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TECHNICAL NOTE 4308

TRANSIENT TEMPERATURE DISTRIBUTION IN A
TWO-COMPONENT SEMI-INFINITE COMPOSITE SLAB OF ARBITRARY
MATERIALS SUBJECTED TO AERODYNAMIC HEATING WITH A
DISCONTINUOUS CHANGE IN EQUILIBRIUM TEMPERATURE
OR HEAT-TRANSFER COEFFICIENT

By Robert L. Trimpi and Robert A. Jones

Langley Aeronautical Laboratory
Langley Field, Va.



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SUMMARY

A solution is obtained to the transient temperature distribution in a semi-infinite two-component composite slab of arbitrary materials subjected to an instantaneous application of aerodynamic heating with constant equilibrium temperature and heat-transfer coefficient. The numerical results are tabulated in a form to permit easy computation of heat-transfer problems typical of aerodynamic testing. The solutions are valid for finite two-component slabs as long as the times considered are small compared with the diffusion time of the backing material.

Analytical results obtained from these solutions can be used to determine (a) the heat-transfer testing time for which the outer skin may be assumed to act as a calorimeter without exceeding a given error or (b) correction curves by which the indicated calorimeter heat-transfer coefficient may be multiplied to obtain the true heat-transfer coefficient. For such a correction curve to be valid, the bond between the two materials must have negligible thermal resistance, a condition difficult to attain if the slab is not composed of two metals.

Since the differential equation for the temperature distribution is linear, the principle of superposition is valid. Consequently, the problem of continuously varying equilibrium temperatures and heat-transfer coefficients may be treated by using the tabulated solutions and considering the continuous variation as a series of superimposed step functions.

INTRODUCTION

The increase in flight speeds for both operational and experimental missiles and airplanes has resulted in renewed interest in the problems involved in fluid- and solid-state heat conduction. The two problems of transient behavior of a two-component slab, consisting of an outer skin with a backing material, exposed to aerodynamic heating that have become of prime importance are: (1) the performance of the surface as measured by the ability of the material to withstand extreme heating rates on the full-scale vehicle during the course of its mission and (2) the transient testing techniques presently in wide use wherein the aerodynamic heating is varied and the resultant time-wise variation of the temperature of a skin, either with or without a backing, is then used to obtain heat-transfer rates and coefficients. For example, these transient testing techniques are used for shock tubes, free-flight pilotless rocket-propelled models, the sudden insertion of a model into a wind tunnel, or the rapid variation of the stagnation conditions in wind tunnels.

In the interest of simplicity, these techniques, except that used for the shock tube, customarily assume that the outer skin of the slab acts as a calorimeter and absorbs all the aerodynamic heat with negligible loss to the backing material. However, even when the backing material has very low conductivity, heat is continually being transferred across the interface and this heat transfer may introduce large errors in the answers obtained by the calorimeter assumption.

Two recent papers (refs. 1 and 2) treat the heat transfer to such a semi-infinite composite slab. Reference 1 is applicable to shock-tube testing wherein a very thin metal plating is formed on an insulator and the temperature response of the metal is used to determine the heat transfer. This study is applicable to arbitrary heat flux rates but assumes the plating to have such negligible thickness that the interface-temperature solution may be found as a perturbation to the surface solution in the absence of the plating. The solutions obtained are valid for times much greater than the diffusion time in the plating.¹ Reference 2 considers the case of the heating of a composite slab in which the heat transfer is proportional to the difference between wall and equilibrium

¹In the discussion of transient heating problems it is convenient to have reference values of time such as "diffusion time" and "relaxation time" for comparison purposes. The diffusion time is the quotient of the square of a typical length (for example, the thickness of a slab) divided by the thermal diffusivity of the material. The relaxation time, applicable in exponential decay behavior, is also called the time constant and is that value of time which makes the absolute value of the exponent of e unity.

temperatures. The equilibrium temperature is a step function. The outer skin has finite thickness but infinite conductivity; thus, the temperature in the skin is uniform at any given time. A solution is presented for the heat conduction to the backing material, no coupling between this loss of heat to the backing and the temperature distribution in the outer skin being assumed. This solution is valid for times that are small compared with the relaxation time of the outer skin. Reference 2 shows that conduction even to relatively good insulators introduces appreciable error.

The present investigation was initiated to obtain solutions for the heat transfer to the composite slab without these restrictions. The magnitude of the errors arising from the calorimetric assumption and possible correction factors were to be determined. When the thickness of the backing material is such that the diffusion time for the backing material is much larger than any of the testing times, the backing material then acts as if it were of infinite extent during the test. Consequently, solutions to the problem of a semi-infinite composite slab would apply under this condition. The problem considered in this report is the transient temperature distribution in a semi-infinite two-component slab subjected to aerodynamic heating at a rate equal to the product of a constant heat-transfer coefficient and the difference between surface and equilibrium temperatures. The equilibrium temperature is assumed to be a step function and the outer skin is assumed to have both finite conductivity and thickness. Solutions are found which are applicable for times varying from much less than to much greater than the skin diffusion time. The superposition principle permits extension of the results to arbitrary timewise variation in heat-transfer coefficient and equilibrium temperature. The investigation reported herein was conducted in the Gas Dynamics branch of the Langley Aeronautical Laboratory.

SYMBOLS

| | |
|-------------|---|
| a,b | constants |
| A,B,C,D | constants |
| c | specific heat |
| G_n | functions defined by equation (23) |
| $H = h/k$ | |
| h | heat-transfer coefficient |
| \hat{h} | indicated heat-transfer coefficient |
| \hat{h}_0 | indicated heat-transfer coefficient at $\xi = 0$; $\rho c l \frac{(dT/dt)_0}{T_e - T_0}$ |

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- \hat{h}_1 indicated heat-transfer coefficient at $\xi = 1$
- J coefficient of ξ in expression for P and equal to +1 or -1
- k thermal conductivity
- $L = \frac{\lambda h}{k_1} = \lambda H_1$
- L_n Laguerre polynomials (see appendix A)
- λ distance from surface to interface
- m, n integers
- P parameter in inverse Laplace transform
- p variable of the Laplace transform
- Q heat flux to surface per unit area per unit time
- $q = \sqrt{p/\alpha}$
- $s = p/4a^2$
- T temperature
- T_e equilibrium temperature
- T_0 temperature at $\xi = 0$
- T_{relax} relaxation temperature
- t_{relax} relaxation time
- t, t', u time variables
- x coordinate measured normal to surface exposed to heating
- α thermal diffusivity, $k/\rho c$
- $\beta = \frac{1 - \sigma}{1 + \sigma}$

$$\epsilon = \phi - 2.5$$

$$\Gamma_s \quad \text{solutions to } \Gamma_s \tan \Gamma_s = L$$

$$\lambda = \sqrt{L} \omega$$

$$\xi = x/l$$

ρ density of material

$$\sigma = \frac{k_2 q_2}{k_1 q_1} = \sqrt{\frac{\rho_2 c_2 k_2}{\rho_1 c_1 k_1}}$$

η, χ, τ dummy variables

$$\phi = \psi + L\omega$$

$$\psi = \frac{P}{2\omega}$$

$$\Omega = \phi^2 - \psi^2$$

$$\omega = \sqrt{\frac{\alpha_1 t}{l^2}}$$

\mathcal{L}^{-1} inverse Laplace transform

A bar over a symbol denotes the Laplace transform. Except as designated in the preceding list, the subscripts 1 or 2 refer to evaluation in region 1 or 2.

THEORY

General Solution

The problem to be solved is the one-dimensional transient temperature distribution in a semi-infinite two-component slab which has one face suddenly exposed to a fluid. This fluid will transfer heat to the exposed face at a rate proportional to the difference between the constant equilibrium temperature of the fluid and the outer surface temperature of the slab. In the interval $0 < x < l$, the slab is composed of a material with properties ρ_1 , c_1 , k_1 , and α_1 ; and, in the interval $l < x < \infty$, it is composed of a material with properties ρ_2 , c_2 , k_2 , and α_2 . No

thermal resistance is assumed at the interface in the slab. The mathematical formulation of this problem is given in terms of the following governing differential equations and boundary conditions (see fig. 1):

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial T_1}{\partial t} = 0 \quad (t > 0, \quad 0 < x < l) \quad (1)$$

$$\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial T_2}{\partial t} = 0 \quad (t > 0, \quad l < x < \infty) \quad (2)$$

$$T_1 = T_2 = 0 \quad (t < 0, \quad \text{all } x) \quad (3)$$

$$T_2 \rightarrow 0 \quad (t > 0, \quad x \rightarrow \infty) \quad (4)$$

$$k_1 \frac{\partial T_1}{\partial x} + h(T_e - T_1) = 0 \quad (x = 0, \quad t > 0) \quad (5)$$

$$T_1 = T_2 \quad (x = l, \quad t > 0) \quad (6)$$

$$k_1 \frac{\partial T_1}{\partial x} = k_2 \frac{\partial T_2}{\partial x} \quad (x = l, \quad t > 0) \quad (7)$$

The Laplace transforms $\bar{T}(p) = \int_0^{\infty} e^{-pt} T(t) dt$ and the definitions of $q \equiv \sqrt{p/\alpha}$ are then employed to reduce equations (1) to (7) to the following forms:

$$\frac{d^2 \bar{T}_1}{dx^2} - q_1^2 \bar{T}_1 = 0 \quad (0 < x < l) \quad (1a)$$

$$\frac{d^2 \bar{T}_2}{dx^2} - q_2^2 \bar{T}_2 = 0 \quad (l < x < \infty) \quad (2a)$$

$$\bar{T}_2 \rightarrow 0 \quad (x \rightarrow \infty) \quad (4a)$$

$$k_1 \frac{d\bar{T}_1}{dx} - h\bar{T}_1 + \frac{hT_e}{p} = 0 \quad (x = 0) \quad (5a)$$

$$\bar{T}_1 = \bar{T}_2 \quad (x = l) \quad (6a)$$

$$k_1 \frac{d\bar{T}_1}{dx} = k_2 \frac{d\bar{T}_2}{dx} \quad (x = l) \quad (7a)$$

The general solutions to equations 1(a) and 2(a) are:

$$\bar{T}_1 = Ae^{-q_1 x} + Be^{q_1 x} \quad (8)$$

$$\bar{T}_2 = Ce^{-q_2 x} + De^{q_2 x} \quad (9)$$

The value of D must be identically zero to satisfy equation 4(a). Application of the boundary conditions (eqs. 6(a) and 7(a)) yields

$$B = \beta Ae^{-2q_1 l} \quad (10)$$

$$C = (1 + \beta)Ae^{-q_1 l + q_2 l} \quad (11)$$

The remaining constant A is then found from equation 5(a) to be

$$A = \frac{H_1 T_e}{p} \frac{1}{q_1 + H_1} \left(1 - \beta \frac{q_1 - H_1}{q_1 + H_1} e^{-2q_1 l} \right)^{-1} \quad (12)$$

The solutions to the differential equations in the transformed plane become

$$\frac{\bar{T}_1}{T_e} = \frac{H_1}{p} \frac{1}{q_1 + H_1} \frac{1}{1 - \beta \frac{q_1 - H_1}{q_1 + H_1} e^{-2q_1 l}} \left[e^{-q_1 x} + \beta e^{-q_1 (2l-x)} \right] \quad (0 < x < l) \quad (13)$$

$$\frac{\bar{T}_2}{T_e} = \frac{H_1}{P} \frac{1}{q_1 + H_1} \frac{1 + \beta}{1 - \beta \frac{q_1 - H_1}{q_1 + H_1} e^{-2q_1 l}} e^{-q_1 \left[l + \frac{k_1}{k_2} \frac{1 - \beta}{1 + \beta} (x - l) \right]} \quad (x > l) \quad (14)$$

The inverse Laplace transform $\mathcal{L}^{-1} \left(\frac{\bar{T}_1}{T_e} \right)$ of equation (13) is known for the special case $\sigma = 0$, $\beta = 1$ which corresponds to a perfect insulator as the backing material. If the exponentials of equation (13) are expressed in hyperbolic form, the inversion on page 259 of reference 3 applies.

$$\mathcal{L}^{-1} \left(\frac{\bar{T}_1}{T_e} \right) = \frac{T_1}{T_e} = 2L \sum_{s=1}^{\infty} \frac{1}{\Gamma_s^2 + L^2 + L} \frac{\cos \Gamma_s (1 - \xi)}{\cos \Gamma_s} \left(1 - e^{-\frac{\Gamma_s^2 \alpha_1 t}{L^2}} \right) \quad (15)$$

where Γ_s are the roots of the equation

$$\Gamma_s \tan \Gamma_s = L \quad (16)$$

If the slab is composed entirely of the same material ($\beta = 0$; $k_1 = k_2$), equations (13) and (14) become identical and have as their transform

$$\frac{T}{T_e} = \operatorname{erfc} \frac{x}{2\sqrt{\alpha t}} - e^{H_1 x + \alpha H_1^2 t} \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} + H_1 \sqrt{\alpha t} \right) \quad (17)$$

For the general case of $\sigma \neq 0$, $\beta \neq 1$, the inverse transform of \bar{T}/T_e as expressed in the form of equations (13) and (14) is neither known nor easily determined. Recourse is then made to expansion of the right-hand side of these equations. (Note that $|\beta| \leq 1$.) An expansion of this type generally results in solutions which will converge more rapidly for small values of time.

Since

$$\left(1 - \beta \frac{q_1 - H_1}{q_1 + H_1} e^{-2q_1 l}\right)^{-1} = \sum_{n=0}^{\infty} \left(\beta \frac{q_1 - H_1}{q_1 + H_1} e^{-2q_1 l}\right)^n \quad (18)$$

substitution of equation (18) into equations (13) and (14) yields

$$\frac{\bar{T}_1}{T_e} = H_1 \sum_{n=0}^{\infty} \frac{\beta^n}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l(2n+\xi)} + \frac{\beta^{n+1}}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l[2(n+1)-\xi]} \quad (19)$$

$$\frac{\bar{T}_2}{T_e} = H_1(1 + \beta) \sum_{n=0}^{\infty} \frac{\beta^n}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l \left[2n+1 + \frac{k_1}{k_2} \frac{1-\beta}{1+\beta} (\xi-1)\right]} \quad (20)$$

If the orders of summation and integration of the inversion process are assumed to be interchangeable, then

$$\mathcal{L}^{-1}\left(\frac{\bar{T}_1}{T_e}\right) = H_1 \sum_{n=0}^{\infty} \mathcal{L}^{-1} \left\{ \frac{\beta^n}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l(2n+\xi)} + \frac{\beta^{n+1}}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l[2(n+1)-\xi]} \right\} \quad (21)$$

$$\mathcal{L}^{-1}\left(\frac{\bar{T}_2}{T_e}\right) = H_1(1 + \beta) \sum_{n=0}^{\infty} \mathcal{L}^{-1} \left\{ \frac{\beta^n}{p} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l \left[2n+1 + \frac{k_1}{k_2} \frac{1-\beta}{1+\beta} (\xi-1)\right]} \right\} \quad (22)$$

Let the function $G_n(P, \omega, L)$ be defined as follows:

$$G_n(P, \omega, L) \equiv \frac{2}{\sqrt{\pi}} \int_{P/2\omega}^{\infty} e^{-x^2} \int_0^{2\omega L \left(x - \frac{P}{2\omega}\right)} e^{-\eta} I_n(2\eta) d\eta dx \quad (23)$$

In appendix A it is shown that the inversion of the general term of the series has the form

$$\mathcal{L}^{-1} \left[\frac{H_1}{P} \frac{(q_1 - H_1)^n}{(q_1 + H_1)^{n+1}} e^{-q_1 l P} \right] = G_n(P, \omega, L) \quad (24)$$

Substitution of equation (24) into equations (21) and (22) produces

$$\frac{T_1}{T_e} = \sum_{n=0}^{\infty} \beta^n G_n(2n+\xi, \omega, L) + \beta^{n+1} G_n[2(n+1)-\xi, \omega, L] \quad (25)$$

$$\frac{T_2}{T_e} = (1 + \beta) \sum_{n=0}^{\infty} \beta^n G_n \left[2n+1 + \frac{k_1}{k_2} \frac{1-\beta}{1+\beta} (\xi-1), \omega, L \right] \quad (26)$$

The formal integrations required for the evaluation of G_n for a range of values of n from 0 to 5 are given in appendix B. The associated timewise and spacewise derivatives $\left(\frac{\partial G_n}{\partial \omega^2} \text{ and } \frac{\partial G_n}{\partial \xi} \right)$ are also presented. These derivatives permit the evaluation of both the correction curves to the indicated calorimetric heat-transfer coefficient and the amount of heat crossing a given station, in order that estimate of the flux through the backing material may be obtained.

Significance of parameters L , λ , and β

The physical significance of the nondimensional parameters L , λ , and β will be briefly discussed at this point. The parameter $L = \frac{lh}{k_1}$ is proportional to the ratio of the temperature difference required to transport a given amount of heat per unit area across the slab to the temperature difference required to transport the same amount of heat per

unit area from the fluid to the slab. This may be illustrated for the steady-flow case as follows:

$$Q = h [T_e - T(0)] = k_1 \frac{T(0) - T(1)}{l}$$

Therefore,

$$\frac{T(0) - T(1)}{T_e - T(0)} = \frac{hl}{k_1} = L$$

Now if L is small, a small temperature difference across the outer skin is indicated; thus, the surface temperature and heat transfer when $T(0) \rightarrow T_e$ will be sensitive to changes in the temperature at the interface. Since the interface temperature is in turn influenced strongly by the backing material, the relative influence of the backing material increases with a decrease in L for times of the order of the relaxation time. (For very large times the backing material must always be the dominating factor.) The significance of L may be realized in an alternate way by considering that, when L is small owing to a thin skin, high thermal conductivity, or a low heat-transfer coefficient, a greater portion of the heat transferred to the surface is available for transfer to the backing material and the backing material can exert a larger influence on the temperature time history of the skin for finite times. A plot of the parameter L obtained for various skin thickness and material combinations with $h = 0.1$ Btu/sq ft-sec- $^{\circ}$ F is shown in figure 2. A range of L from 0.001 to 0.5 appears to cover practical outer skins which might be used for high-speed vehicles.

The significance of the parameter $\lambda = \sqrt{L} \omega = \sqrt{\frac{ht}{\rho cl}}$ is most easily described from consideration of the aerodynamic heating of a slab of thickness l having infinite conductivity ($\partial T / \partial x = 0$). The slab acts as a calorimeter absorbing all the heat input. This slab is initially at $T = 0$ and is subjected to a step input in equilibrium temperature at $t = 0$. The temperature of the slab at time t may be expressed as

$$\frac{T}{T_e} = 1 - e^{-\frac{\omega l}{l^2} t} = 1 - e^{-\lambda^2 t}$$

Thus, the value $\lambda = 1$ corresponds to the relaxation time in such a slab calorimeter.

For a perfectly insulated skin having finite conductivity and thickness, the temperature distribution departs from that of the calorimeter

as L increases from zero. However, at the value $\lambda = 1$, the temperature is approximately the relaxation value given by $T_{\text{relax}} = T_e \left(1 - \frac{1}{e}\right)$. Since $t \propto \lambda^2$, the ratio of any time t to the relaxation time t_{relax} is simply λ^2 .

The parameter β denotes the relative insulating perfection of the backing material. A value of $\beta = 1$ applies to a perfect insulator; $\beta = 0$ describes the homogenous composite slab constructed of the same material throughout; and $\beta = -1$ describes a perfect conductor which maintains a constant temperature at the interface at $\xi = 1$. Values of β appropriate to various combinations of outer skin and backing material are shown in figure 3. The outer skin materials have common abscissas and the intersection of these vertical lines with the curves for the backing materials determine the appropriate value of β . If the materials for skin and backing are interchanged, the sign of β is reversed.

Particular Solutions

Since many aerodynamic heat-transfer experiments are performed with a model which has a skin of high conductivity and density compared with that of the backing material, it is desirable to have a form of solution applicable to these circumstances. The solution for a perfect insulator backing material is either the series given in equation (15) or that given in equation (25) when a value of $\beta = 1$ is used. These alternate solutions converge most rapidly at opposite times. The number of terms required to express the timewise derivative for equation (15) is large as $t \rightarrow 0$ and decreases rapidly with an increase in time; thus for values of t greater than about one-half the relaxation time, only one or two terms are required to give the value of T_1/T_e and its timewise and spacewise derivatives with acceptable accuracy. Conversely, although one term is sufficient for equation (25) when $t \rightarrow 0$, the number of terms required increases with time. A series that will converge rapidly for the times of interest for aerodynamic testing purposes when $\beta \approx 1$ may be obtained by applying the solution of equation (25) as a perturbation to the solution of equation (15). If T_1/T_e for $\beta = 1.0$ is added and subtracted to the right-hand side of equation (25), the resulting equation is

$$\frac{T_1}{T_e} = \left(\frac{T_1}{T_e}\right)_{\beta=1} - \sum_{n=0}^{\infty} \left\{ (1 - \beta^n) G_n(2n+\xi, \omega, L) + (1 - \beta^{n+1}) G_n[2(n+1)-\xi, \omega, L] \right\} \quad (27)$$

where $\left(\frac{T_1}{T_e}\right)_{\beta=1}$ is evaluated from equation (15) for the appropriate values of L , ω , and ξ .

Of particular interest is the temperature history on the surface exposed to the transferring fluid ($\xi = 0$) and that on the interface ($\xi = 1$) between the two different materials. Substitution of $\xi = 0$ into equation (27) produces the following set of equations:

$$\frac{T_1}{T_e}(\xi=0, \omega, L) = \left(\frac{T_1}{T_e}\right)_{\beta=1} - \left\{ \sum_{n=0}^{\infty} (1 - \beta^n) G_n(2n, \omega, L) + \sum_{n=1}^{\infty} (1 - \beta^n) G_{n-1}(2n, \omega, L) \right\} \quad (28)$$

$$\frac{\partial T_1}{\partial \omega^2}(\xi=0, \omega, L) = \left(\frac{\partial T_1}{\partial \omega^2}\right)_{\beta=1} - \left\{ \sum_{n=0}^{\infty} (1 - \beta^n) \frac{\partial G_n}{\partial \omega^2}(2n, \omega, L) + \sum_{n=1}^{\infty} (1 - \beta^n) \frac{\partial G_{n-1}}{\partial \omega^2}(2n, \omega, L) \right\} \quad (29)$$

$$\frac{\partial T_1}{\partial \xi}(\xi=0, \omega, L) = \left(\frac{\partial T_1}{\partial \xi}\right)_{\beta=1} - \left\{ \sum_{n=0}^{\infty} (1 - \beta^n) \left[\frac{\partial G_n}{\partial \xi}(2n, \omega, L) \right]_{j=1} - \sum_{n=1}^{\infty} (1 - \beta^n) \left[\frac{\partial G_{n-1}}{\partial \xi}(2n, \omega, L) \right]_{j=1} \right\} \quad (30)$$

A corresponding set of equations may be obtained for the interface by substituting $\xi = 1$ into equation (27):

$$\frac{T_1}{T_e}(\xi=1, \omega, L) = \left(\frac{T_1}{T_e}\right)_{\beta=1} - \sum_{n=0}^{\infty} [2 - \beta^n(1 + \beta)] G_n(2n+1, \omega, L) \quad (31)$$

$$\frac{\partial T_1}{\partial \omega^2}(\xi=1, \omega, L) = \left(\frac{\partial T_1}{\partial \omega^2}\right)_{\beta=1} - \sum_{n=0}^{\infty} [2 - \beta^n(1 + \beta)] \frac{\partial G_n}{\partial \omega^2}(2n+1, \omega, L) \quad (32)$$

$$\frac{\partial T_1}{\partial \xi}(\xi=1, \omega, L) = (1 - \beta) \sum_{n=0}^{\infty} \beta^n \left[\frac{\partial G_n}{\partial \xi}(2n+1, \omega, L) \right]_{j=1} \quad (33)$$

The evaluation of the terms of equations (25) and (26) becomes progressively more difficult as n increases. Consequently, if the

temperature at the interface ($\xi = 1$) has to be evaluated for any reason or if the values tabulated in this report are to be used, then the temperature distribution in the backing material can advantageously be obtained by Duhamel's method.

Since the temperature distribution in a semi-infinite slab ($l < x < \infty$) when one surface at $x = l$ undergoes a unit-step-function increase in temperature is (ref. 3)

$$T_2(x,t) = \text{erfc} \frac{x-l}{2\sqrt{\alpha_2 t}} \quad (34)$$

Duhamel's technique yields

$$\frac{T_2(x,t)}{T_e} = \frac{(x-l)(1+\beta)}{2\sqrt{\pi\alpha_2}} \int_0^t \frac{e^{-\frac{(x-l)^2}{4\alpha_2(t-\tau)}}}{(t-\tau)^{3/2}} \left[\sum_{n=0}^{\infty} \beta^n G_n \left(2n+1, \frac{\alpha_1 \tau}{l^2}, L \right) \right] d\tau \quad (35)$$

RESULTS AND DISCUSSION

General Results

The values of G_n and its derivatives for $n = 0$ to 5 and $\xi = 0$ and 1 were computed on a card-programed calculator. Details of this procedure are discussed in appendix C. The results of these computations are given in table I for values of L between 0.001 and 0.5 with $0.02 \leq \lambda \leq 8$. Table I is arranged for future computing convenience. Values on the left for $\xi = 0$ are aligned so that pairs in G or its derivatives on the same horizontal line are operated on by the same power of β as prescribed by equations (28) to (30).

As discussed in appendix C the computing procedure at times involved the subtraction of two large and nearly equal numbers obtained by approximations. Certain voids appear in table I where an inspection of the machine results indicated insufficient accuracy. There was no rigorous rule for determining which answers to discard, but all answers with less than four significant figures were usually eliminated. Those with less than three significant figures were always eliminated.

Typical values of G_n and its associated derivatives which are required at $\xi = 0$ are plotted against L for $\lambda = 1.0$. (See fig. 4.) The values shown are those needed in the evaluation of the first series in the braces of equations (28) to (30). It is immediately obvious that

the convergence of the series is much more rapid for large values of L than for small values of L . For example, at $L = 0.5$ the ratio of G_1 to G_0 is of the order of 10^{-6} whereas at $L = 0.001$ the ratio is of the order of one.

An indication of the accuracy which results from consideration of a finite number of terms to represent the infinite series of equation (25) may be obtained from figure 5. In this figure are plotted the values of the ratio of T_1/T_e for $\beta = 1$ when the upper limit of summation in equation (25) is reduced from infinity to 0, 1, 2, 3, 4, or 5 to that obtained from table II. (Table II contains the values of T_1/T_e as determined from equation (15) by using a sufficient number of terms to get the desired accuracy.) The ratio for values of $L = 0.001, 0.1,$ and 0.5 is plotted as a function of λ . The number of terms required to reduce the error to a given limit is seen to increase with an increase in λ at a constant L and to increase with a decrease in L at a constant λ . For a constant L , the behavior of T_1/T_e for any finite number of terms becomes oscillatory as λ increases. The errors for $0 \leq \beta < 1$ at a given λ and L should be less than those indicated in figure 5 for $\beta = 1$ since the terms neglected contain β^n or β^{n+1} . In other words, the neglected terms for $0 \leq \beta < 1$ have a smaller contribution than those for $\beta = 1.0$.

Illustrative Example

The effects of the backing material in a composite slab under typical transient test conditions will be illustrated by the following study. The numerical results obtained are found by using the values of table I for $0 \leq n \leq 5$ and those of table II. The mathematical assumption of an infinite extent of backing material to represent the finite backing of the test is valid for values of β near unity as long as the times considered are much less than the diffusion time of the backing. The diffusion times, which are equal to the square of the thickness of the backing material divided by the diffusivity of the backing material, for 1/4-inch-thick balsa and mahogany are approximately 120 and 170 seconds, respectively. Consider two stainless-steel outer skins ($k = 0.0028$ Btu/sec-ft- $^{\circ}$ F, $\alpha = 5.2 \times 10^{-5}$ sq ft/sec) with thicknesses of 0.060 inch and 0.030 inch backed by either a perfect insulator ($\beta = 1.000$), balsa ($\beta \approx 0.975$), or mahogany ($\beta \approx 0.950$). These composite slabs are subjected to a step input in equilibrium temperature (for example, by sudden exposure to the airstream of a wind tunnel) and the heat-transfer coefficient h is 0.056 Btu/sq ft-sec- $^{\circ}$ F. The values of L are then 0.10 and 0.05 for the 0.060- and 0.030-inch thicknesses, respectively.

The resulting behavior of the outer skin is illustrated in figure 6 which shows the temperature distribution at the outer face ($\xi = 0$) as a function of time. The effect of the heat lost to the backing material is evident in a lower temperature in the outer skin at a given time, a larger effect being observed for the thinner skin.

An indicated heat-transfer coefficient is often obtained from the temperature-time history of such an experiment. This heat-transfer coefficient is determined by assuming that the slab acts as a calorimeter with an infinite value of k ; that is,

$$\hat{h} \equiv \rho c l \frac{dT/dt}{T_e - T}$$

Since the value of \hat{h} depends on the value of ξ at which T is measured for finite k , the term \hat{h}_0 or \hat{h}_1 will be used to designate the values of \hat{h} which would be found when T and dT/dt are evaluated at $\xi = 0$ or l .

The ratio of the indicated heat-transfer coefficient to the true heat-transfer coefficient \hat{h}/h is plotted as a function of time in figures 7 and 8. Figure 7 shows the ratio at $\xi = 0$ and $l.0$ for the 0.030-inch-thick skin. Except for the initial discrepancy at early times the values of \hat{h}_0 and \hat{h}_1 are nearly equal. At $t = 0$, \hat{h}_0 is infinite while \hat{h}_1 is zero, but after about 0.1 second the difference is minor. Figure 8 shows the ratio \hat{h}_0/h as a function of time for both the 0.030-inch and 0.060-inch skin. It is evident that $\hat{h}_0 = h$ only at a single discrete point for each combination of skin thickness and backing material. Furthermore, even the perfect insulator does not give a ratio of unity for \hat{h}/h except at one value of time. It may be shown for the perfect insulator that the asymptotic value of \hat{h}/h is

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\hat{h}}{h} &= \frac{\Gamma_1^2}{L} \\ &= 1 - \frac{1}{3} L \frac{1 + \frac{2}{5} L}{1 + \frac{2}{3} L + \frac{2}{5} L^2} + \dots \\ &\approx 1 - \frac{1}{3} L \quad (L \ll 1) \end{aligned}$$

Thus even the perfect insulator gives errors of 5/3 and 10/3 percent for the cases considered (0.030-inch and 0.060-inch-thick skins). The large increase with time of the deviation of \hat{h}/h from unity for the imperfect insulators is striking. This error increases with time because, as the heat transferred into the skin at $\xi = 0$ decreases monotonically with time, the heat transferred out at $\xi = 1$ first rises with time to a maximum (at approximately $\lambda = 1.0$) and then subsequently decreases but at a rate slower than that at $\xi = 0$. An approximate method for estimating the amount of heat conducted across the interface for the conditions of $\lambda \ll 1$ and $k \rightarrow \infty$ was also presented in reference 2.

If an error of 10 percent were set on the deviation of indicated to true heat-transfer coefficient, the time limit for useful data with the respective backings composed of mahogany and balsa would be about 1 second and 2.5 seconds when $l = 0.030$ inch and 2.4 seconds and 5.7 seconds when $l = 0.060$ inch. If, instead of using a time limit, a temperature rise limit for a 10-percent error were desired, it may be obtained from a plot similar to that of figure 9. For $l = 0.030$ inch, temperature rises greater than 30 percent of T_e for a mahogany backing or 65 percent of T_e for a balsa backing would give values of \hat{h}/h deviating from unity by more than 10 percent. Approximately the same limits also apply for $l = 0.060$ inch.

CONCLUDING REMARKS

A solution has been obtained to the transient temperature distribution in a semi-infinite two-component composite slab of arbitrary materials subjected to an instantaneous application of aerodynamic heating with constant equilibrium temperature and heat-transfer coefficient. The numerical results are tabulated in a form to permit easy computation of heat-transfer problems typical of aerodynamic testing. The solutions are valid for finite two-component slabs as long as the times considered are small compared with the diffusion time of the backing material.

Analytical results obtained from these solutions can be used to determine (a) the heat-transfer testing time for which the outer skin may be assumed to act as a calorimeter without exceeding a given error or (b) correction curves by which the indicated calorimeter heat-transfer coefficient may be multiplied to obtain the true heat-transfer coefficient. For such a correction curve to be valid, the bond between the two materials must have negligible thermal resistance, a condition difficult to attain if the slab is not composed of two metals.

Since the differential equation for the temperature distribution is linear, the principle of superposition is valid. Consequently, the problem

of continuously varying equilibrium temperatures and heat-transfer coefficients may be treated by using the tabulated solutions and considering the continuous variation as a series of superimposed step functions.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 4, 1958.

APPENDIX A

INVERSION OF THE LAPLACE TRANSFORM

The steps required to invert the general terms of the series of equations (21) and (22) will be determined in this section. The inversion of the expression

$$\mathcal{L}^{-1} \left[\frac{ae^{-b\sqrt{p}}}{p} \frac{(\sqrt{p}-a)^n}{(\sqrt{p}+a)^{n+1}} \right] = \mathcal{L}^{-1} \left[\frac{e^{-2ab\sqrt{\frac{p}{4a^2}} \left(2\sqrt{\frac{p}{4a^2}} - 1\right)^n}{4a^2 \left(\frac{p}{4a^2}\right) \left(2\sqrt{\frac{p}{4a^2}} + 1\right)^{n+1}} \right] \quad (A1)$$

will first be found and then the constants a and b will be assigned pertinent values.

Let $s = \frac{p}{4a^2}$. It is then necessary to find the inversion for

$$\mathcal{L}^{-1} \left[\frac{e^{-2ab\sqrt{s}}}{4a^2 s} \frac{(2\sqrt{s}-1)^n}{(2\sqrt{s}+1)^{n+1}} \right] = \mathcal{L}^{-1} \left[\frac{1}{4a^2} \frac{1}{\sqrt{s}} \frac{e^{-2ab\sqrt{s}}}{\sqrt{s}} \frac{(2\sqrt{s}-1)^n}{(2\sqrt{s}+1)^{n+1}} \right] \quad (A2)$$

The square-root image relation (see page 123 of ref. 4) can be used to evaluate equation (A2). Thus if

$$f(t') = \mathcal{L}^{-1} [g(s)]$$

then

$$\frac{1}{\sqrt{\pi t'}} \int_0^\infty e^{-\frac{u^2}{4t'}} f(u) du = \mathcal{L}^{-1} \left[\frac{1}{\sqrt{s}} g(\sqrt{s}) \right] \quad (A3)$$

and the evaluation of the following expression is required:

$$\mathcal{L}^{-1} \left[\frac{e^{-2abs}}{s} \frac{(2s-1)^n}{(2s+1)^{n+1}} \right]$$

The term

$$\left. \begin{aligned} \mathcal{L}^{-1}\left(\frac{e^{-2abs}}{s}\right) &= 0 && (0 < \tau < 2ab) \\ \mathcal{L}^{-1}\left(\frac{e^{-2abs}}{s}\right) &= 1 && (2ab < \tau) \end{aligned} \right\} \quad (A4)$$

and

$$\mathcal{L}^{-1}\left[\frac{(2s - 1)^n}{(2s + 1)^{n+1}}\right] = \frac{1}{2} e^{-\tau/2} L_n(\tau) \quad (A5)$$

(See page 298 of ref. 5 and page 129 of ref. 4.) The symbol $L_n(\tau)$ designates Laguerre polynomials of the following form:

$$\left. \begin{aligned} L_n(\tau) &= \sum_{m=0}^n \binom{n}{n-m} \frac{(-\tau)^m}{m!} \\ L_n(\tau) &= \sum_{m=0}^n \frac{n!}{(n-m)! (m!)^2} (-\tau)^m \end{aligned} \right\} \quad (A6)$$

Application of the convolution theorem to equations (A4) and (A5) results in:

$$\begin{aligned} \mathcal{L}^{-1}\left[\frac{e^{-2abs}}{s} \frac{(2s - 1)^n}{(2s + 1)^{n+1}}\right] &= \frac{1}{2} \int_0^{u-2ab} e^{-\tau/2} L_n(\tau) (1) d\tau + \\ &\quad \frac{1}{2} \int_{u-2ab}^u e^{-\tau/2} L_n(\tau) (0) d\tau \\ &= \frac{1}{2} \int_0^{u-2ab} e^{-\tau/2} L_n(\tau) d\tau \end{aligned} \quad (A7)$$

Combination of equation (A7) and the image formula (A3) yields

$$\mathcal{L}^{-1} \left\{ \frac{1}{\sqrt{s}} \left[\frac{e^{-2ab\sqrt{s}}}{\sqrt{s}} \frac{(2\sqrt{s} - 1)^n}{(2\sqrt{s} + 1)^{n+1}} \right] \right\} = \frac{1}{\sqrt{\pi t'}} \int_{2ab}^{\infty} e^{-\frac{u^2}{4t'}} \int_0^{u-2ab} \frac{1}{2} e^{-\tau/2} L_n(\tau) d\tau du \quad (A8)$$

For $s = cp$, $ct' = t$, it may be shown that, if

$$\mathcal{L}^{-1} [g(s)] = f(t')$$

then

$$\mathcal{L}^{-1} [g(pc)] = \frac{1}{c} f\left(\frac{t}{c}\right) \quad (A9)$$

Consequently, since $c = \frac{1}{4a^2}$, equation (A8) may be operated on to yield:

$$\mathcal{L}^{-1} \left\{ \frac{1}{\sqrt{\frac{p}{4a^2}}} \left[\frac{e^{-2ab\sqrt{\frac{p}{4a^2}}}}{\sqrt{\frac{p}{4a^2}}} \frac{\left(2\sqrt{\frac{p}{4a^2}} - 1\right)^n}{\left(2\sqrt{\frac{p}{4a^2}} + 1\right)^{n+1}} \right] \right\} = \frac{4a^2}{2\sqrt{\pi 4a^2 t}} \int_{2ab}^{\infty} e^{-\frac{u^2}{16a^2 t}} \int_0^{u-2ab} e^{-\frac{\tau}{2}} L_n(\tau) d\tau du$$

or

$$\mathcal{L}^{-1} \left[a \frac{e^{-b\sqrt{p}}}{p} \frac{(\sqrt{p} - a)^n}{(\sqrt{p} + a)^{n+1}} \right] = \frac{1}{4a\sqrt{\pi t}} \int_{2ab}^{\infty} e^{-\frac{u^2}{16a^2 t}} \int_0^{u-2ab} e^{-\frac{\tau}{2}} L_n(\tau) d\tau du \quad (A10)$$

If $a = H \sqrt{\alpha}$ and $b = \frac{Pl}{\sqrt{\alpha}}$, equation (A10) becomes

$$\mathcal{L}^{-1} \left[\frac{e^{-lPq}}{P} \frac{(q-H)^n}{(q+H)^{n+1}} \right] = \frac{1}{4H_1} \frac{1}{\sqrt{\pi\alpha t}} \int_{2lHP}^{\infty} e^{-\frac{u^2}{16H^2\alpha t}} \int_0^{u-2lHP} e^{-\tau/2} L_n(\tau) d\tau du \quad (A11)$$

If

$$\eta = \tau/2$$

and

$$\begin{aligned} x &= \frac{u}{4lH\sqrt{\frac{\alpha t}{l^2}}} \\ &= \frac{u}{4l\omega} \end{aligned}$$

are substituted in the right-hand side of equation (A11), the following equations result:

$$\begin{aligned} \mathcal{L}^{-1} \left[\frac{e^{-lPq_1}}{P} \frac{(q_1-H_1)^n}{(q_1+H_1)^{n+1}} \right] &= \frac{2}{\sqrt{\pi}} \int_{P/2\omega}^{\infty} e^{-x^2} \int_0^{2l\omega(x-\frac{P}{2\omega})} e^{-\eta} L_n(2\eta) d\eta dx \\ &\equiv G_n(P, \omega, L) \end{aligned} \quad (A12)$$

APPENDIX B

EXPRESSIONS FOR THE VARIOUS G_n , $\frac{\partial G_n}{\partial \omega^2}$, AND $\frac{\partial G_n}{\partial \xi}$

FOR VALUES OF n FROM 0 TO 5

Formal integration of the equations for the G_n for values of n from 0 to 5 and subsequent partial differentiation with respect to ω^2 and ξ produced the following expressions:

$$G_0 = \operatorname{erfc} \psi - e^{\Omega} \operatorname{erfc} \phi$$

$$\frac{\partial G_0}{\partial \xi} = -jLe^{\Omega} \operatorname{erfc} \phi$$

$$\frac{\partial G_0}{\partial \omega^2} = \frac{1}{\sqrt{\pi}} \frac{L^{3/2}}{\lambda} e^{-\psi^2} - L^2 e^{\Omega} \operatorname{erfc} \phi$$

$$G_1 = -\operatorname{erfc} \psi + (1 - 4\lambda\sqrt{L}\phi) e^{\Omega} \operatorname{erfc} \phi + \frac{4}{\sqrt{\pi}} \lambda\sqrt{L} e^{-\psi^2}$$

$$\frac{\partial G_1}{\partial \xi} = -Lj(1 + 4\lambda\sqrt{L}\phi) e^{\Omega} \operatorname{erfc} \phi + j \frac{4}{\sqrt{\pi}} \lambda L^{3/2} e^{-\psi^2}$$

$$\frac{\partial G_1}{\partial \omega^2} = -L^2(3 + 4\lambda\sqrt{L}\phi) e^{\Omega} \operatorname{erfc} \phi + \frac{L^{3/2}}{\sqrt{\pi}} \left(\frac{1}{\lambda} + 4\lambda L \right) e^{-\psi^2}$$

$$G_2 = \operatorname{erfc} \psi - (1 + 8\lambda^2 L \phi^2 + 4\lambda^2 L) e^{\Omega} \operatorname{erfc} \phi + \left(\frac{8}{\sqrt{\pi}} \lambda^2 L \phi \right) e^{-\psi^2}$$

$$\frac{\partial G_2}{\partial \xi} = -Lj(1 + 8\lambda\sqrt{L}\phi + 8\lambda^2 L \phi^2 + 4\lambda^2 L) e^{\Omega} \operatorname{erfc} \phi + \frac{8j}{\sqrt{\pi}} \lambda L^{3/2} (1 + \lambda\sqrt{L}\phi) e^{-\psi^2}$$

$$\frac{\partial G_2}{\partial \omega^2} = -L^2(5 + 16\lambda\sqrt{L}\phi + 8\lambda^2 L \phi^2 + 4\lambda^2 L) e^{\Omega} \operatorname{erfc} \phi +$$

$$\frac{L}{\sqrt{\pi}} \left[\frac{\sqrt{L}}{\lambda} + 8\lambda L^{3/2} (2 + \lambda\sqrt{L}\phi) \right] e^{-\psi^2}$$

$$G_3 = -\operatorname{erfc} \psi - e^{\Omega} \operatorname{erfc} \phi \left(-1 + 4\lambda\sqrt{L}\phi + 8\lambda^2 L \phi^2 + 4\lambda^2 L + \frac{32}{3} \lambda^3 L^{3/2} \phi^3 + \right. \\ \left. 16\lambda^3 L^{3/2} \phi \right) + \frac{1}{\sqrt{\pi}} \left(4\lambda\sqrt{L} + 8\lambda^2 L \phi + \frac{32}{3} \lambda^3 L^{3/2} \phi^2 + \frac{32}{3} \lambda^3 L^{3/2} \right) e^{-\psi^2}$$

$$\frac{\partial G_3}{\partial \xi} = -e^{\Omega} \operatorname{erfc} \phi j \left(L + 12\lambda L^{3/2} \phi + 24\lambda^2 L^2 \phi^2 + 12\lambda^2 L^2 + \frac{32}{3} \lambda^3 L^{5/2} \phi^3 + \right. \\ \left. 16\lambda^3 L^{5/2} \phi \right) + j \frac{1}{\sqrt{\pi}} \left(12\lambda L^{3/2} + 24\lambda^2 L^2 \phi + \frac{32}{3} \lambda^3 L^{5/2} + \frac{32}{3} \lambda^3 L^{5/2} \phi^2 \right) e^{-\psi^2}$$

$$\frac{\partial G_3}{\partial \omega^2} = e^{-\psi^2} \frac{1}{\sqrt{\pi}} \left(\frac{L^{3/2}}{\lambda} + 36\lambda L^{5/2} + 40\lambda^2 L^3 \phi + \frac{32}{3} \lambda^3 L^{7/2} \phi^2 + \frac{32}{3} \lambda^3 L^{7/2} \right) - \\ e^{\Omega} \operatorname{erfc} \phi \left(7L^2 + 36\lambda L^{5/2} \phi + 40\lambda^2 L^3 \phi^2 + 20\lambda^2 L^3 + \frac{32}{3} \lambda^3 L^{7/2} \phi^3 + \right. \\ \left. 16\lambda^3 L^{7/2} \phi \right)$$

$$G_4 = \operatorname{erfc} \psi - e^{\Omega} \operatorname{erfc} \phi \left(1 + 16\lambda^2 L \phi^2 + 8\lambda^2 L + \frac{64}{3} \lambda^3 L^{3/2} \phi^3 + \frac{32}{3} \lambda^4 L^2 \phi^4 + \right. \\ \left. 32\lambda^4 L^2 \phi^2 + 8\lambda^4 L^2 + 32\lambda^3 L^{3/2} \phi \right) + \frac{1}{3\sqrt{\pi}} \left(48\lambda^2 L \phi + 64\lambda^3 L^{3/2} \phi^2 + 64\lambda^3 L^{3/2} + \right. \\ \left. 32\lambda^4 L^2 \phi^3 + 80\lambda^4 L^2 \phi \right) e^{-\psi^2}$$

$$\frac{\partial G_4}{\partial \xi} = e^{-\psi^2} \frac{j}{3\sqrt{\pi}} \left(48\lambda L^{3/2} + 144\lambda^2 L^2 \phi + 128\lambda^3 L^{5/2} + 128\lambda^3 L^{5/2} \phi^2 + 32\lambda^4 L^3 \phi^3 + \right. \\ \left. 80\lambda^4 L^3 \phi \right) - e^{\Omega} \operatorname{erfc} \phi j \left(16\lambda L^{3/2} \phi + 48\lambda^2 L^2 \phi^2 + 24\lambda^2 L^2 + \frac{128}{3} \lambda^3 L^{5/2} \phi^3 + \right. \\ \left. 64\lambda^3 L^{5/2} \phi + \frac{32}{3} \lambda^4 L^3 \phi^4 + 32\lambda^4 L^3 \phi^2 + 8\lambda^4 L^3 + L \right)$$

$$\frac{\partial G_4}{\partial \omega^2} = e^{-\psi^2} \frac{1}{\sqrt{\pi}} \left(\frac{1}{\lambda} L^{3/2} + 64\lambda L^{5/2} + 112\lambda^2 L^3 \phi + 64\lambda^3 L^{7/2} \phi^2 + 64\lambda^3 L^{7/2} + \right. \\ \left. \frac{32}{3} \lambda^4 L^4 \phi^3 + \frac{80}{3} \lambda^4 L^4 \phi \right) - e^{\Omega} \operatorname{erfc} \phi \left(9L^2 + 64\lambda L^{5/2} \phi + 112\lambda^2 L^3 \phi^2 + \right. \\ \left. 56\lambda^2 L^3 + 64\lambda^3 L^{7/2} \phi^3 + 96\lambda^3 L^{7/2} \phi + \frac{32}{3} \lambda^4 L^4 \phi^4 + 32\lambda^4 L^4 \phi^2 + 8\lambda^4 L^4 \right)$$

$$G_5 = -\operatorname{erfc} \psi - e^{\Omega} \operatorname{erfc} \phi \left(-1 + 4\lambda \sqrt{L} \phi + 16\lambda^2 L \phi^2 + 8\lambda^2 L + \frac{128}{3} \lambda^3 L^{3/2} \phi^3 + \right. \\ \left. 64\lambda^3 L^{3/2} \phi + 32\lambda^4 L^2 \phi^4 + 96\lambda^4 L^2 \phi^2 + 24\lambda^4 L^2 + \frac{128}{15} \lambda^5 L^{5/2} \phi^5 + \right. \\ \left. \frac{128}{3} \lambda^5 L^{5/2} \phi^3 + 32\lambda^5 L^{5/2} \phi \right) + \frac{1}{\sqrt{\pi}} \left(4\lambda \sqrt{L} + 16\lambda^2 L \phi + \frac{128}{3} \lambda^3 L^{3/2} \phi^2 + \right. \\ \left. \frac{128}{3} \lambda^3 L^{3/2} + 32\lambda^4 L^2 \phi^3 + 80\lambda^4 L^2 \phi + \frac{128}{15} \lambda^5 L^{5/2} \phi^4 + \frac{192}{5} \lambda^5 L^{5/2} \phi^2 + \right. \\ \left. \frac{256}{15} \lambda^5 L^{5/2} \right) e^{-\psi^2}$$

$$\frac{\partial G_5}{\partial \xi} = -e^{\Omega} \operatorname{erfc} \phi \frac{j}{3} \left(60\lambda L^{3/2} \phi + 240\lambda^2 L^2 \phi^2 + 120\lambda^2 L^2 + 320\lambda^3 L^{5/2} \phi^3 + \right. \\ \left. 480\lambda^3 L^{5/2} \phi + 160\lambda^4 L^3 \phi^4 + 480\lambda^4 L^3 \phi^2 + 120\lambda^4 L^3 + \frac{128}{5} \lambda^5 L^{7/2} \phi^5 + \right. \\ \left. 128\lambda^5 L^{7/2} \phi^3 + 96\lambda^5 L^{7/2} \phi + 3L \right) + \frac{j}{3\sqrt{\pi}} \left(60\lambda L^{3/2} + 240\lambda^2 L^2 \phi + 320\lambda^3 L^{5/2} + \right. \\ \left. 320\lambda^3 L^{5/2} \phi^2 + 160\lambda^4 L^3 \phi^3 + 400\lambda^4 L^3 \phi + \frac{128}{5} \lambda^5 L^{7/2} \phi^4 + \frac{576}{5} \lambda^5 L^{7/2} \phi^2 + \right. \\ \left. \frac{256}{5} \lambda^5 L^{7/2} \right) e^{-\psi^2}$$

$$\begin{aligned} \frac{\partial G_5}{\partial \omega^2} = & -e^{-\psi} \operatorname{erfc} \psi \left(11L^2 + 100\lambda L^{5/2}\phi + 240\lambda^2 L^3 \phi^2 + 120\lambda^2 L^3 + \frac{640}{3} \lambda^3 L^{7/2} \phi^3 + \right. \\ & 320\lambda^3 L^{7/2} \phi + \frac{224}{3} \lambda^4 L^4 \phi^4 + 224\lambda^4 L^4 \phi^2 + 56\lambda^4 L^4 + \frac{128}{15} \lambda^5 L^9/2 \phi^5 + \\ & \left. \frac{128}{3} \lambda^5 L^9/2 \phi^3 + 32\lambda^5 L^9/2 \phi \right) + \frac{1}{3\sqrt{\pi}} \left(300\lambda L^{5/2} + 720\lambda^2 L^3 \phi + 640\lambda^3 L^{7/2} \phi^2 + \right. \\ & 640\lambda^3 L^{7/2} + 224\lambda^4 L^4 \phi^3 + 560\lambda^4 L^4 \phi + \frac{128}{5} \lambda^5 L^9/2 \phi^4 + \frac{576}{5} \lambda^5 L^9/2 \phi^2 + \\ & \left. \frac{256}{5} \lambda^5 L^9/2 + 3L^{3/2} \frac{1}{\lambda} \right) e^{-\psi^2} \end{aligned}$$

APPENDIX C

NUMERICAL COMPUTATIONS FOR THE VARIOUS G_n , $\frac{\partial G_n}{\partial w^2}$, AND $j \frac{\partial G_n}{\partial \xi}$

The computations necessary to evaluate the equations of appendix B were carried out on a card-programed calculator. On this machine, the number of significant figures in any computational step was limited to eight, a limitation which resulted in a progressive diminishing of the significant figures in the answers as the value of n increased at a given λ and L . The reason for the loss of significant figures is the fact that each answer (except for $\frac{\partial G_0}{\partial \xi}$) is the difference between two numbers; thus, when these numbers become large and nearly equal, the resultant difference has few significant figures.

