NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2613

DETERMINATION OF INDICIAL LIFT AND MOMENT OF A TWO-DIMENSIONAL PITCHING AIRFOIL AT SUBSONIC MACH NUMBERS FROM OSCILLATORY COEFFICIENTS WITH NUMERICAL CALCULATIONS FOR A MACH NUMBER OF 0.7

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SUMMARY

The reciprocal equations relating the lift and moment acting on an airfoil due to harmonic oscillations in compressible flow to the indicial lift and moment functions for a pitching airfoil are determined. Two indicial functions are required to describe completely the lift or moment on the airfoil; one function is used for describing the effects due to the angular position of the airfoil whereas the second function is used for describing the effects due to pitching velocity.

Calculations for the indicial lift and moment functions are made for a pitching airfoil at a Mach number of 0.7 for which sufficient oscillatory coefficients were available. The growth of lift to the steady-state value is less rapid for compressible flow than for incompressible flow for both indicial functions of the pitching airfoil. Although the circulatory lift due to pitching velocity on an airfoil rotating about its three-quarter-chord point is nonexistent at a Mach number of 0, for a Mach number of 0.7 a time-dependent lift exists because of pitching velocity and approaches zero in approximately 2 chords.

INTRODUCTION

In the study of transient flows, two types of airfoil motions have special significance - a harmonically oscillating airfoil and an airfoil experiencing a sudden change in angle of attack. The reciprocal relations between the indicial lift and the lift associated with a harmonically oscillating airfoil for both sinking and pitching motion are given in reference 1 for incompressible flow. In reference 2 the reciprocal relations were extended to apply to an airfoil suddenly acquiring a vertical velocity in a subsonic compressible flow, and the indicial lift and moment functions were evaluated for a Mach number of 0.7. The present paper is an extension of reference 2 in that the indicial lift and moment functions for a pitching airfoil are determined. The reciprocal equations for the case of a pitching airfoil are indicated, and the indicial lift and moment functions are computed for a Mach number of 0.7.

In a recent paper by Lomax, Heaslet, and Sluder (reference 3), a different method for determining the indicial lift and moment functions is given. Although parts of the indicial functions can be determined readily, the solution for the complete indicial functions is lengthy and tedious, and, consequently, numerical results are given only for a Mach number of 0.8 in reference 3. Part of the solution presented herein for a Mach number of 0.7, however, was determined by the method in reference 3 and was compared with the solution determined in this paper.

SYMBOLS

s	distance traveled, half-chords
ω	angular frequency
V	forward velocity of airfoil
c	chord
x,z	Cartesian coordinates
k	reduced-frequency parameter $(\omega c/2V)$
$L(s)$	lift per unit length of span, positive in downward direction
$L_q(s)$	lift per unit length of span due to pitching velocity, positive in downward direction
$M(s)$	moment per unit length of span about quarter-chord point, positive when moment tends to depress trailing edge
$M_q(s)$	moment per unit length of span due to pitching velocity about the quarter-chord point, positive when moment tends to depress trailing edge
ρ	density

w	normal component of perturbation velocity
h	amplitude of vertical displacement of airfoil, half-chords
θ	angle of pitch, positive when trailing edge is lower than leading edge
q	rate of change of pitch of airfoil with respect to distance traveled in half-chords, positive when trailing edge is falling with reference to leading edge $(\dot{\theta}c/2V)$
$k_1(s)$	indicial lift function for an airfoil experiencing a sudden change in vertical velocity
$m_1(s)$	indicial moment function for an airfoil experiencing a sudden change in vertical velocity where moment is taken about quarter-chord position
$k_{1_q}(s)$	indicial lift function for an airfoil experiencing a sudden change in pitching velocity about its leading edge
$m_{1_q}(s)$	indicial moment function for an airfoil experiencing a sudden change in pitching velocity about its leading edge
$F_c(k) + iG_c(k)$	coefficient of complex compressible oscillatory lift derivative
$M(k) + iN(k)$	coefficient of complex compressible oscillatory moment derivative where moment is taken about quarter-chord point
$F_{c_q}(k) + iG_{c_q}(k)$	coefficient of complex compressible oscillatory lift derivative due to pitching velocity only
$M_q(k) + N_q(k)$	coefficient of complex compressible oscillatory moment derivative due to pitching velocity only

$f(k) = F_c(k) - F_c(\infty)$

$m(k) = M(k) - M(\infty)$

$f_q(k) = F_{c_q}(k) - F_{c_q}(\infty)$

$m_q(k) = M_q(k) - M_q(\infty)$

Z_1,Z_2 — in-phase and out-of-phase lift coefficients, respectively, associated with translation of airfoil

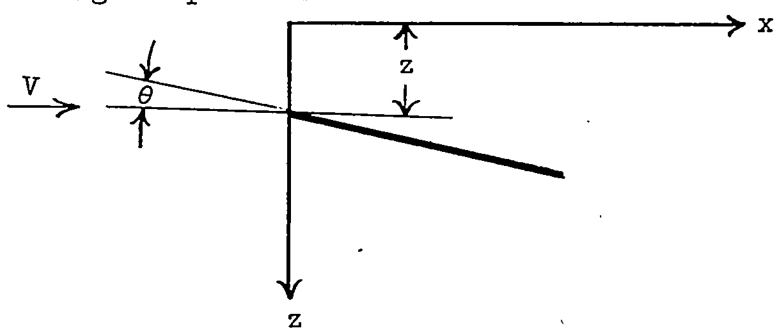
M_1,M_2 — in-phase and out-of-phase moment coefficients, respectively, about quarter-chord point associated with translation of airfoil

Z_3,Z_4 — in-phase and out-of-phase lift coefficients, respectively, associated with pitching motion of airfoil about its leading edge

M_3,M_4 — in-phase and out-of-phase moment coefficients, respectively, about quarter-chord point associated with pitching motion of airfoil about its leading edge

METHOD OF ANALYSIS

Consider the perturbation velocities on a two-dimensional wing (see the following sketch) for the case of combined vertical motion and pitching motion. For convenience the reference axis is taken as the leading edge and z and θ are used to describe, respectively, the vertical and angular positions of the airfoil.



The perturbation velocity (w measured positive upward, z positive downward) may be expressed as

$$w(x,t) = -(V\theta + \dot{z} + x\dot{\theta})$$

If the airfoil is considered to have vertical motion only, then the perturbation velocity will be uniform across the chord and will be of intensity $\dot{z}$. If the case of an airfoil pitching about its leading edge is considered, however, the perturbation velocity will be composed of two parts: a uniform part of intensity $V\theta$ and a linearly varying part of intensity $x\dot{\theta}$. When the lift on an airfoil having arbitrary motions is determined, therefore, these observations mean that only two types of perturbation velocities have to be considered: a uniform distribution which applies either to vertical motion or to angular position alone and a linearly varying distribution which applies to angular velocity of the airfoil.

When unit step motions of the airfoil are considered, it should be apparent that one indicial function is sufficient to define the lift on an airfoil suddenly acquiring a vertical velocity, since only the uniform perturbation velocity is involved. For the case of a pitching airfoil, however, two indicial functions are required: one corresponding to the uniform perturbation velocity associated with angular position and the other corresponding to the linear variation which is associated with angular velocity.

The lift on an airfoil suddenly acquiring a vertical velocity is generally given in terms of an indicial lift function $k_1(s)$ by means of the equation

$$L(s) = -\pi\rho cV^2 \frac{dh}{ds} k_1(s) \tag{1a}$$

where

$$\frac{dh}{ds} = \frac{c}{2V} \dot{h} = \frac{\dot{z}}{V}$$

This equation may be made to apply to an airfoil suddenly acquiring an angular position, because of the similarity that exists between the perturbation velocities for the case of vertical motion and for the case of angular position alone. The lifts for these two cases therefore will be equal if the intensities of perturbation velocity are equal, that is, if $\dot{z} = V\theta$. With this condition and equation (1a), the lift on an airfoil following a sudden change in angular position is

$$L(s) = -\pi\rho cV^2\theta k_1(s) \tag{1b}$$

For the pitching case then, the only indicial lift function that remains to be determined is the function associated with pitching velocity of the airfoil. The equation for lift following a sudden change in angular velocity $\dot{\theta}$, where this angular velocity is, for convenience, taken about the leading edge, may be written similar to equation (1a) or (1b)

$$L_q(s) = -2\pi\rho cV^2 q k_{1_q}(s)$$

where $q = \frac{c}{2V}\dot{\theta}$ and $k_{1_q}(s)$ is the indicial lift function associated with a sudden acquisition of an angular velocity about the leading edge.

