi & \frac{7\pi-10\sqrt{3}}{6} & \frac{\sqrt{15}}{9} & \frac{1}{6} & \frac{\sqrt{15}}{15} & \frac{\sqrt{3}}{18} & \frac{\sqrt{7}}{21} & \cdots
\end{bmatrix}
\begin{Bmatrix} k_1(0) \\ k_1\left(\frac{1}{4}\right) \\ k_1\left(\frac{1}{2}\right) \\ k_1\left(\frac{3}{4}\right) \\ k_1(1) \\ k_1\left(1\frac{1}{4}\right) \\ k_1\left(1\frac{1}{2}\right) \\ k_1\left(1\frac{3}{4}\right) \\ k_1(2) \\ k_1\left(2\frac{1}{4}\right) \\ k_1\left(2\frac{1}{2}\right) \\ k_1\left(2\frac{3}{4}\right) \\ k_1(3) \\ k_1\left(3\frac{1}{4}\right) \\ k_1\left(3\frac{1}{2}\right) \\ k_1\left(3\frac{3}{4}\right) \\ \vdots \end{Bmatrix}
+
\begin{Bmatrix} \frac{1}{\pi} \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}
\qquad \text{(B9)}
$$

In figure 10 the solutions for the $k_2(s)$ function are given for $M = 0.0$ and $M = 0.7$. No discernable difference was present when the calculated values for $M = 0.0$ were compared with the Küssner function.

REFERENCES

1. Wagner, Herbert: Über die Entstehung des dynamischen Auftriebes von Tragflügeln. Z.f.a.M.M., Bd. 5, Heft 1, Feb. 1925, pp. 17-35.

2. Küssner, H. G.: Das zweidimensionale Problem der beliebig bewegten Tragfläche unter Berücksichtigung von Partialbewegungen der Flüssigkeit. Luftfahrtforschung, Bd. 17, Lfg. 11/12, Dec. 10, 1940, pp. 355-361.

3. Garrick, I. E.: On Some Fourier Transforms in the Theory of Non-Stationary Flows. Proc. Fifth Int. Cong. Appl. Mech. (Cambridge, Mass., 1938), John Wiley & Sons, Inc., 1939, pp. 590-593.

4. Lomax, Harvard, Heaslet, Max. A., and Sluder, Loma: The Indicial Lift and Pitching Moment for a Sinking or Pitching Two-Dimensional Wing Flying at Subsonic or Supersonic Speeds. NACA TN 2403, 1951.

5. Possio, Camillo: L'Azione aerodinamica sul profilo oscillante in un fluido compressibile a velocità ipsonora. L'Aerotecnica, vol. XVIII, fasc. 4, April 1938, pp. 441-458. (Available as British Air Ministry Translation No. 830.)

6. Dietze, F.: The Air Forces of the Harmonically Vibrating Wing in Compressible Medium at Subsonic Velocity (Plane Problem). Part II: Numerical Tables and Curves. Translation No. F-TS-948-RE, Air Materiel Command, U. S. Air Force, March 1947.

7. Schade, Th.: Numerical Solution of the Possio Integral Equation of the Oscillating Aerofoil in a Two-Dimensional Subsonic Flow. Part IV - Numerical Part. Reps. and Translations No. 846, British M.A.P. Völkenrode, Feb. 1, 1947.

8. Frazer, R. A., and Skan, Sylvia W.: Influence of Compressibility on the Flexural-Torsional Flutter of Tapered Cantilever Wings. Rep. No. 5916, British A.R.C., June 30, 1942. (Reissued as R. & M. No. 2553.)

9. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. NACA Rep. 496, 1935.

10. Bush, Vannevar: Operational Circuit Analysis. John Wiley & Sons, Inc., 1937, p. 183.

11. Jones, Robert T.: The Unsteady Lift of a Wing of Finite Aspect Ratio. NACA Rep. 681, 1940.

TABLE I.- COEFFICIENTS FOR THE LIFT AND MOMENT DUE TO TRANSLATORY OSCILLATION

k	Z_1 for -			Z_2 for -			M_1 for -			M_2 for -			Reference
	M = 0.5	M = 0.6	M = 0.7	M = 0.5	M = 0.6	M = 0.7	M = 0.5	M = 0.6	M = 0.7	M = 0.5	M = 0.6	M = 0.7	
0	0	0	0	0	0	0	0	0	0	0	0	0	
.02	.00417	.00523	.00711	.04385	.04699	.05177	-.00015	-.00018	-.00026	.000005	.00001	.00003	
.04	.01214	.01508	.02006	.08308	.08808	.09524	-.00059	-.00073	-.00097	.00004	.00008	.00016	
.06	.02119	.02611	.03399	.1183	.1243	.1324	-.00131	-.00158	-.00208	.00010	.00020	.00042	
.08	.03016	.03780	.04722	.1503	.1567	.1651	-.00228	-.00274	-.00355	.00021	.00040	.00083	
.10	.03838	.04642	.0589	.1796	.1863	.1941	-.00352	-.0042	-.0053	.00034	.00066	.00126	6
.20	.0614	.0751	.0945	.3057	.3121	.3186	-.01341	-.0155	-.0186	.00163	.00300	.00574	
.30	.0538	.0723	.0991	.4175	.4255	.4332	-.02944	-.0336	-.0393	.00389	.00712	.01346	
.40	.0198	.0447	.0827	.5286	.5407	.5523	-.0518	-.0585	-.0672	.0074	.01354	.02596	
.50	-.0375	-.0022	.0538	.6447	.6645	.6820	-.0805	-.0902	-.1014	.0126	.02324	.04482	
.60	-.1146	-.0646	.0187	.7698	.8006	.8229	-.1160	-.1280	-.1402	.0201	.03752	.07175	
.70	-.2103	-.1377	-.0154	.9071	.9526	.9751	-.1582	-.1742	-.1813	.0308	.0581	.1090	
.80	-.3207	-.2203	-.0449	1.0590	1.1187	1.1316	-.2069	-.2254	-.2178	.0438	.08185	.1578	7
1.00	-.5857	-.3780	-.0874	1.4211	1.5273	1.4482	-.3285	-.3455	-.2798	.0881	.1718	.2665	
1.50	-------	-------	-.39295	-------	-------	2.311	-------	-------	-.4400	-------	-------	.4896	
2.00	-------	-------	-.6521	-------	-------	3.306	-------	-------	-.5740	-------	-------	.77185	8
2.50	-------	-------	-.6083	-------	-------	4.241	-------	-------	-.4547	-------	-------	1.218	

