NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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A RAPID METHOD FOR PREDICTING ATTACHED-SHOCK SHAPE

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SUMMARY

A method is presented for the rapid prediction of the shape of attached shocks emanating from smoothly contoured axisymmetric and two-dimensional nose shapes. From a practical viewpoint the accuracy of the method is comparable to that of the method of characteristics.

INTRODUCTION

The shape of the attached shock arising from the nose or leading edge of a body at supersonic speeds has long been recognized as a subject of considerable practical importance, particularly in the solution of interference problems, since all such shocks define the beginning of an interference flow field. For the arbitrary nose shape, interference analyses which approximate the shock position by a Mach angle propagation of disturbances and those which employ the straight-shock approximation based on the initial cone (or wedge) angle of the nose are greatly restricted in application. An accurate evaluation of interference requires an accurate representation of the curved exact shock which is the true beginning of the interference field; this exact shock lies between those estimated by the two aforementioned approximations. Accurate shock shape can be particularly helpful in determining the properties of the flow field downstream of the shock. (See ref. 1, for example.)

In addition to its value in interference problems, knowledge of the shape of the shock frequently has a direct bearing upon the choice of maximum model size for a given wind tunnel.

A number of methods are available for calculating shock shape. Of the less laborious methods, the linear-theory approach of Whitham (ref. 2) has perhaps met with as much success and received as much attention as any. However, the range of applicability of these less laborious methods is severely restricted. For example, Whitham has stated (ref. 2) that when the seminose angle is as large as approximately 20° his method does not give accurate quantitative results; he also shows that his method, when applied to the simple case of cones, does not give satisfactory shock angles beyond semicone angles of about 10° and Mach numbers of about 3. (An upper limit in Mach number of about 2.5 would appear more reasonable from the results shown in ref. 2.) When it is recognized that the difference between Mach angle and shock angle for a semicone angle of 5° is no more than 0.16°, and usually less, for Mach numbers from 1.1 to 2.5, the range for which this method has practical use is seen to be small.

For calculating shock shape over wide ranges of Mach number and nose shape, the laborious method of characteristics must be used. For two-dimensional shapes at all Mach numbers and for three-dimensional shapes at high Mach numbers the characteristic calculations can often be handled in a simplified and less tedious form with satisfactory results. (See refs. 3 and 4, for example.) Even in simplified form, however, the calculations are not brief. At the lower supersonic Mach numbers (of the order of 3 or less) the time-consuming axisymmetric method of characteristics must be employed for three-dimensional shapes.

The purpose of this report is to present a rapid method, applicable over a wide range of Mach number and nose shape, for calculating the shape of curved attached shocks emanating from the noses of two- and three-dimensional bodies. The establishment of the method was based upon results obtained from a number of calculations by the method of characteristics. However, although these calculations were essential to the development of the method, similar calculations are not required in the application of the method. The method was not developed for Mach numbers beyond about 5; attached shocks will probably be a rarity at Mach numbers of 5 and higher, since problems associated with aerodynamic heating will quite likely dictate the use of blunted noses. Further, at high Mach numbers the presence of thick boundary layers may significantly alter the effective nose shape.

SYMBOLS

K — scale factor for converting unspecified ordinates to circular-arc ordinates

l — nose length (from tip of nose to station where slope of nose surface becomes zero, measured along x-axis)

M — free-stream Mach number

x, y — orthogonal coordinates having origin at nose

δ — semicone angle or semiwedge angle at $x = 0$

ϵ shock inclination at $x = 0$

μ Mach angle, $\sin^{-1} \frac{1}{M}$

Subscripts:

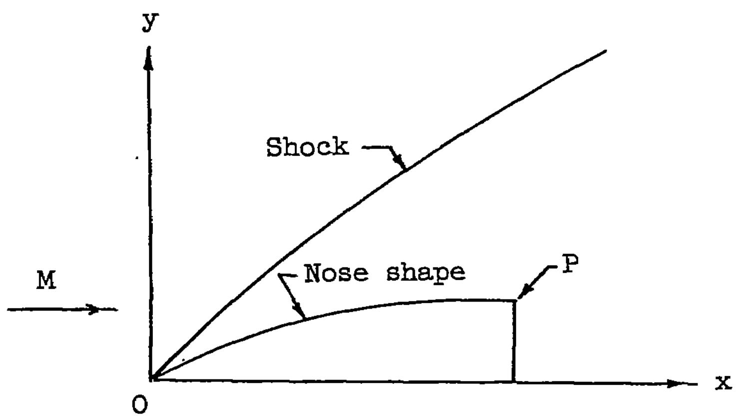
c circular-arc nose

i unspecified scale and shape of nose and unspecified scale of shock

DISCUSSION

Analytical Considerations

Equation for unspecified shock.- Consider a pointed nose shape immersed in a supersonic stream and having an attached, curved nose shock as shown in the following sketch:



The nose shape is a smoothly varying contour having no discontinuities in slope and is assumed to terminate at point P where the slope of the surface is zero. At a particular Mach number, the initial inclination of the shock ϵ must be that dictated by the wedge or cone angle at the tip of the nose $(x = 0)$; the inclination of the shock at $x = \infty$ must be equal to the Mach angle of the free stream. Thus, two general conditions to be satisfied on the shock are:

At $x = 0$,

$$\frac{dy}{dx} = \tan \epsilon \tag{1}$$

At $x = \infty$,

$$\frac{dy}{dx} = \tan \mu \tag{2}$$

It is assumed that an expression $y = f(x)$, which satisfies these conditions and varies from one limit to the other without discontinuity in $\frac{dy}{dx}$ and with $\frac{d^2y}{dx^2}$ always remaining negative (except when $x = \infty$, of course), represents satisfactorily the shape of any shock of unspecified scale from any nose of the type just described that is of unspecified scale and shape but with the same value of δ. It is to be noted that the unspecified scales of the shock and of the nose are not assumed to be identical; they might be, but this remains to be determined. With the subscript i denoting the unspecified conditions of scale and shape, a simple equation that satisfies the aforementioned limits and variations may be written as follows:

$$\left(\frac{dy}{dx}\right)_i = \tan \epsilon\left(\frac{1}{1 + x_i}\right) + \tan \mu\left(\frac{x_i}{1 + x_i}\right) \tag{3}$$

Integration yields

$$y_i = \tan \epsilon\left[\log_e\left(1 + x_i\right)\right] + \tan \mu\left[x_i - \log_e\left(1 + x_i\right)\right] \tag{4}$$

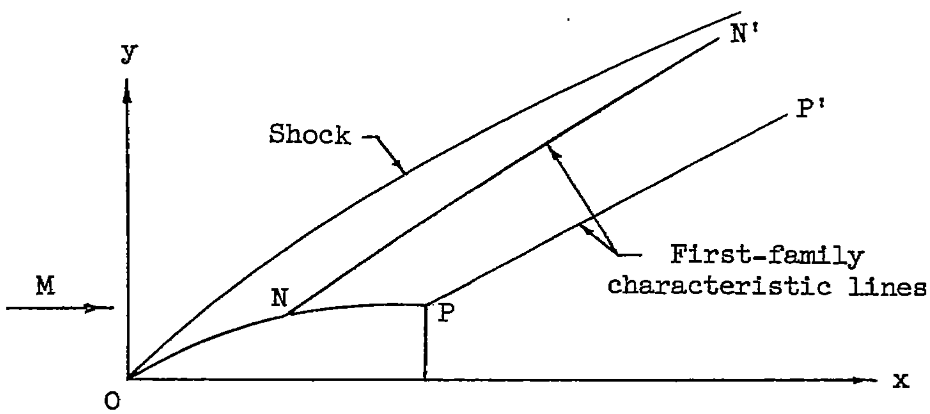
This is the equation of the so-called unspecified shock. It may also be expressed in terms of the parameter $\frac{\tan \epsilon}{\tan \mu}$ as

$$\frac{y_i}{\tan \mu} = x_i + \left(\frac{\tan \epsilon}{\tan \mu} - 1\right)\log_e\left(1 + x_i\right) \tag{5}$$

Curves showing $\frac{y_i}{\tan \mu}$ as a function of x_i for various values of the parameter $\frac{\tan \epsilon}{\tan \mu}$ are given in figure 1; the use of these curves becomes apparent subsequently. The curve for $\frac{\tan \epsilon}{\tan \mu} = 1$ is seen to be a straight line and is the trivial solution of the shock with zero strength, that is, a Mach wave.

The unspecified shock, defined by equation (4), must now be related to specified nose shapes and the scale relation determined. The problem of selecting the specified nose shapes is considered first.

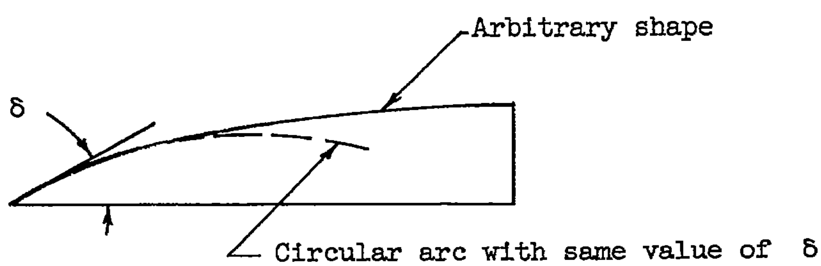
Establishment of important part of nose shape.- For a pointed nose of smooth contour such as that considered herein, the shape of the shock for all practical purposes is determined primarily by the forward portion of the nose rather than the rearward portion. Consider the following simplified sketch of an arbitrary nose shape:



For a two-dimensional nose shape the Mach number on the nose surface at the rear of the nose (point P, where the slope is zero) is only slightly different from the free-stream Mach number; consequently, the influence of point P, which is propagated along the first-family characteristic from P (denoted by PP', which generally has small curvature), is felt at the shock only at extremely large values of x (from a practical viewpoint, only when $x \rightarrow \infty$). Furthermore, at these extreme values of x, the attenuation in shock strength has already reduced the shock to essentially a Mach wave. The rearmost portions of the nose may therefore be regarded as of no concern in the problem. The rearmost region on the nose surface that has significant influence on the shock shape (for instance, in the vicinity of point N with first-family characteristic NN' which in the real case is slightly curved) was indicated in the course of the characteristic calculations for the two-dimensional phase of this study to lie approximately between 40 and 70 percent of the nose length.

For three-dimensional, or axisymmetric, nose shapes, the negligible effect of the rear portion of the nose upon shock shape is recognized even more readily from the fact that the Mach number on the surface at point P is greater than that of the free stream. Consequently, point P and a portion of the surface ahead of this point could never influence shock shape (except through the interaction with recompressions from an afterbody added downstream of P, generally a trivial effect so long as the shock is attached). Thus the rearmost region on an axisymmetric nose that has significant influence on shock shape, that is, in the vicinity of point N, lies well forward on the nose. Some typical illustrations of this are given in figure 2 where characteristic solutions from previous studies of some arbitrary nose shapes are shown. In these examples a first-family characteristic has been selected and labeled NN' to show that points on the nose downstream of the general vicinity of the point N would have no significant influence on shock shape.

Selection of family of nose shapes.- It is thus firmly established that attention should be focused on the forward part of the nose and that the rearward portion of the nose is of little concern in relating the unspecified shock to specified nose shapes. The importance of this observation is that, for pointed noses of smooth contour such as those considered herein, the unspecified shock may be related to a particular family of nose shapes (i.e., parabolas, circular arcs, and so forth). Although this family obviously cannot represent the entire contour of some arbitrary nose shape, it may be adequately fitted to the forward portion of an arbitrary shape in such a way that this forward contour, which for all practical purposes determines the shock shape, is effectively reproduced. Consider, for example, the arbitrary nose shape in the following sketch:



A circular arc with the same value of δ as the arbitrary nose shape, while being a grossly inadequate representation of the rear portion of the arbitrary nose, would duplicate almost exactly the forward 40 to 50 percent of the nose and would therefore be adequate as a means for determining the shock shape of the arbitrary nose.

The selection of the particular family of nose shapes to which the unspecified shock is to be related is governed by two principal factors; namely, this family of shapes should be flexible in order to afford satisfactory fitting to the forward portion of an arbitrary shape, and it should be a family that can be expediently fitted to the arbitrary shape from an engineering viewpoint. With regard to flexibility only, a family defined by a general cubic or quartic equation would certainly suffice. However, the advantage that such a family of shapes has in flexibility over simpler shapes is far overshadowed by the complexities of application, since the fitting of the family shape to the arbitrary shape will always involve a trial-and-error process. Further, and of equal importance, the establishment of the relation between the unspecified shock and the family of shapes requires sufficient calculations by the method of characteristics to cover every possible shape represented by the family chosen. Obviously, the use of cubics, quartics, and the like must be ruled out. Consideration of all factors pointed to a simple family of shapes that could be defined by δ only. Thus, attention was focused on circular arcs and parabolas. The circular arc was chosen in preference to the parabola for the following reasons. Although the value of δ as obtained from the arbitrary nose shape dictates the particular parabola or particular circular arc, it does not define the scale to be used in fitting this family shape to the arbitrary nose shape. This freedom of choice of scale constitutes the trial-and-error process by which the best fit is obtained even though δ is fixed. In the case of the parabola, this trial-and-error process involves replotting of the parabola each time the scale is changed. In the case of the circular arc, the process involves merely the swinging of trial arcs by means of a compass from any point along a line passing through the tip of the nose and normal to the slope of the nose at the tip.