The indicial lift and moment functions due to pitching velocity can be determined with the aid of the appropriate reciprocal equations and flutter coefficients in a manner similar to that shown for the sinking airfoil in reference 2. The available values for the lift and moment on the oscillating airfoil for pitching motion at subsonic Mach numbers, however, have been obtained numerically only for the total lift and moment; that is, the components due to angular position and pitching velocity have not been separated. Fortunately, the component due to angular position may be subtracted out of the total oscillatory coefficients because of the similarity that exists between the lift on a sinking airfoil and the component of lift associated with angular position of a pitching airfoil. The available data for an airfoil oscillating harmonically in vertical motion therefore are used in the reduction of the data for an airfoil oscillating in pitch to give the lift and moment associated with oscillatory pitching velocity alone. These data can then be used to determine the indicial functions associated with pitching velocity.

Reciprocal Equations

The reciprocal relations given by equations (8a) and (8b) in reference 2 between the compressible lift on an airfoil due to harmonic oscillations and the indicial lift on an airfoil experiencing a change in angle of attack due to a sudden acquisition of vertical velocity are as follows:

$$k_1(s) = \frac{2}{\pi}\int_0^{\infty} \frac{F_c(k)\sin ks}{k}\,dk \qquad (s > 0) \tag{2a}$$

and

$$k_1(s) = F_c(0) + \frac{2}{\pi}\int_0^{\infty} \frac{G_c(k)\cos ks}{k}\,dk \qquad (s > 0) \tag{2b}$$

where $F_c(k)$ and $G_c(k)$ are related, respectively, to the in-phase and out-of-phase lift components on an oscillating airfoil and are defined by the following equation:

$$L(s) = -\pi\rho c V^2 e^{iks}(-ikh)\left[F_c(k) + iG_c(k)\right] \tag{3}$$

For the case of a pitching airfoil the component of lift associated with angular position and its corresponding indicial lift function are given by equations (2) and (3), where the term

$$-ikh = \theta \tag{4}$$

a relationship which follows from the fact that the perturbation velocities for vertical motion are equal to the velocities due to angular position alone when $\dot{z} = V\theta$.

In the interpretation of the expression for lift given by equation (3), it should be noted that the expression for the motion of the airfoil is in complex form. If, for example, the actual angular position of the airfoil is denoted by

$$I.P.\theta e^{iks}$$

then the lift is

$$I.P.-\pi\rho cV^2\theta e^{iks}\left[F_c(k) + iG_c(k)\right]$$

or

$$-\pi\rho cV^2\theta\left[F_c(k)^2 + G_c(k)^2\right]^{1/2} \sin\left[ks + \tan^{-1}\frac{G_c(k)}{F_c(k)}\right]$$

Thus, the lift and the angular displacement have the same frequency and both their magnitude and phase are functions of k.

The reciprocal equations for the component of lift due to pitching velocity may be expressed in terms of a harmonically oscillating airfoil and the indicial lift function due to a sudden change in pitching velocity $k_{1_q}(s)$ as follows:

$$k_{1_q}(s) = \frac{2}{\pi}\int_0^\infty \frac{F_{c_q}(k) \sin ks}{k} dk \qquad (s > 0) \tag{5a}$$

and

$$k_{1_q}(s) = F_{c_q}(0) + \frac{2}{\pi}\int_0^\infty \frac{G_{c_q}(k) \cos ks}{k} dk \qquad (s > 0) \tag{5b}$$

where $F_{c_q}(k)$ and $G_{c_q}(k)$ are related, respectively, to the in-phase and out-of-phase lift components on an oscillating airfoil due to pitching velocity about the leading edge and are defined by the equation

$$L_q(s) = -\pi\rho cV^2 e^{iks}(2ik\theta)\left[F_{c_q}(k) + iG_{c_q}(k)\right] \tag{6}$$

In equations (5a) and (5b) the functions $F_{c_q}(k)$ and $G_{c_q}(k)/k$ must be continuous and finite in the interval from $k = 0$ to $k = \infty$.

The available data for evaluating the $F_c(k)$, $G_c(k)$, $F_{c_q}(k)$, and $G_{c_q}(k)$ functions have been given in various forms, beginning with the work of Possio. One form in which all of the available data can be put is as follows:

Translatory motion:

$$L(s) = \pi\rho cV^2 e^{iks}\,\frac{h}{2}\left(Z_1 + iZ_2\right) \tag{7a}$$

$$M(s) = \pi\rho c^2 V^2 e^{iks}\,\frac{h}{2}\left(M_1 + iM_2\right) \tag{7b}$$

Pitching motion (including contribution due to pitching velocity):

$$L(s) = -\pi\rho cV^2 e^{iks}\,\theta\left(Z_3 + iZ_4\right) \tag{8a}$$

$$M(s) = -\pi\rho c^2 V^2 e^{iks}\,\theta\left(M_3 + iM_4\right) \tag{8b}$$

where the moment is taken about the quarter-chord point and the axis of rotation for the harmonically pitching airfoil is located at the leading edge.

In order to make use of the reciprocal equations (5a) and (5b) to determine the indicial lift function due to pitching velocity $k_{1_q}(s)$, the component of the flutter coefficients due to pitching velocity must first be separated from the flutter coefficients given by equations (8a) and (8b) and then converted to the form given by equation (6). If the expression for h determined by equation (4) is substituted into equation (7a) and if the resulting expression is subtracted from equation (8a), the following expression for the lift can be obtained for the component due to pitching velocity on a harmonically oscillating airfoil:

$$L_q(s) = -\pi\rho cV^2 e^{iks}\,\theta\left[Z_3 - \frac{Z_2}{2k} + i\left(Z_4 + \frac{Z_1}{2k}\right)\right] \tag{9}$$

Comparison of equations (6) and (9) leads to the following expressions for the functions $F_{c_q}(k)$ and $G_{c_q}(k)$ in terms of the oscillatory coefficients Z_1, Z_2, Z_3, and Z_4:

$$F_{c_q}(k) = \frac{1}{2k}\left(Z_4 + \frac{Z_1}{2k}\right) \tag{10a}$$

and

$$G_{c_q}(k) = -\frac{1}{2k}\left(Z_3 - \frac{Z_2}{2k}\right) \tag{10b}$$

Substitution of equations (10) into equations (5) then allows for the determination of the indicial function $k_{1_q}(s)$ for pitching velocity.

The procedure shown for determining the indicial lift function due to pitching velocity from the oscillatory coefficients may be applied to determine the indicial moment function due to pitching velocity $m_{1_q}(s)$ defined as follows:

$$M(s) = 2\pi\rho c^2 V^2 q m_{1_q}(s) \tag{11}$$

where q is the magnitude of the pitching velocity. The reciprocal equations can be written for the moment in terms of the oscillatory coefficients and the indicial moment function $m_{1_q}(s)$ as follows:

$$m_{1_q}(s) = \frac{2}{\pi}\int_0^\infty \frac{M_q(k)\ \sin ks}{k}\,dk \qquad (s > 0) \tag{12a}$$

and

$$m_{1_q}(s) = M_q(0) + \frac{2}{\pi}\int_0^\infty \frac{N_q(k)\ \cos ks}{k}\,dk \qquad (s > 0) \tag{12b}$$

where $M_q(k)$ and $N_q(k)$ are, respectively, the in-phase and out-of-phase moment coefficients on an oscillating airfoil due to the angular pitching velocity about the leading edge and are defined by the equation

$$M_q(s) = \pi\rho c^2 V^2 e^{iks}(2ik\theta)\left[M_q(k) + iN_q(k)\right] \tag{13}$$

In equations (12a) and (12b) the functions $M_q(k)$ and $N_q(k)/k$ must be continuous and finite in the interval from $k = 0$ to $k = \infty$.

In a manner similar to that shown for the lift case, expressions for $M_q(k)$ and $N_q(k)$ may be obtained in terms of the oscillatory coefficients M_1, M_2, M_3, and M_4 as follows:

$$M_q(k) = -\frac{1}{2k}\left(M_4 + \frac{M_1}{2k}\right) \tag{14a}$$

and

$$N_q(k) = \frac{1}{2k}\left(M_3 - \frac{M_2}{2k}\right) \tag{14b}$$

Summary of Available Flutter Coefficients and Related Data for a Mach Number of 0.7

The two-dimensional compressible flutter coefficients for the real and imaginary parts of the lift and moment for sinking and pitching motion are given in table I for a Mach number of 0.7. The results given in table I were taken from three sources (references 4 to 6) and have been converted to the form given by equations (7) and (8) for the lift and moment. The range of reduced frequencies taken from each source is indicated in table I.