NACA

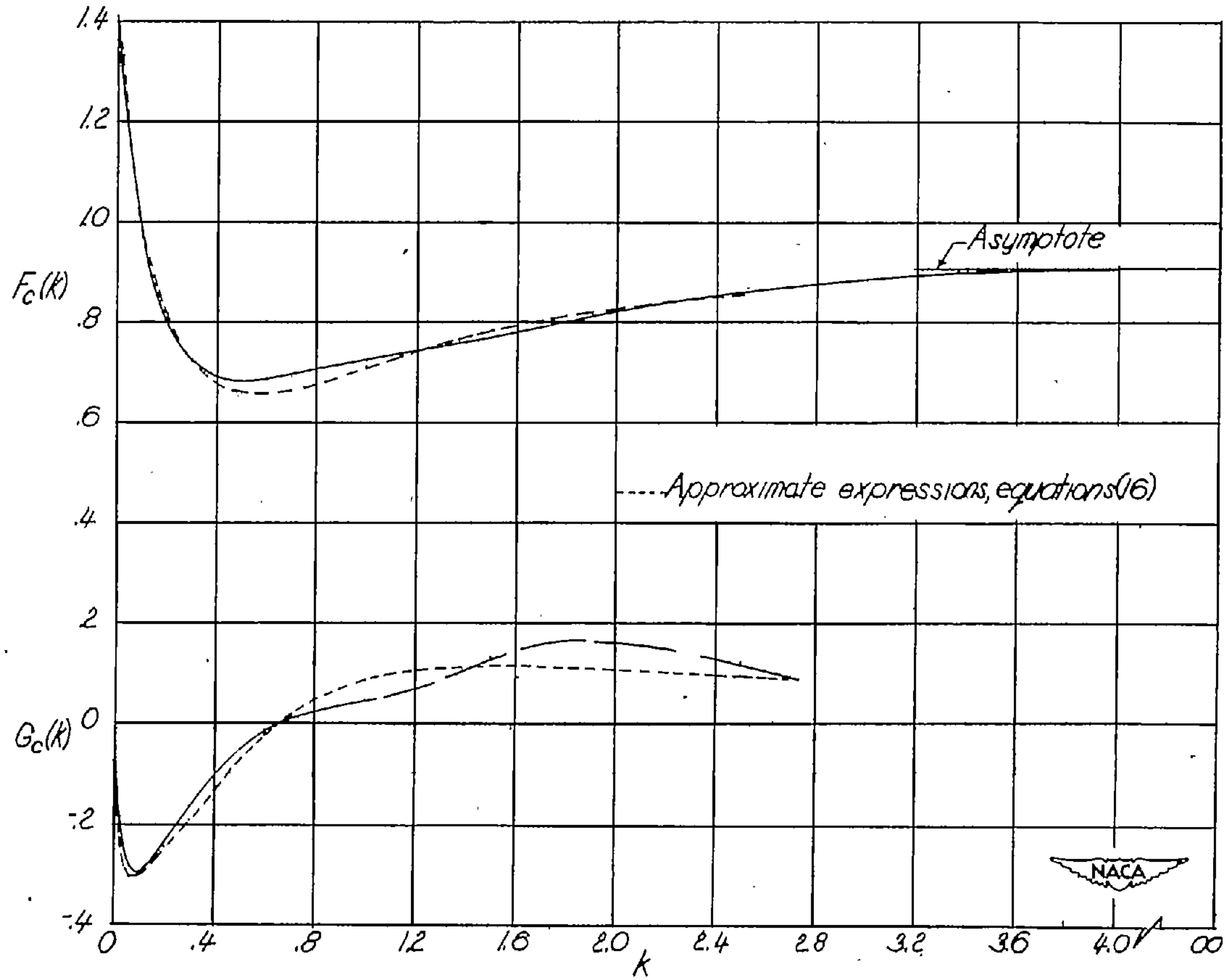

Figure 1.- Lift functions due to translatory oscillation at M = 0.7.

