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TECHNICAL NOTE 2735

AN ANALYSIS OF THE NORMAL ACCELERATIONS AND AIRSPEEDS OF
A TWO-ENGINE TYPE OF TRANSPORT AIRPLANE IN COMMERCIAL
OPERATIONS ON ROUTES IN THE CENTRAL UNITED STATES

FROM 1948 TO 1950

By Walter G. Walker and Paul W. J. Schumacher

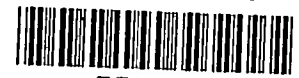
Langley Aeronautical Laboratory
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SUMMARY

Normal-acceleration and airspeed data obtained from NACA V-G recorders installed in a two-engine type of transport airplane, employed in routine commercial operation on a north-south route in the central part of the United States from 1948 to 1950, are analyzed to determine the gust and gust-load experiences of the airplane. The accelerations experienced equaled or exceeded the limit-gust-load-factor increment, on the average, twice (once positive and once negative) in about 2.8×10^6 flight miles, and an effective gust velocity of 30 feet per second was equaled or exceeded, on the average, twice in about 0.7×10^6 flight miles. A comparison of these values with corresponding values from the same type of airplane during past operations indicates that the gust and gust-load experiences for the present operation agree well with the wartime (1941 to 1945) experiences but were about 20 percent larger than the prewar (1937 to 1941) experiences.

INTRODUCTION

A general investigation has been under way for a number of years to determine (from V-G data) the magnitude and frequency of occurrence of gusts and gust loads experienced by transport-type aircraft in routine postwar commercial operation. This investigation was designed to supplement previous research of this type, as it was felt that changes in operating conditions since the prewar period might be reflected in changes in the gust and load experiences. The results of some earlier phases of the current program are reported in references 1 and 2.

This paper presents the results of an analysis of V-G data obtained from a two-engine type of airplane, operated by a commercial transport

airline on north-south routes in the central part of the United States, from 1948 to 1950. The data on the gust, gust-load, and airspeed experiences of these airplanes are compared with similar data obtained from airplanes of the same type during operations from 1937 to 1945. These data are also used to predict the gust-load experience of another type of transport airplane of more recent design flown in somewhat similar airline operations.

SYMBOLS

A	aspect ratio (b^2/S)
b	span, feet
\bar{c}	mean geometric chord, feet
K	gust-alleviation factor (ref. 3, p. 11)
m	slope of lift curve per radian
W	gross weight, pounds
S	wing area, square feet
Δn	normal-acceleration increment, g units
Δn_{LLF}	design limit-gust-load-factor increment, g units
U_e	effective gust velocity, feet per second
V	equivalent airspeed, miles per hour
V_B	design speed for maximum gust intensity, miles per hour (ref. 3, p. 3)
V_C	design cruising speed, miles per hour (ref. 3, p. 3)
V_D	design diving speed, miles per hour (ref. 3, p. 3)
V_L	design maximum level-flight speed, miles per hour
V_{NE}	never-exceed speed, miles per hour (ref. 3, p. 36)

- V_0 indicated airspeed at which maximum positive or negative acceleration increment occurs on a V-G record, miles per hour
- V_p most probable operating airspeed at which maximum acceleration increment occurs in a sample of V-G data, miles per hour
- C_N normal-force coefficient
- τ average flight time per V-G record, hours
- T total flight time for number of V-G records in a sample, hours
- N total number of observations
- P probability that maximum value on a V-G record will equal or exceed a given value
- P_V probability that maximum indicated airspeed for a record will equal or exceed a given value
- $P_{\Delta n}$ probability that maximum positive or negative acceleration increment for a record will equal or exceed a given value
- P_p product probability of acceleration and airspeed
- u location parameter of distribution of extreme values (ref. 4, p. 2)
- α scale parameter of distribution of extreme values (ref. 4, p. 2)
- σ_V, σ_0 standard deviations of distributions of V_{max} and V_0 , respectively (ref. 5, p. 73)
- k_V, k_0 coefficients of skewness of distributions of V_{max} and V_0 , respectively (ref. 5, pp. 74-75)

Subscripts:

- \max maximum value of variable
- 30 denotes an effective gust velocity of 30 feet per second

A bar over a symbol indicates the mean value of the variable for a given set of observations.

SCOPE AND EVALUATION OF RECORDS

The scope of the V-G records analyzed herein is summarized in table I which gives the number of records, the range of record hours, the average hours per record, and the total record hours. The 79 records evaluated represent a total of 23,940 hours of flight during the operations of seven airplanes from 1948 to 1950. During the record collecting period, NACA oil-damped V-G recorders described in reference 6 were substituted for the older friction-damped type. Each type of recorder contributed approximately one-half of the records obtained from each airplane. Table I accordingly gives a breakdown of the records by instrument type for subsequent use in studying the influence of the different instrument characteristics on the measured values of the gust loads and gusts. For studying variations in the gust-load and gust experiences due to seasonal weather changes during the 1948 to 1950 operation, each year was broken down into two periods, and each period contains approximately the same number of records from each type of V-G recorder. As shown in table I, period I (summer) presents a breakdown of the data from April through September and period II (winter) presents the data from October through March in order to examine gustiness during the summer and the winter.

The routes covered by the present operation were over level terrain running in a north-south direction through the central part of the United States. The operating altitudes during the times the records were collected were not supplied; however, most flights are believed to have been at low altitudes (below 10,000 ft) because the routes flown were between closely spaced terminals. Since no detailed information was provided for actual operating weights, an average value of 85 percent of the design gross weight, which is assumed as a representative average, was taken as the weight at which the gust loads and gusts were experienced.

Faulty records and records taken during pilot check flights were not used in the analysis. Information was supplied regarding large acceleration and airspeed occurrences in some records and such information was of appreciable help in evaluating the records. Owing to the limited amount of data available, all records which could be included were used regardless of the average time covered by the record. Since this method of record selection differs from that used in the past (ref. 7), wherein the sample is selected on the basis of near-constant record times, a check was made of the results obtained by the two methods. No significant differences were found in the over-all probability distributions of gust loads and gusts experienced on the basis of the two methods of record selection. This result in the present case appears to be due to the clustering of the majority of record times near the average times of the distributions as shown in figure 1.

The airplane characteristics used in the analysis are as follows:

Design gross weight, W, lb	25,200
Wing area, S, sq ft	987
Wing span, b, ft	95
Mean geometric chord, \bar{c} , ft	10.4
Aspect ratio, A	9.1
Limit-gust-load-factor increment, Δn_{LLF} ,	
g units	2.34
Computed slope of lift curve,	
$m = \frac{6A}{A + 2}$, per radian	4.92
Gust-alleviation factor, K -	
For gross weight	1.100
For 0.85 gross weight	1.064
Maximum normal-force coefficient, C_{Nmax}	1.6
Maximum level-flight speed, V_L , mph	211
Design speed for -	
Maximum gust intensity, V_B , mph	138
Cruising, V_C , mph	211
Diving, V_D , mph	286
Placard never-exceed speed, V_{NE} , mph	257

The Δn_{LLF} value given in this table was computed in accordance with current Civil Aeronautics Administration design requirements, reference 3, by using the design gross weight, an effective gust velocity U_e of 30K feet per second at V_C , and a slope of the lift curve computed from the relation using aspect ratio as recommended in reference 8. The values of C_{Nmax} , V_L , and V_D were obtained from design data of the airplane manufacturer. The design speeds V_B and V_C were obtained in accordance with the requirements of reference 3. The value of V_C was selected as equal to V_L . The value of V_{NE} is equal to $0.9V_D$, which agrees with the value used by the airplane manufacturer.

The procedures used in evaluating the records have been described in detail in reference 2. Briefly, the procedures consisted of reading from each record the maximum positive and negative acceleration increments Δn_{max} , the airspeed V_0 at which these accelerations occurred, and the maximum airspeed V_{max} . In addition, the maximum positive and negative Δn values in each 20-mile-per-hour speed bracket covering the range from 120 to 240 miles per hour were read from each record. The maximum effective gust velocity $U_{e_{max}}$ and the maximum effective gust velocities within each 20-mile-per-hour speed bracket were read

from each record by the use of plotted lines of constant gust velocity computed from the sharp-edge-gust formula (ref. 1, p. 2). The gust velocities obtained in this manner are based on the assumed average airplane weight of 85 percent of the design gross weight. The positive and negative values read from the V-G records were combined for the analysis because past evidence indicates that most of the V-G records obtained in turbulence are essentially symmetric with respect to positive and negative acceleration increments.