Expressions for the end points $F_c(0)$ and $F_c(\infty)$ can be determined independently of the flutter coefficients. The value of $F_c(0)$ which represents the steady-state lift is determined by the Prandtl-Glauert factor $\frac{1}{\sqrt{1 - M^2}}$ (where M refers to the Mach number of the free stream). The value of $F_c(\infty)$ which was shown in reference 2 to correspond to the value of $k_1(s)$ at $s = 0$ can be determined by the following equation given in reference 2 for any Mach number:

$$F_c(\infty) = \frac{2}{\pi M} \tag{15}$$

The corresponding expression for the moment $M(\infty)$ is noted in reference 2 as

$$M(\infty) = -\frac{1}{2\pi M} \tag{16}$$

For the case of a pitching airfoil the expression for the end points for the $F_{c_q}(k)$ and $M_q(k)$ functions may be obtained in a manner similar to that shown for the case of sinking airfoil. For an airfoil experiencing a sudden change in pitching velocity about its leading edge, the steady-state value of the $k_{1_q}(s)$ function (which can be shown to correspond to $F_{c_q}(0)$) can be determined from incompressible flow together with a correction given by the Prandtl-Glauert factor. From the work of either Theodorsen or Wagner (reference 8 or 9) the incompressible steady-state value can be determined, and, together with the Prandtl-Glauert factor, the resulting expression for $F_{c_q}(0)$ for subsonic flow is as follows:

$$F_{c_q}(0) = k_{1_q}(\infty) = \frac{3}{4} \frac{1}{\sqrt{1 - M^2}} \tag{17}$$

Similarly, for the moment the expression for $M_q(0)$ is

$$M_q(0) = m_{1_q}(\infty) = -\frac{1}{16} \frac{1}{\sqrt{1 - M^2}} \tag{18}$$

The values for $F_{c_q}(\infty)$ and $M_q(\infty)$ may be obtained from the starting values of the $k_{1_q}(s)$ and $m_{1_q}(s)$ functions, respectively. The same method of analysis for obtaining the starting values of the $k_1(s)$ and $m_1(s)$ functions given in reference 9 for a sinking airfoil can be applied to the case of a pitching airfoil to obtain the starting values for the $k_{1_q}(s)$ and $m_{1_q}(s)$ functions if a substitution of the boundary conditions due to pitching velocity is made in lieu of sinking velocity as indicated by equation (1). If the perturbation velocity component due to a sudden change in pitching velocity is substituted for the perturbation velocity due to sinking motion in the analysis given in reference 9 for determining $k_1(s)$ and $m_1(s)$ at $s = 0$, the following expressions are obtained for the starting values of the $k_{1_q}(s)$ and $m_{1_q}(s)$ functions

$$k_{1_q}(0) = \frac{1}{\pi M} \tag{19}$$

and

$$m_{1_q}(0) = -\frac{5}{12}\frac{1}{\pi M} \tag{20}$$

The values of $k_{1_q}(0)$ and $m_{1_q}(0)$, however, correspond, respectively, to the values of the flutter coefficients $F_{c_q}(k)$ and $M_q(k)$ at infinite frequency. The proof for this correspondence is identical to the one shown for the sinking airfoil given in appendix A of reference 2. Thus equations (19) and (20) denote, respectively, the values of $F_{c_q}(\infty)$ and $M_q(\infty)$

$$F_{c_q}(\infty) = \frac{1}{\pi M} \tag{21}$$

and

$$M_q(\infty) = -\frac{5}{12}\frac{1}{\pi M} \tag{22}$$

RESULTS AND DISCUSSION

Numerical Solution of the Reciprocal Equations for a Mach Number of 0.7

As noted in reference 2, the indicial lift and moment functions determined by the reciprocal equations containing the in-phase lift or moment functions ($F_c(k)$ or $M(k)$) provided a more reliable and simpler solution than the functions determined by the reciprocal equations containing the out-of-phase components ($G_c(k)$ and $N(k)$). Consequently, the indicial lift and moment functions for the pitching-velocity case were determined numerically by using equations (5a) and (12a) in a manner similar to that shown for the case of the sinking airfoil.

In figure 1 a plot is shown of the complex oscillatory lift functions due to pitching velocity at a Mach number of 0.7 which were evaluated from the flutter coefficients by use of equations (10a) and (10b). If

$$f_q(k) = F_{c_q}(k) - F_{c_q}(\infty) \tag{23}$$

is substituted into equation (5a), the following form which can be evaluated graphically more readily is obtained:

$$k_{1_q}(s) = F_{c_q}(\infty) + \frac{2}{\pi}\int_0^{\infty}\frac{f_q(k)\ \sin ks}{k}\,dk \tag{24}$$

A plot of the function $f_q(k)$ is shown in figure 2. The integrand in equation (24) was graphically evaluated for several values of the parameter s. In figure 3, plots of the indicial lift function $k_{1_q}(s)$ are shown for Mach numbers of 0.7 and 0. The indicial lift function $k_1(s)$ due to angular position alone (corresponding to the case of a sinking airfoil) is plotted in figure 4 as determined from reference 2 for a Mach number of 0.7. Also shown in this figure are the beginning position of the curve for a Mach number of 0.7 as determined by the equations given in reference 3, the curve for a Mach number of 0.8 given in reference 3, and the curve for a Mach number of 0. Comparison of the $k_1(s)$ and $k_{1_q}(s)$ functions at Mach numbers of 0 and 0.7 indicates that the growth of lift to the steady state for a pitching airfoil is less rapid in subsonic compressible flow than for incompressible flow.

The indicial moment function $m_{1_q}(s)$ due to a sudden change in pitching velocity may be obtained in a manner similar to that shown for determining the indicial lift function $k_{1_q}(s)$. In figure 5, plots are shown of the complex moment function due to pitching velocity for a Mach number of 0.7 determined from the flutter coefficients M_1, M_2, M_3, and M_4 by equations (14). The asymptotic value for $M_q(k)$ is determined by equation (22). Because of the nature of the curves at the higher reduced frequencies, the curves are represented by dashed lines to indicate a degree of unreliability. Reference 2, however, indicates that errors in the curves at the higher reduced frequencies would have little effect on the indicial function.

In a manner similar to that shown for the lift case, the $M_q(k)$ function was transformed by the following equation

$$m_q(k) = M_q(k) - M_q(\infty) \tag{25}$$

in order to facilitate the numerical determination of the indicial moment function $m_{1_q}(s)$. A plot of $m_q(k)$ is shown in figure 6. The function $m_{1_q}(s)$ was evaluated graphically by substitution of equation (25) into equation (12a) for several values of the parameters and is plotted in figure 7 together with the solution for a Mach number of 0. In figure 8 a plot of the indicial moment function due to angle-of-attack change (no pitching velocity) $m_1(s)$ is shown for Mach numbers of 0.7 and 0.8 determined, respectively, from references 2 and 3 together with the solution for a Mach number of 0.

References 8 and 9 show that the indicial lift due to pitching velocity on an airfoil pitching about its three-quarter-chord position is impulsive at $s = 0$ and is zero thereafter for incompressible flow. For compressible flow the indicial lift, however, is initially finite and is time-dependent thereafter, as is shown subsequently. When the results presented in figures 3, 4, 7, and 8 are used, the indicial lift and moment functions due to pitching velocity for an airfoil pitching about any axis can be obtained readily by the following general transformations:

$$\left(k_{1_q}\right)_x(s) = k_{1_q}(s) - \frac{x}{c} k_1(s) \tag{26a}$$

and

$$\left(m_{1_q}\right)_x(s) = m_{1_q}(s) - \frac{x}{c} m_1(s) \tag{26b}$$

where x denotes the location of the axis of rotation measured positively to the right from the leading edge of the airfoil (see sketch in section entitled "Method of Analysis"). For the case of an airfoil pitching about its three-quarter-chord location, equations (26a) and (26b) become

$$\left(k_{1_q}\right)_{\frac{3c}{4}}(s) = k_{1_q}(s) - \frac{3}{4} k_1(s) \tag{27a}$$

and

$$\left(m_{1_q}\right)_{\frac{3c}{4}}(s) = m_{1_q}(s) - \frac{3}{4} m_1(s) \tag{27b}$$

The functions $\left(k_{1_q}\right)_{\frac{3c}{4}}(s)$ and $\left(m_{1_q}\right)_{\frac{3c}{4}}(s)$ were determined for a Mach number of 0.7 from equations (27a) and (27b) and from the results plotted in figures 3, 4, 7, and 8. Plots of the functions $\left(k_{1_q}\right)_{\frac{3c}{4}}(s)$

and $\left(m_{1_q}\right)_{\frac{3c}{4}}(s)$ are shown in figures 9 and 10, respectively, together with the indicial functions for the case for a Mach number of 0. Also included in these figures are the solutions for a Mach number of 0.8 and part of the solution for a Mach number of 0.7, both determined from reference 3. A comparison of the $\left(k_{1_q}\right)_{\frac{3c}{4}}(s)$ function for subsonic compressible and incompressible flow indicates that, although this function is impulsive at $s = 0$ and zero for $s > 0$ for incompressible flow, the function is finite at $s = 0$ and decays to zero very rapidly in the compressible case. If for compressible flow the circulation is assumed to be zero for $s > 0$ (as is the case for this component at $M = 0$), the time-dependent function which is present in compressible flow may be attributed to the time-dependent apparent-mass effects. Also, the part of the $k_{1_q}(s)$ function shown in figure 4 for s greater than approximately 4 may be associated only with the lift due to circulation. Comparison of this part of the curve with the curve for a Mach number of 0 in the region $s > 4$ indicates that the growth of lift to the steady state is relatively less rapid for compressible flow than for incompressible flow.