The circular arc is thus selected as the nose shape to which the unspecified shock is to be related. The problem of relation is now, therefore, one of scale only. With the subscript c denoting values applying to the circular arc, the simplest scale relations between the ordinates from the unspecified shock and the shock for the circular-arc nose of length l are

$$\left.\begin{aligned} \left(\frac{x}{l}\right)_c &= Kx_i \\ \left(\frac{y}{l}\right)_c &= Ky_i \end{aligned}\right\} \qquad (6)$$

<u>Determination of values of K</u>.- In order to establish the values of the scale factor K, the shape of the shock for a number of circular-arc noses was calculated by the method of characteristics. There was some question as to whether to include the effects of rotational flow (i.e., vorticity) in these calculations. For exactness in inviscid flow, the inclusion of the effects of vorticity is demanded. However, from a practical viewpoint there were other considerations. First, the effects of vorticity upon attached-shock shape must be insignificant at low Mach numbers inasmuch as the entropy gain across a normal shock at these Mach

numbers is trivial. Second, although the effects of vorticity upon attached-shock shape for a given nose shape increase with Mach number, there was some information available from previous calculations at a Mach number of 2.2 and $\delta = 20^\circ$ (two-dimensional) and at a Mach number of 2.59 and $\delta = 35^\circ$ (axisymmetric) which tended to indicate that the effects of vorticity would be secondary at the higher Mach numbers and nose angles covered in this study. Additional calculations were made in conjunction with the present study to examine the effects of vorticity over the portion of the shock very near the nose, since in this region the greatest change in the effects of vorticity occur; the results of these calculations are shown in figure 3. Figure 4 compiles all of this information in terms of the ratio of the vertical ordinate of the shock when vorticity is included to that when vorticity is neglected. The fact that this ratio reaches unity at $\frac{x}{l} > 0$ (in this case of the order of 0.05) rather than at $\frac{x}{l} = 0$ is an inherent feature of the method of characteristics which of necessity assumes that the tip of the nose is a cone; consequently, the field of flow in the immediate neighborhood of the tip is conical and without vorticity. The results in figure 4 indicate that the effects of vorticity are quite likely secondary at most at the highest Mach numbers and nose angles of this study. When the effects of viscosity are considered together with these indicated small effects of vorticity, there is considerable justification for neglecting vorticity in making calculations for practical applications that fall within the range of variables considered herein. This arises from the effect that the presence of the boundary layer has in making the nose shape effectively slightly bluffer, with the result that the shock is forced slightly outward and in a direction that tends to compensate for the effect that vorticity has in turning the shock slightly inward. Vorticity is neglected in the calculations of figure 2; nevertheless, the calculated shock for which experimental results were available (fig. 2(b)) is seen to be in excellent agreement with the experimental shock. Additional evidence of the compensating tendency of the effects of the boundary layer and of vorticity is given subsequently.

For the reasons contained in the preceding paragraph and because of the belief that the small effects of vorticity may be adequately estimated from the information contained herein, if so desired, the characteristic calculations that were made to establish the values of K did not include the effects of vorticity. The axisymmetric characteristic calculations were performed on electronic computing machines and had a mesh density of the net of the order of 5 times those shown in figure 2; these calculations thus maintained a high degree of accuracy. Most of the two-dimensional characteristic calculations were performed manually and had a mesh density of the order of those shown in the middle and outer portions of the nets in figure 2. Spot checks of these manual calculations by machine calculations revealed only minor differences. Thus, although the two-dimensional calculations do not have the precision of the axisymmetric calculations, they are believed to be sufficiently accurate. A total of 34 two-dimensional characteristic calculations were made which covered Mach numbers from 1.5 to 5.0 and values of δ from 5° to 40°. A total of 42 three-dimensional, or axisymmetric, characteristic calculations were made (excluding calculations to determine effects of vorticity); these covered Mach numbers from 1.5 to 4.0 and values of δ from 10° to 50°. In all of these calculations the ordinates were nondimensionalized with respect to nose length. The value of K thus relates the unspecified shock to a circular-arc nose of unit length.

For a particular combination of M and δ, a trial-and-error graphical superposition at large scale of the unspecified shock given by equation (4) upon the shock from the characteristic solution showed that the simple scale relations of equation (6) were adequate and determined, with good precision, the value of K. Typical illustrations of the precision with which K may be determined are shown in figure 5 together with an indication of the error in shock shape produced by a change in K from the value which was selected as satisfying the shock from the characteristic solution.

The values of K thus determined are given in table I, and the variations in K with M and with δ are shown in figure 6 for two-dimensional circular-arc noses and in figure 7 for axisymmetric circular-arc noses.

For completeness and in order to facilitate the application of the method to arbitrary nose shapes, figures 8 and 9 present curves giving shock angles as a function of M for various values of δ for wedges (i.e., two-dimensional turning) and cones, respectively. These curves were obtained from references 5 and 6.

Predictions by Present Method

Procedure.- The problem is to determine the shock for an arbitrary nose shape. The values of M and δ are known; therefore, μ and ϵ (figs. 8 and 9) are readily obtained. Values of y_i for assumed values of x_i may be obtained by means of figure 1 if a quick approximation is desired, or they may be easily calculated from equation (4) if more accuracy is desired. The appropriate value of K corresponding to the particular combination of M and δ is obtained from figure 6 or 7. Values of x and y are converted to $\left(\frac{x}{l}\right)_c$ and $\left(\frac{y}{l}\right)_c$ by equation (6). Along the line normal to the slope of the nose at the tip and passing through the tip of the nose of the arbitrary shape (see fig. 10), an arc is swung which appears to fit best as much of the forward part of the nose as possible. The appropriate scales $\left(\frac{x}{l}\right)_c$ and $\left(\frac{y}{l}\right)_c$ for the circular-arc nose may then be established and the shock plotted in accordance with these scales. The shock thus obtained is the desired one. An expedient procedure that obviates the necessity of the new scales $\left(\frac{x}{l}\right)_c$ and $\left(\frac{y}{l}\right)_c$ is to note the value of $\frac{x}{l}$ at the intercept of the base of the circular-arc nose on the $\frac{x}{l}$-axis (see fig. 10), multiply the calculated values of $\left(\frac{x}{l}\right)_c$ and $\left(\frac{y}{l}\right)_c$ by this value, and plot the resulting values in accordance with the existing x/l and y/l scales of the arbitrary nose shape.

Comparisons with experiment and characteristic theory.- It is implicit from the development of the present method that the method should give an accurate prediction of the experimental shock for a circular-arc nose. Figures 11 and 12 give examples at $M = 1.62$ that show this to be true. The value of K for the calculation in figure 11 could be determined accurately by interpolation from the curves of figure 7; consequently, the excellent agreement of the prediction with experiment is to be expected. The value of K for the calculation in figure 12 was obtained by extrapolation in a region where the curves of figure 7 are experiencing large changes in slope. A value of $K = 0.35$ is believed to be a reasonable extrapolation; however, as the results of figure 12 show, small changes in K need not cause concern. With either $K = 0.35$ or $K = 0.39$ an excellent prediction is obtained.

Figure 13 gives an example of a prediction at $M = 1.62$ for a nose shape whose contour is of the form $y = a - bx^2 + cx^3 - dx^4$ (with the origin for this contour at the base of the nose). The fitted circular-arc nose by means of which the prediction was made is also shown. The prediction agrees closely with experiment.

Figure 14 gives an example of a prediction at $M = 2.59$ for a nose shape whose contour is of the form $y = ax - bx^2 + cx^3$. Again the fitted circular-arc nose is shown. The present prediction and the prediction by the method of characteristics (vorticity neglected) give, within the accuracy of the presentation, identical shock shapes, and these predictions are in good agreement with the experimental results. The experimental results were obtained at $M = 2.62$ as compared with $M = 2.59$ employed in the predictions. However, this difference in Mach number amounts to a difference in ϵ of only about 0.2^o; this difference in ϵ would cause only a minor difference in the shock shape and would move the shock slightly inward. It is interesting to note, however, that the experimental results at $M = 2.62$ lie above the prediction by the method of characteristics at $M = 2.59$ when the effects of vorticity are included.

This is attributed primarily to the effects of the boundary layer on the nose as discussed previously in this report. These results lend additional support to the choice of neglecting vorticity that was made in the development of the present method.

Although the present method need not be employed when the convexity of the arbitrary nose shape is so small that the nose is essentially a cone (or wedge), it is interesting to note that the method does not break down in the limiting case for which the nose is an exact cone (or wedge). In this case the radius of the fitted circular-arc nose is infinite. Thus, the first calculated point on the shock lies at $\frac{x}{l} = \infty$, the shock is straight, and its inclination is correct.

The preceding results indicate that, from a practical viewpoint, the accuracy of the method proposed herein is comparable to that of the method of characteristics. Differences can be found between the ratio of the radius of curvature of the shock to that of the nose, exactly at the tip of the nose, that may be calculated from the present method and the exact ratio as derived by several investigators (ref. 7, for example). However, a cursory examination indicates that, when these ratios differ to any extent, such large radii of curvature are involved that these differences are mainly of academic interest, since they do not correspond to any significant error in the local inclinations of the shock given by the present method, including that portion of the shock near the tip of the nose.

Errors associated with fitting circular arc.- The errors likely to be encountered as a result of poor fitting of the circular arc or slight differences in fitting are illustrated in figure 15 for the nose shape of figure 14. Two arbitrary fittings are compared with that selected for the prediction of figure 14 (also repeated in fig. 15). The fitting which gives the shortest circular-arc nose is regarded as a hasty choice since it certainly can be bettered. Nevertheless, had this fitting been chosen, the resulting prediction would be negligibly different from the prediction obtained by the fitting employed in figure 14 and would therefore be in close agreement with the experimental shock. The fitting giving the longest circular-arc nose in figure 15 can be regarded only as a poor one, and there is no justification for a misfit of this magnitude; however, it serves to show that even a fitting so poor as this yields surprisingly good results.

From this and similar examinations it may be safely concluded that the differences in predictions that are likely to be introduced by differences in the choice of the circular arc will, in any reasonable application of the method, be insignificant.

CONCLUDING REMARKS

A method has been developed for the rapid prediction of the shape of attached shocks emanating from smoothly contoured axisymmetric and two-dimensional nose shapes. The method covers Mach numbers from about 1.5 to 5.0 and semiapex angles at the tip of the nose up to values near those for sonic conditions behind the shock. From a practical viewpoint the accuracy of the method is comparable to that of the method of characteristics.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 16, 1957.

REFERENCES

1. Uchida, Shigeo, and Yasuhara, Michiru: The Rotational Field Behind a Curved Shock Wave Calculated by the Method of Flux Analysis. Jour. Aero. Sci., vol. 23, no. 9, Sept. 1956, pp. 830-845.

2. Whitham, G. B.: The Flow Pattern of a Supersonic Projectile. Communications on Pure and Appl. Math., vol. V, no. 3, Aug. 1952, pp. 301-348.

3. Eggers, A. J., Jr., and Savin, Raymond C.: A Unified Two-Dimensional Approach to the Calculation of Three-Dimensional Hypersonic Flows, With Application to Bodies of Revolution. NACA Rep. 1249, 1955. (Supersedes NACA TN 2811.)

4. Eggers, A. J., Jr., Syvertson, Clarence A., and Kraus, Samuel: A Study of Inviscid Flow About Airfoils at High Supersonic Speeds. NACA Rep. 1123, 1953. (Supersedes NACA TN 2646 by Eggers and Syvertson and NACA TN 2729 by Kraus.)

5. Neice, Mary M.: Tables and Charts of Flow Parameters Across Oblique Shocks. NACA TN 1673, 1948.

6. Staff of the Computing Section, Center of Analysis (Under Direction of Zdeněk Kopal): Tables of Supersonic Flow Around Cones. Tech. Rep. No. 1 (NOrd Contract No. 9169), M.I.T., 1947.

7. Schaefer, M.: The Relation Between Wall Curvature and Shock Front Curvature in Two-Dimensional Gas Flow. Tech. Rep. No. F-TS-1202-IA, Air Materiel Command, U. S. Air Force, Jan. 1949.

