NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4363

SIMPLIFIED METHOD FOR DETERMINATION OF CRITICAL HEIGHT OF DISTRIBUTED ROUGHNESS PARTICLES FOR BOUNDARY-LAYER TRANSITION AT MACH NUMBERS FROM 0 TO 5

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SUMMARY

A simplified method has been devised for determination of the critical height of three-dimensional roughness particles required to promote premature transition of a laminar boundary layer on models of airplanes or airplane components in a wind tunnel with zero heat transfer. A single equation is derived which relates the roughness height to a Reynolds number based on the roughness height and on local flow conditions at the height of the roughness, and charts are presented from which the critical roughness height can be easily obtained for Mach numbers from 0 to 5. A discussion of the use of these charts is presented with consideration of various model configurations.

The method has been applied to various types of configurations in several wind-tunnel investigations conducted by the National Advisory Committee for Aeronautics at Mach numbers up to 4, and in all cases the calculated roughness height caused premature boundary-layer transition for the range of test conditions.

INTRODUCTION

In wind-tunnel investigations with models of airplanes or airplane components, it is often desirable to locate artificially the position of boundary-layer transition from laminar to turbulent flow by some method that will result in a negligible increase in drag other than that due to the change in the transition location. One satisfactory method of initiating transition is with the use of a strip of distributed particles of roughness. A correlation of the minimum roughness size required to initiate transition has been accomplished in references 1 and 2 at subsonic and supersonic speeds on the basis of a critical roughness Reynolds number formulated with the local flow conditions about the particles. Roughness particles smaller than the critical size have been found to introduce no disturbances of sufficient magnitude to influence transition, whereas roughness particles equal to the critical size initiate the formation of turbulent spots at the roughness that coalesce into a continuously turbulent flow somewhat downstream of the roughness. Only a small increase in roughness Reynolds number above the critical value is required to move the fully developed turbulent boundary layer substantially up to the roughness particles.

Determination of the critical roughness height for transition may be accomplished through a trial-and-error procedure with the use of the critical roughness Reynolds number and local variations of velocity and temperature through the boundary layer. It is the purpose of this report, however, to present a direct approach, based on some simplifying assumptions, to the calculation of the size of roughness particles required for transition without the need for individual calculations of the velocity and temperature profiles. Charts required for the application of this method are presented for Mach numbers from 0 to 5.

SYMBOLS

C	constant of proportionality, $\sqrt{\frac{T_w}{T_o}}\left(\frac{T_o + S}{T_w + S}\right)$, in the assumed viscosity relationship $\frac{\mu}{\mu_o} = C\frac{T}{T_o}$
k	height of roughness particles
M	Mach number
R_k	Reynolds number based on roughness height and local flow conditions at top of roughness, $u_k k/\nu_k$
R_x	Reynolds number based on length of x from leading edge to roughness station and on conditions outside boundary layer, Ux/ν_o
S	Sutherland's constant, 216° R
T	local absolute temperature, °R

T_w	wall equilibrium temperature, $T_o\left[1 + 0.845\left(\frac{\gamma - 1}{2} M_o^2\right)\right]$, °R
U	local streamwise component of velocity outside boundary layer
u	local streamwise component of velocity inside boundary layer
x	surface distance measured streamwise from leading edge to roughness station
y	distance normal to surface
γ	ratio of specific heat at constant pressure to specific heat at constant volume
η	nondimensional height in boundary layer based on distance above surface, $\frac{y}{2x}\sqrt{R_x}$
η_k	nondimensional height in boundary layer based on roughness height, $\frac{k}{2x}\sqrt{R_x}$
μ	coefficient of absolute viscosity
ν	coefficient of kinematic viscosity
ρ	local mass density

Subscripts:

k	conditions at top of roughness particle
o	conditions outside boundary layer
t	conditions at which transition occurs
2-D	two-dimensional flat-plate surface
3-D	three-dimensional cone surface

DEVELOPMENT OF METHOD

The method presented in this report for determining the critical roughness height required for premature boundary-layer transition at Mach numbers from 0 to 5 relates by means of charts the roughness height to the roughness Reynolds number based on the local flow conditions at the top of the particle for a given Mach number and roughness location. The relations presented have been derived for zero pressure gradient and zero heat transfer at the surface by the following procedure.

A given roughness particle may be represented nondimensionally by the parameter η_k, which is defined as

$$\eta_k = \frac{k}{2x} \sqrt{R_x} \tag{1}$$

The roughness Reynolds number, based on the particle height and the local flow conditions in the boundary layer at the top of the particle, is defined as

$$R_k = \frac{u_k k}{\nu_k} \tag{2}$$

The roughness Reynolds number, however, may be expressed as a function of station Reynolds number R_x based on the chordwise location of the roughness and local flow conditions outside the boundary layer:

$$R_k = \frac{k}{x} R_x \left(\frac{u_k}{U}\right)\left(\frac{\nu_o}{\nu_k}\right) \tag{3}$$

Equation (3) may be rewritten in the form

$$\frac{R_k}{\sqrt{R_x}} = \left(\frac{k}{x} \sqrt{R_x}\right)\left(\frac{u_k}{U}\right)\left(\frac{\nu_o}{\nu_k}\right) \tag{4}$$