Approximation of Indicial Lift and Moment Functions

by Analytic Expressions

Since the exponential function has a simple operational equivalent and it has been found convenient to approximate the $k_1(s)$ function at a Mach number of 0 (see reference 10), a limited series of such functions were chosen to approximate the indicial lift and moment functions for sinking and pitching motion at a Mach number of 0.7. The functions were found to fit these curves quite well and are:

$$k_1(s) = 1.4\left(1 - 0.364e^{-0.0536s} - 0.405e^{-0.357s} + 0.419e^{-0.902s}\right) \tag{28}$$

$$\left(k_{1_q}\right)_{\frac{3c}{4}}(s) = -0.083e^{-0.800s} - 0.293e^{-1.565s} + 0.149e^{-2.44s} \tag{29}$$

$$m_1(s) = -0.2425e^{-0.974s} + 0.084e^{-0.668s} - 0.069e^{-0.438s} \tag{30}$$

$$\left(m_{1_q}\right)_{\frac{3c}{4}}(s) = -0.0875\left(1 + 0.1141e^{-0.1865s} - 1.233e^{-1.141s} + 0.3337e^{-4.04s}\right) \tag{31}$$

These analytic approximations given by equations (28), (29), (30), and (31) are plotted in figures 4, 9, 8, and 10, respectively. Comparison of the approximation with the actual curves in these figures indicates the order of good agreement reached.

The corresponding approximate expressions for the harmonically oscillating airfoil can be found from the reciprocal form of equations (2) and (5) where the harmonically oscillating functions are expressed in terms of their corresponding indicial function. For the case of the sinking airfoil, the reciprocal form of equation (2) is

$$F_c(k) + iG_c(k) = ik\int_0^\infty k_1(s)e^{-iks}\,ds \tag{32}$$

If ik is considered as the operator in the Laplace transformation, then equation (32) is simply ik times the Laplace transformation of $k_1(s)$. Therefore $F_c(k)$ and $G_c(k)$ are, respectively, the real and imaginary parts of ik times the Laplace transform of $k_1(s)$. Substitution of equation (28) into equation (32) leads to the following expressions for $F_c(k)$ and $G_c(k)$:

$$\left.\begin{aligned}F_c(k) &= 1.4\left[1 - \frac{0.364k^2}{(0.0536)^2 + k^2} - \frac{0.405k^2}{(0.357)^2 + k^2} + \frac{0.419k^2}{(0.902)^2 + k^2}\right]\\ G_c(k) &= 1.4\left[-\frac{(0.364)(0.0536)k}{(0.0536)^2 + k^2} - \frac{(0.405)(0.357)k}{(0.357)^2 + k^2} + \frac{(0.419)(0.902)k}{(0.902)^2 + k^2}\right]\end{aligned}\right\} \tag{33}$$

In a manner similar to that shown for the lift case, the following approximate expressions for $M(k)$ and $N(k)$ can be obtained:

$$\left.\begin{aligned} M(k) &= \frac{-0.2425k^2}{(0.974)^2 + k^2} + \frac{0.084k^2}{(0.668)^2 + k^2} - \frac{0.069k^2}{(0.438)^2 + k^2} \\ N(k) &= \frac{(-0.2425)(0.974)k}{(0.974)^2 + k^2} + \frac{(0.084)(0.668)k}{(0.668)^2 + k^2} - \frac{(0.069)(0.438)k}{(0.438)^2 + k^2} \end{aligned}\right\} \quad (34)$$

The expressions for the $\left(F_{c_q}\right)_{\frac{3c}{4}}(k)$ and $\left(G_{c_q}\right)_{\frac{3c}{4}}(k)$ lift functions due to pitching velocity are

$$\left.\begin{aligned} \left(F_{c_q}\right)_{\frac{3c}{4}}(k) &= \frac{-0.083k^2}{(0.800)^2 + k^2} - \frac{0.293k^2}{(1.565)^2 + k^2} + \frac{0.149k^2}{(2.44)^2 + k^2} \\ \left(G_{c_q}\right)_{\frac{3c}{4}}(k) &= \frac{(-0.083)(0.800)k}{(0.800)^2 + k^2} - \frac{(0.293)(1.565)k}{(1.565)^2 + k^2} + \frac{(0.149)(2.44)k}{(2.44)^2 + k^2} \end{aligned}\right\} \quad (35)$$

and the expressions for the $\left(M_q\right)_{\frac{3c}{4}}(k)$ and $\left(N_q\right)_{\frac{3c}{4}}(k)$ moment functions are

$$\left.\begin{aligned} \left(M_q\right)_{\frac{3c}{4}}(k) &= -0.0875\left[1 + \frac{0.1141k^2}{(0.1865)^2 + k^2} - \frac{1.233k^2}{(1.141)^2 + k^2} + \frac{0.3337k^2}{(4.04)^2 + k^2}\right] \\ \left(N_q\right)_{\frac{3c}{4}}(k) &= -0.0875\left[\frac{(0.1141)(0.1865)k}{(0.1865)^2 + k^2} - \frac{(1.233)(1.141)k}{(1.141)^2 + k^2} + \right. \\ &\qquad \left. \frac{(0.3337)(4.04)k}{(4.04)^2 + k^2}\right] \end{aligned}\right\} \quad (36)$$

CONCLUDING REMARKS

The indicial lift and moment functions due to a sudden change in pitching velocity have been obtained from the available data on a harmonically pitching airfoil at a Mach number of 0.7 by the use of reciprocal relations. Comparison of the results obtained for the case of a Mach number of 0.7 with incompressible flow indicates that the growth of lift to the steady state appears to be less rapid for compressible flow than for incompressible flow for a pitching airfoil. Although the circulatory lift component due to pitching velocity for an airfoil rotating about its three-quarter-chord point was zero for incompressible flow, a time-dependent component of lift was found to exist for compressible flow and decayed to a negligible value in approximately 2 chords.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., October 30, 1951

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TABLE I

LIFT AND MOMENT OSCILLATORY COEFFICIENTS FOR SINKING AND PITCHING MOTION AT A MACH NUMBER OF 0.7

Reference	k	Z_1	Z_2	Z_3	Z_4	M_1	M_2	M_3	M_4
4	0	0	0	1.4003	0	0	0	0	0
	.02	.00711	.05177	1.2985	-.13891	-.00026	.00003	.000574	.010002
	.04	.02006	.09524	1.2059	-.17914	-.00097	.00016	.001452	.019332
	.06	.03399	.1324	1.1304	-.18304	-.00208	.00042	.002319	.028182
	.08	.04722	.1651	1.0698	-.17128	-.00355	.00083	.003090	.036758
	.10	.0589	.1941	1.0216	-.14992	-.0053	.00126	.003675	.045114
	.20	.0945	.3186	.8869	-.003247	-.0186	.00574	.004301	.086501
	.30	.0991	.4332	.8364	.14614	-.0393	.01346	.002632	.12953
	.40	.0827	.5523	.8199	.28495	-.0672	.02596	.000512	.17541
	.50	.0538	.6820	.8239	.4135	-.1014	.04482	.000436	.2242
	.60	.0187	.8229	.8399	.5294	-.1402	.07175	.000573	.27498
	.70	-.0154	.9751	.8669	.6355	-.1813	.1090	.010257	.32692
5	.80	-.0449	1.1316	.8934	.7191	-.2178	.1578	.021912	.37005
	1.00	-.0874	1.4482	.9391	.8607	-.2798	.2665	.059166	.45875
6	1.50	-.39295	2.311	.8625	1.3512	-.4400	.4896	-.001739	.58628
	2.00	-.6521	3.306	.8110	1.8258	-.5740	.77185	-.062784	.73568
	2.50	-.6083	4.241	.8566	2.0913	-.4547	1.218	.090049	.93504