TABLE I.- VALUES OF K

(a) For two-dimensional nose shapes

M	Values of K for δ =							
	5°	10°	15°	20°	25°	30°	35°	40°
1.5	8.0	5.0	---	---	----	----	----	----
2.0	8.0	5.0	3.0	1.5	----	----	----	----
2.6	8.0	5.0	3.0	2.0	1.15	0.50	----	----
3.3	8.0	5.0	3.0	2.0	1.50	1.10	0.45	----
4.0	8.0	5.0	3.0	2.0	1.50	1.10	.70	----
5.0	8.0	5.0	3.0	2.0	1.55	1.20	.85	0.43

(b) For axisymmetric nose shapes

M	Values of K for δ =								
	10°	15°	20°	25°	30°	35°	40°	45°	50°
1.5	6.80	1.47	0.710	0.350	-----	-----	-----	-----	-----
2.0	2.85	1.23	.765	.585	0.435	0.297	-----	-----	-----
2.5	1.95	1.03	.750	.615	.506	.405	0.300	-----	-----
3.0	1.50	.914	.730	.620	.540	.462	.370	0.250	-----
3.5	1.24	.850	.720	.630	.575	.505	.417	.305	-----
4.0	1.10	.810	.705	.640	.586	.525	.450	.350	0.190

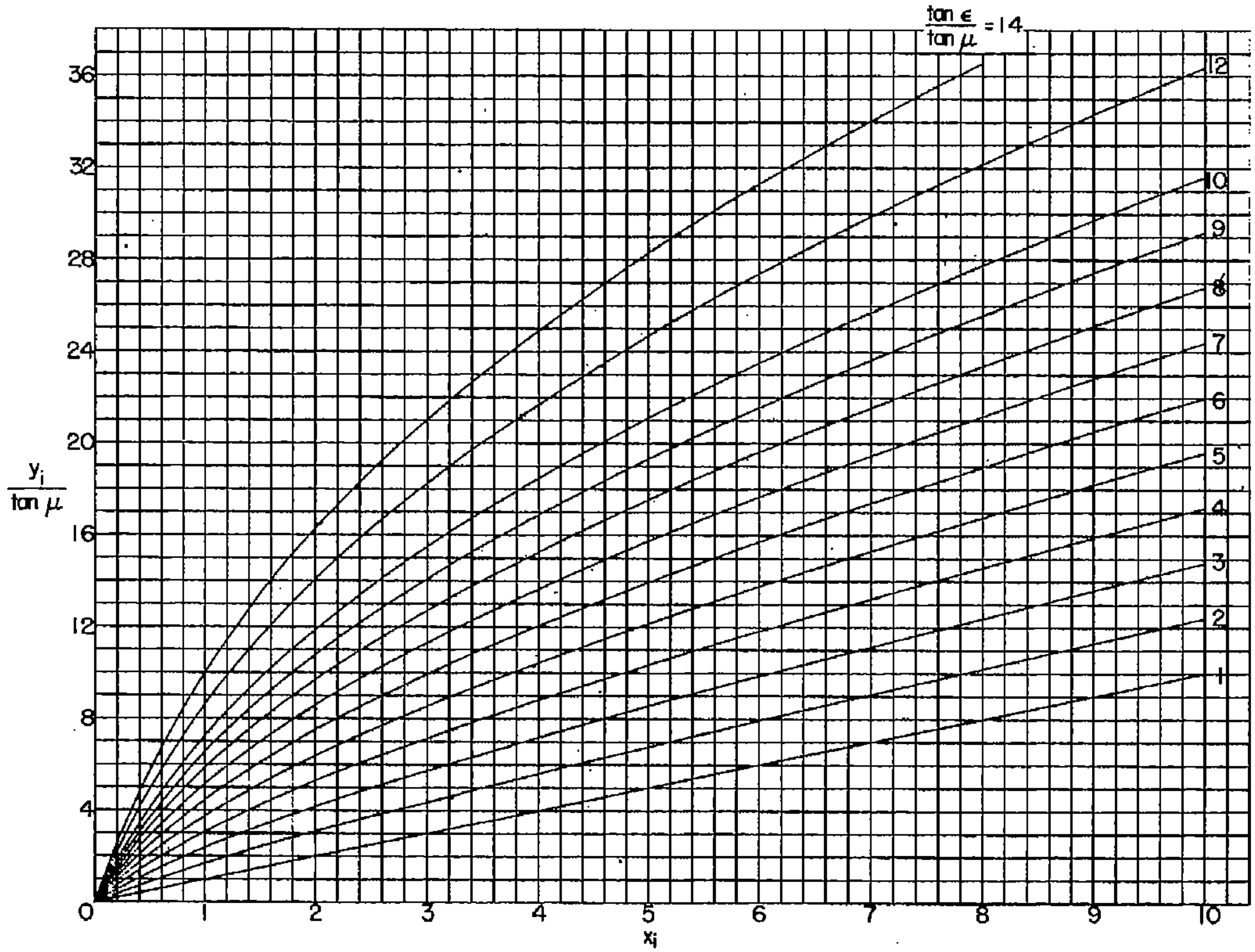
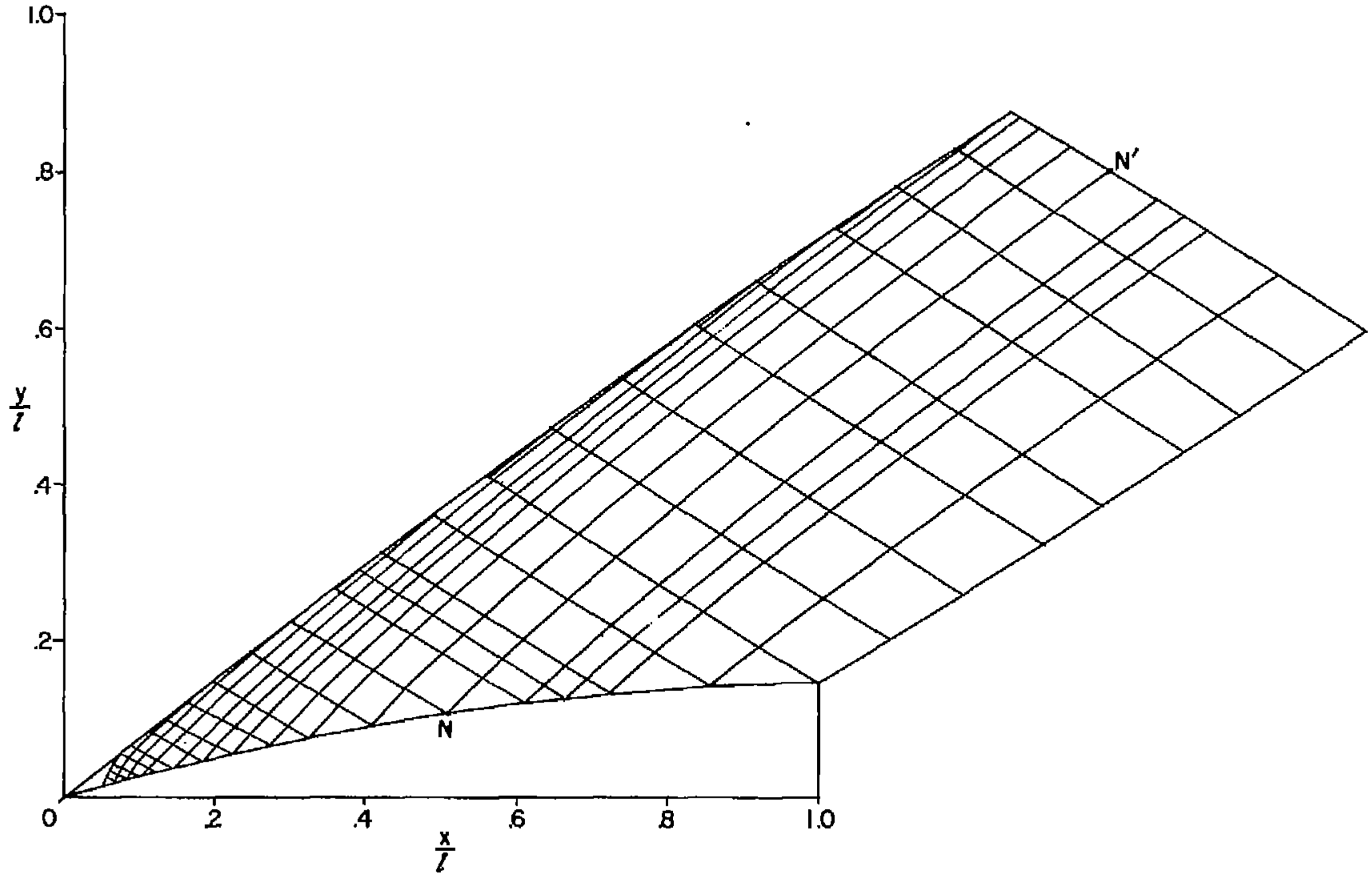
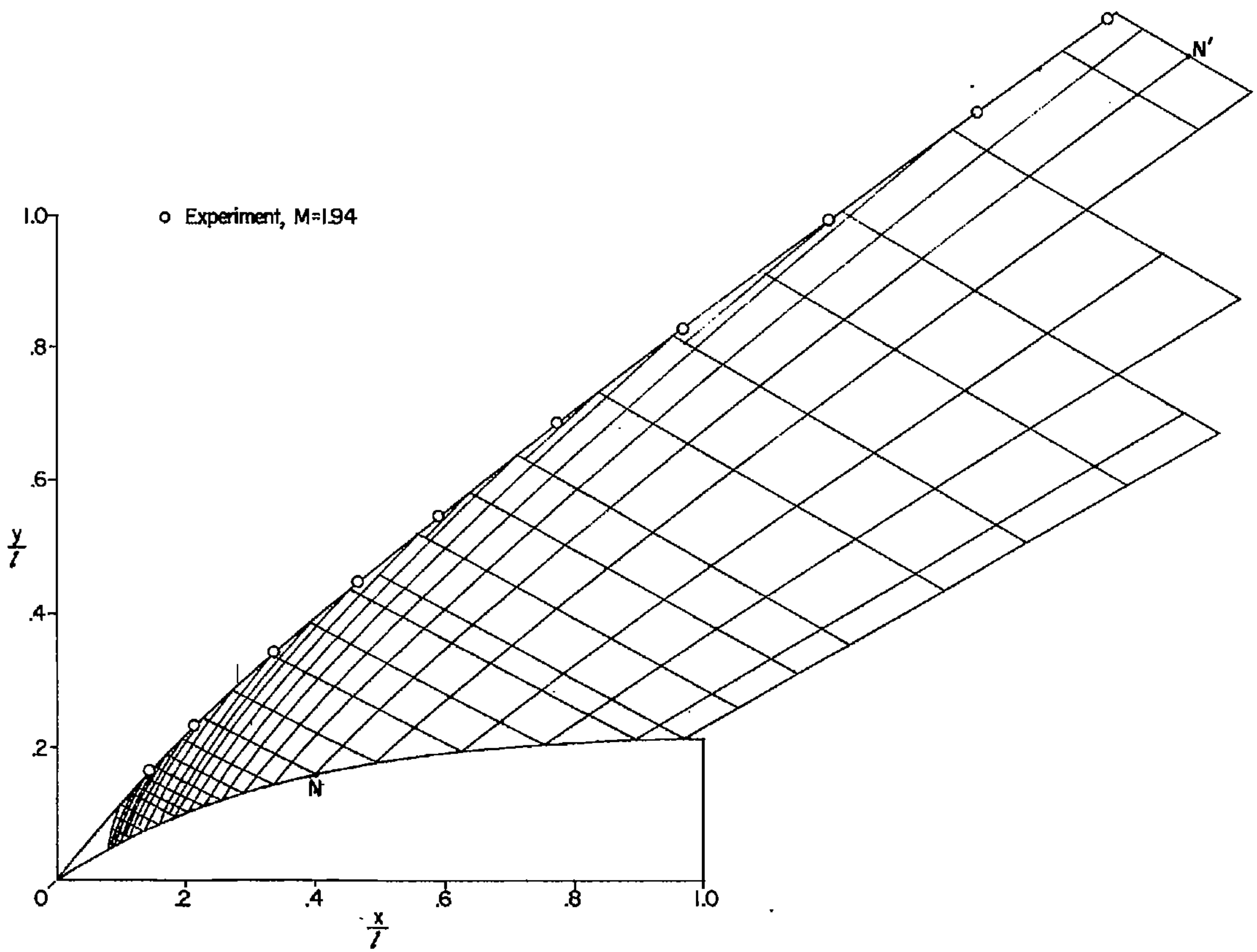


Figure 1.- Chart giving unspecified shock shapes in the form of $\frac{y_i}{\tan \mu}$ as a function of x_i for various values of the parameter $\frac{\tan \epsilon}{\tan \mu}$.



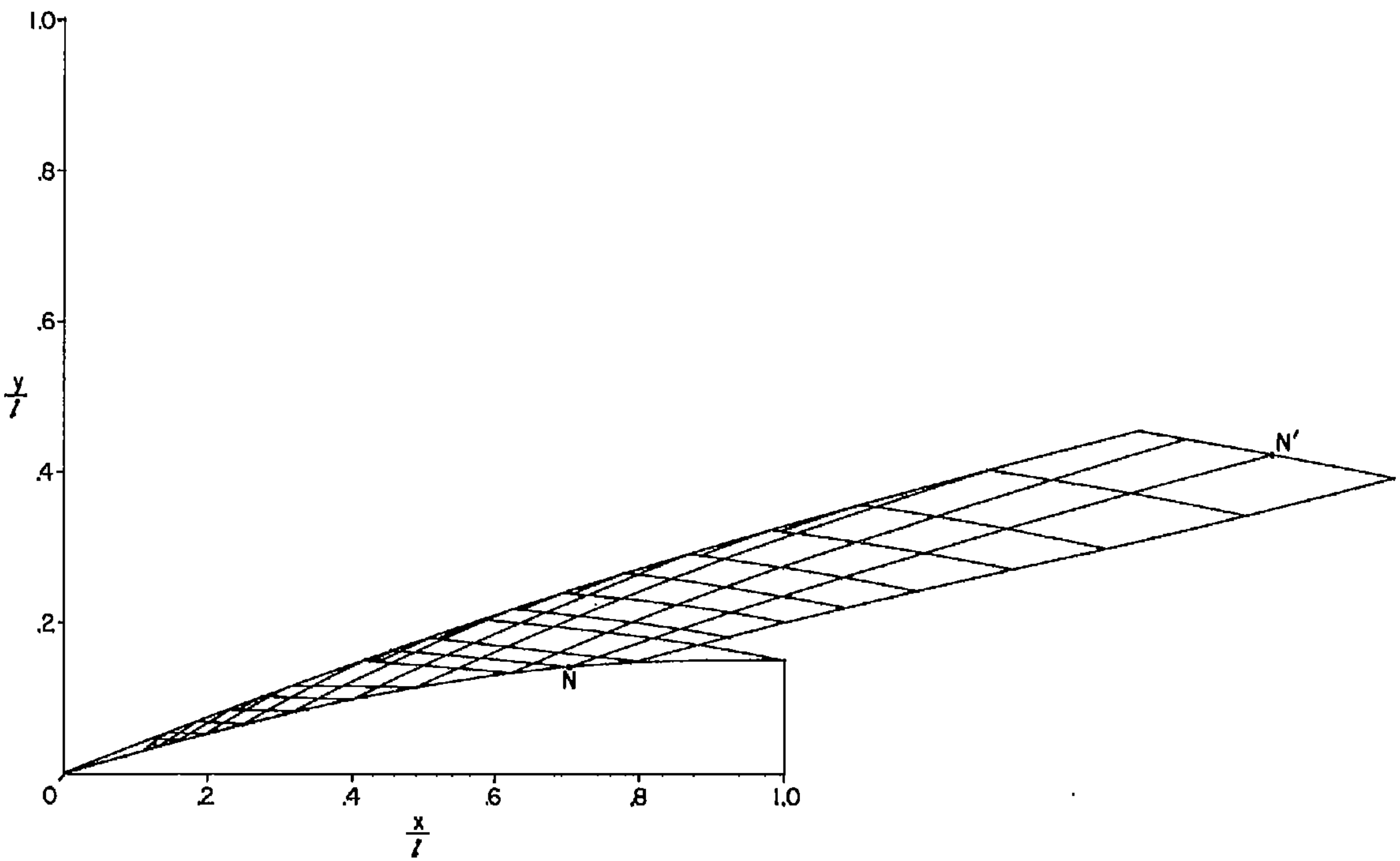
(a) M = 1.88.

