NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2565

A THEORETICAL ANALYSIS OF THE EFFECT OF SEVERAL AUXILIARY DAMPING DEVICES ON THE LATERAL STABILITY AND CONTROLLABILITY OF A HIGH-SPEED AIRCRAFT

By Ordway B. Gates, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
December 1951

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2565

A THEORETICAL ANALYSIS OF THE EFFECT OF SEVERAL AUXILIARY DAMPING DEVICES ON THE LATERAL STABILITY AND CONTROLLABILITY OF A HIGH-SPEED AIRCRAFT

By Ordway B. Gates, Jr.

SUMMARY

A theoretical analysis has been made of the effect of several auxiliary damping devices on the lateral stability and controllability of a high-speed aircraft. The systems investigated included stabilization devices which deflect the rudder or an auxiliary surface proportional to the yawing velocity, rolling velocity, or rolling acceleration and one which deflects both aileron and rudder proportional to the rolling velocity. An idealized control system without phase lag was assumed for the analysis.

The present investigation indicated that each of the assumed stabilization systems is capable of improving the damping of the lateral oscillations of the assumed aircraft. The system which deflected the rudder proportional to yawing velocity made necessary increased pedal forces in steady turns, and the systems which deflected the rudder or rudder and ailerons proportional to rolling velocity required unnatural rudder deflections to maintain zero sideslip subsequent to an applied rolling moment. The system which deflected the rudder proportional to rolling acceleration introduced adverse yaw subsequent to applied yawing or rolling moments.

INTRODUCTION

Recently much interest has been shown in automatic stabilization devices as a means of improving the damping of the lateral oscillation of some aircraft designed for transonic and supersonic flight. The investigations reported in references 1 to 3 were concerned primarily with the effect of these devices on the damping of the aircraft lateral oscillation, with little or no emphasis on the problem of lateral controllability. Investigation, therefore, of the effect of a number of stabilization systems on the lateral controllability, as well as oscillatory damping, of present-day high-speed aircraft seemed desirable since both factors are significant in a pilot's evaluation of the flying qualities of an aircraft equipped with a particular stabilization system.

The type of stabilization devices which are analyzed are those which deflect a control surface proportional to the angular velocity in either yaw or roll, or to one of their time derivatives. The assumption is made that there is zero phase shift in the stabilization system, and, that the stabilization system gain is independent of frequency.

The results of this investigation are presented in the form of aircraft motions subsequent to rudder or aileron deflections, comparisons of the time to damp to half amplitude and the period of the lateral modes of motions, and plots of the rudder motion required to perform a perfectly coordinated turn for a given aileron deflection, for each stabilization system discussed. In addition, the effect of each assumed stabilization system on the ratio of aileron deflection to rudder deflection required for a steady turning maneuver is discussed.

SYMBOLS AND COEFFICIENTS

ϕ	angle of roll, radians
ψ	angle of yaw, radians
β	angle of sideslip, radians (v/V)
$r,\dot{\psi}$	yawing angular velocity, radians per second $(d\psi/dt)$
$p,\dot{\phi}$	rolling angular velocity, radians per second $(d\phi/dt)$
v	sideslip velocity along Y-axis, feet per second
V	airspeed, feet per second
ρ	mass density of air, slugs per cubic foot
q	dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$
b	wing span, feet
S	wing area, square feet
W	weight of airplane, pounds

m mass of airplane, slugs (W/g)

g acceleration due to gravity, feet per second per second

μ_b relative-density factor $(m/\rho Sb)$

η inclination of principal longitudinal axis of airplane with respect to flight path, positive when principal axis is above flight path at nose, degrees

γ angle of flight path to horizontal axis, positive in climb, degrees

k_{X_0} radius of gyration in roll about principal longitudinal axis, feet

k_{Z_0} radius of gyration in yaw about principal vertical axis, feet

K_{X_0} nondimensional radius of gyration in roll about principal longitudinal axis $\left(k_{X_0}/b\right)$

K_{Z_0} nondimensional radius of gyration in yaw about principal vertical axis $\left(k_{Z_0}/b\right)$

K_X nondimensional radius of gyration in roll about longitudinal stability axis $\left(\sqrt{{K_{X_0}}^2\cos^2\eta + {K_{Z_0}}^2\sin^2\eta}\right)$

K_Z nondimensional radius of gyration in yaw about vertical stability axis $\left(\sqrt{{K_{Z_0}}^2\cos^2\eta + {K_{X_0}}^2\sin^2\eta}\right)$

K_{XZ} nondimensional product-of-inertia parameter $\left(\left({K_{Z_0}}^2 - {K_{X_0}}^2\right)\sin\eta\cos\eta\right)$

$\left(\Delta K_{XZ}\right)_{YM}$ increment to K_{XZ} in yawing-moment equation due to stabilization system

C_L trim lift coefficient $\left(\frac{\text{Lift}}{qS}\right)$

C_l rolling-moment coefficient $\left(\frac{\text{Rolling moment}}{qSb}\right)$

C_n yawing-moment coefficient $\left(\frac{\text{Yawing moment}}{qSb}\right)$

C_Y lateral-force coefficient $\left(\frac{\text{Lateral force}}{qS}\right)$

C_{l_β} effective-dihedral derivative, rate of change of rolling-moment coefficient with angle of sideslip, per radian $\left(\partial C_l/\partial\beta\right)$

C_{n_β} directional-stability derivative, rate of change of yawing-moment coefficient with angle of sideslip, per radian $\left(\partial C_n/\partial\beta\right)$

C_{Y_β} lateral-force derivative, rate of change of lateral-force coefficient with angle of sideslip, per radian $\left(\partial C_Y/\partial\beta\right)$

C_{n_r} damping-in-yaw derivative, rate of change of yawing-moment coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_n/\partial\frac{rb}{2V}\right)$

ΔC_{n_r} increment to C_{n_r} due to stabilization system

C_{n_p} rate of change of yawing-moment coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_n/\partial\frac{pb}{2V}\right)$

ΔC_{n_p} increment to C_{n_p} due to stabilization system

C_{l_p} damping-in-roll derivative, rate of change of rolling-moment coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_l/\partial\frac{pb}{2V}\right)$

ΔC_{l_p} increment to C_{l_p} due to stabilization system

C_{Y_p} rate of change of lateral-force coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_Y/\partial\frac{pb}{2V}\right)$

C_{Y_r} rate of change of lateral-force coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_Y/\partial\frac{rb}{2V}\right)$

C_{l_r} rate of change of rolling-moment coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_l / \frac{\partial rb}{2V}\right)$

δ_r rudder deflection, radians

δ_a aileron deflection, radians

$C_{n_{\delta_r}}$ rate of change of yawing-moment coefficient with rudder deflection, per radian $\left(\frac{\partial C_n}{\partial \delta_r}\right)$

$C_{l_{\delta_r}}$ rate of change of rolling-moment coefficient with rudder deflection, per radian $\left(\frac{\partial C_l}{\partial \delta_r}\right)$

$C_{Y_{\delta_r}}$ rate of change of lateral-force coefficient with rudder deflection, per radian $\left(\frac{\partial C_Y}{\partial \delta_r}\right)$

$C_{n_{\delta_a}}$ rate of change of yawing-moment coefficient with aileron deflection, per radian $\left(\frac{\partial C_n}{\partial \delta_a}\right)$

$C_{l_{\delta_a}}$ rate of change of rolling-moment coefficient with aileron deflection, per radian $\left(\frac{\partial C_l}{\partial \delta_a}\right)$

$C_{Y_{\delta_a}}$ rate of change of lateral-force coefficient with aileron deflection, per radian $\left(\frac{\partial C_Y}{\partial \delta_a}\right)$

$\frac{\delta_r}{\dot{\psi}}$ control-gearing ratio, rate of change of rudder deflection with yawing angular velocity

$\frac{\delta_a}{\dot{\phi}}$ control-gearing ratio, rate of change of aileron deflection with rolling angular velocity

$\frac{\delta_r}{\dot{\phi}}$ control-gearing ratio, rate of change of rudder deflection with rolling angular velocity

$\frac{\delta_r}{\ddot{\phi}}$ control-gearing ratio, rate of change of rudder deflection with rolling angular acceleration

t time, seconds

s_b nondimensional time parameter based on span (Vt/b)

D_b differential operator $\left(\frac{d}{ds_b}\right)$

λ root of characteristic stability equation

P period of oscillation, seconds

$T_{1/2}$ time for amplitude of lateral oscillation or an aperiodic mode to decrease by factor of 2

T_2 time for amplitude of lateral oscillation or an aperiodic mode to increase by factor of 2

A,B,C,D,E coefficients of lateral-stability equation

EQUATIONS OF MOTION

The linearized equations of motion, referred to stability axes, for any flight conditions are:

Rolling

$$2\mu_b\left(K_X{}^2 D_b{}^2\phi + K_{XZ} D_b{}^2\psi\right) = C_{l_\beta}\beta + \frac{1}{2}C_{l_p}D_b\phi + \frac{1}{2}C_{l_r}D_b\psi + C_{l_{\delta_a}}\delta_a$$

Yawing

$$2\mu_b\left(K_Z{}^2 D_b{}^2\psi + K_{XZ} D_b{}^2\phi\right) = C_{n_\beta}\beta + \frac{1}{2}C_{n_p}D_b\phi + \frac{1}{2}C_{n_r}D_b\psi + C_{n_{\delta_r}}\delta_r \qquad (1)$$

Sideslipping

$$2\mu_b\left(D_b\psi + D_b\beta\right) = C_{Y_\beta}\beta + \frac{1}{2}C_{Y_p}D_b\phi + \frac{1}{2}C_{Y_r}D_b\psi + C_L\phi + (C_L \tan\gamma)\psi$$

The control derivatives $C_{n_{\delta_a}}$, $C_{l_{\delta_r}}$, $C_{Y_{\delta_a}}$, and $C_{Y_{\delta_r}}$ are assumed to be zero and have been neglected in equations (1).