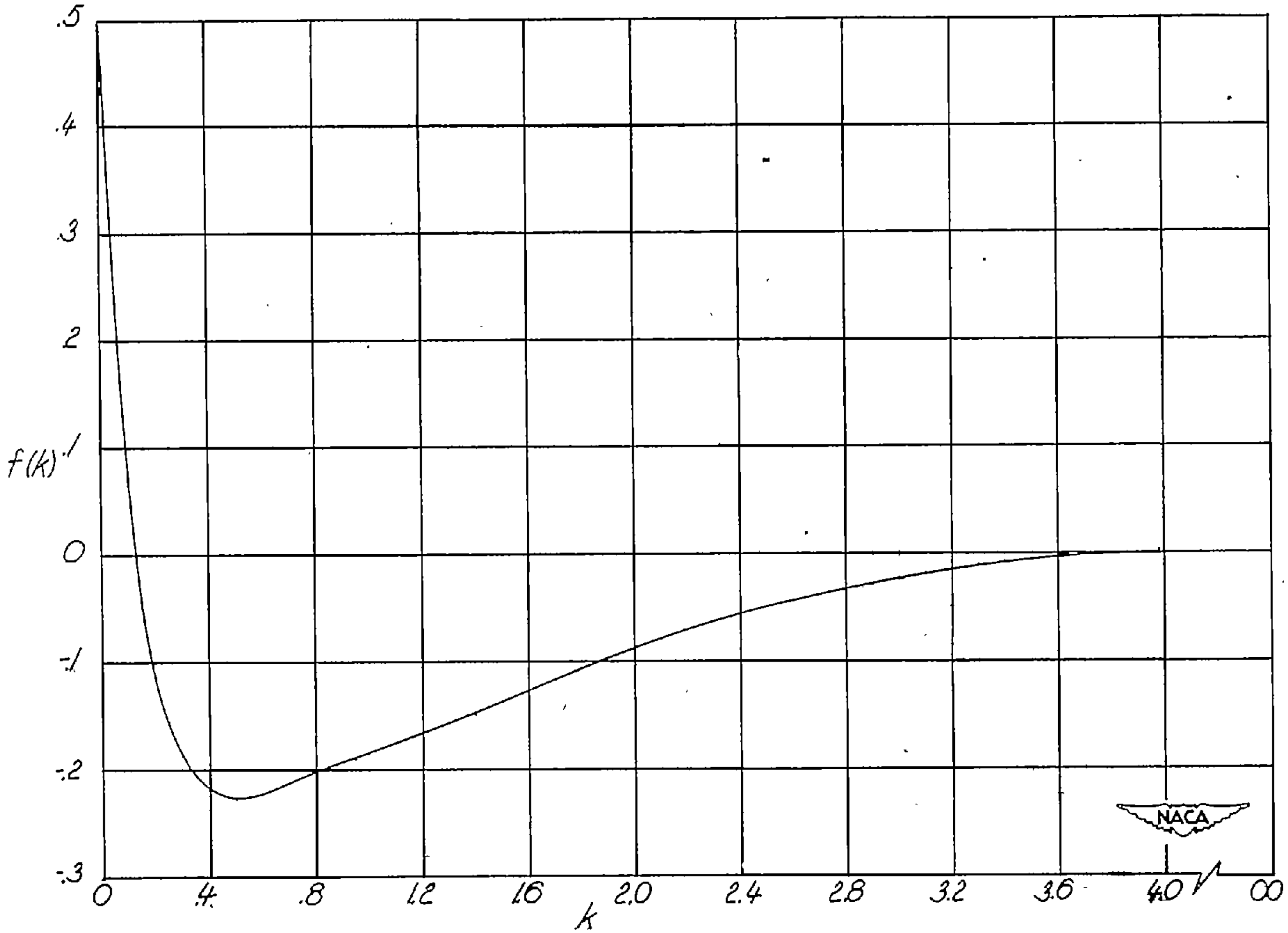

Figure 2.- Plot of f(k).