Figure 2.- Illustration of the general location of the first-family-characteristic line beyond the general vicinity of which there is no significant influence of body shape on shock shape.



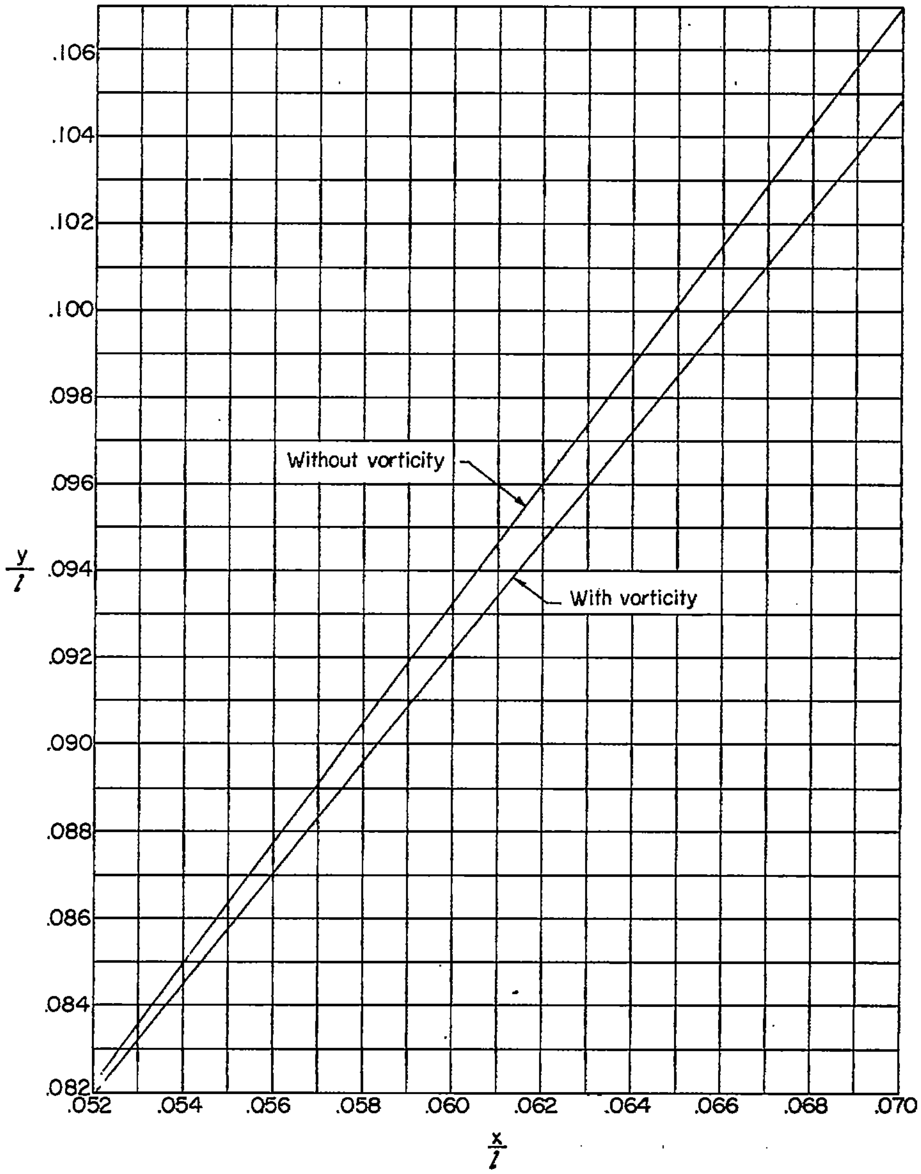
(b) M = 1.95.

Figure 2.- Continued.



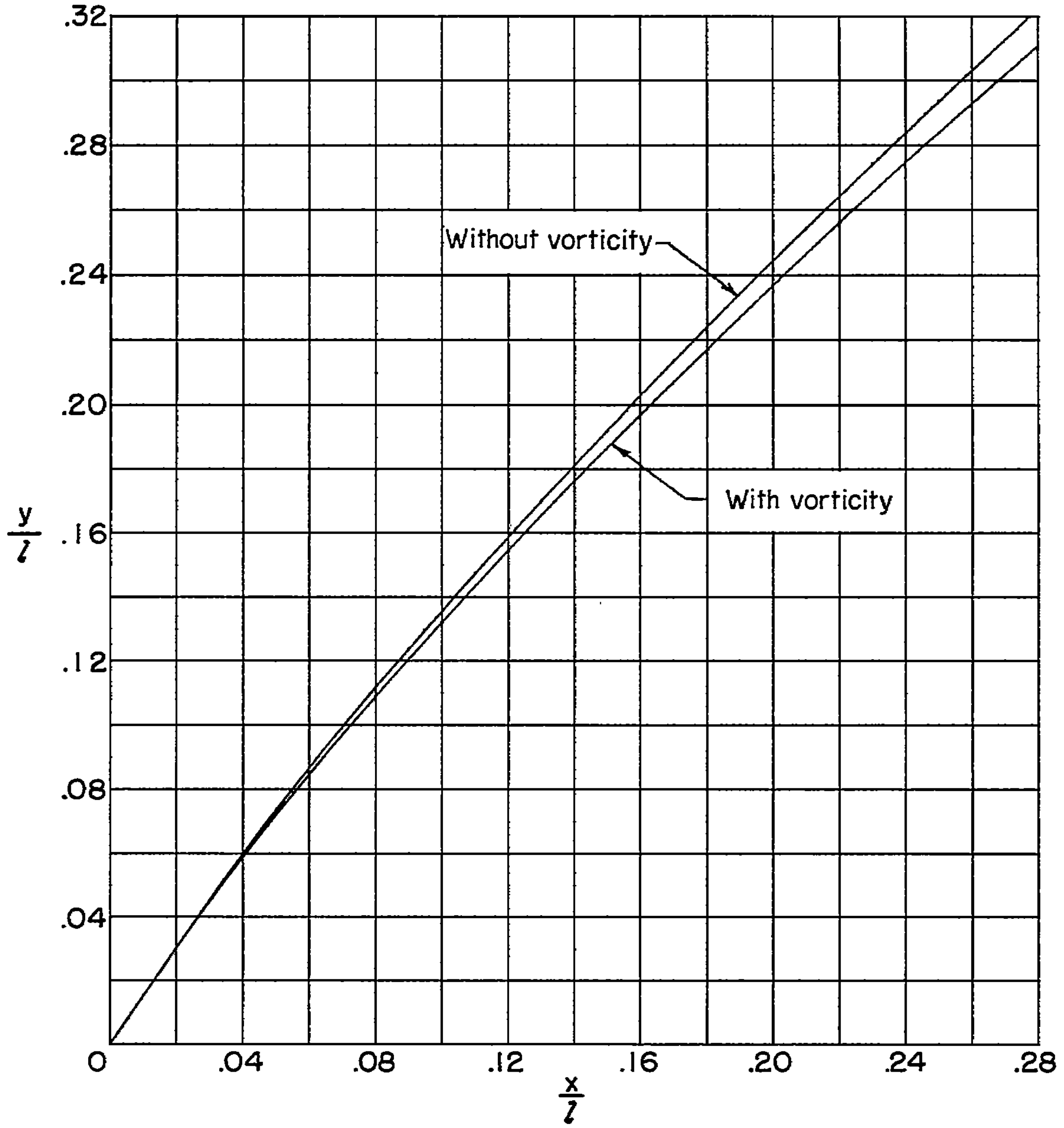
(c) M = 5.05.

Figure 2.- Concluded.



(a) $M = 3.0$; $\delta = -45^{\circ}$. (Interval between calculated points on shock ranges from $\frac{x}{l} \approx 0.0005$ at lowest values of $\frac{x}{l}$ to $\frac{x}{l} \approx 0.001$ at highest values of $\frac{x}{l}$.)

Figure 3.- Effects of vorticity on shock shape over the portion of the shock very near the nose. Axisymmetric circular-arc nose.



(b) $M = 3.5$; $\delta = 45^{\circ}$. (Interval between calculated points on shock ranges from $\frac{x}{l} \approx 0.005$ at lowest values of $\frac{x}{l}$ to $\frac{x}{l} \approx 0.01$ at highest values of $\frac{x}{l}$.)

Figure 3.- Concluded.