If the viscosity relationship

$$\frac{\mu}{\mu_o} = C \frac{T}{T_o} \tag{5}$$

is assumed, the kinematic-viscosity ratio may be written as

$$\frac{\nu_o}{\nu_k} = \frac{\mu_o}{\mu_k}\frac{\rho_k}{\rho_o} = \frac{1}{C}\left(\frac{T_k}{T_o}\right)^{-2} \tag{6}$$

inasmuch as the pressure gradient normal to the surface in the boundary layer is negligible. A complete discussion of the validity of the assumption inherent in equation (5) is given in reference 3. When equation (6) is substituted into equation (4) and the factor $\frac{k}{x}\sqrt{R_x}$ is rewritten as $2\eta_k$, the final expression for $\frac{R_k}{\sqrt{R_x}}$ becomes

$$\frac{R_k}{\sqrt{R_x}} = \frac{2}{C}\,\eta_k\,\frac{u_k}{U}\left(\frac{T_k}{T_o}\right)^{-2} \tag{7}$$

In order to evaluate equation (7), the variation of velocity and temperature through the boundary layer must be determined. The velocity and temperature distribution through the boundary layer, of course, is dependent on whether the flow is of the two- or three-dimensional type. Two-dimensional distributions for a flat plate determined by the method of reference 3 and three-dimensional distributions for a cone obtained by simply applying Mangler's transformation (ref. 4) to the two-dimensional results are presented in figures 1 and 2 for Mach numbers from 0 to 5. Substitution of values of velocity and temperature ratio as well as a value of C into equation (7) permits determination of the variation of the nondimensional roughness height η_k with $\frac{R_k}{\sqrt{R_x}}$ for a selected Mach number as presented in figures 3 and 4. The values of C used were determined for stagnation temperatures of 120° F and 160° F for Mach numbers from 0 to 2 and from 2.25 to 5, respectively, and the equilibrium wall temperature T_w was calculated for each Mach number by using the laminar "recovery" factor. The selected stagnation temperatures are representative of those for wind tunnels operating in the Mach number range included. Slight variations in the stagnation temperature introduce only second-order effects in the results.

APPLICATION OF RESULTS

Calculation of Critical Roughness Height

Inherent in the application of these charts for the determination of the critical roughness height required to cause premature transition is the selection of a value of the critical roughness Reynolds number $R_{k,t}$. Experimental values of $R_{k,t}$ between about 250 and 600 for subsonic and supersonic speeds up to a Mach number of 2 are presented in references 1 and 2. Further research is required for determination of $R_{k,t}$ at higher Mach numbers, but until such results are available a value of $R_{k,t}$ of 600 at the higher Mach numbers appears to be a reasonable value. It should be recalled that the critical roughness Reynolds number $R_{k,t}$ has been defined as the value at which turbulent spots are initiated at the roughness and that a small increase in roughness Reynolds number R_k above this value is required to move the fully developed turbulent boundary layer substantially up to the roughness particles. Consequently, for investigations in which a fully developed turbulent boundary layer occurring at the roughness is desired, values of $R_{k,t}$ slightly larger than 600 should be used.

For a selected value of R_k for transition, then, and for a given Mach number, unit Reynolds number, and roughness location, the value of the nondimensional roughness height η_k may be determined from figure 3 or 4 with the use of the calculated value of the Reynolds number ratio $\frac{R_k}{\sqrt{R_x}}$. The roughness height is then calculated with this value of η_k. As pointed out in references 1 and 2, the concept of a constant value of the critical roughness Reynolds number applies only to the case of roughness submerged in the boundary layer; therefore, the foregoing procedure for the estimation of the critical roughness height should be applied only for that particular case. The height of the roughness particles compared with the boundary-layer thickness at the location of the roughness can be obtained with use of the nondimensional roughness height η_k and the boundary-layer velocity profiles of figure 1.

Minimization of Roughness Drag

In order to minimize the drag contribution of the roughness particles themselves, that is, the drag associated with the roughness particles other than the increment due to a forward movement in the location of transition, the roughness particles should be spread thinly in a narrow band. Photographs of satisfactory strips of distributed granular-type roughness particles are shown in figure 5. Sometimes it is desirable to use roughness particles larger than the critical height, because the critical height is too small to be manageable (<0.001 inch) or because of reasons such as those discussed subsequently; for example, the application of a single roughness size which will initiate transition through a Mach number and/or Reynolds number range. In these cases, roughness particles somewhat larger than the critical height can be used with no measurable roughness drag if the requirement of only a narrow strip of thinly spread roughness is maintained.

Limitations

As previously mentioned, figures 1 to 4 apply to conditions of zero pressure gradient on surfaces at equilibrium temperature. For the case of heat transfer, therefore, the simplified procedure presented in this paper is inapplicable, and calculations of the boundary-layer velocity and temperature profiles for the specific conditions considered are required. For surfaces at equilibrium temperature, however, deviations from zero pressure gradient found on supersonic-airplane configurations for conditions where laminar flow is possible are most often small enough to permit successful application of the proposed method.