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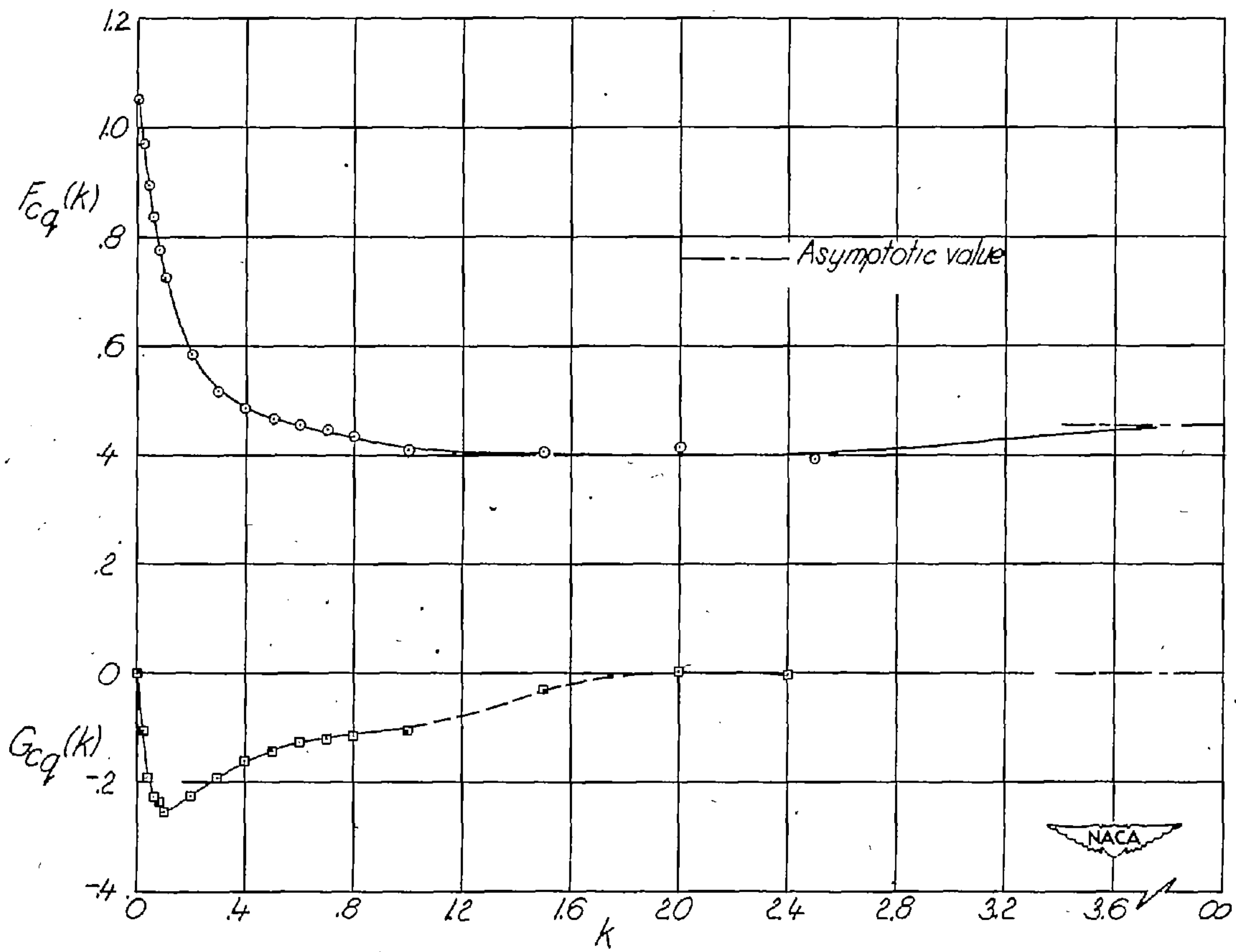

Figure 1.- Oscillatory lift functions due to pitching velocity for airfoil rotating about its leading edge. M = 0.7.

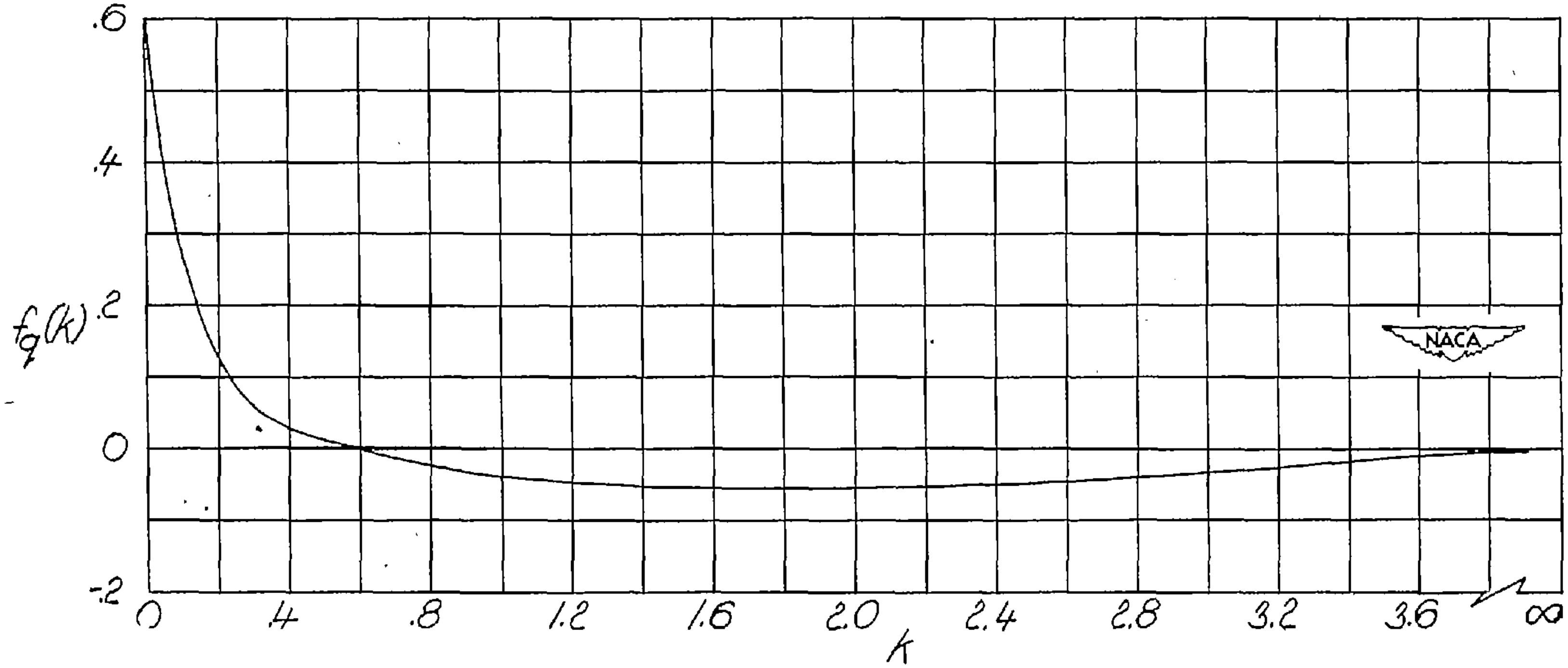

Figure 2.- Plot of $f_q(k)$.

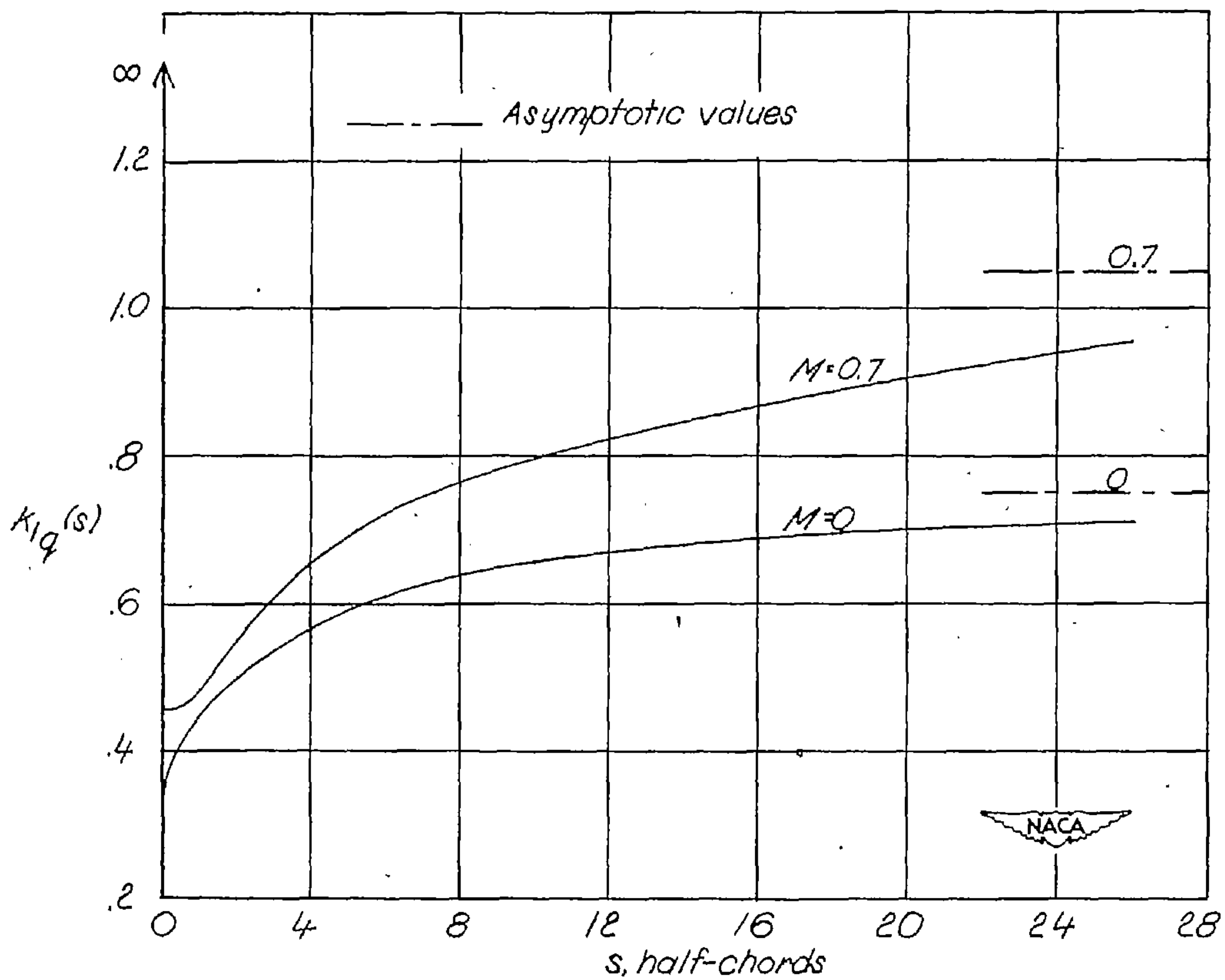

Figure 3.- Comparison of indicial lift functions due to a sudden change in pitching velocity for an airfoil rotating about its leading edge at M = 0 and M = 0.7.