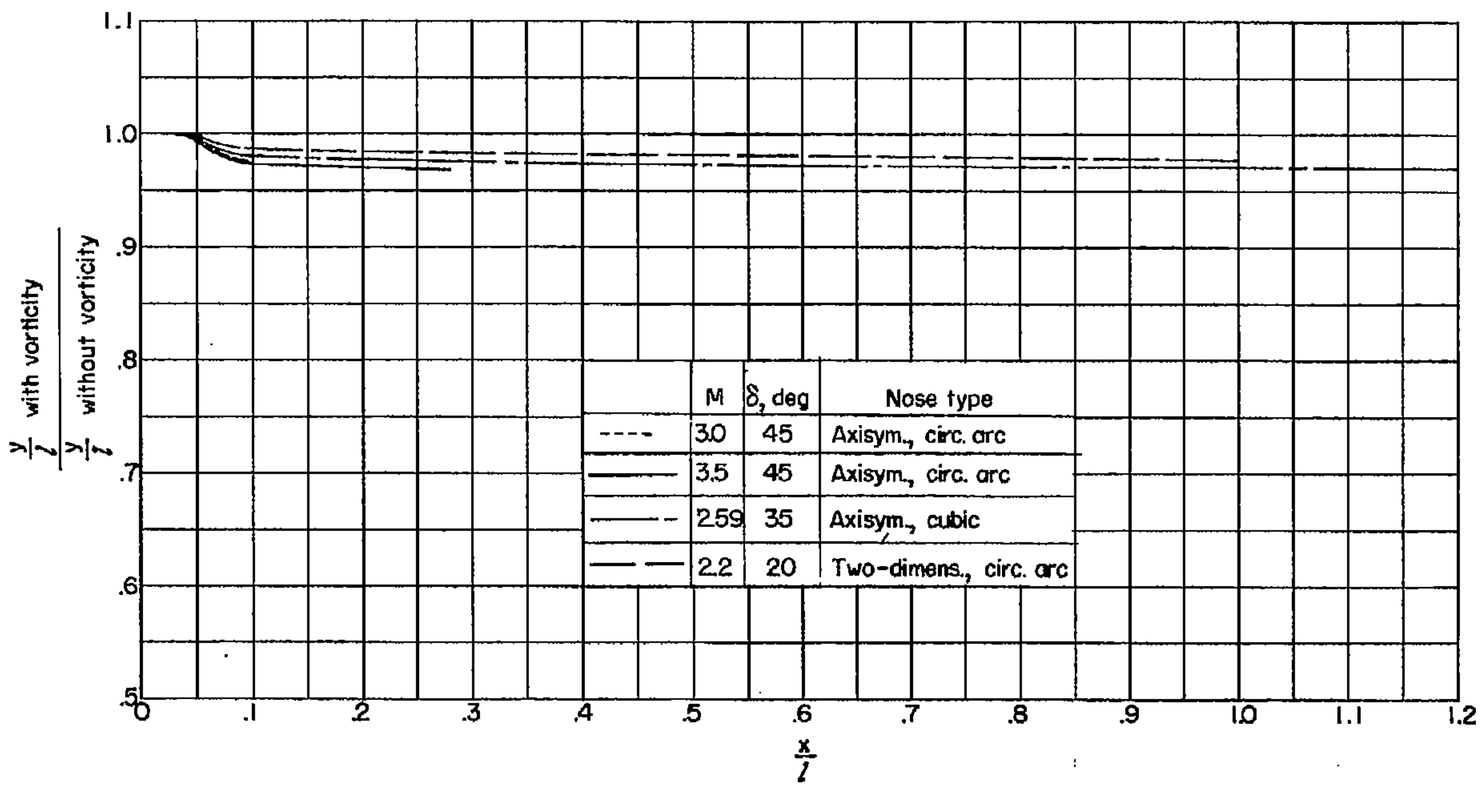
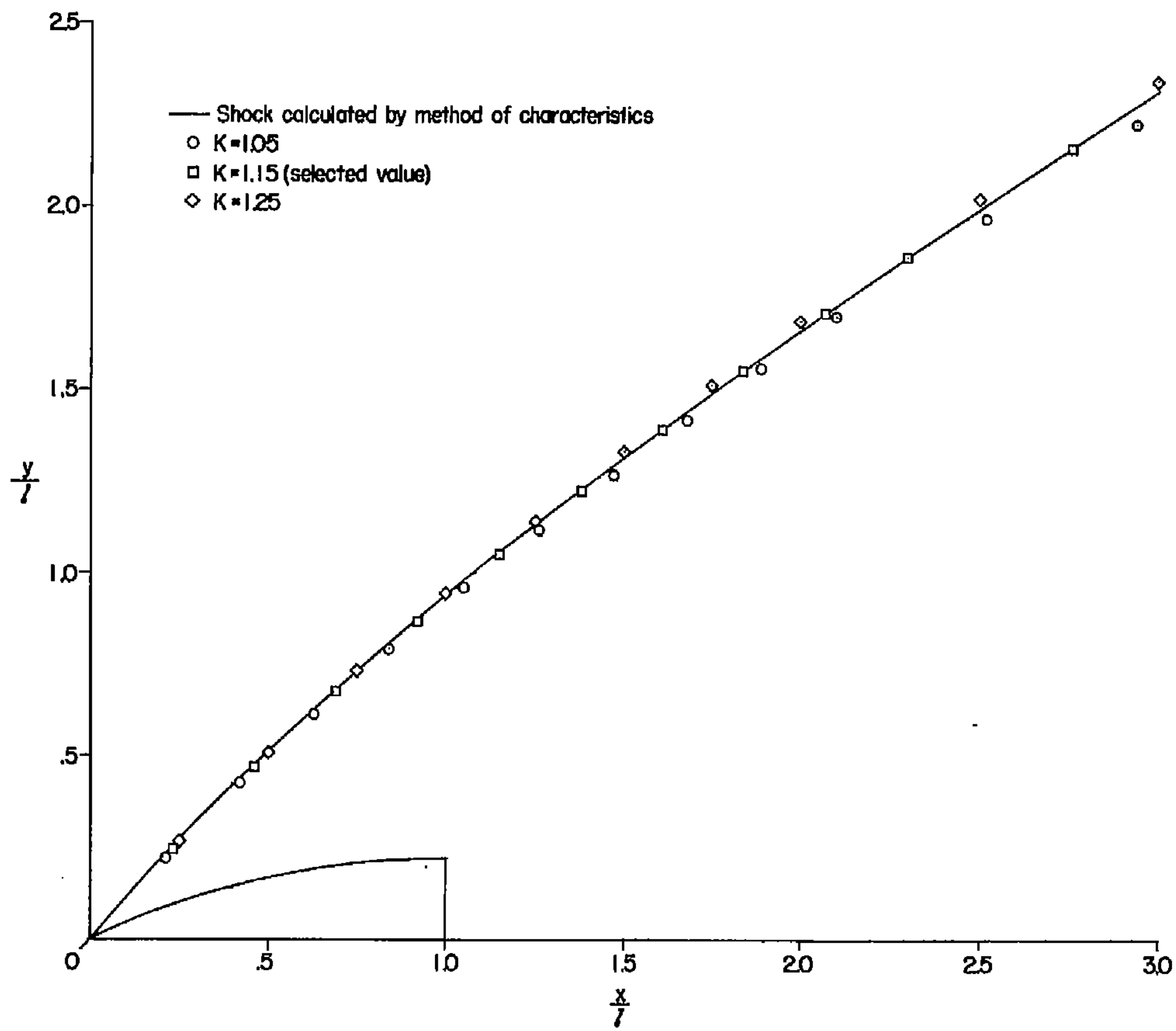
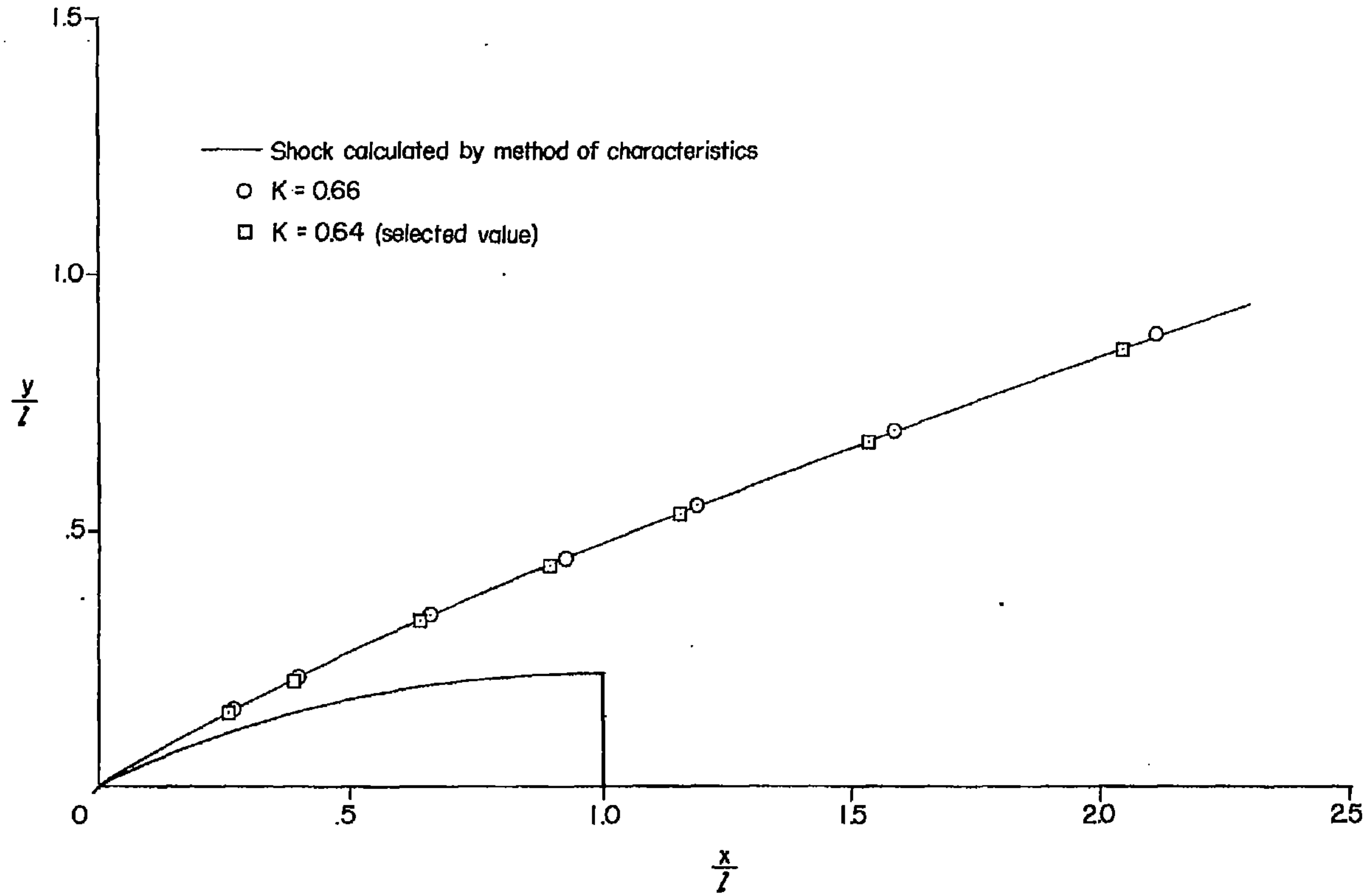


Figure 4.- Compilation of effects of vorticity upon shock ordinates from several calculations.



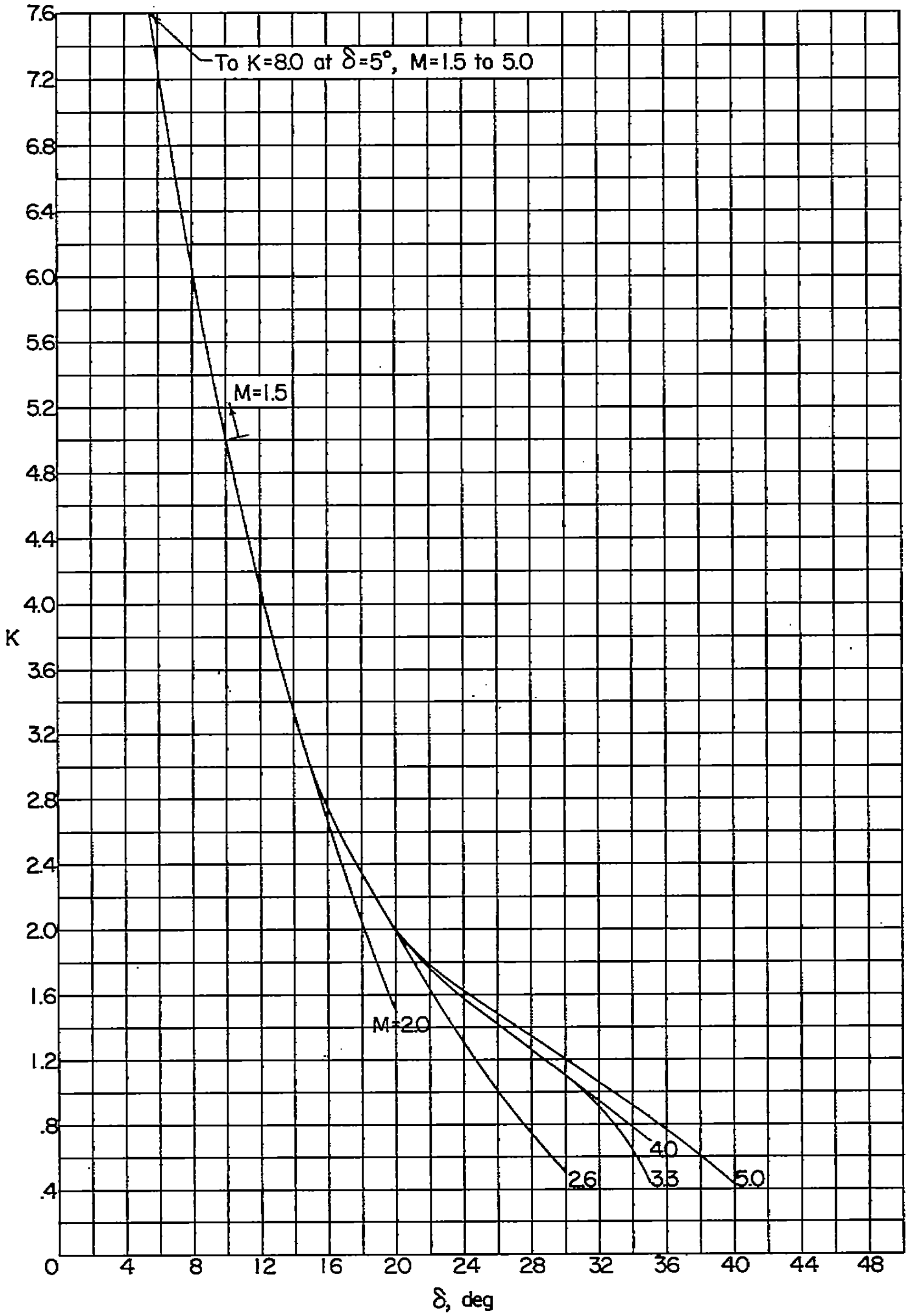
(a) Two-dimensional circular-arc nose. $M = 2.6$; $\delta = 25^{\circ}$.

Figure 5.- Illustration of the precision with which the values of K may be determined.



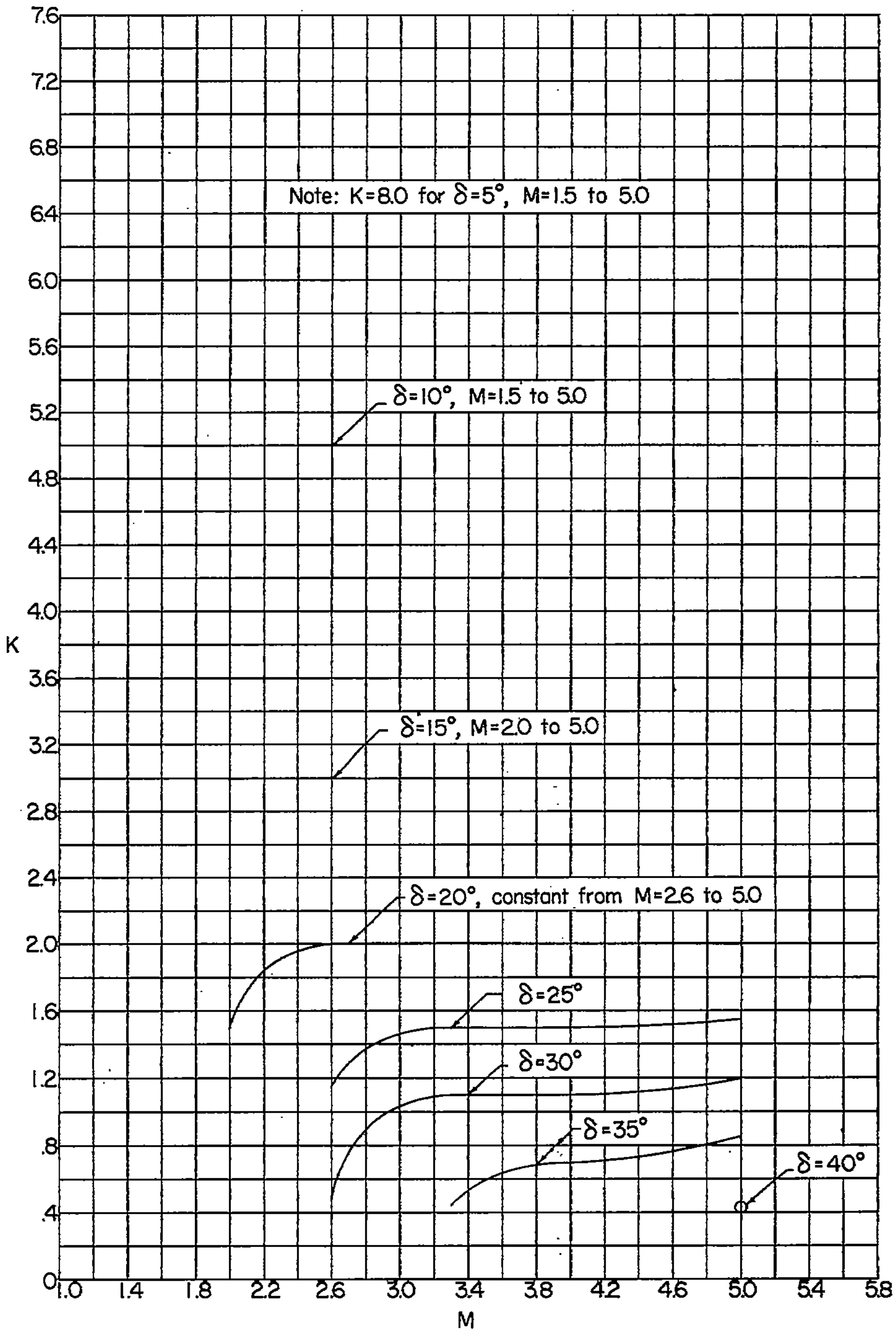
(b) Axisymmetric circular-arc nose. $M = 4.0$; $\delta = 25^{\circ}$.