Simplifying Considerations

In applying this method to various types of configurations in several wind-tunnel investigations conducted by the National Advisory Committee for Aeronautics at subsonic speeds and at supersonic speeds up to a Mach number of 4, further simplifying considerations, which yielded successful results in all cases, were found to be expedient. A brief discussion of some of these considerations appears warranted. For tests of models through a range of Reynolds number and Mach number, it was desirable to eliminate the need for changes in the roughness arrangement. This elimination was accomplished by using a roughness size determined for the combination of test Reynolds number and Mach number which required the largest roughness size - usually at the smallest Reynolds number and largest Mach number condition. For tapered wings, roughness was applied at a constant percentage of the local chord (usually at about 5 percent); and in order to permit use of a single grain size across the span, the roughness was calculated for the largest chord. For a wing with a sharp supersonic leading edge, roughness was applied to both wing surfaces in a size calculated by the use of the Mach number and unit Reynolds number based on the flow outside the boundary layer on the upper surface at the maximum test angle of attack, which is the condition for which the critical roughness size was greatest. For a sharp subsonic leading-edge wing or for a round leading-edge wing, whether or not swept behind the Mach line, the freestream Mach number and unit Reynolds number were used. It is obvious that, with these simplifying procedures which permit use of a single grain size along the span of both wing surfaces, roughness particles larger than the minimum required to cause transition are used on many of the roughness strips for some of the test conditions; however, as pointed out previously, careful application of a sparse distribution of roughness particles to a narrow strip will minimize the drag contribution of the roughness itself.

In order to initiate transition near the nose of slender fuselages, the charts of figure 4 were used. Although these charts were computed for boundary-layer flow over a cone, deviations of the boundary-layer growth over the nose regions of slender fuselages from boundary-layer growth on a cone were usually small enough to permit application of these charts with reasonable accuracy. For fuselages with blunt noses, however, the calculations of reference 5 for hemispherical and flat noses appear to provide more accurate estimates of the critical roughness than the cone charts even though the methods of reference 5 were derived on the basis of incompressible flow. For supersonic Mach numbers, the flow behind the bowwave is, of course, subsonic and estimates of the critical roughness based on these subsonic conditions and on the methods of reference 5 seem reasonable.

CONCLUDING REMARKS

A simplified method has been presented for determination of the critical height of three-dimensional roughness particles required to promote premature boundary-layer transition on models of airplanes or airplane components at equilibrium conditions (zero heat transfer) for Mach numbers from 0 to 5. Application of this method to various types of configurations in several wind-tunnel investigations conducted by the National Advisory Committee for Aeronautics at Mach numbers up to 4 has in all cases resulted in the successful initiation of transition at the roughness strips.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 21, 1958.