The V-G records used in this analysis were obtained near the center of gravity of the airplane. Although it is recognized that dynamic response of the aircraft structure may significantly influence the accelerations measured at the center of gravity, dynamic-response effects are not known for this type of airplane and are not accounted for in the gust-load and gust experiences presented herein.

ANALYSIS AND RESULTS

The frequency distributions of Δn_{\max} , U_{\max} , V_0 , and V_{\max} obtained from the values read from the V-G records are shown in table II. Table III shows the frequency distributions of Δn obtained from the values read from the airspeed brackets of each record. The parameter values given in tables II(a), II(b), and III for the acceleration and gust-velocity distributions are the mean value of the distribution and the location and scale parameters u and α , respectively, which were obtained by applying the statistical procedures outlined in reference 4. Extreme-value distributions (ref. 4) were fitted to the Δn_{\max} , the U_{\max} , and the Δn by speed-bracket distributions by using the parameters of these distributions in accordance with the method of reference 4 to obtain the probability P of equaling or exceeding given values of the respective variables. The parameter values given in table II(c) for the airspeed distributions - the mean values \bar{V}_0 and \bar{V}_{\max} , the standard deviations σ_0 and σ_v , and the coefficients of skewness k_0 and k_v - were obtained by applying the statistical procedures given in reference 5 to these distributions. Since extreme-value distributions were not applicable to distributions of maximum airspeeds, as reference 4 points out, Pearson Type III distributions (ref. 9) which in past work had yielded reasonable representations of maximum airspeeds were used. The V_{\max} distribution of table II(c) was fitted with a Pearson Type III distribution by using the parameters of the V_{\max} distribution and the procedures outlined in reference 7 for obtaining the probability P of equaling or exceeding given values of maximum airspeed. The probable speed V_p given in table II(c) is a measure of the most frequent operating airspeed at which the largest accelerations were experienced.

The value of V_p was computed from the parameters of the V_0 distribution by using the equation for the mode given in reference 9, page 92.

The theoretical probability distributions fitted to the observed distributions of Δn_{max} , U_{max} , and V_{max} were transformed to curves of average flight miles to equal or exceed given values of the variable by using the relation

$$\text{Flight miles} = 0.8V_C \tau \left(\frac{1}{P} \right) \quad (1)$$

The value of $0.8V_C$ used in this relation is assumed as an average speed for these operations.

Figure 2 shows the average flight miles to equal or exceed given values of Δn_{max} twice (once positive and once negative) for the data from the NACA oil-damped and friction-damped recorders. Figure 3 shows the average flight miles to equal or exceed given values of Δn_{max} twice for the data taken during the period I (summer) and period II (winter) operations. Similarly, figures 4, 5, and 6 show the average flight miles to equal or exceed the total Δn_{max} , U_{max} , and V_{max} values, respectively.

Confidence bands obtained by using the method of reference 10 are shown in figures 2 to 5 to aid in interpreting the results. These confidence bands can be taken as a measure of the range within which, for a given probability level (a probability level of 95 percent is used herein), the true value may be expected to lie. The portions of the curves beyond the data points have been extrapolated as indicated by the dashed lines in the figures.

Figures 7 and 8 show a comparison of average values of maximum-acceleration-increment and of maximum-effective-gust-velocity ratios, respectively, for the present data and for data representing the same type of airplane operated on three different routes, and also shown in these figures are the 95-percent confidence bands about the ratio values. The letter and Roman numeral designations used in figures 7 and 8 were taken from reference 2 to represent the airlines and routes given therein. For the present operation the ratios were obtained from the values indicated in figures 4 and 5 at 10^7 flight miles, chosen herein as representing a reasonable average flight distance for the operational life of the airplane. For comparing the ratio values of any two samples shown in figures 7 and 8, the difference between them is considered as real (not due to sampling variations) if each value is not enclosed by the confidence band of the other value.

A summary of the operating speeds and the gust loads is presented in table IV. The most probable speeds at which the largest accelerations occurred are presented in the form of the nondimensional speed ratio and are directly comparable with the corresponding speed ratios from reference 2 (presented as V_p/V_L therein) since V_C is equal to V_L . The average flight miles to equal or exceed Δn_{LLF} and U_{e30} , as obtained from figures 4 and 5, and V_{NE} , as would be obtained from an extrapolation of the curve in figure 6, are compared in this table with the corresponding values from reference 2.

Figure 9 shows calculated flight envelopes of effective gust velocities representing the present operation and the 1937 to 1941 and 1941 to 1945 operations of the same type of airplane, and also shown is a design gust diagram for this type of airplane. The calculated envelope was obtained in each case for a value of 10^7 flight miles by transforming this value to the corresponding probability value P by the use of equation (1). For this probability value the corresponding Δn values for each speed bracket were read from the speed-bracket probability curves of Δn corresponding to each of the three different operating periods. These Δn values were converted to effective gust velocities by use of the sharp-edge-gust relation and plotted at the midpoint speed values in each speed bracket up to 220 miles per hour. Above this speed the flight envelope is constructed by using the product probability of given values of acceleration and airspeed. If it is assumed that the airspeeds and accelerations at the high-speed end are independent, the product probability is obtained from the relation

$$P_p = P_v P_{\Delta n} = \frac{T}{T} \quad (2)$$

For a given value of airspeed V , the probability P_v is determined from the probability curve for V_{max} . The value of Δn corresponding to this speed is determined from the equation

$$P_{\Delta n} = \frac{1}{P_v} \left(\frac{T}{T} \right) \quad (3)$$

and from the probability curve for $P_{\Delta n}$. The probability $P_{\Delta n}$ was obtained from the Δn distribution for the airspeed bracket from 200 to 220 miles per hour and is also assumed to apply to the airspeed range from 220 to 240 miles per hour. The values of Δn (or U_e) are plotted

at the corresponding values of V . The completed envelope consists of a curve which has been faired through the plotted points. From the envelope calculated on this basis it is expected that, for the given value of flight miles, an average of one positive and one negative effective gust velocity will exceed the envelope in each speed bracket and one maximum airspeed occurrence will exceed the maximum speed of the calculated envelope.

PRECISION

The performance of the NACA friction-damped and oil-damped V-G recorders are compared in reference 6. The errors inherent in either type of recorder are assumed not to exceed a maximum value of $\pm 0.2g$ or 3 percent of the maximum-airspeed range. For the large number of V-G recorders used in obtaining the present data, random positive and negative errors between instruments would tend to reduce the error of the final results.

Errors in reading the records may have occurred during the evaluation but are believed to be random and to balance out. Sampling errors are a function of the sample size and an assessment of such errors is made by the use of confidence bands.

DISCUSSION

Figure 2 shows that the acceleration experiences based on the data obtained from the NACA friction-damped and oil-damped V-G recorders were about the same. Figure 3 indicates that the acceleration experiences for different periods also were about the same. When the same sets of data were analyzed for the gust experiences, the effective gust velocities averaged about 10 percent higher for the friction-damped than for the oil-damped recorders. This 10-percent difference cannot be attributed to instrumental effects alone, however, because the records cover different periods of operation spread over two different years and no clearly defined separation of instrumental and seasonal effects is possible. The agreement in the acceleration experiences can be accounted for by the differences in the operating speeds at which the larger gusts were encountered. Since figures 2 and 3 indicate no real differences in the basic acceleration data, the samples of data were combined for obtaining the over-all gust-load and gust experiences.

Acceleration experience.- Figure 4 indicates that the over-all incremental accelerations experienced in the present operation equaled or exceeded Δn_{LLF} , on the average, twice (once positive and once negative)

in about 2.8×10^6 flight miles. A comparison of this value with the values of flight miles to equal or exceed the Δn_{LLF} experienced on the same type of airplane operated over three different transcontinental routes, see table IV, indicates that the present acceleration experience and that for the E-I wartime operation closely agree but some difference is apparent between the present acceleration experience and the experiences for the prewar operations. Inspecting the acceleration results at 10^7 flight miles shown in figure 7 indicates that the value of acceleration ratio for the present operation is not significantly different from the acceleration ratios for the E-I wartime operation or for the E-V prewar operation but is about 20 percent larger than the ratios for the E-I and E-VI prewar operations. These 20-percent differences, although perhaps not individually statistically significant, as a group are indicative of more severe load experiences in the postwar than in the prewar period. Thus the accelerations experienced in the present operation agree well with those experienced during wartime (1941 to 1945) and were more severe than those experienced on two out of three transcontinental routes during operations of the same type of airplane in the prewar period (1937 to 1941).