It was also found that the complimentary error functions had to be computed to within a very small fraction of a percent of the correct value in order to obtain usable accuracy. The above-mentioned subtraction of two large but nearly equal numbers, one of which contains the complimentary error function as a multiplier, is the reason for the particular accuracy required. The following expressions were used to compute the complimentary error function of ϕ and ψ in the ranges of the arguments listed (ϕ is taken as the illustrative argument):

$$\operatorname{erfc} \phi = \frac{2}{\sqrt{\pi}} e^{-\phi^2} (a_1 \eta + a_2 \eta^2 + a_3 \eta^3 + a_4 \eta^4 + a_5 \eta^5) \quad (0 \leq \phi < 2) \quad (C1)$$

where

$$\eta = \frac{1}{1 + 0.3275911\phi}$$

$$a_1 = 0.22583685$$

$$a_2 = -0.25212867$$

$$a_3 = 1.2596951$$

$$a_4 = -1.2878225$$

$$a_5 = 0.94064607$$

$$\operatorname{erfc} \phi = 0.00040695202 - 0.0021782842\epsilon \left(1 - 2.5\epsilon + 3.8333333\epsilon^2 - 3.9583333\epsilon^3 + 2.8083333\epsilon^4 - 1.2847222\epsilon^5 + 0.24900794\epsilon^6 + 0.11966766\epsilon^7 - 0.11490024\epsilon^8 + 0.036175871\epsilon^9 \right) \quad (2 \leq \phi \leq 2.8) \quad (C2)$$

where $\epsilon = \phi - 2.5$ and

$$\operatorname{erfc} \phi = \frac{1}{\sqrt{\pi}} \frac{e^{-\phi^2}}{\phi} \left[1 - \frac{1}{x} + \frac{1 \cdot 3}{x^2} - \frac{1 \cdot 3 \cdot 5}{x^3} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{x^4} - \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{x^5} + \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{x^6} - \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13}{x^7} \right] \left[1 + e^{-\phi^2} \left(14.695 - \frac{76.331}{\phi} + \frac{104.18}{\phi^2} \right) \right] \quad (2.8 < \phi < \infty) \quad (C3)$$

where $x = 2\phi^2$.

Equation (C1) is the approximation found on page 169 of reference 6. The errors introduced by this approximation became unacceptable in this application above a value of the argument of 2. Equation (C3) was originally thought to be useful in the range of arguments from 2 to ∞ since the product of the first bracketed expression and $\frac{1}{\sqrt{\pi}} \frac{e^{-\phi^2}}{\phi}$ represent the first eight terms of the semiconvergent series for $\operatorname{erfc} \phi$ valid for large ϕ and the constants of the second bracketed expression are chosen to give true answers at $\phi = 2, 2.5, \text{ and } 3$. However, it was discovered that this equation was also unacceptable in the range $2 < \phi < 2.8$. A usable equation (eq. (C2)) for this range was found by employing the first 10 terms of the Taylor expansion of $\operatorname{erfc} \phi$ about the point $\phi = 2.5$.

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TABLE I.- COMPUTED VALUES OF Q_n , $\int \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND INFINITE

VALUES OF P FOR VARIOUS VALUES OF L AND λ

(a) $L = 0.001$

| P | n | $Q_n(P)$ | $\int \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $\int \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $\int \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|------------------|---|-----------------------------|--|---|----|---|-----------------------------|--|---|----|---|-----------------------------|--|---|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.72330000 \times 10^{-3}$ | $-0.99986667 \times 10^{-3}$ | $0.89106275 \times 10^{-3}$ | 2 | 0 | $0.76840000 \times 10^{-3}$ | $-0.25339356 \times 10^{-4}$ | $0.73199982 \times 10^{-4}$ | 1 | 0 | $0.11842000 \times 10^{-3}$ | $-0.26543419 \times 10^{-3}$ | $0.47723504 \times 10^{-3}$ |
| 2 | 1 | $.76778800 \times 10^{-3}$ | $-.25323594 \times 10^{-4}$ | $.73148921 \times 10^{-4}$ | 3 | 1 | $.18904400 \times 10^{-6}$ | $-.79967516 \times 10^{-6}$ | $.32148902 \times 10^{-5}$ | 3 | 1 | $.18904400 \times 10^{-6}$ | $-.79967516 \times 10^{-6}$ | $.32148902 \times 10^{-5}$ |
| 4 | 2 | $.14221489 \times 10^{-8}$ | $-.77370913 \times 10^{-8}$ | $.40460842 \times 10^{-7}$ | 6 | 2 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | 5 | 2 | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-9})$ |
| 6 | 3 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | 6 | 2 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | 5 | 2 | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-9})$ |
| $\lambda = 0.06$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.22376000 \times 10^{-2}$ | $-0.99786253 \times 10^{-3}$ | $0.29635615 \times 10^{-3}$ | 2 | 0 | $0.70860000 \times 10^{-3}$ | $-0.43534779 \times 10^{-5}$ | $0.22477996 \times 10^{-3}$ | 1 | 0 | $0.12860700 \times 10^{-2}$ | $-0.70810226 \times 10^{-3}$ | $0.27869705 \times 10^{-3}$ |
| 2 | 1 | $.70883860 \times 10^{-3}$ | $-.43539323$ | $.22587069$ | 4 | 1 | $.16011080$ | $-.13535612$ | $.97479522 \times 10^{-4}$ | 3 | 1 | $.35402510 \times 10^{-3}$ | $-.26648872$ | $.15837327$ |
| 4 | 2 | $.15961845 \times 10^{-3}$ | $-.23523649$ | $.97208758 \times 10^{-4}$ | 6 | 2 | $.25754408 \times 10^{-4}$ | $-.25229222 \times 10^{-4}$ | $.24281578$ | 5 | 2 | $.6490876 \times 10^{-4}$ | $-.62022052 \times 10^{-4}$ | $.5208384 \times 10^{-4}$ |
| 6 | 3 | $.23513870 \times 10^{-4}$ | $-.25182152 \times 10^{-4}$ | $.24251467$ | 8 | 3 | $.21788900 \times 10^{-5}$ | $-.28538446 \times 10^{-5}$ | $.34719858 \times 10^{-5}$ | 7 | 3 | $.79987300 \times 10^{-5}$ | $-.90342133 \times 10^{-5}$ | $.98327275 \times 10^{-5}$ |
| 8 | 4 | $.2179620 \times 10^{-5}$ | $-.28494702 \times 10^{-5}$ | $.34662824 \times 10^{-5}$ | 10 | 4 | $.12373466 \times 10^{-6}$ | $-.19282515 \times 10^{-6}$ | $.28490121 \times 10^{-6}$ | 9 | 4 | $.57222765 \times 10^{-6}$ | $-.73124666 \times 10^{-6}$ | $.10652846$ |
| 10 | 5 | $.12376640 \times 10^{-6}$ | $-.19287787 \times 10^{-6}$ | $.28451641 \times 10^{-6}$ | 12 | 5 | $.42228900 \times 10^{-8}$ | $-.76973870 \times 10^{-8}$ | $.13414922 \times 10^{-7}$ | 11 | 5 | $.24412690 \times 10^{-7}$ | $-.41124890 \times 10^{-7}$ | $.66222684 \times 10^{-7}$ |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.35522000 \times 10^{-2}$ | $-0.99644183 \times 10^{-3}$ | $0.17741597 \times 10^{-3}$ | 2 | 0 | $0.19145900 \times 10^{-2}$ | $-0.63220633 \times 10^{-3}$ | $0.16078111 \times 10^{-3}$ | 1 | 0 | $0.26501400 \times 10^{-2}$ | $-0.8204324 \times 10^{-3}$ | $0.17328700 \times 10^{-3}$ |
| 2 | 1 | $.19024806$ | $-.6488630$ | $.12547962$ | 4 | 1 | $.90186750 \times 10^{-3}$ | $-.5683841$ | $.11284849$ | 3 | 1 | $.13333392$ | $-.49822303$ | $.14096493$ |
| 4 | 2 | $.89803062 \times 10^{-3}$ | $-.56698052$ | $.11774972$ | 6 | 2 | $.36906693$ | $-.17789215$ | $.71643366 \times 10^{-4}$ | 5 | 2 | $.58622586 \times 10^{-3}$ | $-.26606667$ | $.94187200 \times 10^{-4}$ |
| 6 | 3 | $.36773830$ | $-.17712234$ | $.71283376 \times 10^{-4}$ | 8 | 3 | $.12384870$ | $-.72744693 \times 10^{-4}$ | $.35066951$ | 7 | 3 | $.22202210$ | $-.11595378$ | $.5192926$ |
| 8 | 4 | $.12342302$ | $-.72464999 \times 10^{-4}$ | $.35363461$ | 10 | 4 | $.38920474 \times 10^{-4}$ | $-.24994698$ | $.14412469$ | 9 | 4 | $.72443140 \times 10^{-4}$ | $-.43354728 \times 10^{-4}$ | $.23154795$ |
| 10 | 5 | $.38815300 \times 10^{-4}$ | $-.24516962$ | $.14368956$ | 12 | 5 | $.98730000 \times 10^{-5}$ | $-.71810940 \times 10^{-5}$ | $.47593290 \times 10^{-5}$ | 11 | 5 | $.19992220$ | $-.13684710$ | $.85113798 \times 10^{-5}$ |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.70568000 \times 10^{-2}$ | $-0.9929218 \times 10^{-3}$ | $0.88215301 \times 10^{-4}$ | 2 | 0 | $0.52862200 \times 10^{-2}$ | $-0.81777696 \times 10^{-3}$ | $0.86128925 \times 10^{-4}$ | 1 | 0 | $0.61476600 \times 10^{-2}$ | $-0.90483250 \times 10^{-3}$ | $0.87745583 \times 10^{-4}$ |
| 2 | 1 | $.52812250$ | $-.80725792$ | $.84560892$ | 4 | 1 | $.37832780$ | $-.64369737$ | $.78722211$ | 3 | 1 | $.44668300$ | $-.72322602$ | $.82137441$ |
| 4 | 2 | $.37469821$ | $-.63278711$ | $.77429498$ | 6 | 2 | $.26520051$ | $-.48906724$ | $.68753544$ | 5 | 2 | $.31491684$ | $-.56222628$ | $.73462288$ |
| 6 | 3 | $.26015410$ | $-.48334071$ | $.67782636$ | 8 | 3 | $.17622774$ | $-.35257626$ | $.57243907$ | 7 | 3 | $.21507130$ | $-.41856677$ | $.62691733$ |
| 8 | 4 | $.17478963$ | $-.35506977$ | $.56330327$ | 10 | 4 | $.11434307$ | $-.25908067$ | $.45446442$ | 9 | 4 | $.142201494$ | $-.30129096$ | $.51000073$ |
| 10 | 5 | $.11345121$ | $-.25060271$ | $.44920737$ | 12 | 5 | $.71277010 \times 10^{-3}$ | $-.17168225$ | $.34336927$ | 11 | 5 | $.90530270 \times 10^{-3}$ | $-.22839126$ | $.39292900$ |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.01114500 | $-0.98980251 \times 10^{-3}$ | $0.43617216 \times 10^{-4}$ | 2 | 0 | 0.012229720 | $-0.89874974 \times 10^{-3}$ | $0.43426928 \times 10^{-4}$ | 1 | 0 | 0.013150230 | $-0.94226965 \times 10^{-3}$ | $0.43991198 \times 10^{-4}$ |
| 2 | 1 | $.011968211$ | $-.87455211$ | $.41631136$ | 4 | 1 | $.010920264$ | $-.79172326$ | $.41085040$ | 3 | 1 | $.011114098$ | $-.82299330$ | $.41436546$ |
| 4 | 2 | $.010088664$ | $-.77133312$ | $.39221921$ | 6 | 2 | $.86643227 \times 10^{-2}$ | $-.69521340$ | $.3892510$ | 5 | 2 | $.92371168 \times 10^{-2}$ | $-.73200108$ | $.39122039$ |
| 6 | 3 | $.84524140 \times 10^{-2}$ | $-.67607734$ | $.37222319$ | 8 | 3 | $.71739420$ | $-.60279478$ | $.3562217$ | 7 | 3 | $.77543770$ | $-.63911074$ | $.36690328$ |
| 8 | 4 | $.70333971$ | $-.5822243$ | $.34771467$ | 10 | 4 | $.59267762$ | $-.52054665$ | $.33222444$ | 9 | 4 | $.64612221$ | $-.55416731$ | $.34046926$ |
| 10 | 5 | $.52164040$ | $-.50671118$ | $.32192137$ | 12 | 5 | $.48622260$ | $-.44612547$ | $.30407274$ | 11 | 5 | $.53223950$ | $-.47700692$ | $.31341979$ |

TABLE I.- COEFFICIENT VALUES OF C_{D_n} , $J \frac{dC_{D_n}}{d\lambda}$, AND $\frac{dC_{D_n}}{d\lambda}$ FOR $n = 0$ TO 5 AND INTERIM

VALUES OF F FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.001$ - Continued

| F | n | $C_{D_n}(F)$ | $J \frac{dC_{D_n}}{d\lambda}$ | $\frac{dC_{D_n}}{d\lambda}$ | F | n | $C_{D_n}(F)$ | $J \frac{dC_{D_n}}{d\lambda}$ | $\frac{dC_{D_n}}{d\lambda}$ | F | n | $C_{D_n}(F)$ | $J \frac{dC_{D_n}}{d\lambda}$ | $\frac{dC_{D_n}}{d\lambda}$ |
|------------------|-----|--------------|-------------------------------|-----------------------------|------|-----|--------------|-------------------------------|-----------------------------|------|-----|--------------|-------------------------------|-----------------------------|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0.0 | 0 | 0.02024700 | $-0.9789728 \times 10^{-3}$ | $0.28726426 \times 10^{-4}$ | 2.0 | 0 | 0.012079180 | $-0.9814928 \times 10^{-3}$ | $0.28723421 \times 10^{-4}$ | 11.0 | 0 | 0.020250000 | $-0.9501817 \times 10^{-3}$ | $0.28726426 \times 10^{-4}$ |
| 2.1 | 1 | 0.01212700 | -0.8874882 | 0.26966380 | 4.1 | 1 | 0.016207985 | -0.82094020 | 0.26828280 | 3.1 | 1 | 0.017632800 | -0.8588120 | 0.26978225 |
| 4.2 | 2 | 0.016206660 | -0.7822454 | 0.25282454 | 6.2 | 2 | 0.024794829 | -0.74049220 | 0.25027028 | 5.2 | 2 | 0.022021449 | -0.77152551 | 0.25154387 |
| 6.3 | 3 | 0.024794006 | -0.71747200 | 0.23663180 | 8.3 | 3 | 0.034794829 | -0.67049220 | 0.23407028 | 7.3 | 3 | 0.032782216 | -0.6928047 | 0.23504525 |
| 8.4 | 4 | 0.034792229 | -0.69492226 | 0.22111825 | 10.4 | 4 | 0.046997229 | -0.60097229 | 0.21860229 | 9.4 | 4 | 0.043869400 | -0.6280226 | 0.22002200 |
| 10.5 | 5 | 0.046991222 | -0.70822200 | 0.20822228 | 12.5 | 5 | 0.060972229 | -0.52774222 | 0.20569119 | 11.5 | 5 | 0.057222218 | -0.55802254 | 0.20742225 |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0.0 | 0 | 0.027232220 | $-0.97822221 \times 10^{-3}$ | $0.22222229 \times 10^{-4}$ | 2.0 | 0 | 0.026622220 | $-0.88222222 \times 10^{-3}$ | $0.22222222 \times 10^{-4}$ | 11.0 | 0 | 0.026622220 | $-0.9507446 \times 10^{-3}$ | $0.22222222 \times 10^{-4}$ |
| 2.1 | 1 | 0.026622220 | -0.87822222 | 0.20222222 | 4.1 | 1 | 0.032222220 | -0.82222222 | 0.18222222 | 3.1 | 1 | 0.030222220 | -0.85222222 | 0.20222222 |
| 4.2 | 2 | 0.032222220 | -0.79222222 | 0.18222222 | 6.2 | 2 | 0.040222220 | -0.72222222 | 0.16222222 | 5.2 | 2 | 0.038222220 | -0.75222222 | 0.18222222 |
| 6.3 | 3 | 0.040222220 | -0.71222222 | 0.16222222 | 8.3 | 3 | 0.050222220 | -0.65222222 | 0.14222222 | 7.3 | 3 | 0.048222220 | -0.68222222 | 0.16222222 |
| 8.4 | 4 | 0.050222220 | -0.68222222 | 0.12222222 | 10.4 | 4 | 0.062222220 | -0.60222222 | 0.12222222 | 9.4 | 4 | 0.060222220 | -0.63222222 | 0.14222222 |
| 10.5 | 5 | 0.062222220 | -0.68222222 | 0.08222222 | 12.5 | 5 | 0.076222220 | -0.58222222 | 0.08222222 | 11.5 | 5 | 0.074222220 | -0.61222222 | 0.10222222 |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0.0 | 0 | 0.034722220 | $-0.96222222 \times 10^{-3}$ | $0.16222222 \times 10^{-4}$ | 2.0 | 0 | 0.032222220 | $-0.82222222 \times 10^{-3}$ | $0.16222222 \times 10^{-4}$ | 11.0 | 0 | 0.032222220 | $-0.94222222 \times 10^{-3}$ | $0.16222222 \times 10^{-4}$ |
| 2.1 | 1 | 0.032222220 | -0.82222222 | 0.14222222 | 4.1 | 1 | 0.038222220 | -0.78222222 | 0.12222222 | 3.1 | 1 | 0.036222220 | -0.81222222 | 0.14222222 |
| 4.2 | 2 | 0.038222220 | -0.74222222 | 0.12222222 | 6.2 | 2 | 0.046222220 | -0.68222222 | 0.10222222 | 5.2 | 2 | 0.044222220 | -0.71222222 | 0.12222222 |
| 6.3 | 3 | 0.046222220 | -0.68222222 | 0.08222222 | 8.3 | 3 | 0.056222220 | -0.62222222 | 0.08222222 | 7.3 | 3 | 0.054222220 | -0.65222222 | 0.08222222 |
| 8.4 | 4 | 0.056222220 | -0.65222222 | 0.06222222 | 10.4 | 4 | 0.068222220 | -0.58222222 | 0.06222222 | 9.4 | 4 | 0.066222220 | -0.61222222 | 0.06222222 |
| 10.5 | 5 | 0.068222220 | -0.65222222 | 0.04222222 | 12.5 | 5 | 0.082222220 | -0.54222222 | 0.04222222 | 11.5 | 5 | 0.080222220 | -0.57222222 | 0.04222222 |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0.0 | 0 | 0.041222220 | $-0.95222222 \times 10^{-3}$ | $0.12222222 \times 10^{-4}$ | 2.0 | 0 | 0.038222220 | $-0.82222222 \times 10^{-3}$ | $0.12222222 \times 10^{-4}$ | 11.0 | 0 | 0.038222220 | $-0.94222222 \times 10^{-3}$ | $0.12222222 \times 10^{-4}$ |
| 2.1 | 1 | 0.038222220 | -0.82222222 | 0.10222222 | 4.1 | 1 | 0.044222220 | -0.78222222 | 0.08222222 | 3.1 | 1 | 0.042222220 | -0.81222222 | 0.10222222 |
| 4.2 | 2 | 0.044222220 | -0.74222222 | 0.08222222 | 6.2 | 2 | 0.052222220 | -0.68222222 | 0.06222222 | 5.2 | 2 | 0.050222220 | -0.71222222 | 0.08222222 |
| 6.3 | 3 | 0.052222220 | -0.68222222 | 0.06222222 | 8.3 | 3 | 0.062222220 | -0.62222222 | 0.04222222 | 7.3 | 3 | 0.060222220 | -0.65222222 | 0.06222222 |
| 8.4 | 4 | 0.062222220 | -0.65222222 | 0.04222222 | 10.4 | 4 | 0.074222220 | -0.58222222 | 0.04222222 | 9.4 | 4 | 0.072222220 | -0.61222222 | 0.04222222 |
| 10.5 | 5 | 0.074222220 | -0.65222222 | 0.02222222 | 12.5 | 5 | 0.088222220 | -0.54222222 | 0.02222222 | 11.5 | 5 | 0.086222220 | -0.57222222 | 0.02222222 |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0.0 | 0 | 0.046222220 | $-0.94222222 \times 10^{-3}$ | $0.08222222 \times 10^{-4}$ | 2.0 | 0 | 0.042222220 | $-0.82222222 \times 10^{-3}$ | $0.08222222 \times 10^{-4}$ | 11.0 | 0 | 0.042222220 | $-0.94222222 \times 10^{-3}$ | $0.08222222 \times 10^{-4}$ |
| 2.1 | 1 | 0.042222220 | -0.82222222 | 0.06222222 | 4.1 | 1 | 0.048222220 | -0.78222222 | 0.04222222 | 3.1 | 1 | 0.046222220 | -0.81222222 | 0.06222222 |
| 4.2 | 2 | 0.048222220 | -0.74222222 | 0.04222222 | 6.2 | 2 | 0.056222220 | -0.68222222 | 0.02222222 | 5.2 | 2 | 0.054222220 | -0.71222222 | 0.04222222 |
| 6.3 | 3 | 0.056222220 | -0.68222222 | 0.02222222 | 8.3 | 3 | 0.066222220 | -0.62222222 | 0.00222222 | 7.3 | 3 | 0.064222220 | -0.65222222 | 0.02222222 |
| 8.4 | 4 | 0.066222220 | -0.65222222 | 0.00222222 | 10.4 | 4 | 0.078222220 | -0.58222222 | 0.00222222 | 9.4 | 4 | 0.076222220 | -0.61222222 | 0.00222222 |
| 10.5 | 5 | 0.078222220 | -0.65222222 | 0.00222222 | 12.5 | 5 | 0.092222220 | -0.54222222 | 0.00222222 | 11.5 | 5 | 0.090222220 | -0.57222222 | 0.00222222 |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INCREASING

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.001$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ |
|-----------------|----|-------------|---|---|---|----|-------------|---|---|---|----|-------------|---|---|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.094626800 | $-0.94777281 \times 10^{-3}$ | $0.10207401 \times 10^{-4}$ | 2 | 0 | 0.052779600 | $-0.92494756 \times 10^{-3}$ | $0.10222476 \times 10^{-4}$ | 1 | 0 | 0.095689580 | $-0.99516565 \times 10^{-3}$ | $0.10214523 \times 10^{-4}$ |
| | 1 | .04822350 | -.82397725 | .84729540 $\times 10^{-5}$ | | 1 | .052779600 | -.82397725 | .84729540 $\times 10^{-5}$ | | 1 | .047926610 | -.81549879 | .84960450 $\times 10^{-5}$ |
| | 2 | .04629996 | -.71789096 | .69925770 | | 2 | .04629996 | -.80689599 | .85173940 $\times 10^{-5}$ | | 2 | .041899244 | -.70284079 | .70278980 |
| | 3 | .05761980 | -.62110947 | .57310470 | | 3 | .041131780 | -.70797715 | .70799470 | | 3 | .036999680 | -.61995045 | .57744270 |
| | 4 | .052197146 | -.54407490 | .46921500 | | 4 | .056328280 | -.61299923 | .58197810 | | 4 | .038609767 | -.59999170 | .47079990 |
| | 5 | .029271620 | -.47390210 | .37479100 | | 5 | .038078734 | -.53469940 | .47298800 | | 5 | .028800910 | -.46999940 | .38029940 |
| | 10 | | | | | 10 | .028332900 | -.46970500 | .38950800 | | 10 | | | |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.061122900 | $-0.92887769 \times 10^{-3}$ | $0.89729228 \times 10^{-5}$ | 2 | 0 | 0.09222220 | $-0.92091590 \times 10^{-3}$ | $0.89878298 \times 10^{-5}$ | 1 | 0 | 0.066187820 | $-0.92990093 \times 10^{-3}$ | $0.89811398 \times 10^{-5}$ |
| | 1 | .059336310 | -.80611734 | .72587940 | | 1 | .09222220 | -.80611734 | .72587940 | | 1 | .052731110 | -.80084777 | .72802280 |
| | 2 | .046282708 | -.69474029 | .58112990 | | 2 | .059336310 | -.73299790 | .73001940 | | 2 | .046190744 | -.68891074 | .58349260 |
| | 3 | .041048070 | -.59650111 | .49999990 | | 3 | .049904736 | -.68509390 | .58732440 | | 3 | .040455880 | -.59128731 | .46999410 |
| | 4 | .039920598 | -.511139940 | .39722210 | | 4 | .039884968 | -.58729991 | .46709260 | | 4 | .039422971 | -.50784460 | .36669110 |
| | 5 | .031146990 | -.43784340 | .27181990 | | 5 | .030916668 | -.50420210 | .36622490 | | 5 | .031009980 | -.42999990 | .27666990 |
| | 10 | | | | | 10 | .030976930 | -.438231060 | .28138200 | | 10 | | | |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.067947800 | $-0.99492820 \times 10^{-3}$ | $0.79821222 \times 10^{-5}$ | 2 | 0 | 0.066998740 | $-0.91646133 \times 10^{-3}$ | $0.80019228 \times 10^{-5}$ | 1 | 0 | 0.066619280 | $-0.92446044 \times 10^{-3}$ | $0.79996074 \times 10^{-5}$ |
| | 1 | .058666690 | -.79209690 | .62999723 | | 1 | .066998740 | -.79209690 | .62999723 | | 1 | .057877400 | -.78979930 | .63188981 |
| | 2 | .050262217 | -.67149002 | .48004169 | | 2 | .057094870 | -.71947109 | .69923980 | | 2 | .050272228 | -.66699908 | .49900076 |
| | 3 | .041129080 | -.56789610 | .37070640 | | 3 | .049992138 | -.66169226 | .49969998 | | 3 | .043611280 | -.56413257 | .37822970 |
| | 4 | .038200941 | -.47900680 | .27190460 | | 4 | .049094980 | -.56097197 | .37764290 | | 4 | .037822704 | -.47664990 | .27767880 |
| | 5 | .039920598 | -.40899980 | .19999220 | | 5 | .037397430 | -.47349990 | .28136040 | | 5 | .038799610 | -.40099920 | .19819970 |
| | 10 | | | | | 10 | .038996080 | -.39899940 | .20257990 | | 10 | | | |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.089999600 | $-0.91669041 \times 10^{-3}$ | $0.62128097 \times 10^{-5}$ | 2 | 0 | 0.061488470 | $-0.90429940 \times 10^{-3}$ | $0.62111256 \times 10^{-5}$ | 1 | 0 | 0.082999980 | $-0.91046998 \times 10^{-3}$ | $0.62297494 \times 10^{-5}$ |
| | 1 | .070674470 | -.72807846 | .49747980 | | 1 | .061488470 | -.72807846 | .49747980 | | 1 | .069922730 | -.74749999 | .49904209 |
| | 2 | .056694021 | -.61286922 | .38481996 | | 2 | .05777920 | -.64289791 | .46094976 | | 2 | .052842196 | -.61060971 | .38711962 |
| | 3 | .050600190 | -.49808800 | .28786169 | | 3 | .050633280 | -.60728247 | .38990279 | | 3 | .050109990 | -.49988720 | .28144969 |
| | 4 | .042697462 | -.40139160 | .23499000 | | 4 | .049610970 | -.49969940 | .28409214 | | 4 | .042296661 | -.39999900 | .19999900 |
| | 5 | .039920598 | -.32079290 | .18999990 $\times 10^{-6}$ | | 5 | .041897008 | -.39860090 | .18409980 | | 5 | .039990790 | -.32009150 | .16089000 $\times 10^{-6}$ |
| | 10 | | | | | 10 | .039991040 | -.31998010 | .17409980 $\times 10^{-6}$ | | 10 | | | |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.098601900 | $-0.90134292 \times 10^{-3}$ | $0.50497315 \times 10^{-5}$ | 2 | 0 | 0.096699810 | $-0.89244744 \times 10^{-3}$ | $0.50391219 \times 10^{-5}$ | 1 | 0 | 0.097992470 | $-0.89690023 \times 10^{-3}$ | $0.50906190 \times 10^{-5}$ |
| | 1 | .081562990 | -.71826264 | .34910985 | | 1 | .096699810 | -.71826264 | .34910985 | | 1 | .080991450 | -.70996870 | .34638992 |
| | 2 | .067222699 | -.60999990 | .28118482 | | 2 | .080049890 | -.60999990 | .28118482 | | 2 | .066669904 | -.59999999 | .22222222 |
| | 3 | .059206910 | -.53879960 | .22609999 | | 3 | .066102861 | -.53807474 | .22472279 | | 3 | .054774900 | -.49140880 | .18811898 |
| | 4 | .042697462 | -.49909110 | .18084460 $\times 10^{-6}$ | | 4 | .054344190 | -.430019730 | .15017999 | | 4 | .044891682 | -.39013960 | .15900940 $\times 10^{-6}$ |
| | 5 | .038200941 | -.44822140 | .12190000 $\times 10^{-6}$ | | 5 | .044901661 | -.38996990 | .12499920 $\times 10^{-6}$ | | 5 | .039999990 | -.38866990 | .09999990 $\times 10^{-7}$ |
| | 10 | | | | | 10 | .036699940 | -.24822240 | .01671000 $\times 10^{-7}$ | | 10 | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{dQ_n}{d\lambda}$, AND $\frac{dQ_n}{d\lambda}$ FOR $n = 0$ TO 5 AND LIFTING

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) L = 0.001 - Continued

| P | n | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ | P | n | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ | P | n | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ |
|------------------|---|------------------------------|------------------------------|-----------------------------|----|---|------------------------------|------------------------------|-----------------------------|----|---|------------------------------|------------------------------|-----------------------------|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.11758340 | $-0.88641162 \times 10^{-3}$ | $0.42110698 \times 10^{-3}$ | 0 | 0 | 0.11182375 | $-0.87792139 \times 10^{-3}$ | $0.42193000 \times 10^{-3}$ | 0 | 0 | 0.11270408 | $-0.88219846 \times 10^{-3}$ | $0.42151949 \times 10^{-3}$ |
| 2 | 1 | 0.091400140 | -0.67472740 | .26663613 | 2 | 1 | 0.090026040 | -0.66940966 | .26872280 | 2 | 1 | 0.090726820 | -0.67202575 | .26762948 |
| 4 | 2 | 0.073172772 | -0.50617490 | .15117735 | 4 | 2 | 0.072165266 | -0.50312260 | .15404665 | 4 | 2 | 0.072667215 | -0.50469920 | .15366297 |
| 6 | 3 | 0.058422250 | -0.37271680 | .06646230 $\times 10^{-6}$ | 6 | 3 | 0.057452526 | -0.37132510 | .06972270 $\times 10^{-6}$ | 6 | 3 | 0.057870020 | -0.37204400 | .06892060 $\times 10^{-6}$ |
| 8 | 4 | 0.046049786 | -0.26730710 | .037940700 $\times 10^{-7}$ | 8 | 4 | 0.045458190 | -0.26723510 | .041440100 $\times 10^{-7}$ | 8 | 4 | 0.045781722 | -0.26774030 | .040745000 $\times 10^{-7}$ |
| 10 | 5 | 0.036122850 | -0.18601320 | .026224000 $\times 10^{-6}$ | 10 | 5 | 0.035213899 | -0.18670770 | .028967770 $\times 10^{-6}$ | 10 | 5 | 0.0355942140 | -0.18696890 | .028490700 $\times 10^{-6}$ |
| 12 | 5 | | | | 12 | 5 | 0.035755220 | -0.18670690 | .028964000 $\times 10^{-6}$ | 12 | 5 | | | |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12213360 | $-0.87126639 \times 10^{-3}$ | $0.37884438 \times 10^{-3}$ | 0 | 0 | 0.11639707 | $-0.86162266 \times 10^{-3}$ | $0.37993492 \times 10^{-3}$ | 0 | 0 | 0.12272522 | $-0.86627263 \times 10^{-3}$ | $0.37919644 \times 10^{-3}$ |
| 2 | 1 | 0.10024694 | -0.63803905 | .20926277 | 2 | 1 | 0.098975220 | -0.63529613 | .21107285 | 2 | 1 | 0.099609890 | -0.63794125 | .21012700 |
| 4 | 2 | 0.077246706 | -0.45722130 | .10193777 | 4 | 2 | 0.076294220 | -0.45492150 | .10471027 | 4 | 2 | 0.077390230 | -0.45598880 | .10321722 |
| 6 | 3 | 0.059893040 | -0.31002730 | .07006390 $\times 10^{-6}$ | 6 | 3 | 0.058944220 | -0.30792150 | .07381027 $\times 10^{-6}$ | 6 | 3 | 0.059611010 | -0.31120290 | .07201450 $\times 10^{-6}$ |
| 8 | 4 | 0.04661022 | -0.22224700 | .052224000 | 8 | 4 | 0.04593440 | -0.21792150 | .053883770 $\times 10^{-6}$ | 8 | 4 | 0.046441214 | -0.22274420 | .053014200 |
| 10 | 5 | 0.034399020 | -0.15341360 | .034964000 | 10 | 5 | 0.033829860 | -0.15300980 | .038227770 | 10 | 5 | 0.034229210 | -0.15396430 | .037429500 |
| 12 | 5 | | | | 12 | 5 | 0.034090920 | -0.15340140 | .038100900 | 12 | 5 | | | |
| $\lambda = 5.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12223190 | $-0.81734213 \times 10^{-3}$ | $0.21961920 \times 10^{-3}$ | 0 | 0 | 0.116102122 | $-0.81302144 \times 10^{-3}$ | $0.22064269 \times 10^{-3}$ | 0 | 0 | 0.12279921 | $-0.81519028 \times 10^{-3}$ | $0.22052677 \times 10^{-3}$ |
| 2 | 1 | 0.12223190 | -0.50212226 | .08221260 $\times 10^{-6}$ | 2 | 1 | 0.116102122 | -0.50212226 | .08221260 $\times 10^{-6}$ | 2 | 1 | 0.12279921 | -0.50212226 | .08221260 $\times 10^{-6}$ |
| 4 | 2 | 0.08612290 | -0.29122220 | .07222220 $\times 10^{-7}$ | 4 | 2 | 0.08612290 | -0.29122220 | .07222220 $\times 10^{-7}$ | 4 | 2 | 0.08612290 | -0.29122220 | .07222220 $\times 10^{-7}$ |
| 6 | 3 | 0.058944220 | -0.1899920 | .07122220 $\times 10^{-6}$ | 6 | 3 | 0.058944220 | -0.1899920 | .07122220 $\times 10^{-6}$ | 6 | 3 | 0.058944220 | -0.1899920 | .07122220 $\times 10^{-6}$ |
| 8 | 4 | 0.046049786 | $-0.12222220 \times 10^{-4}$ | .026702200 | 8 | 4 | 0.046049786 | $-0.12222220 \times 10^{-4}$ | .026702200 | 8 | 4 | 0.046049786 | $-0.12222220 \times 10^{-4}$ | .026702200 |
| 10 | 5 | 0.02213360 | $-0.05020000 \times 10^{-6}$ | .026902200 | 10 | 5 | 0.02213360 | $-0.05020000 \times 10^{-6}$ | .026902200 | 10 | 5 | 0.02213360 | $-0.05020000 \times 10^{-6}$ | .026902200 |
| 12 | 5 | | | | 12 | 5 | 0.02213360 | $-0.05020000 \times 10^{-6}$ | .026902200 | 12 | 5 | | | |
| $\lambda = 8.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.22122220 | $-0.76213320 \times 10^{-3}$ | $0.14622220 \times 10^{-3}$ | 0 | 0 | 0.22022220 | $-0.76222220 \times 10^{-3}$ | $0.14622220 \times 10^{-3}$ | 0 | 0 | 0.22122220 | $-0.76222220 \times 10^{-3}$ | $0.14622220 \times 10^{-3}$ |
| 2 | 1 | 0.14622220 | -0.39222220 | .06222220 $\times 10^{-6}$ | 2 | 1 | 0.14622220 | -0.39222220 | .06222220 $\times 10^{-6}$ | 2 | 1 | 0.14622220 | -0.39222220 | .06222220 $\times 10^{-6}$ |
| 4 | 2 | 0.08222220 | -0.22222220 | .04222220 $\times 10^{-7}$ | 4 | 2 | 0.08222220 | -0.22222220 | .04222220 $\times 10^{-7}$ | 4 | 2 | 0.08222220 | -0.22222220 | .04222220 $\times 10^{-7}$ |
| 6 | 3 | 0.04622220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ | 6 | 3 | 0.04622220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ | 6 | 3 | 0.04622220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ |
| 8 | 4 | 0.02222220 | -0.02222220 | .04222220 $\times 10^{-6}$ | 8 | 4 | 0.02222220 | -0.02222220 | .04222220 $\times 10^{-6}$ | 8 | 4 | 0.02222220 | -0.02222220 | .04222220 $\times 10^{-6}$ |
| 10 | 5 | $0.72122220 \times 10^{-2}$ | 0.22222220 | .02222220 $\times 10^{-2}$ | 10 | 5 | $0.72122220 \times 10^{-2}$ | 0.22222220 | .02222220 $\times 10^{-2}$ | 10 | 5 | $0.72122220 \times 10^{-2}$ | 0.22222220 | .02222220 $\times 10^{-2}$ |
| 12 | 5 | | | | 12 | 5 | $0.72122220 \times 10^{-2}$ | 0.22222220 | .02222220 $\times 10^{-2}$ | 12 | 5 | | | |
| $\lambda = 12.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.31622220 | $-0.62222220 \times 10^{-3}$ | $0.80222220 \times 10^{-6}$ | 0 | 0 | 0.31222220 | $-0.62222220 \times 10^{-3}$ | $0.80222220 \times 10^{-6}$ | 0 | 0 | 0.31622220 | $-0.62222220 \times 10^{-3}$ | $0.80422220 \times 10^{-6}$ |
| 2 | 1 | 0.12222220 | -0.22222220 | .02222220 $\times 10^{-7}$ | 2 | 1 | 0.12222220 | -0.22222220 | .02222220 $\times 10^{-7}$ | 2 | 1 | 0.12222220 | -0.22222220 | .02222220 $\times 10^{-7}$ |
| 4 | 2 | 0.06222220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ | 4 | 2 | 0.06222220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ | 4 | 2 | 0.06222220 | $-0.12222220 \times 10^{-4}$ | .02222220 $\times 10^{-6}$ |
| 6 | 3 | 0.02222220 | -0.02222220 | .02222220 $\times 10^{-6}$ | 6 | 3 | 0.02222220 | -0.02222220 | .02222220 $\times 10^{-6}$ | 6 | 3 | 0.02222220 | -0.02222220 | .02222220 $\times 10^{-6}$ |
| 8 | 4 | $-0.36222220 \times 10^{-2}$ | 0.62222220 | .02222220 $\times 10^{-2}$ | 8 | 4 | $-0.36222220 \times 10^{-2}$ | 0.62222220 | .02222220 $\times 10^{-2}$ | 8 | 4 | $-0.36222220 \times 10^{-2}$ | 0.62222220 | .02222220 $\times 10^{-2}$ |
| 10 | 5 | -0.92222220 | 0.72222220 | .02222220 $\times 10^{-7}$ | 10 | 5 | -0.92222220 | 0.72222220 | .02222220 $\times 10^{-7}$ | 10 | 5 | -0.92222220 | 0.72222220 | .02222220 $\times 10^{-7}$ |
| 12 | 5 | | | | 12 | 5 | -0.92222220 | 0.72222220 | .02222220 $\times 10^{-7}$ | 12 | 5 | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(b) $L = 0.005$

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | | | | | |
|------------------|---|-----------------------------|---------------------------------------|---------------------------------------|------------------|---|-----------------------------|---------------------------------------|---------------------------------------|----|---|-----------------------------|---------------------------------------|---------------------------------------|--|--|--|--|--|
| $\lambda = 0.02$ | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | $0.15939000 \times 10^{-2}$ | $-0.49920308 \times 10^{-2}$ | $0.99485970 \times 10^{-2}$ | 2 | 0 | $0.21379000 \times 10^{-9}$ | $-0.28654382 \times 10^{-8}$ | $0.37153660 \times 10^{-7}$ | 1 | 0 | $0.80110000 \times 10^{-5}$ | $-0.62056725 \times 10^{-4}$ | $0.43789710 \times 10^{-3}$ | | | | | |
| 2 | 1 | $.21362600 \times 10^{-9}$ | $-.28633011 \times 10^{-8}$ | $.37122016 \times 10^{-7}$ | 4 | 1 | $0(10^{-26})$ | $0(10^{-25})$ | $0(10^{-23})$ | 3 | 1 | $0(10^{-16})$ | $0(10^{-15})$ | $0(10^{-14})$ | | | | | |
| 4 | 2 | $0(10^{-26})$ | $0(10^{-25})$ | $0(10^{-23})$ | $\lambda = 0.05$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.47696000 \times 10^{-2}$ | $-0.49761522 \times 10^{-2}$ | $0.32996382 \times 10^{-2}$ | 2 | 0 | $0.23740100 \times 10^{-5}$ | $-0.47671619 \times 10^{-5}$ | $0.32292238 \times 10^{-5}$ | 1 | 0 | $0.13750500 \times 10^{-2}$ | $-0.20163040 \times 10^{-2}$ | $0.23391229 \times 10^{-2}$ | | | | | |
| 2 | 1 | $.23632080 \times 10^{-5}$ | $-.47434799 \times 10^{-5}$ | $.32183707 \times 10^{-5}$ | 4 | 1 | $.13389940 \times 10^{-5}$ | $-.42704791 \times 10^{-5}$ | $.12788140 \times 10^{-4}$ | 3 | 1 | $.23922080 \times 10^{-4}$ | $-.61737130 \times 10^{-4}$ | $.14521467 \times 10^{-3}$ | | | | | |
| 4 | 2 | $.13350168 \times 10^{-5}$ | $-.42571094 \times 10^{-5}$ | $.12745502 \times 10^{-4}$ | 6 | 2 | $.63812230 \times 10^{-9}$ | $-.28505125 \times 10^{-8}$ | $.12317874 \times 10^{-7}$ | 5 | 2 | $.40185341 \times 10^{-7}$ | $-.15333516 \times 10^{-6}$ | $.56084111 \times 10^{-6}$ | | | | | |
| 6 | 3 | $.63876200 \times 10^{-9}$ | $-.28441389 \times 10^{-8}$ | $.12289401 \times 10^{-7}$ | 8 | 3 | $0(10^{-13})$ | $0(10^{-12})$ | $0(10^{-10})$ | 7 | 3 | $0(10^{-11})$ | $0(10^{-10})$ | $.13478721 \times 10^{-9}$ | | | | | |
| 8 | 4 | $0(10^{-13})$ | $0(10^{-12})$ | $0(10^{-10})$ | $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.79831000 \times 10^{-2}$ | $-0.49603345 \times 10^{-2}$ | $0.19693096 \times 10^{-2}$ | 2 | 0 | $0.16587800 \times 10^{-2}$ | $-0.15782589 \times 10^{-2}$ | $0.12019626 \times 10^{-2}$ | 1 | 0 | $0.39532200 \times 10^{-2}$ | $-0.30696994 \times 10^{-2}$ | $0.17449981 \times 10^{-2}$ | | | | | |
| 2 | 1 | $.16438811$ | $-.15611749$ | $.11862621$ | 4 | 1 | $.16809710 \times 10^{-3}$ | $-.22496788 \times 10^{-3}$ | $.22665338 \times 10^{-3}$ | 3 | 1 | $.37924330 \times 10^{-3}$ | $-.65933777 \times 10^{-3}$ | $.63763982 \times 10^{-3}$ | | | | | |
| 4 | 2 | $.16695473 \times 10^{-3}$ | $-.22329842 \times 10^{-3}$ | $.26432809 \times 10^{-3}$ | 6 | 2 | $.75423461 \times 10^{-5}$ | $-.13309220 \times 10^{-4}$ | $.21284442 \times 10^{-4}$ | 5 | 2 | $.39483409 \times 10^{-4}$ | $-.61102502 \times 10^{-4}$ | $.86102008 \times 10^{-4}$ | | | | | |
| 6 | 3 | $.75017200 \times 10^{-5}$ | $-.13240000 \times 10^{-4}$ | $.21691326 \times 10^{-4}$ | 8 | 3 | $.14080570 \times 10^{-6}$ | $-.31174766 \times 10^{-6}$ | $.69215300 \times 10^{-6}$ | 7 | 3 | $.11502610 \times 10^{-5}$ | $-.22857023 \times 10^{-5}$ | $.42272803 \times 10^{-5}$ | | | | | |
| 8 | 4 | $.14012202 \times 10^{-6}$ | $-.31034302 \times 10^{-6}$ | $.69504435 \times 10^{-6}$ | 10 | 4 | $.10220223 \times 10^{-8}$ | $-.22187822 \times 10^{-8}$ | $.73056941 \times 10^{-8}$ | 9 | 4 | $.13641615 \times 10^{-7}$ | $-.33374428 \times 10^{-7}$ | $.72403905 \times 10^{-7}$ | | | | | |
| 10 | 5 | $.10483080 \times 10^{-8}$ | $-.22022206 \times 10^{-8}$ | $.72775592 \times 10^{-8}$ | 12 | 5 | $0(10^{-11})$ | $0(10^{-11})$ | $0(10^{-10})$ | 11 | 5 | $0(10^{-10})$ | $-.18634761 \times 10^{-9}$ | $.52214237 \times 10^{-9}$ | | | | | |
| $\lambda = 0.20$ | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.015759900 | $-0.49212008 \times 10^{-2}$ | $0.97274972 \times 10^{-3}$ | 2 | 0 | $0.72288600 \times 10^{-2}$ | $-0.30462212 \times 10^{-2}$ | $0.26493227 \times 10^{-3}$ | 1 | 0 | 0.011323140 | $-0.39563212 \times 10^{-2}$ | $0.94688867 \times 10^{-3}$ | | | | | |
| 2 | 1 | $.76640470 \times 10^{-2}$ | $-.29687667$ | $.83489717$ | 4 | 1 | $.32454830$ | $-.15373088$ | $.58134001$ | 3 | 1 | $.50942230 \times 10^{-2}$ | $-.22188994$ | $.71949644$ | | | | | |
| 4 | 2 | $.31849563$ | $-.15021664$ | $.36632763$ | 6 | 2 | $.11274468$ | $-.63955149 \times 10^{-3}$ | $.30746736$ | 5 | 2 | $.19402730$ | $-.10069986$ | $.43083744$ | | | | | |
| 6 | 3 | $.11098660$ | $-.62616499 \times 10^{-3}$ | $.30112998$ | 8 | 3 | $.32284360 \times 10^{-3}$ | $-.21289730$ | $.22722366$ | 7 | 3 | $.61461820 \times 10^{-3}$ | $-.37892939 \times 10^{-3}$ | $.60206960$ | | | | | |
| 8 | 4 | $.31944772 \times 10^{-3}$ | $-.21258083$ | $.12908069$ | 10 | 4 | $.75946180 \times 10^{-4}$ | $-.38666097 \times 10^{-4}$ | $.41107023 \times 10^{-4}$ | 9 | 4 | $.15992932$ | $-.11483898$ | $.74018341 \times 10^{-4}$ | | | | | |
| 10 | 5 | $.75055000 \times 10^{-4}$ | $-.57811094 \times 10^{-4}$ | $.40524958 \times 10^{-4}$ | 12 | 5 | $.14415640$ | $-.12268295$ | $.10360025$ | 11 | 5 | $.33798020 \times 10^{-4}$ | $-.27880972 \times 10^{-4}$ | $.21148694$ | | | | | |
| $\lambda = 0.40$ | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0.051139400 | $-0.48443283 \times 10^{-2}$ | $0.47445616 \times 10^{-3}$ | 2 | 0 | 0.02290210 | $-0.39009859 \times 10^{-2}$ | $0.46383020 \times 10^{-3}$ | 1 | 0 | 0.022929050 | $-0.43699923 \times 10^{-2}$ | $0.47284713 \times 10^{-3}$ | | | | | |
| 2 | 1 | $.021376114$ | $-.36821542$ | $.42521450$ | 4 | 1 | $.014847700$ | $-.22561223$ | $.32722167$ | 3 | 1 | $.017909009$ | $-.32251588$ | $.41406389$ | | | | | |
| 4 | 2 | $.014248168$ | $-.27108297$ | $.36728893$ | 6 | 2 | $.9514047 \times 10^{-2}$ | $-.22162432$ | $.28502061$ | 5 | 2 | $.011694783$ | $-.23227777$ | $.34723469$ | | | | | |
| 6 | 3 | $.91421920 \times 10^{-2}$ | $-.19235742$ | $.30354831$ | 8 | 3 | $.32683730$ | $-.13677421$ | $.29091617$ | 7 | 3 | $.73662880 \times 10^{-2}$ | $-.16324930$ | $.27830843$ | | | | | |
| 8 | 4 | $.36669566$ | $-.13101172$ | $.23722888$ | 10 | 4 | $.54744369$ | $-.88765840 \times 10^{-3}$ | $.12470621$ | 9 | 4 | $.44602264$ | $-.10856400$ | $.21128221$ | | | | | |
| 10 | 5 | $.33995140$ | $-.87347660 \times 10^{-3}$ | $.17600231$ | 12 | 5 | $.19729380$ | $-.54969450$ | $.12292291$ | 11 | 5 | $.25921130$ | $-.68968880 \times 10^{-3}$ | $.13170220$ | | | | | |