The characteristic control-fixed stability equation, obtained by expanding the determinant of equations (1) is of the form

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \tag{2}$$

The coefficients A, B, C, D, and E are:

$$A = 8\mu_b^3\left(K_X^2K_Z^2 - K_{XZ}^2\right)$$

$$B = -2\mu_b^2\left(2K_X^2K_Z^2C_{Y_\beta} + K_X^2C_{n_r} + K_Z^2C_{l_p} - 2K_{XZ}^2C_{Y_\beta} - K_{XZ}C_{l_r} - K_{XZ}C_{n_p}\right)$$

$$C = \mu_b\Big(K_X^2C_{n_r}C_{Y_\beta} + 4\mu_bK_X^2C_{n_\beta} + K_Z^2C_{l_p}C_{Y_\beta} + \frac{1}{2}C_{n_r}C_{l_p} - K_{XZ}C_{l_r}C_{Y_\beta} -$$

$$4\mu_bK_{XZ}C_{l_\beta} - \frac{1}{2}C_{n_p}C_{l_r} - C_{n_p}K_{XZ}C_{Y_\beta} + K_{XZ}C_{n_\beta}C_{Y_p} - K_Z^2C_{Y_p}C_{l_\beta} -$$

$$K_X^2C_{Y_r}C_{n_\beta} + K_{XZ}C_{Y_r}C_{l_\beta}\Big)$$

$$D = -\frac{1}{4}C_{n_r}C_{l_p}C_{Y_\beta} - \mu_bC_{l_p}C_{n_\beta} + \frac{1}{4}C_{n_p}C_{l_r}C_{Y_\beta} + \mu_bC_{n_p}C_{l_\beta} + 2\mu_bC_LK_{XZ}C_{n_\beta} -$$

$$2\mu_bC_LK_Z^2C_{l_\beta} - 2\mu_bK_X^2C_{n_\beta}C_L\tan\gamma + 2\mu_bK_{XZ}C_{l_\beta}C_L\tan\gamma + \frac{1}{4}C_{l_p}C_{n_\beta}C_{Y_r} -$$

$$\frac{1}{4}C_{n_p}C_{l_\beta}C_{Y_r} - \frac{1}{4}C_{l_r}C_{n_\beta}C_{Y_p} + \frac{1}{4}C_{n_r}C_{l_\beta}C_{Y_p}$$

$$E = \frac{1}{2}C_L\left[C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta} + \tan\gamma\left(C_{l_p}C_{n_\beta} - C_{n_p}C_{l_\beta}\right)\right]$$

If an auxiliary damping device with zero phase lag which applies rudder control proportional to the nth derivative of the yawing or rolling displacement is assumed installed in the aircraft, the equation for δ_r as a function of s_b is

$$\delta_r(s_b) = \frac{\partial \delta_r}{\partial D_b{}^n \psi} D_b{}^n \psi + \frac{\partial \delta_r}{\partial D_b{}^n \phi} D_b{}^n \phi \tag{3}$$

The terms $\frac{\partial \delta_r}{\partial D_b{}^n \psi}$ and $\frac{\partial \delta_r}{\partial D_b{}^n \phi}$ are the control gearing ratios of the autopilot.

Similarly, if an auxiliary damping device with zero phase lag which applies aileron control proportional to the nth derivative of the yawing or rolling displacement is assumed installed in the aircraft, the equation for δ_a as a function of s_b is

$$\delta_a(s_b) = \frac{\partial \delta_a}{\partial D_b{}^n \psi} D_b{}^n \psi + \frac{\partial \delta_a}{\partial D_b{}^n \phi} D_b{}^n \phi \tag{4}$$

The auxiliary dampers which were investigated include the following:

Rudder control applied proportional to yawing velocity.- The equation for δ_r from equation (3) is $\delta_r = \frac{\partial \delta_r}{\partial D_b \psi} D_b \psi$. If this value of δ_r is substituted into equations (1), the term $C_{n_{\delta_r}} \frac{\partial \delta_r}{\partial D_b \psi} D_b \psi$ is introduced into the yawing-moment equation. This term effectively changes the stability derivative C_{n_r} by the increment $\Delta C_{n_r} = 2 \frac{\partial \delta_r}{\partial D_b \psi} C_{n_{\delta_r}}$ (hereinafter called C_{n_r} damper). The following terms are introduced into the coefficients of equation (2):

$$\Delta A = 0$$

$$\Delta B = -2\mu_b{}^2 K_X{}^2 \Delta C_{n_r}$$

$$\Delta C = \mu\left(K_X{}^2 C_{Y_\beta} + \frac{1}{2} C_{l_p}\right)\Delta C_{n_r}$$

$$\Delta D = \frac{1}{4}\left(C_{l_\beta} C_{Y_p} - C_{l_p} C_{Y_\beta}\right)\Delta C_{n_r}$$

$$\Delta E = \frac{1}{2} C_L C_{l_\beta} \Delta C_{n_r}$$

Rudder control applied proportional to rolling velocity.- The equation for δ_r from equation (3) is $\delta_r = \frac{\partial \delta_r}{\partial D_b \phi} D_b \phi$. Substitution of this value of δ_r into equations (1) effectively changes the derivative C_{n_p} by the amount $\Delta C_{n_p} = 2 \frac{\partial \delta_r}{\partial D_b \phi} C_{n_{\delta_r}}$ (hereinafter called C_{n_p} damper). The coefficients of equation (2) are changed by the amounts:

$$\Delta A = 0$$

$$\Delta B = 2\mu^2 K_{XZ} \Delta C_{n_p}$$

$$\Delta C = -\mu_b\left(\frac{1}{2} C_{l_r} + K_{XZ} C_{Y_\beta}\right)\Delta C_{n_p}$$

$$\Delta D = \left(\mu_b C_{l_\beta} + \frac{1}{4} C_{l_r} C_{Y_\beta} - \frac{1}{4} C_{l_\beta} C_{Y_r}\right)\Delta C_{n_p}$$

$$\Delta E = -\left(\frac{1}{2} C_{l_\beta} C_L \tan \gamma\right)\Delta C_{n_p}$$

Aileron control applied proportional to rolling velocity.- The equation for δ_a from equation (4) is $\delta_a = \frac{\partial \delta_a}{\partial D_b \phi} D_b \phi$. Substitution of this value of δ_a into equations (1) introduces an increment to the stability derivative C_{l_p} which is $\Delta C_{l_p} = 2 \frac{\partial \delta_a}{\partial D_b \phi} C_{l_{\delta_a}}$ (hereinafter called C_{l_p} damper). The following terms are added to the coefficients of equation (2):

$$\Delta A = 0$$

$$\Delta B = -2\mu_b^{\,2} K_Z^{\,2} \Delta C_{l_p}$$

$$\Delta C = \mu_b\left(K_Z^{\,2} C_{Y_\beta} + \frac{1}{2} C_{n_r}\right)\Delta C_{l_p}$$

$$\Delta D = \left(\frac{1}{4} C_{n_\beta} C_{Y_r} - \frac{1}{4} C_{n_r} C_{Y_\beta} - \mu_b C_{n_\beta}\right)\Delta C_{l_p}$$

$$\Delta E = \left(\frac{1}{2} C_{n_\beta} C_L \tan \gamma\right)\Delta C_{l_p}$$

Both aileron and rudder control proportional to rolling velocity.- The equations for δ_a and δ_r, respectively, are the same as for the ΔC_{l_p} damper and the ΔC_{n_p} damper. Thus, the derivative C_{l_p} is changed by the amount $\Delta C_{l_p} = 2 \frac{\partial \delta_a}{\partial D_b \phi} C_{l_{\delta_a}}$, and the derivative C_{n_p} is changed by the amount $\Delta C_{n_p} = 2 \frac{\partial \delta_r}{\partial D_b \phi} C_{n_{\delta_r}}$ (hereinafter called $C_{l_p}C_{n_p}$ damper). To the coefficients of equation (2) are added the terms listed for both the ΔC_{n_p} and ΔC_{l_p} dampers.

Rudder control applied proportional to rolling acceleration.- The equation for δ_r is $\delta_r = \frac{\partial \delta_r}{\partial {D_b}^2 \phi} {D_b}^2 \phi$. This auxiliary damper therefore changes the parameter K_{XZ} in the yawing-moment equation by the amount $\Delta K_{XZ} = -\frac{1}{2\mu_b} C_{n_{\delta_r}} \frac{\partial \delta_r}{\partial {D_b}^2 \phi}$ (hereinafter called $(K_{XZ})_{YM}$ damper). The parameter K_{XZ} in the rolling-moment equation is unaltered. The following terms are added to the coefficients of equation (2):

$$\Delta A = -8{\mu_b}^3 K_{XZ} \left(\Delta K_{XZ}\right)_{YM}$$

$$\Delta B = 2{\mu_b}^2 \left(2K_{XZ} C_{Y_\beta} + C_{l_r}\right) \left(\Delta K_{XZ}\right)_{YM}$$

$$\Delta C = \mu_b \left(C_{Y_r} C_{l_\beta} - 4\mu_b C_{l_\beta} - C_{l_r} C_{Y_\beta}\right) \left(\Delta K_{XZ}\right)_{YM}$$

$$\Delta D = \left(2\mu_b C_{l_\beta} C_L \tan \gamma\right) \left(\Delta K_{XZ}\right)_{YM}$$

$$\Delta E = 0$$

RESULTS AND DISCUSSION

The mass and aerodynamic characteristics of the aircraft selected for the calculations are presented in table I.

Effect of Assumed Auxiliary Stabilization Systems on Period and Damping of Lateral Motions

C_{n_r} damper.- The variation of the damping of the aperiodic modes of motion and the period and damping of the lateral oscillation as ΔC_{n_r} is increased from 0 to -3.20 are presented in table II. The condition $\Delta C_{n_r} = 0$ corresponds to the aircraft with no auxiliary stabilization. The damping of the lateral oscillation continues to improve throughout the range of ΔC_{n_r} investigated. For $\Delta C_{n_r} = 0$, one of the aperiodic modes is approximately neutrally stable. This mode is generally referred to as the spiral mode of motion and, as ΔC_{n_r} is increased, the damping of this mode becomes more positive. The damping of the remaining aperiodic mode is relatively insensitive to changes in ΔC_{n_r}. An upper limit of ΔC_{n_r} may be reached, however, beyond which oscillatory instability will exist. This result is due to the fact that, as ΔC_{n_r} becomes increasingly larger, the degree of freedom in yaw is effectively eliminated, and the aircraft stability characteristics approach those obtained by assuming freedom only in roll and sideslip. The characteristic equation of this system is of the form

$$a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

The condition for oscillatory stability of a cubic, that is, Routh's discriminant for a cubic equation is

$$R = \left(a_2a_3 - a_1a_4\right) > 0$$

For the case being considered, the coefficients are defined as follows:

$$a_1 = 4\mu_b{}^2K_X{}^2$$

$$a_2 = -\mu_b\left(C_{l_p} + 2K_X{}^2C_{Y_\beta}\right)$$

$$a_3 = \frac{1}{2}\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right)$$

$$a_4 = -C_LC_{l_\beta}$$

Since μ_b, C_L, and $K_X{}^2$ are positive, C_{l_p} and C_{Y_β} are negative, and $C_{Y_p} = 0$ for the airplane discussed, R will be negative if C_{l_β} is more negative than

$$C_{l_\beta} = \frac{C_{l_p} C_{Y_\beta}}{8\mu_b C_L K_X{}^2}\left(C_{l_p} + 2K_X{}^2 C_{Y_\beta}\right)$$

For the aircraft discussed in this paper, this limiting value is $C_{l_\beta} = -0.115$. Since the value of C_{l_β} used in the calculations is -0.126, the system would approach an unstable condition as ΔC_{n_r} became very large.