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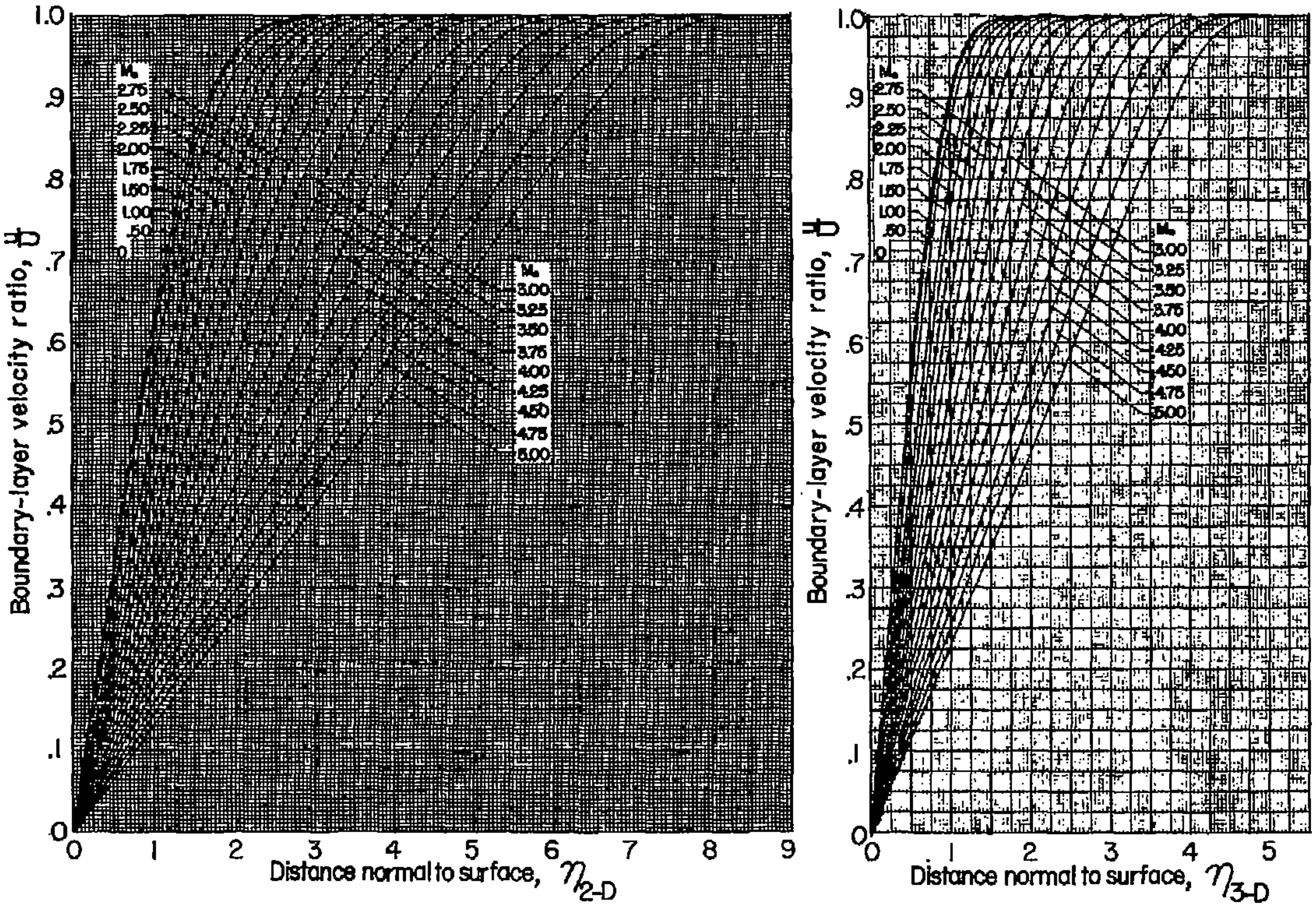
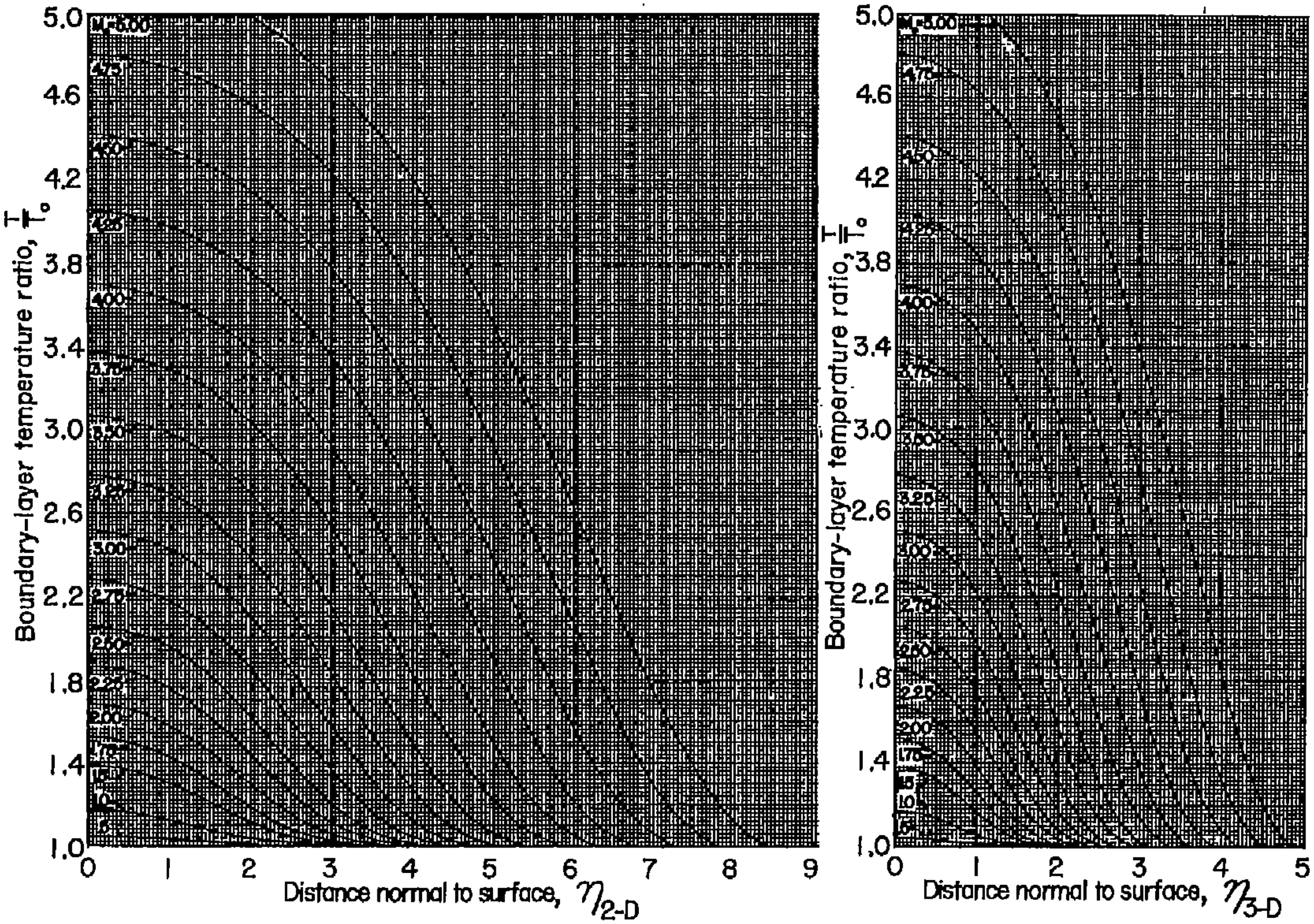


(a) Two-dimensional flow for flat plate.

(b) Three-dimensional flow for cone.