Figure 5.- Concluded.



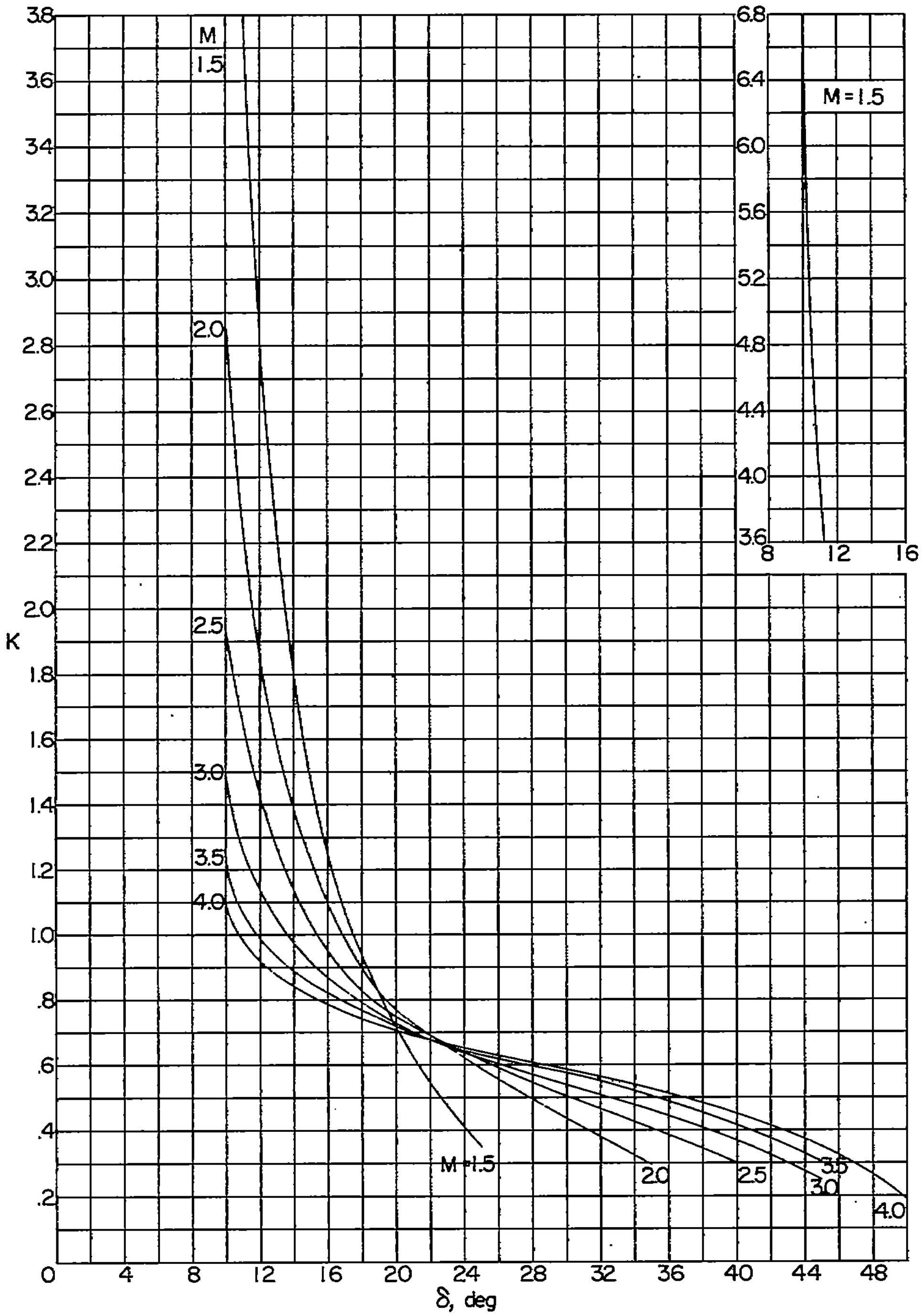
(a) Variation with δ for various values of M.

Figure 6.- Values of K for two-dimensional circular-arc noses.



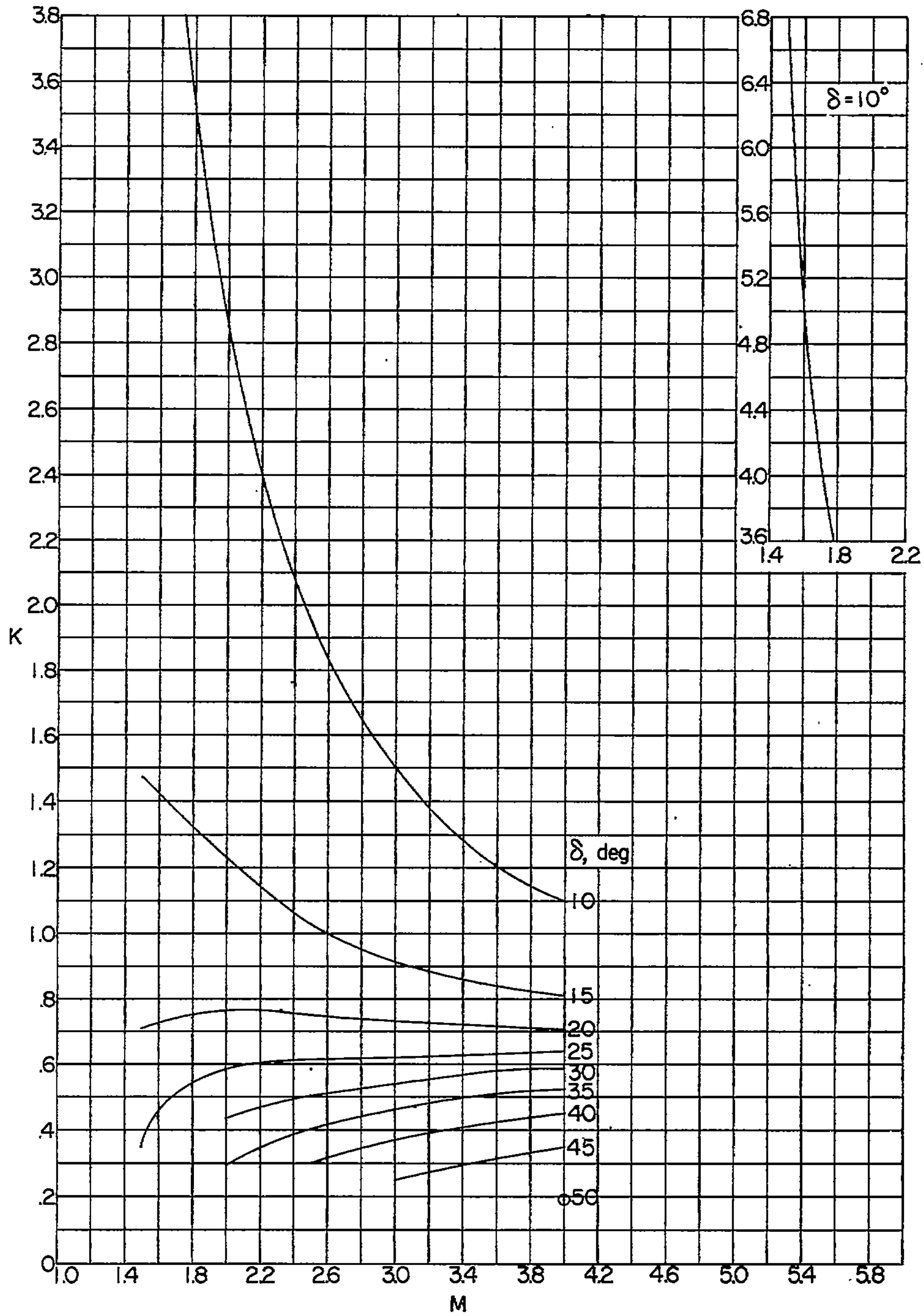
(b) Variation with M for various values of δ.

Figure 6.- Concluded.



(a) Variation with δ for various values of M.

Figure 7.- Values of K for axisymmetric circular-arc noses.



(b) Variation with M for various values of δ.

Figure 7.- Concluded.

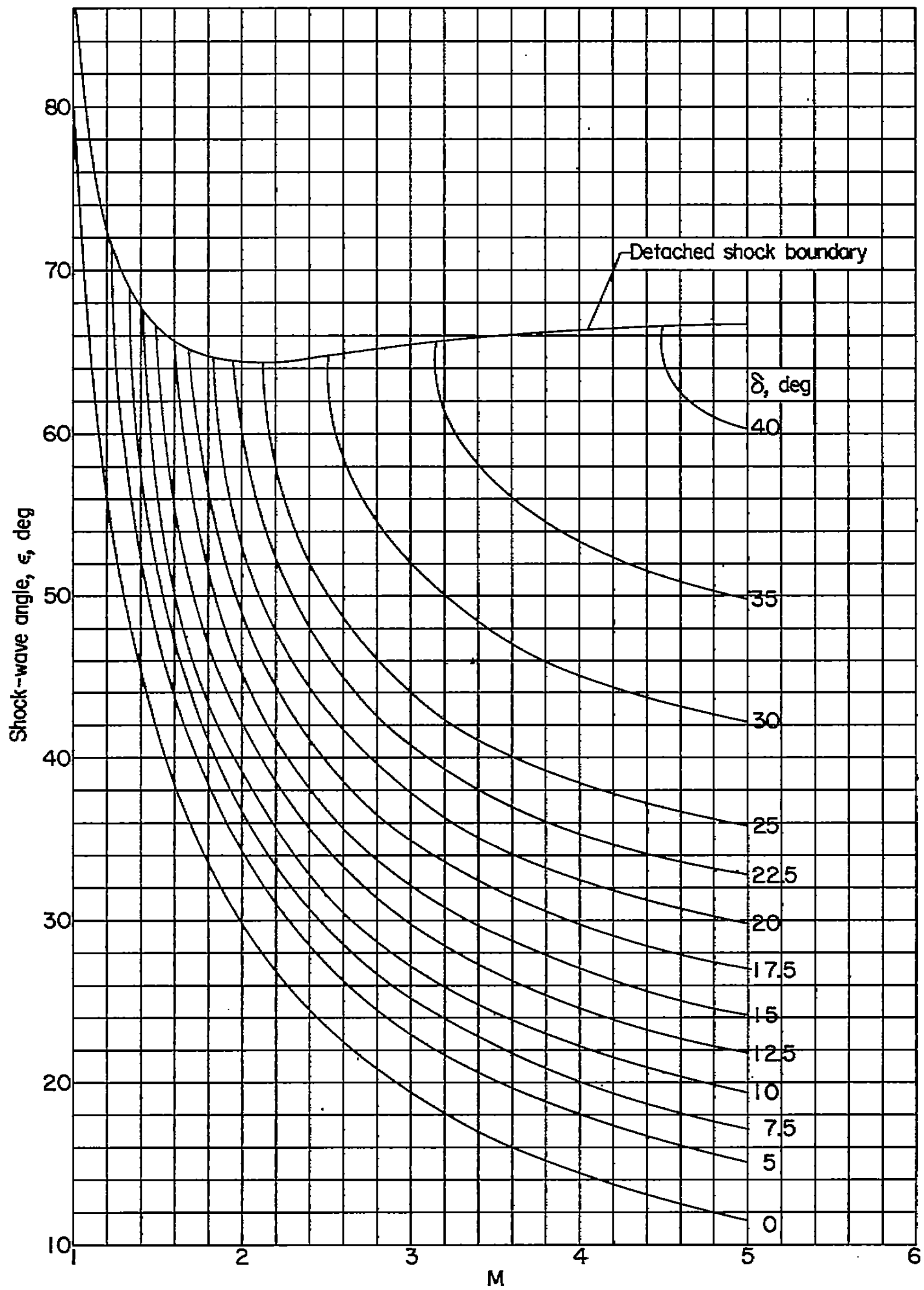


Figure 8.- Variation of shock-wave angle with Mach number for various semiwedge angles (two-dimensional turning).