Since the acceleration experience is principally a function of the gust velocity and the corresponding speed at the time the gust is encountered, the present data were examined for determining to what degree variations in the accelerations experienced were contributed by the gusts and by the operating speeds.

Gust experience.- Table IV indicates that an effective gust velocity of 30 feet per second was encountered in the present operation, on the average, twice in about 0.7×10^6 flight miles. This value agrees well with the value of flight miles to equal or exceed U_{e30} shown in table IV for the E-I wartime operation but differs somewhat from each of the values shown for the prewar operations. The gust experiences in each of these cases appear directly reflected in the corresponding acceleration experiences. Examination of figure 8 with respect to the prewar, wartime, and present operations indicates that the present operation and the wartime operation may be classed as having the same gust experiences but both of these experiences are significantly greater than the prewar gust experiences. The more severe turbulence encountered during the present operation than that encountered during prewar operations apparently was the predominating factor in the higher level of gust-load experience.

Average operating airspeed.- Inasmuch as the average operating speed under normal conditions is usually taken as $0.8V_C$, an average value of 0.8 is assumed for the ratio V_p/V_C at which the Δn_{max} values

were experienced and on this basis the speed-ratio value of 0.75 shown in table IV for the present operation is about 6 percent lower than the average. It is also indicated in table IV that this value of 0.75 for the present operation is conservative compared with the corresponding speed-ratio values indicated in three out of four cases for the prewar and wartime operations from reference 2. This reduction in operating speed in rough air from the average might be considered as a moderating factor on the acceleration experience of the present operation.

Maximum airspeed.- Table IV indicates that the placard never-exceed speed V_{NE} would be equaled or exceeded once in about 10^9 miles of flight and this value is conservative in comparison with most of the corresponding V_{NE} values shown from reference 2. The placard never-exceed speed V_{NE} was not exceeded during the present operation and the available data indicate that the likelihood of reaching the placard speed appears remote for the present type of operation.

Comparison of predicted gust envelopes.- Inspection of figure 9 indicates that over the operating speed range from V_B to about $0.8V_C$ the design gust diagram agrees well with the calculated gust-velocity envelope representing 10^7 miles of flight for the present operation and exceeds somewhat the calculated envelopes for the prewar and the wartime periods. At speeds greater than $0.8V_C$ up to the design gust value of 15 feet per second at V_D the design diagram exceeds all three predicted gust envelopes. The tapering-off of the gust values above $0.8V_C$ indicates that these airplanes operated such a small part of the total flight time above this speed that they have not yet encountered the larger gusts in this range. The results for the present operation indicate that the expected maximum gusts for 10^7 miles of operation approximate the design gust requirements in the speed range from V_B to $0.8V_C$. The 15-foot-per-second design gust at V_D appears to be a conservative gust standard for this airplane in view of the results shown in figure 9 for the prewar, wartime, and postwar operations.

Predicted gust-load envelopes.- The gust experience of the present operation has been used for predicting the gust-load experience on another type of airplane. The data used for the comparison were obtained during operation on a northern transcontinental route of a two-engine type of transport airplane of more recent design. The airplane characteristics for the two types are somewhat similar. Both types were operated at altitudes generally below 10,000 feet. Speed differences between these airplanes are accounted for by using the nondimensional speed

ratio V_p/V_C to effect the load transfer. The predicted flight envelopes are based on the following assumptions:

- (1) The gust experiences were the same
- (2) The airspeed practices in turbulence were the same
- (3) Dynamic-response effects were negligible in the acceleration data of the present operation but are accounted for in the operation compared

The measured composite V-G envelope representing 7.9×10^6 miles of operation on the northern transcontinental route is shown in figure 10 in comparison with two predicted flight envelopes developed from the data of the present operation to represent the same flight distance. These predicted envelopes were calculated by using five airspeed brackets and thus indicate that, on the average, 10 accelerations and one maximum airspeed can be expected to exceed the envelope. One predicted envelope includes 20-percent amplification due to dynamic response, since the results of reference 11 indicate that dynamic response of about 20 percent was present in accelerations measured at the center of gravity of this type of airplane. For comparison, an envelope assuming no dynamic response is also shown.

Figure 10 indicates reasonably good agreement between the composite V-G envelope and the predicted flight envelope containing 20-percent dynamic response. The measured composite exceeds the predicted envelope based on no dynamic response with a greater frequency than expected. The composite V-G record shown would agree more closely with the predicted envelopes at the highest speeds shown if one record which contains an excessive speed was not included in the operation compared. Thus, it appears noteworthy that, despite differences in route, period, operator, and sampling variations, reasonable agreement is obtained between the measured composite V-G record for one type of modern aircraft and the predicted flight envelope based on data collected on another older type of airplane of somewhat similar design.

SUMMARY OF RESULTS

The analysis of the acceleration and airspeed data obtained on a two-engine type of transport airplane operated on commercial airline routes during the period from 1948 to 1950 indicates:

1. The acceleration increments equaled or exceeded the limit-gust-load-factor increment Δn_{LIF} , on the average, twice in about

2.8×10^6 flight miles. The acceleration experience appeared to be about the same as that indicated for operation of the same type of airplane on other routes during the wartime (1941 to 1945) period but was about 20 percent greater than the acceleration experiences in two out of three cases during the prewar (1937 to 1941) operations.

2. An effective gust velocity of 30 feet per second was encountered, on the average, twice in about 0.7×10^6 flight miles. The gusts experienced in the present operation were about the same as those experienced during the wartime (1941 to 1945) operation but were more severe than the gusts experienced during the prewar (1937 to 1941) operations.

3. The more severe turbulence encountered during the present operation than that encountered during prewar operations apparently was the predominating factor in the higher level of gust-load experience. The slightly lower than average operating speed in rough air during the present operation might be considered as a moderating factor on the gust-load experience.

4. The placard never-exceed speed was not exceeded during the present operation and the available data indicate that the likelihood of reaching the placard speed appears remote for the present type of operation.

5. The use of the predicted envelopes of gust-load experience for the present operation yielded estimates of gust loads for another similar type of transport airplane that were in reasonable agreement with those measured.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., March 24, 1952

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TABLE I

SUMMARY OF V-G RECORDS ANALYZED FROM 1948 TO 1950

	NACA friction-damped recorder		NACA oil-damped recorder		All records
	Period I (Apr.-Sept.)	Period II (Oct.-Mar.)	Period I (Apr.-Sept.)	Period II (Oct.-Mar.)	
Number of records	22	17	23	17	79
Range of record hours	95 to 719	88 to 618	47 to 621	119 to 748	47 to 748
Average hours per record, τ	324	343	282	264	303
Total record hours	7,131	5,826	6,491	4,492	23,940



TABLE II

FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS

(a) Δn_{max}

Δn_{max} (g units)	Frequency distribution for -				
	Instrument		Period		Total
	Friction-damped	Oil-damped	I (Apr.-Sept.)	II (Oct.-Mar.)	
0.6 to 0.7	-	2	-	2	2
.7 to .8	1	4	5	0	5
.8 to .9	2	5	4	3	7
.9 to 1.0	7	10	12	5	17
1.0 to 1.1	10	9	9	10	19
1.1 to 1.2	7	6	8	5	13
1.2 to 1.3	11	9	12	8	20
1.3 to 1.4	12	11	12	11	23
1.4 to 1.5	5	5	7	3	10
1.5 to 1.6	5	4	4	5	9
1.6 to 1.7	5	4	3	6	9
1.7 to 1.8	3	1	2	2	4
1.8 to 1.9	1	4	2	3	5
1.9 to 2.0	3	0	3	0	3
2.0 to 2.1	1	3	2	2	4
2.1 to 2.2	1	2	3	0	3
2.2 to 2.3	3	0	1	2	3
2.3 to 2.4	0	0	0	0	0
2.4 to 2.5	0	1	1	0	1
2.5 to 2.6	0	-	-	0	0
2.6 to 2.7	0	-	-	0	0
2.7 to 2.8	1	-	-	1	1
Total, N	78	80	90	68	158
$\bar{\Delta n}_{max}$, g units	1.38	1.28	1.31	1.35	1.33
u	1.16	1.11	1.14	1.18	1.16
α	3.34	3.33	3.34	3.38	3.36