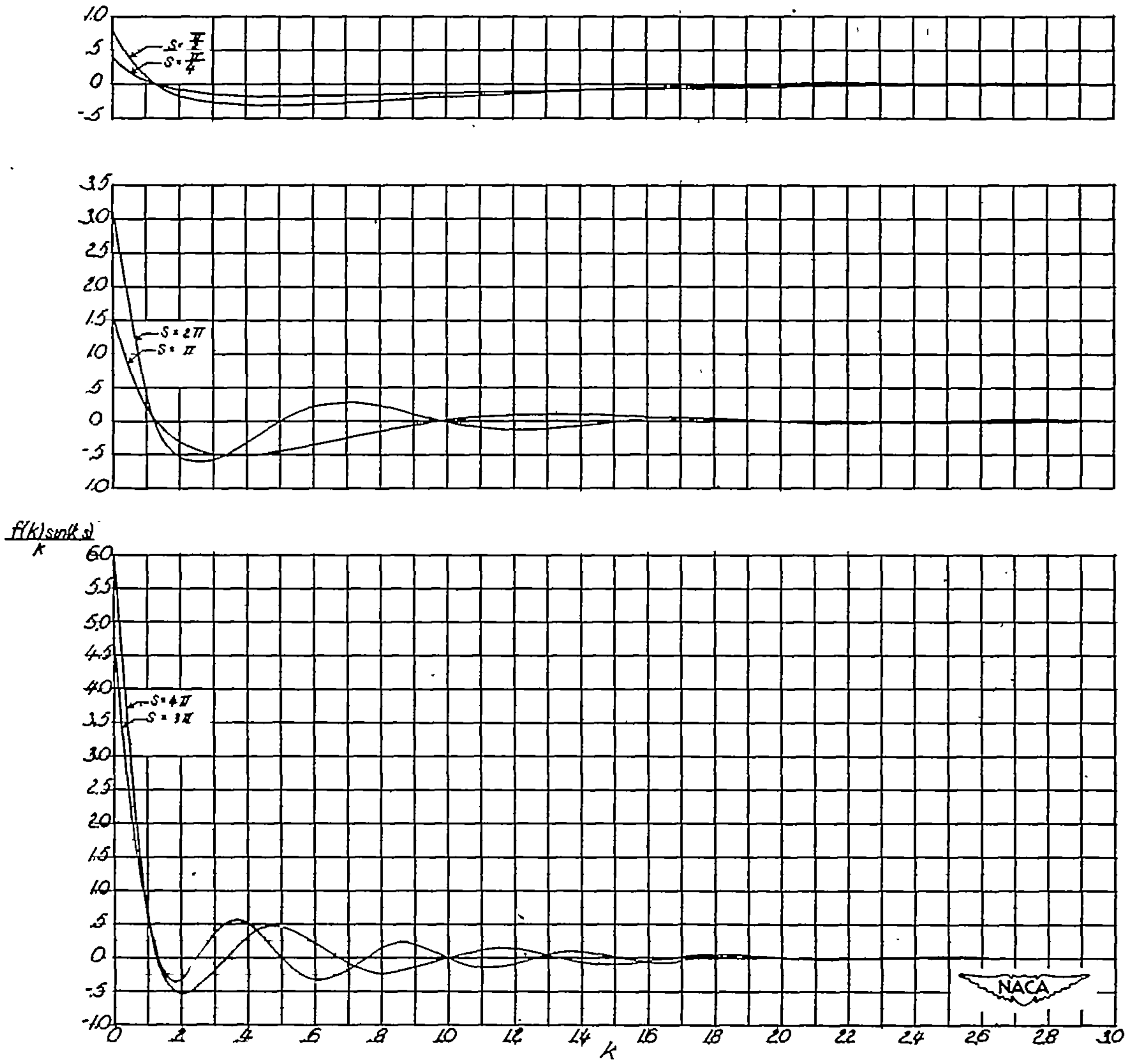

Figure 3.- Plots of the integrand in equation (14) for determining the indicial lift $k_1(s)$ at $M = 0.7$.

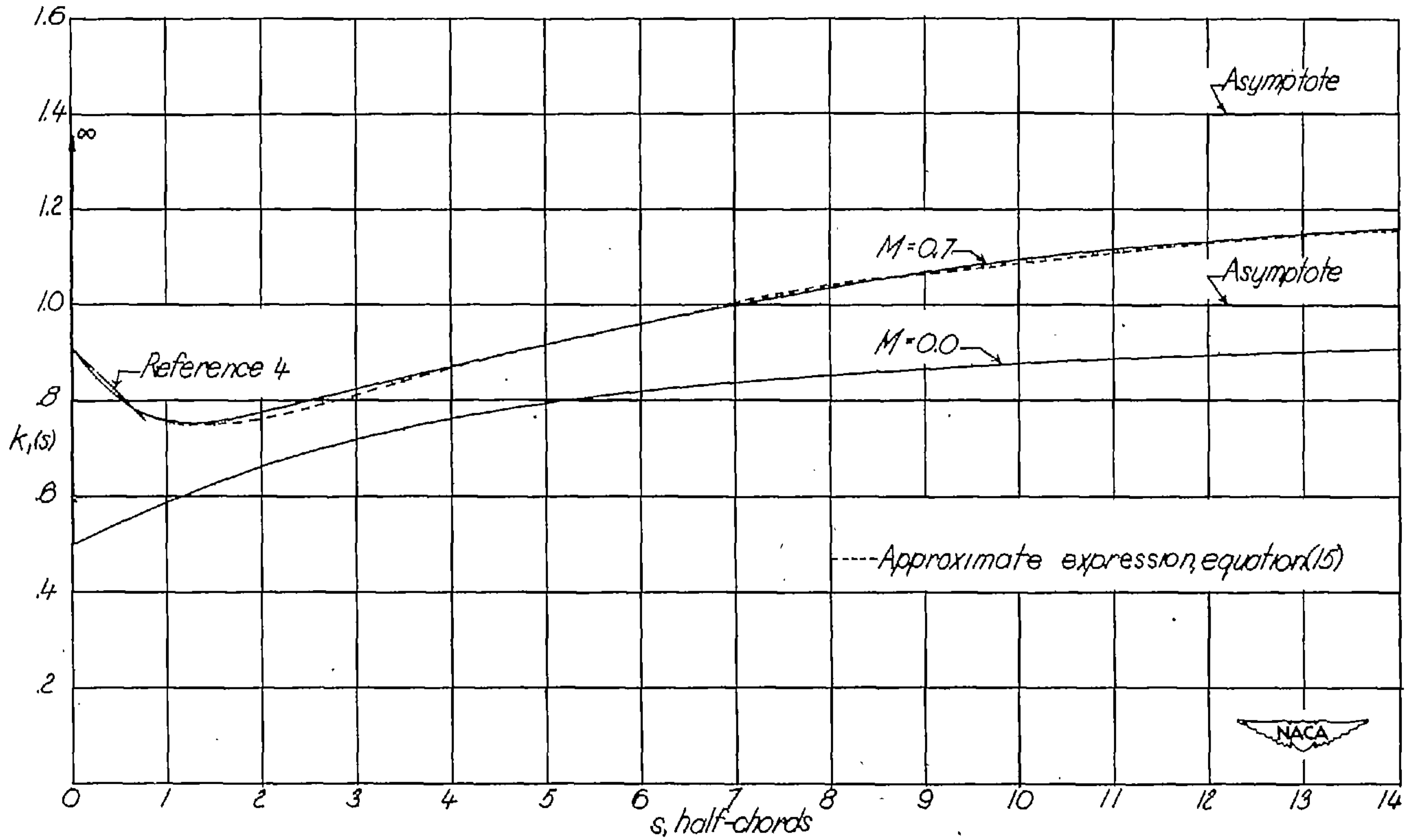

Figure 4.- Comparison of the indicial lift functions due to a sudden acquisition of vertical velocity at M = 0.0 and M = 0.7.