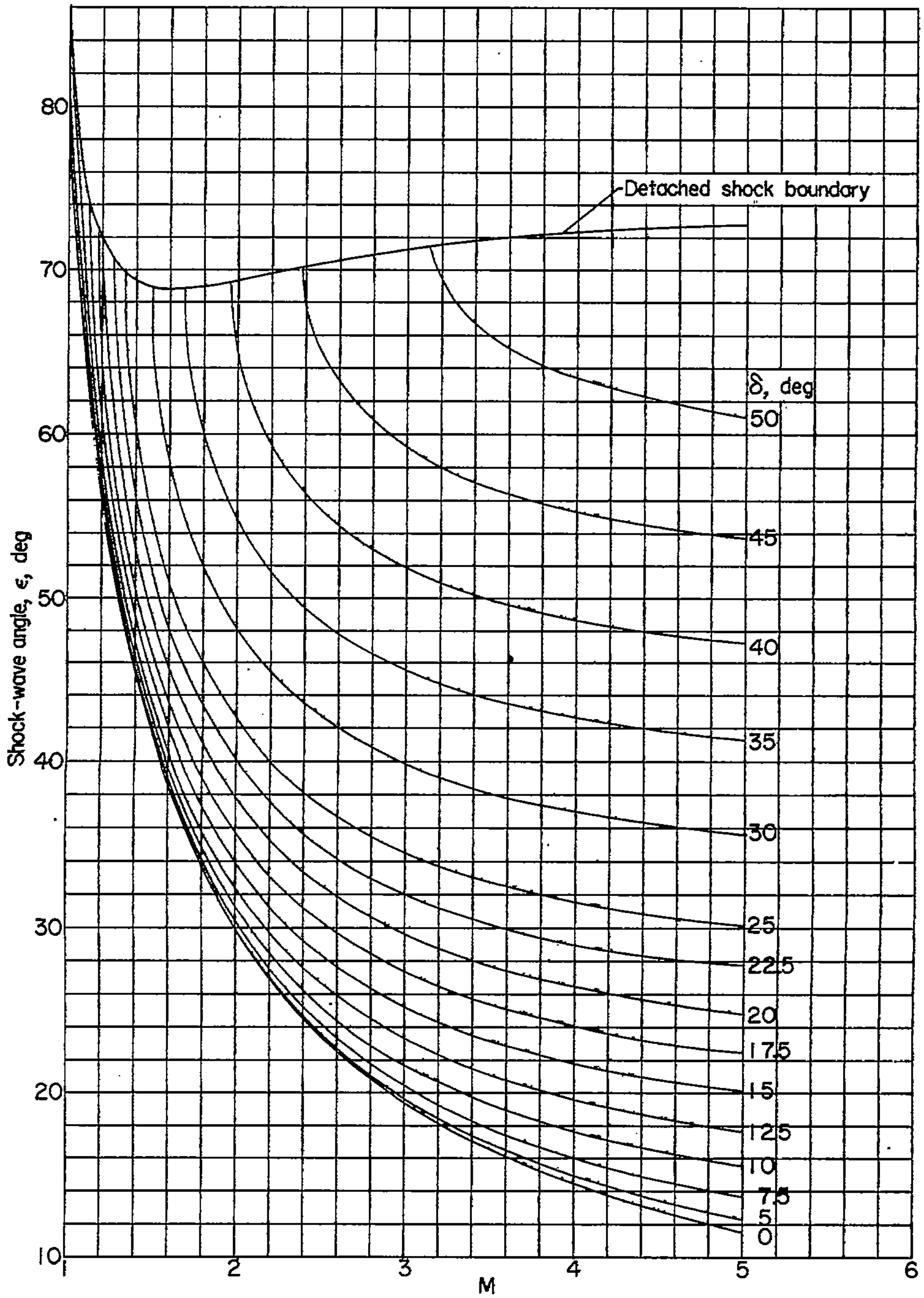


Figure 9.- Variation of shock-wave angle with Mach number for various semiapex angles of cones.

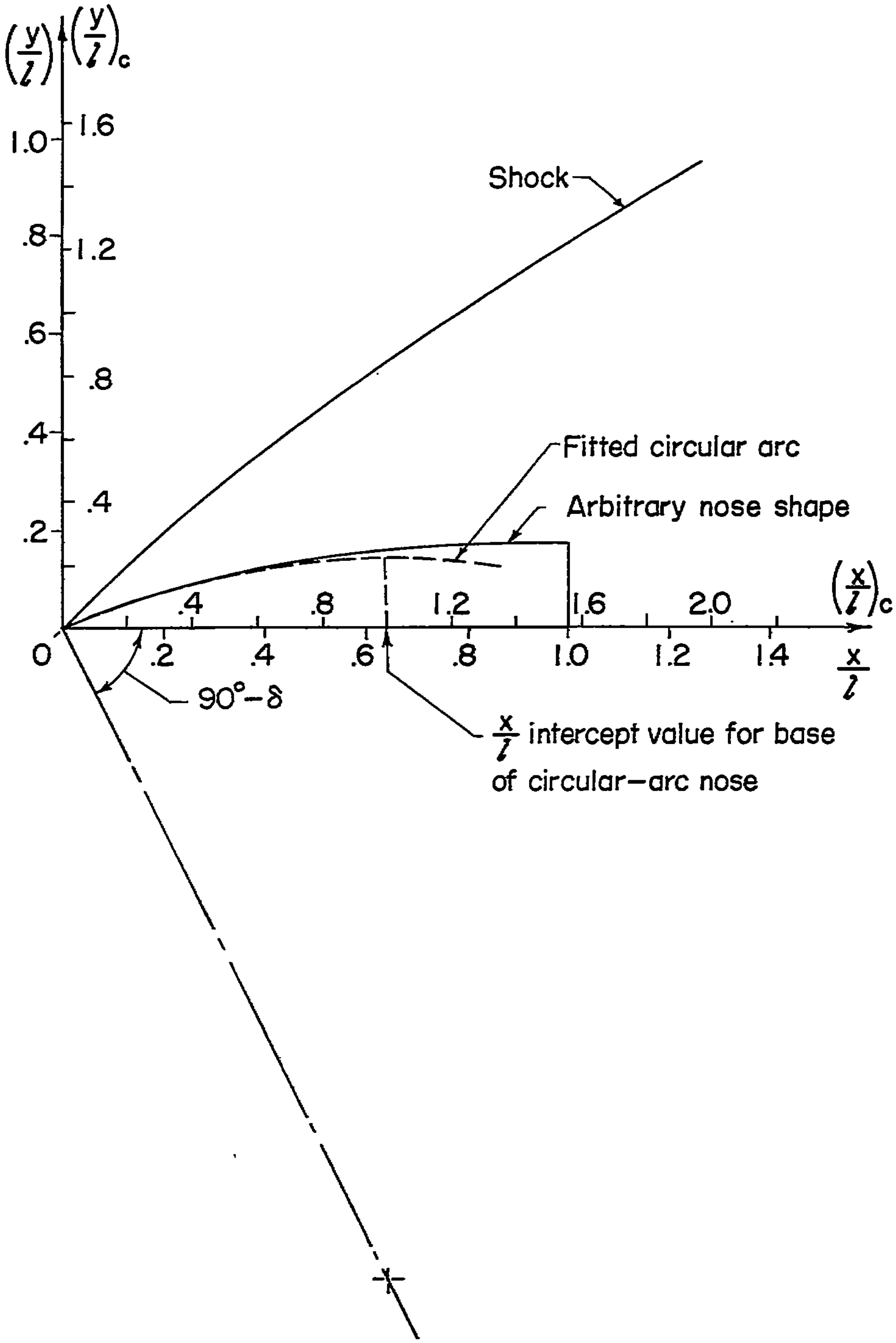


Figure 10.- Sketch of procedure for obtaining shock for arbitrary nose shape.

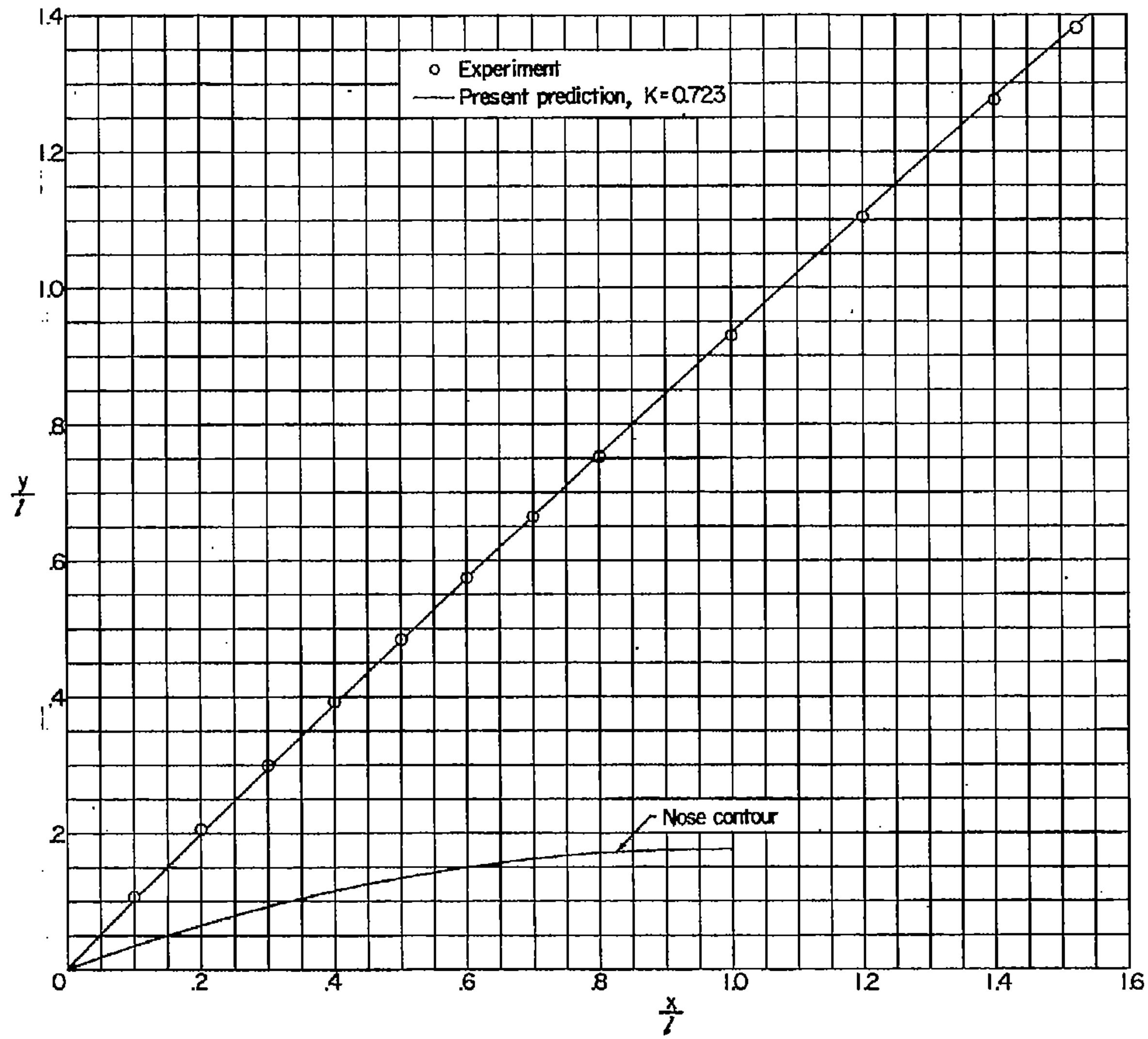


Figure 11.- Comparison of predicted and experimental shock for axisymmetric circular-arc nose. $M = 1.62$; $\delta = 20.25^{\circ}$.