<u>C_{n_p} damper.</u>- The stability derivative C_{n_p} has been shown to have a significant effect on the damping of the lateral oscillation (references 4 and 5). In view of this fact, an auxiliary damper which effectively varies C_{n_p} was investigated. The effect on the aperiodic and periodic modes of motion as ΔC_{n_p} is varied from -0.38 to 1.02 is presented in table III. As ΔC_{n_p} is increased positively, the damping of the lateral oscillation continues to improve; whereas the period is relatively unchanged. The effect on the aperiodic modes is such that, for some value of $0.62 < \Delta C_{n_p} < 0.82$, these modes combine to form a long-period oscillation which very rapidly becomes unstable for additional increases in ΔC_{n_p}. The formation of this second oscillation is discussed in detail in reference 4. Thus, this type of system is very effective in increasing the damping of the lateral oscillation, but care should be taken not to use gearing ratios which will result in values of ΔC_{n_p} greater than the value required for formation of the long-period oscillation, since for further increases in ΔC_{n_p} this long-period oscillation becomes unstable and subsequently breaks down to form two unstable aperiodic modes, one of which rapidly becomes highly divergent.

<u>C_{l_p} damper.</u>- The results presented in reference 1 indicated that the derivative C_{l_p} is ineffective as a means of improving the damping of the lateral oscillation. For increases in C_{l_p}, however, the damping

of one of the aperiodic modes was found to increase almost in direct proportion to increases in C_{l_p}. These results were verified for the aircraft considered in the calculations for this paper but are not presented. From these results, it appeared probable that, if C_{l_p} were increased simultaneously with C_{n_p}, the formation of the long-period oscillation discussed in the section entitled "C_{n_p} damper" would be delayed to larger values of ΔC_{n_p}, with a resulting increase in the damping of the short-period oscillation. Consequently, a configuration which increased both C_{n_p} and C_{l_p} was investigated.

$C_{n_p}C_{l_p}$ damper.- For $\Delta C_{l_p} = -0.40$, the effect of ΔC_{n_p} on the stability of the lateral modes of motion as ΔC_{n_p} is varied from -0.38 to 1.82 is presented in table IV. As was predicted, the formation of the long-period oscillation was delayed to a considerably higher value of ΔC_{n_p} and the obtainable damping of the short-period oscillation was also increased.

$(K_{XZ})_{YM}$ damper.- As was pointed out in the section of the paper entitled "Equations of Motion," this type of stabilization system effectively increases the value of K_{XZ} in the yawing-moment equation. The results presented in references 6 and 7 indicated that K_{XZ} (product-of-inertia parameter) has a stabilizing effect on the damping of the lateral oscillation if the principal longitudinal axis is inclined above the flight path at the nose of the airplane $\left(K_{XZ} > 0\right)$. The results of reference 6 also indicated that the parameter involving K_{XZ} in the yawing-moment equation was primarily responsible for the stabilizing effect. It was believed, therefore, that if the rudder were deflected proportional to the rolling acceleration an appreciable stabilizing effect on the damping of the lateral oscillation would result since the value of K_{XZ} in the yawing-moment equation would be increased. The ratio B/A, where A and B are coefficients of the characteristic equation of the system (see equation (2)), is the negative sum of the damping in the system, and, for the $\left(K_{XZ}\right)_{YM}$ damper, this ratio is

$$\frac{B}{A} = -\frac{1}{4\mu_b}\left[2C_{Y_\beta} + \frac{{K_X}^2C_{n_r} + {K_Z}^2C_{l_p} - \left(\Delta K_{XZ}\right)_{YM}C_{l_r} - K_{XZ}\left(C_{l_r} + C_{n_p}\right)}{\left({K_X}^2{K_Z}^2 - {K_{XZ}}^2\right) - K_{XZ}\left(\Delta K_{XZ}\right)_{YM}}\right]$$

The quantities μ_b, K_X^2, K_Z^2, C_{l_r}, and $\left(\Delta K_{XZ}\right)_{YM}$ are positive, and C_{Y_β}, C_{n_r}, C_{l_p}, and C_{n_p} are negative for the airplane configuration discussed in this paper. In addition $\left(C_{l_r} + C_{n_p}\right) > 0$ for this airplane and the quantity $\left(K_X^2K_Z^2 - K_{XZ}^2\right) > 0$ and is equal to $K_{X_0}^2K_{Z_0}^2$ for all flight conditions. The parameter K_{XZ} is positive for flight conditions where the principal axis is above the flight path at the nose of the airplane and negative if the principal axis is below the flight path. As $\left(\Delta K_{XZ}\right)_{YM}$ approaches infinity, the ratio B/A approaches the value

$$\lim_{\left(\Delta K_{XZ}\right)_{YM}\to\infty} \frac{B}{A} = -\frac{1}{4\mu_b}\left(2C_{Y_\beta} + \frac{C_{l_r}}{K_{XZ}}\right)$$

if $\left|\frac{C_{l_r}}{K_{XZ}}\right| > 2\left|C_{Y_\beta}\right|$, the ratio B/A is negative for $K_{XZ} > 0$; hence, the system is unstable as $\left(\Delta K_{XZ}\right)_{YM}$ becomes large. In fact, as can be seen from the general expression for B/A, this ratio shifts from positive infinity to negative infinity as $\left(\Delta K_{XZ}\right)_{YM}$ passes through the value $\left(\Delta K_{XZ}\right)_{YM} = \frac{K_X^2K_Z^2 - K_{XZ}^2}{K_{XZ}}$, $K_{XZ} > 0$, and will remain negative unless $\left|\frac{C_{l_r}}{K_{XZ}}\right| < 2\left|C_{Y_\beta}\right|$. Thus, it is necessary to use a gearing such that, for the highest angle-of-attack condition anticipated, the coefficient A will still be positive, since, if $A < 0$, the system will definitely be unstable if the other coefficients are positive. For flight conditions where $K_{XZ} < 0$, the ratio B/A will remain positive as $\left(\Delta K_{XZ}\right)_{YM}$ is increased without limit.

The roots of the characteristic stability equation as $\left(\Delta K_{XZ}\right)_{YM}$ approaches infinity can be shown to be two zero roots and the roots

$$\lambda = \frac{1}{4\mu_b K_{XZ}}\left[\left(K_{XZ}C_{Y_\beta} + \frac{1}{2}C_{l_r}\right) \pm \sqrt{\left(K_{XZ}C_{Y_\beta} + \frac{1}{2}C_{l_r}\right)^2 - 2K_{XZ}\left(C_{l_r}C_{Y_\beta} + 4\mu_b C_{l_\beta}\right)}\right]$$

If $K_{XZ} > 0$, one root is negative, and one is positive. If $K_{XZ} < 0$, the roots are a complex pair with the real part negative, or two negative real roots.

The principal-axis location for the airplane described in table I is 2° below the flight path, and, hence, $K_{XZ} < 0$. For this principal-axis location, the effect on the stability of the lateral modes of motions as $(\Delta K_{XZ})_{YM}$ is varied from 0 to 0.40 is shown in table V. The damping of the oscillatory mode increases very rapidly as $(\Delta K_{XZ})_{YM}$ is increased, but the period of the oscillation becomes increasingly shorter. One of the aperiodic modes (spiral mode) is essentially insensitive to changes in $(\Delta K_{XZ})_{YM}$; whereas the remaining aperiodic mode becomes considerably less damped as this parameter is increased. For purposes of comparison, the principal-axis location was arbitrarily assumed to be 2° above the flight path $(K_{XZ} > 0)$, and the effect on the stability of the lateral modes of motion, for this principal-axis location as $(\Delta K_{XZ})_{YM}$ varies from 0 to 10.00, is shown in table VI. The value of $(\Delta K_{XZ})_{YM}$ for which the coefficient A changes sign is 0.34. For $0.05 < \Delta K_{XZ} < 0.30$ the damping of the oscillation improves more rapidly with changes in $(\Delta K_{XZ})_{YM}$ for this principal-axis location than for $\eta = -2^{\circ}$ and, correspondingly, the period of the oscillation decreases much more rapidly than for the previous condition. As was noted before, one of the aperiodic modes is relatively insensitive to variations in $(\Delta K_{XZ})_{YM}$; whereas the remaining aperiodic mode becomes considerably less stable as $(\Delta K_{XZ})_{YM}$ is increased. For $(\Delta K_{XZ})_{YM} = 0.33$, the oscillation has become two stable aperiodic modes and, for $(\Delta K_{XZ})_{YM} > 0.34$, one of the newly formed aperiodic modes becomes highly divergent. As this parameter is increased beyond 0.40, the original aperiodic modes combine to form a long-period oscillation, which, as $(\Delta K_{XZ})_{YM}$ approaches infinity, approaches the condition of zero damping and zero frequency. The remaining aperiodic modes, as $(\Delta K_{XZ})_{YM}$ approaches infinity, approach the values discussed previously and, for this flight condition, are $T_2 = 0.0296$ second and $T_{1/2} = 0.0371$ second. The results shown in table VI for $(\Delta K_{XZ})_{YM} = 10.00$ verify these conclusions.

Effect of Assumed Auxiliary Stabilization Systems on Aircraft Lateral Motions Subsequent to an Applied Yawing or Rolling Moment

General characteristics of lateral motions.- Each of the stabilization systems discussed in the previous section gave an appreciable increase in the damping of the lateral oscillation for the range of parameters investigated. As was mentioned previously, however, the acceptability of each assumed auxiliary damping device would depend, to a large extent, on the lateral-response characteristics of the automatically stabilized system subsequent to control deflections. The lateral motions subsequent to a constant-step rudder deflection of -3.5° $(C_n = 0.01)$ and to a constant-step aileron deflection of approximately -6° $\left(C_l = 0.01\right)$ therefore were calculated for the basic aircraft with no automatic stabilization and for the aircraft equipped with each of the discussed stabilization systems. The control and stabilization systems parameters assumed for the calculations are as follows:

$$C_{n_{\delta_r}} = -0.163$$

$$C_{l_{\delta_a}} = -0.10$$

$$\Delta C_{n_r} = -0.80 \quad \left(C_{n_r} \text{ damper}\right)$$

$$\Delta C_{n_p} = 0.62 \quad \left(C_{n_p} \text{ damper}\right)$$

$$\Delta C_{n_p} = 0.62; \quad \Delta C_{l_p} = -0.40 \quad \left(C_{n_p} C_{l_p} \text{ damper}\right)$$

$$\left(\Delta K_{XZ}\right)_{YM} = 0.025 \quad \left(\left(K_{XZ}\right)_{YM} \text{ damper}\right)$$

The lateral motions were calculated by the methods of reference 8, and the general form of the solutions for the type of disturbances under discussion are:

$$\left.\begin{aligned}
\beta &= \beta_o + \sum_{n=1}^{m} \beta_n e^{\lambda_n s_b} \\
D_b\psi &= \left(D_b\psi\right)_o + \sum_{n=1}^{m} \left(D_b\psi\right)_n e^{\lambda_n s_b} \\
D_b\phi &= \sum_{n=1}^{m} \left(D_b\phi\right)_n e^{\lambda_n s_b}
\end{aligned}\right\} \tag{5}$$

where β_o, β_n, $\left(D_b\psi\right)_o$, $\left(D_b\psi\right)_n$, and $\left(D_b\phi\right)_n$ are constants, and the λ_n's are the linear and distinct roots of the characteristic equation (equation (2)) of the system set equal to zero. For a completely stable system, the real λ_n's are all less than zero and the complex λ_n's all have real parts which are less than zero. Thus, it is evident from equations (5) that as s_b approaches infinity, β approaches β_o, $D_b\psi$ approaches $(D_b\psi)_o$, and $D_b\phi$ approaches 0 for a completely stable system.