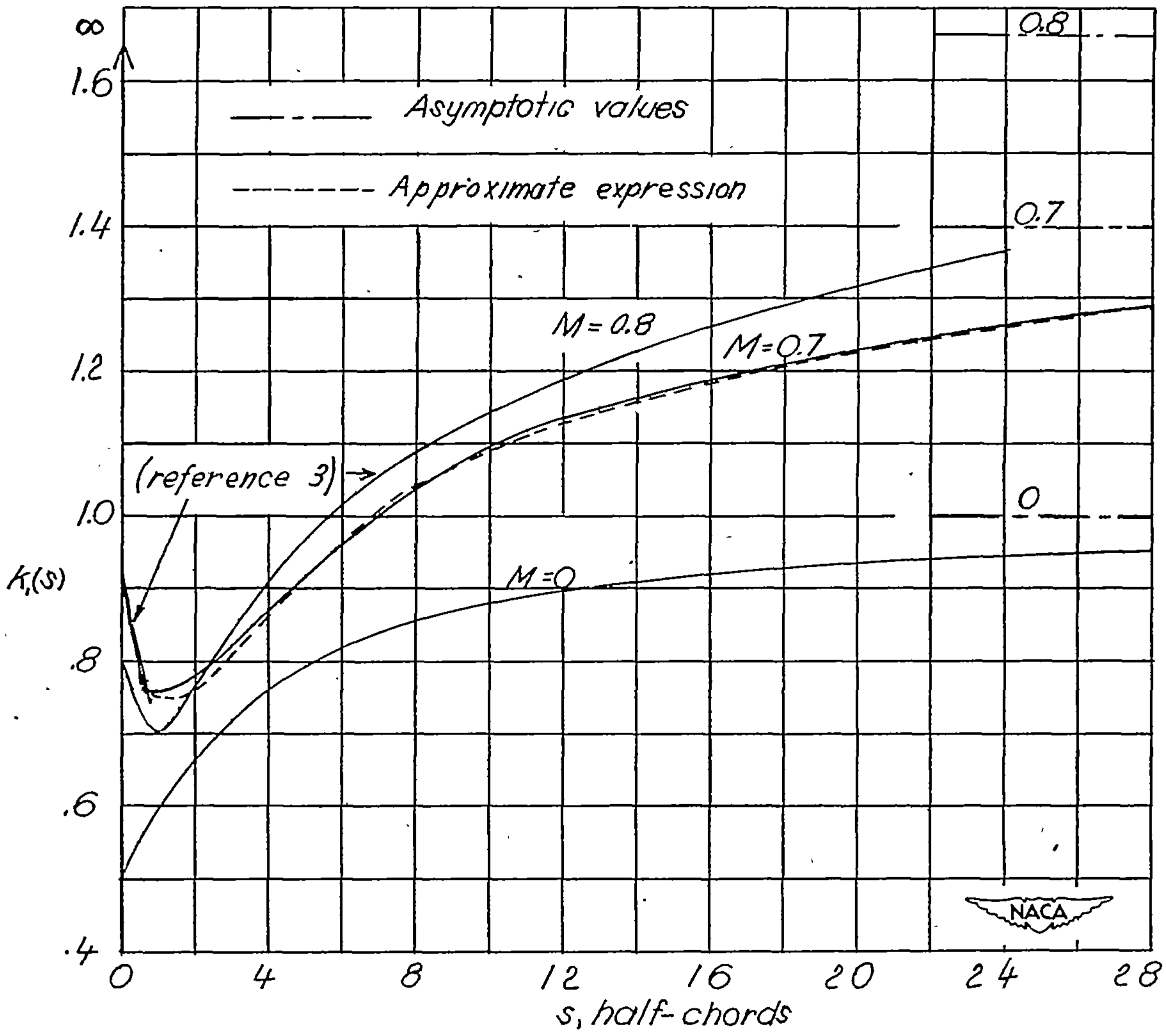

Figure 4.- Comparison of indicial lift functions due to a sudden change in angle of attack (without pitching motion) at M = 0, M = 0.7, and M = 0.8.

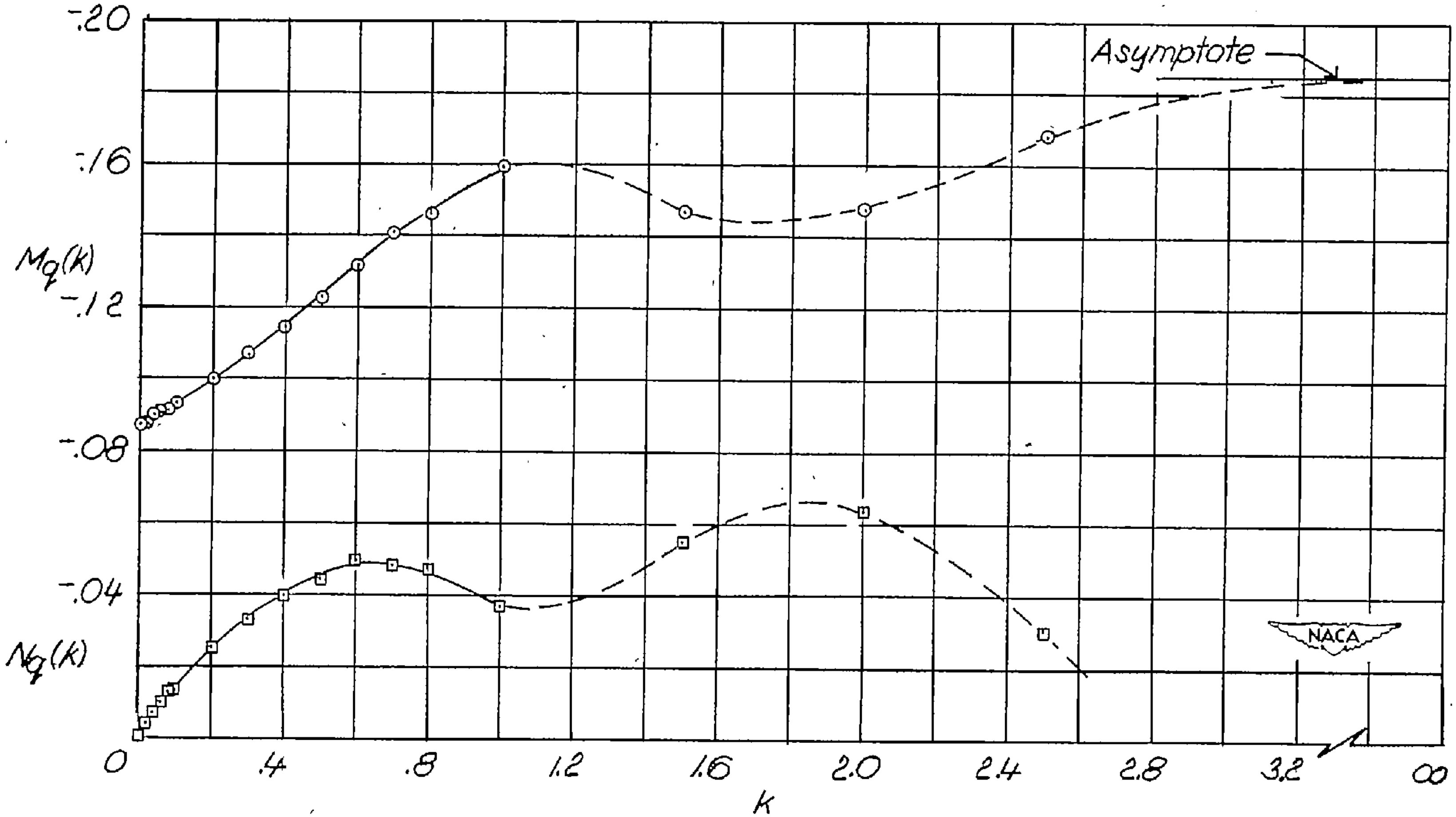

Figure 5.- Oscillatory moment function due to pitching velocity for an airfoil pitching about its leading edge at M = 0.7. (Moment taken about quarter-chord point.)