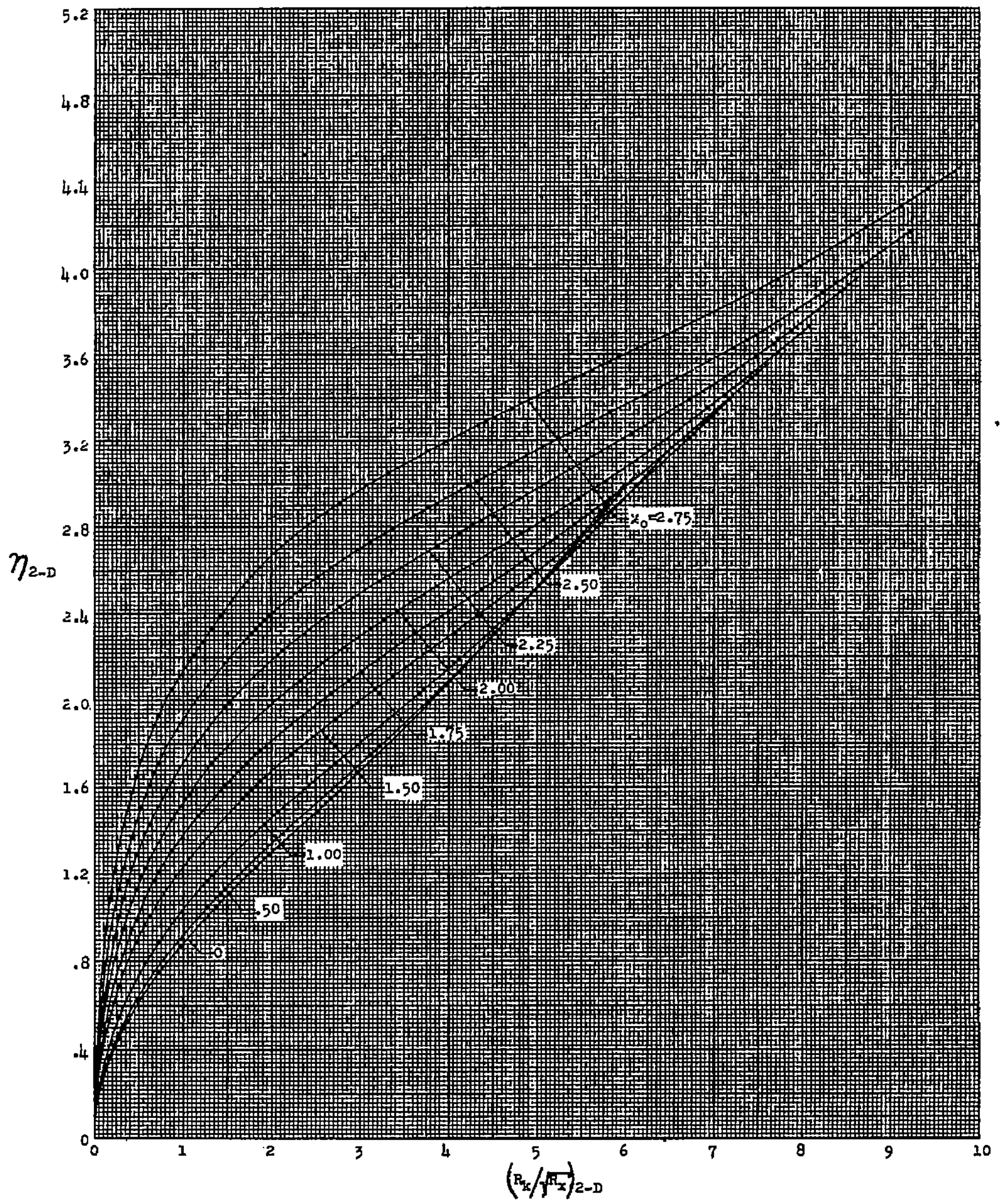
Figure 1.- Nondimensional velocity distribution within laminar boundary layer for Mach numbers from 0 to 5.



(a) Two-dimensional flow for flat plate.

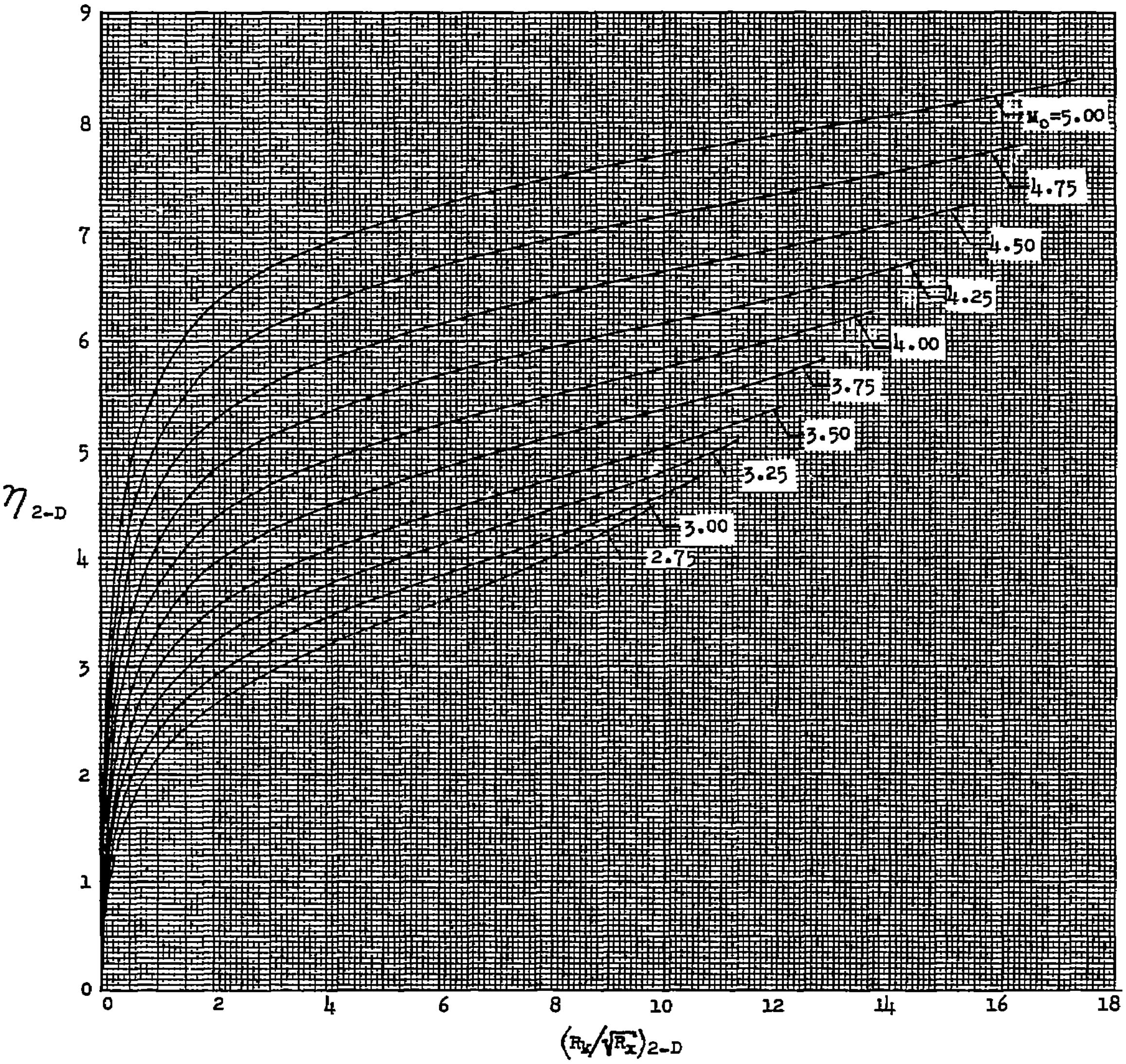
(b) Three-dimensional flow for cone.