For the response to a yawing-moment coefficient C_n, the steady-state values of β_o/C_n and $D_b\psi_o\,C_n$ are

$$\frac{\beta_o}{C_n} = \frac{C_{l_r}}{C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta}}$$

$$\frac{D_b\psi_o}{C_n} = \frac{-2C_{l_\beta}}{C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta}}$$

and, for the response to a rolling-moment coefficient C_l, the steady-state value of β_o/C_l and $D_b\psi_o/C_l$ are

$$\frac{\beta_o}{C_l} = \frac{-C_{n_r}}{C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta}}$$

$$\frac{D_b\psi_o}{C_l} = \frac{2C_{n_\beta}}{C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta}}$$

Therefore, of the stabilization systems discussed in this paper, only the C_{n_r} system affects the steady-state values of β and $D_b\psi$. This result is discussed in more detail in a subsequent section of the paper.

The initial yawing and rolling accelerations due to a step deflection of the rudder are:

$$\frac{d^2\psi}{dt^2} = \frac{V^2}{b^2}\frac{K_X^2}{2\mu_b}\frac{C_n}{\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}}$$

$$\frac{d^2\phi}{dt^2} = -\frac{V^2}{b^2}\frac{K_{XZ}}{2\mu_b}\frac{C_n}{\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}}$$

The parameter $\left(\Delta K_{XZ}\right)_{YM}$ is the increment to K_{XZ} in the yawing-moment equation introduced by the $\left(K_{XZ}\right)_{YM}$ auxiliary damper and is equal to zero for any other system discussed in this paper. Thus, only the $\left(K_{XZ}\right)_{YM}$ system affects the initial yawing and rolling accelerations when a yawing moment is applied. For the basic system (no automatic stabilization) and for the other configurations considered, the initial yawing acceleration is of the same sign algebraically as the applied yawing moment. The rolling acceleration is seen to depend on the sign of both the applied yawing moment and the parameter K_{XZ}. As mentioned previously, K_{XZ} is negative if the principal longitudinal axis is below the flight path at the nose of the airplane; hence, for the airplane flight condition of table I, the initial rolling acceleration is also of the same sign as the applied yawing moment. For the $\left(K_{XZ}\right)_{YM}$ damper the initial accelerations are also algebraically the same as the applied yawing moment $\left(\text{since } K_{XZ} < 0\right)$, but the magnitude of the accelerations is reduced because of the increased value of the term $\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}$, which appears in the denominator of the expressions for both the initial yawing and rolling acceleration. The initial yawing acceleration for all the dampers considered, with the exception of the $\left(K_{XZ}\right)_{YM}$ damper, is independent of principal-axis inclination since the factor $K_X^2K_Z^2 - K_{XZ}^2$ is equal to $K_{X_0}^2K_{Z_0}^2$ which is a constant for a given airplane. The initial rolling acceleration is seen to depend directly on principal-axis inclination because of the parameter K_{XZ} in the expression for the initial rolling acceleration. Thus, if the $\left(K_{XZ}\right)_{YM}$ system is to be used for automatic stabilization, it is necessary to choose a value of $\left(\Delta K_{XZ}\right)_{YM}$ such that, for the highest angle-of-attack condition anticipated, the factor $\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}$ will not be algebraically negative since, if this change occurs, the initial accelerations not only are reversed but, as was pointed out previously, the system shifts from a very stable to a very unstable condition.

Similarly, the initial rolling and yawing accelerations subsequent to application of a rolling moment are:

$$\frac{d^2\phi}{dt^2} = \frac{V^2}{b^2}\frac{K_Z^2}{2\mu_b}\frac{C_l}{\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}}$$

$$\frac{d^2\psi}{dt^2} = -\frac{V^2}{b^2}\frac{C_l}{2\mu_b}\frac{K_{XZ} + \left(\Delta K_{XZ}\right)_{YM}}{\left(K_X^2K_Z^2 - K_{XZ}^2\right) - \left(\Delta K_{XZ}\right)_{YM}K_{XZ}}$$

Therefore, in every system except the $(K_{XZ})_{YM}$ system, the initial rolling acceleration is independent of principal-axis location; whereas the initial yawing acceleration depends directly on the principal-axis location through the parameter K_{XZ}. For the $(K_{XZ})_{YM}$ system, the initial rolling acceleration decreases in magnitude as $(\Delta K_{XZ})_{YM}$ increases if $K_{XZ} < 0$, and, for $K_{XZ} > 0$, the rolling acceleration increases as $(\Delta K_{XZ})_{YM}$ increases and, for this airplane, approaches infinity as this parameter approaches 0.34. Beyond this value, the accelerations are different in sign than for values less than 0.34, and, as before, the airplane is highly unstable for $(\Delta K_{XZ})_{YM} > 0.34$. The same general conclusions apply for the initial yawing acceleration except that, for $K_{XZ} < 0$, the yawing acceleration is positive if $\left|(\Delta K_{XZ})_{YM}\right| < \left|K_{XZ}\right|$ and negative if the inequality is reversed. In addition, the algebraic sign of the yawing acceleration changes again for $(\Delta K_{XZ})_{YM} = 0.34$.

Lateral responses to $C_n = 0.01$.- The yawing velocity, rolling velocity, and sideslip responses subsequent to application of a constant yawing-moment coefficient C_n equal to 0.01 are presented in figure 1 for the aircraft equipped with each of the assumed stabilization systems. The results are plotted in terms of time in seconds instead of the non-dimensional time parameter s_b. The motions are plotted for 3 seconds only since the time immediately subsequent to application of control is of most interest from the consideration of lateral controllability. The application of a positive yawing moment initially introduces positive yawing acceleration and positive rolling acceleration since $K_{XZ} < 0$. The initial peak yawing velocity for the basic system shown in figure 1(a) is about 10.5° per second. A reduction in the peak velocity is evident for both the C_{n_r} and $(K_{XZ})_{YM}$ dampers; whereas an increased yawing velocity is noted for the C_{n_p} and the $C_{n_p}C_{l_p}$ configurations. The reduced velocity for the C_{n_r} and $(K_{XZ})_{YM}$ systems is due to the fact that the yawing moments introduced by these systems tend to oppose the initial yawing acceleration. The increased yawing velocity for the C_{n_p} and the $C_{n_p}C_{l_p}$ systems results from the yawing moment due to rolling velocity, which is in the same direction as the initial yawing acceleration. The peak yawing velocity for the $C_{n_p}C_{l_p}$ damper, although higher than the basic system, is less than that noted for the C_{n_p} system, since the rolling velocity for the $C_{n_p}C_{l_p}$ damper is reduced appreciably because of the increased damping in roll introduced by the increment to C_{l_p} and, therefore, the magnitude of the yawing moment due to rolling is somewhat less.

The rolling-velocity responses are presented in figure 1(b). The rolling velocities obtained with the C_{n_p} auxiliary stabilization system are much higher than for any of the other configurations investigated. This result can be attributed to the continuous reinforcement of the yawing motion by the C_{n_p} damper, which results in an increased rolling moment due to yawing velocity and also an increased rolling moment due to sideslip. The peak rolling velocity obtained for the C_{n_r} system is less than the basic system since the yawing velocity is reduced somewhat and, hence, the rolling moment due to yawing is smaller than for the basic system. In addition, the rolling moment due to sideslip is also less since the sideslip is reduced for the C_{n_r} damper (fig. 1(c)). The decrease in rolling velocity for the $C_{n_p}C_{l_p}$ damper, as mentioned previously, is attributed to the increased damping in roll due to the increment to the derivative C_{l_p}. The rolling velocity for the $(K_{XZ})_{YM}$ system is less because of the decreased rolling moment due to yawing and also because of the reduced rolling moment due to sideslip. In general, the aircraft equipped with the C_{n_p} damper responds more quickly to rudder deflections than any of the configurations investigated; whereas a definite reduction in the magnitude of the aircraft motions is noted for the $(K_{XZ})_{YM}$ system. The aircraft equipped with the C_{n_r} damper appears to behave similar to the basic configuration and the motions are considerably better damped.

Lateral responses to $C_l = 0.01$.- The lateral responses $d\phi/dt$, β, and $d\psi/dt$ subsequent to a constant-step rolling-moment coefficient equal to 0.01 are presented in figure 2. The application of a positive rolling moment initially introduces positive rolling acceleration but, as pointed out previously, the initial yawing acceleration is dependent upon the value of K_{XZ}. Since, for the flight condition discussed, K_{XZ} is negative (airplane principal axis below the flight path) the initial yawing acceleration is positive for every system except the $(K_{XZ})_{YM}$ system. The value of $(\Delta K_{XZ})_{YM}$ chosen for the calculations is such that $\left(K_{XZ} + (\Delta K_{XZ})_{YM}\right) > 0$; hence, the initial yawing acceleration is negative for this system. This conclusion is illustrated in figure 2(c).

From figure 2(a), the peak rolling velocity for the C_{n_p} system is much higher than for the other systems investigated. The higher peak velocity is due, as explained previously, to the continuous reinforcement of the yawing motion by the C_{n_p} damper and the accompanying increased rolling moments due to yawing and sideslip.

The motions for the C_{n_r} system do not differ greatly from the basic system, with the exception of the improved damping. The motions for the $\left(K_{XZ}\right)_{YM}$ system appear definitely less desirable than those of the other system, and from figure 2(c), the yaw is seen to be adverse; that is, negative yawing motion is coupled with positive rolling. The rolling motion for the $C_{n_p}C_{l_p}$ system is somewhat less than the basic system, and this condition is undoubtedly due to the higher damping in roll supplied by the increased value of C_{l_p}. Thus, as was the case for the lateral motions subsequent to an applied yawing moment, the C_{n_p} auxiliary damper is the one which responds the fastest to an applied rolling moment.