TABLE I.- COMPOUND VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF F FOR VARIOUS VALUES OF L AND λ - Continued

(b) $L = 0.005$ - Continued

| F | n | $Q_n(F)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | F | n | $Q_n(F)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | F | n | $Q_n(F)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|------------------|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.046129200 | $-0.47699743 \times 10^{-2}$ | $0.36860713 \times 10^{-3}$ | 0 | 0 | 0.037207880 | $-0.41921281 \times 10^{-2}$ | $0.30710205 \times 10^{-3}$ | 1 | 0 | 0.043333970 | $-0.44609360 \times 10^{-2}$ | $0.30099775 \times 10^{-3}$ |
| 2 | 1 | 0.04650021 | -0.37929321 | .86738097 | 2 | 0 | 0.037207880 | $-0.41921281 \times 10^{-2}$ | $0.30710205 \times 10^{-3}$ | 3 | 1 | 0.03970665 | -0.35299955 | .26612430 |
| 4 | 2 | 0.04754419 | -0.29947548 | .83146972 | 4 | 1 | 0.027771148 | -0.28614027 | .68671051 | 5 | 2 | 0.02877894 | -0.27646562 | .28843406 |
| 6 | 3 | 0.04844486 | -0.2346952 | .39921036 | 6 | 2 | 0.020221029 | -0.25389267 | .22521647 | 7 | 3 | 0.026700741 | -0.21457308 | .19453415 |
| 8 | 4 | 0.049768974 | -0.18120612 | .16952133 | 8 | 3 | 0.014824440 | -0.19541630 | .12835265 | 9 | 4 | 0.022040693 | -0.16453370 | .16370049 |
| 10 | 5 | 0.05146020 $\times 10^{-2}$ | -0.13833196 | .14889278 | 10 | 4 | 0.010476080 | -0.14870731 | .12665774 | 11 | 5 | 0.01613970 $\times 10^{-2}$ | -0.12842995 | .13566920 |
| 12 | 5 | | | | 12 | 5 | 0.00797610 $\times 10^{-2}$ | -0.11122740 | .12816895 | | | | | |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.060762400 | $-0.44691882 \times 10^{-2}$ | $0.28282778 \times 10^{-3}$ | 0 | 0 | 0.051282120 | $-0.42439067 \times 10^{-2}$ | $0.22618101 \times 10^{-3}$ | 1 | 0 | 0.056178840 | $-0.44699283 \times 10^{-2}$ | $0.22870277 \times 10^{-3}$ |
| 2 | 1 | 0.047002820 | -0.37499734 | .12661672 | 2 | 0 | 0.051282120 | $-0.42439067 \times 10^{-2}$ | $0.22618101 \times 10^{-3}$ | 3 | 1 | 0.043392660 | -0.35296699 | .12700214 |
| 4 | 2 | 0.04681931 | -0.29948367 | .35496667 | 4 | 1 | 0.036833360 | -0.33756650 | .12684928 | 5 | 2 | 0.032594875 | -0.2764434 | .13565279 |
| 6 | 3 | 0.04720290 | -0.23922553 | .13001977 | 6 | 2 | 0.030603399 | -0.28838333 | .13535922 | 7 | 3 | 0.027594080 | -0.22809792 | .13033179 |
| 8 | 4 | 0.021409925 | -0.19067643 | .10928294 | 8 | 3 | 0.023398360 | -0.23208025 | .12897026 | 9 | 4 | 0.02527869 | -0.17970466 | .10922674 |
| 10 | 5 | 0.016347280 | -0.15169403 | .92822270 $\times 10^{-4}$ | 10 | 4 | 0.017814320 | -0.16977530 | .10884373 | 11 | 5 | 0.014876970 | -0.12846292 | .9269240 $\times 10^{-4}$ |
| 12 | 5 | | | | 12 | 5 | 0.013498730 | -0.13524210 | .91278030 $\times 10^{-4}$ | | | | | |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.075042600 | $-0.46247869 \times 10^{-2}$ | $0.17654721 \times 10^{-3}$ | 0 | 0 | 0.065146210 | $-0.42709958 \times 10^{-2}$ | $0.17712135 \times 10^{-3}$ | 1 | 0 | 0.072058530 | $-0.44420216 \times 10^{-2}$ | $0.17698125 \times 10^{-3}$ |
| 2 | 1 | 0.056462820 | -0.36472282 | .13722676 | 2 | 0 | 0.065146210 | $-0.42709958 \times 10^{-2}$ | $0.17712135 \times 10^{-3}$ | 3 | 1 | 0.048217580 | -0.35094725 | .13770921 |
| 4 | 2 | 0.047569943 | -0.28834960 | .10821193 | 4 | 1 | 0.051447220 | -0.33705233 | .13542232 | 5 | 2 | 0.044722004 | -0.27704923 | .10971946 |
| 6 | 3 | 0.039523970 | -0.22020447 | .89922820 $\times 10^{-4}$ | 6 | 2 | 0.040017326 | -0.26662113 | .11076940 | 7 | 3 | 0.032786620 | -0.22216847 | .87973590 $\times 10^{-4}$ |
| 8 | 4 | 0.027690427 | -0.18194430 | .69026640 | 8 | 3 | 0.031121070 | -0.21132025 | .88921370 $\times 10^{-4}$ | 9 | 4 | 0.02909628 | -0.17499948 | .70272270 |
| 10 | 5 | 0.02576930 | -0.14499896 | .56052040 | 10 | 4 | 0.024191280 | -0.16764273 | .71692090 | 11 | 5 | 0.022152200 | -0.13592167 | .57423040 |
| 12 | 5 | | | | 12 | 5 | 0.018791320 | -0.13542613 | .58227070 | | | | | |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.089811100 | $-0.45770943 \times 10^{-2}$ | $0.14345048 \times 10^{-3}$ | 0 | 0 | 0.080128940 | $-0.42671421 \times 10^{-2}$ | $0.14444108 \times 10^{-3}$ | 1 | 0 | 0.084497860 | $-0.44113330 \times 10^{-2}$ | $0.14428207 \times 10^{-3}$ |
| 2 | 1 | 0.06908920 | -0.35222596 | .10772221 | 2 | 0 | 0.080128940 | $-0.42671421 \times 10^{-2}$ | $0.14444108 \times 10^{-3}$ | 3 | 1 | 0.06224360 | -0.34220097 | .10662222 |
| 4 | 2 | 0.053692137 | -0.27290600 | .77421240 $\times 10^{-4}$ | 4 | 1 | 0.062198940 | -0.33080705 | .10760690 | 5 | 2 | 0.050199979 | -0.28079998 | .79224470 $\times 10^{-4}$ |
| 6 | 3 | 0.041690980 | -0.21210417 | .56963470 | 6 | 2 | 0.048302221 | -0.27110382 | .80422870 $\times 10^{-4}$ | 7 | 3 | 0.039598780 | -0.22251021 | .50764690 |
| 8 | 4 | 0.032462795 | -0.16594390 | .42023770 | 8 | 3 | 0.037521000 | -0.22035542 | .60119020 | 9 | 4 | 0.03029072 | -0.18104884 | .43862900 |
| 10 | 5 | 0.025307250 | -0.12850973 | .31128790 | 10 | 4 | 0.028242278 | -0.16582092 | .45463110 | 11 | 5 | 0.024201270 | -0.12209287 | .32954060 |
| 12 | 5 | | | | 12 | 5 | 0.022789760 | -0.12272022 | .34492900 | | | | | |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.10292820 | $-0.44870976 \times 10^{-2}$ | $0.12004410 \times 10^{-3}$ | 0 | 0 | 0.092834840 | $-0.42426090 \times 10^{-2}$ | $0.12222636 \times 10^{-3}$ | 1 | 0 | 0.096161380 | $-0.43667438 \times 10^{-2}$ | $0.12207484 \times 10^{-3}$ |
| 2 | 1 | 0.078720910 | -0.35221202 | .82740410 $\times 10^{-4}$ | 2 | 0 | 0.092834840 | $-0.42426090 \times 10^{-2}$ | $0.12222636 \times 10^{-3}$ | 3 | 1 | 0.072369990 | -0.32893392 | .83524430 $\times 10^{-4}$ |
| 4 | 2 | 0.060459139 | -0.27292721 | .56076150 | 4 | 1 | 0.072111210 | -0.32294085 | .84222970 $\times 10^{-4}$ | 5 | 2 | 0.05725008 | -0.24956928 | .57215970 |
| 6 | 3 | 0.046466560 | -0.21287534 | .37169260 | 6 | 2 | 0.054462970 | -0.24579777 | .58945210 | 7 | 3 | 0.043780000 | -0.18295119 | .38874790 |
| 8 | 4 | 0.032820377 | -0.14722449 | .26399480 | 8 | 3 | 0.042717520 | -0.21849519 | .40431900 | 9 | 4 | 0.032360478 | -0.13524288 | .26594470 |
| 10 | 5 | 0.027210930 | -0.11032037 | .14692220 | 10 | 4 | 0.028941289 | -0.14060116 | .27236690 | 11 | 5 | 0.022950930 | -0.10277069 | .16379970 |
| 12 | 5 | | | | 12 | 5 | 0.022432670 | -0.10709225 | .17946970 | | | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND THEREAFTER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(b) L = 0.005 - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ |
|-----------------|---|------------|---------------------------------------|---------------------------------------|----|---|------------|---------------------------------------|---------------------------------------|----|---|-------------|---------------------------------------|---------------------------------------|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.11587430 | $-0.44206286 \times 10^{-2}$ | $0.10256632 \times 10^{-3}$ | 2 | 0 | 0.10723832 | $-0.42186299 \times 10^{-2}$ | $0.10335305 \times 10^{-3}$ | 1 | 0 | 0.11150485 | $-0.43178261 \times 10^{-2}$ | $0.10301948 \times 10^{-3}$ |
| 2 | 1 | 0.08770010 | -0.36409871 | $.66076970 \times 10^{-4}$ | 4 | 1 | 0.08122244 | -0.31063433 | $.68092190 \times 10^{-4}$ | 3 | 1 | 0.08436869 | -0.31739694 | $.67141360 \times 10^{-4}$ |
| 4 | 2 | 0.06197915 | -0.29592416 | .40714270 | 6 | 2 | 0.05427932 | -0.22507756 | .43378370 | 5 | 2 | 0.05834937 | -0.22272298 | .42100730 |
| 6 | 3 | 0.05005998 | -0.17870600 | .25322900 | 8 | 3 | 0.04669426 | -0.16774068 | .26267870 | 7 | 3 | 0.04834467 | -0.17022697 | .26841470 |
| 8 | 4 | 0.07871996 | -0.12947765 | .11606960 | 10 | 4 | 0.07538767 | -0.12229300 | .13295700 | 9 | 4 | 0.06623111 | -0.12429979 | .13136960 |
| 10 | 5 | 0.02865610 | $-0.90226580 \times 10^{-3}$ | $1.99017100 \times 10^{-5}$ | 12 | 5 | 0.02895830 | $-0.89734970 \times 10^{-3}$ | $1.67621600 \times 10^{-5}$ | 11 | 5 | 0.02799990 | $-0.90562330 \times 10^{-3}$ | 1.9689900×10^{-5} |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12228487 | $-0.43337378 \times 10^{-2}$ | $0.89058310 \times 10^{-4}$ | 2 | 0 | 0.12031390 | $-0.41769005 \times 10^{-2}$ | $0.89761910 \times 10^{-4}$ | 1 | 0 | 0.12439910 | $-0.42665111 \times 10^{-2}$ | $0.89442010 \times 10^{-4}$ |
| 2 | 1 | 0.09560090 | -0.30971262 | .53391770 | 4 | 1 | 0.08505380 | -0.22224992 | .55223780 | 3 | 1 | 0.09256967 | -0.30432201 | .54346720 |
| 4 | 2 | 0.07090923 | -0.21261406 | .25250720 | 6 | 2 | 0.06597981 | -0.12500022 | .31794190 | 5 | 2 | 0.06875890 | -0.21251711 | .30977040 |
| 6 | 3 | 0.05291190 | -0.15294639 | .13961770 | 8 | 3 | 0.04980070 | -0.12499924 | .16040740 | 7 | 3 | 0.05098860 | -0.15253373 | .14780610 |
| 8 | 4 | 0.05884309 | -0.10583468 | .52642000 $\times 10^{-5}$ | 10 | 4 | 0.05725948 | -0.10432277 | .58091600 $\times 10^{-5}$ | 9 | 4 | 0.05777788 | -0.10544310 | .57512200 $\times 10^{-5}$ |
| 10 | 5 | 0.02867200 | $-0.72223990 \times 10^{-3}$ | -0.02290000 | 12 | 5 | 0.02722970 | $-0.72590960 \times 10^{-3}$ | $-0.64948000 \times 10^{-6}$ | 11 | 5 | 0.027999160 | $-0.72867040 \times 10^{-3}$ | -0.122294900 |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12122240 | $-0.42222511 \times 10^{-2}$ | $0.72273931 \times 10^{-4}$ | 2 | 0 | 0.11909290 | $-0.41351495 \times 10^{-2}$ | $0.72992636 \times 10^{-4}$ | 1 | 0 | 0.12726665 | $-0.42132255 \times 10^{-2}$ | $0.72654707 \times 10^{-4}$ |
| 2 | 1 | 0.10295986 | -0.29548884 | .49425090 | 4 | 1 | 0.09713204 | -0.22222270 | .51338610 | 3 | 1 | 0.10002276 | -0.29109719 | .44339710 |
| 4 | 2 | 0.07467824 | -0.20071449 | .20771940 | 6 | 2 | 0.07070709 | -0.12694626 | .22278760 | 5 | 2 | 0.07262180 | -0.19289311 | .21244930 |
| 6 | 3 | 0.05297910 | -0.13400924 | .63612800 $\times 10^{-5}$ | 8 | 3 | 0.05123040 | -0.12499972 | .89281900 $\times 10^{-5}$ | 7 | 3 | 0.05222220 | -0.13330025 | .74940900 $\times 10^{-5}$ |
| 8 | 4 | 0.05222220 | $-0.7417990 \times 10^{-3}$ | -0.24020100 | 10 | 4 | 0.05771102 | $-0.72222220 \times 10^{-3}$ | $-0.22222200 \times 10^{-6}$ | 9 | 4 | 0.05729797 | $-0.72604050 \times 10^{-3}$ | -0.12122200 |
| 10 | 5 | 0.02722220 | -0.55222200 | -0.74006600 | 12 | 5 | 0.02722220 | -0.56422200 | $-0.54202200 \times 10^{-5}$ | 11 | 5 | 0.02722220 | -0.55222200 | -0.6403900 |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.11712220 | $-0.41405224 \times 10^{-2}$ | $0.59082221 \times 10^{-4}$ | 2 | 0 | 0.11677327 | $-0.40215924 \times 10^{-2}$ | $0.59216667 \times 10^{-4}$ | 1 | 0 | 0.12722220 | $-0.40210924 \times 10^{-2}$ | $0.59267040 \times 10^{-4}$ |
| 2 | 1 | 0.11222220 | -0.26116136 | .26450605 | 4 | 1 | 0.11209434 | -0.22274123 | .27777021 | 3 | 1 | 0.11262225 | -0.25947411 | .27102106 |
| 4 | 2 | 0.08042602 | -0.15297127 | .69942000 $\times 10^{-5}$ | 6 | 2 | 0.07722220 | -0.12741220 | .89212000 $\times 10^{-5}$ | 5 | 2 | 0.07889969 | -0.15223361 | .77666300 $\times 10^{-5}$ |
| 6 | 3 | 0.05412220 | $-0.92699970 \times 10^{-3}$ | -0.32341700 | 8 | 3 | 0.05222220 | $-0.92322220 \times 10^{-3}$ | -0.24222200 | 7 | 3 | 0.05222220 | $-0.92222220 \times 10^{-3}$ | -0.31794600 |
| 8 | 4 | 0.05222220 | -0.82222200 | -0.94511000 | 10 | 4 | 0.05222220 | -0.82222200 | -0.11109000 | 9 | 4 | 0.05222220 | -0.82222200 | -0.87796000 |
| 10 | 5 | 0.02222220 | -0.20994600 | $-0.11659720 \times 10^{-4}$ | 12 | 5 | 0.02222220 | -0.21143000 | $-0.10992220 \times 10^{-4}$ | 11 | 5 | 0.02222220 | -0.20994600 | $-0.11099600 \times 10^{-4}$ |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12002220 | $-0.39967229 \times 10^{-2}$ | $0.48906616 \times 10^{-4}$ | 2 | 0 | 0.11927497 | $-0.39222229 \times 10^{-2}$ | $0.46926965 \times 10^{-4}$ | 1 | 0 | 0.11966722 | $-0.39901332 \times 10^{-2}$ | $0.48730445 \times 10^{-4}$ |
| 2 | 1 | 0.12222220 | -0.22222220 | .12522220 | 4 | 1 | 0.12222220 | -0.22222220 | .12522220 | 3 | 1 | 0.12722220 | -0.22222220 | .12522220 |
| 4 | 2 | 0.08222220 | -0.12222220 | .45980000 $\times 10^{-6}$ | 6 | 2 | 0.07971333 | -0.12222229 | .66272000 $\times 10^{-6}$ | 5 | 2 | 0.08079424 | -0.12222220 | .11004000 $\times 10^{-6}$ |
| 6 | 3 | 0.05222220 | $-0.57222220 \times 10^{-3}$ | $-0.21222220 \times 10^{-5}$ | 8 | 3 | 0.04971333 | $-0.52222220 \times 10^{-3}$ | $-0.70060000 \times 10^{-5}$ | 7 | 3 | 0.04999401 | $-0.52072220 \times 10^{-3}$ | $-0.70099000 \times 10^{-5}$ |
| 8 | 4 | 0.02999998 | -0.12222220 | $-0.11222220 \times 10^{-4}$ | 10 | 4 | 0.02999998 | -0.12222220 | $-0.10999998 \times 10^{-4}$ | 9 | 4 | 0.02999998 | -0.12222220 | $-0.10999998 \times 10^{-4}$ |
| 10 | 5 | 0.02681774 | $0.19036000 \times 10^{-4}$ | -0.11560920 | 12 | 5 | 0.02681774 | $0.19036000 \times 10^{-4}$ | $-0.10999998 \times 10^{-4}$ | 11 | 5 | 0.02681774 | $0.19036000 \times 10^{-4}$ | -0.11062220 |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND DEGREE

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(b) $L = 0.005$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|------------------|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 5.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.22778840 | $-0.56610881 \times 10^{-2}$ | $0.57686511 \times 10^{-4}$ | 2 | 0 | 0.22015973 | $-0.57855553 \times 10^{-2}$ | $0.58041750 \times 10^{-4}$ | 1 | 0 | 0.22355973 | $-0.58253101 \times 10^{-2}$ | $0.57865388 \times 10^{-4}$ |
| 2 | 1 | .13753181 | $-.19970137$ | $-.61298880 \times 10^{-5}$ | 4 | 1 | .13375680 | $-.19779413$ | $-.62947820 \times 10^{-5}$ | 3 | 1 | .13553594 | $-.19876866$ | $-.6287150 \times 10^{-5}$ |
| 4 | 2 | .08025265 | $-.50269670 \times 10^{-3}$ | $-.42155500$ | 6 | 2 | .07840556 | $-.51720070 \times 10^{-3}$ | $-.36965100$ | 5 | 2 | .07920462 | $-.51300750 \times 10^{-3}$ | $-.41037500$ |
| 6 | 3 | .04453526 | $-.30849100$ | $-.97545900$ | 8 | 3 | .04390550 | $-.32140600$ | $-.91219500$ | 7 | 3 | .04420620 | $-.31211600$ | $-.94078900$ |
| 8 | 4 | .02211000 | $-.12211000 \times 10^{-1}$ | $-.10687850 \times 10^{-1}$ | 10 | 4 | .02211000 | $-.12211000 \times 10^{-1}$ | $-.10191730 \times 10^{-1}$ | 9 | 4 | .02211000 | $-.12211000 \times 10^{-1}$ | $-.10430540 \times 10^{-1}$ |
| 10 | 5 | .01002900 | $-.15557100 \times 10^{-3}$ | $-.94571100 \times 10^{-5}$ | 12 | 5 | .01031750 | $-.15694200 \times 10^{-3}$ | $-.91688800 \times 10^{-5}$ | 11 | 5 | .01017800 | $-.14617900 \times 10^{-3}$ | $-.95215100 \times 10^{-5}$ |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.25344570 | $-0.57367717 \times 10^{-2}$ | $0.51205925 \times 10^{-4}$ | 2 | 0 | 0.24604256 | $-0.56700515 \times 10^{-2}$ | $0.51521894 \times 10^{-4}$ | 1 | 0 | 0.24978855 | $-0.57014500 \times 10^{-2}$ | $0.51356455 \times 10^{-4}$ |
| 2 | 1 | .14250141 | $-.17273416$ | $-.45148800 \times 10^{-5}$ | 4 | 1 | .13906624 | $-.17176728$ | $-.51435040 \times 10^{-5}$ | 3 | 1 | .14077659 | $-.17288667$ | $-.4854590 \times 10^{-5}$ |
| 4 | 2 | .07525550 | $-.6426560 \times 10^{-3}$ | $-.66528800$ | 6 | 2 | .074041170 | $-.65539590 \times 10^{-3}$ | $-.6467600$ | 5 | 2 | .075283440 | $-.6450560 \times 10^{-3}$ | $-.63486700$ |
| 6 | 3 | .03711590 | $-.96588000 \times 10^{-4}$ | $-.98066600$ | 8 | 3 | .036907190 | $-.11575000$ | $-.92643000$ | 7 | 3 | .03712840 | $-.10685900$ | $-.95047600$ |
| 8 | 4 | .015353420 | $-.14547500 \times 10^{-3}$ | $-.98158400$ | 10 | 4 | .015251300 | $-.12731300$ | $-.89507500$ | 9 | 4 | .015480650 | $-.13652800$ | $-.90231100$ |
| 10 | 5 | .57591000 $\times 10^{-2}$ | $-.22457000$ | $-.71921500$ | 12 | 5 | .42339000 $\times 10^{-2}$ | $-.21008800$ | $-.70610000$ | 11 | 5 | .40201000 $\times 10^{-2}$ | $-.21780800$ | $-.71378900$ |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.34331890 | $-0.52854055 \times 10^{-2}$ | $0.16822165 \times 10^{-4}$ | 2 | 0 | 0.33675578 | $-0.52445387 \times 10^{-2}$ | $0.16992658 \times 10^{-4}$ | 1 | 0 | 0.34004420 | $-0.53266550 \times 10^{-2}$ | $0.16911350 \times 10^{-4}$ |
| 2 | 1 | .17560442 | $-.85743270 \times 10^{-3}$ | $-.75954640 \times 10^{-5}$ | 4 | 1 | .17786224 | $-.87452600 \times 10^{-3}$ | $-.53003050 \times 10^{-5}$ | 3 | 1 | .17875495 | $-.87115150 \times 10^{-3}$ | $-.74464560 \times 10^{-5}$ |
| 4 | 2 | .042806720 | $-.40319000 \times 10^{-4}$ | $-.74608500$ | 6 | 2 | .042873580 | $-.42066000 \times 10^{-4}$ | $-.72842800$ | 5 | 2 | .042844250 | $-.33568000 \times 10^{-4}$ | $-.73767000$ |
| 6 | 3 | .58220000 $\times 10^{-2}$ | $-.28194400 \times 10^{-3}$ | $-.57452800$ | 8 | 3 | .58431000 | $-.27110900 \times 10^{-3}$ | $-.56894500$ | 7 | 3 | .61007000 $\times 10^{-2}$ | $-.27921600 \times 10^{-3}$ | $-.57782500$ |
| 8 | 4 | .74928000 | $-.26451000$ | $-.50165000$ | 10 | 4 | .69711000 | $-.25845100$ | $-.50428000$ | 9 | 4 | .72298000 | $-.26149500$ | $-.50300400$ |
| 10 | 5 | -.57771000 | $-.17570800$ | $-.87210000 \times 10^{-6}$ | 12 | 5 | -.52274000 | $-.17590500$ | $-.95440000 \times 10^{-6}$ | 11 | 5 | -.54012000 | $-.17484200$ | $-.95040000 \times 10^{-6}$ |
| $\lambda = 8.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.41649450 | $-0.25115187 \times 10^{-2}$ | $0.10546398 \times 10^{-4}$ | 2 | 0 | 0.41060200 | $-0.24967236 \times 10^{-2}$ | $0.10448327 \times 10^{-4}$ | 1 | 0 | 0.41352382 | $-0.24971465 \times 10^{-2}$ | $0.10597674 \times 10^{-4}$ |
| 2 | 1 | .11260530 | $-.27588630 \times 10^{-3}$ | $-.54547100 \times 10^{-5}$ | 4 | 1 | .11221500 | $-.29064100 \times 10^{-3}$ | $-.52841260 \times 10^{-5}$ | 3 | 1 | .11240900 | $-.28928590 \times 10^{-3}$ | $-.55994070 \times 10^{-5}$ |
| 4 | 2 | .55555700 $\times 10^{-2}$ | $-.5176900$ | $-.51320700$ | 6 | 2 | .510788510 | $-.50955700$ | $-.50915400$ | 5 | 2 | .510276380 | $-.31465900$ | $-.51151700$ |
| 6 | 3 | -.014871900 | $-.28628900$ | $-.21004300$ | 8 | 3 | -.014866000 | $-.28405400$ | $-.21260000$ | 7 | 3 | -.014551400 | $-.28617500$ | $-.21144500$ |
| 8 | 4 | -.014112100 | $-.14216800$ | | 10 | 4 | -.01428900 | $-.14212900$ | $-.28300000 \times 10^{-7}$ | 9 | 4 | -.013965500 | $-.14217100$ | $-.16400000 \times 10^{-7}$ |
| 10 | 5 | -.80190000 $\times 10^{-2}$ | $-.52900000 \times 10^{-4}$ | $-.85650000 \times 10^{-6}$ | 12 | 5 | -.75900000 $\times 10^{-2}$ | $-.54550000 \times 10^{-4}$ | $-.80250000 \times 10^{-6}$ | 11 | 5 | -.79500000 $\times 10^{-2}$ | $-.55740000 \times 10^{-4}$ | $-.81260000 \times 10^{-6}$ |
| $\lambda = 12.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.52719590 | $-0.25640307 \times 10^{-2}$ | $0.14824410 \times 10^{-5}$ | 2 | 0 | 0.52247551 | $-0.25543782 \times 10^{-2}$ | $0.149001470 \times 10^{-5}$ | 1 | 0 | 0.52402809 | $-0.25522167 \times 10^{-2}$ | $0.14825680 \times 10^{-5}$ |
| 2 | 1 | .028841700 | $-.52220500 \times 10^{-3}$ | $-.49607280$ | 4 | 1 | .027616500 | $-.52035590 \times 10^{-3}$ | $-.49443490$ | 3 | 1 | .027231400 | $-.52072800 \times 10^{-3}$ | $-.49525740$ |
| 4 | 2 | -.056457400 | $-.55812900$ | $-.15120500$ | 6 | 2 | -.057794100 | $-.55448600$ | $-.13277500$ | 5 | 2 | -.056129900 | $-.55620500$ | $-.15529200$ |
| 6 | 3 | -.012174000 | $-.62590000 \times 10^{-4}$ | $-.66240000 \times 10^{-6}$ | 8 | 3 | -.012668000 | $-.65990000 \times 10^{-4}$ | $-.61118000 \times 10^{-6}$ | 7 | 3 | -.012751000 | $-.65200000 \times 10^{-4}$ | $-.61280000 \times 10^{-6}$ |
| 8 | 4 | -.29510000 $\times 10^{-2}$ | $-.44250000$ | $-.75550000$ | 10 | 4 | -.29590000 $\times 10^{-2}$ | $-.42740000$ | $-.75170000$ | 9 | 4 | -.30000000 $\times 10^{-2}$ | $-.43550000$ | $-.75490000$ |
| 10 | 5 | .24750000 | $-.45660000$ | $-.28960000$ | 12 | 5 | .22760000 | $-.45230000$ | $-.29280000$ | 11 | 5 | .24300000 | $-.45950000$ | $-.28900000$ |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INTERMEDIATE

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(c) $L = 0.01$

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ |
|------------------|---|-----------------------------|---------------------------------------|---------------------------------------|----|---|-----------------------------|---------------------------------------|---------------------------------------|----|---|-----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.22589000 \times 10^{-2}$ | $-0.99774711 \times 10^{-2}$ | 0.0280109704 | 2 | 0 | $0(10^{-15})$ | $0(10^{-15})$ | $0(10^{-12})$ | 1 | 0 | $0.28686000 \times 10^{-6}$ | $-0.40666516 \times 10^{-5}$ | $0.54416437 \times 10^{-4}$ |
| 2 | 1 | $0(10^{-15})$ | $0(10^{-15})$ | $0(10^{-12})$ | 4 | 1 | | | | 3 | 1 | $0(10^{-29})$ | $0(10^{-27})$ | $0(10^{-25})$ |
| $\lambda = 0.06$ | | | | | | | | | | | | | | |
| 0 | 0 | $0.67343000 \times 10^{-2}$ | $-0.99326574 \times 10^{-2}$ | $0.99098333 \times 10^{-2}$ | 2 | 0 | $0.52367000 \times 10^{-4}$ | $-0.18369726 \times 10^{-3}$ | $0.58881888 \times 10^{-3}$ | 1 | 0 | $0.99117000 \times 10^{-5}$ | $-0.25760172 \times 10^{-2}$ | $0.46717237 \times 10^{-2}$ |
| 2 | 1 | $.52103410 \times 10^{-4}$ | $-.18265236 \times 10^{-3}$ | $-.57912537 \times 10^{-3}$ | 4 | 1 | $.40264100 \times 10^{-8}$ | $-.24163560 \times 10^{-7}$ | $.13920689 \times 10^{-6}$ | 3 | 1 | $.85596000 \times 10^{-6}$ | $-.40437722 \times 10^{-5}$ | $.18907012 \times 10^{-4}$ |
| 4 | 2 | $.41309448 \times 10^{-8}$ | $-.24083166 \times 10^{-7}$ | $.13998442 \times 10^{-6}$ | 6 | 2 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | 5 | 2 | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-9})$ |
| 6 | 3 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | | | | | | | | | | |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.011184600 | $-0.98801539 \times 10^{-2}$ | $0.99430143 \times 10^{-2}$ | 2 | 0 | $0.99945000 \times 10^{-3}$ | $-0.15629988 \times 10^{-2}$ | $0.20999078 \times 10^{-2}$ | 1 | 0 | $0.39690100 \times 10^{-2}$ | $-0.47993492 \times 10^{-2}$ | $0.43463999 \times 10^{-2}$ |
| 2 | 1 | $.98818120 \times 10^{-3}$ | $-.15431225$ | $.20288466$ | 4 | 1 | $.19951800 \times 10^{-4}$ | $-.46194286 \times 10^{-4}$ | $.10194131 \times 10^{-3}$ | 3 | 1 | $.17007140 \times 10^{-3}$ | $-.59381534 \times 10^{-3}$ | $.58496822 \times 10^{-3}$ |
| 4 | 2 | $.19186800 \times 10^{-4}$ | $-.49809149 \times 10^{-4}$ | $.10102128 \times 10^{-3}$ | 6 | 2 | $.66124650 \times 10^{-7}$ | $-.21752600 \times 10^{-6}$ | $.68990669 \times 10^{-6}$ | 5 | 2 | $.14133274 \times 10^{-3}$ | $-.39989807 \times 10^{-3}$ | $.10689799 \times 10^{-4}$ |
| 6 | 3 | $.69743900 \times 10^{-7}$ | $-.21629599 \times 10^{-6}$ | $.68096837 \times 10^{-6}$ | 8 | 3 | $0(10^{-10})$ | $0(10^{-9})$ | $0(10^{-9})$ | 7 | 3 | $.19423000 \times 10^{-8}$ | $-.78998090 \times 10^{-8}$ | $.28481881 \times 10^{-7}$ |
| 8 | 4 | $0(10^{-10})$ | $0(10^{-9})$ | $0(10^{-9})$ | | | | | | 9 | 4 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-11})$ |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.022173500 | $-0.97782647 \times 10^{-2}$ | $0.27251693 \times 10^{-2}$ | 2 | 0 | $0.78731000 \times 10^{-2}$ | $-0.47162489 \times 10^{-2}$ | $0.21497940 \times 10^{-2}$ | 1 | 0 | 0.013746840 | $-0.70992678 \times 10^{-2}$ | $0.25790424 \times 10^{-2}$ |
| 2 | 1 | $.78766130 \times 10^{-2}$ | $-.45609912$ | $.20970222$ | 4 | 1 | $.19431720$ | $-.15138076$ | $.99156890 \times 10^{-3}$ | 3 | 1 | $.40982610 \times 10^{-2}$ | $-.87622560$ | $.15227946$ |
| 4 | 2 | $.18995799$ | $-.14729821$ | $.96167670 \times 10^{-3}$ | 6 | 2 | $.32921843 \times 10^{-3}$ | $-.32211176 \times 10^{-3}$ | $.28031860$ | 5 | 2 | $.83174484 \times 10^{-3}$ | $-.72896214 \times 10^{-3}$ | $.59386889 \times 10^{-3}$ |
| 6 | 3 | $.92913690 \times 10^{-3}$ | $-.31998820 \times 10^{-3}$ | $.27444120$ | 8 | 3 | $.37028900 \times 10^{-4}$ | $-.44113978 \times 10^{-4}$ | $.48488944 \times 10^{-4}$ | 7 | 3 | $.11942220 \times 10^{-3}$ | $-.12496922$ | $.12290748$ |
| 8 | 4 | $.96460200 \times 10^{-4}$ | $-.43979086 \times 10^{-4}$ | $.47614012 \times 10^{-4}$ | 10 | 4 | $.27021290 \times 10^{-3}$ | $-.38189134 \times 10^{-3}$ | $.90997821 \times 10^{-3}$ | 9 | 4 | $.10499143 \times 10^{-4}$ | $-.13601500 \times 10^{-4}$ | $.16984299 \times 10^{-4}$ |
| 10 | 5 | $.26660900 \times 10^{-5}$ | $-.7662232 \times 10^{-5}$ | $.90191366 \times 10^{-5}$ | 12 | 5 | $.12984060 \times 10^{-6}$ | $-.20661097 \times 10^{-6}$ | $.32469964 \times 10^{-6}$ | 11 | 5 | $.61942600 \times 10^{-6}$ | $-.99622990 \times 10^{-6}$ | $.13989220 \times 10^{-5}$ |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.043982200 | $-0.99641784 \times 10^{-2}$ | $0.13148322 \times 10^{-2}$ | 2 | 0 | 0.027071890 | $-0.69660179 \times 10^{-2}$ | $0.12999974 \times 10^{-2}$ | 1 | 0 | 0.034675490 | $-0.82900894 \times 10^{-2}$ | $0.13061199 \times 10^{-2}$ |
| 2 | 1 | $.027406974$ | $-.64412299$ | $.11212849$ | 4 | 1 | $.01681668$ | $-.49974964$ | $.96651020 \times 10^{-3}$ | 3 | 1 | $.019917099$ | $-.94492516$ | $.10969160$ |
| 4 | 2 | $.019870293$ | $-.40919772$ | $.87841540 \times 10^{-3}$ | 6 | 2 | $.79999999 \times 10^{-2}$ | $-.24999999$ | $.66963697$ | 5 | 2 | $.010241973$ | $-.92210977$ | $.78046072 \times 10^{-3}$ |
| 6 | 3 | $.70251970 \times 10^{-2}$ | $-.29911615$ | $.62117199$ | 8 | 3 | $.32287690$ | $-.19191992$ | $.41287697$ | 7 | 3 | $.49679920 \times 10^{-2}$ | $-.17831421$ | $.51262762$ |
| 8 | 4 | $.22747170$ | $-.12461189$ | $.39282999$ | 10 | 4 | $.14547089$ | $-.62727190 \times 10^{-3}$ | $.29999996$ | 9 | 4 | $.22104240$ | $-.89669090 \times 10^{-3}$ | $.30769497$ |
| 10 | 5 | $.139999790$ | $-.99877060 \times 10^{-3}$ | $.22079992$ | 12 | 5 | $.96136100 \times 10^{-3}$ | $-.27040229$ | $.11999497$ | 11 | 5 | $.89676900 \times 10^{-3}$ | $-.40829910$ | $.16225112$ |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND DIFFERENT

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(c) $L = 0.01$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|------------------|-----|-----------------------------|---|---|-----|-----|----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.064259300 | $-0.59574070 \times 10^{-2}$ | $0.84674192 \times 10^{-3}$ | 2 | 0 | 0.04784040 | $-0.7664250 \times 10^{-2}$ | $0.03752382 \times 10^{-3}$ | 1 | 0 | 0.055525210 | $-0.85085988 \times 10^{-2}$ | $0.84878264 \times 10^{-3}$ |
| 2 | 1 | 0.048762130 | -0.67642085 | $.69562878$ | 2 | 1 | 0.050610610 | -0.55966667 | $.66871009$ | 3 | 1 | 0.056544070 | -0.60734519 | $.68287404$ |
| 4 | 2 | 0.087691413 | -0.8112864 | $.56483498$ | 4 | 1 | 0.036510610 | -0.77192806 | $.58153995$ | 5 | 2 | 0.023559905 | -0.42943721 | $.54740285$ |
| 6 | 3 | 0.117754050 | -0.75478887 | $.45066285$ | 6 | 2 | 0.029575113 | -0.8466485 | $.59662876$ | 7 | 3 | 0.014657760 | -0.29086574 | $.46668585$ |
| 8 | 4 | 0.12008847 | -0.82670833 | $.34898641$ | 8 | 3 | 0.015257707 | -0.16246986 | $.29122931$ | 9 | 4 | $0.89119420 \times 10^{-2}$ | -1.9513710 | $.52176504$ |
| 10 | 5 | $0.66170650 \times 10^{-2}$ | -0.14871051 | $.26017787$ | 10 | 4 | 0.1364800×10^{-2} | -0.10209464 | $.20255779$ | 11 | 5 | 0.2225870 | -0.2802823 | $.25280591$ |
| 12 | 5 | 0.1272970 | | | 12 | 5 | 0.1272970 | | | | | | | |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.084256200 | $-0.91576505 \times 10^{-2}$ | $0.61366060 \times 10^{-3}$ | 2 | 0 | 0.067252830 | $-0.75203140 \times 10^{-2}$ | $0.61505080 \times 10^{-3}$ | 1 | 0 | 0.075507890 | $-0.85418214 \times 10^{-2}$ | $0.61706923 \times 10^{-3}$ |
| 2 | 1 | 0.058592130 | -0.66677950 | $.46911908$ | 2 | 1 | 0.04606270 | -0.57248293 | $.47102650$ | 3 | 1 | 0.052268960 | -0.61968894 | $.47263978$ |
| 4 | 2 | 0.047691413 | -0.4856645 | $.36522700$ | 4 | 1 | 0.031573328 | -0.41252176 | $.36442688$ | 5 | 2 | 0.028802279 | -0.34000595 | $.36711594$ |
| 6 | 3 | 0.029871200 | -0.35234656 | $.28789506$ | 6 | 2 | 0.023388330 | -0.29237760 | $.28521202$ | 7 | 3 | 0.014657760 | -0.24814353 | $.28744064$ |
| 8 | 4 | 0.02896611 | -0.25199965 | $.22814029$ | 8 | 3 | 0.014535854 | -0.21020506 | $.21976926$ | 9 | 4 | 0.016613710 | -0.20249009 | $.22042661$ |
| 10 | 5 | 0.02806680 | -0.18306150 | $.18044305$ | 10 | 5 | 0.02806680 | -0.18306150 | $.18044305$ | 11 | 5 | 0.02806680 | -0.18306150 | $.18044305$ |
| 12 | 5 | 0.02806680 | | | 12 | 5 | 0.02806680 | | | | | | | |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.10574300 | $-0.85045696 \times 10^{-2}$ | $0.47454388 \times 10^{-3}$ | 2 | 0 | 0.086667840 | $-0.80096979 \times 10^{-2}$ | $0.47847880 \times 10^{-3}$ | 1 | 0 | 0.094816150 | $-0.84881160 \times 10^{-2}$ | $0.47789970 \times 10^{-3}$ |
| 2 | 1 | 0.072783430 | -0.61611706 | $.35422011$ | 2 | 1 | 0.066292820 | -0.57566385 | $.34574121$ | 3 | 1 | 0.066337810 | -0.60787584 | $.34020285$ |
| 4 | 2 | 0.051192887 | -0.46129628 | $.25992190$ | 4 | 1 | 0.042450444 | -0.41252176 | $.25078206$ | 5 | 2 | 0.04701167 | -0.37402555 | $.24660592$ |
| 6 | 3 | 0.056012990 | -0.35412284 | $.17610928$ | 6 | 2 | 0.026590740 | -0.29775776 | $.18517781$ | 7 | 3 | 0.02760280 | -0.31619684 | $.18222512$ |
| 8 | 4 | 0.025211687 | -0.24275740 | $.13212611$ | 8 | 3 | 0.023388330 | -0.21020506 | $.13039684$ | 9 | 4 | 0.02250921 | -0.22227720 | $.13720092$ |
| 10 | 5 | 0.017759680 | -0.17089920 | $.10116925$ | 10 | 5 | 0.017759680 | -0.17089920 | $.10116925$ | 11 | 5 | 0.017759680 | -0.17089920 | $.10116925$ |
| 12 | 5 | 0.017759680 | | | 12 | 5 | 0.017759680 | | | | | | | |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.12220870 | $-0.87775131 \times 10^{-2}$ | $0.38297885 \times 10^{-3}$ | 2 | 0 | 0.10544135 | $-0.80076404 \times 10^{-2}$ | $0.38222790 \times 10^{-3}$ | 1 | 0 | 0.11362277 | $-0.85595072 \times 10^{-2}$ | $0.38742277 \times 10^{-3}$ |
| 2 | 1 | 0.025360960 | -0.60992173 | $.28575532$ | 2 | 1 | 0.07360600 | -0.55964318 | $.27661669$ | 3 | 1 | 0.073544550 | -0.58064422 | $.28094516$ |
| 4 | 2 | 0.057755880 | -0.46220500 | $.15005804$ | 4 | 1 | 0.052581270 | -0.39517029 | $.17136283$ | 5 | 2 | 0.057597219 | -0.41022011 | $.16539287$ |
| 6 | 3 | 0.041972970 | -0.29561660 | $.10220987$ | 6 | 2 | 0.036195900 | -0.27776122 | $.11965187$ | 7 | 3 | 0.050228740 | -0.28922760 | $.10596189$ |
| 8 | 4 | 0.025342510 | -0.21222490 | $.66692470 \times 10^{-4}$ | 8 | 3 | 0.023388330 | -0.21020506 | $.75179660 \times 10^{-4}$ | 9 | 4 | 0.027405662 | -0.20000222 | $.73461240 \times 10^{-4}$ |
| 10 | 5 | 0.02039420 | -0.15102610 | $.44334200$ | 10 | 5 | 0.02039420 | -0.15102610 | $.44334200$ | 11 | 5 | 0.02039420 | -0.15102610 | $.44334200$ |
| 12 | 5 | 0.02039420 | | | 12 | 5 | 0.02039420 | | | | | | | |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.14026030 | $-0.85973569 \times 10^{-2}$ | $0.31701258 \times 10^{-3}$ | 2 | 0 | 0.12570224 | $-0.79285748 \times 10^{-2}$ | $0.32122817 \times 10^{-3}$ | 1 | 0 | 0.13122148 | $-0.82789613 \times 10^{-2}$ | $0.31966984 \times 10^{-3}$ |
| 2 | 1 | 0.056408730 | -0.57777409 | $.18222225$ | 2 | 1 | 0.025699940 | -0.557776318 | $.19472265$ | 3 | 1 | 0.020744130 | -0.55700853 | $.18996047$ |
| 4 | 2 | 0.056424260 | -0.38606900 | $.10225943$ | 4 | 1 | 0.028217056 | -0.36423430 | $.11558218$ | 5 | 2 | 0.06211922 | -0.37047690 | $.10592075$ |
| 6 | 3 | 0.045855890 | -0.25945120 | $.53021600 \times 10^{-4}$ | 6 | 2 | 0.040212740 | -0.24745410 | $.66628640 \times 10^{-4}$ | 7 | 3 | 0.043519340 | -0.25377410 | $.60006580 \times 10^{-4}$ |
| 8 | 4 | 0.021789890 | -0.17425120 | $.84022260$ | 8 | 3 | 0.028212740 | -0.18870480 | $.36417210$ | 9 | 4 | 0.030254730 | -0.27211750 | $.30525940$ |
| 10 | 5 | 0.022096930 | -0.11252250 | $.77050000 \times 10^{-3}$ | 10 | 5 | 0.022096930 | -0.11252250 | $.77050000$ | 11 | 5 | 0.022096930 | -0.11252250 | $.77050000$ |
| 12 | 5 | 0.022096930 | | | 12 | 5 | 0.022096930 | | | | | | | |