Figure 2.- Nondimensional temperature distribution within laminar boundary layer for Mach numbers from 0 to 5.



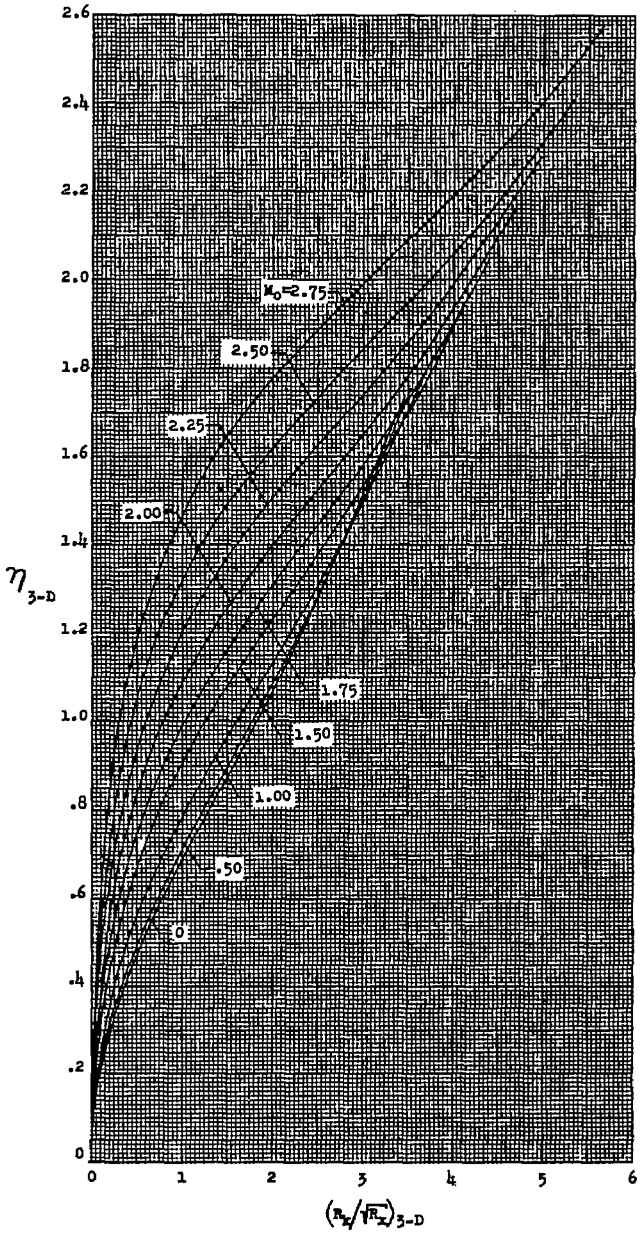
(a) $0 \leqq M_0 \leqq 2.75$.

Figure 3.- Variation of nondimensional height in boundary layer with ratio of roughness Reynolds number to square root of station Reynolds number for two-dimensional flat-plate bodies at Mach numbers from 0 to 5.