TABLE II - Continued

FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS

(b) $U_{e_{max}}$

$U_{e_{max}}$ (fps)	Frequency distribution for -				
	Instrument		Period		Total
	Friction-damped	Oil-damped	I (Apr.-Sept.)	II (Oct.-Mar.)	
8 to 10	-	1	-	1	1
10 to 12	-	4	3	1	4
12 to 14	2	6	5	3	8
14 to 16	7	9	11	5	16
16 to 18	12	12	10	14	24
18 to 20	8	12	12	8	20
20 to 22	15	15	18	12	30
22 to 24	13	5	11	7	18
24 to 26	6	4	6	4	10
26 to 28	3	5	2	6	8
28 to 30	2	3	3	2	5
30 to 32	5	1	4	2	6
32 to 34	1	2	3	0	3
34 to 36	2	0	1	1	2
36 to 38	0	0	0	0	0
38 to 40	0	1	1	0	1
40 to 42	2	-	-	2	2
Total, N	78	80	90	68	158
$\bar{U}_{e_{max}}$, fps	22.18	19.83	20.89	21.12	20.99
u	19.51	17.27	18.28	18.37	18.33
α	0.22	0.23	0.22	0.21	0.22

TABLE II - Concluded

FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS

(c) V_o and V_{max}

Airspeed (mph)	Frequency distribution of V_o at Δn_{max}	Airspeed (mph)	Frequency distribution of V_{max}
120 to 130	11	190 to 195	1
130 to 140	15	195 to 200	0
140 to 150	14	200 to 205	16
150 to 160	38	205 to 210	17
160 to 170	48	210 to 215	17
170 to 180	15	215 to 220	13
180 to 190	13	220 to 225	9
190 to 200	1	225 to 230	6
200 to 210	1		
210 to 220	2		
Total, N	158	Total, N	79
\bar{V}_o , mph	158.9	\bar{V}_{max} , mph	212.3
σ_o	17.57	σ_v	7.95
k_o	0.16	k_v	0.22
V_p , mph	157.5		



TABLE III
 FREQUENCY DISTRIBUTIONS OF ACCELERATION INCREMENTS
 BY AIRSPEED BRACKETS

Δn (g units)	Frequency distributions for -					
	120 to 140 mph	140 to 160 mph	160 to 180 mph	180 to 200 mph	200 to 220 mph	220 to 240 mph
0 to 0.2	-	-	-	-	12	8
.2 to .4	-	-	-	13	46	7
.4 to .6	13	6	3	19	44	3
.6 to .8	31	22	24	39	19	-
.8 to 1.0	47	35	41	40	12	-
1.0 to 1.2	34	41	34	27	3	-
1.2 to 1.4	21	33	23	15	4	-
1.4 to 1.6	3	9	13	5	1	-
1.6 to 1.8	5	5	8	-	0	-
1.8 to 2.0	2	5	3	-	0	-
2.0 to 2.2	1	1	6	-	1	-
2.2 to 2.4	1	1	1	-	-	-
2.4 to 2.6	-	-	1	-	-	-
2.6 to 2.8	-	-	1	-	-	-
Total, N	158	158	158	158	142	18
$\bar{\Delta n}$, g units	0.99	1.10	1.15	0.84	0.52	-
u	0.84	0.95	0.97	0.71	0.38	-
α	3.95	3.86	3.16	4.29	4.13	-



TABLE IV
 SUMMARY OF GUST LOADS AND OPERATING AIRSPEEDS

Operation	Dates of operation	V_p/V_C at Δn_{max}	Average flight miles to equal or exceed -			
			Δn_{LLF} (twice)	U_{e30} (twice)	V_{NE} (once)	
Present (postwar)	Feb. 1948 to Feb. 1950	0.75	2.8×10^6	0.7×10^6	$>1,000 \times 10^6$	
E-I (wartime, from ref. 2)	July 1941 to Dec. 1944	.85	2.7	.8	15.5	
E-I	Dec. 1937 to Dec. 1941	.84	10.9	5.2	80.0	
E-V	(prewar, from ref. 2)	Feb. 1937 to Oct. 1939	.77	4.6	1.3	30.0
E-VI		Sept. 1938 to Oct. 1940	.68	13.4	3.6	$>1,000$



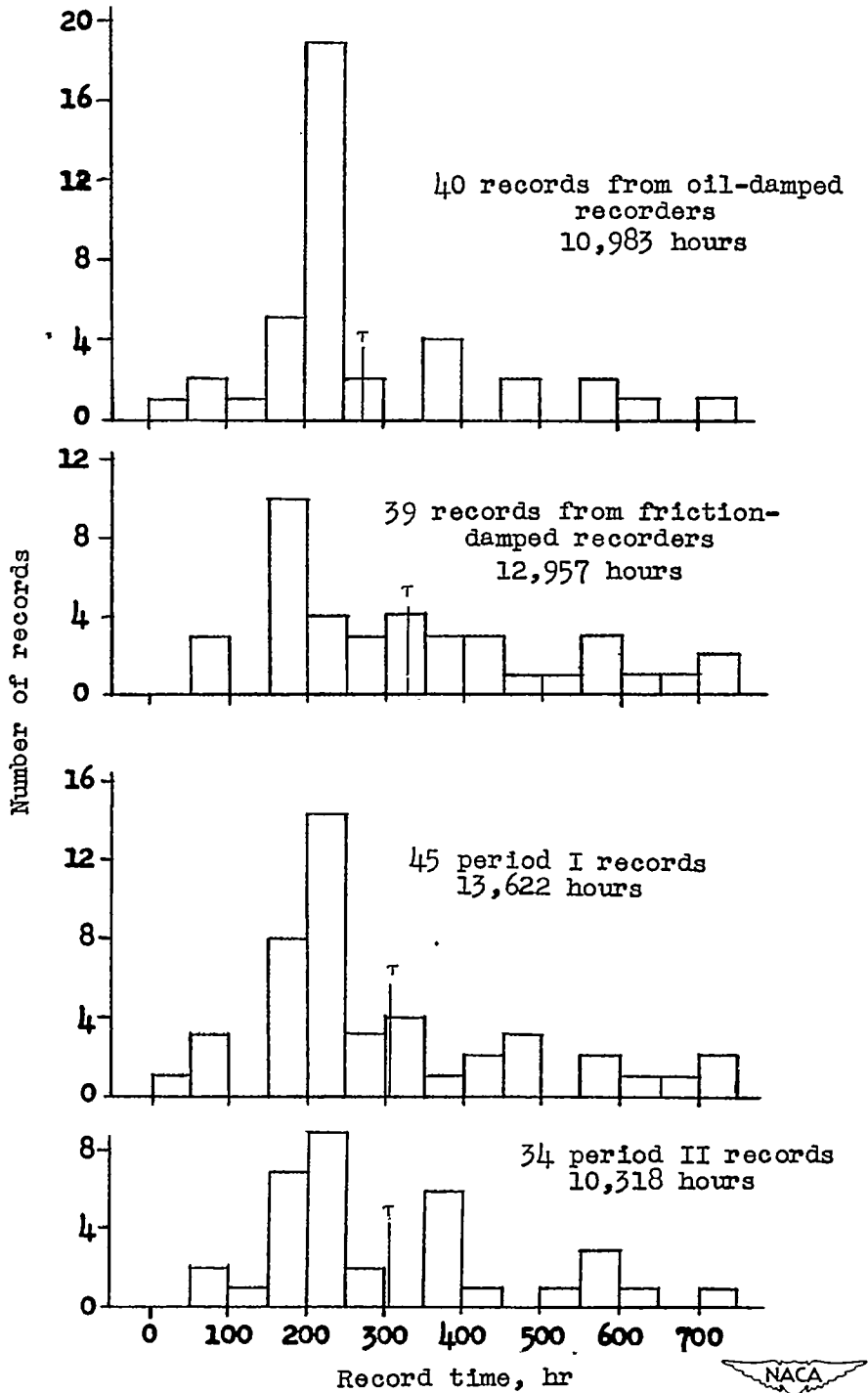


Figure 1.- Distribution of V-G records for flight time per record.