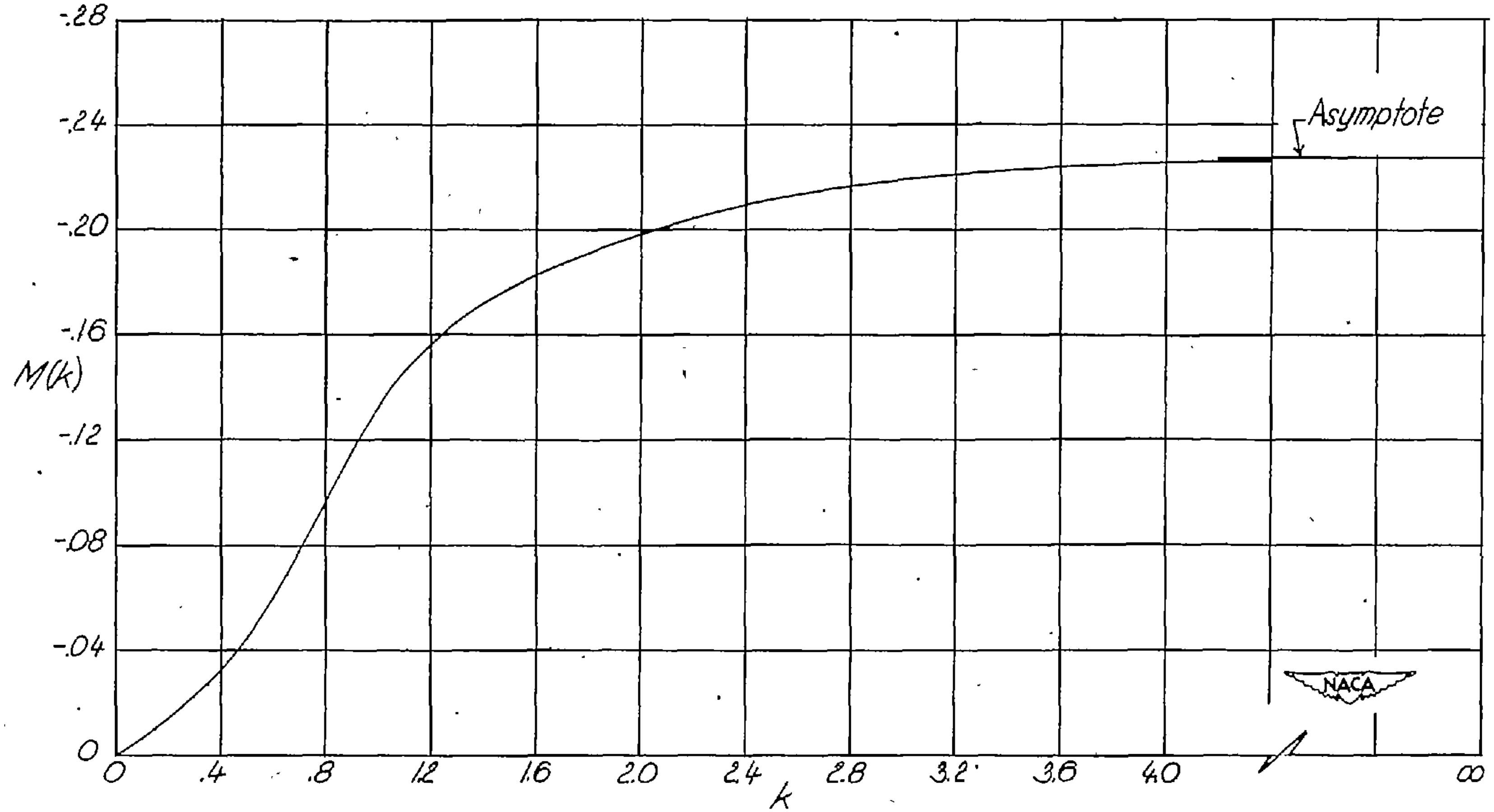

Figure 5.- Function for the real part of the moment about the quarter chord due to translatory oscillation at M = 0.7.