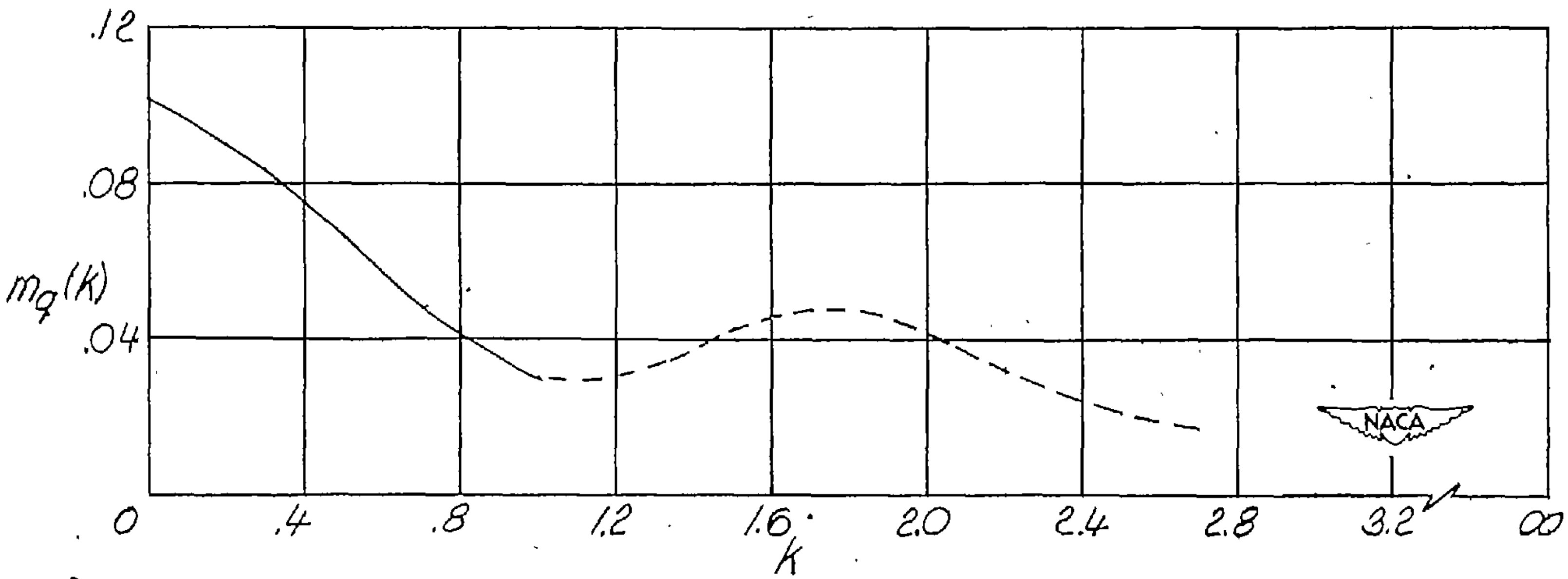

Figure 6.- Plot of $m_q(k)$.

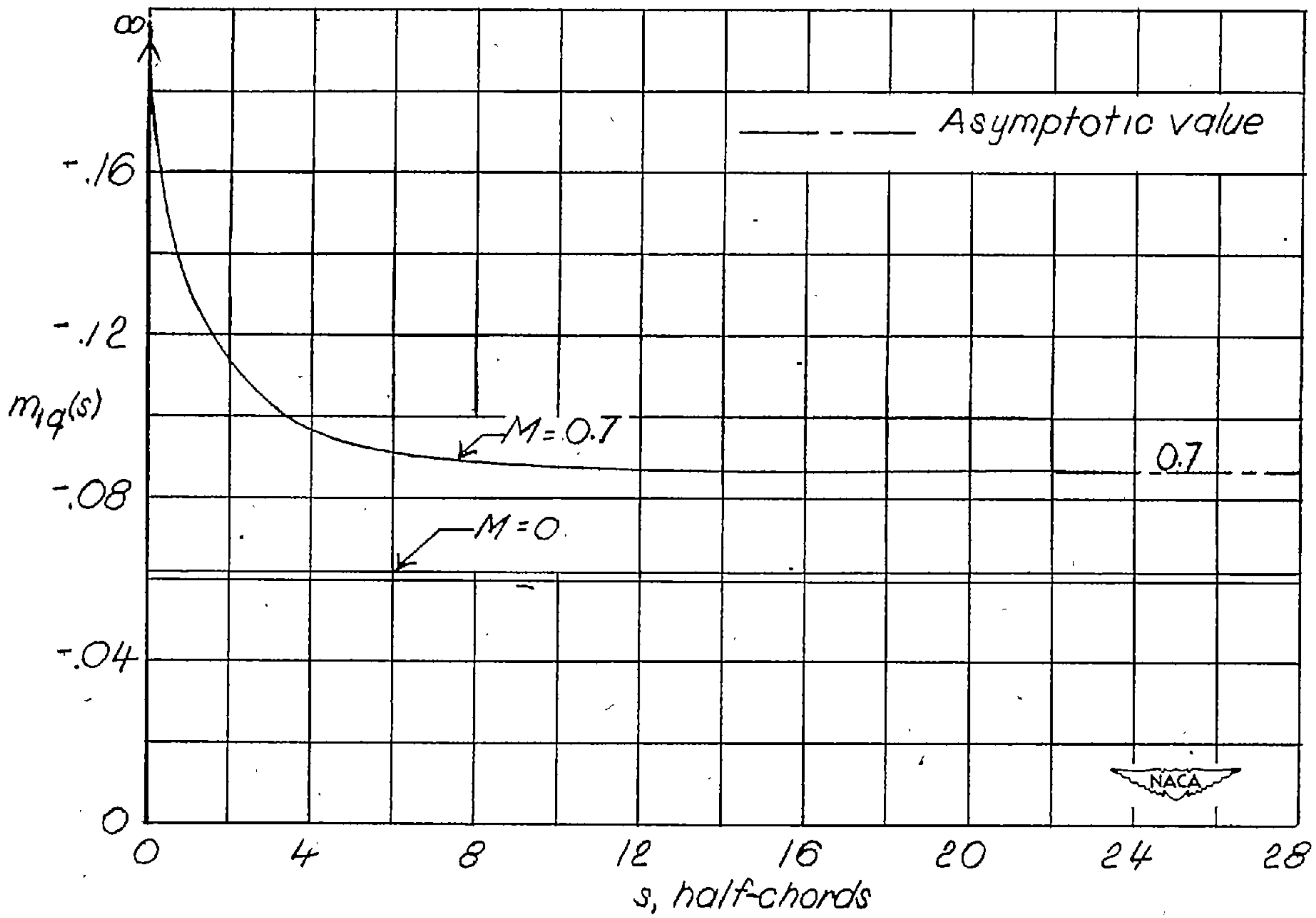

Figure 7.- Comparison of indicial moment functions due to a sudden change in pitching velocity for an airfoil rotating about its leading edge at $M = 0$ and $M = 0.7$. (Moment taken about quarter-chord point.)