TABLE I.- COMPUTED VALUES OF q_n , $J \frac{\partial q_n}{\partial \epsilon}$, AND $\frac{\partial q_n}{\partial \epsilon^2}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF F FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.01$ - Continued

| F | n | $q_n(F)$ | $J \frac{\partial q_n}{\partial \epsilon}$ | $\frac{\partial q_n}{\partial \epsilon^2}$ | F | n | $q_n(F)$ | $J \frac{\partial q_n}{\partial \epsilon}$ | $\frac{\partial q_n}{\partial \epsilon^2}$ | F | n | $q_n(F)$ | $J \frac{\partial q_n}{\partial \epsilon}$ | $\frac{\partial q_n}{\partial \epsilon^2}$ |
|-----------------|-----|-----------------------------|--|--|------|-----|-----------------------------|--|--|------|-----|-----------------------------|--|--|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.15772320 | $-0.64227685 \times 10^{-2}$ | $0.26839061 \times 10^{-3}$ | 2 0 | 0 | 0.14141736 | $-0.76815067 \times 10^{-2}$ | $0.27248770 \times 10^{-3}$ | 1 0 | 0 | 0.14943482 | $-0.81253482 \times 10^{-2}$ | $0.27074895 \times 10^{-3}$ |
| 2 1 | 1 | 0.10602960 | -0.54070971 | .13934266 | 4 1 | 1 | 0.092422680 | -0.51178705 | .14927655 | 3 1 | 1 | 0.10683736 | -0.52049180 | .14471347 |
| 4 2 | 2 | 0.071310779 | -0.34436100 | $.69799540 \times 10^{-4}$ | 6 2 | 2 | 0.064348704 | -0.35105280 | $.75338600 \times 10^{-4}$ | 5 2 | 2 | 0.078989810 | -0.32810300 | $.69729220 \times 10^{-4}$ |
| 6 3 | 3 | 0.048000000 | -0.21250580 | .20322210 | 8 3 | 3 | 0.043678720 | -0.21222410 | .32113540 | 7 3 | 3 | 0.049866540 | -0.21619550 | .28477060 |
| 8 4 | 4 | 0.029342660 | -0.13720291 | $-0.89873000 \times 10^{-5}$ | 10 4 | 4 | 0.026601170 | -0.13667950 | $.79098600 \times 10^{-5}$ | 9 4 | 4 | 0.029700050 | -0.13721150 | $.89769000 \times 10^{-5}$ |
| 10 5 | 5 | 0.02182070 | $-0.09284000 \times 10^{-3}$ | $-0.14288800 \times 10^{-4}$ | 12 5 | 5 | 0.020953440 | $-0.09152900 \times 10^{-3}$ | -0.14874000 | 11 5 | 5 | 0.020961680 | $-0.09442400 \times 10^{-3}$ | -0.14657000 |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.17462210 | $-0.82537790 \times 10^{-2}$ | $0.25090087 \times 10^{-3}$ | 2 0 | 0 | 0.15857887 | $-0.77079771 \times 10^{-2}$ | $0.23459500 \times 10^{-3}$ | 1 0 | 0 | 0.16642392 | $-0.80217995 \times 10^{-2}$ | $0.23097891 \times 10^{-3}$ |
| 2 1 | 1 | 0.114301174 | -0.50991710 | .10612132 | 4 1 | 1 | 0.10401150 | -0.48787928 | .11488099 | 3 1 | 1 | 0.10926268 | -0.49507201 | .11070697 |
| 4 2 | 2 | 0.074627110 | -0.30476050 | $.36086040 \times 10^{-4}$ | 6 2 | 2 | 0.068611959 | -0.29290970 | $.46516370 \times 10^{-4}$ | 5 2 | 2 | 0.074588690 | -0.30089900 | $.41232100 \times 10^{-4}$ |
| 6 3 | 3 | 0.04904640 | -0.17930730 | $-0.12046900 \times 10^{-5}$ | 8 3 | 3 | 0.045002890 | -0.17894120 | $.87512400 \times 10^{-5}$ | 7 3 | 3 | 0.046792090 | -0.17251760 | $.89700000 \times 10^{-5}$ |
| 8 4 | 4 | 0.03328330 | -0.10200980 | $-0.19903800 \times 10^{-4}$ | 10 4 | 4 | 0.029437700 | -0.10496600 | $-0.10380100 \times 10^{-4}$ | 9 4 | 4 | 0.030499420 | -0.10771070 | $-0.10748800 \times 10^{-4}$ |
| 10 5 | 5 | 0.020384350 | $-0.53129700 \times 10^{-3}$ | -0.06389500 | 12 5 | 5 | 0.019294220 | $-0.52639400 \times 10^{-3}$ | -0.06328900 | 11 5 | 5 | 0.019220690 | $-0.52771000 \times 10^{-3}$ | -0.06345800 |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.19098090 | $-0.80901070 \times 10^{-2}$ | $0.40119222 \times 10^{-3}$ | 2 0 | 0 | 0.17220462 | $-0.76842513 \times 10^{-2}$ | $0.20454812 \times 10^{-3}$ | 1 0 | 0 | 0.18299081 | $-0.78280592 \times 10^{-2}$ | $0.20509801 \times 10^{-3}$ |
| 2 1 | 1 | 0.12133543 | -0.47128908 | .80517500 $\times 10^{-4}$ | 4 1 | 1 | 0.11200401 | -0.43497902 | .82319900 $\times 10^{-4}$ | 3 1 | 1 | 0.11697736 | -0.46526698 | .84528460 $\times 10^{-4}$ |
| 4 2 | 2 | 0.076973016 | -0.26637710 | .16198980 | 6 2 | 2 | 0.071225980 | -0.26221300 | .25098990 | 5 2 | 2 | 0.073925349 | -0.26430790 | .20779400 |
| 6 3 | 3 | 0.047902120 | -0.14304130 | -0.15431590 | 8 3 | 3 | 0.045001592 | -0.14322250 | $-0.70810800 \times 10^{-5}$ | 7 3 | 3 | 0.046464400 | -0.14437060 | -0.11120690 |
| 8 4 | 4 | 0.026663000 | $-0.70609700 \times 10^{-3}$ | -0.28669700 | 10 4 | 4 | 0.023198670 | $-0.75616100 \times 10^{-3}$ | $-0.21544800 \times 10^{-4}$ | 9 4 | 4 | 0.023943350 | $-0.73289500 \times 10^{-3}$ | -0.23045900 |
| 10 5 | 5 | 0.012128760 | -0.28228600 | -0.32015700 | 12 5 | 5 | 0.017422610 | -0.25110000 | -0.36232900 | 11 5 | 5 | 0.017220920 | -0.22244000 | -0.29098700 |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.22963340 | $-0.77094661 \times 10^{-2}$ | $0.14866117 \times 10^{-3}$ | 2 0 | 0 | 0.21437570 | $-0.74094331 \times 10^{-2}$ | $0.13128071 \times 10^{-3}$ | 1 0 | 0 | 0.22220448 | $-0.77541096 \times 10^{-2}$ | $0.13004448 \times 10^{-3}$ |
| 2 1 | 1 | 0.15404225 | -0.59172526 | $.38070860 \times 10^{-4}$ | 4 1 | 1 | 0.12628756 | -0.32734879 | $.45904990 \times 10^{-4}$ | 3 1 | 1 | 0.13014474 | -0.32779800 | $.41022980 \times 10^{-4}$ |
| 4 2 | 2 | 0.07641742 | -0.28084330 | -0.22552220 | 6 2 | 2 | 0.072779670 | -0.18274080 | $-0.64819000 \times 10^{-5}$ | 5 2 | 2 | 0.074623222 | -0.18194340 | $-0.54715000 \times 10^{-5}$ |
| 6 3 | 3 | 0.042162190 | $-0.67799000 \times 10^{-3}$ | -0.15452190 | 8 3 | 3 | 0.040786500 | $-0.72598400 \times 10^{-3}$ | $-0.20401990 \times 10^{-4}$ | 7 3 | 3 | 0.041469090 | $-0.70225400 \times 10^{-3}$ | $-0.22992400 \times 10^{-4}$ |
| 8 4 | 4 | 0.022193230 | -0.10699500 | -0.34911500 | 10 4 | 4 | 0.021874040 | -0.17202800 | -0.31145600 | 9 4 | 4 | 0.022211160 | -0.14029500 | -0.33029700 |
| 10 5 | 5 | 0.010728900 | $.15309400$ | -0.51348000 | 12 5 | 5 | 0.010923200 | $.28951000 \times 10^{-4}$ | -0.28792800 | 11 5 | 5 | 0.010876000 | $.122277000$ | -0.30024000 |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.26340060 | $-0.7349944 \times 10^{-2}$ | $0.11460325 \times 10^{-3}$ | 2 0 | 0 | 0.25099908 | $-0.71146222 \times 10^{-2}$ | $0.11670215 \times 10^{-3}$ | 1 0 | 0 | 0.25611186 | $-0.72208327 \times 10^{-2}$ | $0.11570265 \times 10^{-3}$ |
| 2 1 | 1 | 0.18073177 | -0.51977117 | $.13784620 \times 10^{-4}$ | 4 1 | 1 | 0.12430699 | -0.31661041 | $.17979900 \times 10^{-4}$ | 3 1 | 1 | 0.13776110 | -0.31230049 | $.15201460 \times 10^{-4}$ |
| 4 2 | 2 | 0.070978290 | -0.11128990 | -0.28400700 | 6 2 | 2 | 0.062706960 | -0.11079590 | -0.20729770 | 5 2 | 2 | 0.06924120 | -0.11252270 | -0.22740220 |
| 6 3 | 3 | 0.028992220 | $-0.12304800 \times 10^{-3}$ | -0.3712940 | 8 3 | 3 | 0.022637600 | $-0.20775000 \times 10^{-3}$ | -0.30780050 | 7 3 | 3 | 0.032227800 | $-0.17402700 \times 10^{-3}$ | -0.32229590 |
| 8 4 | 4 | 0.013077980 | $.25260700$ | -0.30922500 | 10 4 | 4 | 0.013522120 | $.13221700$ | -0.22627400 | 9 4 | 4 | 0.013314960 | $.22212200$ | -0.29434000 |
| 10 5 | 5 | 0.22900000 $\times 10^{-2}$ | $.36022100$ | -0.25092900 | 12 5 | 5 | 0.25098000 $\times 10^{-2}$ | $.31205100$ | -0.22207800 | 11 5 | 5 | 0.25096000 $\times 10^{-2}$ | $.29714700$ | -0.22234700 |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.01$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ |
|-----------------|---|-----------------------------|---------------------------------------|---------------------------------------|----|---|-----------------------------|---------------------------------------|---------------------------------------|----|---|-----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.29509770 | $-0.70149626 \times 10^{-2}$ | $0.91047400 \times 10^{-4}$ | 2 | 0 | 0.28469689 | $-0.68311237 \times 10^{-2}$ | $0.92744250 \times 10^{-4}$ | 11 | 0 | 0.29153423 | $-0.69244636 \times 10^{-2}$ | $0.91284750 \times 10^{-4}$ |
| 2 | 1 | .14211443 | $-.25594100$ | $-.11511000 \times 10^{-3}$ | 4 | 1 | .13755556 | $-.25953510$ | $-.22062400 \times 10^{-3}$ | 13 | 1 | .13555469 | $-.25597050$ | $-.24453000 \times 10^{-3}$ |
| 4 | 2 | .062064780 | $-.56372700 \times 10^{-3}$ | $-.29014310 \times 10^{-4}$ | 6 | 2 | .060881400 | $-.61901200 \times 10^{-3}$ | $-.26273700 \times 10^{-4}$ | 15 | 2 | .061486810 | $-.59805100 \times 10^{-3}$ | $-.27640710 \times 10^{-4}$ |
| 6 | 3 | .022505500 | .21486900 | $-.30513800$ | 8 | 3 | .022877100 | .15785500 | $-.28659100$ | 17 | 3 | .022706700 | .18496800 | $-.29511000$ |
| 8 | 4 | $.44473000 \times 10^{-2}$ | .42909500 | $-.22285400$ | 10 | 4 | $.52604000 \times 10^{-2}$ | .38404900 | $-.22190400$ | 19 | 4 | $.48646000 \times 10^{-2}$ | .40640200 | $-.22228400$ |
| 10 | 5 | $-.27568000$ | .40909000 | $-.14255200$ | 12 | 5 | $-.19668000$ | .58058000 | $-.14286700$ | 21 | 5 | $-.23547000$ | .59482000 | $-.14264400$ |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.3222250 | $-0.67078774 \times 10^{-2}$ | $0.73968650 \times 10^{-4}$ | 2 | 0 | 0.31594522 | $-0.65255091 \times 10^{-2}$ | $0.75744180 \times 10^{-4}$ | 11 | 0 | 0.3224130 | $-0.66334555 \times 10^{-2}$ | $0.74609900 \times 10^{-4}$ |
| 2 | 1 | .14221548 | $-.19969000$ | $-.10179920$ | 4 | 1 | .13620301 | $-.20149480$ | $-.25800300 \times 10^{-3}$ | 13 | 1 | .13221582 | $-.20064260$ | $-.28712200 \times 10^{-3}$ |
| 4 | 2 | .051071170 | $-.14190500 \times 10^{-3}$ | $-.29146390$ | 6 | 2 | .050730550 | $-.19856800 \times 10^{-3}$ | $-.27514680 \times 10^{-4}$ | 15 | 2 | .0505015000 | $-.17059700 \times 10^{-3}$ | $-.28231520 \times 10^{-4}$ |
| 6 | 3 | .012131200 | .45025500 | $-.24955800$ | 8 | 3 | .012242200 | .58106900 | $-.24180100$ | 17 | 3 | .012249700 | .40545000 | $-.24554600$ |
| 8 | 4 | $-.26855000 \times 10^{-2}$ | .42650100 | $-.15553400$ | 10 | 4 | $-.17556000 \times 10^{-2}$ | .45250600 | $-.15453000$ | 19 | 4 | $-.22157000 \times 10^{-2}$ | .46807200 | $-.15501500$ |
| 10 | 5 | $-.66252000$ | .36813000 | $-.72459000 \times 10^{-3}$ | 12 | 5 | $-.59714000$ | .25553000 | $-.75267000 \times 10^{-3}$ | 21 | 5 | $-.65228000$ | .36075000 | $-.74009000 \times 10^{-3}$ |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.43215550 | $-0.56780450 \times 10^{-2}$ | $0.37221149 \times 10^{-4}$ | 2 | 0 | 0.42051400 | $-0.56028115 \times 10^{-2}$ | $0.37577373 \times 10^{-4}$ | 11 | 0 | 0.42655615 | $-0.56406099 \times 10^{-2}$ | $0.37618974 \times 10^{-4}$ |
| 2 | 1 | .10554850 | $-.32812200 \times 10^{-3}$ | $-.21622660$ | 4 | 1 | .10278980 | $-.40041400 \times 10^{-3}$ | $-.20600190$ | 13 | 1 | .10317950 | $-.37255900 \times 10^{-3}$ | $-.21155550$ |
| 4 | 2 | $.35746000 \times 10^{-2}$ | .66525000 | $-.17972000$ | 6 | 2 | $.48646000 \times 10^{-2}$ | .62743000 | $-.17705050$ | 15 | 2 | $.42288000 \times 10^{-2}$ | .64528000 | $-.17840070$ |
| 6 | 3 | $-.016122500$ | .51428500 | $-.62778000 \times 10^{-3}$ | 8 | 3 | $-.015029000$ | .50224500 | $-.64672000 \times 10^{-3}$ | 17 | 3 | $-.015608900$ | .50665500 | $-.65755000 \times 10^{-3}$ |
| 8 | 4 | $-.012935500$ | .22127000 | $.77431000 \times 10^{-6}$ | 10 | 4 | $-.012849700$ | .22310000 | $.48640000 \times 10^{-6}$ | 19 | 4 | $-.012712100$ | .22256000 | $.62270000 \times 10^{-6}$ |
| 10 | 5 | $-.64980000 \times 10^{-2}$ | $.35240000 \times 10^{-4}$ | $.30450000 \times 10^{-5}$ | 12 | 5 | $-.64260000 \times 10^{-2}$ | $.59150000 \times 10^{-4}$ | $.28410000 \times 10^{-5}$ | 21 | 5 | $-.64600000 \times 10^{-2}$ | $.36520000 \times 10^{-4}$ | $.29480000 \times 10^{-5}$ |
| $\lambda = 8.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.51089950 | $-0.42910074 \times 10^{-2}$ | $0.21613624 \times 10^{-4}$ | 2 | 0 | 0.50116056 | $-0.42473527 \times 10^{-2}$ | $0.22052055 \times 10^{-4}$ | 11 | 0 | 0.50601909 | $-0.42692872 \times 10^{-2}$ | $0.21222076 \times 10^{-4}$ |
| 2 | 1 | .045622400 | $.60077700 \times 10^{-3}$ | $-.22426510$ | 4 | 1 | .044113500 | $.56057600 \times 10^{-3}$ | $-.19973820$ | 13 | 1 | .044242800 | $.58046100 \times 10^{-3}$ | $-.22220050$ |
| 4 | 2 | $-.050075100$ | .70776100 | $-.72925000 \times 10^{-3}$ | 6 | 2 | $-.0228674000$ | .65309100 | $-.73775000 \times 10^{-3}$ | 15 | 2 | $-.022571000$ | .70044200 | $-.72566000 \times 10^{-3}$ |
| 6 | 3 | $-.020051700$ | .20605000 | .16155000 | 8 | 3 | $-.019618800$ | .20901000 | .14192000 | 17 | 3 | $-.019225400$ | .20759000 | .15159000 |
| 8 | 4 | $-.56540000 \times 10^{-2}$ | $-.43750000 \times 10^{-4}$ | .50710000 | 10 | 4 | $-.57550000 \times 10^{-2}$ | $-.37720000 \times 10^{-4}$ | $.29700000$ | 19 | 4 | $-.56280000 \times 10^{-2}$ | $-.40560000 \times 10^{-4}$ | .30250000 |
| 10 | 5 | $.84000000 \times 10^{-3}$ | $-.82900000$ | $-.17250000$ | 12 | 5 | $-.77200000 \times 10^{-2}$ | $-.29700000$ | $-.84700000$ | 21 | 5 | $.76000000 \times 10^{-3}$ | $-.84700000$ | $-.17510000$ |

TABLE I.-- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial t}$, AND $\frac{\partial Q_n}{\partial t^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(d) $L = 0.05$

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial t}$ | $\frac{\partial Q_n}{\partial t^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial t}$ | $\frac{\partial Q_n}{\partial t^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial t}$ | $\frac{\partial Q_n}{\partial t^2}$ |
|------------------|---|-----------------------------|-------------------------------------|-------------------------------------|------|---|-----------------------------|-------------------------------------|-------------------------------------|------|---|-----------------------------|-------------------------------------|-------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 0 | | $0.50263000 \times 10^{-2}$ | -0.049748683 | 0.31290414 | 2 0 | | | | | 3 0 | | $0(10^{-17})$ | $0(10^{-15})$ | $0(10^{-14})$ |
| 2 1 | | | | | 4 1 | | | | | 5 1 | | | | |
| $\lambda = 0.06$ | | | | | | | | | | | | | | |
| 0 0 | | 0.014960800 | -0.042251961 | 0.10266732 | 2 0 | | $0.45217000 \times 10^{-9}$ | $-0.67210950 \times 10^{-8}$ | $0.97333490 \times 10^{-7}$ | 3 0 | | $0.49346100 \times 10^{-4}$ | $-0.41799690 \times 10^{-3}$ | $0.32431921 \times 10^{-2}$ |
| 2 1 | | $.45225200 \times 10^{-9}$ | $-.67234238 \times 10^{-8}$ | $.96677662 \times 10^{-7}$ | 4 1 | | $0(10^{-28})$ | $0(10^{-26})$ | $0(10^{-25})$ | 5 1 | | $0(10^{-17})$ | $0(10^{-15})$ | $0(10^{-14})$ |
| 4 2 | | $0(10^{-28})$ | $0(10^{-26})$ | $0(10^{-25})$ | 6 2 | | | | | 7 2 | | | | |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 0 | | 0.024739800 | -0.048793008 | 0.06060165 | 2 0 | | $0.15538600 \times 10^{-4}$ | $-0.77602210 \times 10^{-4}$ | $0.42213835 \times 10^{-3}$ | 3 0 | | $0.15183000 \times 10^{-2}$ | $-0.36167995 \times 10^{-2}$ | 0.017731418 |
| 2 1 | | $.13143250 \times 10^{-4}$ | $-.76277119 \times 10^{-4}$ | $.41344439 \times 10^{-3}$ | 4 1 | | $0(10^{-11})$ | $0(10^{-10})$ | $.12212435 \times 10^{-9}$ | 5 1 | | $.12747350 \times 10^{-7}$ | $-.10324420 \times 10^{-6}$ | $.80487849 \times 10^{-6}$ |
| 4 2 | | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-9})$ | 6 2 | | | | $0(10^{-20})$ | 7 2 | | $0(10^{-16})$ | $0(10^{-15})$ | $0(10^{-14})$ |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 0 | | 0.048528100 | -0.04737394 | 0.029160477 | 2 0 | | $0.30007500 \times 10^{-2}$ | $-0.57422770 \times 10^{-2}$ | $0.87590051 \times 10^{-2}$ | 3 0 | | 0.013000050 | -0.020709759 | 0.022059054 |
| 2 1 | | $.28589600 \times 10^{-2}$ | $-.52492916 \times 10^{-2}$ | $.82194266 \times 10^{-2}$ | 4 1 | | $.29695020 \times 10^{-4}$ | $-.74357020 \times 10^{-4}$ | $.22109768 \times 10^{-3}$ | 5 1 | | $.35333960 \times 10^{-3}$ | $-.85089560 \times 10^{-3}$ | $.17658376 \times 10^{-2}$ |
| 4 2 | | $.24832710 \times 10^{-4}$ | $-.71212640 \times 10^{-4}$ | $.19279040 \times 10^{-3}$ | 6 2 | | $.24442540 \times 10^{-7}$ | $-.98720180 \times 10^{-7}$ | $.38479576 \times 10^{-6}$ | 7 2 | | $.10388920 \times 10^{-5}$ | $-.3522538 \times 10^{-5}$ | $.11852066 \times 10^{-4}$ |
| 6 3 | | $.23874000 \times 10^{-7}$ | $-.96594050 \times 10^{-7}$ | $.37503836 \times 10^{-6}$ | 8 3 | | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-10})$ | 9 3 | | $.31360600 \times 10^{-9}$ | $-.14513660 \times 10^{-8}$ | $.65312996 \times 10^{-8}$ |
| 8 4 | | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-10})$ | 10 4 | | | | | 11 4 | | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 0 | | 0.095433100 | -0.04932345 | 0.015903161 | 2 0 | | 0.029126520 | -0.020005335 | 0.010557104 | 3 0 | | 0.054866820 | -0.051882226 | 0.012950066 |
| 2 1 | | $.029739070$ | $-.017260257$ | $.86739840 \times 10^{-2}$ | 4 1 | | $.53194970 \times 10^{-2}$ | $-.48400606 \times 10^{-2}$ | $.57361367 \times 10^{-2}$ | 5 1 | | $.612421268$ | $-.97772690 \times 10^{-2}$ | $.62085268 \times 10^{-2}$ |
| 4 2 | | $.48226890 \times 10^{-2}$ | $-.45329920 \times 10^{-2}$ | $.3274881$ | 6 2 | | $.61569300 \times 10^{-3}$ | $-.71276280 \times 10^{-3}$ | $.74835490 \times 10^{-3}$ | 7 2 | | $.12409766 \times 10^{-2}$ | $-.12950717 \times 10^{-2}$ | $.17036187$ |
| 6 3 | | $.56899900 \times 10^{-3}$ | $-.6562420 \times 10^{-3}$ | $.67972050 \times 10^{-3}$ | 8 3 | | $.61569300 \times 10^{-3}$ | $-.61442680 \times 10^{-4}$ | $.81967840 \times 10^{-4}$ | 9 4 | | $.16735300 \times 10^{-3}$ | $-.22641690 \times 10^{-3}$ | $.25629990 \times 10^{-3}$ |
| 8 4 | | $.40145800 \times 10^{-4}$ | $-.57291340 \times 10^{-4}$ | $.76051130 \times 10^{-4}$ | 10 4 | | $.17347400 \times 10^{-5}$ | $-.29750440 \times 10^{-5}$ | $.48530590 \times 10^{-5}$ | 11 5 | | $.89691300 \times 10^{-5}$ | $-.14087260 \times 10^{-4}$ | $.20831384 \times 10^{-4}$ |
| 10 5 | | $.16405900 \times 10^{-5}$ | $-.28070770 \times 10^{-5}$ | $.45638950 \times 10^{-5}$ | 12 5 | | $.39904300 \times 10^{-7}$ | $-.79990500 \times 10^{-7}$ | $.15457672 \times 10^{-6}$ | 13 5 | | $.27577800 \times 10^{-6}$ | $-.31158800 \times 10^{-6}$ | $.52015680 \times 10^{-6}$ |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial^2 Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND INCHES

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.05$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial^2 Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial^2 Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial^2 Q_n}{\partial \lambda^2}$ |
|------------------|-----|----------------------------|---|---|-----|-----|----------------------------|---|---|-----|-----|----------------------------|---|---|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.13509490 | -0.04384787 | $0.83906890 \times 10^{-2}$ | 8 | 0 | 0.00594980 | -0.08660596 | $0.76177401 \times 10^{-2}$ | 1 | 0 | 0.09608980 | -0.05480685 | $0.64136690 \times 10^{-2}$ |
| 2 | 1 | .023899040 | -.020711169 | .34301981 | 4 | 1 | 0.08210590 | -.010861084 | .41609509 | 3 | 1 | .023132380 | -.015454288 | .49800251 |
| 4 | 2 | .018298625 | -.88282780 $\times 10^{-2}$ | .31764678 | 6 | 2 | .63906010 $\times 10^{-2}$ | -.37706130 $\times 10^{-2}$ | .18310882 | 5 | 2 | .011207439 | -.59618040 $\times 10^{-2}$ | .29328706 |
| 6 | 3 | .55087100 $\times 10^{-2}$ | -.31794990 | .15037764 | 8 | 3 | .12384130 | -.10740506 | .62528440 $\times 10^{-3}$ | 7 | 3 | .8997100 $\times 10^{-2}$ | -.1208230 | .10929020 |
| 8 | 4 | .12500040 | -.88296400 $\times 10^{-3}$ | .25280560 $\times 10^{-3}$ | 10 | 4 | .50014800 $\times 10^{-3}$ | -.24459810 $\times 10^{-3}$ | .10030780 | 9 | 4 | .65747800 $\times 10^{-3}$ | -.49404600 $\times 10^{-3}$ | .38928280 $\times 10^{-3}$ |
| 10 | 5 | .86785900 $\times 10^{-3}$ | -.01660780 | .15728270 | 12 | 5 | .45971300 $\times 10^{-4}$ | -.43977200 $\times 10^{-4}$ | .38116500 $\times 10^{-4}$ | 11 | 5 | .11596900 | -.10404030 | .80517950 $\times 10^{-4}$ |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.17269490 | -0.041310225 | $0.78190884 \times 10^{-2}$ | 8 | 0 | 0.10289484 | -0.029486925 | $0.76117818 \times 10^{-2}$ | 1 | 0 | 0.15222037 | -0.022999825 | $0.75688249 \times 10^{-2}$ |
| 2 | 1 | .07704060 | -.020490780 | .33190191 | 4 | 1 | .04876490 | -.012816325 | .38028826 | 3 | 1 | .02822270 | -.01218441 | .37780280 |
| 4 | 2 | .052924540 | -.010028258 | .8213283 | 6 | 2 | .61981720 $\times 10^{-2}$ | -.61981720 $\times 10^{-2}$ | .17322148 | 5 | 2 | .025928780 | -.80432540 $\times 10^{-2}$ | .19278202 |
| 6 | 3 | .013328050 | -.46912130 $\times 10^{-2}$ | .11984758 | 8 | 3 | .61460300 $\times 10^{-2}$ | -.29908010 | .87772130 $\times 10^{-3}$ | 7 | 3 | .98068000 $\times 10^{-2}$ | -.12608200 | .10928000 |
| 8 | 4 | .45904300 $\times 10^{-2}$ | -.80333790 | .64648890 $\times 10^{-3}$ | 10 | 4 | .80237000 | -.98634600 $\times 10^{-3}$ | .3921120 | 9 | 4 | .32603800 | -.14471630 | .52381780 $\times 10^{-3}$ |
| 10 | 5 | .17046100 | -.75843100 $\times 10^{-3}$ | .30987540 | 12 | 5 | .61819000 $\times 10^{-3}$ | -.32688700 | .15987030 | 11 | 5 | .10469700 | -.58288100 $\times 10^{-3}$ | .28895280 |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.20562920 | -0.029518848 | $0.43218892 \times 10^{-2}$ | 8 | 0 | 0.13943927 | -0.020619214 | $0.44692201 \times 10^{-2}$ | 1 | 0 | 0.17200885 | -0.020102358 | $0.44743075 \times 10^{-2}$ |
| 2 | 1 | .02619960 | -.018866258 | .19949177 | 4 | 1 | .06218020 | -.014525275 | .22560254 | 3 | 1 | .077786690 | -.018769867 | .21770672 |
| 4 | 2 | .043611310 | -.98294940 $\times 10^{-2}$ | .10430820 | 6 | 2 | .022273260 | -.69700690 $\times 10^{-2}$ | .11916975 | 5 | 2 | .034221740 | -.81528270 $\times 10^{-2}$ | .11624348 |
| 6 | 3 | .019728220 | -.46158890 | .61899900 $\times 10^{-3}$ | 8 | 3 | .012784980 | -.52116970 | .69540680 $\times 10^{-3}$ | 7 | 3 | .019427200 | -.59729940 | .66119600 $\times 10^{-3}$ |
| 8 | 4 | .87846700 $\times 10^{-2}$ | -.22861780 | .37551500 | 10 | 4 | .49117200 $\times 10^{-2}$ | -.12540160 | .32729600 | 9 | 4 | .66502400 $\times 10^{-2}$ | -.19054140 | .38025100 |
| 10 | 5 | .37502300 | -.11019100 | .22960000 | 12 | 5 | .19597100 | -.68127800 $\times 10^{-3}$ | .18636000 | 11 | 5 | .27569000 | -.82168800 $\times 10^{-3}$ | .21222200 |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.24308100 | -0.027843521 | $0.33642206 \times 10^{-2}$ | 8 | 0 | 0.17468623 | -0.020089059 | $0.32348875 \times 10^{-2}$ | 1 | 0 | 0.20694177 | -0.024411354 | $0.34905168 \times 10^{-2}$ |
| 2 | 1 | .10908406 | -.016724344 | .11516072 | 4 | 1 | .073802880 | -.014046649 | .14739070 | 3 | 1 | .022971400 | -.015467459 | .11307397 |
| 4 | 2 | .049675670 | -.76286710 $\times 10^{-2}$ | .39434800 $\times 10^{-3}$ | 6 | 2 | .022273260 | -.69700690 $\times 10^{-2}$ | .66407200 $\times 10^{-3}$ | 5 | 2 | .042249010 | -.7173740 $\times 10^{-2}$ | .22528000 $\times 10^{-3}$ |
| 6 | 3 | .088907730 | -.36494600 | .12582600 | 8 | 3 | .016065140 | -.51388490 | .58868300 | 7 | 3 | .019226280 | -.34337800 | .29226000 |
| 8 | 4 | .010642570 | -.150325600 | .81561000 $\times 10^{-4}$ | 10 | 4 | .78804800 $\times 10^{-2}$ | -.15874400 | .17879600 | 9 | 4 | .88913700 $\times 10^{-2}$ | -.16896900 | .14187100 |
| 10 | 5 | .49482000 $\times 10^{-2}$ | -.91602000 $\times 10^{-3}$ | .56628000 | 12 | 5 | .32718000 | -.74676000 $\times 10^{-3}$ | .10204600 | 11 | 5 | .40669000 | -.84294000 $\times 10^{-3}$ | .86369000 $\times 10^{-4}$ |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.27428250 | -0.026802777 | $0.485013090 \times 10^{-2}$ | 8 | 0 | 0.20780077 | -0.020702810 | $0.28569822 \times 10^{-2}$ | 1 | 0 | 0.22926465 | -0.023329784 | $0.28001646 \times 10^{-2}$ |
| 2 | 1 | .11732062 | -.014444744 | .59959060 $\times 10^{-3}$ | 4 | 1 | .090474380 | -.01282043 | .28442210 $\times 10^{-3}$ | 3 | 1 | .10380781 | -.01379096 | .78866300 $\times 10^{-3}$ |
| 4 | 2 | .021459880 | -.57970260 $\times 10^{-2}$ | -.10077000 $\times 10^{-4}$ | 6 | 2 | .040021640 | -.52228790 $\times 10^{-2}$ | .2847720 | 5 | 2 | .042568060 | -.57828660 $\times 10^{-2}$ | .14195000 |
| 6 | 3 | .088907730 | -.23787700 | -.15011000 $\times 10^{-3}$ | 8 | 3 | .017978470 | -.24356600 | .60515000 $\times 10^{-4}$ | 7 | 3 | .080484900 | -.24549100 | .25166000 $\times 10^{-4}$ |
| 8 | 4 | .010528400 | -.10191200 | -.11242000 | 10 | 4 | .81856000 $\times 10^{-2}$ | -.11149500 | .77800000 $\times 10^{-3}$ | 9 | 4 | .98939000 $\times 10^{-2}$ | -.10971100 | -.45836000 |
| 10 | 5 | .47992000 $\times 10^{-2}$ | -.46929000 $\times 10^{-3}$ | -.71254000 $\times 10^{-4}$ | 12 | 5 | .37760000 | -.53124000 $\times 10^{-3}$ | -.10900000 $\times 10^{-6}$ | 11 | 5 | .42019000 | -.21654000 $\times 10^{-3}$ | -.31470000 |

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TABLE I.- COMPUTED VALUES OF C_m , $J \frac{\partial C_m}{\partial \alpha}$, AND $\frac{\partial C_m}{\partial \alpha^2}$ FOR $\alpha = 0$ TO 5 AND VARIOUS

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(3) $L = 0.05$ - Continued

| P | α | $C_m(P)$ | $J \frac{\partial C_m}{\partial \alpha}$ | $\frac{\partial C_m}{\partial \alpha^2}$ | P | α | $C_m(P)$ | $J \frac{\partial C_m}{\partial \alpha}$ | $\frac{\partial C_m}{\partial \alpha^2}$ | P | α | $C_m(P)$ | $J \frac{\partial C_m}{\partial \alpha}$ | $\frac{\partial C_m}{\partial \alpha^2}$ |
|-----------------|----------|-----------------------------|--|--|----|----------|----------------------------|--|--|----|----------|-----------------------------|--|--|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.30542770 | -0.09428325 | 0.2200965×10^{-2} | 8 | 0 | 0.8589692 | -0.090251105 | 0.2722717×10^{-2} | 16 | 0 | 0.86971686 | -0.092578189 | 0.2294824×10^{-2} |
| 2 | 1 | .18270894 | -.012201112 | $-.2309450 \times 10^{-3}$ | 8 | 1 | .028989280 | -.011423185 | $-.2841718 \times 10^{-3}$ | 16 | 1 | .11069091 | -.01188482 | $-.3922040 \times 10^{-3}$ |
| 4 | 2 | .049775690 | -.3992290 $\times 10^{-2}$ | -.2469490 | 4 | 1 | .028989280 | -.011423185 | $-.2841718 \times 10^{-3}$ | 8 | 2 | .045640620 | -.4178490 $\times 10^{-2}$ | -.11940600 |
| 6 | 3 | .020274100 | -.11284800 | -.27692800 | 6 | 2 | .041824700 | -.25273540 $\times 10^{-2}$ | -.74910000 $\times 10^{-3}$ | 6 | 3 | .018997700 | -.13860600 | -.19159700 |
| 8 | 4 | .8599000 $\times 10^{-2}$ | -.24445000 $\times 10^{-3}$ | -.20445700 | 8 | 3 | .017288800 | -.15987000 | -.11228200 $\times 10^{-3}$ | 8 | 4 | .80197000 $\times 10^{-2}$ | -.42122000 $\times 10^{-3}$ | -.15189700 |
| 10 | 5 | .51111000 | | -.15069400 | 10 | 4 | .77513000 $\times 10^{-2}$ | -.54831000 $\times 10^{-3}$ | -.10941000 | 10 | 5 | .34575000 | -.11089000 | -.98744000 $\times 10^{-4}$ |
| | | | | | 12 | 5 | .53087000 | -.19459000 | -.69941000 $\times 10^{-4}$ | | | | | |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.33068440 | -0.092465779 | $0.18910618 \times 10^{-2}$ | 8 | 0 | 0.28724104 | -0.089621077 | $0.19681320 \times 10^{-2}$ | 16 | 0 | 0.29814813 | -0.091598788 | $0.18118804 \times 10^{-2}$ |
| 2 | 1 | .12401578 | -.010074346 | -.12439100 $\times 10^{-4}$ | 8 | 1 | .10400888 | -.08436280 $\times 10^{-2}$ | -.2673680 $\times 10^{-3}$ | 16 | 1 | .11399993 | -.020089977 | -.11712600 $\times 10^{-3}$ |
| 4 | 2 | .045292870 | -.29720900 $\times 10^{-2}$ | -.37827900 $\times 10^{-3}$ | 4 | 1 | .10400888 | -.08436280 $\times 10^{-2}$ | -.2673680 $\times 10^{-3}$ | 8 | 2 | .042890690 | -.26928000 $\times 10^{-2}$ | -.27515600 |
| 6 | 3 | .016044700 | -.18017000 $\times 10^{-3}$ | -.33200900 | 6 | 2 | .040021290 | -.29239800 | -.17980300 | 6 | 3 | .025768700 | -.42027000 $\times 10^{-3}$ | -.26969500 |
| 8 | 4 | .5718000 $\times 10^{-2}$ | .56916000 | -.22899800 | 8 | 3 | .017288800 | -.66920000 $\times 10^{-3}$ | -.21026700 | 8 | 4 | .56720000 $\times 10^{-2}$ | .16209000 | -.16941800 |
| 10 | 5 | .16409000 | .37769000 | -.13716000 | 10 | 4 | .77083000 $\times 10^{-2}$ | -.98000000 $\times 10^{-3}$ | -.19474600 | 10 | 5 | .19543000 | .25050000 | -.11778000 |
| | | | | | 12 | 5 | .21119000 | .12869000 $\times 10^{-3}$ | -.92890000 $\times 10^{-4}$ | | | | | |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.37621190 | -0.09189404 | $0.15444495 \times 10^{-2}$ | 8 | 0 | 0.29901111 | -0.08967804 | $0.16669949 \times 10^{-2}$ | 16 | 0 | 0.32480681 | -0.090609024 | 0.1615684×10^{-2} |
| 2 | 1 | .12240895 | -.80994990 $\times 10^{-2}$ | -.18700290 $\times 10^{-3}$ | 8 | 1 | .10592748 | -.08491690 $\times 10^{-2}$ | -.30782100 $\times 10^{-4}$ | 16 | 1 | .11417516 | -.22228190 $\times 10^{-2}$ | -.77129000 $\times 10^{-4}$ |
| 4 | 2 | .059199540 | -.99901800 $\times 10^{-3}$ | -.43132100 | 4 | 1 | .10592748 | -.08491690 $\times 10^{-2}$ | -.30782100 $\times 10^{-4}$ | 8 | 2 | .027999690 | -.13894000 | -.39926100 $\times 10^{-3}$ |
| 6 | 3 | .010970700 | .66813000 | -.33139400 | 6 | 2 | .09644980 | -.17028700 | -.27970700 $\times 10^{-3}$ | 6 | 3 | .011499800 | .39769000 $\times 10^{-3}$ | -.2894200 |
| 8 | 4 | .20776000 $\times 10^{-2}$ | .77708000 | -.20416600 | 8 | 3 | .011673500 | .09190000 $\times 10^{-4}$ | -.24741900 | 8 | 4 | .27520000 $\times 10^{-2}$ | .52827000 | -.1806200 |
| 10 | 5 | .29400000 $\times 10^{-3}$ | .79790000 | -.11428000 | 10 | 4 | .32513000 $\times 10^{-2}$ | .40946000 $\times 10^{-3}$ | -.16275600 | 10 | 5 | .20960000 $\times 10^{-3}$ | .44720000 | -.10714000 |
| | | | | | 12 | 5 | .60780000 $\times 10^{-3}$ | .74690000 | -.92690000 $\times 10^{-4}$ | | | | | |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.4139980 | -0.09352012 | $0.10969919 \times 10^{-2}$ | 8 | 0 | 0.37687420 | -0.087125459 | $0.11462928 \times 10^{-2}$ | 16 | 0 | 0.32845639 | -0.08289927 | $0.11029721 \times 10^{-2}$ |
| 2 | 1 | .12089960 | -.58767710 $\times 10^{-2}$ | -.40513690 $\times 10^{-3}$ | 8 | 1 | .099997900 | -.45277280 $\times 10^{-2}$ | -.29920010 $\times 10^{-3}$ | 16 | 1 | .10799980 | -.42431240 $\times 10^{-2}$ | -.32996200 $\times 10^{-3}$ |
| 4 | 2 | .019922900 | .14213000 | -.41962100 | 4 | 1 | .099997900 | -.45277280 $\times 10^{-2}$ | -.29920010 $\times 10^{-3}$ | 8 | 2 | .020202200 | .12226900 | -.52027700 |
| 6 | 3 | .12420000 $\times 10^{-2}$ | .16208000 | -.22891000 | 6 | 2 | .021640900 | .68103000 $\times 10^{-3}$ | -.34400700 | 6 | 3 | -.30730000 $\times 10^{-3}$ | .14228900 | -.22892100 |
| 8 | 4 | .46604000 | .10502600 | -.99280000 $\times 10^{-4}$ | 8 | 3 | .10130000 $\times 10^{-2}$ | .12130000 $\times 10^{-2}$ | -.22821600 | 8 | 4 | -.36760000 $\times 10^{-2}$ | .99592000 $\times 10^{-3}$ | -.97720000 $\times 10^{-4}$ |
| 10 | 5 | .54360000 | .98800000 $\times 10^{-3}$ | -.29700000 | 10 | 4 | -.27900000 | .09669000 $\times 10^{-3}$ | -.97730000 $\times 10^{-4}$ | 10 | 5 | -.8940000 | .49900000 | -.3420000 |
| | | | | | 12 | 5 | -.24440000 | .49730000 | -.37100000 | | | | | |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.4621130 | -0.08687433 | 0.7263980×10^{-3} | 8 | 0 | 0.4102179 | -0.082921681 | $0.28697760 \times 10^{-3}$ | 16 | 0 | 0.42991720 | -0.086102901 | $0.79496790 \times 10^{-3}$ |
| 2 | 1 | .085608000 | -.60022000 $\times 10^{-3}$ | -.16282770 | 8 | 1 | .021299400 | -.11424120 $\times 10^{-2}$ | -.37244690 | 16 | 1 | .02781200 | -.10449270 $\times 10^{-2}$ | -.42027190 |
| 4 | 2 | -.69920000 $\times 10^{-3}$ | .26040000 $\times 10^{-2}$ | -.31615800 | 4 | 1 | .021299400 | -.11424120 $\times 10^{-2}$ | -.37244690 | 8 | 2 | .18445000 $\times 10^{-2}$ | -.22940000 | -.30969800 |
| 6 | 3 | -.010866100 | .16739600 | -.10704900 | 6 | 2 | .39680000 $\times 10^{-2}$ | .19972800 | -.22998000 | 6 | 3 | -.98669000 | .19441100 | -.11122200 |
| 8 | 4 | .72370000 $\times 10^{-2}$ | .62120000 $\times 10^{-3}$ | | 8 | 3 | -.77186000 | .14219000 | -.11378900 | 8 | 4 | -.67940000 | .66990000 $\times 10^{-3}$ | -.12940000 $\times 10^{-4}$ |
| 10 | 5 | .34960000 | .18490000 | | 10 | 4 | -.58990000 | .01130000 $\times 10^{-3}$ | -.21760000 $\times 10^{-4}$ | 10 | 5 | -.32840000 | .20520000 | .14900000 |
| | | | | | 12 | 5 | -.30210000 | .21660000 | -.98700000 $\times 10^{-5}$ | | | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INFINITE

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.05$ - Concluded

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ |
|-----------------|-----|-----------------------------|---------------------------------------|---------------------------------------|-----|-----|-----------------------------|---------------------------------------|---------------------------------------|-----|-----|-----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.50482570 | -0.024758725 | $0.56450150 \times 10^{-5}$ | 1 | 0 | 0.48055333 | -0.024180701 | $0.59036440 \times 10^{-5}$ | 2 | 0 | 0.45647275 | -0.02376752 | $0.61607680 \times 10^{-5}$ |
| 2 | 1 | .052956000 | $.18946520 \times 10^{-2}$ | -.46205110 | 3 | 1 | .054621600 | $.14420490 \times 10^{-2}$ | -.43707660 | 4 | 1 | .057505000 | $.10204980 \times 10^{-2}$ | -.40602830 |
| 4 | 2 | -.017306700 | .29377000 | -.20812200 | 5 | 2 | -.014672200 | .27300400 | -.20708400 | 6 | 2 | -.012045400 | .22240500 | -.20470600 |
| 6 | 3 | -.014678900 | .11878100 | $-.19111000 \times 10^{-4}$ | 7 | 3 | -.013500300 | .11649000 | $-.26786000 \times 10^{-4}$ | 8 | 3 | -.012349200 | .11547800 | $-.33504000 \times 10^{-4}$ |
| 8 | 4 | -.61340000 $\times 10^{-2}$ | $.21050000 \times 10^{-3}$ | .33780000 | 9 | 4 | -.59010000 $\times 10^{-2}$ | $.24190000 \times 10^{-3}$ | .27740000 | 10 | 4 | -.56450000 $\times 10^{-2}$ | $.26740000 \times 10^{-3}$ | .22070000 |
| 10 | 5 | -.15950000 | $-.94500000 \times 10^{-4}$ | -.30720000 | 11 | 5 | -.1570000 | $-.69700000 \times 10^{-4}$ | -.28040000 | 12 | 5 | -.15890000 | $-.37800000 \times 10^{-4}$ | -.27340000 |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.54175370 | -0.022912316 | $0.43134200 \times 10^{-5}$ | 1 | 0 | 0.51906060 | -0.022470426 | $0.45220510 \times 10^{-5}$ | 2 | 0 | 0.49681948 | -0.022008990 | $0.47161820 \times 10^{-5}$ |
| 2 | 1 | .018818500 | $.37735000 \times 10^{-2}$ | -.44012700 | 3 | 1 | .022574200 | $.33435700 \times 10^{-2}$ | -.41968260 | 4 | 1 | .025125000 | $.29342670 \times 10^{-2}$ | -.39890590 |
| 4 | 2 | -.023452200 | .27382800 | -.11327700 | 5 | 2 | -.020722600 | .26207500 | -.11956600 | 6 | 2 | -.024190700 | .24995600 | -.122285600 |
| 6 | 3 | -.013950000 | $.39250000 \times 10^{-3}$ | $-.31750000 \times 10^{-4}$ | 7 | 3 | -.013359000 | $.62090000 \times 10^{-3}$ | $-.24610000 \times 10^{-4}$ | 8 | 3 | -.012705000 | $.62250000 \times 10^{-3}$ | $-.17950000 \times 10^{-4}$ |
| 8 | 4 | -.30920000 $\times 10^{-2}$ | -.14730000 | .42690000 | 9 | 4 | -.32110000 $\times 10^{-2}$ | -.10570000 | .39540000 | 10 | 4 | -.32750000 $\times 10^{-2}$ | $-.66500000 \times 10^{-4}$ | .36070000 |
| 10 | 5 | | | | 11 | 5 | | | | 12 | 5 | | | |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.65045420 | -0.017477262 | $0.17744210 \times 10^{-5}$ | 1 | 0 | 0.65306776 | -0.017295451 | $0.18616880 \times 10^{-5}$ | 2 | 0 | 0.61585676 | -0.017105023 | $0.19439510 \times 10^{-5}$ |
| 2 | 1 | -.12385290 | $.74375660 \times 10^{-2}$ | -.22987340 | 3 | 1 | -.11649960 | $.72116900 \times 10^{-2}$ | -.22202550 | 4 | 1 | -.10942830 | $.69517100 \times 10^{-2}$ | -.27790610 |
| 4 | 2 | -.026715000 | $.12461000 \times 10^{-3}$ | $.74915000 \times 10^{-4}$ | 5 | 2 | -.026554000 | $.19649000 \times 10^{-3}$ | $.69097000 \times 10^{-4}$ | 6 | 2 | -.026526000 | $.26266000 \times 10^{-3}$ | $.65457000 \times 10^{-4}$ |
| 6 | 3 | .66650000 $\times 10^{-2}$ | -.72040000 | .40600000 | 7 | 3 | .59570000 $\times 10^{-2}$ | -.68040000 | .40120000 | 8 | 3 | .52810000 $\times 10^{-2}$ | -.64160000 | .39570000 |
| 8 | 4 | | | | 9 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 11 | 5 | | | | 12 | 5 | | | |