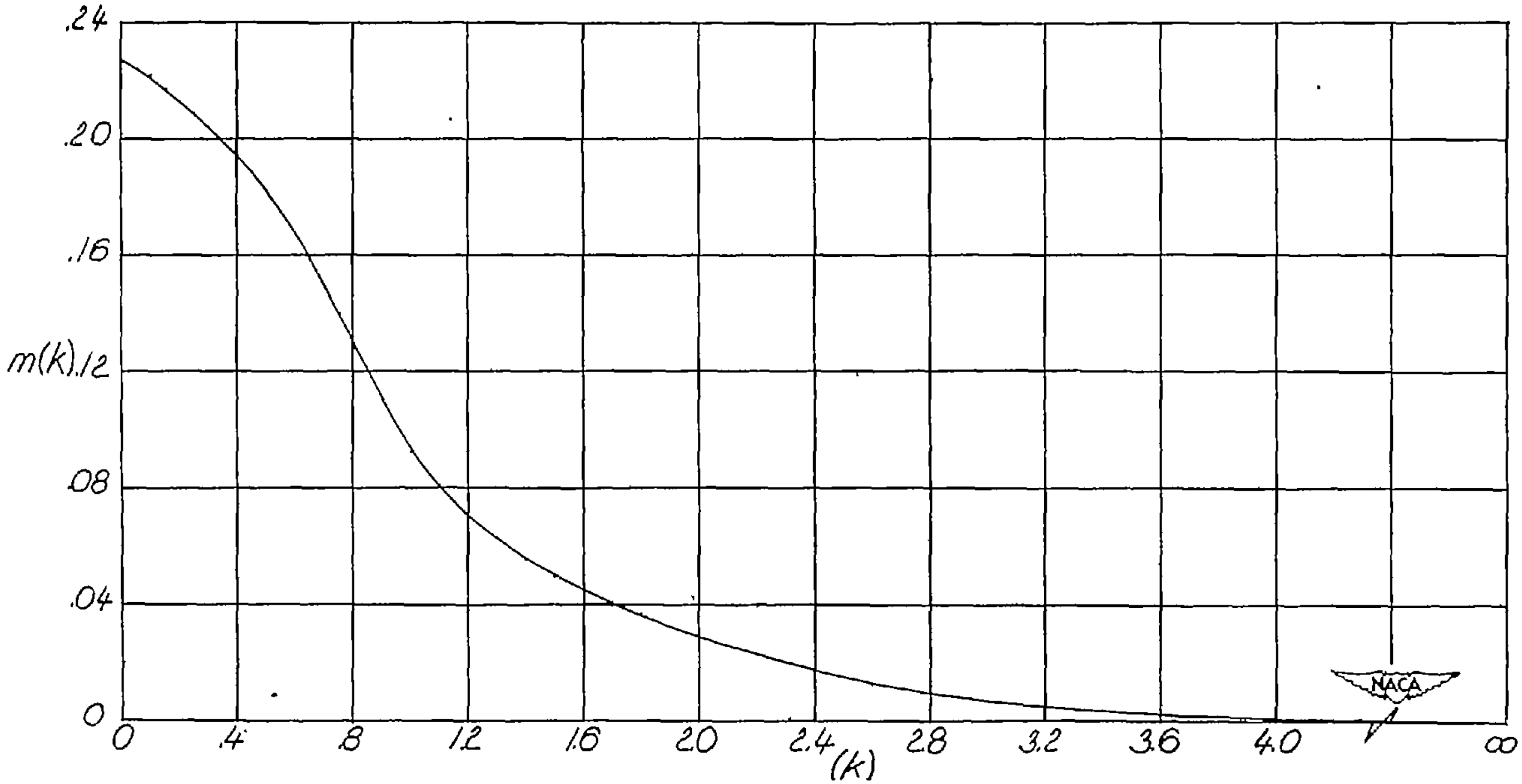

Figure 6.- Plot of m(k).

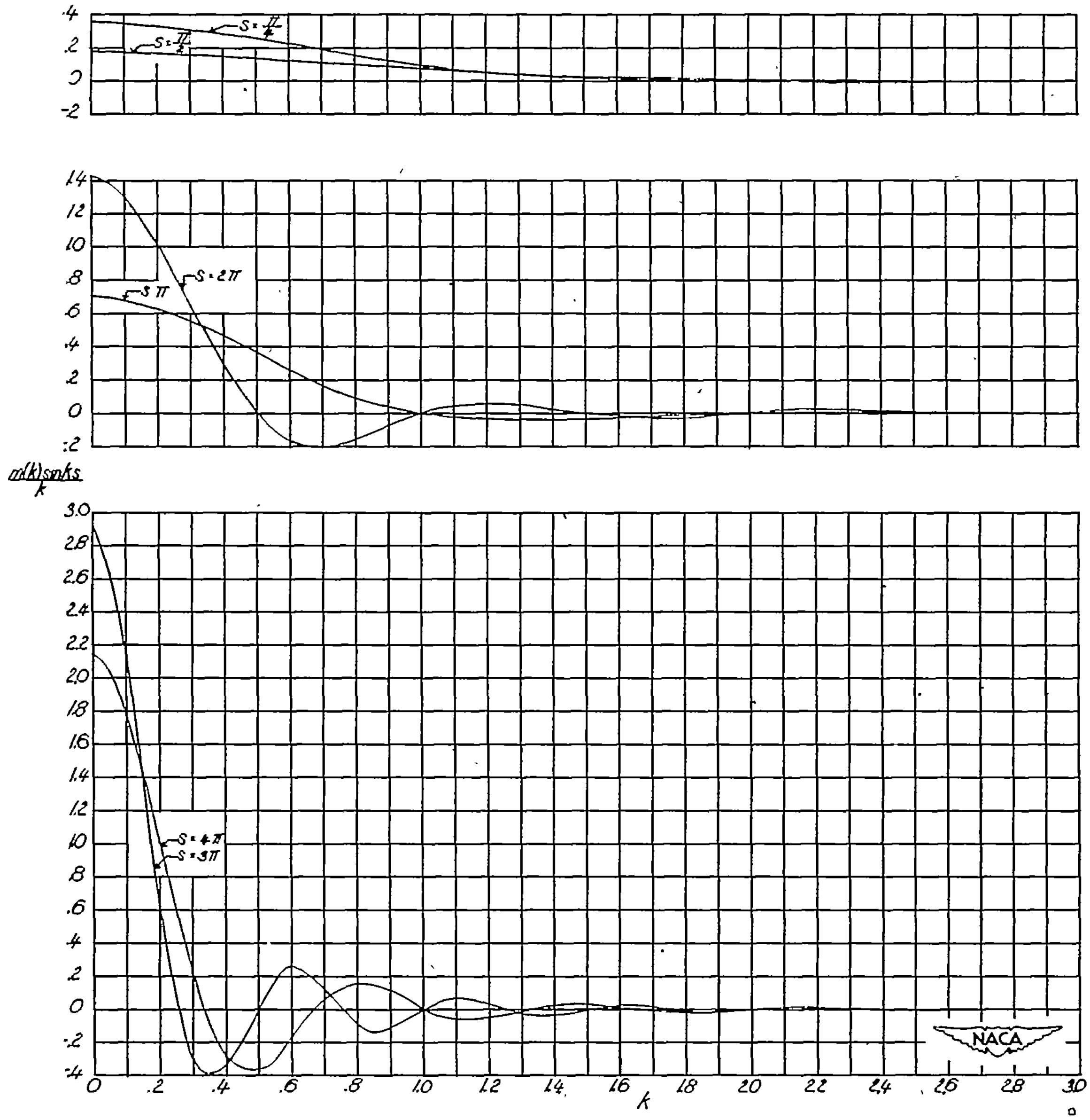

Figure 7.- Plots of the integrand in equation (21) for determining the indicial moment $m_1(s)$ about the quarter chord at M = 0.7.