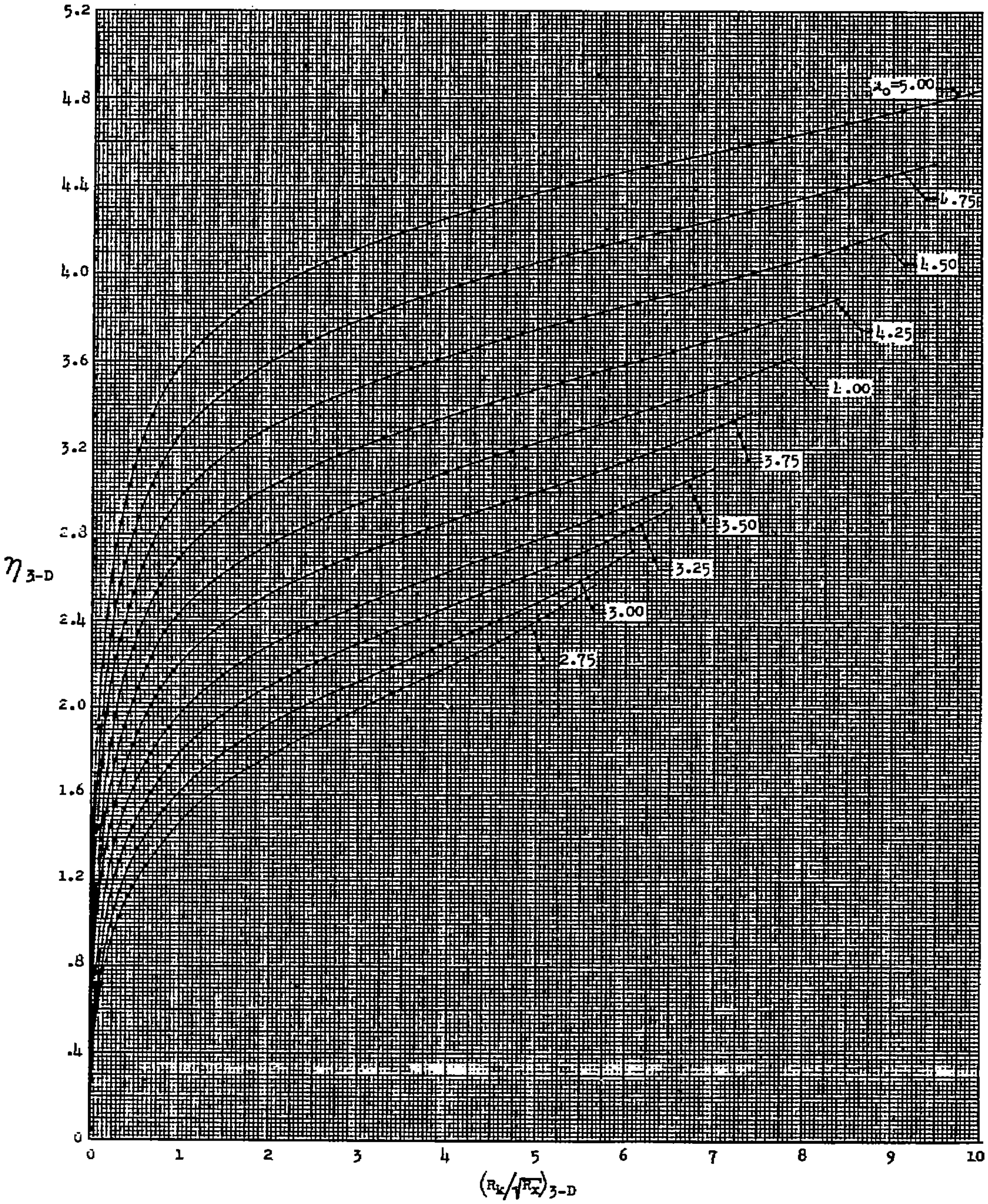
(b) $3 \leqq M_o \leqq 5$.

Figure 3.- Concluded.



(a) $0 \leqq M_0 \leqq 2.75$.

Figure 4.- Variation of nondimensional height in boundary layer with ratio of roughness Reynolds number to square root of station Reynolds number for three-dimensional conical bodies at Mach numbers from 0 to 5.