TABLE I.- COMPUTED VALUES OF q_n , $\int \frac{\partial q_n}{\partial \xi}$, AND $\frac{\partial q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.1$

| P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ | P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ |
|------------------|-----|-----------------------------|--|---------------------------------------|------|-----|-----------------------------|--|---------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | |
| 0 0 | 0 | $0.70968000 \times 10^{-2}$ | -0.099290518 | 0.88215301 | 1 0 | 0 | $0(10^{-31})$ | $0(10^{-29})$ | $0(10^{-27})$ |
| 2 1 | 1 | | | | 5 1 | 1 | | | |
| 4 1 | 1 | | | | 4 1 | 1 | | | |
| $\lambda = 0.05$ | | | | | | | | | |
| 0 0 | 0 | 0.021054700 | -0.097694552 | 0.28756456 | 1 0 | 0 | $0.12564700 \times 10^{-5}$ | $-0.19270524 \times 10^{-4}$ | $0.28471514 \times 10^{-3}$ |
| 2 1 | 1 | $0(10^{-15})$ | $0(10^{-14})$ | $0(10^{-12})$ | 3 1 | 1 | $0(10^{-30})$ | $0(10^{-29})$ | $0(10^{-27})$ |
| 4 1 | 1 | | | | 5 1 | 1 | | | |
| $\lambda = 0.10$ | | | | | | | | | |
| 0 0 | 0 | 0.054705900 | -0.096929409 | 0.16875947 | 1 0 | 0 | $0.38869800 \times 10^{-9}$ | $-0.24958542 \times 10^{-8}$ | 0.014555399 |
| 2 1 | 1 | $.69277600 \times 10^{-7}$ | $-.75588950 \times 10^{-6}$ | $.78710965 \times 10^{-5}$ | 2 0 | 0 | $0.70516600 \times 10^{-7}$ | $-0.76736864 \times 10^{-6}$ | $0.80251722 \times 10^{-5}$ |
| 4 2 | 2 | $0(10^{-20})$ | $0(10^{-19})$ | $0(10^{-18})$ | 4 1 | 1 | $0(10^{-20})$ | $0(10^{-19})$ | $0(10^{-18})$ |
| $\lambda = 0.20$ | | | | | | | | | |
| 0 0 | 0 | 0.067547800 | -0.095245220 | 0.075881682 | 1 0 | 0 | 0.011397310 | -0.025215530 | 0.045227094 |
| 2 1 | 1 | $.72586900 \times 10^{-3}$ | $-.25090265 \times 10^{-2}$ | $.55999759 \times 10^{-2}$ | 2 0 | 0 | $0.76695400 \times 10^{-3}$ | $-0.24580586 \times 10^{-2}$ | $0.70766822 \times 10^{-2}$ |
| 4 2 | 2 | $.15022740 \times 10^{-6}$ | $-.70644990 \times 10^{-6}$ | $.56806285 \times 10^{-5}$ | 4 1 | 1 | $.13495370 \times 10^{-6}$ | $-.73296610 \times 10^{-6}$ | $.58245700 \times 10^{-5}$ |
| 6 3 | 3 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-10})$ | 6 2 | 2 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-10})$ |
| 7 3 | 3 | | | | 7 3 | 3 | $0(10^{-16})$ | $0(10^{-15})$ | $0(10^{-14})$ |
| $\lambda = 0.40$ | | | | | | | | | |
| 0 0 | 0 | 0.12313560 | -0.087126659 | 0.055884458 | 1 0 | 0 | 0.058927680 | -0.051722226 | 0.052976851 |
| 2 1 | 1 | $.028756260$ | $-.020089475$ | $.017033541$ | 3 1 | 1 | $.56705170 \times 10^{-2}$ | $-.74891410 \times 10^{-2}$ | $.84406050 \times 10^{-2}$ |
| 4 2 | 2 | $.11831120 \times 10^{-2}$ | $-.12523037 \times 10^{-2}$ | $.25781956 \times 10^{-2}$ | 5 2 | 2 | $.21566120 \times 10^{-3}$ | $-.59688070 \times 10^{-3}$ | $.67058040 \times 10^{-3}$ |
| 6 3 | 3 | $.27157500 \times 10^{-4}$ | $-.57385150 \times 10^{-4}$ | $.11549885 \times 10^{-3}$ | 7 3 | 3 | $.68408200 \times 10^{-5}$ | $-.68256560 \times 10^{-5}$ | $.15572987 \times 10^{-4}$ |
| 8 4 | 4 | $.20671500 \times 10^{-6}$ | $-.55551400 \times 10^{-6}$ | $.14327048 \times 10^{-5}$ | 9 4 | 4 | $.12109800 \times 10^{-7}$ | $-.56131600 \times 10^{-7}$ | $.10430800 \times 10^{-6}$ |
| 10 5 | 5 | $.45565000 \times 10^{-9}$ | $-.16251200 \times 10^{-8}$ | $.51851410 \times 10^{-8}$ | 11 5 | 5 | $0(10^{-10})$ | $0(10^{-10})$ | $.20071240 \times 10^{-9}$ |
| 12 5 | 5 | | | | 12 5 | 5 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-11})$ |

TABLE I.- COMPUTED VALUES OF C_u , $J \frac{\partial C_u}{\partial \lambda}$, AND $\frac{\partial C_u}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INFINITE

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.1$ - Continued

| P | n | $C_u(P)$ | $J \frac{\partial C_u}{\partial \lambda}$ | $\frac{\partial C_u}{\partial \lambda}$ | P | n | $C_u(P)$ | $J \frac{\partial C_u}{\partial \lambda}$ | $\frac{\partial C_u}{\partial \lambda}$ | P | n | $C_u(P)$ | $J \frac{\partial C_u}{\partial \lambda}$ | $\frac{\partial C_u}{\partial \lambda}$ |
|------------------|-----|-------------------------------|---|---|-----|-----|-------------------------------|---|---|-----|-----|-------------------------------|---|---|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.1862190 | -0.00175613 | 0.00256190 | 0 | 0 | 0.056613780 | -0.022344807 | 0.012529102 | 1 | 0 | 0.11189555 | -0.02779438 | 0.002762771 |
| 1 | 1 | 0.04769460 | -0.002312695 | 0.011821506 | 1 | 1 | 0.011728440 | -0.02268310 x 10 ⁻² | 0.04269280 x 10 ⁻² | 3 | 1 | 0.08976990 | -0.017502364 | 0.00208470 x 10 ⁻² |
| 4 | 4 | 0.0402480 x 10 ⁻² | -0.74113280 x 10 ⁻² | 0.4712170 x 10 ⁻² | 4 | 4 | 0.01728440 | -0.02268310 x 10 ⁻² | 0.04269280 x 10 ⁻² | 5 | 4 | 0.0402700 x 10 ⁻² | -0.2664370 x 10 ⁻² | 0.0023275 |
| 6 | 5 | 0.1204180 | -0.12627490 | 0.11829021 | 6 | 5 | 0.1254710 x 10 ⁻² | -0.12792340 | 0.12368471 | 7 | 5 | 0.03861400 x 10 ⁻² | -0.49212600 x 10 ⁻² | 0.0118820 x 10 ⁻² |
| 8 | 4 | 0.11368400 x 10 ⁻² | -0.14842150 x 10 ⁻² | 0.16270860 x 10 ⁻² | 8 | 3 | 0.13182900 x 10 ⁻² | -0.1627290 x 10 ⁻² | 0.12444200 x 10 ⁻² | 9 | 4 | 0.04015100 x 10 ⁻² | -0.4822500 x 10 ⁻² | 0.0428420 x 10 ⁻² |
| 10 | 5 | 0.63047000 x 10 ⁻² | -0.04682000 x 10 ⁻² | 0.13408200 x 10 ⁻² | 10 | 4 | 0.71423000 x 10 ⁻² | -0.1021190 x 10 ⁻² | 0.12926280 x 10 ⁻² | 11 | 5 | 0.13025600 x 10 ⁻² | -0.2154100 x 10 ⁻² | 0.0351220 x 10 ⁻² |
| 12 | 5 | 0.23736000 x 10 ⁻² | -0.4196800 x 10 ⁻² | 0.71319200 x 10 ⁻² | 12 | 5 | 0.23736000 x 10 ⁻² | -0.4196800 x 10 ⁻² | 0.71319200 x 10 ⁻² | | | | | |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.23186430 | -0.078211250 | 0.01466096 | 0 | 0 | 0.10222256 | -0.046772438 | 0.014577283 | 1 | 0 | 0.16222722 | -0.061731688 | 0.015274023 |
| 1 | 1 | 0.071304790 | -0.082629403 | 0.68262990 x 10 ⁻² | 1 | 1 | 0.08262910 | -0.02518415 | 0.5562210 x 10 ⁻² | 3 | 1 | 0.04794410 | -0.021504275 | 0.69274860 x 10 ⁻² |
| 4 | 4 | 0.081207670 | -0.012020632 | 0.34224460 | 4 | 4 | 0.08262910 | -0.02518415 | 0.5562210 x 10 ⁻² | 5 | 4 | 0.04606760 | -0.69241020 x 10 ⁻² | 0.2927700 |
| 6 | 5 | 0.20208200 x 10 ⁻² | -0.21292070 x 10 ⁻² | 0.12928290 | 6 | 2 | 0.70890000 x 10 ⁻² | -0.12928210 x 10 ⁻² | 0.20202090 | 7 | 5 | 0.20215000 x 10 ⁻² | -0.12928210 x 10 ⁻² | 0.12928290 |
| 8 | 4 | 0.30837300 | -0.78168200 x 10 ⁻² | 0.47844000 x 10 ⁻² | 8 | 3 | 0.12877800 | -0.10222256 | 0.6945090 x 10 ⁻² | 9 | 4 | 0.20202000 x 10 ⁻² | -0.40200700 x 10 ⁻² | 0.28111500 x 10 ⁻² |
| 10 | 5 | 0.17808000 x 10 ⁻² | -0.13262900 | 0.11681900 | 10 | 4 | 0.28077000 x 10 ⁻² | -0.19827100 x 10 ⁻² | 0.12134990 | 11 | 5 | 0.78168200 x 10 ⁻² | -0.68019000 x 10 ⁻² | 0.57924500 x 10 ⁻² |
| 12 | 5 | 0.27482000 x 10 ⁻² | -0.27482000 x 10 ⁻² | 0.27482000 x 10 ⁻² | 12 | 5 | 0.27482000 x 10 ⁻² | -0.27482000 x 10 ⁻² | 0.27482000 x 10 ⁻² | | | | | |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.27642190 | -0.072227048 | 0.010620436 | 0 | 0 | 0.12279120 | -0.020022327 | 0.011134186 | 1 | 0 | 0.20221024 | -0.061322214 | 0.011262220 |
| 1 | 1 | 0.021200790 | -0.082222222 | 0.20222220 x 10 ⁻² | 1 | 1 | 0.12279120 | -0.020022327 | 0.011134186 | 3 | 1 | 0.02222070 | -0.020222270 | 0.42121600 x 10 ⁻² |
| 4 | 4 | 0.022220790 | -0.022220790 x 10 ⁻² | 0.16020060 | 4 | 4 | 0.04022170 | -0.01747727 | 0.42020470 x 10 ⁻² | 5 | 4 | 0.02222190 | -0.77222200 x 10 ⁻² | 0.12647270 |
| 6 | 5 | 0.04677700 x 10 ⁻² | -0.222220100 | 0.65117100 x 10 ⁻² | 6 | 2 | 0.014416200 | -0.22222250 x 10 ⁻² | 0.12942160 | 7 | 5 | 0.04222000 x 10 ⁻² | -0.27119000 | 0.65408400 x 10 ⁻² |
| 8 | 4 | 0.27222000 | -0.12642200 | 0.40222000 | 8 | 3 | 0.02222000 x 10 ⁻² | -0.12222000 | 0.22122000 x 10 ⁻² | 9 | 4 | 0.12222000 | -0.27119000 x 10 ⁻² | 0.54422000 |
| 10 | 5 | 0.72022000 x 10 ⁻² | -0.22222000 x 10 ⁻² | 0.12222000 | 10 | 4 | 0.10177200 | -0.26217000 x 10 ⁻² | 0.22122000 | 11 | 5 | 0.22222000 x 10 ⁻² | -0.27119000 | 0.12222000 |
| 12 | 5 | 0.22222000 x 10 ⁻² | -0.22222000 x 10 ⁻² | 0.22222000 x 10 ⁻² | 12 | 5 | 0.22222000 x 10 ⁻² | -0.22222000 x 10 ⁻² | 0.22222000 x 10 ⁻² | | | | | |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.31628000 | -0.06211968 | 0.02222000 x 10 ⁻² | 0 | 0 | 0.12222000 | -0.02222000 | 0.01222000 | 1 | 0 | 0.22222000 | -0.02222000 | 0.02222000 x 10 ⁻² |
| 1 | 1 | 0.01222000 | -0.02222000 | 0.12222000 | 1 | 1 | 0.12222000 | -0.02222000 | 0.01222000 | 3 | 1 | 0.02222000 | -0.02222000 | 0.22222000 |
| 4 | 4 | 0.02222000 | -0.02222000 x 10 ⁻² | 0.12222000 x 10 ⁻² | 4 | 4 | 0.02222000 | -0.02222000 | 0.12222000 | 5 | 4 | 0.02222000 | -0.72222000 x 10 ⁻² | 0.71748000 x 10 ⁻² |
| 6 | 5 | 0.11122000 | -0.22222000 | 0.12222000 | 6 | 2 | 0.02222000 | -0.22222000 x 10 ⁻² | 0.12222000 | 7 | 5 | 0.02222000 x 10 ⁻² | -0.22222000 | 0.12222000 |
| 8 | 4 | 0.22222000 x 10 ⁻² | -0.12222000 | 0.22222000 x 10 ⁻² | 8 | 3 | 0.02222000 | -0.22222000 | 0.12222000 | 9 | 4 | 0.02222000 | -0.22222000 x 10 ⁻² | 0.12222000 |
| 10 | 5 | 0.12222000 | -0.12222000 x 10 ⁻² | 0.12222000 | 10 | 4 | 0.02222000 | -0.22222000 x 10 ⁻² | 0.12222000 | 11 | 5 | 0.02222000 x 10 ⁻² | -0.22222000 | 0.12222000 x 10 ⁻² |
| 12 | 5 | 0.02222000 x 10 ⁻² | -0.22222000 x 10 ⁻² | 0.02222000 x 10 ⁻² | 12 | 5 | 0.02222000 x 10 ⁻² | -0.22222000 x 10 ⁻² | 0.02222000 x 10 ⁻² | | | | | |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.22222000 | -0.02222000 | 0.02222000 x 10 ⁻² | 0 | 0 | 0.02222000 | -0.02222000 | 0.02222000 | 1 | 0 | 0.02222000 | -0.02222000 | 0.02222000 x 10 ⁻² |
| 1 | 1 | 0.02222000 | -0.02222000 | 0.02222000 x 10 ⁻² | 1 | 1 | 0.02222000 | -0.02222000 | 0.02222000 | 3 | 1 | 0.02222000 | -0.02222000 | 0.02222000 x 10 ⁻² |
| 4 | 4 | 0.02222000 | -0.02222000 x 10 ⁻² | 0.02222000 x 10 ⁻² | 4 | 4 | 0.02222000 | -0.02222000 | 0.02222000 | 5 | 4 | 0.02222000 | -0.02222000 | 0.02222000 x 10 ⁻² |
| 6 | 5 | 0.02222000 | -0.02222000 | 0.02222000 | 6 | 2 | 0.02222000 | -0.02222000 | 0.02222000 | 7 | 5 | 0.02222000 x 10 ⁻² | -0.02222000 | 0.02222000 x 10 ⁻² |
| 8 | 4 | 0.02222000 x 10 ⁻² | -0.02222000 x 10 ⁻² | 0.02222000 | 8 | 3 | 0.02222000 x 10 ⁻² | -0.02222000 | 0.02222000 | 9 | 4 | 0.02222000 | -0.02222000 x 10 ⁻² | 0.02222000 x 10 ⁻² |
| 10 | 5 | 0.02222000 | -0.02222000 | 0.02222000 | 10 | 4 | 0.02222000 | -0.02222000 | 0.02222000 | 11 | 5 | 0.02222000 | -0.02222000 | 0.02222000 |
| 12 | 5 | 0.02222000 x 10 ⁻² | -0.02222000 x 10 ⁻² | 0.02222000 x 10 ⁻² | 12 | 5 | 0.02222000 x 10 ⁻² | -0.02222000 x 10 ⁻² | 0.02222000 x 10 ⁻² | | | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(e) L = 0.1 - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ |
|-----------------|---|-----------------------------|---|---|----|---|-----------------------------|---|---|----|---|-----------------------------|---|---|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.38739490 | -0.061264905 | $0.50843240 \times 10^{-2}$ | 8 | 0 | 0.2748917 | -0.090442755 | $0.56799480 \times 10^{-2}$ | 10 | 0 | 0.38867929 | -0.056017484 | $0.54446630 \times 10^{-2}$ |
| 2 | 1 | .10436620 | -.018463120 | $-.61128200 \times 10^{-3}$ | 8 | 1 | .079912000 | -.012515980 | $-.48877100 \times 10^{-3}$ | 10 | 1 | .091701500 | -.012762530 | $-.79990000 \times 10^{-3}$ |
| 4 | 2 | .028141250 | $-.17998800 \times 10^{-2}$ | -.94868800 | 4 | 1 | .079912000 | -.012515980 | $-.48877100 \times 10^{-3}$ | 8 | 2 | .029529310 | $-.29658100 \times 10^{-2}$ | $-.58762200 \times 10^{-3}$ |
| 6 | 3 | $-.77048000 \times 10^{-2}$ | $-.90200000 \times 10^{-4}$ | -.57029000 | 6 | 2 | .085128200 | $-.89928600 \times 10^{-2}$ | -.28067100 | 6 | 3 | $-.75489000 \times 10^{-2}$ | $-.38749000 \times 10^{-3}$ | $-.38896000$ |
| 8 | 4 | .82124000 | $-.88490000 \times 10^{-3}$ | -.27909000 | 8 | 3 | .68999000 $\times 10^{-2}$ | $-.69225000 \times 10^{-3}$ | -.23205000 | 8 | 4 | .23158000 | | $-.19246000$ |
| 10 | 5 | $-.71790000 \times 10^{-3}$ | .18970000 | -.12288000 | 10 | 4 | .22200000 | -.16420000 | -.31844000 | 10 | 5 | $-.77720000 \times 10^{-3}$ | | $-.82960000 \times 10^{-4}$ |
| | | | | | 12 | 5 | $-.75970000 \times 10^{-3}$ | $-.47200000 \times 10^{-4}$ | $-.50430000 \times 10^{-4}$ | | | | | |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.41214260 | -0.050267799 | $0.40992266 \times 10^{-2}$ | 8 | 0 | 0.31042601 | -0.049935884 | $0.46769697 \times 10^{-2}$ | 10 | 0 | 0.36206679 | -0.059906994 | $0.44449200 \times 10^{-2}$ |
| 2 | 1 | .092562200 | $-.84567600 \times 10^{-2}$ | -.13028990 | 8 | 1 | .079912000 | $-.96832600 \times 10^{-2}$ | $-.16770800 \times 10^{-3}$ | 10 | 1 | .089443000 | $-.90413000 \times 10^{-2}$ | $-.60228800 \times 10^{-3}$ |
| 4 | 2 | .021070600 | $-.43512000 \times 10^{-3}$ | -.10949220 | 4 | 1 | .079912000 | $-.96832600 \times 10^{-2}$ | $-.16770800 \times 10^{-3}$ | 8 | 2 | .020983500 | $-.54499000 \times 10^{-3}$ | $-.82811300 \times 10^{-3}$ |
| 6 | 3 | $-.36638000 \times 10^{-2}$ | $-.11226900 \times 10^{-2}$ | $-.59264000 \times 10^{-3}$ | 6 | 2 | .0200657000 | -.12472500 | -.38144400 | 6 | 3 | .45151000 $\times 10^{-2}$ | .59113000 | $-.47850000$ |
| 8 | 4 | $-.27200000 \times 10^{-3}$ | $-.68990000 \times 10^{-3}$ | -.27546000 | 8 | 3 | .48909000 $\times 10^{-2}$ | $-.17326000 \times 10^{-3}$ | -.36178000 | 8 | 4 | .69900000 $\times 10^{-3}$ | $-.45990000 \times 10^{-3}$ | $-.28634000$ |
| 10 | 5 | -.12400000 | .54520000 | -.11201000 | 10 | 4 | .11744000 | .83850000 | -.17621000 | 10 | 5 | .14800000 | .25120000 | $-.97640000 \times 10^{-4}$ |
| | | | | | 12 | 5 | $-.30700000 \times 10^{-3}$ | -.13870000 | $-.78760000 \times 10^{-4}$ | | | | | |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.44659380 | -0.053160617 | $0.55849587 \times 10^{-2}$ | 8 | 0 | 0.3428394 | -0.046022414 | $0.38961289 \times 10^{-2}$ | 10 | 0 | 0.39277022 | -0.051820084 | $0.56830320 \times 10^{-2}$ |
| 2 | 1 | .088771200 | $-.48613100 \times 10^{-2}$ | -.13902440 | 8 | 1 | .076812100 | $-.68326600 \times 10^{-2}$ | $-.61127100 \times 10^{-3}$ | 10 | 1 | .083284700 | $-.60449200 \times 10^{-2}$ | $-.98230000 \times 10^{-3}$ |
| 4 | 2 | .012720900 | -.21131100 | -.10839910 | 4 | 1 | .076812100 | $-.68326600 \times 10^{-2}$ | $-.61127100 \times 10^{-3}$ | 8 | 2 | .014328400 | .11225600 | -.90100000 |
| 6 | 3 | $-.57880000 \times 10^{-3}$ | -.17578000 | $-.51268000 \times 10^{-3}$ | 6 | 2 | .019028200 | $-.51222000 \times 10^{-3}$ | -.7194300 | 6 | 3 | $-.96800000 \times 10^{-3}$ | .12754000 | $-.45206000$ |
| 8 | 4 | $-.12290000 \times 10^{-2}$ | $-.89990000 \times 10^{-3}$ | -.20739000 | 8 | 3 | .20002000 $\times 10^{-2}$ | .87780000 | -.38370000 | 8 | 4 | | $-.45990000$ | $-.19201000$ |
| 10 | 5 | | .41480000 | | 10 | 4 | | .51890000 | -.16927000 | 10 | 5 | | | |
| | | | | | 12 | 5 | | -.25920000 | | | | | | |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.50761900 | -0.049282109 | $0.22126996 \times 10^{-2}$ | 8 | 0 | 0.41338888 | -0.044418872 | $0.28315542 \times 10^{-2}$ | 10 | 0 | 0.45922203 | -0.046920751 | $0.24159288 \times 10^{-2}$ |
| 2 | 1 | .079319500 | $-.29469400 \times 10^{-2}$ | -.16296990 | 8 | 1 | .079319100 | $-.42704000 \times 10^{-3}$ | -.11328990 | 10 | 1 | .079399000 | $-.84131000 \times 10^{-3}$ | $-.13389840$ |
| 4 | 2 | $-.85702000 \times 10^{-2}$ | .42940900 | $-.78560000 \times 10^{-3}$ | 4 | 1 | .079319100 | $-.42704000 \times 10^{-3}$ | -.11328990 | 8 | 2 | $-.44909000 \times 10^{-2}$ | $-.39497400 \times 10^{-2}$ | $-.72109000 \times 10^{-3}$ |
| 6 | 3 | -.83712000 | .18946000 | -.18922000 | 6 | 2 | $-.12904000 \times 10^{-2}$ | $-.88569000 \times 10^{-2}$ | $-.69432700 \times 10^{-3}$ | 6 | 3 | -.65680000 | .16695000 | $-.20698000$ |
| 8 | 4 | -.37730000 | $-.61110000 \times 10^{-3}$ | | 8 | 3 | .49720000 | .14859000 | -.21466000 | 8 | 4 | -.52560000 | $-.59760000 \times 10^{-3}$ | $-.25200000 \times 10^{-4}$ |
| 10 | 5 | -.11620000 | | | 10 | 4 | -.29730000 | $-.96220000 \times 10^{-3}$ | | 10 | 5 | | | |
| | | | | | 12 | 5 | | | | | | | | |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.59797890 | -0.044202170 | $0.15226670 \times 10^{-2}$ | 8 | 0 | 0.47261911 | -0.040267834 | $0.17949837 \times 10^{-2}$ | 10 | 0 | 0.51496405 | -0.042601995 | $0.18709840 \times 10^{-2}$ |
| 2 | 1 | $-.97990000 \times 10^{-2}$ | $-.79907000 \times 10^{-2}$ | -.15721940 | 8 | 1 | .021621900 | $-.45649600 \times 10^{-2}$ | -.12706170 | 10 | 1 | .016996900 | $-.99067500 \times 10^{-2}$ | $-.12130090$ |
| 4 | 2 | -.02206000 | .43415000 | $-.57997000 \times 10^{-3}$ | 4 | 1 | .021621900 | $-.45649600 \times 10^{-2}$ | -.12706170 | 8 | 2 | $-.019706100$ | .59229000 | $-.59690000 \times 10^{-3}$ |
| 6 | 3 | $-.28490000 \times 10^{-2}$ | $-.97310000 \times 10^{-3}$ | | 6 | 2 | $-.019949900$ | $-.40299000 \times 10^{-3}$ | -.40299000 | 6 | 3 | $-.28470000 \times 10^{-2}$ | .10070000 | $-.16990000 \times 10^{-4}$ |
| 8 | 4 | -.22220000 | | | 8 | 3 | $-.78740000 \times 10^{-2}$ | .10099000 | $-.11400000 \times 10^{-4}$ | 8 | 4 | -.22130000 | | |
| 10 | 5 | | | | 10 | 4 | -.22140000 | $-.14400000 \times 10^{-3}$ | -.57100000 | 10 | 5 | | | |
| | | | | | 12 | 5 | | | | | | | | |

TABLE I.- COMPILED VALUES OF q_n , $\int \frac{\partial q_n}{\partial \xi}$, AND $\frac{\partial q_n}{\partial \xi^2}$ FOR $n=0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(e) $L = 0.1$ - Concluded

| P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ | P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ | P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ |
|-----------------|-----|-----------------------------|--|---------------------------------------|-----|-----|-----------------------------|--|---------------------------------------|-----|-----|-----------------------------|--|---------------------------------------|
| $\lambda = 5.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.59922770 | -0.040007426 | 0.10967548 $\times 10^{-2}$ | 2 | 0 | 0.52224412 | -0.07760261 | 0.18922285 $\times 10^{-2}$ | 1 | 0 | 0.56048449 | -0.038277451 | 0.12032260 $\times 10^{-2}$ |
| 2 | 1 | -.092035100 | .010204640 | -.13841330 | 4 | 1 | -.019043900 | .82115000 $\times 10^{-2}$ | -.12169040 | 3 | 1 | -.027878500 | .94711100 $\times 10^{-2}$ | -.13022300 |
| 4 | 2 | -.030211500 | .32259000 $\times 10^{-2}$ | -.72760000 $\times 10^{-4}$ | 6 | 2 | -.0245222500 | .30075000 | -.14139000 $\times 10^{-3}$ | 5 | 2 | -.027627100 | .31339000 | -.11061000 $\times 10^{-3}$ |
| 6 | 3 | -.61620000 $\times 10^{-2}$ | -.64600000 $\times 10^{-4}$ | .15226000 $\times 10^{-5}$ | 8 | 3 | -.60300000 $\times 10^{-2}$ | .19010000 $\times 10^{-3}$ | .10350000 | 7 | 3 | -.61640000 $\times 10^{-2}$ | .74800000 $\times 10^{-4}$ | .12733000 |
| 8 | 4 | | | | 10 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | | 11 | 5 | | | |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.69922770 | -0.076473225 | 0.81228790 $\times 10^{-3}$ | 2 | 0 | 0.56407056 | -0.074692750 | 0.96322770 $\times 10^{-3}$ | 1 | 0 | 0.59921426 | -0.075620570 | 0.89120360 $\times 10^{-3}$ |
| 2 | 1 | -.086945200 | .013077850 | -.11982400 $\times 10^{-2}$ | 4 | 1 | -.062222900 | .010777340 | -.10997510 $\times 10^{-2}$ | 3 | 1 | -.073858600 | .011905000 | -.11509480 $\times 10^{-2}$ |
| 4 | 2 | -.029210000 | .16040000 $\times 10^{-2}$ | -.19297000 $\times 10^{-3}$ | 6 | 2 | -.027785000 | .17999000 $\times 10^{-2}$ | -.60220000 $\times 10^{-4}$ | 5 | 2 | -.027544000 | .17213000 $\times 10^{-2}$ | -.97260000 $\times 10^{-4}$ |
| 6 | 3 | | -.76620000 $\times 10^{-3}$ | .16150000 | 8 | 3 | -.14222000 $\times 10^{-2}$ | -.50150000 $\times 10^{-3}$ | .13400000 $\times 10^{-3}$ | 7 | 3 | -.81700000 $\times 10^{-3}$ | -.69660000 $\times 10^{-3}$ | .14790000 $\times 10^{-3}$ |
| 8 | 4 | | | | 10 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | | 11 | 5 | | | |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.73318250 | -0.026621767 | 0.30536540 $\times 10^{-3}$ | 2 | 0 | 0.62046041 | -0.026011559 | 0.36413620 $\times 10^{-3}$ | 1 | 0 | 0.70669777 | -0.026961359 | 0.35954000 $\times 10^{-3}$ |
| 2 | 1 | -.25015850 | .016019450 | -.65907500 | 4 | 1 | -.22929550 | .014764130 | -.62851200 | 3 | 1 | -.24446100 | .015384640 | -.63324000 |
| 4 | 2 | .037722000 | -.12871000 $\times 10^{-2}$ | .38220000 | 6 | 2 | -.027322000 | -.32920000 $\times 10^{-2}$ | .34975000 | 5 | 2 | .031360000 | -.14210000 $\times 10^{-2}$ | .36928000 |
| 6 | 3 | | | | 8 | 3 | | | | 7 | 3 | | | |
| 8 | 4 | | | | 10 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | | 11 | 5 | | | |

TABLE I.- COMPUTED VALUES OF G_n , $J \frac{\partial G_n}{\partial \epsilon}$, AND $\frac{\partial G_n}{\partial \epsilon^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(x) L = 0.2

| P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \epsilon}$ | $\frac{\partial G_n}{\partial \epsilon^2}$ | P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \epsilon}$ | $\frac{\partial G_n}{\partial \epsilon^2}$ | P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \epsilon}$ | $\frac{\partial G_n}{\partial \epsilon^2}$ |
|------------------|---|----------------------------|--|--|-----|---|-----------------------------|--|--|-----|---|-----------------------------|--|--|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.010013400 | -0.19799735 | 2.4877530 | 2 0 | 0 | | | | 1 0 | 0 | | | |
| 2 1 | 1 | | | | 4 1 | 1 | | | | 3 1 | 1 | | | |
| $\lambda = 0.06$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.069772100 | -0.19400558 | 0.80822707 | 2 0 | 0 | $0(10^{-27})$ | $0(10^{-20})$ | $0(10^{24})$ | 1 0 | 0 | $0.91336000 \times 10^{-9}$ | $-0.27053342 \times 10^{-7}$ | $0.77613372 \times 10^{-6}$ |
| 2 1 | 1 | $0(10^{-27})$ | $0(10^{-27})$ | $0(10^{-24})$ | 4 1 | 1 | | | | 3 1 | 1 | | | |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.048286100 | -0.19029438 | 0.46626763 | 2 0 | 0 | $0(11^{-11})$ | $0(10^{-10})$ | $0.10500303 \times 10^{-8}$ | 1 0 | 0 | $0.26935400 \times 10^{-4}$ | $-0.30777974 \times 10^{-3}$ | $0.33387902 \times 10^{-2}$ |
| 2 1 | 1 | $0(10^{-11})$ | $0(10^{-10})$ | $.10101161 \times 10^{-8}$ | 4 1 | 1 | | | $0(10^{-33})$ | 3 1 | 1 | $0(10^{-22})$ | $0(10^{-21})$ | $0(10^{-19})$ |
| 4 2 | 2 | | | $0(10^{-33})$ | 6 2 | 2 | | | | 5 2 | 2 | | | |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.093432100 | -0.18131378 | 0.21609057 | 2 0 | 0 | $0.32171400 \times 10^{-4}$ | $-0.3029614 \times 10^{-3}$ | 0.1695439×10^{-2} | 1 0 | 0 | $0.38679300 \times 10^{-2}$ | -0.061596700 | 0.067969610 |
| 2 1 | 1 | $.48879100 \times 10^{-4}$ | $-.28243604 \times 10^{-3}$ | $.15225274 \times 10^{-2}$ | 4 1 | 1 | $0(10^{-11})$ | $0(10^{-10})$ | $.4902861 \times 10^{-9}$ | 3 1 | 1 | $.48330300 \times 10^{-7}$ | $-.39034313 \times 10^{-6}$ | $.30597637 \times 10^{-5}$ |
| 4 2 | 2 | $0(10^{-11})$ | $0(10^{-10})$ | $.47169282 \times 10^{-9}$ | 6 2 | 2 | | | $0(10^{-20})$ | 5 2 | 2 | $0(10^{-16})$ | $0(10^{-15})$ | $0(10^{-14})$ |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 0 | 0 | 0.17569490 | -0.16526102 | 0.055104430 | 2 0 | 0 | 0.011804680 | -0.020768222 | 0.052098612 | 1 0 | 0 | 0.059009870 | -0.074833076 | 0.077331248 |
| 2 1 | 1 | $.92062300 \times 10^{-2}$ | $-.016446141$ | $.024647920$ | 4 1 | 1 | $.88757000 \times 10^{-4}$ | $-.25487430 \times 10^{-3}$ | $.68192129 \times 10^{-3}$ | 3 1 | 1 | $.11861800 \times 10^{-2}$ | $-.27482920 \times 10^{-2}$ | $.57218882 \times 10^{-2}$ |
| 4 2 | 2 | $.77761600 \times 10^{-4}$ | $-.22157050 \times 10^{-3}$ | $.58662230 \times 10^{-3}$ | 6 2 | 2 | $.81770400 \times 10^{-7}$ | $-.32829660 \times 10^{-6}$ | $.12700463 \times 10^{-5}$ | 5 2 | 2 | $.33766700 \times 10^{-5}$ | $-.11572990 \times 10^{-4}$ | $.37622039 \times 10^{-4}$ |
| 6 3 | 3 | $.74873000 \times 10^{-7}$ | $-.29708730 \times 10^{-6}$ | $.11449697 \times 10^{-5}$ | 8 3 | 3 | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-9})$ | 7 3 | 3 | $.10059300 \times 10^{-8}$ | $-.46271300 \times 10^{-8}$ | $.20337543 \times 10^{-7}$ |
| 8 4 | 4 | $0(10^{-11})$ | $0(10^{-10})$ | $.18280190 \times 10^{-9}$ | | | | | | 9 4 | 4 | $0(10^{-13})$ | $0(10^{-12})$ | $0(10^{-12})$ |

TABLE I.- COMPUTED VALUES OF C_m , $J \frac{dC_m}{d\lambda}$, AND $\frac{dC_m}{d\lambda}$ FOR $\alpha = 0$ TO 5 AND $\lambda = 0.5$

VALUES OF F FOR VARIOUS VALUES OF L AND λ - Continued

(F) L = 0.2 - Continued

| F | α | $C_m(F)$ | $J \frac{dC_m}{d\lambda}$ | $\frac{dC_m}{d\lambda}$ | F | α | $C_m(F)$ | $J \frac{dC_m}{d\lambda}$ | $\frac{dC_m}{d\lambda}$ | F | α | $C_m(F)$ | $J \frac{dC_m}{d\lambda}$ | $\frac{dC_m}{d\lambda}$ |
|------------------|----------|-----------------------------|------------------------------|------------------------------|----|----------|-----------------------------|------------------------------|-----------------------------|----|----------|-----------------------------|------------------------------|-----------------------------|
| $\lambda = 0.50$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.0430610 | -0.1513550 | 0.05584768 | 0 | 0 | 0.04681510 | -0.04669886 | 0.05892799 | 1 | 0 | 0.11951827 | -0.09579884 | 0.054092196 |
| 1 | 1 | 0.04195550 | -0.08078470 | 0.082304781 | 1 | 1 | 0.04681510 | -0.04669886 | 0.05892799 | 2 | 1 | 0.11951827 | -0.09579884 | 0.054092196 |
| 2 | 2 | 0.5959210×10^{-2} | $-3.4717070 \times 10^{-2}$ | 0.1059240×10^{-2} | 2 | 2 | 3.6979400×10^{-2} | $-1.4690080 \times 10^{-2}$ | 0.7786080×10^{-2} | 3 | 2 | 5.6956800×10^{-2} | $-0.9618800 \times 10^{-2}$ | 0.1358619×10^{-2} |
| 3 | 3 | 0.8095800×10^{-4} | $-1.9007500 \times 10^{-3}$ | 0.2667980×10^{-3} | 3 | 3 | 9.9006600×10^{-4} | $-2.8661670 \times 10^{-3}$ | 0.3800960×10^{-3} | 4 | 3 | 0.1109100×10^{-4} | $-0.2389900 \times 10^{-4}$ | 0.4691600×10^{-4} |
| 4 | 4 | $0.99577000 \times 10^{-6}$ | $-0.2783500 \times 10^{-5}$ | 0.5306040×10^{-5} | 4 | 4 | 0.1168800×10^{-5} | $-0.2785900 \times 10^{-5}$ | 0.233370×10^{-5} | 5 | 4 | 0.8200000×10^{-7} | $-0.2134300 \times 10^{-6}$ | 0.4397100×10^{-6} |
| 5 | 5 | 0.4460000×10^{-8} | $-1.0847800 \times 10^{-7}$ | 0.3604500×10^{-7} | 5 | 5 | 0.5084000×10^{-8} | $-0.1475000 \times 10^{-7}$ | 0.1026700×10^{-7} | 6 | 5 | 0.1040000×10^{-9} | $-0.0769800 \times 10^{-8}$ | 0.0805500×10^{-8} |
| | | | | | 10 | | $0(10^{-11})$ | $0(10^{-10})$ | $0(10^{-10})$ | 11 | | | | |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.30342770 | -0.13951446 | 0.05281541 | 0 | 0 | 0.05891850 | -0.06609594 | 0.05892009 | 1 | 0 | 0.12895486 | -0.10801968 | 0.05795595 |
| 1 | 1 | 0.04646690 | -0.04278360 | 0.08111299 | 1 | 1 | 0.05891850 | -0.06609594 | 0.05892009 | 2 | 1 | 0.09304860 | -0.08180510 | 0.08181956 |
| 2 | 2 | 0.9056370×10^{-2} | $-0.7827840 \times 10^{-2}$ | 0.1444520×10^{-2} | 2 | 2 | 0.0774810×10^{-2} | $-0.0117418 \times 10^{-2}$ | 0.0820040×10^{-2} | 3 | 2 | 0.5764600×10^{-2} | $-0.5779810 \times 10^{-2}$ | 0.0839780×10^{-2} |
| 3 | 3 | 0.9049000×10^{-4} | -0.1080010 | 0.0881700×10^{-3} | 3 | 3 | 0.1701100×10^{-2} | $-0.1864280 \times 10^{-2}$ | 0.1489610 | 4 | 3 | 0.3096750×10^{-3} | $-0.3760000 \times 10^{-3}$ | 0.4084600×10^{-3} |
| 4 | 4 | 0.6464000×10^{-4} | $-0.6971000 \times 10^{-4}$ | 0.1059780 | 4 | 4 | 0.0821000×10^{-4} | $-0.1169000 \times 10^{-4}$ | 0.1469000×10^{-4} | 5 | 4 | 0.1508000×10^{-4} | $-0.2349000 \times 10^{-4}$ | 0.2825600×10^{-4} |
| 5 | 5 | 0.2586000×10^{-5} | $-0.4182500 \times 10^{-5}$ | 0.6441000×10^{-5} | 5 | 5 | 0.3642000×10^{-5} | $-0.5790000 \times 10^{-5}$ | 0.8747000×10^{-5} | 6 | 5 | 0.4731000×10^{-6} | $-0.8280000 \times 10^{-6}$ | 0.1431700×10^{-5} |
| | | | | | 10 | | 0.7867000×10^{-7} | $-1.1955000 \times 10^{-6}$ | 0.0007000×10^{-6} | 11 | | | | |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.37661190 | -0.16877782 | 0.084711127 | 0 | 0 | 0.15190876 | -0.079056074 | 0.086308111 | 1 | 0 | 0.08049574 | -0.10887403 | 0.08746772 |
| 1 | 1 | 0.071109170 | -0.089438490 | 0.13345980 $\times 10^{-2}$ | 1 | 1 | 0.15190876 | -0.079056074 | 0.086308111 | 2 | 1 | 0.047276880 | -0.08546140 | 0.1045690 $\times 10^{-2}$ |
| 2 | 2 | 0.015048790 | $-0.17228400 \times 10^{-2}$ | 0.21568900 | 2 | 2 | 0.07698150 | -0.01610400 | 0.0886650×10^{-2} | 3 | 2 | 0.8688400×10^{-2} | $-0.5648700 \times 10^{-2}$ | 0.2004500 |
| 3 | 3 | $0.86771900 \times 10^{-2}$ | -2.7417900 | 0.0809900×10^{-3} | 3 | 3 | $0.44993100 \times 10^{-2}$ | $-0.1688500 \times 10^{-2}$ | 0.18081660 | 4 | 3 | 0.13033400 | -0.10071100 | $0.08871300 \times 10^{-3}$ |
| 4 | 4 | 0.5827000×10^{-3} | $-0.5148800 \times 10^{-3}$ | 0.21529000 | 4 | 4 | 0.2523500×10^{-3} | $-0.30841000 \times 10^{-3}$ | $0.37990500 \times 10^{-3}$ | 5 | 4 | $0.25841000 \times 10^{-3}$ | $-0.14346000 \times 10^{-3}$ | 0.1214800 |
| 5 | 5 | 0.1010000×10^{-4} | $-0.4190000 \times 10^{-4}$ | 0.3785000×10^{-4} | 5 | 5 | 0.5871000×10^{-4} | $-0.6180000 \times 10^{-4}$ | 0.7798000×10^{-4} | 6 | 5 | $0.14890000 \times 10^{-4}$ | $-0.3789000 \times 10^{-4}$ | 0.1662000×10^{-4} |
| | | | | | 10 | | 0.7806000×10^{-5} | $0.57705000 \times 10^{-5}$ | | 11 | | | | |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.4028510 | -0.11647381 | 0.081277447 | 0 | 0 | 0.08020788 | -0.078970748 | 0.080804990 | 1 | 0 | 0.08274694 | -0.099880174 | 0.08061087 |
| 1 | 1 | 0.079921400 | -0.08210940 | 0.14630000 $\times 10^{-3}$ | 1 | 1 | 0.08020788 | -0.078970748 | 0.080804990 | 2 | 1 | 0.08448280 | -0.08054660 | 0.1216500 $\times 10^{-2}$ |
| 2 | 2 | 0.017088790 | $-0.51593900 \times 10^{-2}$ | -0.10869000 | 2 | 2 | 0.03990480 | -0.01977940 | 0.0846910×10^{-2} | 3 | 2 | 0.01806470 | $-0.1799700 \times 10^{-2}$ | 0.0879500×10^{-3} |
| 3 | 3 | $0.37909000 \times 10^{-2}$ | -2.4021500 | 0.18279000 | 3 | 3 | 0.7808400×10^{-2} | $-0.37109000 \times 10^{-2}$ | 0.11456000 | 4 | 3 | 0.0175000×10^{-2} | -0.11641100 | 0.38489000 |
| 4 | 4 | $0.76650000 \times 10^{-3}$ | $-0.37480000 \times 10^{-3}$ | 0.10189000 | 4 | 4 | 0.14390400 | $-0.21597000 \times 10^{-3}$ | $0.34004000 \times 10^{-3}$ | 5 | 4 | $0.44510000 \times 10^{-3}$ | $-0.26510000 \times 10^{-3}$ | 0.11349000 |
| 5 | 5 | 0.17710000 | $-0.25900000 \times 10^{-4}$ | $0.39600000 \times 10^{-4}$ | 5 | 5 | $0.25520000 \times 10^{-3}$ | -0.26039000 | $0.08870000 \times 10^{-4}$ | 6 | 5 | $0.69800000 \times 10^{-4}$ | $-0.50680000 \times 10^{-4}$ | $0.09900000 \times 10^{-4}$ |
| | | | | | 10 | | $0.38720000 \times 10^{-4}$ | $-0.86250000 \times 10^{-4}$ | 0.18940000 | 11 | | | | |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.44268110 | -0.11128778 | 0.013791194 | 0 | 0 | 0.02147995 | -0.079995970 | 0.016949472 | 1 | 0 | 0.3996068 | -0.09680028 | 0.01282897 |
| 1 | 1 | 0.07786690 | -0.04840800 | 0.08979430 $\times 10^{-2}$ | 1 | 1 | 0.02147995 | -0.079995970 | 0.016949472 | 2 | 1 | 0.08235200 | -0.01280180 | 0.1362500 $\times 10^{-3}$ |
| 2 | 2 | 0.014753800 | $-0.10181000 \times 10^{-2}$ | -0.14982700 | 2 | 2 | 0.04749480 | -0.1425090 | 0.1719200×10^{-2} | 3 | 2 | 0.02378600 | $-0.4769200 \times 10^{-2}$ | 0.14292000 |
| 3 | 3 | $0.31922000 \times 10^{-2}$ | $-0.32210000 \times 10^{-3}$ | $-0.47250000 \times 10^{-3}$ | 3 | 3 | 0.9420000×10^{-2} | $-0.0899500 \times 10^{-2}$ | $0.17990000 \times 10^{-3}$ | 4 | 3 | 0.0675000×10^{-2} | $-0.64970000 \times 10^{-3}$ | -0.12290000 |
| 4 | 4 | $0.68750000 \times 10^{-3}$ | -0.16480000 | -0.11569000 | 4 | 4 | 0.19436000 | $-0.08697000 \times 10^{-3}$ | $0.47660000 \times 10^{-4}$ | 5 | 4 | $0.08760000 \times 10^{-3}$ | -0.18840000 | $0.10840000 \times 10^{-4}$ |
| 5 | 5 | 0.17850000 | $-0.51100000 \times 10^{-4}$ | $-0.11600000 \times 10^{-4}$ | 5 | 5 | $0.40730000 \times 10^{-3}$ | -0.26840000 | 0.38410000 | 6 | 5 | 0.10800000 | $-0.58400000 \times 10^{-4}$ | $0.49100000 \times 10^{-5}$ |
| | | | | | 10 | | $0.75900000 \times 10^{-4}$ | $-0.40100000 \times 10^{-4}$ | 0.13930000 | 11 | | | | |

TABLE I.- COMPUTED VALUES OF C_n , $J \frac{\partial C_n}{\partial \lambda}$, AND $\frac{\partial C_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INCREASING
 VALUES OF P FOR VARIOUS VALUES OF λ AND λ - Continued.