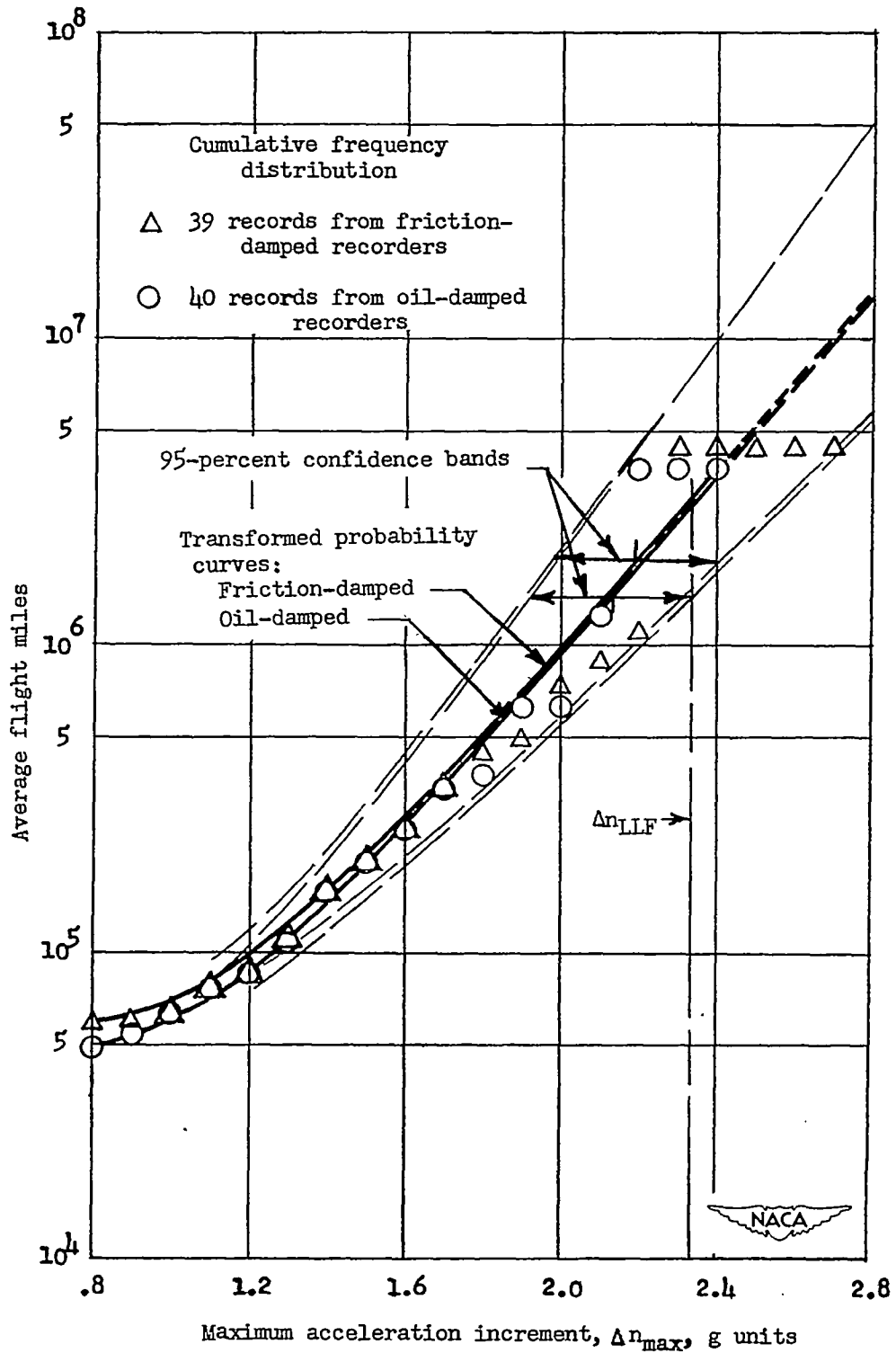


Figure 2.- Average flight miles for a maximum positive and negative acceleration increment to equal or exceed a given value for different instruments.

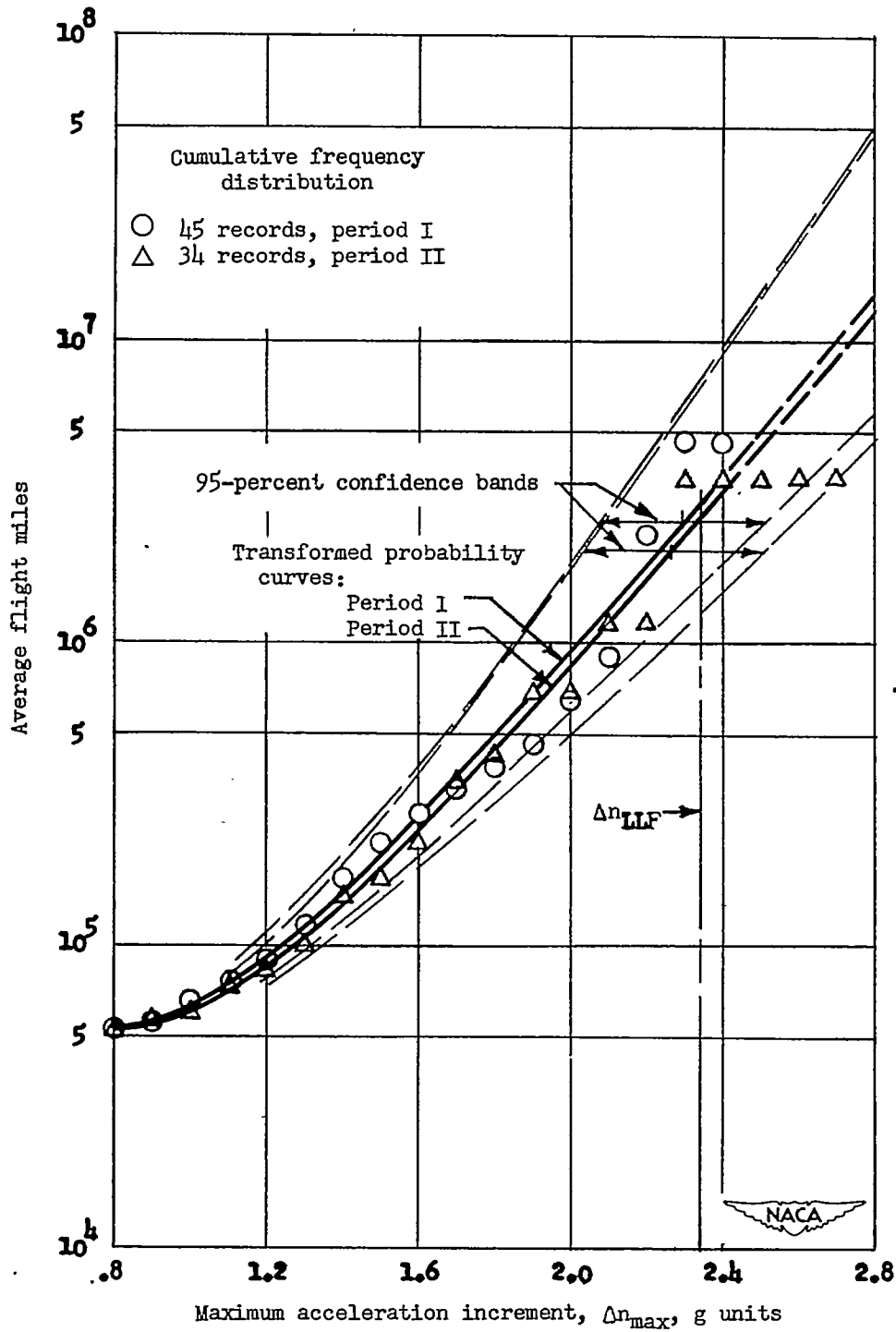


Figure 3.- Average flight miles for a maximum positive and negative acceleration increment to equal or exceed a given value during different periods.