Determination of Rudder Deflection Necessary to Maintain Zero Sideslip Subsequent to an Aileron Deflection

Additional calculations were made for the aircraft equipped with each of the discussed stabilization systems to determine the rudder motion necessary to maintain zero sideslip subsequent to an aileron deflection. This condition is necessary for a perfectly coordinated turn, and the ease with which this maneuver can be executed should have appreciable bearing on the pilot's opinion of the flying qualities of each of the systems. If it is assumed that the respective damping devices are geared to the aircraft control surfaces, the total control-surface motion is a superposition of the motions obtained from these calculations and the motions induced by the auxiliary damper. For irreversible control systems, or if the damper is geared to an auxiliary surface, this component of the motion will not be apparent to the pilot and, hence, will have no influence on his opinion of the flying qualities of a particular system. Thus, only that part of the control-surface motion which must be induced by the pilot is considered. In order to determine the rudder motion necessary for $\beta = 0$, this condition was substituted into equations (1). The rudder deflection δ_r was assumed to be a variable and the forcing function was the aileron rolling-moment coefficient C_l. The resulting equations written in determinant form are for the condition that $C_{l_{\delta_r}}$, $C_{Y_{\delta_r}}$, $C_{Y_{\delta_a}}$, $C_{n_{\delta_a}}$, C_{Y_p}, and C_{Y_r} equal zero:

$$\begin{bmatrix} 2\mu_b K_Z{}^2 D_b{}^2 - \frac{1}{2} C_{n_r} D_b & 2\mu_b K_{XZ} D_b{}^2 - \frac{1}{2} C_{n_p} D_b & -C_{n_{\delta_r}} \\ 2\mu_b K_{XZ} D_b{}^2 - \frac{1}{2} C_{l_r} D_b & 2\mu_b K_X{}^2 D_b{}^2 - \frac{1}{2} C_{l_p} D_b & 0 \\ 2\mu_b D_b & -C_L & 0 \end{bmatrix} \begin{matrix} \psi \\ \phi \\ \delta_r \end{matrix} = \begin{bmatrix} 0 \\ C_l \\ 0 \end{bmatrix} \tag{6}$$

For the parameters given in table I, and the stabilization system derivatives given in the section entitled "Effect of Assumed Auxiliary Stabilization Systems on Aircraft Lateral Motions Subsequent to an Applied Yawing or Rolling Moment," the rudder motion δ_r was calculated from these equations for each assumed configuration. The aileron rolling-moment coefficient C_l was taken as 0.01. These rudder time histories are presented in figure 3. The δ_r motion required to maintain $\beta = 0$ is considerably different for each of the stabilization systems investigated. The C_{n_r} system, however, differs very little from the basic case. Also, subsequent to $t = 1$ second, the δ_r motion for the $\left(K_{XZ}\right)_{YM}$ system is similar to the C_{n_r} and the basic system. The large negative rudder deflections required initially for the $\left(K_{XZ}\right)_{YM}$ system are due to the adverse sideslip noted in figure 2(b). The relatively large deflections required for the C_{n_p} configuration, as well as the $C_{n_p}C_{l_p}$ system, are consistent with the β motions presented in figure 2(b) for these systems. The rudder deflections required for the C_{n_p} damper, although higher than for the other systems, are not believed to be excessive and, in addition, subsequent to $t = 1$ second, the rudder deflection is essentially constant. One disadvantage, from a pilot's viewpoint, would be that the rudder motion required to maintain $\beta = 0$ for the C_{n_p} and $C_{n_p}C_{l_p}$ dampers is similar to the motion which would increase the sideslip for the aircraft without automatic stabilization.

Rudder and Aileron Deflections Necessary to Perform Steady Turning Maneuvers

When the lateral controllability of an aircraft is analyzed, an investigation of the combinations of rudder and aileron deflections required to perform steady-state turning maneuvers is often useful. In such a maneuver, $D_b\phi$, $D_b{}^2\phi$, $D_b{}^2\psi$, and $D_b\beta$ are all zero for a completely stable airplane. For a perfectly coordinated turn, another condition which must exist is that $\beta = 0$. For these assumptions and the conditions noted for $C_{l_{\delta_r}}$, $C_{Y_{\delta_r}}$, $C_{n_{\delta_a}}$, $C_{Y_{\delta_a}}$, C_{Y_p}, and C_{Y_r}, equations (1) reduce to the following:

$$\frac{1}{2} C_{n_r} D_b\psi + C_{n_{\delta_r}}\delta_r = 0 \tag{7a}$$

$$\frac{1}{2} C_{l_r} D_b\psi + C_{l_{\delta_a}}\delta_a = 0 \tag{7b}$$

$$2\mu_b D_b\psi - C_L\phi = 0 \tag{7c}$$

From equation 7(c)

$$D_b\psi = \frac{C_L\phi}{2\mu_b}$$

Substitution of this value for $D_b\psi$ into equations 7(a) and 7(b) results in the expressions:

$$\left.\begin{aligned} C_{n_{\delta_r}}\delta_r &= \frac{-C_L\phi C_{n_r}}{4\mu_b} \\ C_{l_{\delta_a}}\delta_a &= \frac{-C_L\phi C_{l_r}}{4\mu_b} \\ \text{or} \qquad \frac{C_{n_{\delta_r}}\delta_r}{C_{l_{\delta_a}}\delta_a} &= \frac{C_{n_r}}{C_{l_r}} \end{aligned}\right\} \tag{8}$$

The stability derivatives C_{n_r} and C_{l_r} are negative and positive, respectively. Hence, for a steady turn with zero sideslip, the rudder moment and the aileron moment must be opposite in sign; that is, for a perfect steady turn the rudder must be held into the turn to balance out the damping in yaw, and the aileron must be applied against the turn to counteract the rolling moment due to the yawing velocity.

The $\Delta \dot{C}_{n_r}$ stabilization system discussed in this paper effectively increases the derivative C_{n_r} and, according to reference 2, one of the objectionable features of this type of system is the increased rudder pedal force during the steady part of turns. This increase in pedal force is predicted by equations (8), but it might be added that the steady turn could also be executed with no increase in rudder force and a decrease in aileron control since the ratio $\frac{C_{n_{\delta_r}}\delta_r}{C_{l_{\delta_a}}\delta_a}$ can be increased by decreasing δ_a as well as by increasing δ_r. It is conceded, however, that the steady yawing velocity and the steady angle of bank are less if the ratio is increased in this fashion rather than by keeping the ratio $\frac{C_{n_{\delta_r}}\delta_r}{C_{n_r}}$ constant. The values of the steady-state yawing velocity and the steady angle of bank as obtained from equations (7) are:

$$D_b\psi = \frac{-2C_{n_{\delta_r}}\delta_r}{C_{n_r}} \equiv \frac{-2C_{l_{\delta_a}}\delta_a}{C_{l_r}}$$

$$\phi = \frac{2\mu_b}{C_L} D_b\psi \tag{9}$$

From these equations, it is apparent that, if C_{n_r} is increased artificially, the steady yawing velocity is decreased if δ_r is constant. The value of δ_a, however, must be reduced to satisfy the condition for a steady turn $\frac{C_{n_{\delta_r}}\delta_r}{C_{l_{\delta_a}}\delta_a} = \frac{C_{n_r}}{C_{l_r}}$. Also, this reduction in $D_b\psi$ results in a smaller angle of bank in the steady turn.

The ratio of rudder deflection to aileron deflection required for a steady turn is readily seen to be the same for each of the other discussed stabilization systems as for the aircraft with no automatic stabilization; therefore, the problem of increased pedal forces in steady turns would not arise with these configurations.

CONCLUSIONS

The following conclusions were obtained from a theoretical analysis of the effect of various types of automatic stabilization systems on the lateral stability and controllability of a present-day high-speed aircraft:

1. Each of the stabilization systems investigated resulted in increased damping of the lateral oscillations of the assumed aircraft.

2. The lateral motions of each configuration investigated subsequent to rudder or aileron deflections indicated that a device which deflected the rudder proportional to rolling velocity (C_{n_p} damper) increased considerably the sensitivity of the aircraft to control deflections; whereas a device which deflected the rudder proportional to yawing velocity (C_{n_r} damper) affected only slightly the aircraft-response characteristics. The lateral responses calculated for the system where the rudder was assumed to be deflected proportional to the rolling acceleration ($(K_{XZ})_{YM}$ damper) were considered unsatisfactory because of the presence of a large amount of adverse yaw subsequent to aileron deflections.

3. Calculations made to determine the rudder motion required to maintain zero sideslip subsequent to an aileron deflection indicated that an increased rudder motion is necessary for each damper investigated compared with the aircraft with no automatic stabilization. The C_{n_p} damper required the largest rudder deflections; whereas the deflections for the C_{n_r} damper were only slightly different from those of the basic configuration. The rudder deflection required for the $(K_{XZ})_{YM}$ damper is very high initially, but after about 1 second it was similar to the basic and C_{n_r} systems.

4. An analysis of the ratio of rudder angle to aileron angle required for steady turning maneuvers indicated that use of the C_{n_r} damper would result in increased rudder deflections if it were desired to obtain the same steady rate of yaw and bank angle as for the basic system. The

remaining configurations investigated would have no effect on the ratio of rudder deflection to aileron deflection $\left(\frac{\delta_r}{\delta_a}\right)$ required to perform steady turning maneuvers.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., August 29, 1951

REFERENCES

1. Sternfield, Leonard: Effect of Automatic Stabilization on the Lateral Oscillatory Stability of a Hypothetical Airplane at Supersonic Speeds. NACA TN 1818, 1949.

2. White, Roland J.: Investigation of Lateral Dynamic Stability in the XB-47 Airplane. Jour. Aero. Sci. vol. 17, no. 3, March 1950, pp. 133-148.

3. Beckhardt, Arnold R.: A Theoretical Investigation of the Effect on the Lateral Oscillations of an Airplane of an Automatic Control Sensitive to Yawing Accelerations. NACA TN 2006, 1950.

4. Sternfield, Leonard, and Gates, Ordway B., Jr.: A Simplified Method for the Determination and Analysis of the Neutral-Lateral-Oscillatory-Stability Boundary. NACA Rep. 943, 1949. (Formerly NACA TN 1727.)

5. Johnson, Joseph L., and Sternfield, Leonard: A Theoretical Investigation of the Effect of Yawing Moment Due to Rolling on Lateral Oscillatory Stability. NACA TN 1723, 1948.