(c) $\lambda = 0.2$ - Continued

| P | n | $C_n(P)$ | $J \frac{\partial C_n}{\partial \lambda}$ | $\frac{\partial C_n}{\partial \lambda}$ | P | n | $C_n(P)$ | $J \frac{\partial C_n}{\partial \lambda}$ | $\frac{\partial C_n}{\partial \lambda}$ | P | n | $C_n(P)$ | $J \frac{\partial C_n}{\partial \lambda}$ | $\frac{\partial C_n}{\partial \lambda}$ |
|-----------------|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.4800700 | -0.1059791 | 0.010743274 | 0 | 0 | 0.48598827 | -0.07934934 | 0.013300787 | 1 | 0 | 0.58181865 | -0.088301878 | 0.012468879 |
| 2 | 1 | .067707300 | -.66138200 $\times 10^{-2}$ | -.38901250 $\times 10^{-2}$ | 2 | 1 | .049944400 | -.010431750 | -.24374400 $\times 10^{-3}$ | 3 | 1 | .059710400 | -.94377500 $\times 10^{-2}$ | -.18296870 $\times 10^{-2}$ |
| 4 | 2 | .92950000 $\times 10^{-2}$ | .18589600 | -.20793000 | 4 | 2 | .87435000 $\times 10^{-2}$ | -.12736000 $\times 10^{-2}$ | -.60881000 | 5 | 2 | .96188000 $\times 10^{-2}$ | -.35480000 $\times 10^{-3}$ | -.12528500 |
| 6 | 3 | .13808000 | .75900000 $\times 10^{-3}$ | -.71376000 $\times 10^{-3}$ | 6 | 3 | .16639000 | -.18110000 $\times 10^{-3}$ | -.28935000 | 7 | 3 | .17843000 | .15780000 | -.44030000 $\times 10^{-3}$ |
| 8 | 4 | .15100000 $\times 10^{-3}$ | .18160000 | -.28580000 | 8 | 4 | .36480000 $\times 10^{-3}$ | -.59600000 $\times 10^{-4}$ | -.77500000 $\times 10^{-4}$ | 9 | 4 | .12700000 $\times 10^{-3}$ | -.41500000 $\times 10^{-4}$ | -.15660000 |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| 12 | 5 | | | | 12 | 5 | | | | | | | | |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.51851770 | -0.097476488 | 0.03335110 $\times 10^{-2}$ | 0 | 0 | 0.53664963 | -0.077755134 | 0.010809987 | 1 | 0 | 0.41967210 | -0.088174134 | 0.03706630 $\times 10^{-2}$ |
| 2 | 1 | .052907700 | -.17734000 $\times 10^{-2}$ | -.47011250 | 2 | 1 | .046073300 | -.59804500 $\times 10^{-2}$ | -.16230990 $\times 10^{-2}$ | 3 | 1 | .051017600 | -.36749900 $\times 10^{-2}$ | -.50752300 |
| 4 | 2 | .80129000 $\times 10^{-2}$ | .56274000 $\times 10^{-2}$ | -.20217500 | 4 | 2 | .78135000 $\times 10^{-2}$ | .50973000 $\times 10^{-3}$ | -.10267000 | 5 | 2 | .46960000 $\times 10^{-2}$ | .15118000 | -.15966700 |
| 6 | 3 | -.10779000 | .14368000 | -.69400000 $\times 10^{-3}$ | 6 | 3 | .61270000 $\times 10^{-3}$ | .39970000 | -.36820000 $\times 10^{-3}$ | 7 | 3 | .38600000 $\times 10^{-4}$ | .87690000 $\times 10^{-3}$ | -.50660000 $\times 10^{-3}$ |
| 8 | 4 | -.58300000 $\times 10^{-3}$ | .40440000 $\times 10^{-3}$ | -.20820000 | 8 | 4 | | .19810000 | -.107150000 | 9 | 4 | .45700000 $\times 10^{-3}$ | .18660000 | -.19460000 |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| 12 | 5 | | | | 12 | 5 | | | | | | | | |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.54173370 | -0.091099862 | 0.03014730 $\times 10^{-2}$ | 0 | 0 | 0.57304670 | -0.07596614 | 0.08814780 $\times 10^{-2}$ | 1 | 0 | 0.45976995 | -0.084119408 | 0.80940140 $\times 10^{-2}$ |
| 2 | 1 | .034883900 | .60294800 $\times 10^{-2}$ | -.50318900 | 2 | 1 | .052026800 | -.14501900 $\times 10^{-2}$ | -.25270960 | 3 | 1 | .038088800 | .16609700 $\times 10^{-2}$ | -.57201900 |
| 4 | 2 | -.54429000 $\times 10^{-2}$ | .50665000 | -.18037900 | 4 | 2 | .14554000 $\times 10^{-2}$ | .20408000 | -.11638100 | 5 | 2 | -.12759000 $\times 10^{-2}$ | .33997000 | -.15189600 |
| 6 | 3 | -.52070000 | .16896000 | -.44790000 $\times 10^{-3}$ | 6 | 3 | -.82500000 $\times 10^{-3}$ | .81870000 $\times 10^{-3}$ | -.38700000 $\times 10^{-3}$ | 7 | 3 | -.17850000 | .12854000 | -.42400000 $\times 10^{-3}$ |
| 8 | 4 | -.18060000 | .42100000 $\times 10^{-3}$ | -.18060000 | 8 | 4 | | | | 9 | 4 | -.11285000 | .18900000 $\times 10^{-3}$ | -.16610000 |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| 12 | 5 | | | | 12 | 5 | | | | | | | | |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.6085740 | -0.079472928 | 0.14802740 $\times 10^{-2}$ | 0 | 0 | 0.45331008 | -0.069274336 | 0.56704910 $\times 10^{-2}$ | 1 | 0 | 0.52944702 | -0.074773270 | 0.50683710 $\times 10^{-2}$ |
| 2 | 1 | -.022076200 | .01624470 | -.48379000 | 2 | 1 | .88739000 $\times 10^{-2}$ | -.86214000 $\times 10^{-2}$ | -.34490700 | 3 | 1 | -.73840000 $\times 10^{-2}$ | .012938800 | -.41462000 |
| 4 | 2 | -.012823400 | .52789000 $\times 10^{-2}$ | -.67666000 $\times 10^{-3}$ | 4 | 2 | -.012844200 | .77848000 | -.78137000 $\times 10^{-3}$ | 5 | 2 | -.013894400 | .45840000 $\times 10^{-2}$ | -.76498000 $\times 10^{-3}$ |
| 6 | 3 | -.47430000 $\times 10^{-2}$ | .76600000 $\times 10^{-3}$ | .12510000 | 6 | 3 | -.34130000 $\times 10^{-2}$ | .75180000 $\times 10^{-3}$ | -.70800000 $\times 10^{-4}$ | 7 | 3 | -.40500000 $\times 10^{-2}$ | .80230000 $\times 10^{-3}$ | .14700000 $\times 10^{-4}$ |
| 8 | 4 | | | | 8 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| 12 | 5 | | | | 12 | 5 | | | | | | | | |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.63045480 | -0.069909048 | 0.28890730 $\times 10^{-2}$ | 0 | 0 | 0.51701941 | -0.062801912 | 0.58102870 $\times 10^{-2}$ | 1 | 0 | 0.58002667 | -0.066798860 | 0.53679280 $\times 10^{-2}$ |
| 2 | 1 | -.083797000 | .025446520 | -.11449000 | 2 | 1 | -.044833000 | .012851360 | -.34087100 | 3 | 1 | -.068470000 | .019493900 | -.37823000 |
| 4 | 2 | -.081128000 | .25669000 $\times 10^{-2}$ | .38800000 $\times 10^{-3}$ | 4 | 2 | -.016036000 | .27468000 $\times 10^{-2}$ | -.79400000 $\times 10^{-4}$ | 5 | 2 | -.018760000 | .87600000 $\times 10^{-2}$ | -.73900000 $\times 10^{-4}$ |
| 6 | 3 | -.13290000 $\times 10^{-2}$ | -.73800000 $\times 10^{-3}$ | .38240000 | 6 | 3 | -.25950000 $\times 10^{-2}$ | .11050000 $\times 10^{-3}$ | -.11050000 $\times 10^{-3}$ | 7 | 3 | -.28810000 $\times 10^{-2}$ | -.55800000 $\times 10^{-3}$ | .21750000 $\times 10^{-3}$ |
| 8 | 4 | -.42800000 | -.18130000 $\times 10^{-2}$ | -.38100000 | 8 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| 12 | 5 | | | | 12 | 5 | | | | | | | | |

TABLE I.-- COMPILED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND INCREASING

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(x) $L = 0.2$ - Concluded

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|-----------------|-----|----------------------------|---|---|------|-----|----------------------------|---|---|------|-----|----------------------------|---|---|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 0 | | 0.68877640 | -0.062244728 | $0.19689740 \times 10^{-2}$ | 2 0 | | 0.56872200 | -0.077776242 | $0.66691700 \times 10^{-2}$ | 1 0 | | 0.6277947 | -0.060087784 | $0.23480150 \times 10^{-2}$ |
| 2 1 | | -.14507890 | .027152070 | $-.34156700 \times 10^{-2}$ | 4 1 | | -.097797600 | .022669730 | $-.30417200 \times 10^{-2}$ | 3 1 | | -.11961990 | .0228117500 | $-.32446100 \times 10^{-3}$ |
| 4 2 | | -.011596000 | -.11287000 $\times 10^{-2}$ | $.86640000 \times 10^{-3}$ | 6 2 | | -.0122973000 | .15470000 $\times 10^{-2}$ | $.46820000 \times 10^{-3}$ | 5 2 | | -.012406000 | $-.38770000 \times 10^{-3}$ | $.65520000 \times 10^{-3}$ |
| 6 3 | | | -.20090000 | | 8 3 | | | | .22660000 | 7 3 | | .25100000 $\times 10^{-2}$ | $-.10520000 \times 10^{-2}$ | .25640000 |
| 8 4 | | | | | 10 4 | | | | | 9 4 | | | | |
| 10 5 | | | | | 12 5 | | | | | 11 5 | | | | |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 0 | | 0.72005290 | -0.055997430 | $0.14169770 \times 10^{-2}$ | 2 0 | | 0.61129986 | -0.072222426 | $0.19328640 \times 10^{-2}$ | 1 0 | | 0.66479518 | -0.074432802 | $0.16887410 \times 10^{-2}$ |
| 2 1 | | -.20890730 | .029041490 | -.17829200 | 4 1 | | -.15021470 | .022291090 | -.26297800 | 3 1 | | -.17229500 | .026291310 | -.27024600 |
| 4 2 | | .01810000 $\times 10^{-2}$ | -.47950000 $\times 10^{-2}$ | .11753000 | 6 2 | | .75800000 $\times 10^{-3}$ | -.27511000 $\times 10^{-2}$ | .04720000 $\times 10^{-3}$ | 5 2 | | .39540000 $\times 10^{-2}$ | $-.36721000 \times 10^{-2}$ | .10054000 |
| 6 3 | | | -.11780000 | | 8 3 | | | | | 7 3 | | | -.11010000 | |
| 8 4 | | | | | 10 4 | | | | | 9 4 | | | | |
| 10 5 | | | | | 12 5 | | | | | 11 5 | | | | |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 0 | | 0.60203230 | -0.039599740 | $0.42173380 \times 10^{-3}$ | 2 0 | | 0.7299224 | -0.058419746 | $0.67929790 \times 10^{-3}$ | 1 0 | | 0.76259689 | -0.039074066 | $0.52799660 \times 10^{-3}$ |
| 2 1 | | -.38810460 | .028749800 | $-.12540900 \times 10^{-2}$ | 4 1 | | -.33312360 | .026205800 | $-.12829500 \times 10^{-2}$ | 3 1 | | -.39988670 | .027484100 | $-.12726000 \times 10^{-2}$ |
| 4 2 | | .13120000 | -.014060000 | .11405000 | 6 2 | | .10743900 | -.011966000 | .10744000 | 5 2 | | .11805000 | -.012295000 | .10872000 |
| 6 3 | | | | | 8 3 | | | | | 7 3 | | | | |
| 8 4 | | | | | 10 4 | | | | | 9 4 | | | | |
| 10 5 | | | | | 12 5 | | | | | 11 5 | | | | |

TABLE I.- COMPUTED VALUES OF G_n , $\int \frac{\partial G_n}{\partial \xi}$, AND $\frac{\partial G_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(g) $L = 0.5$

| P | n | $G_n(P)$ | $\int \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P | n | $G_n(P)$ | $\int \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P | n | $G_n(P)$ | $\int \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ |
|------------------|---|----------------------------|--|---------------------------------------|---|---|------------------------------|--|---------------------------------------|---|---|-----------------------------|--|---------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.012241900 | -0.29632742 | 4.5463923 | 1 | 0 | | | | 1 | 0 | | | |
| 2 | 1 | | | | 2 | 0 | | | | 3 | 1 | | | |
| 4 | 1 | | | | 4 | 1 | | | | | | | | |
| $\lambda = 0.06$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.096028700 | -0.28919145 | 1.4583394 | 2 | 0 | $0(10^{-39})$ | $0(10^{-37})$ | $0(10^{-36})$ | 1 | 0 | $0(10^{-12})$ | $0(10^{-10})$ | $0.13743829 \times 10^{-8}$ |
| 2 | 1 | $0(10^{-39})$ | $0(10^{-37})$ | $0(10^{-36})$ | 4 | 1 | | | | 3 | 1 | | | |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.096923400 | -0.28822239 | 0.84236119 | 2 | 0 | $0(10^{-16})$ | $0(10^{-14})$ | $0(10^{-13})$ | 1 | 0 | $0.18968600 \times 10^{-5}$ | $-0.31684236 \times 10^{-4}$ | $0.50223608 \times 10^{-3}$ |
| 2 | 1 | $0(10^{-16})$ | $0(10^{-14})$ | $0(10^{-13})$ | 4 | 1 | | | | 3 | 1 | $0(10^{-32})$ | $0(10^{-31})$ | $0(10^{-29})$ |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.11232910 | -0.26624126 | 0.38369667 | 2 | 0 | $0.57508700 \times 10^{-24}$ | $-0.31134095 \times 10^{-14}$ | $0.24702046 \times 10^{-3}$ | 1 | 0 | $0.29548720 \times 10^{-2}$ | -0.014955749 | 0.066597754 |
| 2 | 1 | 34823300×10^{-3} | -28970133×10^{-4} | $.22899919 \times 10^{-3}$ | 4 | 1 | $0(10^{-19})$ | $0(10^{-14})$ | $0(10^{-13})$ | 3 | 1 | $.14716100 \times 10^{-9}$ | $-1.7431589 \times 10^{-8}$ | $.20121778 \times 10^{-7}$ |
| 4 | 2 | $0(10^{-15})$ | $0(10^{-14})$ | $0(10^{-13})$ | 4 | 2 | | | | 5 | 2 | $0(10^{-29})$ | $0(10^{-22})$ | $0(10^{-20})$ |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.20610960 | -0.23816711 | 0.16031440 | 2 | 0 | $0.56209790 \times 10^{-2}$ | -0.044359319 | 0.031293464 | 1 | 0 | 0.047947870 | -0.089492185 | 0.11950690 |
| 2 | 1 | $.45190060 \times 10^{-2}$ | $-.011113926$ | $.023714510$ | 4 | 1 | $.62831500 \times 10^{-5}$ | $-.26054600 \times 10^{-4}$ | $.10232289 \times 10^{-3}$ | 3 | 1 | $.25676840 \times 10^{-5}$ | $-.84297850 \times 10^{-5}$ | $.25511841 \times 10^{-2}$ |
| 4 | 2 | $.54719600 \times 10^{-5}$ | $-.22906960 \times 10^{-4}$ | $.87760290 \times 10^{-4}$ | 6 | 2 | $.24769200 \times 10^{-9}$ | $-.14603260 \times 10^{-8}$ | $.85904970 \times 10^{-8}$ | 5 | 2 | $.37537200 \times 10^{-7}$ | $-.28777810 \times 10^{-6}$ | $.13864366 \times 10^{-5}$ |
| 6 | 3 | $.22424000 \times 10^{-9}$ | $-.15189640 \times 10^{-8}$ | $.75566980 \times 10^{-8}$ | 8 | 3 | $0(10^{-15})$ | $0(10^{-14})$ | $0(10^{-13})$ | 7 | 3 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-10})$ |
| 8 | 4 | $0(10^{-15})$ | $0(10^{-14})$ | $0(10^{-13})$ | 8 | 4 | | | | 9 | 4 | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{dQ_n}{d\lambda}$, AND $\frac{dQ_n}{d\lambda}$ FOR $n=0$ TO 5 AND INVERSE

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(a) $L = 0.3$ - Continued

| P | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ | P | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ | P | $Q_n(P)$ | $J \frac{dQ_n}{d\lambda}$ | $\frac{dQ_n}{d\lambda}$ |
|------------------|------------------------------|--------------------------------|-------------------------------|------|------------------------------|-------------------------------|------------------------------|------|-------------------------------|-------------------------------|-------------------------------|
| $\lambda = 0.60$ | | | | | | | | | | | |
| 0 0 | 0.8846040 | -0.8146001 | 0.03048950 | 0 0 | 0.05607870 | -0.04818886 | 0.02869887 | 1 0 | 0.11789510 | -0.18040496 | 0.06977050 |
| 2 1 | .048054120 | -.050137480 | .029199100 | 2 0 | .05607870 | -0.04818886 | 0.02869887 | 3 1 | .079667000 x 10 ⁻² | -.08096150 x 10 ⁻² | .018284915 |
| 4 2 | .72196600 x 10 ⁻³ | -.13920480 x 10 ⁻² | .24137990 x 10 ⁻² | 4 1 | .95879300 x 10 ⁻³ | -.18998515 x 10 ⁻² | .34080410 x 10 ⁻² | 5 2 | .89920600 x 10 ⁻⁴ | -.19977080 x 10 ⁻³ | .13029540 x 10 ⁻³ |
| 6 3 | .99871000 x 10 ⁻³ | -.120887800 x 10 ⁻² | .58178980 x 10 ⁻² | 6 2 | .69021300 x 10 ⁻³ | -.18773350 x 10 ⁻² | .18519840 x 10 ⁻² | 7 3 | .31084000 x 10 ⁻⁶ | -.97780000 x 10 ⁻⁶ | .8040680 x 10 ⁻⁵ |
| 8 4 | .97780000 x 10 ⁻⁶ | -.93738000 x 10 ⁻⁷ | .11328140 x 10 ⁻⁶ | 8 3 | .11470000 x 10 ⁻⁷ | -.10098800 x 10 ⁻⁷ | .13566870 x 10 ⁻⁶ | 9 4 | .84347000 x 10 ⁻⁹ | -.94618000 x 10 ⁻⁹ | .59812500 x 10 ⁻⁸ |
| 10 5 | 0(10 ⁻¹¹) | 0(10 ⁻¹⁰) | 0(10 ⁻¹⁰) | 10 4 | 0(10 ⁻¹¹) | 0(10 ⁻¹⁰) | 0(10 ⁻¹⁰) | 11 5 | 0(10 ⁻¹³) | 0(10 ⁻¹²) | 0(10 ⁻¹²) |
| $\lambda = 0.80$ | | | | | | | | | | | |
| 0 0 | 0.75118560 | -0.19484400 | 0.077489060 | 0 0 | 0.08497870 | -0.07438715 | 0.02068881 | 1 0 | 0.18721465 | -0.19482284 | 0.069770860 |
| 2 1 | .045881690 | -.092126840 | .017499360 | 2 0 | .08497870 | -0.07438715 | 0.02068881 | 3 1 | .018289040 | -.018289040 | .018289040 |
| 4 2 | .40965600 x 10 ⁻² | -.149218000 x 10 ⁻² | .40968250 x 10 ⁻² | 4 1 | .69366800 x 10 ⁻² | -.77418740 x 10 ⁻² | .77050440 x 10 ⁻² | 5 2 | .18888800 x 10 ⁻² | -.16166170 x 10 ⁻² | .1877880 x 10 ⁻² |
| 6 3 | .19944000 x 10 ⁻³ | -.29739400 x 10 ⁻³ | .42674000 x 10 ⁻³ | 6 2 | .8803100 x 10 ⁻³ | -.14401460 x 10 ⁻³ | .6862880 x 10 ⁻³ | 7 3 | .3794000 x 10 ⁻⁴ | -.6698000 x 10 ⁻⁴ | .1005000 x 10 ⁻³ |
| 8 4 | .14750000 x 10 ⁻⁵ | -.09109000 x 10 ⁻⁵ | .13681900 x 10 ⁻⁴ | 8 3 | .58801000 x 10 ⁻⁵ | -.11970100 x 10 ⁻⁴ | .2170680 x 10 ⁻⁴ | 9 4 | .92110000 x 10 ⁻⁶ | -.11910000 x 10 ⁻⁵ | .29007400 x 10 ⁻⁵ |
| 10 5 | .13660000 x 10 ⁻⁷ | -.10094000 x 10 ⁻⁶ | .29999000 x 10 ⁻⁶ | 10 4 | .94160000 x 10 ⁻⁷ | -.13050000 x 10 ⁻⁶ | .30288000 x 10 ⁻⁶ | 11 5 | .94100000 x 10 ⁻⁸ | -.08880000 x 10 ⁻⁸ | .22808000 x 10 ⁻⁷ |
| $\lambda = 1.0$ | | | | | | | | | | | |
| 0 0 | 0.40798170 | -0.17760949 | 0.09984165 | 0 0 | 0.14011338 | -0.089230996 | 0.041818870 | 1 0 | 0.25160943 | -0.19407788 | 0.049789887 |
| 2 1 | .098019950 | -.089684750 | .60917500 x 10 ⁻² | 2 0 | .14011338 | -0.089230996 | 0.041818870 | 3 1 | .093425840 | -.081584950 | .30878700 x 10 ⁻² |
| 4 2 | .81349800 x 10 ⁻² | -.75128400 x 10 ⁻² | .23140000 | 4 1 | .016583380 | -.012870500 | .78290050 x 10 ⁻² | 5 2 | .97899000 x 10 ⁻² | -.91089000 x 10 ⁻² | .20967100 |
| 6 3 | .86187000 x 10 ⁻³ | -.79947000 x 10 ⁻³ | .98210000 x 10 ⁻³ | 6 2 | .15094500 x 10 ⁻² | -.15090600 x 10 ⁻² | .12818790 | 7 3 | .31840000 x 10 ⁻³ | -.54343000 x 10 ⁻³ | .36941600 x 10 ⁻³ |
| 8 4 | .61060000 x 10 ⁻⁴ | -.73970000 x 10 ⁻⁴ | .78840000 x 10 ⁻⁴ | 8 3 | .96090000 x 10 ⁻⁴ | -.12108900 x 10 ⁻³ | .19733000 x 10 ⁻³ | 9 4 | .16760000 x 10 ⁻⁴ | -.25060000 x 10 ⁻⁴ | .29099000 x 10 ⁻⁴ |
| 10 5 | .29430000 x 10 ⁻⁵ | -.58760000 x 10 ⁻⁵ | .96880000 x 10 ⁻⁵ | 10 4 | .99980000 x 10 ⁻⁵ | -.99990000 x 10 ⁻⁵ | .86999000 x 10 ⁻⁵ | 11 5 | .61800000 x 10 ⁻⁶ | -.87900000 x 10 ⁻⁶ | .14938000 x 10 ⁻⁵ |
| $\lambda = 1.2$ | | | | | | | | | | | |
| 0 0 | 0.43684450 | -0.18694670 | 0.08978021 | 0 0 | 0.19918628 | -0.097843619 | 0.055612931 | 1 0 | 0.30926168 | -0.19128782 | 0.097947973 |
| 2 1 | .082067600 | -.019887450 | -.14677900 x 10 ⁻² | 2 0 | .19918628 | -0.097843619 | 0.055612931 | 3 1 | .042409410 | -.018646410 | .39009000 x 10 ⁻² |
| 4 2 | .95117000 x 10 ⁻² | -.97781100 x 10 ⁻² | -.29994000 x 10 ⁻³ | 4 1 | .082868880 | -.018186590 | .90310090 x 10 ⁻² | 5 2 | .60213400 x 10 ⁻² | -.91284000 x 10 ⁻² | .84980000 x 10 ⁻³ |
| 6 3 | .14489000 | -.18660000 x 10 ⁻³ | .15100000 | 6 2 | .93397400 x 10 ⁻² | -.21948900 x 10 ⁻² | .10400800 | 7 3 | .78960000 x 10 ⁻³ | -.94086000 x 10 ⁻³ | .29780000 |
| 8 4 | .12700000 x 10 ⁻³ | -.13700000 | .70300000 x 10 ⁻⁴ | 8 3 | .56829000 x 10 ⁻³ | -.30928000 x 10 ⁻³ | .80299000 x 10 ⁻³ | 9 4 | .29300000 x 10 ⁻⁴ | -.75410000 x 10 ⁻⁴ | .99200000 x 10 ⁻⁴ |
| 10 5 | ----- | -.17280000 x 10 ⁻⁴ | .14110000 | 10 4 | .38230000 x 10 ⁻³ | -.98690000 x 10 ⁻³ | .28050000 x 10 ⁻³ | 11 5 | ----- | ----- | ----- |
| $\lambda = 1.4$ | | | | | | | | | | | |
| 0 0 | 0.44921920 | -0.19084489 | 0.081144979 | 0 0 | 0.24716166 | -0.099872919 | 0.086898840 | 1 0 | 0.36045010 | -0.18288799 | 0.09786160 |
| 2 1 | .099860100 | -.91458100 x 10 ⁻² | -.98466000 x 10 ⁻² | 2 0 | .24716166 | -0.099872919 | 0.086898840 | 3 1 | .044001800 | -.018469920 | .11100000 x 10 ⁻² |
| 4 2 | .78870000 x 10 ⁻² | -.92990000 x 10 ⁻² | -.20994700 | 4 1 | .091385610 | -.018011150 | .16884900 x 10 ⁻² | 5 2 | .61814000 x 10 ⁻² | -.16278600 x 10 ⁻² | -.60999000 x 10 ⁻³ |
| 6 3 | .11188000 | -.12790000 | -.11430000 x 10 ⁻³ | 6 2 | .43686000 x 10 ⁻² | -.18008600 x 10 ⁻² | .17906000 x 10 ⁻³ | 7 3 | .92960000 x 10 ⁻³ | -.98460000 x 10 ⁻³ | .69400000 x 10 ⁻⁴ |
| 8 4 | .81540000 x 10 ⁻³ | ----- | ----- | 8 3 | .68680000 x 10 ⁻³ | -.91100000 x 10 ⁻³ | .69880000 x 10 ⁻³ | 9 4 | .14770000 | ----- | ----- |
| 10 5 | ----- | ----- | ----- | 10 4 | .82800000 x 10 ⁻⁴ | -.99900000 x 10 ⁻⁴ | .22970000 | 11 5 | ----- | ----- | ----- |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(g) $L = 0.3$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ |
|-----------------|-----|--------------------------------|---|---|-----|-----|-------------------------------|---|---|-------|-------|-------------------------------|---|---|
| $\lambda = 1.6$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.59611610 | -0.13906716 | 0.006191598 | 2 | 0 | 0.2922941 | -0.099917008 | 0.001952572 | 10 | 0 | 0.40277148 | -0.18088873 | 0.000001765 |
| 2 | 1 | .040999800 | .94068000 x 10 ⁻² | -.81949500 x 10 ⁻² | 4 | 1 | .051204100 | -.76902400 x 10 ⁻² | -.11667800 x 10 ⁻² | 5 | 1 | .058599900 | -.51096000 x 10 ⁻² | -.41150700 x 10 ⁻² |
| 4 | 2 | -.040999800 x 10 ⁻² | -.89999000 x 10 ⁻² | -.266801000 | 6 | 2 | -.50040000 x 10 ⁻² | -.79999000 x 10 ⁻² | -.67844000 x 10 ⁻³ | 7 | 2 | -.98999000 x 10 ⁻² | -.79010000 x 10 ⁻² | -.15564400 |
| 6 | 3 | -.17900000 x 10 ⁻⁴ | .99670000 x 10 ⁻³ | -.66140000 x 10 ⁻³ | 8 | 3 | -.99999000 x 10 ⁻³ | -.99999000 x 10 ⁻³ | -.18270000 | 9 | 3 | .90680000 x 10 ⁻³ | .17690000 | -.9460000 x 10 ⁻⁵ |
| 8 | 4 | ----- | .11120000 | -.12730000 | 10 | 4 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- |
| $\lambda = 1.8$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.5625920 | -0.1894407 | 0.008071211 | 2 | 0 | 0.33919691 | -0.095999898 | 0.007446745 | 10 | 0 | 0.44607799 | -0.11908905 | 0.00799001 |
| 2 | 1 | .001189500 | .97829000 x 10 ⁻² | -.92864000 x 10 ⁻² | 4 | 1 | .008729400 | -.88494000 x 10 ⁻² | -.28412600 x 10 ⁻² | 5 | 1 | .008997400 | .22761700 x 10 ⁻² | -.92077700 x 10 ⁻² |
| 4 | 2 | -.37721000 x 10 ⁻² | .46929000 | -.87072000 | 6 | 2 | -.16020000 x 10 ⁻² | -.80000000 x 10 ⁻³ | -.11894400 | 7 | 2 | -.58900000 x 10 ⁻⁴ | .05040000 | -.18486600 |
| 6 | 3 | -.14780000 | .10179000 | -.98790000 x 10 ⁻³ | 8 | 3 | ----- | .89130000 | -.27160000 x 10 ⁻³ | 9 | 3 | -.99800000 x 10 ⁻³ | .69680000 x 10 ⁻³ | -.42510000 x 10 ⁻³ |
| 8 | 4 | -.26600000 x 10 ⁻³ | .28960000 x 10 ⁻³ | -.11970000 | 10 | 4 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- |
| $\lambda = 2.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.59715270 | -0.18079418 | 0.00996692 | 2 | 0 | 0.37908949 | -0.095897197 | 0.008291667 | 10 | 0 | 0.44810992 | -0.10999046 | 0.00867472 |
| 2 | 1 | -.05999000 x 10 ⁻² | .01719000 | -.98469000 x 10 ⁻² | 4 | 1 | .016682600 | -.97788000 x 10 ⁻² | -.46024000 x 10 ⁻² | 5 | 1 | .00084000 | .90849000 x 10 ⁻² | -.68112000 x 10 ⁻² |
| 4 | 2 | -.92224000 | .99979000 x 10 ⁻² | -.19099000 | 6 | 2 | -.16999000 x 10 ⁻² | .28996000 | -.33141000 | 7 | 2 | -.49999000 x 10 ⁻² | .97790000 | -.16748000 |
| 6 | 3 | ----- | ----- | -.9927000 x 10 ⁻³ | 8 | 3 | -.89999000 x 10 ⁻³ | -.94000000 x 10 ⁻³ | -.26990000 x 10 ⁻³ | 9 | 3 | -.13870000 | .89460000 x 10 ⁻³ | -.29980000 x 10 ⁻³ |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- |
| $\lambda = 2.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.6796270 | -0.2099420 | 0.0090090 x 10 ⁻² | 2 | 0 | 0.46341005 | -0.09798005 | 0.009490000 x 10 ⁻² | 10 | 0 | 0.77969999 | -0.09096047 | 0.0792800 x 10 ⁻² |
| 2 | 1 | -.09999000 | .09009490 | -.89994700 | 4 | 1 | -.08498000 | .01987000 | -.98988000 | 5 | 1 | -.09991900 | .00299430 | -.7466000 |
| 4 | 2 | -.06180000 | .97940000 x 10 ⁻² | ----- | 6 | 2 | -.90040000 x 10 ⁻² | -.91092000 x 10 ⁻² | -.99000000 x 10 ⁻³ | 7 | 2 | -.01829000 | -.94999000 x 10 ⁻² | -.98740000 x 10 ⁻³ |
| 6 | 3 | -.16990000 x 10 ⁻² | ----- | ----- | 8 | 3 | -.14840000 | ----- | ----- | 9 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- |
| $\lambda = 3.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.7029120 | -0.09988631 | 0.3999940 x 10 ⁻² | 2 | 0 | 0.98991697 | -0.07990968 | 0.99029640 x 10 ⁻² | 10 | 0 | 0.61868004 | -0.06999909 | 0.9087990 x 10 ⁻² |
| 2 | 1 | -.19999980 | .09788600 | -.89999000 | 4 | 1 | -.07760460 | .08499270 | -.98999600 | 5 | 1 | -.10996960 | .09088000 | -.89049800 |
| 4 | 2 | -.86900000 x 10 ⁻² | -.11990000 x 10 ⁻² | -.14412000 | 6 | 2 | -.87710000 x 10 ⁻² | .99000000 x 10 ⁻³ | -.94900000 x 10 ⁻³ | 7 | 2 | -.91800000 x 10 ⁻² | .14900000 x 10 ⁻⁴ | -.94180000 x 10 ⁻³ |
| 6 | 3 | -.88200000 | -.12170000 | .40000000 x 10 ⁻³ | 8 | 3 | ----- | ----- | ----- | 9 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | ----- | ----- | ----- | ----- | ----- |

TABLE I.- COMPUTED VALUES OF G_n , $J \frac{\partial G_n}{\partial \xi}$, AND $\frac{\partial G_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(g) $L = 0.3$ - Continued

| P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P | n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ |
|-----------------|-----|------------|---------------------------------------|---------------------------------------|-----|-----|----------------------------|---------------------------------------|---------------------------------------|-----|-----|----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.7544240 | -0.079567277 | $0.26771910 \times 10^{-2}$ | 2 | 0 | 0.58507895 | -0.072520940 | $0.40872040 \times 10^{-2}$ | 1 | 0 | 0.65754269 | -0.076505770 | $0.34545720 \times 10^{-2}$ |
| 2 | 1 | -.20560970 | .040709900 | -.545909500 | 4 | 1 | -.15480070 | .050287400 | -.49030600 | 3 | 1 | -.16759640 | .055538900 | -.52259700 |
| 4 | 2 | .011583000 | -.66730000 $\times 10^{-2}$ | .21756000 | 6 | 2 | .20120000 $\times 10^{-2}$ | -.31710000 $\times 10^{-2}$ | .13587000 | 5 | 2 | .59180000 $\times 10^{-2}$ | -.47210000 $\times 10^{-2}$ | .17446000 |
| 6 | 3 | ----- | ----- | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.76256890 | -0.070225225 | $0.18975950 \times 10^{-2}$ | 2 | 0 | 0.62622869 | -0.066066928 | $0.29259850 \times 10^{-2}$ | 1 | 0 | 0.69568092 | -0.068755620 | $0.24415890 \times 10^{-2}$ |
| 2 | 1 | -.26628470 | .041916700 | -.45189800 | 4 | 1 | -.19099720 | .053440700 | -.41122000 | 3 | 1 | -.22251880 | .057626800 | -.42490800 |
| 4 | 2 | .040278000 | -.011595000 | .24415000 | 6 | 2 | .022155000 | -.73280000 $\times 10^{-2}$ | .18016000 | 5 | 2 | .050454000 | -.95160000 $\times 10^{-2}$ | .21140000 |
| 6 | 3 | ----- | ----- | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.75556320 | -0.049591057 | $0.65569700 \times 10^{-3}$ | 2 | 0 | 0.73809610 | -0.047754861 | $0.99628700 \times 10^{-3}$ | 1 | 0 | 0.78632219 | -0.048663695 | $0.81970500 \times 10^{-3}$ |
| 2 | 1 | -.45089780 | .058404700 | -.18087800 $\times 10^{-2}$ | 4 | 1 | -.37781100 | .054699900 | -.19104500 $\times 10^{-2}$ | 3 | 1 | -.41541720 | .056960000 | -.18750400 $\times 10^{-2}$ |
| 4 | 2 | .18927000 | -.021901000 | .19090000 | 6 | 2 | .12871000 | -.018543000 | .17660000 | 5 | 2 | .16810000 | -.020025000 | .18440000 |
| 6 | 3 | ----- | .011680000 | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | .010470000 | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |

TABLE I.- COMPUTED VALUES OF G_n , $J \frac{\partial G_n}{\partial \xi}$, AND $\frac{\partial G_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L' AND λ - Continued

(h) $L = 0.4$

| P n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ | P n | $G_n(P)$ | $J \frac{\partial G_n}{\partial \xi}$ | $\frac{\partial G_n}{\partial \xi^2}$ |
|------------------|----------------------------|---------------------------------------|---------------------------------------|-----|-----------------------------|---------------------------------------|---------------------------------------|-----|-----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | | | |
| 0 0 | 0.014114500 | -0.39435420 | 6.9787549 | 2 0 | | | | 1 0 | | | |
| 2 1 | | | | 4 1 | | | | 3 1 | | | |
| $\lambda = 0.05$ | | | | | | | | | | | |
| 0 0 | 0.041419500 | -0.58743250 | 2.2254792 | 2 0 | | | | 1 0 | $0(10^{-15})$ | $0(10^{-13})$ | $0(10^{-11})$ |
| 2 1 | | | | 4 1 | | | | 3 1 | | | |
| $\lambda = 0.10$ | | | | | | | | | | | |
| 0 0 | 0.067547800 | -0.37296088 | 1.2781070 | 2 0 | $0(10^{-20})$ | $0(10^{-18})$ | $0(10^{-17})$ | 1 0 | $0.13980440 \times 10^{-6}$ | $-0.30417355 \times 10^{-5}$ | 0.6952270×10^{-4} |
| 2 1 | $0(10^{-20})$ | $0(10^{-18})$ | $0(10^{-17})$ | 4 1 | | | | 3 1 | $0(10^{-18})$ | $0(10^{-16})$ | $0(10^{-15})$ |
| $\lambda = 0.20$ | | | | | | | | | | | |
| 0 0 | 0.12815360 | -0.34874656 | 0.57415104 | 2 0 | $0.27481410 \times 10^{-6}$ | $-0.29877354 \times 10^{-5}$ | $0.31204555 \times 10^{-4}$ | 1 0 | $0.14914160 \times 10^{-2}$ | $-0.9542226 \times 10^{-2}$ | 0.054765007 |
| 2 1 | $.2377700 \times 10^{-6}$ | $-.27754588 \times 10^{-5}$ | $.2889258 \times 10^{-4}$ | 4 1 | $0(10^{-20})$ | $0(10^{-18})$ | $0(10^{-17})$ | 3 1 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-9})$ |
| 4 2 | $0(10^{-20})$ | $0(10^{-18})$ | $0(10^{-17})$ | 6 2 | | | | 5 2 | | | |
| $\lambda = 0.40$ | | | | | | | | | | | |
| 0 0 | 0.25186450 | -0.50725420 | 0.25592514 | 2 0 | 0.2222000×10^{-2} | $-0.90076160 \times 10^{-2}$ | 0.02568825 | 1 0 | 0.040899160 | -0.08502380 | 0.15537004 |
| 2 1 | $.22404120 \times 10^{-2}$ | $-.69811720 \times 10^{-2}$ | $.019291809$ | 4 1 | $.46080100 \times 10^{-6}$ | $-.24826510 \times 10^{-5}$ | $.12856410 \times 10^{-4}$ | 3 1 | $.56255400 \times 10^{-4}$ | $-.24129210 \times 10^{-3}$ | $.99737140 \times 10^{-5}$ |
| 4 2 | $.39977300 \times 10^{-6}$ | $-.21441900 \times 10^{-5}$ | $.11043450 \times 10^{-4}$ | 6 2 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-10})$ | 5 2 | $.10216700 \times 10^{-8}$ | $-.67092400 \times 10^{-8}$ | $.42854720 \times 10^{-7}$ |
| 6 3 | $0(10^{-12})$ | $0(10^{-11})$ | $0(10^{-10})$ | 8 3 | | | | 7 3 | $0(10^{-15})$ | $0(10^{-14})$ | $0(10^{-13})$ |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda^2}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(b) L = 0.4 - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda^2}$ |
|------------------|---|------------------------------|---|---|----|---|-------------------------------|---|---|----|---|------------------------------|---|---|
| $\lambda = 0.60$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.31688050 | -0.27284787 | 0.12859406 | 0 | 0 | 0.026481560 | -0.043022228 | 0.060780573 | 1 | 0 | 0.11179778 | -0.13771944 | 0.12510047 |
| 2 | 1 | .017059950 | -.026413770 | .0526886180 | 2 | 1 | .28550100 x 10 ⁻³ | -.78751600 x 10 ⁻³ | .17070210 x 10 ⁻² | 3 | 1 | .28500660 x 10 ⁻² | -.77811510 x 10 ⁻² | .010233357 |
| 4 | 2 | .21195400 x 10 ⁻³ | -.28059500 x 10 ⁻³ | .12047620 x 10 ⁻² | 4 | 2 | .50145000 x 10 ⁻⁵ | -.17820750 x 10 ⁻⁵ | .60704540 x 10 ⁻⁵ | 5 | 2 | .13445500 x 10 ⁻⁴ | -.40646900 x 10 ⁻⁴ | .11973500 x 10 ⁻³ |
| 6 | 3 | .40599000 x 10 ⁻⁵ | -.14198900 x 10 ⁻⁵ | .47896800 x 10 ⁻⁵ | 6 | 3 | .11795000 x 10 ⁻⁹ | -.24438950 x 10 ⁻⁹ | .24438950 x 10 ⁻⁸ | 7 | 3 | .90530000 x 10 ⁻⁸ | -.36790000 x 10 ⁻⁷ | .14479000 x 10 ⁻⁶ |
| 8 | 4 | 0(10 ⁻¹⁰) | -.45877000 x 10 ⁻⁹ | .20436200 x 10 ⁻⁸ | 8 | 4 | 0(10 ⁻¹⁴) | 0(10 ⁻¹³) | 0(10 ⁻¹²) | 9 | 4 | 0(10 ⁻¹²) | 0(10 ⁻¹¹) | 0(10 ⁻¹⁰) |
| 10 | 5 | ----- | 0(10 ⁻¹³) | 0(10 ⁻¹³) | 10 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.38735490 | -0.24509802 | 0.080589200 | 0 | 0 | 0.071721640 | -0.076720568 | 0.064809139 | 1 | 0 | 0.11899684 | -0.13606601 | 0.090177800 |
| 2 | 1 | .056145500 | -.053761620 | .022696540 | 2 | 1 | .31172170 x 10 ⁻² | -.47722500 x 10 ⁻² | .28267750 x 10 ⁻² | 3 | 1 | .012564870 | -.015041220 | .014745222 |
| 4 | 2 | .15279500 x 10 ⁻² | -.27222400 x 10 ⁻² | .32615900 x 10 ⁻² | 4 | 2 | .60659000 x 10 ⁻⁴ | -.12529220 x 10 ⁻³ | .27222220 x 10 ⁻³ | 5 | 2 | .39915000 x 10 ⁻³ | -.69067500 x 10 ⁻³ | .10717950 x 10 ⁻² |
| 6 | 3 | .41692000 x 10 ⁻⁴ | -.22921000 x 10 ⁻⁴ | .15079600 x 10 ⁻³ | 6 | 3 | .41817000 x 10 ⁻⁶ | -.10881500 x 10 ⁻⁵ | .27207400 x 10 ⁻⁵ | 7 | 3 | .48906000 x 10 ⁻⁵ | -.11210900 x 10 ⁻⁴ | .24520400 x 10 ⁻⁴ |
| 8 | 4 | .51550000 x 10 ⁻⁶ | -.75490000 x 10 ⁻⁶ | .19677000 x 10 ⁻⁵ | 8 | 4 | -.30443000 x 10 ⁻⁸ | -.94008000 x 10 ⁻⁸ | 0(10 ⁻¹¹) | 9 | 4 | .20120000 x 10 ⁻⁷ | -.77730000 x 10 ⁻⁷ | .16154000 x 10 ⁻⁶ |
| 10 | 5 | .74200000 x 10 ⁻⁹ | -.25450000 x 10 ⁻⁸ | .72970000 x 10 ⁻⁸ | 10 | 5 | ----- | ----- | ----- | 11 | 5 | 0(10 ⁻¹⁰) | 0(10 ⁻¹⁰) | .50960000 x 10 ⁻⁹ |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.44659580 | -0.22244247 | 0.054152940 | 0 | 0 | 0.12660600 | -0.097794556 | 0.056998687 | 1 | 0 | 0.25476510 | -0.15998511 | 0.065154140 |
| 2 | 1 | .047781500 | -.028029770 | .022228900 x 10 ⁻² | 2 | 1 | .90242700 x 10 ⁻² | -.97106700 x 10 ⁻² | .76941470 x 10 ⁻² | 3 | 1 | .025946950 | -.018972720 | .012265950 |
| 4 | 2 | .46002000 x 10 ⁻² | -.39408900 x 10 ⁻² | .22255500 | 4 | 2 | .55089000 x 10 ⁻³ | -.68869800 x 10 ⁻³ | .78246600 x 10 ⁻³ | 5 | 2 | .17470600 x 10 ⁻² | -.12229500 x 10 ⁻² | .16662400 x 10 ⁻² |
| 6 | 3 | .29548000 x 10 ⁻³ | -.35814000 x 10 ⁻³ | .36574000 x 10 ⁻³ | 6 | 3 | .16757000 x 10 ⁻⁴ | -.27589000 x 10 ⁻⁴ | .22205000 x 10 ⁻⁴ | 7 | 3 | .78960000 x 10 ⁻⁴ | -.11259000 x 10 ⁻³ | .14528200 x 10 ⁻³ |
| 8 | 4 | .10690000 x 10 ⁻⁴ | -.16612000 x 10 ⁻⁴ | .24550000 x 10 ⁻⁴ | 8 | 4 | .28920000 x 10 ⁻⁶ | -.59410000 x 10 ⁻⁶ | .10865000 x 10 ⁻⁵ | 9 | 4 | .19910000 x 10 ⁻⁵ | -.35540000 x 10 ⁻⁵ | .59080000 x 10 ⁻⁵ |
| 10 | 5 | ----- | ----- | .72950000 x 10 ⁻⁶ | 10 | 5 | .25400000 x 10 ⁻⁸ | -.54100000 x 10 ⁻⁸ | .12810000 x 10 ⁻⁷ | 11 | 5 | ----- | ----- | .10820000 x 10 ⁻⁶ |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.49653600 | -0.22046960 | 0.058977570 | 0 | 0 | 0.12955590 | -0.10220900 | 0.046410552 | 1 | 0 | 0.31614552 | -0.15722712 | 0.040045220 |
| 2 | 1 | .042441800 | -.016618720 | -.35207500 x 10 ⁻² | 2 | 1 | .017186550 | -.011622160 | .59222400 x 10 ⁻² | 3 | 1 | .051546150 | -.016524750 | .55722900 x 10 ⁻² |
| 4 | 2 | .56162000 x 10 ⁻² | -.25212000 x 10 ⁻² | -.34008000 x 10 ⁻³ | 4 | 2 | .14221000 x 10 ⁻² | -.12702000 x 10 ⁻² | .22896000 x 10 ⁻² | 5 | 2 | .32003400 x 10 ⁻² | -.21450500 x 10 ⁻² | .20910000 x 10 ⁻³ |
| 6 | 3 | .61610000 x 10 ⁻³ | -.42770000 x 10 ⁻³ | .14956000 | 6 | 3 | .10238000 x 10 ⁻³ | -.10928000 x 10 ⁻³ | .10299000 | 7 | 3 | .27551000 x 10 ⁻³ | -.25002000 x 10 ⁻³ | .17345000 |
| 8 | 4 | .55220000 x 10 ⁻⁴ | -.47450000 x 10 ⁻⁴ | .40050000 x 10 ⁻⁴ | 8 | 4 | .55400000 x 10 ⁻⁵ | -.61200000 x 10 ⁻⁵ | .20620000 x 10 ⁻⁵ | 9 | 4 | .17680000 x 10 ⁻⁴ | -.19640000 x 10 ⁻⁴ | .20820000 x 10 ⁻⁴ |
| 10 | 5 | ----- | ----- | ----- | 10 | 5 | ----- | ----- | ----- | 11 | 5 | .16220000 | .95600000 x 10 ⁻⁵ | .20460000 x 10 ⁻⁵ |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.55296440 | -0.12441226 | 0.026184250 | 0 | 0 | 0.25749289 | -0.11422406 | 0.057415955 | 1 | 0 | 0.37025975 | -0.15165524 | 0.056217554 |
| 2 | 1 | .059223100 | -.57976400 x 10 ⁻² | -.92167500 x 10 ⁻² | 2 | 1 | .021624090 | -.92197800 x 10 ⁻² | .15577000 x 10 ⁻² | 3 | 1 | .051688700 | -.96574900 x 10 ⁻² | -.24325100 x 10 ⁻² |
| 4 | 2 | .57690000 x 10 ⁻² | .55740000 x 10 ⁻³ | -.22922000 | 4 | 2 | .21292000 x 10 ⁻² | -.11275000 | .128744000 x 10 ⁻³ | 5 | 2 | .55186000 x 10 ⁻² | -.99180000 x 10 ⁻³ | -.52855000 x 10 ⁻³ |
| 6 | 3 | .51870000 x 10 ⁻³ | -.45700000 x 10 ⁻⁴ | -.22150000 x 10 ⁻³ | 6 | 3 | .22040000 x 10 ⁻³ | -.14670000 x 10 ⁻³ | .52200000 x 10 ⁻⁴ | 7 | 3 | .52860000 x 10 ⁻³ | -.17540000 | -.21200000 x 10 ⁻⁴ |
| 8 | 4 | ----- | ----- | ----- | 8 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 10 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |