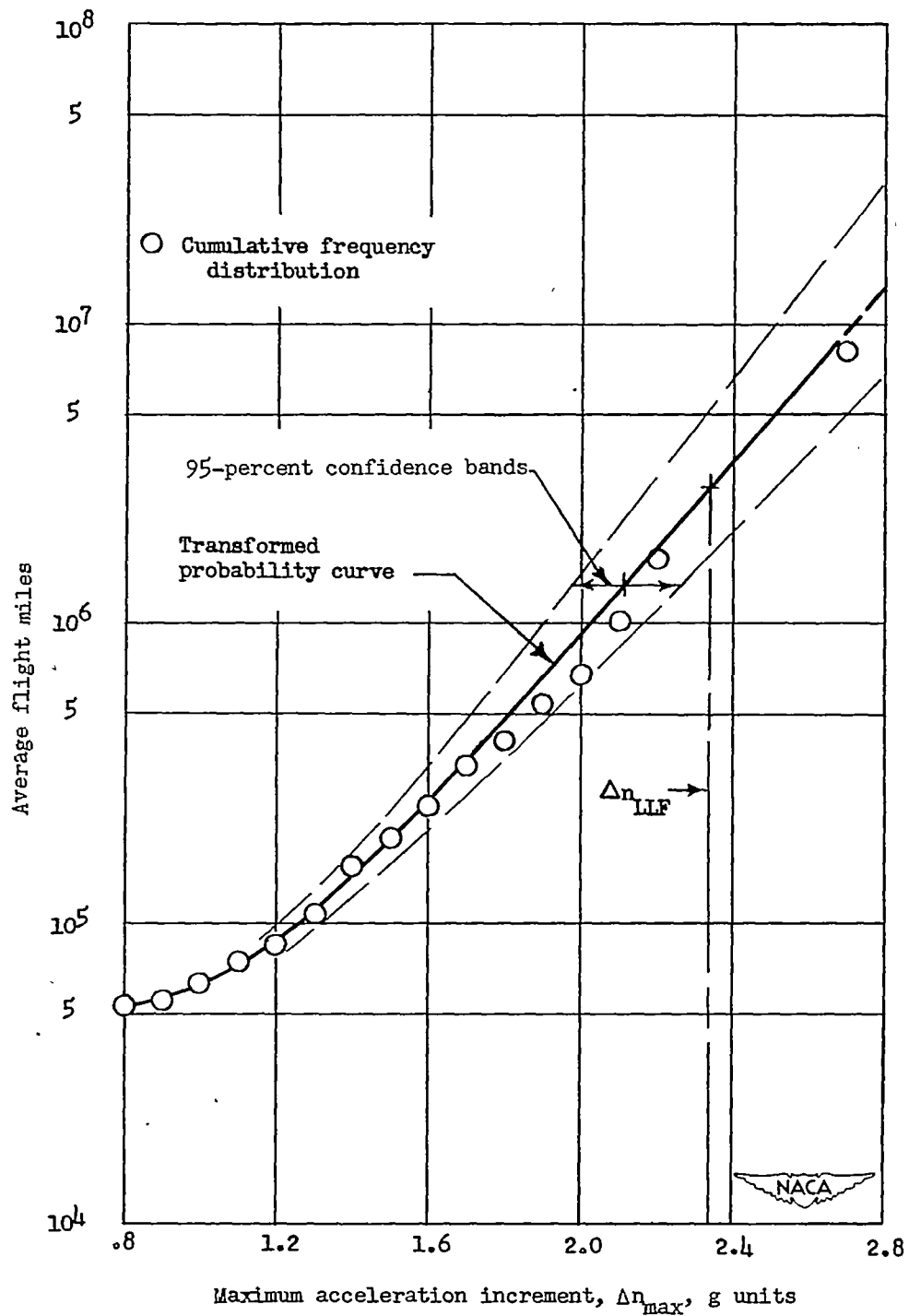


Figure 4.- Average flight miles for a maximum positive and negative acceleration increment to equal or exceed a given value.

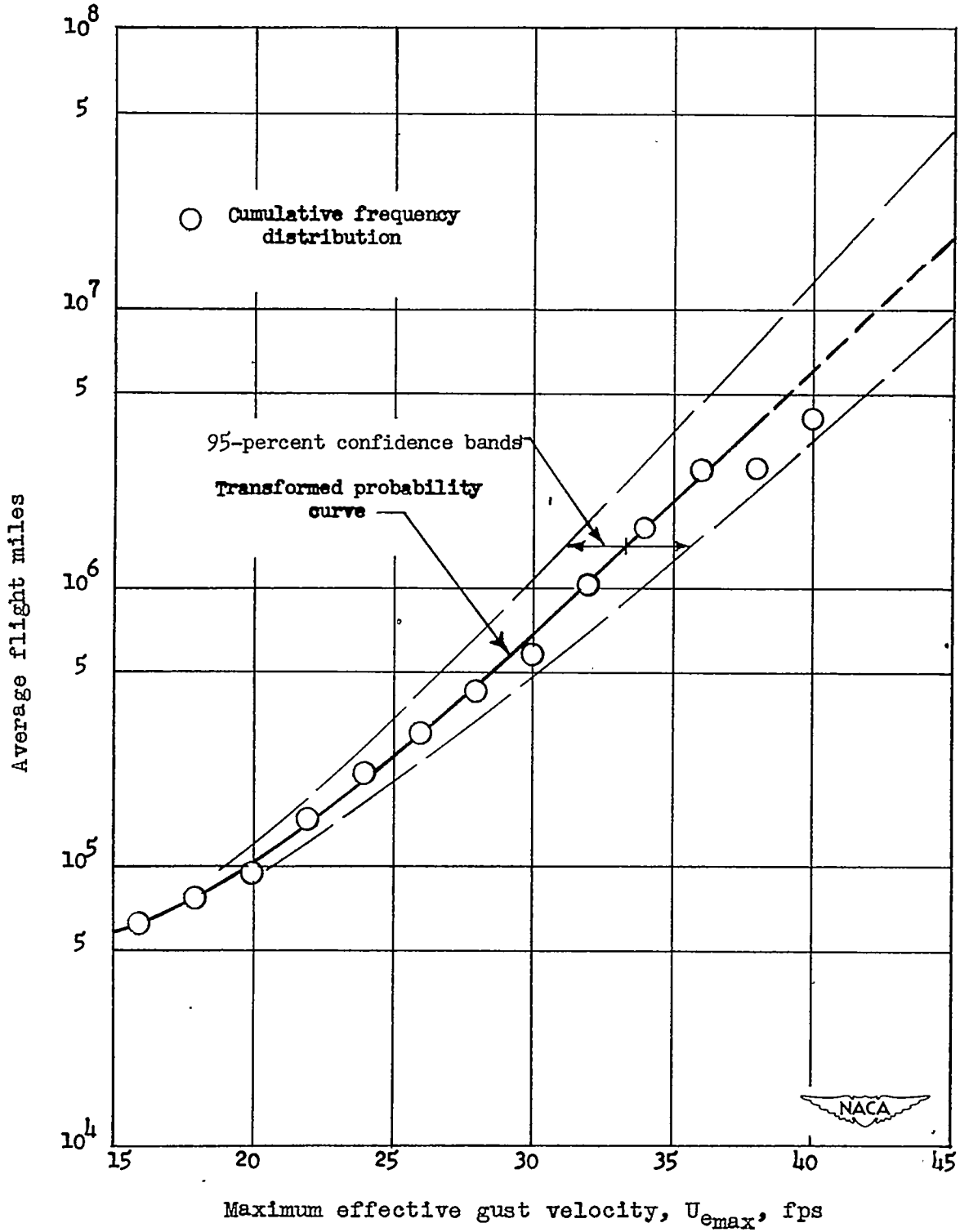


Figure 5.- Average flight miles for a maximum positive and negative effective gust velocity to equal or exceed a given value.