6. Sternfield, Leonard: Effect of Product of Inertia on Lateral Stability. NACA TN 1193, 1947.

7. Sternfield, Leonard: Some Considerations of the Lateral Stability of High-Speed Aircraft. NACA TN 1282, 1947.

8. Mokrzycki, G. A.: Application of the Laplace Transformation to the Solution of the Lateral and Longitudinal Stability Equations. NACA TN 2002, 1950.

TABLE I

MASS AND AERODYNAMIC CHARACTERISTICS OF ASSUMED

HIGH-SPEED AIRCRAFT

Altitude, ft	30,000
W/S, lb/ft^2	65
S, ft^2	130
b, ft	28
ρ, slugs/ft^3	0.000889
V, ft/sec	797
γ, deg	0
C_L	0.23
μ_b	80.7
$K_X{}^2$	0.00967
$K_Z{}^2$	0.0513
K_{XZ}	-0.00145
η, deg	-2.0
C_{l_p}, per radian	-0.40
C_{l_r}, per radian	0.08
C_{n_p}, per radian	-0.02
C_{n_r}, per radian	-0.40
C_{Y_p}, per radian	0
C_{Y_r}, per radian	0
C_{Y_β}, per radian	-1.0
C_{n_β}, per radian	0.25
C_{l_β}, per radian	-0.126
$C_{n_{\delta_r}}$, per radian	-0.163
$C_{l_{\delta_a}}$, per radian	-0.10

NACA

TABLE II

EFFECT OF C_{n_r} DAMPER ON PERIOD AND DAMPING

OF LATERAL MOTIONS

ΔC_{n_r}	Lateral oscillation		Aperiodic modes $T_{1/2}$
	$T_{1/2}$	P	
0	2.58	1.29	59.2, 0.175
-.20	1.60	1.25	32.4, .174
-.40	1.16	1.30	22.3, .174
-.80	.75	1.32	13.7, .173
-1.60	.44	1.38	7.7, .172
-3.20	.24	1.70	4.0, .166

NACA

TABLE III

EFFECT OF C_{n_p} DAMPER ON PERIOD AND DAMPING OF LATERAL MOTIONS

ΔC_{n_p}	Lateral oscillation			Aperiodic modes	
	$T_{1/2}$	T_2	P	$T_{1/2}$	T_2
-0.38	-----	4.39	1.19	{87.20, 0.14}	-----
-.13	6.97	----	1.25	{69.30, 0.16}	-----
0	2.58	----	1.29	{59.20, 0.18}	-----
.12	1.58	----	1.32	{51.40, 0.19}	-----
.62	.44	----	1.44	{15.30, 0.61}	-----
.82	{.33	----	{1.40	-----	-----
	12.80}	----	29.70}	-----	-----
.92	.30	----	{1.37	-----	-----
	-----	4.20	51.90}	-----	-----
1.02	.28	----	1.34	------	{11.6, 1.1}

NACA

TABLE IV

EFFECT OF $C_{n_p}C_{l_p}$ DAMPER ON PERIOD AND DAMPING

OF LATERAL MOTIONS

ΔC_{l_p}	ΔC_{n_p}	Lateral oscillation		Aperiodic modes	
		$T_{1/2}$	P	$T_{1/2}$	T_2
-0.40	-0.38	7.7	1.20	145.0 0.086	-----
-.40	.12	1.79	1.30	108.8 0.094	-----
-.40	.62	.86	1.50	73.1 0.11	-----
-.40	1.02	.50	1.83	44.5 0.14	-----
-.40	1.42	.22	2.20	15.5 0.53	-----
-.40	1.62	.18 12.90	1.84 31.90	------- -------	----- -----
-.40	1.82	.17	1.63	-------	11.20 1.25

NACA

TABLE V

EFFECT OF $(K_{XZ})_{YM}$ DAMPER ON PERIOD AND DAMPING

OF LATERAL MOTIONS

$[\eta = -2^{\circ}]$

$(\Delta K_{XZ})_{YM}$	Lateral oscillation		Aperiodic modes $T_{1/2}$
	$T_{1/2}$	P	
0	2.58	1.29	59.2, 0.175
.0082	.89	1.14	59.2, .23
.0250	.51	.92	59.0, .39
.0410	.42	.79	58.9, .55
.0820	.36	.63	58.5, .95
.4000	.30	.40	55.1, 4.35

NACA

TABLE VI

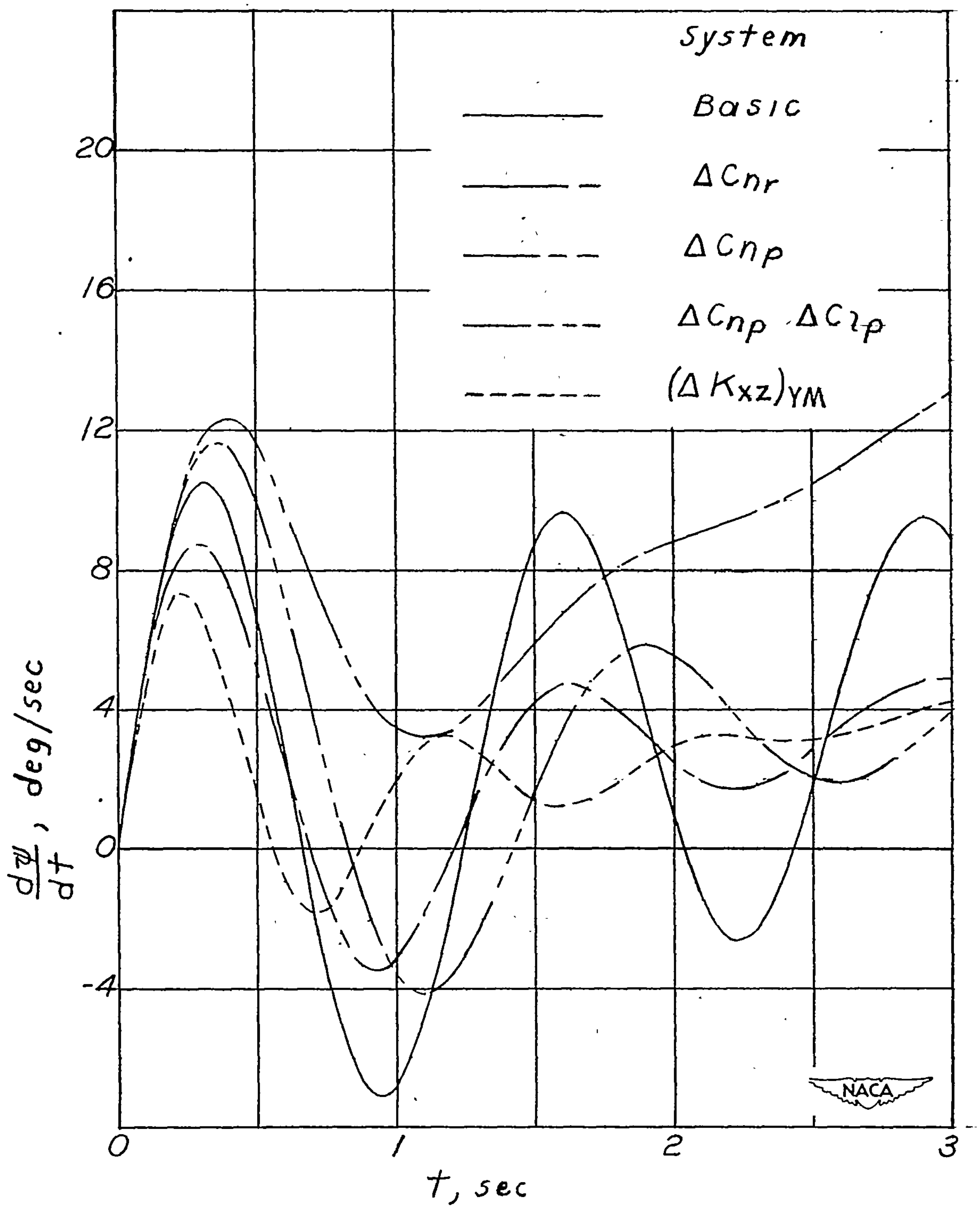
EFFECT OF $(K_{XZ})_{YM}$ DAMPER ON PERIOD AND DAMPING

OF LATERAL MOTIONS

$[\eta = 2^{\circ}]$

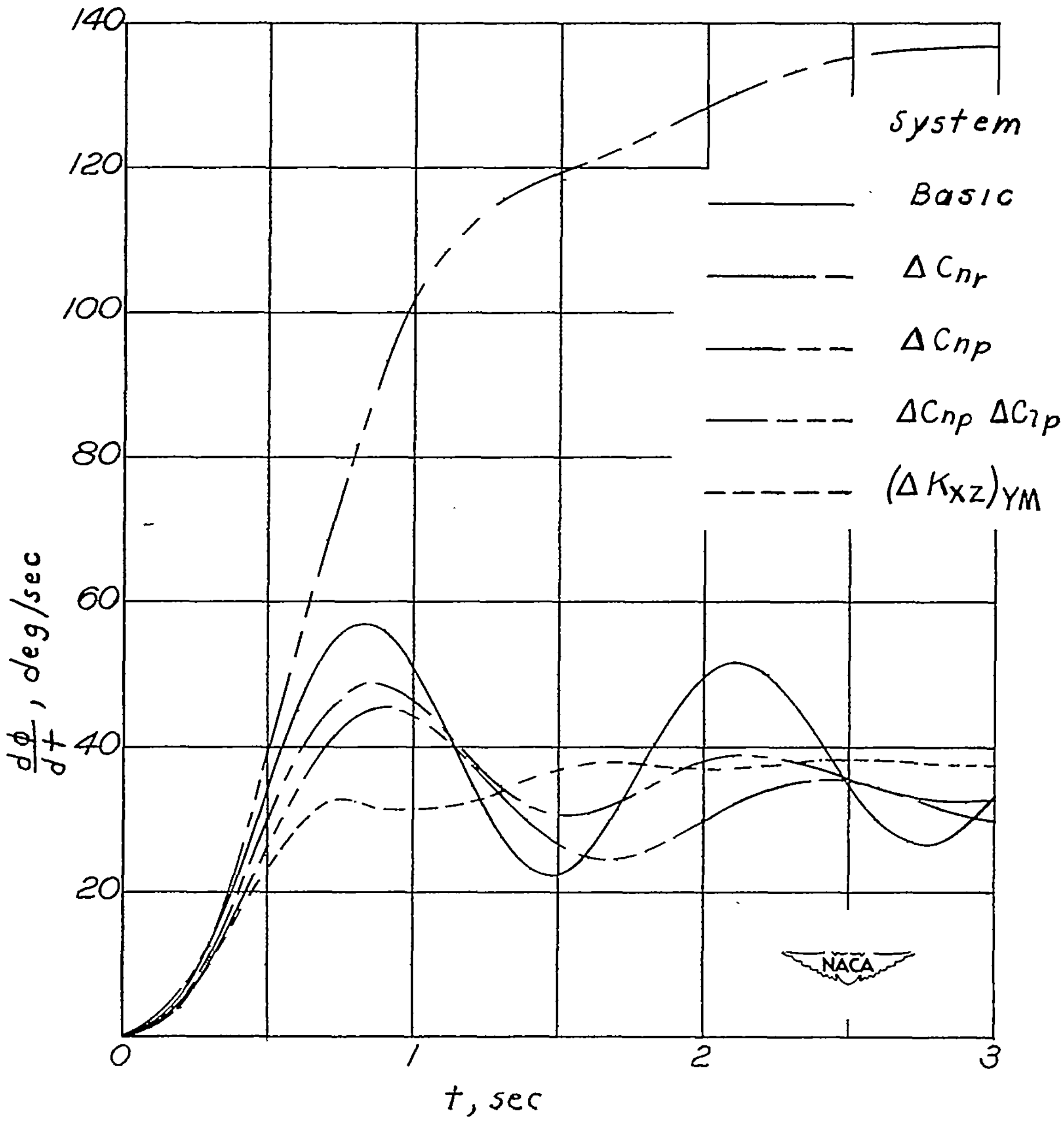
$(\Delta K_{XZ})_{YM}$	Lateral oscillation			Aperiodic modes $T_{1/2}$
	$T_{1/2}$	T_2	P	
0	1.46	-------	1.23	59.1, 0.19
.05	.27	-------	.63	58.7, .69
.10	.18	-------	.44	58.2, 1.16
.20	.08	-------	.25	57.1, 2.16
.30	.02	-------	.14	56.1, 3.20
.33	{.011, .0027}	-------	----	55.8, 3.51
.40	.020	0.0076	----	55.1, 4.27
10.00	.0366	.0289	----	$T_{1/2}$ = 188, P = 740