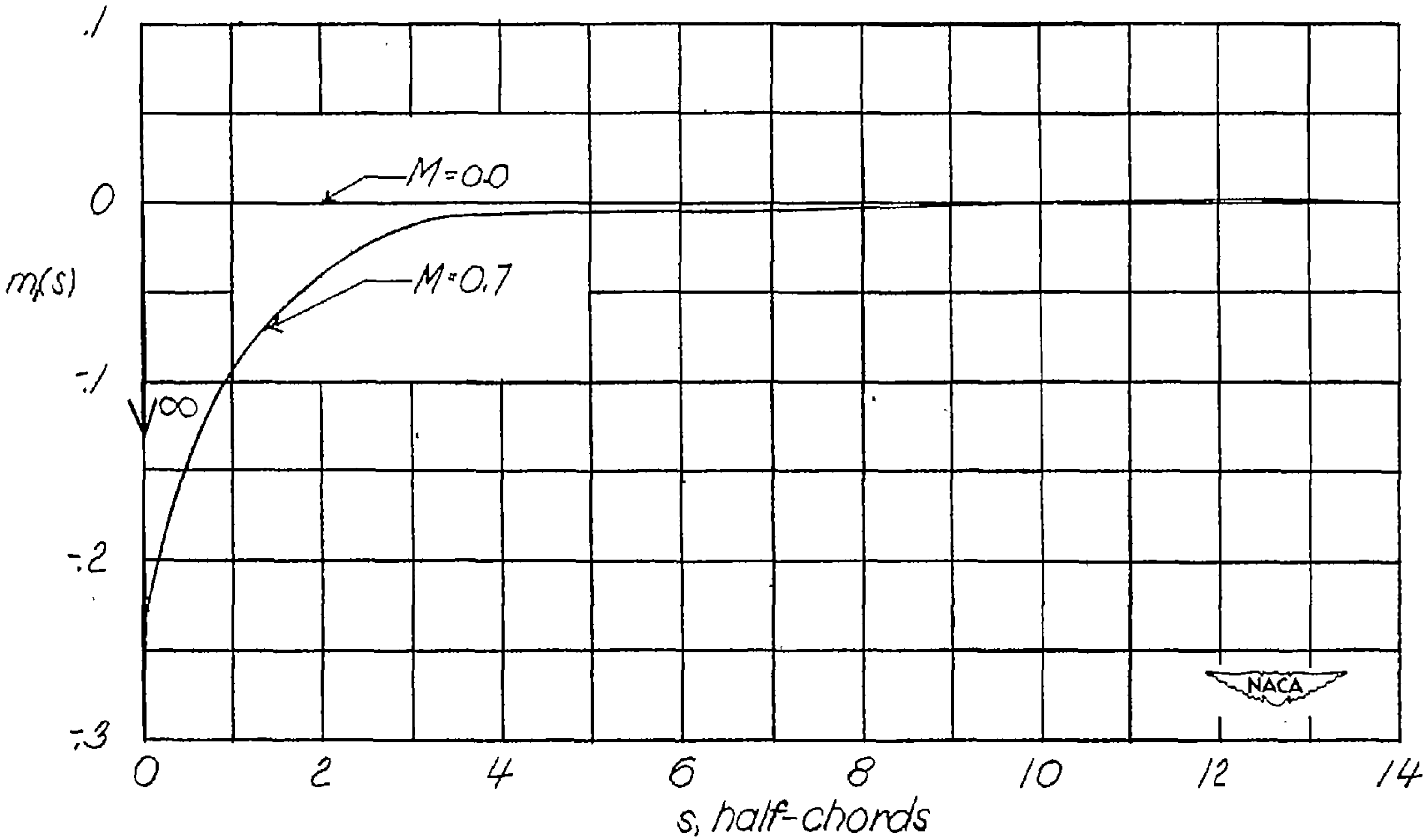

Figure 8.- Comparison of the indicial moment function $m_1(s)$ due to a sudden acquisition of vertical velocity at $M = 0.0$ and $M = 0.7$. (Moment taken about quarter chord.)