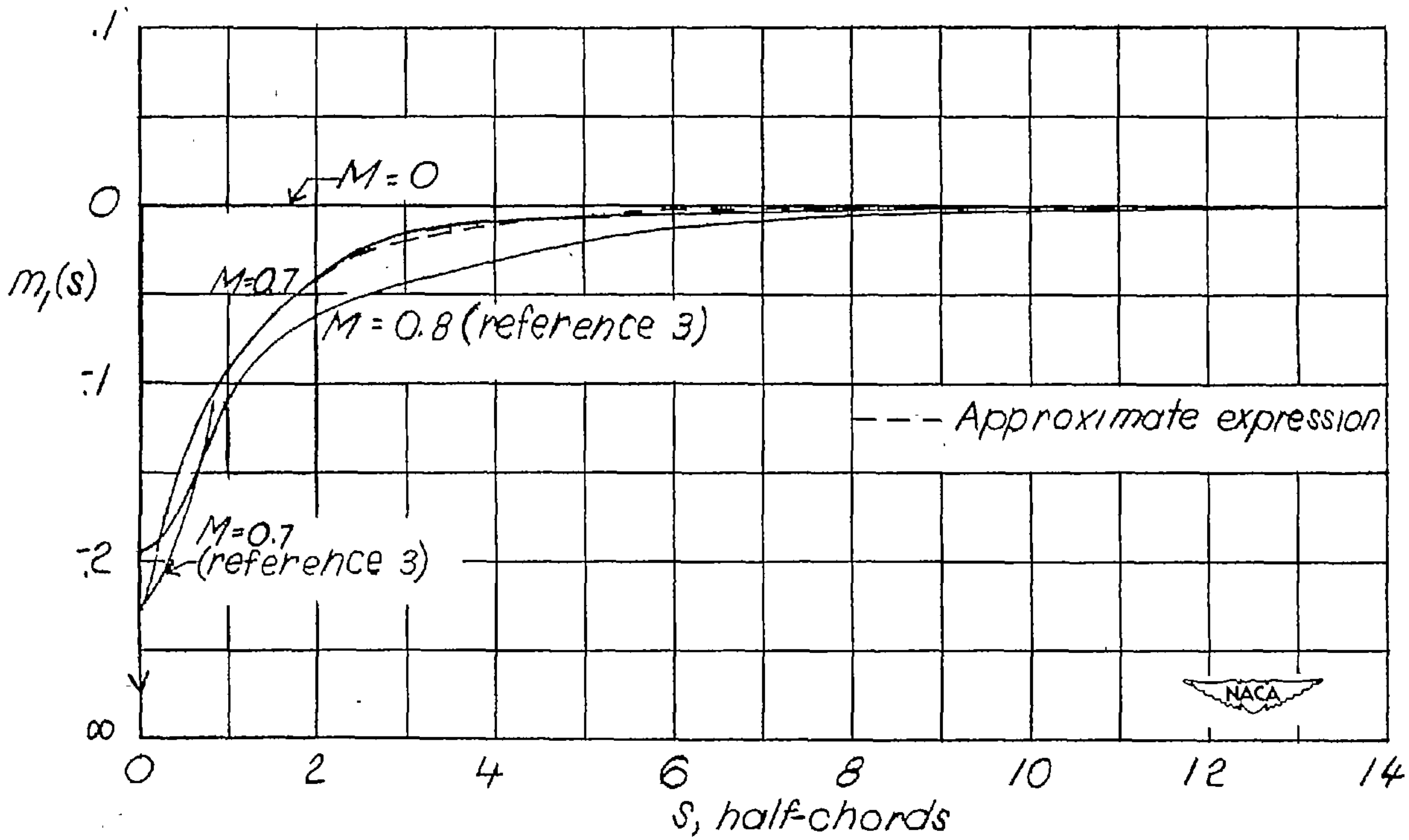

Figure 8.- Comparison of indicial moment functions due to a sudden change in angle of attack (no pitching motion) at $M = 0$, $M = 0.7$, and $M = 0.8$. (Moment taken about quarter-chord point.)

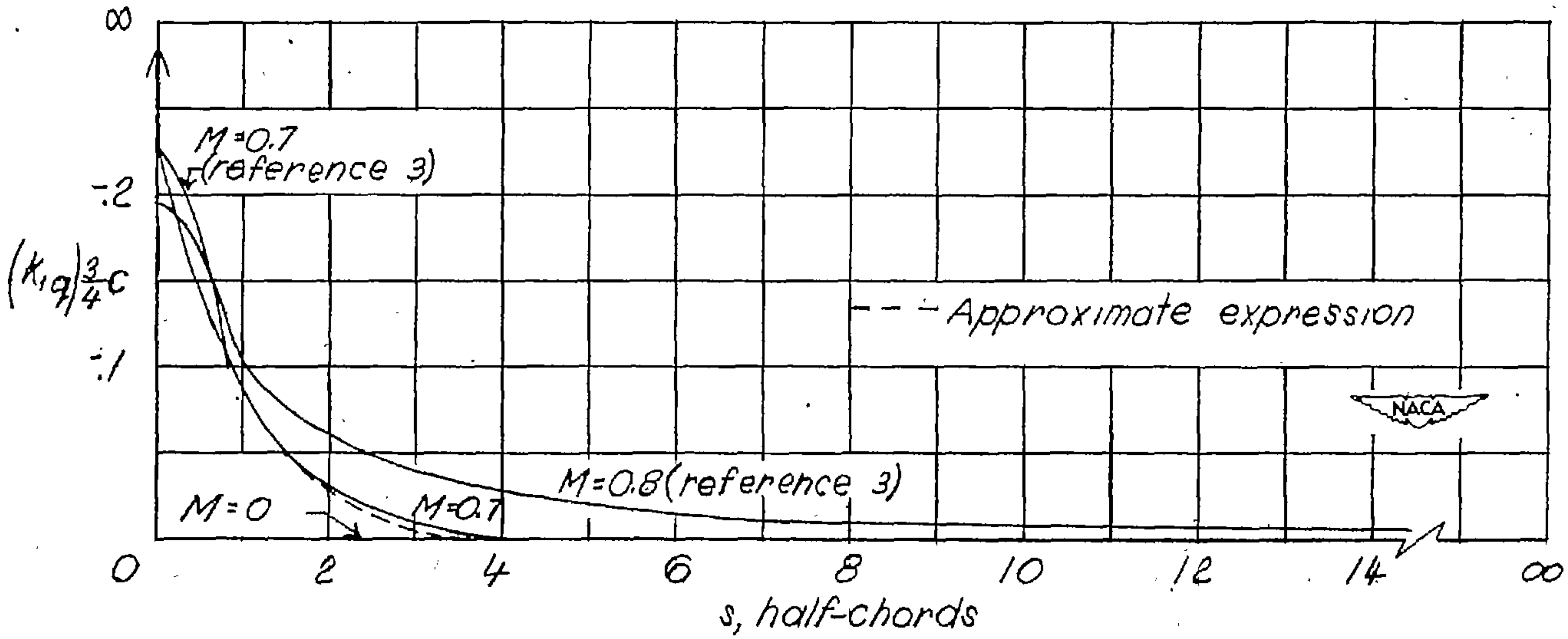

Figure 9.- Comparison of indicial lift functions due to a sudden change in pitching velocity for an airfoil rotating about its three-quarter-chord point at M = 0, M = 0.7, and M = 0.8.

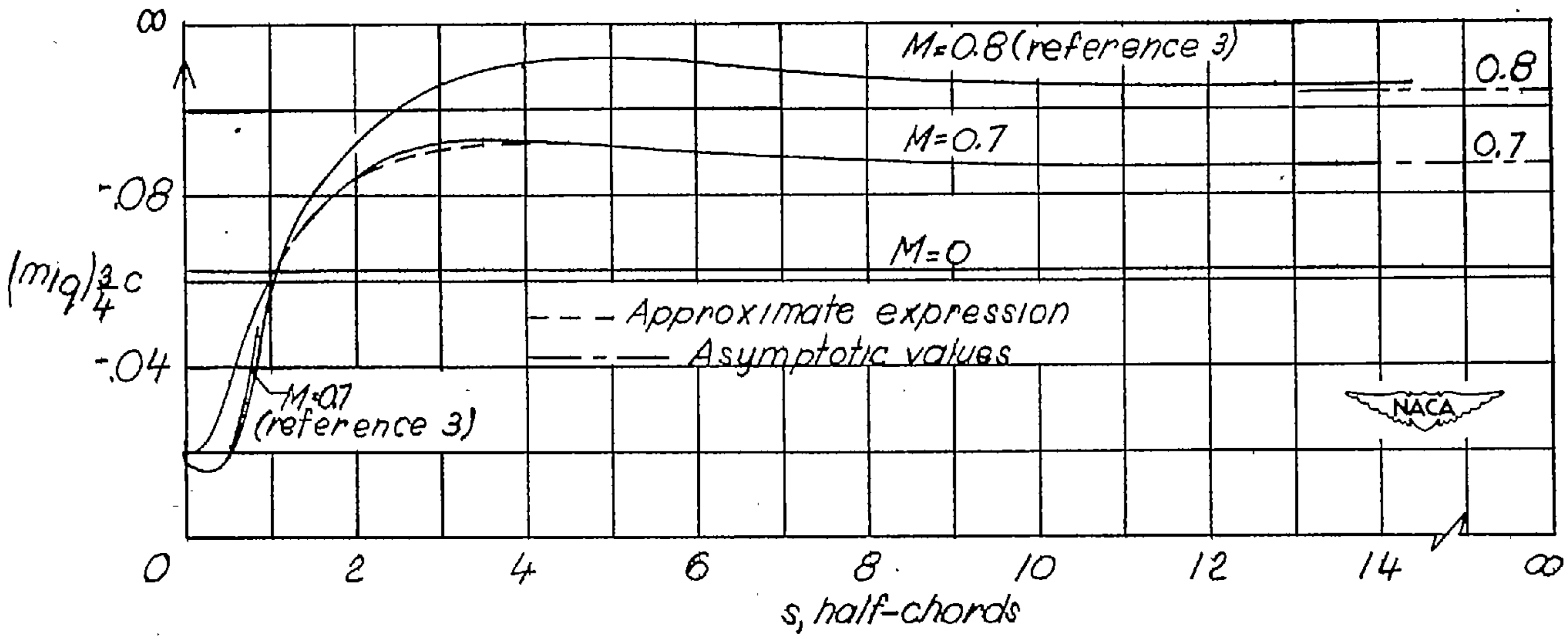

Figure 10.- Comparison of indicial moment functions due to a sudden change in pitching velocity for an airfoil rotating about its three-quarter-chord point of $M = 0$, $M = 0.7$, and $M = 0.8$. (Moment taken about quarter-chord point.)

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