NACA



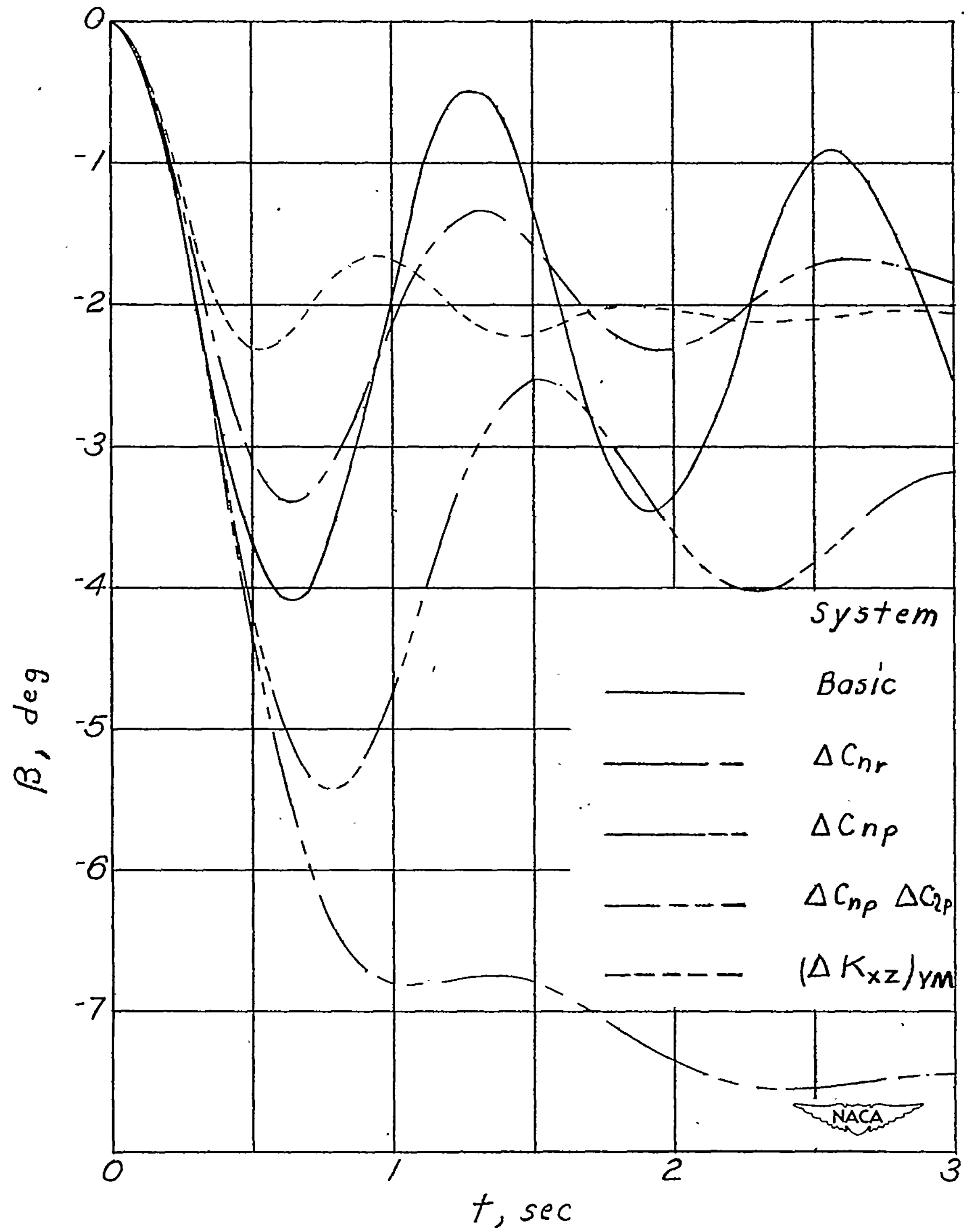
(a) Yawing velocity.

Figure 1.- Lateral responses subsequent to $C_n = 0.01$.



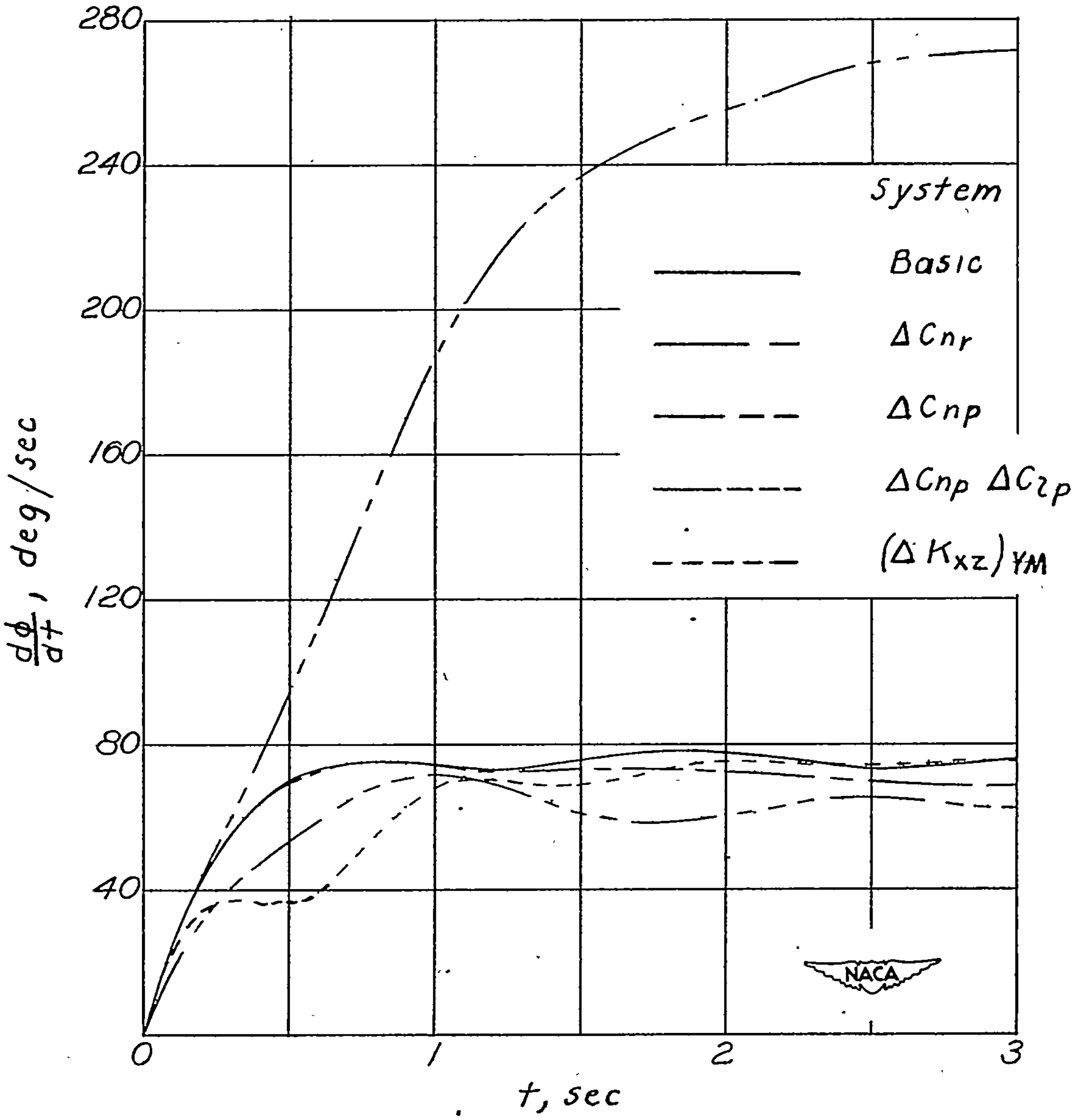
(b) Rolling velocity.

Figure 1.- Continued.