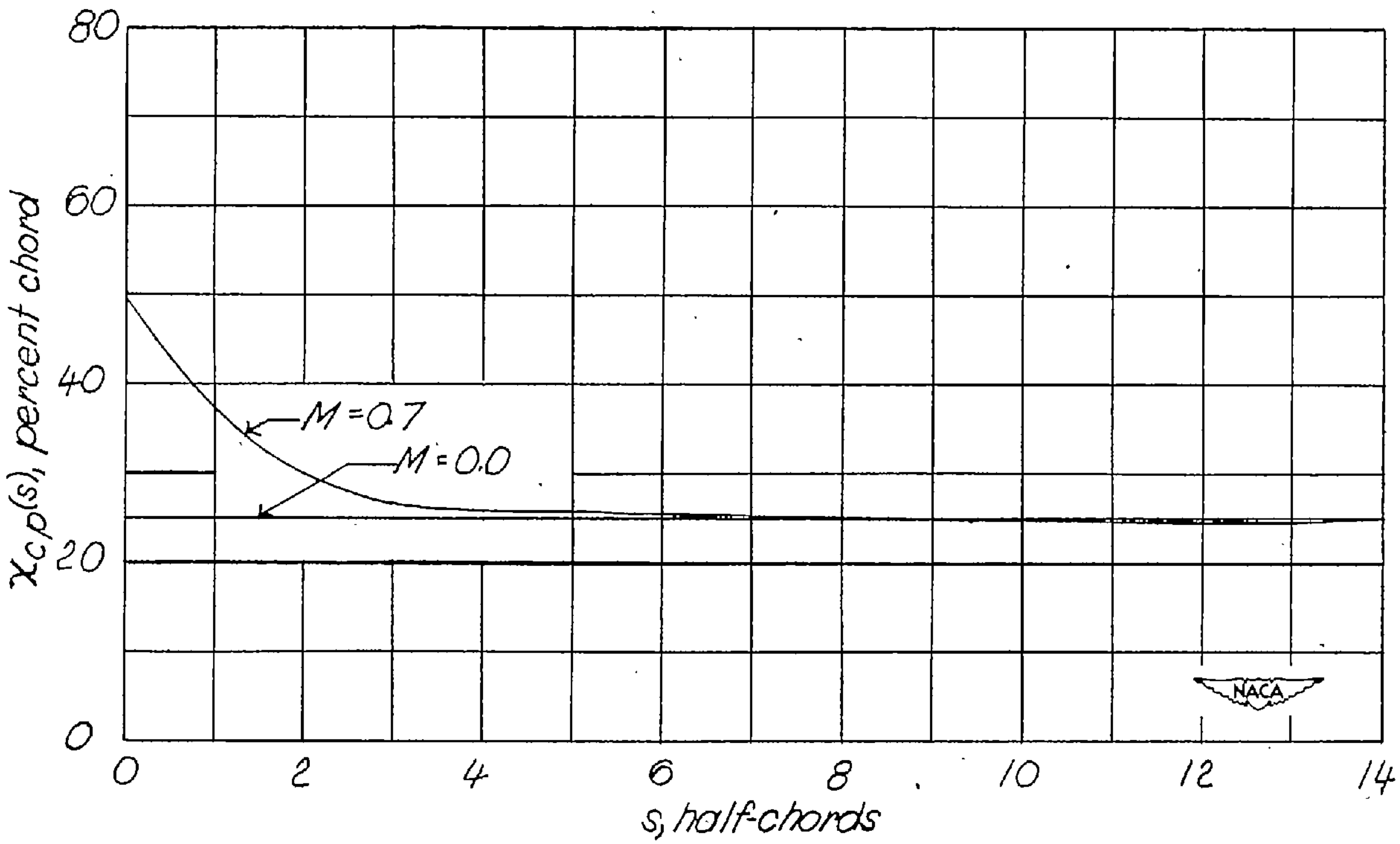

Figure 9.- Comparison of center-of-pressure variation from leading edge of airfoil due to a sudden acquisition in vertical velocity at M = 0.0 and M = 0.7.

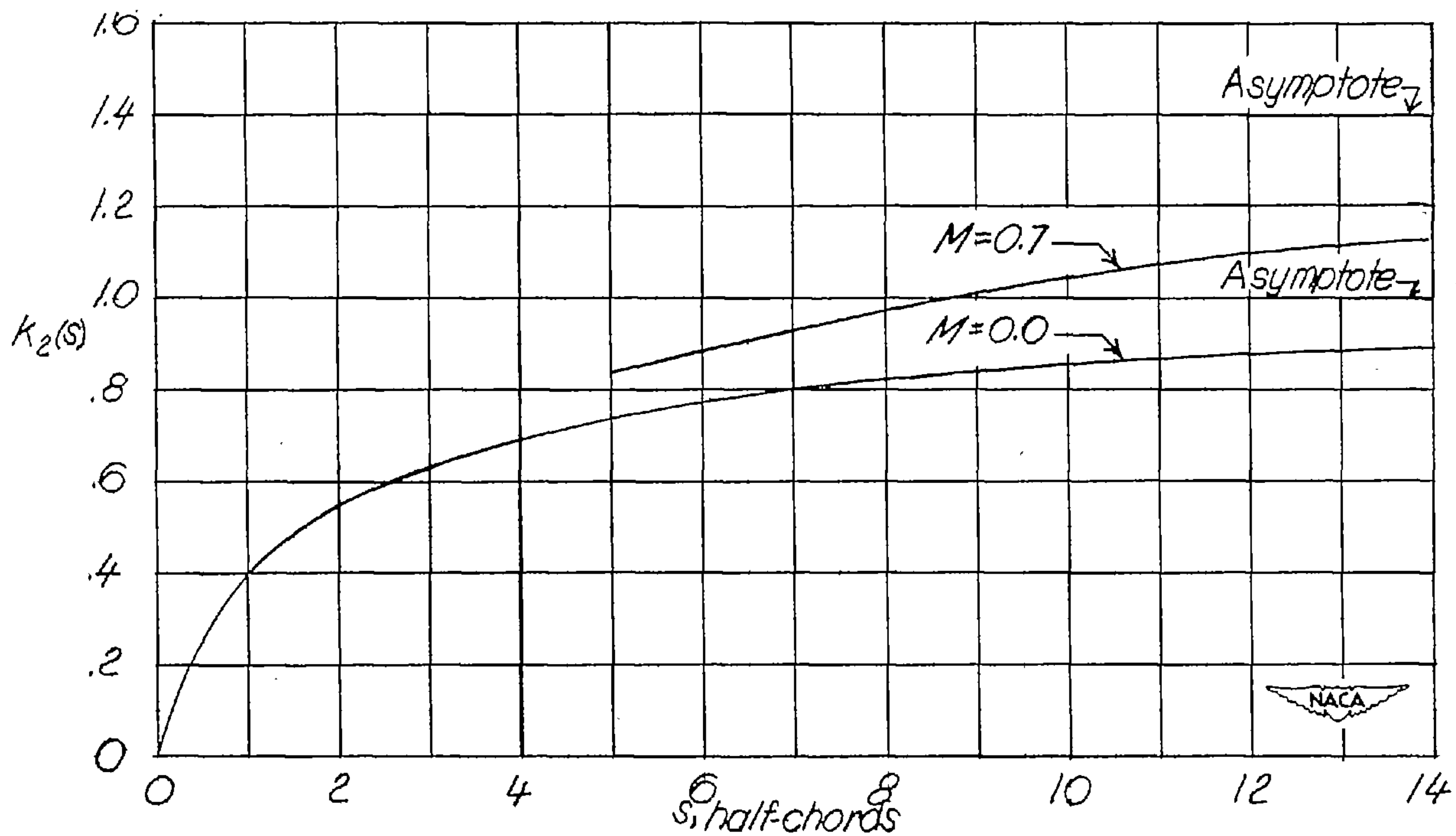

Figure 10.- Comparison of the indicial lift function due to penetration of a sharp-edge gust at M = 0.0 and M = 0.7.

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