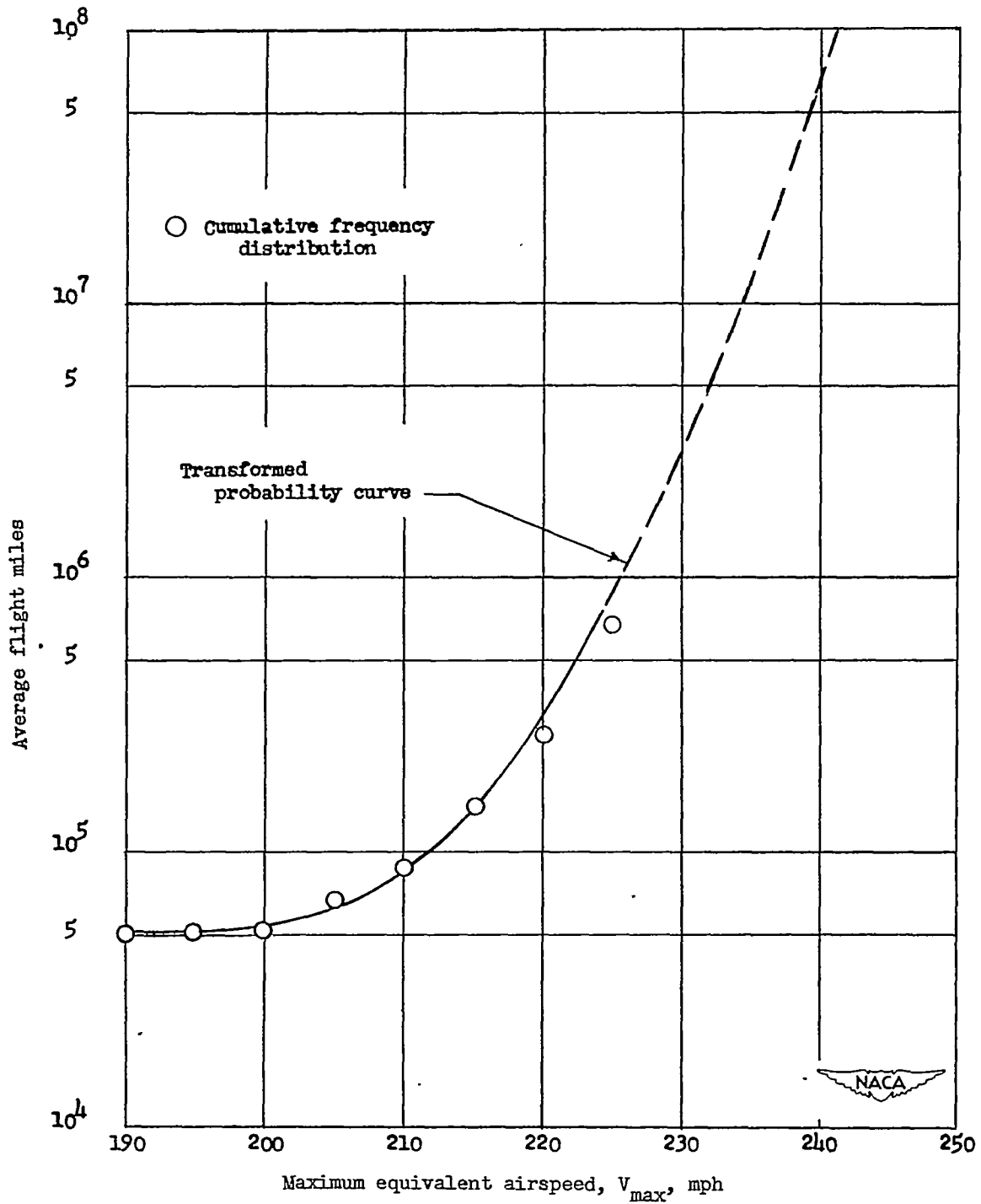


Figure 6.- Average flight miles for maximum equivalent airspeed to equal or exceed a given value.

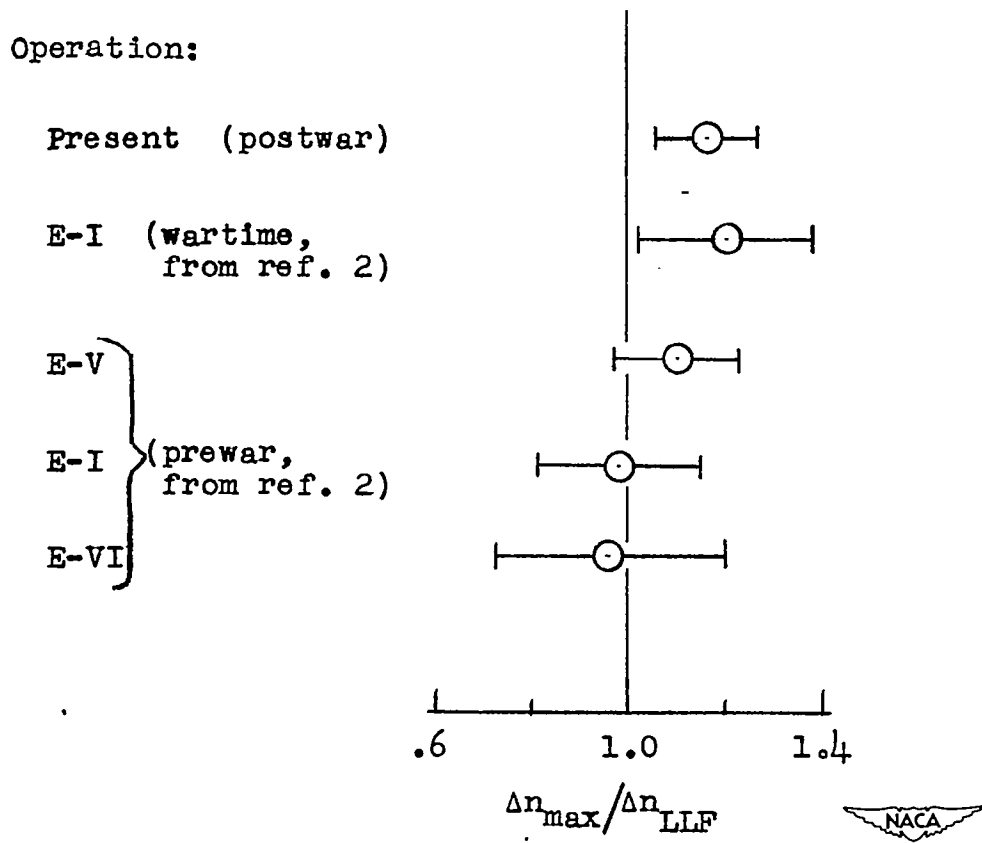


Figure 7.- Comparison of average values of maximum-acceleration-increment ratio at 10^7 flight miles and the 95-percent confidence bands about the values.

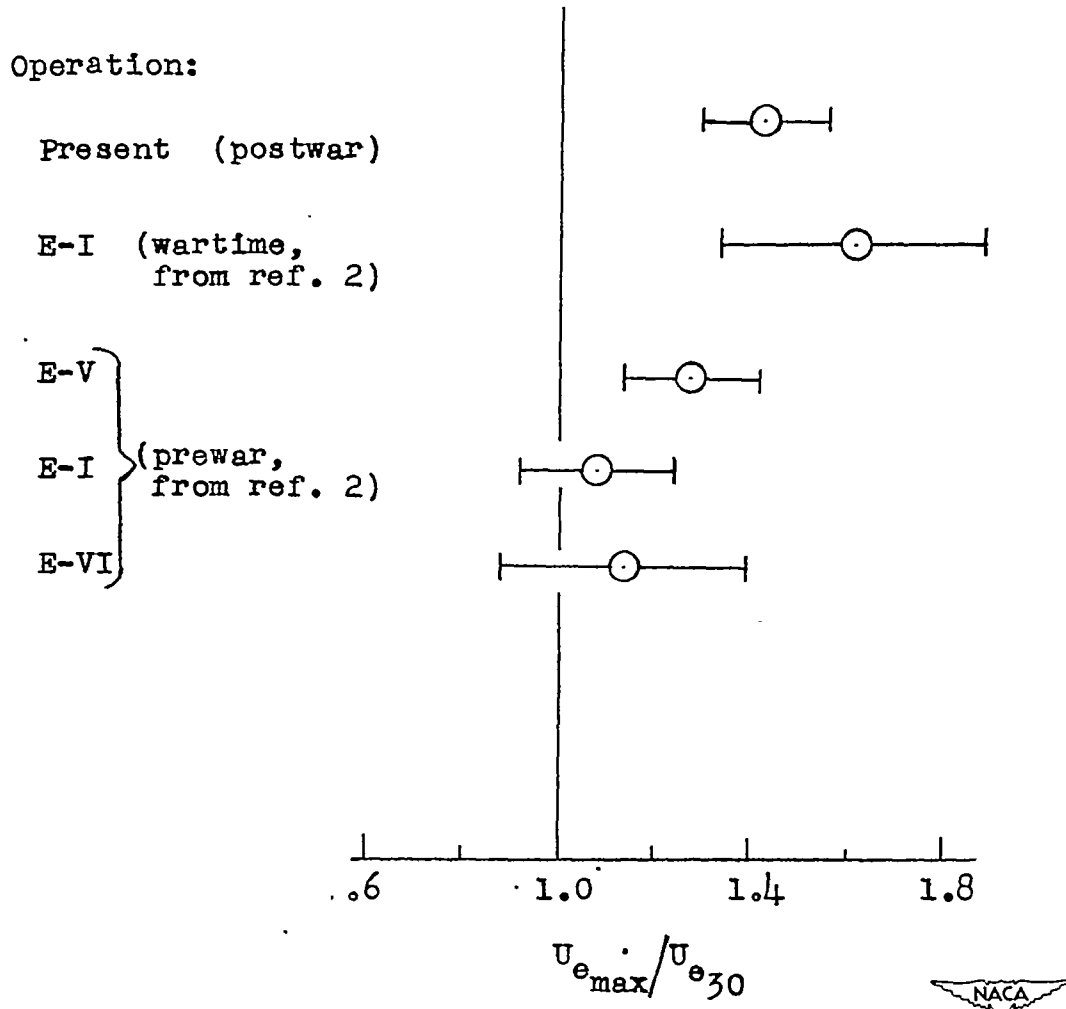


Figure 8.- Comparison of average values of maximum-effective-gust-velocity ratio at 10^7 flight miles and the 95-percent confidence bands about the values.