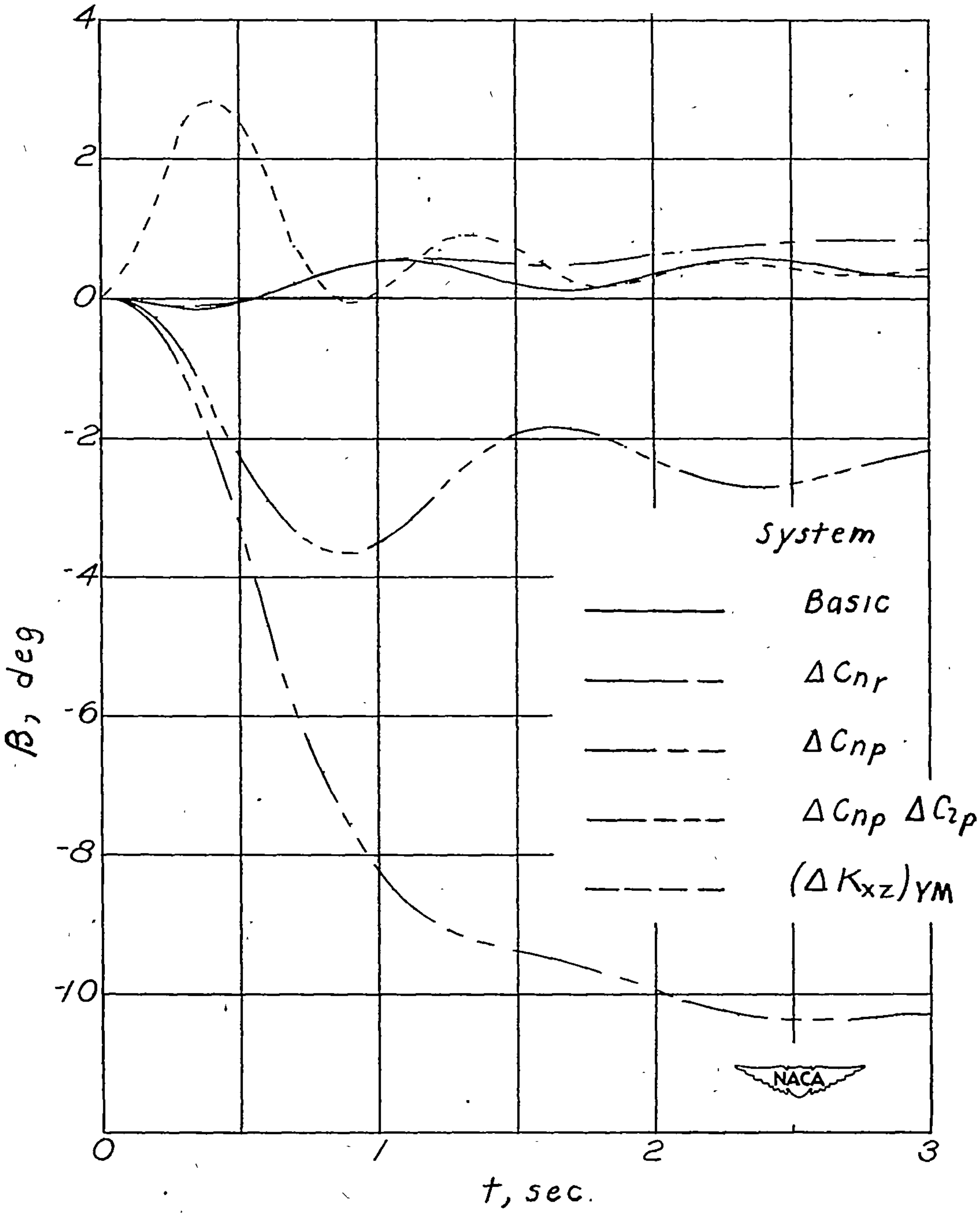
(c) Sideslip.

Figure 1.- Concluded.



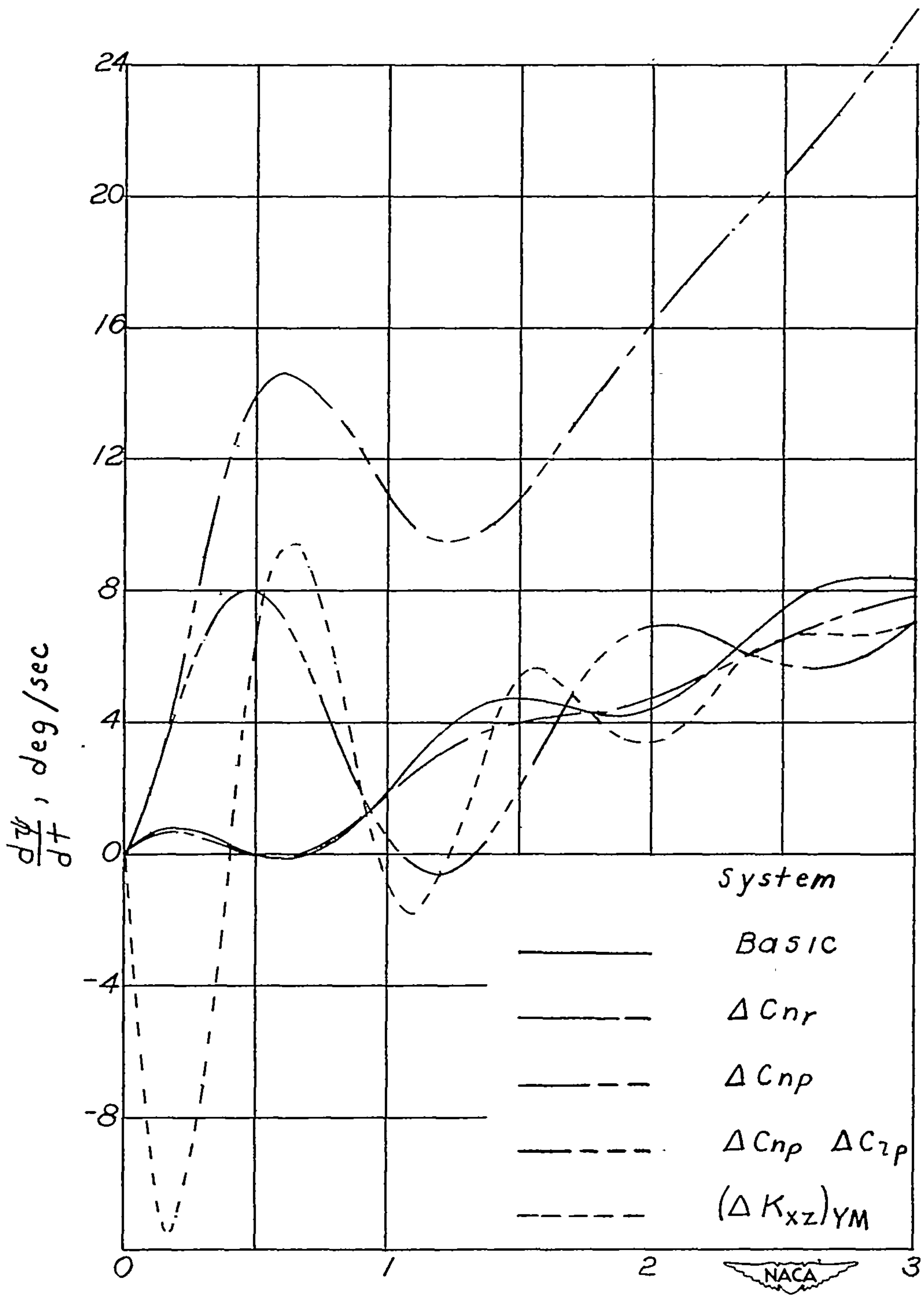
(a) Rolling velocity.

Figure 2.- Lateral responses subsequent to $C_l = 0.01$.



(b) Sideslip.

Figure 2.- Continued.



(c) Yawing velocity.

Figure 2.- Concluded.

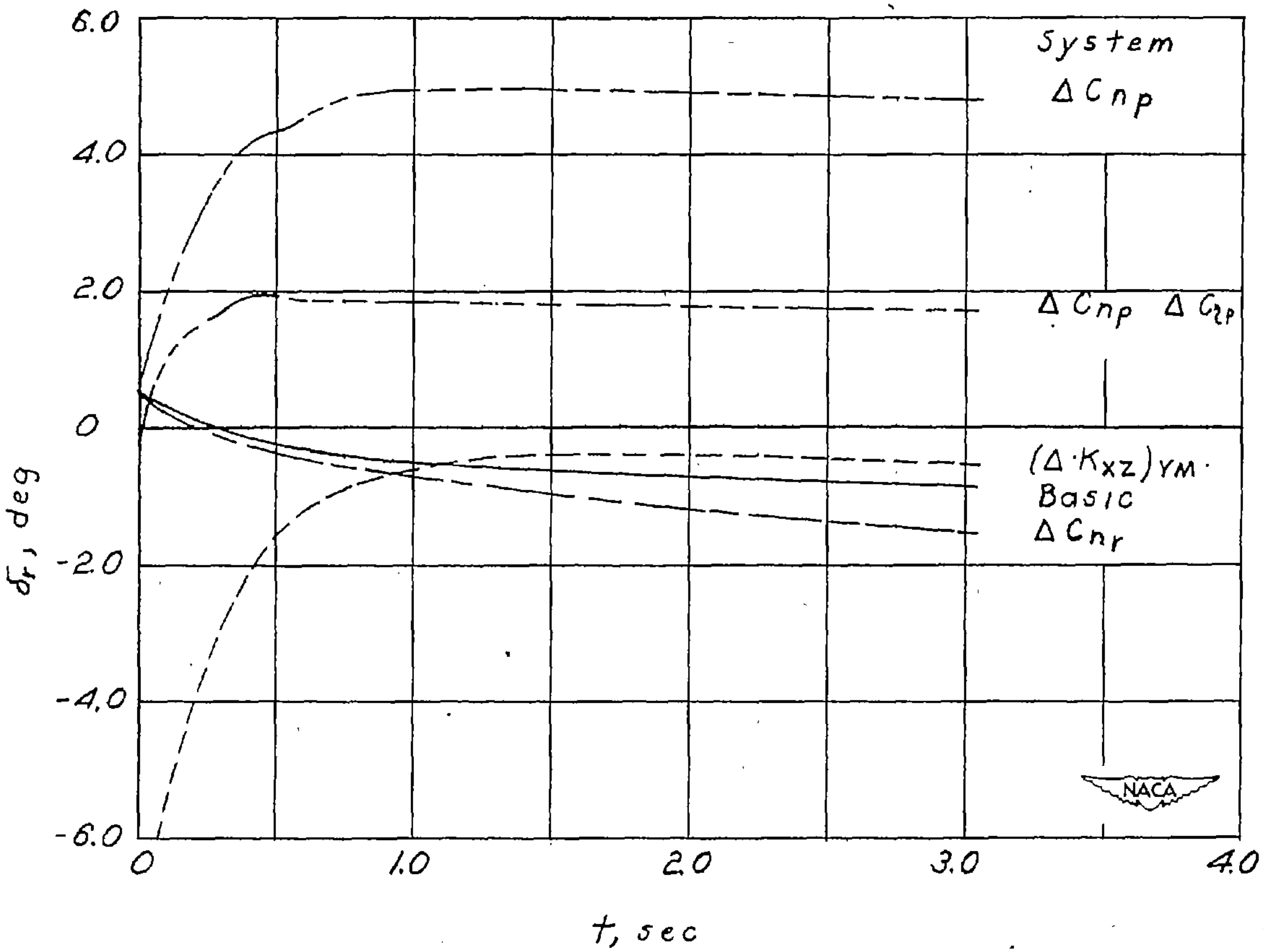


Figure 3.- Rudder deflection required to maintain $\beta = 0$ for $C_l = 0.01$.

NACA-Langley - 12-6-51 - 1000