TABLE I.-- COMPOUND VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$, AND $\frac{\partial Q_n}{\partial \xi}$ FOR $n = 0$ TO 5 AND LIPPER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(h) $L = 0.4$ - Continued

| P | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi}$ | P | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi}$ | P | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi}$ |
|-----------------|-----------------------------|---------------------------------------|-------------------------------------|------|-----------------------------|---------------------------------------|-------------------------------------|------|-----------------------------|---------------------------------------|-------------------------------------|
| $\lambda = 1.6$ | | | | | | | | | | | |
| 0 0 | 0.57565290 | -0.16972862 | 0.021310728 | 2 0 | 0.20721417 | -0.11545428 | 0.070120985 | 1 0 | 0.41785502 | -0.14480667 | 0.027865708 |
| 2 1 | .021750500 | .28516900 $\times 10^{-2}$ | -.012760970 | 4 1 | .021115400 | -.52812200 $\times 10^{-2}$ | -.20251700 $\times 10^{-2}$ | 3 1 | .024759200 | -.12141700 $\times 10^{-2}$ | -.64428900 $\times 10^{-2}$ |
| 4 2 | -.27060000 $\times 10^{-3}$ | .50966000 | -.29129000 $\times 10^{-2}$ | 6 2 | .124399000 $\times 10^{-2}$ | -.20480000 $\times 10^{-3}$ | -.61451000 $\times 10^{-3}$ | 5 2 | .16047000 $\times 10^{-2}$ | .25480000 $\times 10^{-5}$ | -.15708000 |
| 6 3 | -.11200000 | .48000000 $\times 10^{-3}$ | -.50060000 $\times 10^{-3}$ | 8 3 | .20740000 $\times 10^{-3}$ | | | 7 3 | .17260000 $\times 10^{-3}$ | .12950000 | -.24340000 $\times 10^{-3}$ |
| 8 4 | | | | 10 4 | | | | 9 4 | | | |
| 10 5 | | | | 12 5 | | | | 11 5 | | | |
| $\lambda = 1.8$ | | | | | | | | | | | |
| 0 0 | 0.60747590 | -0.15710465 | 0.016490221 | 2 0 | 0.55955622 | -0.11222798 | 0.024369957 | 1 0 | 0.45970722 | -0.13765021 | 0.021832107 |
| 2 1 | -.11022000 $\times 10^{-2}$ | .012669870 | -.012970260 | 4 1 | .0130222600 | .67720000 $\times 10^{-3}$ | -.47497700 $\times 10^{-2}$ | 3 1 | .011505900 | .72824000 $\times 10^{-2}$ | -.88429400 $\times 10^{-2}$ |
| 4 2 | -.52971000 | .47275000 $\times 10^{-2}$ | -.29840000 $\times 10^{-2}$ | 6 2 | .29220000 $\times 10^{-3}$ | .30228000 $\times 10^{-2}$ | -.11328000 | 5 2 | -.14200000 $\times 10^{-2}$ | .25262000 | -.12650000 |
| 6 3 | -.22100000 $\times 10^{-3}$ | .78300000 $\times 10^{-3}$ | -.40620000 $\times 10^{-3}$ | 8 3 | | | -.12490000 $\times 10^{-3}$ | 7 3 | -.32400000 $\times 10^{-3}$ | .42200000 $\times 10^{-3}$ | -.30710000 $\times 10^{-3}$ |
| 8 4 | | | | 10 4 | | | | 9 4 | | | |
| 10 5 | | | | 12 5 | | | | 11 5 | | | |
| $\lambda = 2.0$ | | | | | | | | | | | |
| 0 0 | 0.65926720 | -0.14282229 | 0.013007609 | 2 0 | 0.57221522 | -0.11176222 | 0.018262562 | 1 0 | 0.43667522 | -0.12055522 | 0.017521225 |
| 2 1 | -.027622200 | .027515200 | -.013911150 | 4 1 | .42655000 $\times 10^{-2}$ | .70521200 $\times 10^{-2}$ | -.69715900 $\times 10^{-2}$ | 3 1 | -.66220000 $\times 10^{-2}$ | .01559470 | -.01029970 |
| 4 2 | -.22296000 $\times 10^{-2}$ | .50241000 $\times 10^{-2}$ | -.17132000 $\times 10^{-2}$ | 6 2 | -.22125000 | .20722000 | -.12297000 | 5 2 | -.47612000 | .34921000 $\times 10^{-2}$ | -.15800000 $\times 10^{-2}$ |
| 6 3 | -.143590000 | | | 8 3 | -.32800000 $\times 10^{-3}$ | -.32900000 $\times 10^{-3}$ | -.17250000 $\times 10^{-3}$ | 7 3 | -.22200000 $\times 10^{-3}$ | .52100000 $\times 10^{-5}$ | -.19550000 $\times 10^{-5}$ |
| 8 4 | | | | 10 4 | | | | 9 4 | | | |
| 10 5 | | | | 12 5 | | | | 11 5 | | | |
| $\lambda = 2.5$ | | | | | | | | | | | |
| 0 0 | 0.69220750 | -0.12251702 | 0.78231600 $\times 10^{-2}$ | 2 0 | 0.46512475 | -0.10292219 | 0.012379222 | 1 0 | 0.572201711 | -0.11440436 | 0.010224928 |
| 2 1 | -.10112310 | .041262900 | -.012245610 | 4 1 | -.059021100 | .021472000 | -.22311300 $\times 10^{-2}$ | 3 1 | -.057069500 | .030297700 | -.010297720 |
| 4 2 | -.010253000 | .25922000 $\times 10^{-2}$ | .97890000 $\times 10^{-3}$ | 6 2 | -.62090000 $\times 10^{-2}$ | .21507000 $\times 10^{-2}$ | -.20420000 $\times 10^{-3}$ | 5 2 | -.26560000 $\times 10^{-2}$ | .21694000 $\times 10^{-2}$ | .22960000 $\times 10^{-3}$ |
| 6 3 | | | | 8 3 | | | | 7 3 | | | |
| 8 4 | | | | 10 4 | | | | 9 4 | | | |
| 10 5 | | | | 12 5 | | | | 11 5 | | | |
| $\lambda = 3.0$ | | | | | | | | | | | |
| 0 0 | 0.73212250 | -0.10297207 | 0.42892150 $\times 10^{-2}$ | 2 0 | 0.55120300 | -0.09547662 | 0.21177590 $\times 10^{-2}$ | 1 0 | 0.62221700 | -0.10021212 | 0.62220670 $\times 10^{-2}$ |
| 2 1 | -.17221150 | .042222100 | -.59222200 | 4 1 | -.024970200 | .051222600 | -.72221500 | 3 1 | -.12022200 | .099222000 | -.01222200 |
| 4 2 | .51120000 $\times 10^{-2}$ | -.51270000 $\times 10^{-2}$ | -.27222000 | 6 2 | -.22700000 $\times 10^{-2}$ | -.13920000 $\times 10^{-2}$ | .12419000 | 5 2 | -.72000000 $\times 10^{-3}$ | -.27220000 $\times 10^{-2}$ | .12222000 |
| 6 3 | | | | 8 3 | | | | 7 3 | | | |
| 8 4 | | | | 10 4 | | | | 9 4 | | | |
| 10 5 | | | | 12 5 | | | | 11 5 | | | |

TABLE I.- COMPUTED VALUES OF a_n , $J \frac{da_n}{d\lambda}$, AND $\frac{da_n}{d\lambda^2}$ FOR $n = 0$ TO 5 AND INTEGERS

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(h) $L = 0.4$ - Concluded

| P | n | $a_n(P)$ | $J \frac{da_n}{d\lambda}$ | $\frac{da_n}{d\lambda^2}$ | P | n | $a_n(P)$ | $J \frac{da_n}{d\lambda}$ | $\frac{da_n}{d\lambda^2}$ | P | n | $a_n(P)$ | $J \frac{da_n}{d\lambda}$ | $\frac{da_n}{d\lambda^2}$ |
|-----------------|-----|------------|---------------------------|-----------------------------|-----|-----|------------|-----------------------------|-----------------------------|-----|-----|------------|-----------------------------|------------------------------|
| $\lambda = 3.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.76504840 | -0.095740696 | $0.32837250 \times 10^{-2}$ | 2 | 0 | 0.56637848 | -0.084767812 | $0.55627690 \times 10^{-2}$ | 1 | 0 | 0.67376228 | -0.089228220 | $0.432181100 \times 10^{-2}$ |
| 2 | 1 | -.24401060 | .052175900 | -.74725900 | 4 | 1 | -.15423350 | .037841200 | -.67476400 | 3 | 1 | -.19253020 | .044827300 | -.71984500 |
| 4 | 2 | .029714000 | -.011967000 | .56030000 | 6 | 2 | .012204000 | -.61490000 $\times 10^{-2}$ | .22649000 | 5 | 2 | .009413000 | -.87300000 $\times 10^{-2}$ | .28940000 |
| 6 | 3 | ----- | ----- | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.79128890 | -0.085444460 | $0.25046980 \times 10^{-2}$ | 2 | 0 | 0.63027929 | -0.077113656 | $0.39560280 \times 10^{-2}$ | 1 | 0 | 0.70224672 | -0.080692956 | $0.31829920 \times 10^{-2}$ |
| 2 | 1 | -.30576970 | .032290200 | -.58133500 | 4 | 1 | -.21194920 | .041191400 | -.56054700 | 3 | 1 | -.25598380 | .046885800 | -.57737600 |
| 4 | 2 | .064877000 | -.017655000 | .50190000 | 6 | 2 | .036015000 | -.011213000 | .27720000 | 5 | 2 | .048752000 | -.014209000 | .32710000 |
| 6 | 3 | ----- | ----- | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.85602580 | -0.057590456 | $0.75212500 \times 10^{-3}$ | 2 | 0 | 0.74275869 | -0.055503520 | $0.13240680 \times 10^{-2}$ | 1 | 0 | 0.75885853 | -0.056690128 | $0.10462260 \times 10^{-2}$ |
| 2 | 1 | -.42809060 | .046555700 | -.25590000 $\times 10^{-2}$ | 4 | 1 | -.40022580 | .043450000 | -.25448000 | 3 | 1 | -.44295060 | .045555500 | -.24561000 |
| 4 | 2 | .22520000 | -.020560000 | .26140000 | 6 | 2 | .17281000 | -.029620000 | .24260000 | 5 | 2 | .19779000 | -.026050000 | .25320000 |
| 6 | 3 | ----- | ----- | ----- | 8 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 10 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |

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TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \xi}$ AND $\frac{\partial Q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INTEGER

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(1). $L = 0.5$

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \xi}$ | $\frac{\partial Q_n}{\partial \xi^2}$ |
|------------------|-----|-----------------------------|---------------------------------------|---------------------------------------|-----|-----|-----------------------------|---------------------------------------|---------------------------------------|-----|-----|-----------------------------|---------------------------------------|---------------------------------------|
| $\lambda = 0.02$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.01775900 | -0.49218008 | 9.7274972 | 2 | 0 | | | | 1 | 0 | | | |
| 2 | 1 | | | | 4 | 1 | | | | 3 | 1 | | | |
| $\lambda = 0.05$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.046199200 | -0.47695545 | 3.0860513 | 2 | 0 | | | | 1 | 0 | $0(10^{-18})$ | $0(10^{-16})$ | $0(10^{-14})$ |
| 2 | 1 | | | | 4 | 1 | | | | 3 | 1 | | | |
| $\lambda = 0.10$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.075042600 | -0.46247869 | 1.7654721 | 2 | 0 | $0(10^{-24})$ | $0(10^{-23})$ | $0(10^{-21})$ | 1 | 0 | $0.10508080 \times 10^{-7}$ | $-0.28139968 \times 10^{-6}$ | 0.7298974×10^{-5} |
| 2 | 1 | $0(10^{-24})$ | $0(10^{-23})$ | $0(10^{-21})$ | 4 | 1 | | | | 3 | 1 | | | |
| $\lambda = 0.20$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.14158040 | -0.44965581 | 0.7827521 | 2 | 0 | $0.20656860 \times 10^{-7}$ | $-0.27655829 \times 10^{-6}$ | $0.35786326 \times 10^{-5}$ | 1 | 0 | $0.75610200 \times 10^{-3}$ | $-0.58316270 \times 10^{-2}$ | 0.040904225 |
| 2 | 1 | $-1.9196400 \times 10^{-7}$ | $-2.5641568 \times 10^{-6}$ | $-3.5122586 \times 10^{-5}$ | 4 | 1 | $0(10^{-24})$ | $0(10^{-23})$ | $0(10^{-21})$ | 3 | 1 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ |
| 4 | 2 | $0(10^{-24})$ | $0(10^{-23})$ | $0(10^{-21})$ | | | | | | | | | | |
| $\lambda = 0.40$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.25344770 | -0.37327717 | 0.31203926 | 2 | 0 | $0.14309200 \times 10^{-2}$ | $-0.54942180 \times 10^{-2}$ | 0.019263260 | 1 | 0 | 0.034585840 | -0.08827520 | 0.18413242 |
| 2 | 1 | $.11211810 \times 10^{-2}$ | $-.42181690 \times 10^{-2}$ | $.014307067$ | 4 | 1 | $.34481500 \times 10^{-7}$ | $-.22952240 \times 10^{-6}$ | $.14769211 \times 10^{-5}$ | 3 | 1 | $.12835240 \times 10^{-4}$ | $-.66780320 \times 10^{-4}$ | $.32674690 \times 10^{-3}$ |
| 4 | 2 | $.29797800 \times 10^{-7}$ | $-.19739240 \times 10^{-6}$ | $.12524585 \times 10^{-5}$ | 6 | 1 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | 5 | 2 | $0(10^{-10})$ | $-.15129270 \times 10^{-9}$ | $.12027576 \times 10^{-8}$ |
| 6 | 3 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-12})$ | | | | | | | | | | |

TABLE I.- COMPUTED VALUES OF Q_n , $J \frac{\partial Q_n}{\partial \lambda}$, AND $\frac{\partial Q_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INCREASING

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued

(i) $L = 0.5$ - Continued

| P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ | P | n | $Q_n(P)$ | $J \frac{\partial Q_n}{\partial \lambda}$ | $\frac{\partial Q_n}{\partial \lambda}$ |
|------------------|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 0.50$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.34331890 | -0.34834077 | 0.16282165 | 0 | 0 | 0.035394577 | -0.028252891 | 0.063830930 | 1 | 0 | 0.10909566 | -0.14978049 | 0.16009630 |
| 2 | 1 | 0.018127510 | -0.08234510 | 0.05663700 | 2 | 1 | 0.053110000 | -0.06823160 | 0.12086360 | 3 | 1 | 0.24072500 | -0.34710030 | 0.78796830 |
| 4 | 2 | $0.63944800 \times 10^{-4}$ | $-1.95894370 \times 10^{-3}$ | $0.34898530 \times 10^{-2}$ | 4 | 2 | $0.85431100 \times 10^{-4}$ | $-1.6567730 \times 10^{-3}$ | $0.69921300 \times 10^{-2}$ | 5 | 2 | 0.2531700×10^{-2} | $-0.0098800 \times 10^{-2}$ | 0.8237110×10^{-4} |
| 6 | 3 | $0.29904000 \times 10^{-7}$ | $-1.3008800 \times 10^{-6}$ | 0.3469300×10^{-5} | 6 | 3 | $0.37825000 \times 10^{-7}$ | $-1.6367730 \times 10^{-6}$ | $0.69921300 \times 10^{-5}$ | 7 | 3 | $0.27144000 \times 10^{-2}$ | $-1.1621900 \times 10^{-2}$ | 0.7129900×10^{-8} |
| 8 | 4 | $0(10^{-11})$ | $0(10^{-11})$ | $0(10^{-10})$ | 8 | 4 | $0(10^{-11})$ | $0(10^{-11})$ | $0(10^{-10})$ | 9 | 4 | $0(10^{-14})$ | $0(10^{-13})$ | $0(10^{-13})$ |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| $\lambda = 0.80$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.41649630 | -0.29175187 | 0.10346298 | 0 | 0 | 0.060199310 | -0.072230825 | 0.076380250 | 1 | 0 | 0.18192678 | -0.17500705 | 0.11799708 |
| 2 | 1 | 0.088662140 | -0.031116660 | 0.085047080 | 2 | 1 | 0.15631300 | -0.08470870 | 0.6214790 | 3 | 1 | 0.80400000 | -0.011723110 | 0.04077667 |
| 4 | 2 | $0.98254000 \times 10^{-3}$ | $-1.16012500 \times 10^{-2}$ | $0.28853100 \times 10^{-1}$ | 4 | 2 | $0.15631300 \times 10^{-2}$ | $-0.08470870 \times 10^{-2}$ | 0.6214790×10^{-2} | 5 | 2 | $0.13573500 \times 10^{-3}$ | $-0.0076200 \times 10^{-3}$ | $0.75678600 \times 10^{-3}$ |
| 6 | 3 | $0.91630000 \times 10^{-5}$ | $-0.00240000 \times 10^{-4}$ | $0.11390000 \times 10^{-4}$ | 6 | 3 | $0.13433400 \times 10^{-5}$ | $-0.33643700 \times 10^{-5}$ | $0.75108000 \times 10^{-5}$ | 7 | 3 | $0.64800000 \times 10^{-6}$ | $-0.16330000 \times 10^{-6}$ | $0.80000000 \times 10^{-9}$ |
| 8 | 4 | $0.28370000 \times 10^{-7}$ | $-0.78290000 \times 10^{-7}$ | $0.28449000 \times 10^{-6}$ | 8 | 4 | $0.30738000 \times 10^{-7}$ | $-0.99235000 \times 10^{-7}$ | $0.51048900 \times 10^{-6}$ | 9 | 4 | $0.75900000 \times 10^{-9}$ | $-0.07800000 \times 10^{-8}$ | $0.94900000 \times 10^{-2}$ |
| 10 | 5 | $0(10^{-10})$ | $0(10^{-10})$ | $0.00330000 \times 10^{-9}$ | 10 | 5 | $0(10^{-10})$ | $0(10^{-10})$ | $0(10^{-9})$ | 11 | 5 | | | |
| $\lambda = 1.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.47684390 | -0.26157888 | 0.068680000 | 0 | 0 | 0.113392311 | -0.10199875 | 0.070006060 | 1 | 0 | 0.02387288 | -0.18160091 | 0.089238210 |
| 2 | 1 | 0.038801100 | -0.020362100 | 0.10065100 | 2 | 1 | 0.60184900 | -0.71339400 | 0.69232800 | 3 | 1 | 0.01728540 | -0.01666660 | 0.010741690 |
| 4 | 2 | $0.26984400 \times 10^{-2}$ | $-0.87790000 \times 10^{-2}$ | 0.00866600 | 4 | 2 | $0.19214700 \times 10^{-3}$ | $-0.30678100 \times 10^{-3}$ | $0.32880000 \times 10^{-3}$ | 5 | 2 | $0.02480000 \times 10^{-3}$ | $-0.10663000 \times 10^{-2}$ | 0.00018000 |
| 6 | 3 | $0.10660000 \times 10^{-3}$ | $-0.15690000 \times 10^{-3}$ | $0.00350000 \times 10^{-3}$ | 6 | 3 | $0.30380000 \times 10^{-3}$ | $-0.61886000 \times 10^{-3}$ | $0.11842900 \times 10^{-3}$ | 7 | 3 | $0.02470000 \times 10^{-4}$ | $-0.36838000 \times 10^{-4}$ | $0.99845000 \times 10^{-4}$ |
| 8 | 4 | $0.00800000 \times 10^{-5}$ | $-0.29970000 \times 10^{-5}$ | $0.69800000 \times 10^{-5}$ | 8 | 4 | $0.20540000 \times 10^{-7}$ | $-0.90640000 \times 10^{-7}$ | $0.12399000 \times 10^{-6}$ | 9 | 4 | $0.02050000 \times 10^{-6}$ | $-0.49660000 \times 10^{-6}$ | $0.10856000 \times 10^{-3}$ |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| $\lambda = 1.2$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.52719390 | -0.25640907 | 0.048024410 | 0 | 0 | 0.17000194 | -0.11738725 | 0.028799600 | 1 | 0 | 0.32804690 | -0.17945672 | 0.058668800 |
| 2 | 1 | 0.038060000 | -0.032856590 | 0.01828900 | 2 | 1 | 0.011640600 | -0.05602900 | 0.56838000 | 3 | 1 | 0.02927280 | -0.01111090 | 0.38020000 |
| 4 | 2 | $0.34746000 \times 10^{-2}$ | $-0.18087000 \times 10^{-2}$ | $0.00804000 \times 10^{-2}$ | 4 | 2 | $0.69839000 \times 10^{-3}$ | $-0.74720000 \times 10^{-3}$ | $0.61037000 \times 10^{-3}$ | 5 | 2 | $0.19685900 \times 10^{-2}$ | $-0.14599900 \times 10^{-2}$ | $0.72409000 \times 10^{-3}$ |
| 6 | 3 | $0.27700000 \times 10^{-3}$ | $-0.04040000 \times 10^{-3}$ | 0.12661000 | 6 | 3 | $0.29600000 \times 10^{-4}$ | $-0.39190000 \times 10^{-4}$ | $0.47990000 \times 10^{-4}$ | 7 | 3 | $0.01900000 \times 10^{-5}$ | $-0.11831000 \times 10^{-5}$ | 0.06099000 |
| 8 | 4 | $0.16100000 \times 10^{-4}$ | $-0.16310000 \times 10^{-4}$ | $0.19870000 \times 10^{-4}$ | 8 | 4 | $0.84000000 \times 10^{-6}$ | $-0.11790000 \times 10^{-5}$ | $0.00880000 \times 10^{-5}$ | 9 | 4 | | | |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |
| $\lambda = 1.4$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.56969490 | -0.20117875 | 0.024099000 | 0 | 0 | 0.28234875 | -0.12890392 | 0.047940160 | 1 | 0 | 0.57443077 | -0.17287718 | 0.04708810 |
| 2 | 1 | 0.065445000 | 0.11888000 | 0.03911040 | 2 | 1 | 0.015234350 | -0.79212700 | 0.12693700 | 3 | 1 | 0.085099700 | -0.74638000 | 0.58757000 |
| 4 | 2 | $0.80204000 \times 10^{-2}$ | $0.05660000 \times 10^{-2}$ | $0.00869000 \times 10^{-2}$ | 4 | 2 | $0.11421000 \times 10^{-2}$ | $-0.78269000 \times 10^{-2}$ | $0.12949000 \times 10^{-2}$ | 5 | 2 | $0.88180000 \times 10^{-2}$ | $-0.69400000 \times 10^{-2}$ | $0.50680000 \times 10^{-3}$ |
| 6 | 3 | $0.28300000 \times 10^{-3}$ | $-0.04800000 \times 10^{-3}$ | $0.13890000 \times 10^{-3}$ | 6 | 3 | $0.08800000 \times 10^{-4}$ | $-0.69430000 \times 10^{-4}$ | $0.40850000 \times 10^{-4}$ | 7 | 3 | $0.16940000 \times 10^{-3}$ | | |
| 8 | 4 | | | | 8 | 4 | | | | 9 | 4 | | | |
| 10 | 5 | | | | 10 | 5 | | | | 11 | 5 | | | |

TABLE I.- COMPUTED VALUES OF q_n , $J \frac{\partial q_n}{\partial \lambda}$, AND $\frac{\partial q_n}{\partial \lambda}$ FOR $n = 0$ TO 5 AND INFINITY

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Continued.

(1) $L = 0.5$ - Continued

| P | n | $q_n(P)$ | $J \frac{\partial q_n}{\partial \lambda}$ | $\frac{\partial q_n}{\partial \lambda}$ | P | n | $q_n(P)$ | $J \frac{\partial q_n}{\partial \lambda}$ | $\frac{\partial q_n}{\partial \lambda}$ |
|-----------------|-----|-----------------------------|---|---|-----|-----|-----------------------------|---|---|
| $\lambda = 1.6$ | | | | | | | | | |
| 0 | 0 | 0.60781340 | -0.19709371 | 0.026128810 | 2 | 0 | 0.27692378 | -0.12749933 | 0.052803530 |
| 2 | 1 | $-.77366000 \times 10^{-2}$ | .014867600 | -.017509470 | 4 | 1 | $-.54262100 \times 10^{-2}$ | -.27975400 $\times 10^{-3}$ | -.27975400 $\times 10^{-2}$ |
| 4 | 2 | $-.12302000$ | $-.30651000 \times 10^{-2}$ | $-.80133000 \times 10^{-2}$ | 6 | 2 | $-.94190000 \times 10^{-3}$ | $-.68600000 \times 10^{-4}$ | $-.51440000 \times 10^{-3}$ |
| 6 | 3 | $-.18800000 \times 10^{-3}$ | $-.34040000 \times 10^{-3}$ | $-.37810000 \times 10^{-3}$ | 8 | 3 | | | |
| 8 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | |
| $\lambda = 1.8$ | | | | | | | | | |
| 0 | 0 | 0.63688220 | -0.18159888 | 0.020937860 | 2 | 0 | 0.38447254 | -0.12702111 | 0.051459541 |
| 2 | 1 | $-.017129700$ | .026692900 | -.018784870 | 4 | 1 | $-.80929000 \times 10^{-2}$ | $-.29367400 \times 10^{-2}$ | $-.60408000 \times 10^{-2}$ |
| 4 | 2 | $-.44662000 \times 10^{-2}$ | $-.44019000 \times 10^{-2}$ | $-.84219000 \times 10^{-2}$ | 6 | 2 | $-.14470000 \times 10^{-3}$ | $-.90750000 \times 10^{-3}$ | $-.10113000$ |
| 6 | 3 | | | | 8 | 3 | | $.18880000$ | $-.10270000 \times 10^{-3}$ |
| 8 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | |
| $\lambda = 2.0$ | | | | | | | | | |
| 0 | 0 | 0.66379700 | -0.16810143 | 0.013624632 | 2 | 0 | 0.36764079 | -0.12478475 | 0.025633876 |
| 2 | 1 | $-.043598100$ | .026899000 | -.012639550 | 4 | 1 | $-.29668000 \times 10^{-2}$ | $-.97280700 \times 10^{-2}$ | $-.83902000 \times 10^{-2}$ |
| 4 | 2 | $-.78040000 \times 10^{-2}$ | $-.42627000 \times 10^{-2}$ | $-.18696000 \times 10^{-2}$ | 6 | 2 | $-.17956000$ | $.17899000$ | $-.10888000$ |
| 6 | 3 | | | | 8 | 3 | | | |
| 8 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | |
| $\lambda = 2.5$ | | | | | | | | | |
| 0 | 0 | 0.71732670 | -0.14133166 | $0.51226620 \times 10^{-2}$ | 2 | 0 | 0.45229920 | -0.11545372 | 0.017927184 |
| 2 | 1 | $-.12516900$ | .022188900 | -.013710900 | 4 | 1 | $-.047077400$ | $.023717900$ | $-.010202200$ |
| 4 | 2 | $-.59270000 \times 10^{-2}$ | $-.61900000 \times 10^{-3}$ | $-.20499000 \times 10^{-2}$ | 6 | 2 | $-.43800000 \times 10^{-2}$ | $-.12680000 \times 10^{-2}$ | $-.35600000 \times 10^{-3}$ |
| 6 | 3 | | | | 8 | 3 | | | |
| 8 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | |
| $\lambda = 3.0$ | | | | | | | | | |
| 0 | 0 | 0.73697210 | -0.12151594 | $0.57254100 \times 10^{-2}$ | 2 | 0 | 0.32890761 | -0.10498759 | 0.010405412 |
| 2 | 1 | $-.19981250$ | .029560000 | -.012310400 | 4 | 1 | $-.10999770$ | $-.057046400$ | $-.92589000 \times 10^{-2}$ |
| 4 | 2 | $-.014435000$ | $-.87380000 \times 10^{-2}$ | $-.41970000 \times 10^{-2}$ | 6 | 2 | $-.17900000 \times 10^{-2}$ | $-.27220000 \times 10^{-2}$ | $.19600000$ |
| 6 | 3 | | | | 8 | 3 | | | |
| 8 | 4 | | | | 10 | 4 | | | |
| 10 | 5 | | | | 12 | 5 | | | |

TABLE I.- COMPUTED VALUES OF q_n , $\int \frac{\partial q_n}{\partial \xi}$, AND $\frac{\partial q_n}{\partial \xi^2}$ FOR $n = 0$ TO 5 AND INCREASING

VALUES OF P FOR VARIOUS VALUES OF L AND λ - Concluded

(1) $L = 0.5$ - Concluded

| P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ | P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ | P | n | $q_n(P)$ | $\int \frac{\partial q_n}{\partial \xi}$ | $\frac{\partial q_n}{\partial \xi^2}$ |
|-----------------|-----|------------|--|---------------------------------------|-----|-----|------------|--|---------------------------------------|-----|-----|------------|--|---------------------------------------|
| $\lambda = 5.5$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.78730970 | -0.10634515 | $0.38191760 \times 10^{-2}$ | 1 | 0 | 0.66318291 | -0.10161006 | $0.56081350 \times 10^{-2}$ | 1 | 0 | 0.66318291 | -0.10161006 | $0.56081350 \times 10^{-2}$ |
| 2 | 1 | -.26955650 | .062112400 | -.94603000 | 3 | 1 | -.21255550 | .072777700 | -.91563000 | 3 | 1 | -.21255550 | .072777700 | -.91563000 |
| 4 | 2 | .045237000 | -.016605000 | .50880000 | 5 | 2 | .028794000 | -.012151000 | .40440000 | 5 | 2 | .028794000 | -.012151000 | .40440000 |
| 6 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- |
| $\lambda = 4.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.81117950 | -0.094410250 | $0.26626600 \times 10^{-2}$ | 1 | 0 | 0.71851608 | -0.091103715 | $0.39278510 \times 10^{-2}$ | 1 | 0 | 0.71851608 | -0.091103715 | $0.39278510 \times 10^{-2}$ |
| 2 | 1 | -.35215900 | .061994200 | -.72735000 | 3 | 1 | -.27581140 | .054694300 | -.72862000 | 3 | 1 | -.27581140 | .054694300 | -.72862000 |
| 4 | 2 | .011710000 | -.025012000 | .51610000 | 5 | 2 | .061135000 | -.012209000 | .44840000 | 5 | 2 | .061135000 | -.012209000 | .44840000 |
| 6 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- |
| $\lambda = 6.0$ | | | | | | | | | | | | | | |
| 0 | 0 | 0.87044140 | -0.064773280 | $0.85553000 \times 10^{-5}$ | 1 | 0 | 0.80616026 | -0.063713050 | $0.12734310 \times 10^{-2}$ | 1 | 0 | 0.80616026 | -0.063713050 | $0.12734310 \times 10^{-2}$ |
| 2 | 1 | -.51224020 | .055214300 | -.28471000 $\times 10^{-2}$ | 3 | 1 | -.46048540 | .050268100 | -.30551000 | 3 | 1 | -.46048540 | .050268100 | -.30551000 |
| 4 | 2 | .24894000 | -.034250000 | .32770000 | 5 | 2 | .21626000 | -.031040000 | .51950000 | 5 | 2 | .21626000 | -.031040000 | .51950000 |
| 6 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- | 7 | 3 | ----- | ----- | ----- |
| 8 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- | 9 | 4 | ----- | ----- | ----- |
| 10 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- | 11 | 5 | ----- | ----- | ----- |
| 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- | 12 | 5 | ----- | ----- | ----- |

TABLE II.- VALUES OF $\frac{T_1}{T_e}$, $\frac{\partial T_1}{\partial \xi}$, AND $\frac{\partial T_1}{\partial \omega^2}$ AT $\xi = 0$ AND 1.0 FOR PERFECTLY
 INSULATED PLATE AT VARIOUS VALUES OF L AND λ

| λ | $\xi = 0$ | | | $\xi = 1.0$ | |
|-----------|-------------------|-------------------------------------|--|-------------------|--|
| | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial \xi}$ | $\frac{\partial T_1}{\partial \omega^2}$ | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial \omega^2}$ |
| L = 0.001 | | | | | |
| 0.1 | 0.01026 | -9.8974×10^{-4} | 9.8938×10^{-4} | 0.00978 | 9.8986×10^{-4} |
| .2 | .03950 | -9.6050 | 9.6014 | .03902 | 9.6062 |
| .4 | .14808 | -8.5192 | 8.5160 | .14766 | 8.5202 |
| .6 | .30244 | -6.9756 | 6.9730 | .30210 | 6.9764 |
| .8 | .47274 | -5.2726 | 5.2706 | .47248 | 5.2732 |
| 1.0 | .63210 | -3.6790 | 3.6776 | .63192 | 3.6794 |
| 1.2 | .76502 | -2.3698 | 2.3690 | .76290 | 2.3702 |
| 1.4 | .85906 | -1.4092 | 1.4086 | .85901 | 1.4093 |
| 1.6 | .92265 | -7.7350×10^{-5} | 7.7322×10^{-5} | .92261 | 7.7360×10^{-5} |
| 1.8 | .96080 | -3.9198 | 3.9184 | .96078 | 3.9204 |
| 2.0 | .98166 | -1.8337 | 1.8330 | .98163 | 1.8340 |
| 2.5 | .99807 | -1.9343×10^{-6} | 1.9336×10^{-6} | .99806 | 1.9343×10^{-6} |
| 3.0 | .99988 | -1.2378×10^{-7} | 1.2374×10^{-7} | .99986 | 1.2380×10^{-7} |
| 3.5 | 1.0 | -4.8076×10^{-9} | 4.8058×10^{-9} | 1.0 | 4.8082×10^{-9} |
| 4.0 | 1.0 | -1.1317×10^{-10} | 1.1313×10^{-10} | 1.0 | 1.1319×10^{-10} |
| L = 0.005 | | | | | |
| 0.1 | 0.01164 | -4.9418×10^{-3} | 4.9337×10^{-3} | 0.00919 | 4.9439×10^{-3} |
| .2 | .04076 | -4.7962 | 4.7883 | .03838 | 4.8002 |
| .4 | .14907 | -4.2546 | 4.2477 | .14696 | 4.2582 |
| .6 | .30308 | -3.4846 | 3.4789 | .30135 | 3.4875 |
| .8 | .47304 | -2.6348 | 2.6305 | .47173 | 2.6370 |
| 1.0 | .63214 | -1.8393 | 1.8363 | .63123 | 1.8408 |
| 1.2 | .76290 | -1.1855 | 1.1836 | .76231 | 1.1863 |
| 1.4 | .85893 | -7.0535×10^{-4} | 7.0419×10^{-4} | .85858 | 7.0594×10^{-4} |
| 1.6 | .92250 | -3.8750 | 3.8730 | .92231 | 3.8782 |
| 1.8 | .96069 | -1.9653 | 1.9621 | .96060 | 1.9669 |
| 2.0 | .98159 | -9.2030×10^{-5} | 9.1879×10^{-5} | .98155 | 9.2109×10^{-5} |
| 2.5 | .99805 | -9.7345×10^{-6} | 9.7185×10^{-6} | .99805 | 9.7430×10^{-6} |
| 3.0 | .99987 | -6.2520×10^{-7} | 6.2417×10^{-7} | .99987 | 6.2572×10^{-7} |
| 3.5 | 1.0 | -2.4368×10^{-8} | 2.4328×10^{-8} | 1.0 | 2.4388×10^{-8} |
| 4.0 | 1.0 | -5.7665×10^{-10} | 5.7570×10^{-10} | 1.0 | 5.7710×10^{-10} |

TABLE II.- VALUES OF $\frac{T_1}{T_e}$, $\frac{\partial \frac{T_1}{T_e}}{\partial \xi}$, AND $\frac{\partial^2 \frac{T_1}{T_e}}{\partial \xi^2}$ AT $\xi = 0$ AND 1.0 FOR PERFECTLY

INSULATED PLATE AT VARIOUS VALUES OF L AND λ - Continued

| λ | $\xi = 0$ | | | $\xi = 1.0$ | |
|-----------|-------------------|---|---|-------------------|---|
| | $\frac{T_1}{T_e}$ | $\frac{\partial \frac{T_1}{T_e}}{\partial \xi}$ | $\frac{\partial^2 \frac{T_1}{T_e}}{\partial \xi^2}$ | $\frac{T_1}{T_e}$ | $\frac{\partial \frac{T_1}{T_e}}{\partial \xi}$ |
| L = 0.1 | | | | | |
| 0.1 | 0.03472 | -9.6528×10^{-2} | 0.16877 | 0.00078 | 2.1252×10^{-2} |
| .2 | .06906 | -9.3094 | 9.3568×10^{-2} | .02286 | 9.1054 |
| .4 | .17142 | -8.2858 | 8.0172 | .12964 | 8.4214 |
| .6 | .31720 | -6.8280 | 6.6066 | .28278 | 6.9396 |
| .8 | .47924 | -5.2076 | 5.0388 | .45300 | 5.2926 |
| 1.0 | .63242 | -3.6758 | 3.5566 | .61388 | 3.7360 |
| 1.2 | .75986 | -2.4014 | 2.3236 | .74776 | 2.4406 |
| 1.4 | .85481 | -1.4519 | 1.4048 | .84750 | 1.4756 |
| 1.6 | .91875 | -8.1250×10^{-3} | 7.8616×10^{-3} | .91465 | 8.2580×10^{-3} |
| 1.8 | .95792 | -4.2078 | 4.0714 | .95580 | 4.2768 |
| 2.0 | .97983 | -2.0170 | 1.9516 | .97881 | 2.0502 |
| 2.5 | .99771 | -2.2868×10^{-4} | 2.2126×10^{-4} | .99760 | 2.3242×10^{-4} |
| 3.0 | .99984 | -1.5983×10^{-5} | 1.5465×10^{-5} | .99983 | 1.6244×10^{-5} |
| 3.5 | .99999 | -6.8846×10^{-7} | 6.6614×10^{-7} | .99999 | 6.9972×10^{-7} |
| 4.0 | 1.0 | -1.8286×10^{-8} | 1.7693×10^{-8} | 1.0 | 1.8585×10^{-8} |
| L = 0.2 | | | | | |
| 0.1 | 0.04832 | -0.19030 | 0.46656 | 0.00004 | 0.00667 |
| .2 | .09356 | -.18129 | .21921 | .01172 | .13594 |
| .4 | .19428 | -.16114 | .15106 | .11244 | .16607 |
| .6 | .33192 | -.13362 | .12516 | .26408 | .13788 |
| .8 | .48608 | -.10278 | 9.6284×10^{-2} | .43384 | .10607 |
| 1.0 | .63412 | -7.3176×10^{-2} | 6.8548 | .59592 | 7.5704×10^{-2} |
| 1.2 | .75708 | -4.8584 | 4.5512 | .73240 | 5.0136 |
| 1.4 | .85076 | -2.9849 | 2.7961 | .83360 | 3.0801 |
| 1.6 | .91493 | -1.7014 | 1.5938 | .90628 | 1.7558 |
| 1.8 | .95501 | -8.9984×10^{-3} | 8.4292×10^{-3} | .95044 | 9.2860×10^{-3} |
| 2.0 | .97792 | -4.4156 | 4.1364 | .97568 | 4.5564 |
| 2.5 | .99732 | -5.3660×10^{-4} | 5.0269×10^{-4} | .99704 | 5.5372×10^{-4} |
| 3.0 | .99980 | -4.0820×10^{-5} | 3.8237×10^{-5} | .99978 | 4.2120×10^{-5} |
| 3.5 | .99999 | -1.9445×10^{-6} | 1.8215×10^{-6} | .99999 | 2.0065×10^{-6} |
| 4.0 | 1.0 | -5.7956×10^{-8} | 5.4292×10^{-8} | 1.0 | 5.9808×10^{-8} |

TABLE II.- VALUES OF $\frac{T_1}{T_e}$, $\frac{\partial T_1}{\partial \xi}$, AND $\frac{\partial T_1}{\partial w^2}$ AT $\xi = 0$ AND 1.0 FOR PERFECTLY
 INSULATED PLATE AT VARIOUS VALUES OF L AND λ - Continued

| λ | $\xi = 0$ | | | $\xi = 1.0$ | |
|-----------------|-------------------|-------------------------------------|-------------------------------------|-------------------|-------------------------------------|
| | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial \xi}$ | $\frac{\partial T_1}{\partial w^2}$ | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial w^2}$ |
| L = 0.01 | | | | | |
| 0.1 | 0.01326 | -9.8674×10^{-3} | 9.8358×10^{-3} | 0.00834 | 9.8848×10^{-3} |
| .2 | .04234 | -9.5766 | 9.5460 | .03756 | 9.5936 |
| .4 | .15030 | -8.4970 | 8.4698 | .14606 | 8.5120 |
| .6 | .30388 | -6.9612 | 6.9390 | .30040 | 6.9736 |
| .8 | .47864 | -5.2136 | 5.1970 | .47604 | 5.2228 |
| 1.0 | .63218 | -3.6782 | 3.6664 | .63034 | 3.6848 |
| 1.2 | .76278 | -2.3722 | 2.3664 | .76160 | 2.3764 |
| 1.4 | .85873 | -1.4127 | 1.4082 | .85802 | 1.4152 |
| 1.6 | .92232 | -7.7678×10^{-4} | 7.7430×10^{-4} | .92193 | 7.7816×10^{-4} |
| 1.8 | .96096 | -3.9402 | 3.9314 | .96036 | 3.9510 |
| 2.0 | .98154 | -1.8459 | 1.8400 | .98142 | 1.8522 |
| 2.5 | .99804 | -1.9628×10^{-5} | 1.9566×10^{-5} | .99803 | 1.9633×10^{-5} |
| 3.0 | .99987 | -1.2659×10^{-6} | 1.2618×10^{-6} | .99987 | 1.2682×10^{-6} |
| 3.5 | 1.0 | -4.9586×10^{-8} | 4.9428×10^{-8} | 1.0 | 4.9674×10^{-8} |
| 4.0 | 1.0 | -1.1805×10^{-9} | 1.1767×10^{-9} | 1.0 | 1.1826×10^{-9} |
| L = 0.05 | | | | | |
| 0.1 | 0.02484 | -4.8758×10^{-2} | 6.1436×10^{-2} | 0.00308 | 3.5384×10^{-2} |
| .2 | .05446 | -4.7277 | 4.6533 | .03070 | 4.7633 |
| .4 | .15972 | -4.2014 | 4.1322 | .13860 | 4.2361 |
| .6 | .30976 | -3.4512 | 3.3944 | .29241 | 3.4794 |
| .8 | .47593 | -2.6204 | 2.5772 | .46276 | 2.6420 |
| 1.0 | .63219 | -1.8390 | 1.8088 | .62294 | 1.8543 |
| 1.2 | .76140 | -1.1930 | 1.1734 | .75540 | 1.2029 |
| 1.4 | .85692 | -7.1540×10^{-3} | 7.0362×10^{-3} | .85333 | 7.2128×10^{-3} |
| 1.6 | .92070 | -3.9648 | 3.8996 | .91871 | 3.9975 |
| 1.8 | .95937 | -2.0313 | 1.9979 | .95835 | 2.0481 |
| 2.0 | .98076 | -9.6190×10^{-4} | 9.4607×10^{-4} | .98028 | 9.6982×10^{-4} |
| 2.5 | .99790 | -1.0522 | 1.0349 | .99784 | 1.0608 |
| 3.0 | .99986 | -7.0375×10^{-6} | 6.9217×10^{-6} | .99986 | 7.0933×10^{-6} |
| 3.5 | .99999 | -2.8798×10^{-7} | 2.8324×10^{-7} | .99999 | 2.9036×10^{-7} |
| 4.0 | 1.0 | -7.2015×10^{-9} | 7.0830×10^{-9} | 1.0 | 7.2610×10^{-9} |

TABLE II.- VALUES OF $\frac{T_1}{T_e}$, $\frac{\partial \frac{T_1}{T_e}}{\partial \xi}$, AND $\frac{\partial \frac{T_1}{T_e}}{\partial w^2}$ AT $\xi = 0$ AND 1.0 FOR PERFECTLY

INSULATED PLATE AT VARIOUS VALUES OF L AND λ - Continued

| λ | $\xi = 0$ | | | $\xi = 1.0$ | |
|-----------|-------------------|---|---|-------------------|---|
| | $\frac{T_1}{T_e}$ | $\frac{\partial \frac{T_1}{T_e}}{\partial \xi}$ | $\frac{\partial \frac{T_1}{T_e}}{\partial w^2}$ | $\frac{T_1}{T_e}$ | $\frac{\partial \frac{T_1}{T_e}}{\partial w^2}$ |
| L = 0.3 | | | | | |
| 0.1 | 0.05890 | -0.28233 | 0.84238 | 0 | 0.00103 |
| .2 | .11248 | -.26626 | .38414 | .00586 | .13318 |
| .4 | .21628 | -.23512 | .21551 | .09634 | .24388 |
| .6 | .34654 | -.19604 | .17791 | .24622 | .20522 |
| .8 | .49315 | -.15205 | .13799 | .41534 | .15918 |
| 1.0 | .63441 | -.10968 | 9.9534×10^{-2} | .57829 | .11482 |
| 1.2 | .75476 | -7.3572×10^{-2} | 6.6768 | .71712 | 7.7016×10^{-2} |
| 1.4 | .84703 | -4.5891 | 4.1648 | .82355 | 4.8041 |
| 1.6 | .91126 | -2.6622 | 2.4160 | .89763 | 2.7870 |
| 1.8 | .95212 | -1.4363 | 1.3035 | .94477 | 1.5036 |
| 2.0 | .97598 | -7.2066×10^{-3} | 6.5400×10^{-3} | .97229 | 7.5444×10^{-3} |
| 2.5 | .99688 | -9.3516×10^{-4} | 8.4870×10^{-4} | .99640 | 9.7896×10^{-4} |
| 3.0 | .99974 | -7.7094×10^{-5} | 6.9966×10^{-5} | .99970 | 8.0706×10^{-5} |
| 3.5 | .99999 | -4.0385×10^{-6} | 3.6650×10^{-6} | .99998 | 4.2277×10^{-6} |
| 4.0 | 1.0 | -1.3423×10^{-7} | 1.2182×10^{-7} | 1.0 | 1.4052×10^{-7} |
| L = 0.4 | | | | | |
| 0.1 | 0.06752 | -0.37299 | 1.2780 | 0 | 0.00017 |
| .2 | .12808 | -.34877 | .57422 | .00288 | .10955 |
| .4 | .23690 | -.30524 | .27893 | .08192 | .31266 |
| .6 | .36087 | -.25565 | .22498 | .22917 | .27124 |
| .8 | .50043 | -.19982 | .17582 | .39748 | .21205 |
| 1.0 | .63605 | -.14558 | .12809 | .56105 | .15448 |
| 1.2 | .75287 | -9.8848×10^{-2} | 8.6968×10^{-2} | .70194 | .10490 |
| 1.4 | .84362 | -6.2554 | 5.5036 | .81138 | 6.6380×10^{-2} |
| 1.6 | .90775 | -3.6899 | 3.2465 | .88874 | 3.9154 |
| 1.8 | .94929 | -2.0286 | 1.7848 | .93883 | 2.1527 |
| 2.0 | .97402 | -1.0394 | 9.1448×10^{-3} | .96866 | 1.1030 |
| 2.5 | .99641 | -1.4357×10^{-3} | 1.2631 | .99567 | 1.5235×10^{-3} |
| 3.0 | .99968 | -1.2773×10^{-4} | 1.1238×10^{-4} | .99961 | 1.3554×10^{-4} |
| 3.5 | .99998 | -7.3174×10^{-6} | 6.4381×10^{-6} | .99998 | 7.7650×10^{-6} |
| 4.0 | 1.0 | -2.7017×10^{-7} | 2.3770×10^{-7} | 1.0 | 2.8670×10^{-7} |

TABLE II.- VALUES OF $\frac{T_1}{T_e}$, $\frac{\partial T_1}{\partial \xi}$, AND $\frac{\partial T_1}{\partial \omega^2}$ AT $\xi = 0$ AND 1.0 FOR PERFECTLY

INSULATED PLATE AT VARIOUS VALUES OF L AND λ - Concluded

| λ | $\xi = 0$ | | | $\xi = 1.0$ | |
|-----------|-------------------|-------------------------------------|--|-------------------|--|
| | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial \xi}$ | $\frac{\partial T_1}{\partial \omega^2}$ | $\frac{T_1}{T_e}$ | $\frac{\partial T_1}{\partial \omega^2}$ |
| $L = 0.5$ | | | | | |
| 0.1 | 0.07504 | -0.46248 | 1.7633 | 0.00003 | 0 |
| .2 | .14151 | -.42924 | .78278 | .00152 | .08179 |
| .4 | .25600 | -.37200 | .34551 | .06922 | .36890 |
| .6 | .37499 | -.31250 | .26710 | .21303 | .33547 |
| .8 | .50787 | -.24606 | .21002 | .38028 | .26447 |
| 1.0 | .63796 | -.18102 | .15450 | .54409 | .19456 |
| 1.2 | .75139 | -.12430 | .10610 | .68693 | .13361 |
| 1.4 | .84049 | -7.9755×10^{-2} | 6.8072×10^{-2} | .79914 | 8.5719×10^{-2} |
| 1.6 | .90442 | -4.7792 | 4.0791 | .87964 | 5.1365 |
| 1.8 | .94650 | -2.6748 | 2.2830 | .93264 | 2.8748 |
| 2.0 | .97204 | -1.3982 | 1.1933 | .96479 | 1.5027 |
| 2.5 | .99510 | -2.0490×10^{-3} | 1.7488×10^{-3} | .99484 | 2.2022×10^{-3} |
| 3.0 | .99961 | -1.9596×10^{-4} | 1.6725×10^{-4} | .99951 | 2.1061×10^{-4} |
| 3.5 | .99998 | -1.2226×10^{-5} | 1.0436×10^{-5} | .99997 | 1.3141×10^{-5} |
| 4.0 | 1.0 | -4.9836×10^{-7} | 4.2536×10^{-7} | 1.0 | 5.3563×10^{-7} |

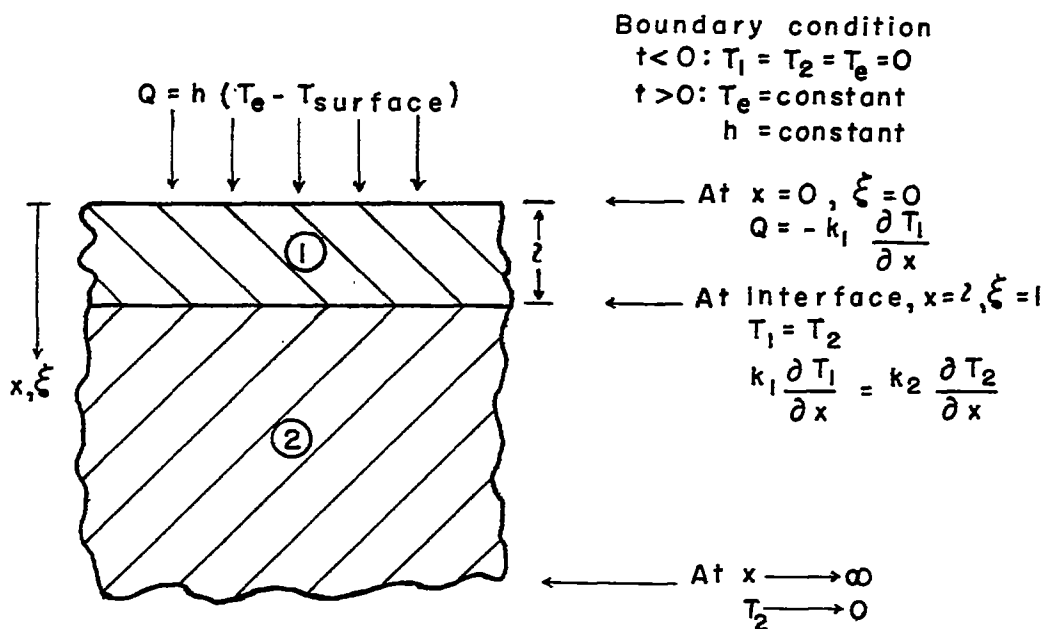


Figure 1.- Sketch of the composite slab showing pertinent boundary conditions.