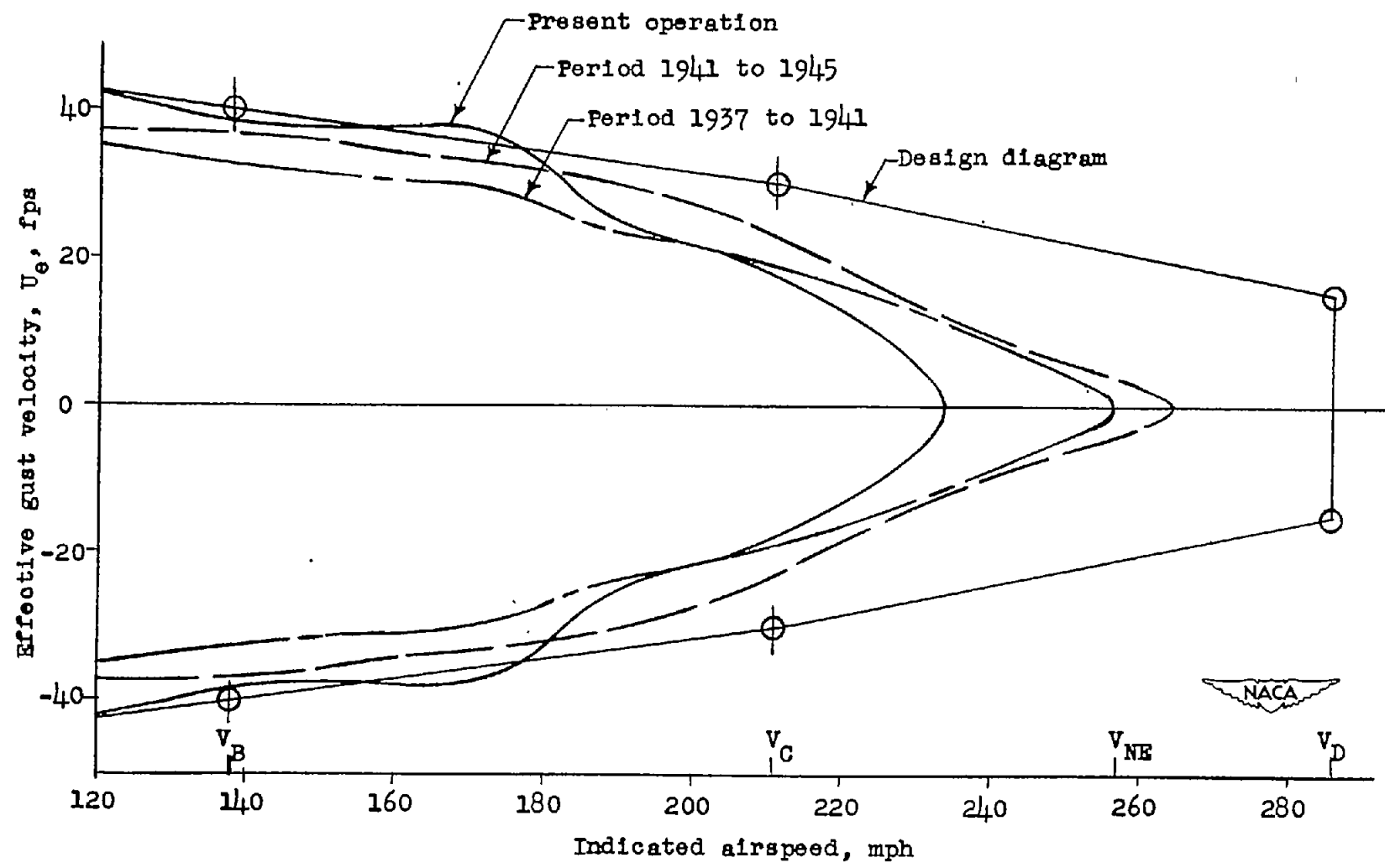


Figure 9.- Comparison of calculated gust-velocity envelopes each representing 10^7 miles of flight during different periods and the design gust diagram.

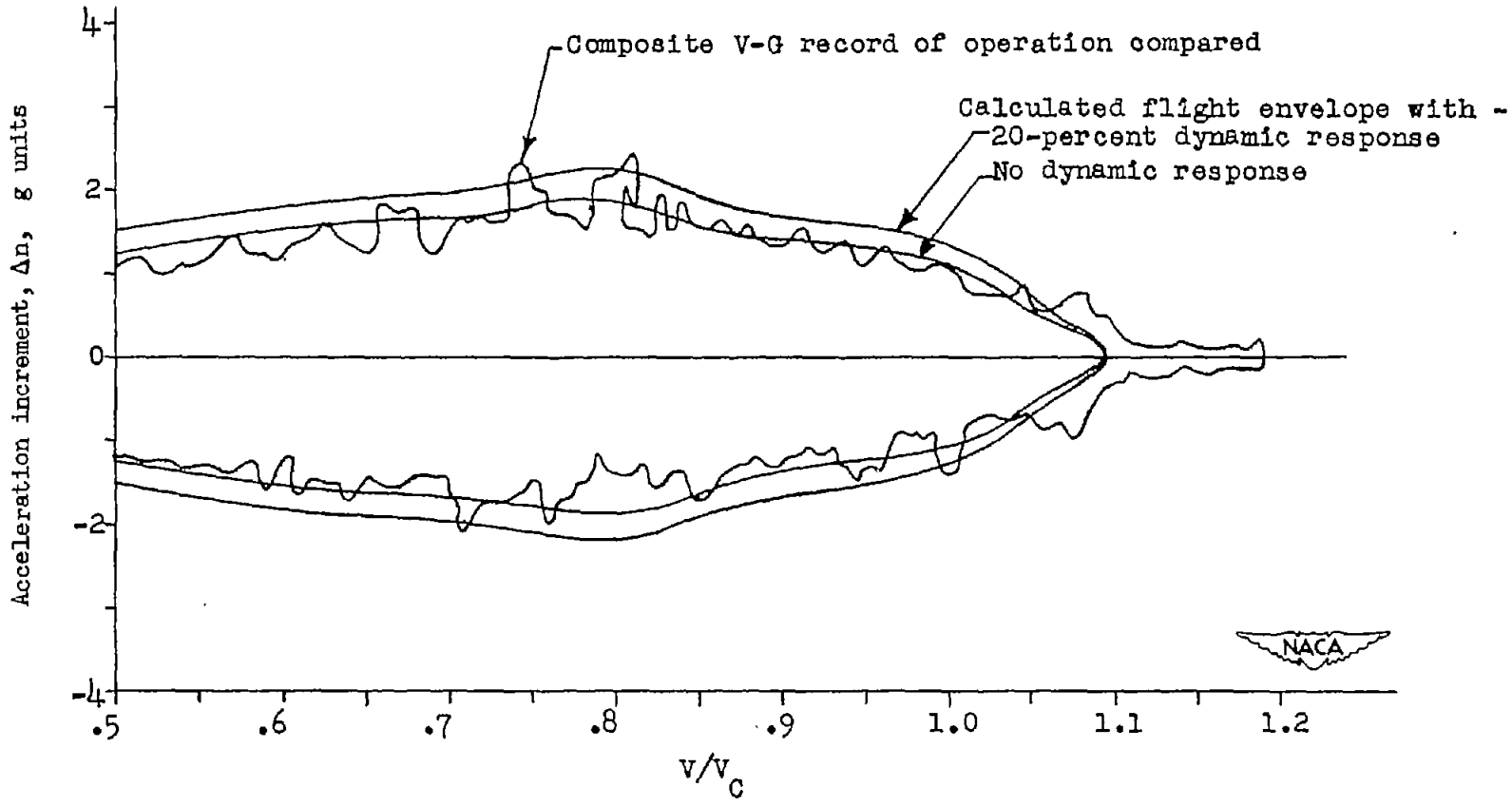


Figure 10.- Comparison of the composite V-G record representing 7.9×10^6 miles of flight for another type of two-engine transport airplane and the corresponding flight envelopes predicted from the present data.