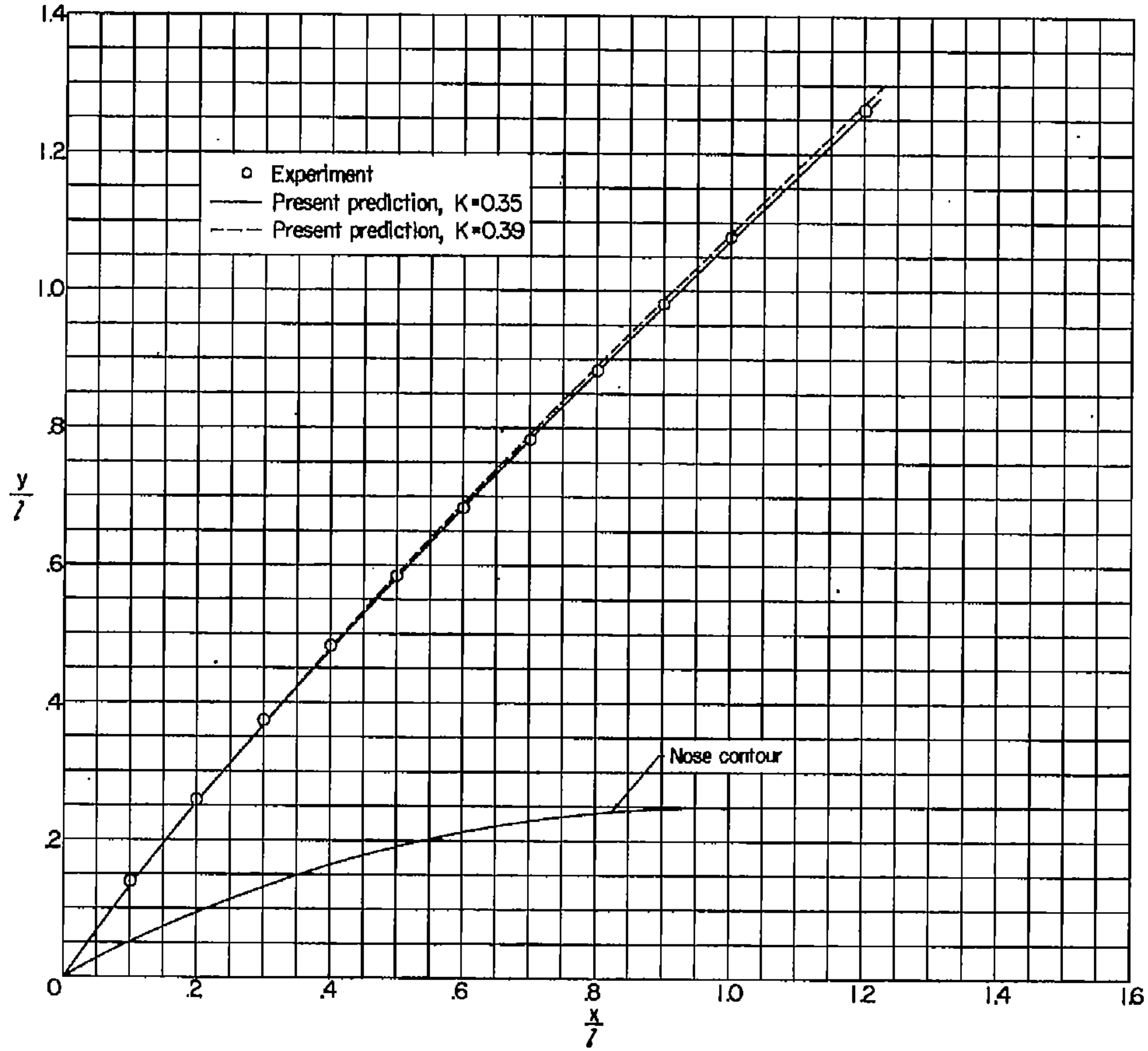


Figure 12.- Comparison of predicted and experimental shock for axisymmetric circular-arc nose showing small effect of difference in extrapolated values of K. $M = 1.62$; $\delta = 28^{\circ}$.

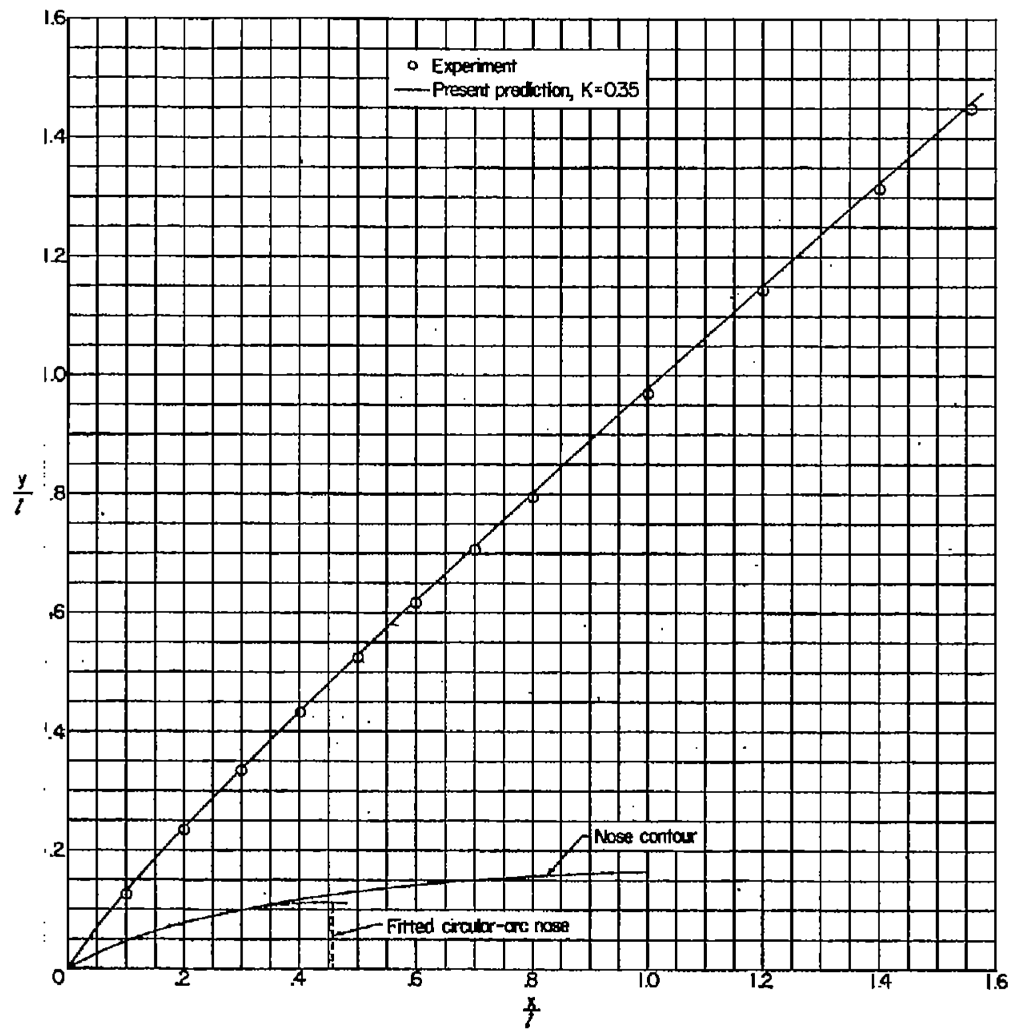


Figure 13.- Comparison of predicted and experimental shock for axisymmetric nose with contour of the form $y = a - bx^2 + cx^3 - dx^4$ (with origin for nose contour at base of nose). $M = 1.62$; $\delta = 28^{\circ}$.

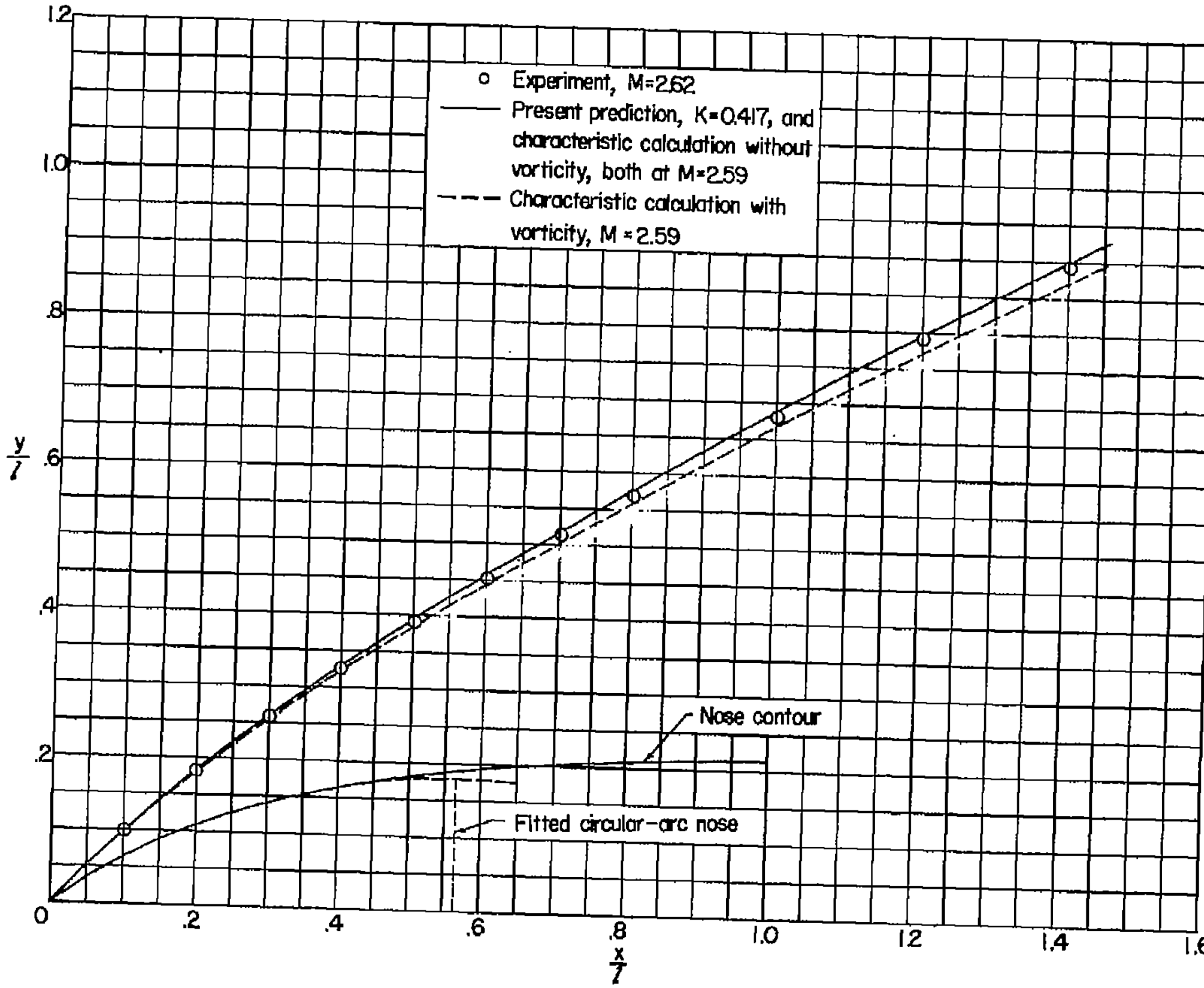


Figure 14.- Comparison of predicted shock with experimental shock and with shocks calculated by method of characteristics with and without vorticity for axisymmetric nose with contour of the form $y = ax - bx^2 + cx^3$. $M \approx 2.6$; $\delta = 35^\circ$.

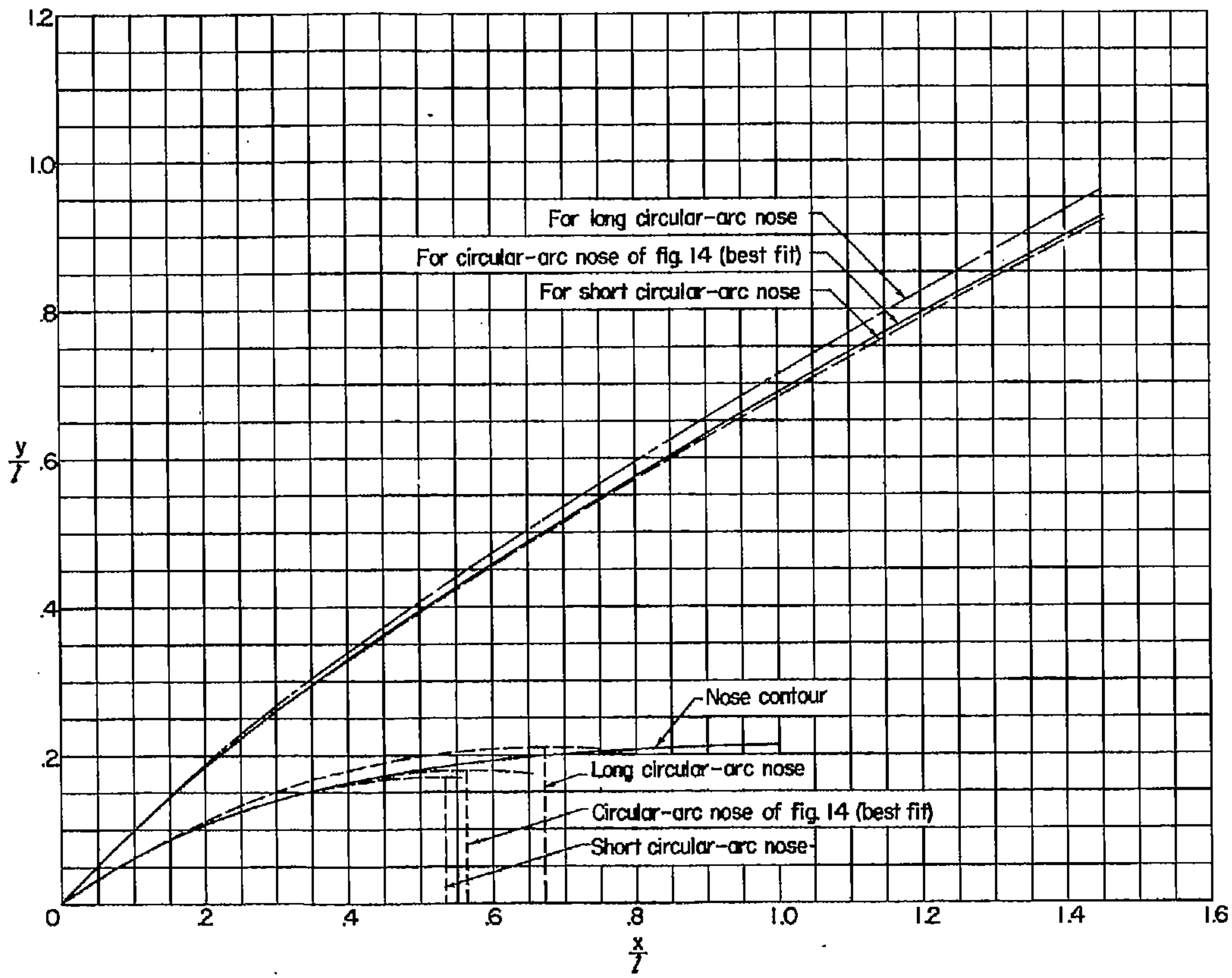


Figure 15.- Example of errors that would occur in prediction as a result of poor fitting of the circular arc. Same nose shape as in figure 14. All predictions are at M = 2.59.