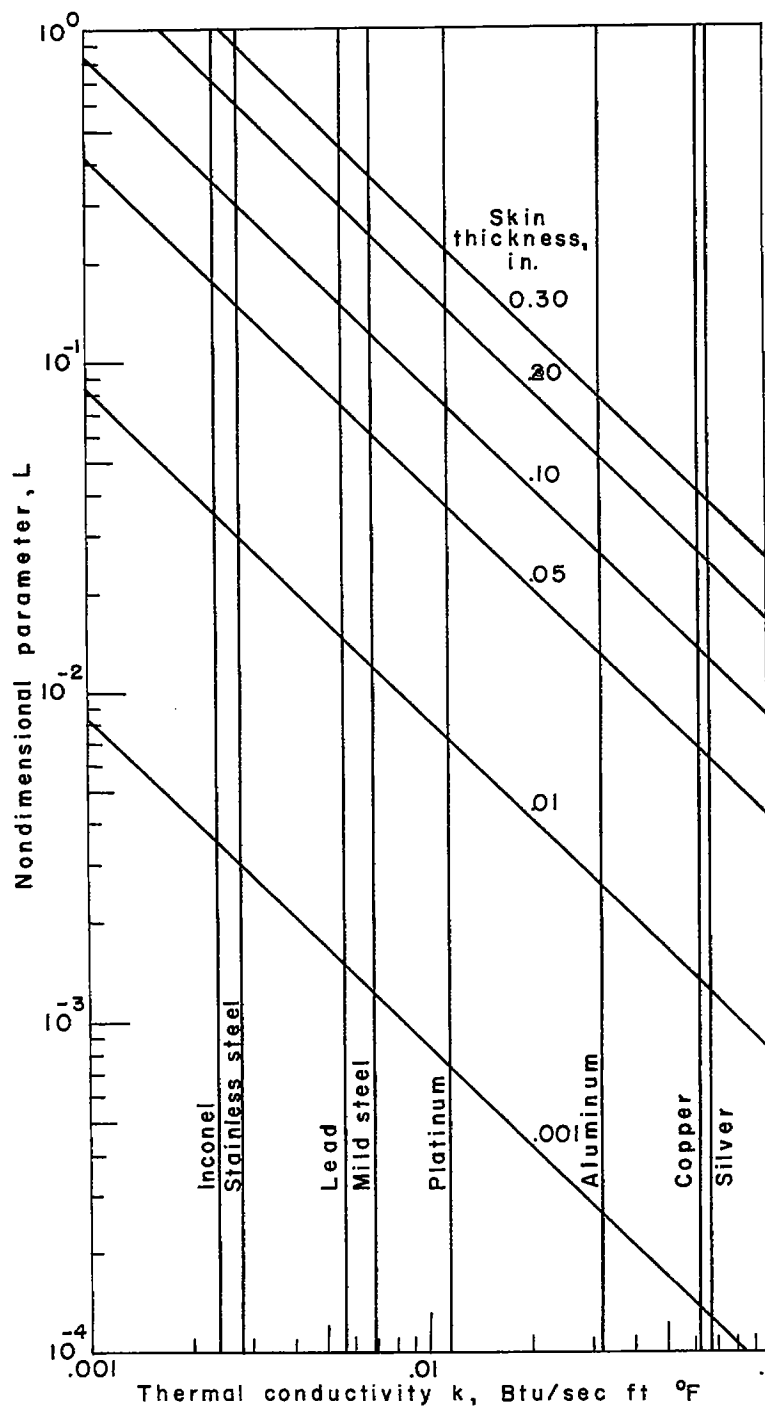


Figure 2.- Plot of the nondimensional parameter L as a function of skin material and thickness for $h = 0.1$ Btu/sq ft-°F-sec.

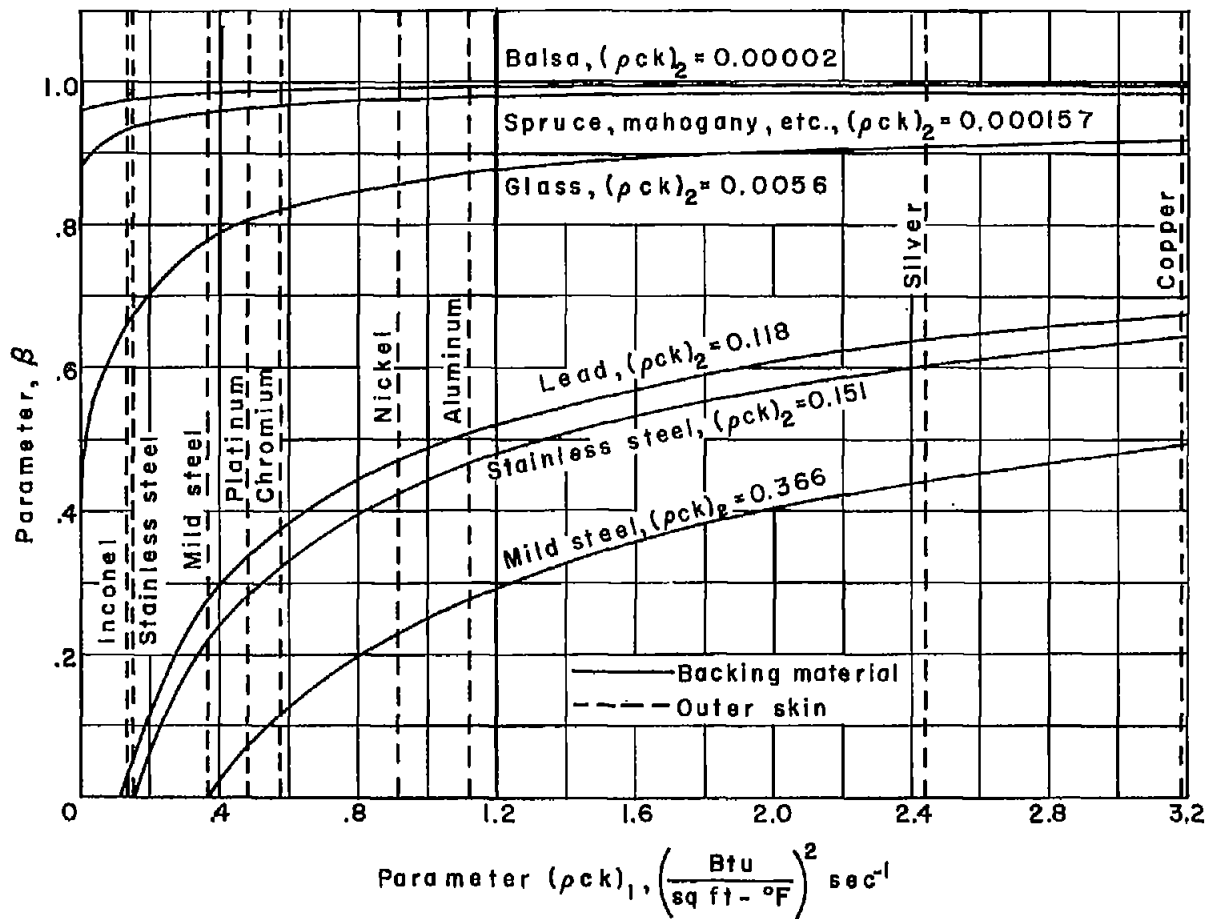
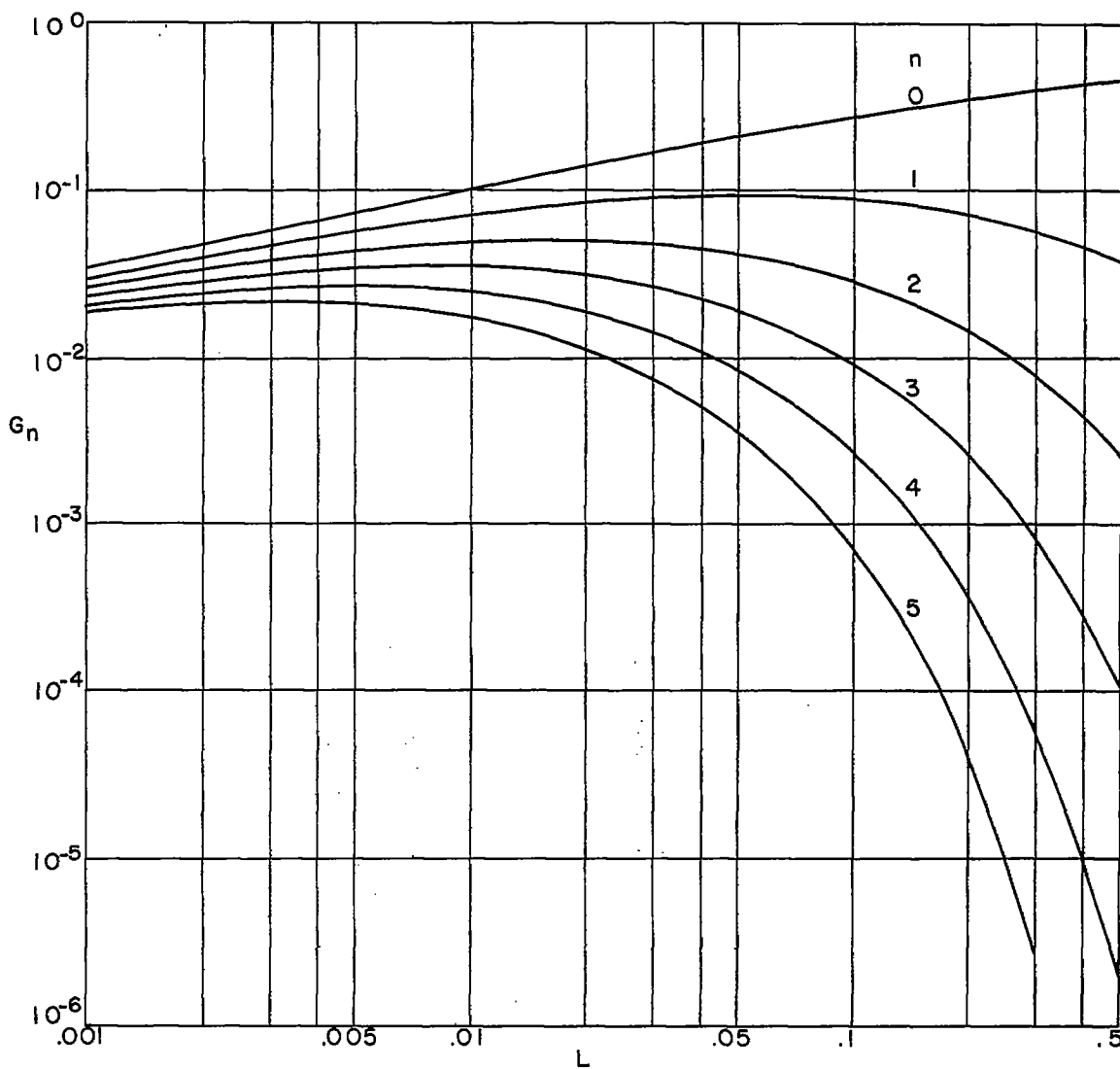
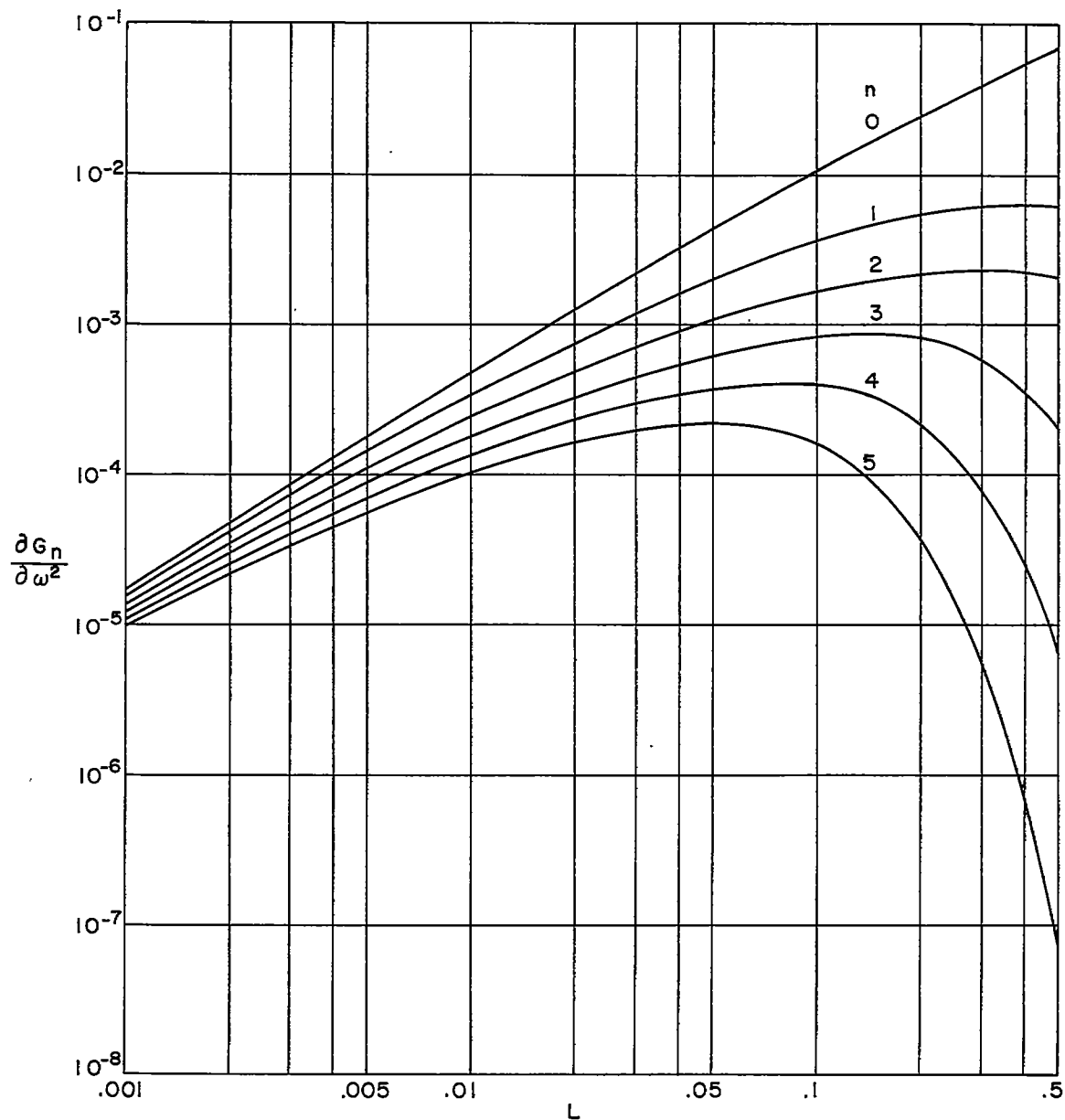


Figure 3.- Variation of the parameter β with the parameter $(\rho ck)_1$ for various outer skins and backing materials. The appropriate value of β is determined by the intersection of the backing-material curve with the abscissa for the outer skin. If the outer skin and backing material are interchanged, the sign of β is reversed.



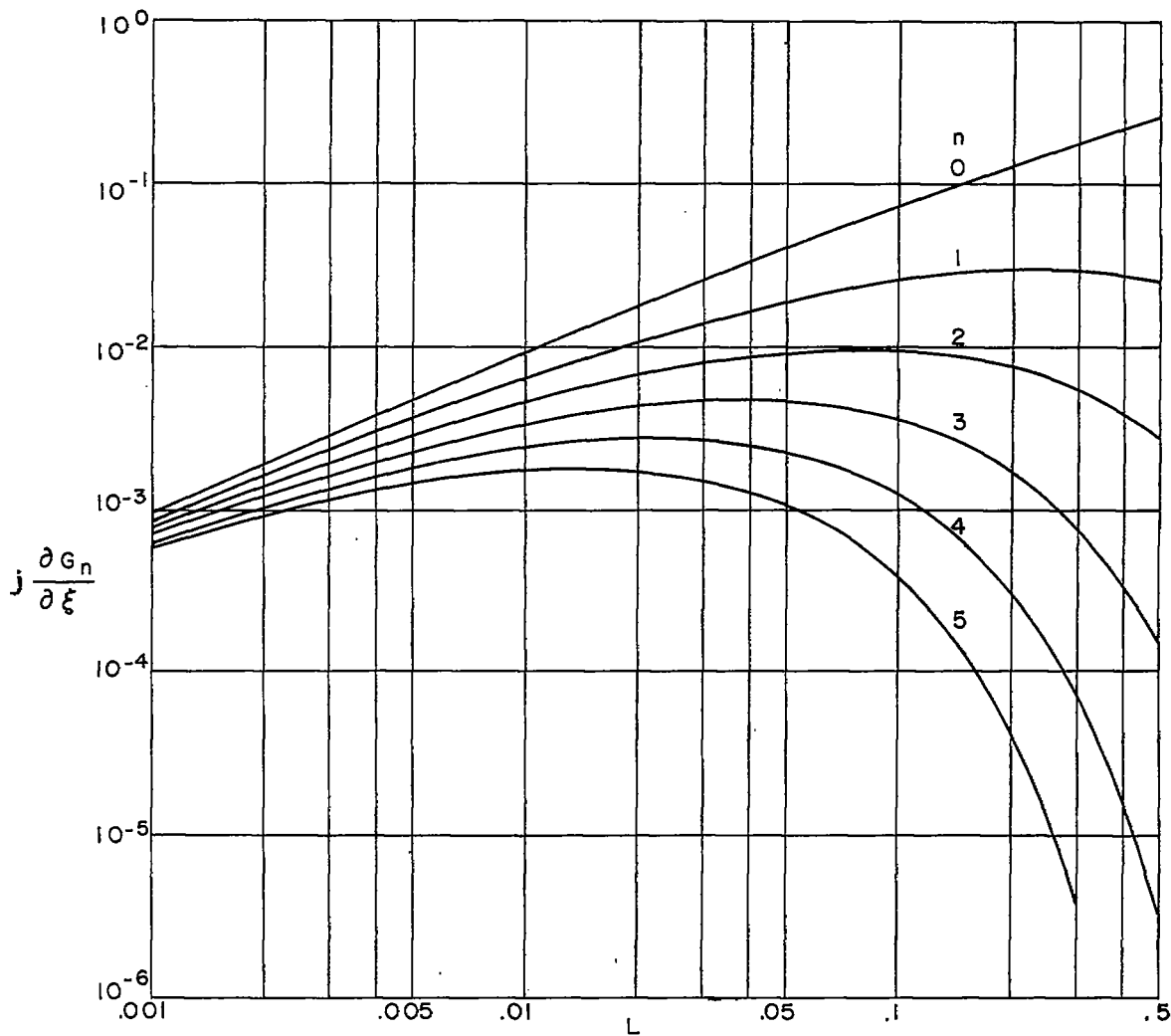
(a) G_n .

Figure 4.- Values of G_n , $\frac{\partial G_n}{\partial \omega^2}$, and $j \frac{\partial G_n}{\partial \xi}$ applicable at $\xi = 0$ plotted against L for $\lambda = 1.0$. $P = 2n$.



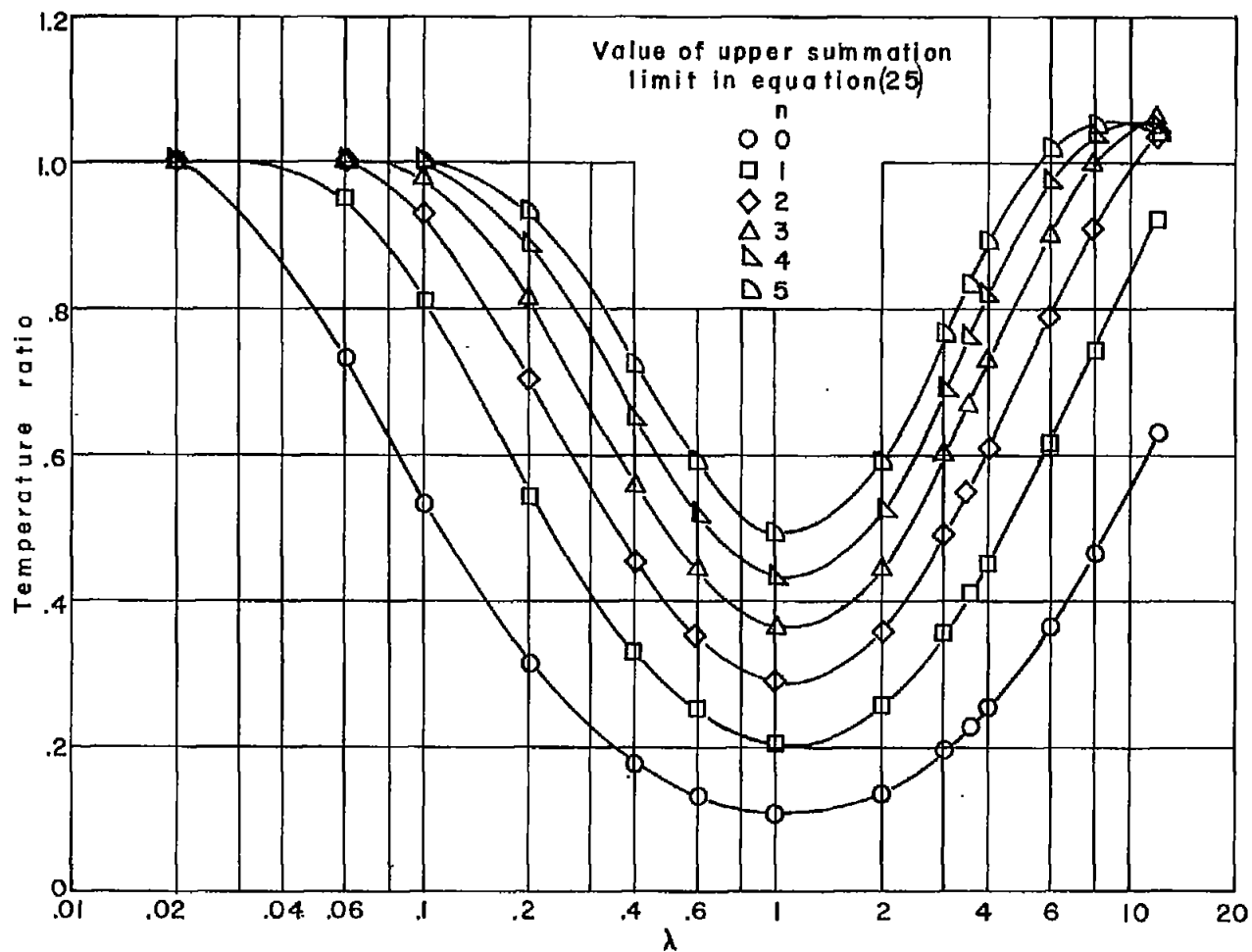
(b) $\frac{\partial G_n}{\partial \omega^2}$.

Figure 4.- Continued.



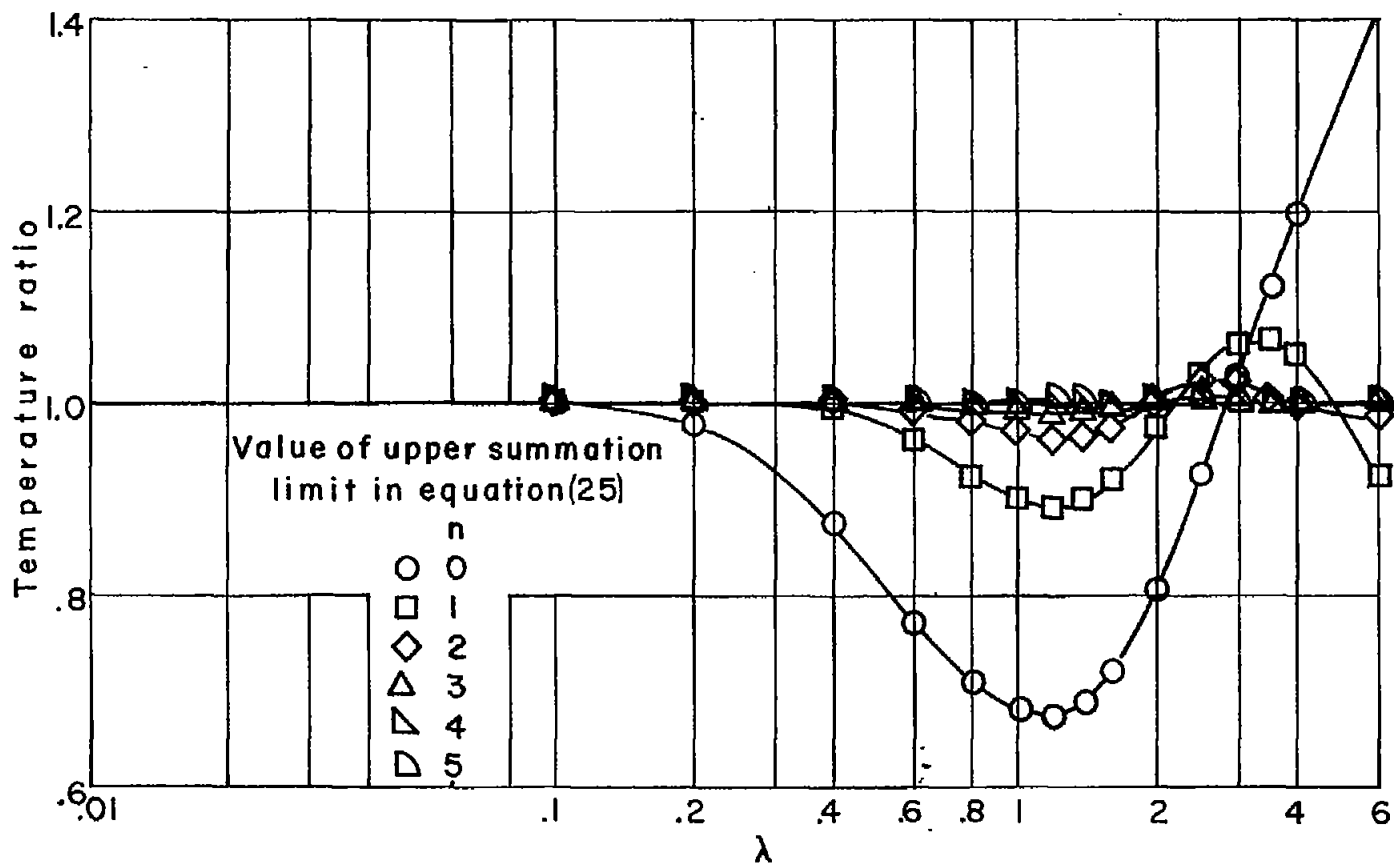
(c) $j \frac{\partial G_n}{\partial \xi}$.

Figure 4.- Concluded.



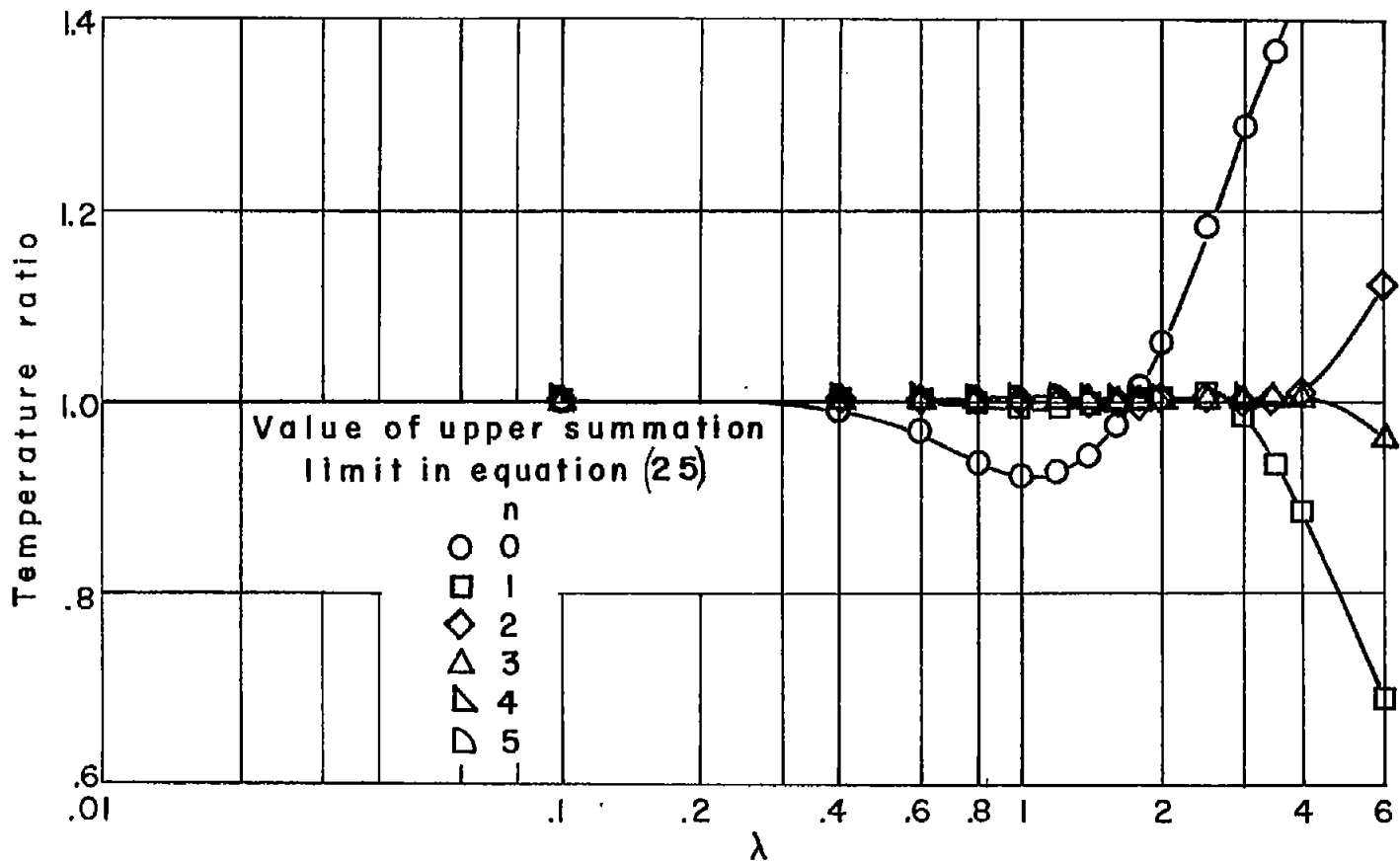
(a) $L = 0.001$.

Figure 5.- Ratio of T_1/T_e at $\xi = 0$ for a finite number of terms in equation (25) to T_1/T_e from equation (15) plotted against the parameter λ . $\beta = 1$.



(b) $L = 0.1.$

Figure 5.- Continued.



(c) $L = 0.5$.

Figure 5.- Concluded.

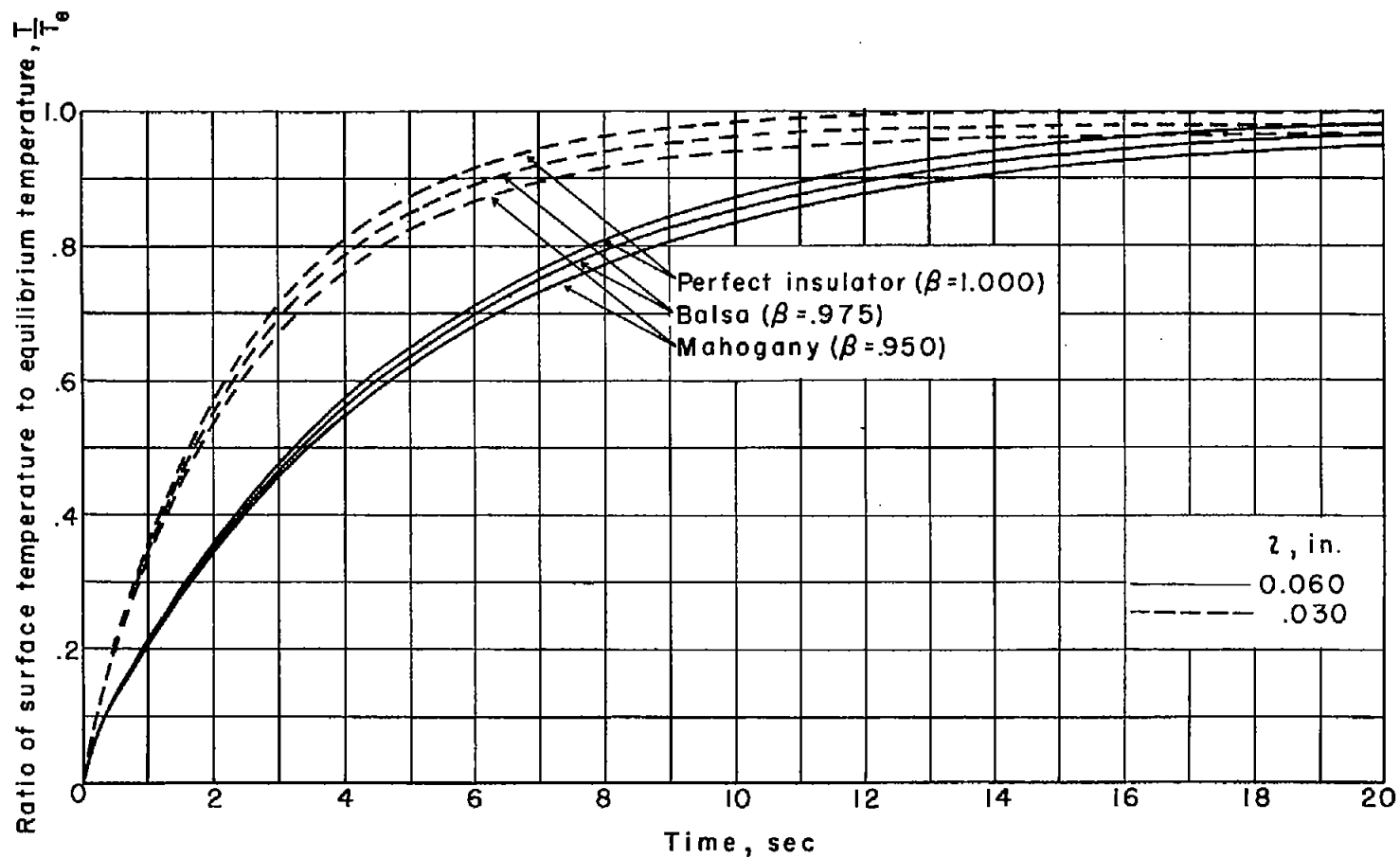


Figure 6.- Temperature distribution at the outer face plotted against time for composite slabs having stainless-steel skins suddenly exposed to aerodynamic heating at $t = 0$. The skin thicknesses are 0.060 and 0.030 inch and the insulating backing materials are mahogany, balsa, and a perfect insulator. $h = 0.056$ Btu/sq ft- $^{\circ}$ F-sec.

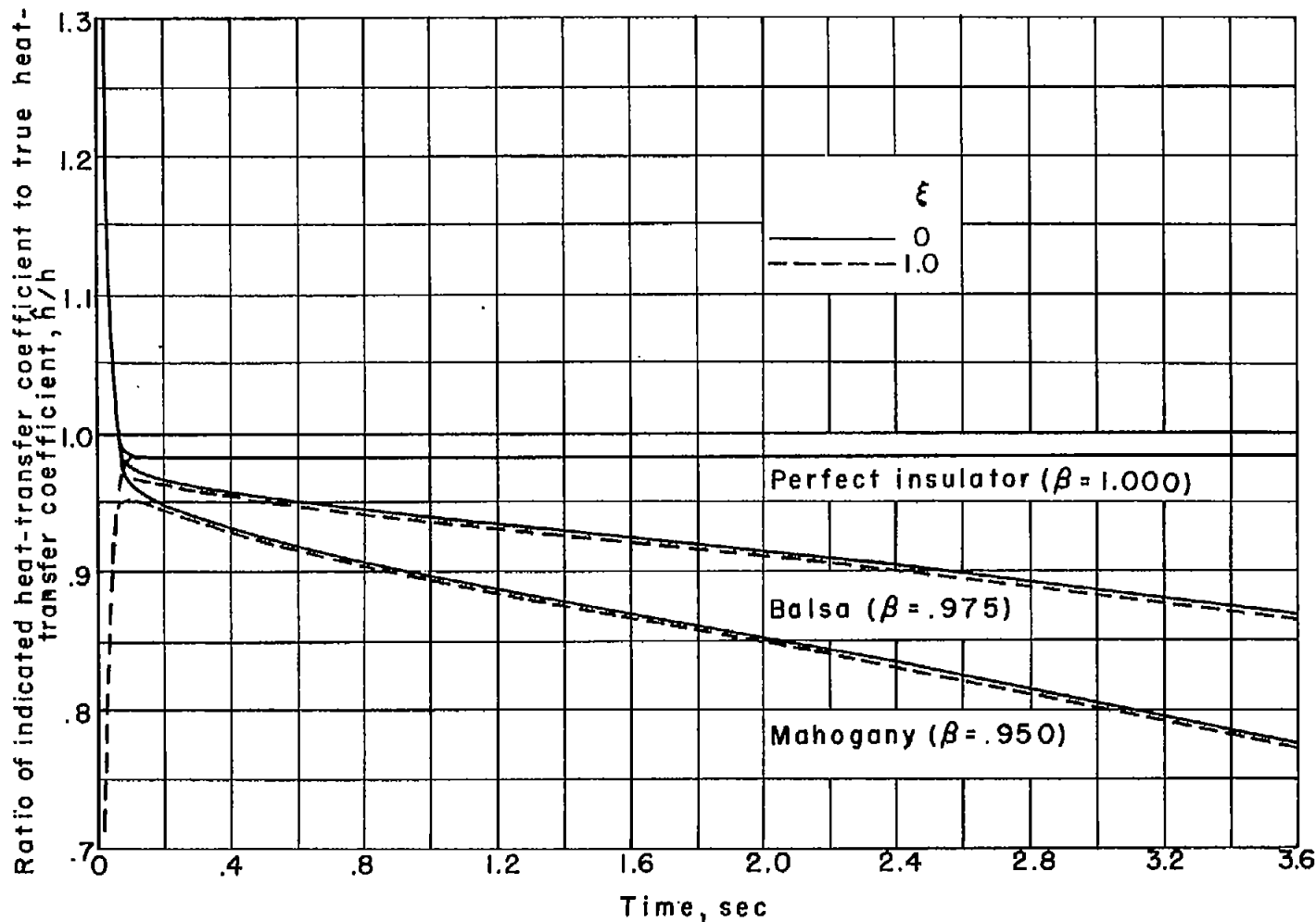


Figure 7.- Ratio of indicated heat-transfer coefficient to true heat-transfer coefficient plotted against time for a composite slab having a 0.030-inch-thick stainless-steel skin and various insulating backing materials. $h = 0.056$ Btu/sq ft- $^{\circ}$ F-sec.

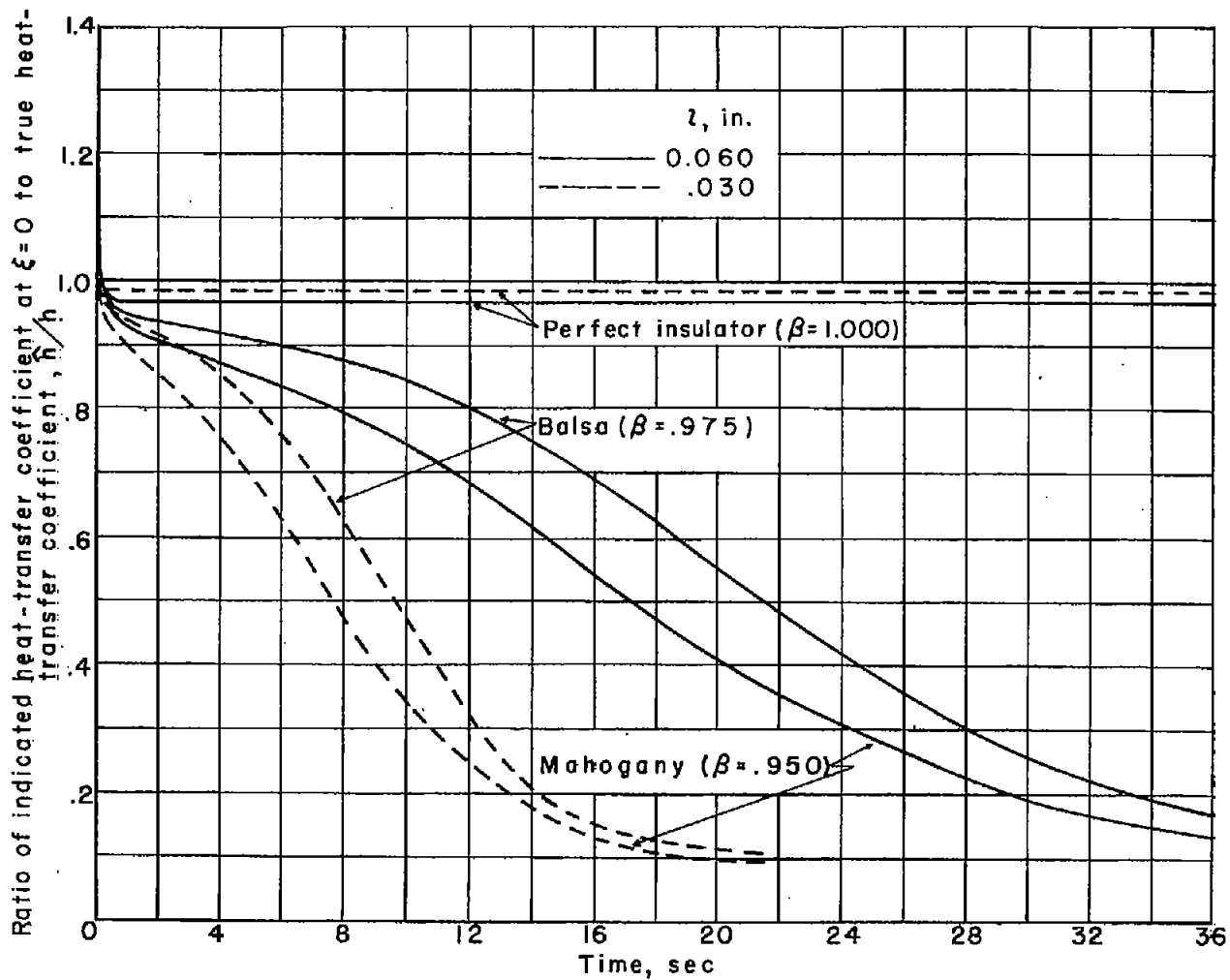


Figure 8.- Ratio of indicated heat-transfer coefficient at $\xi = 0$ to true heat-transfer coefficient plotted against time for composite slabs having stainless-steel skins of thickness l and various insulating backing materials. $h = 0.056$ Btu/sq ft- $^{\circ}$ F-sec.

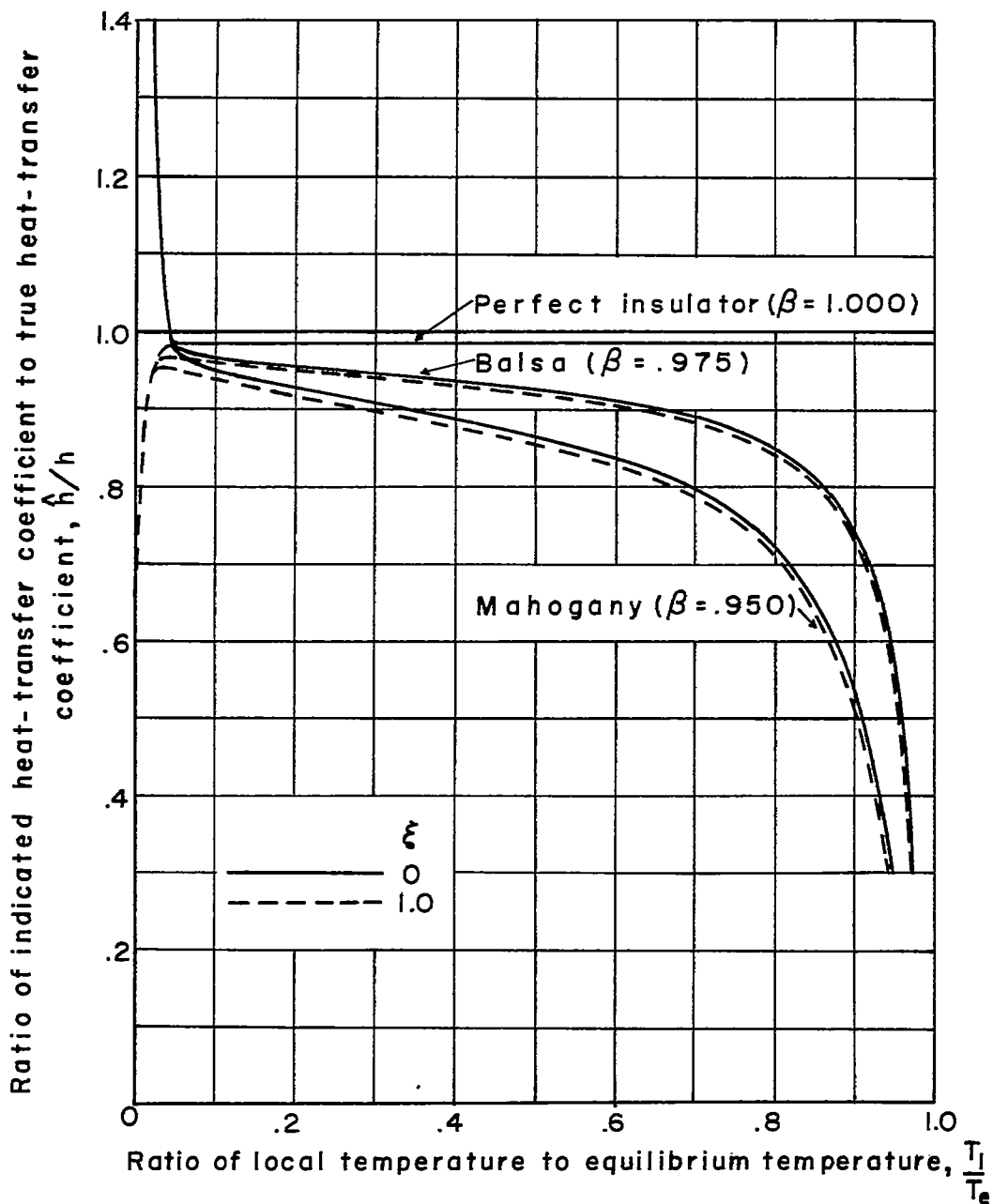


Figure 9.- Ratio of indicated heat-transfer coefficient to true heat-transfer coefficient plotted against ratio of local temperature to equilibrium temperature for composite slab having a 0.030-inch-thick stainless-steel outer skin and various insulating backing materials. $h = 0.056$ Btu/sq ft- $^{\circ}$ F-sec.