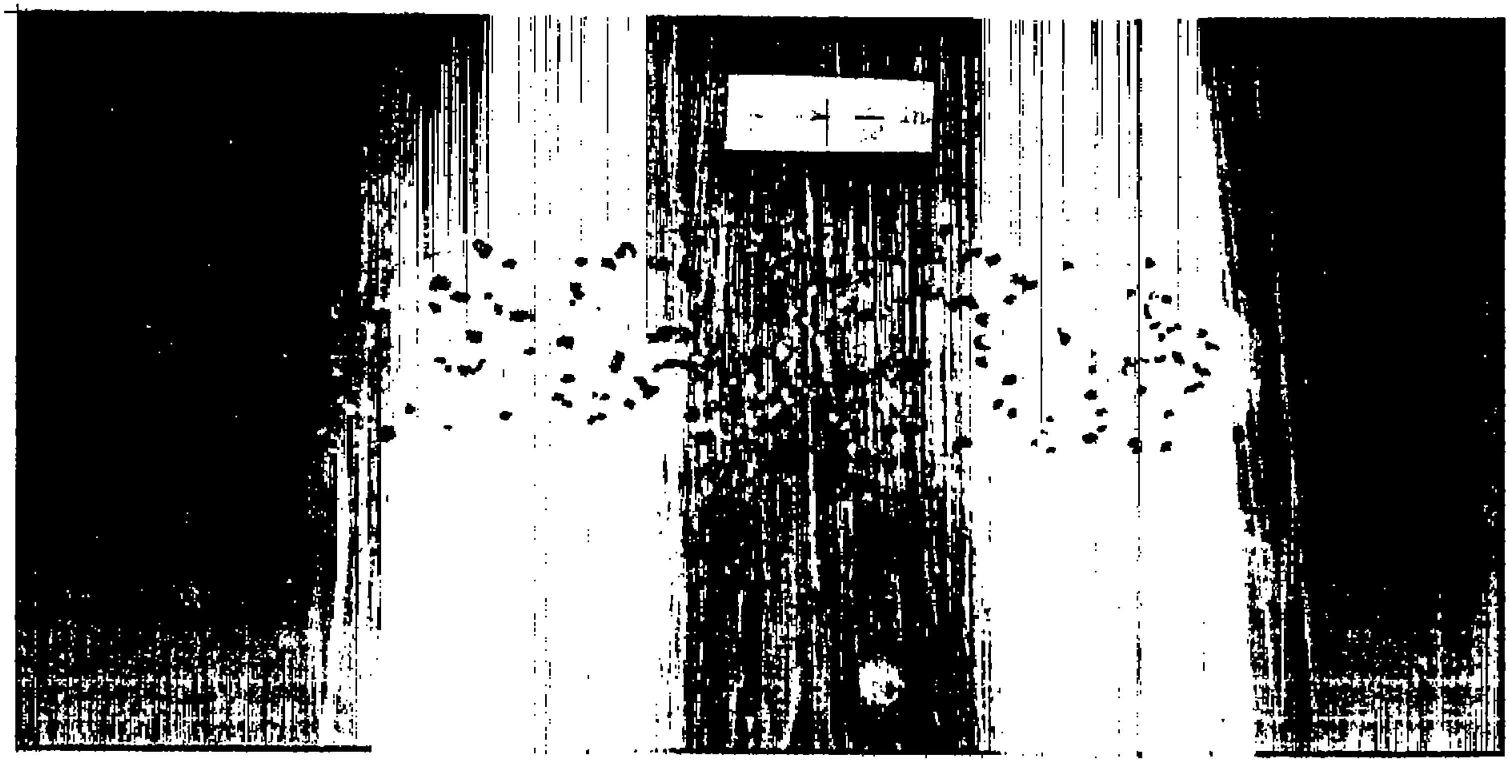


(b) $3 \leqq M_0 \leqq 5$.

Figure 4.- Concluded.

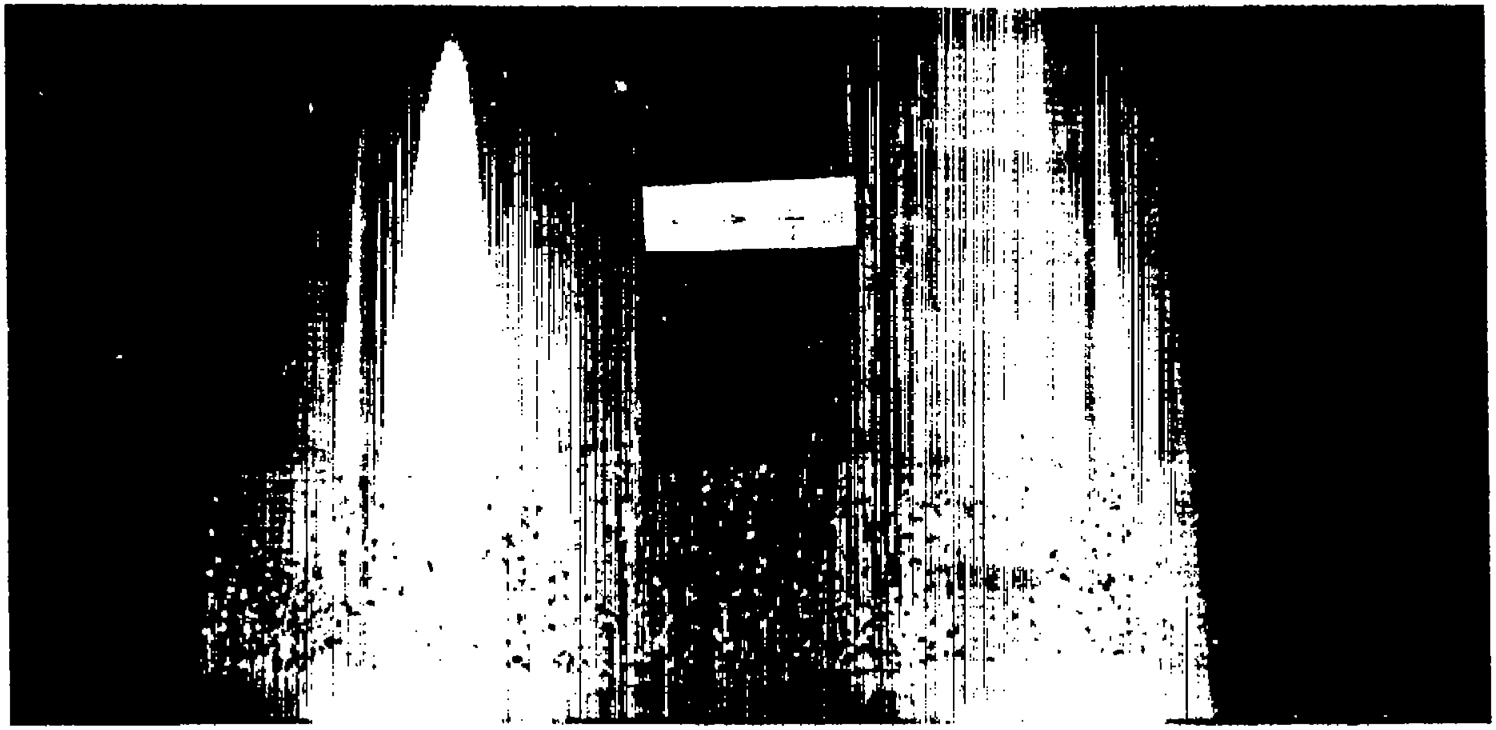
(a) No. 60 carborundum grains. L-57-4944.1

Figure 5.- Closeup photographs of representative strips of distributed granular-type roughness particles.



(b) No. 80 carborundum grains. L-57-4945.1

Figure 5.- Continued.



(c) No. 180 carborundum grains. L-57-4946.1

Figure 5.- Concluded.

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