

*Library L. M. A. 2*

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

---

No. 441

---

ROLLING, YAWING, AND HINGE MOMENTS PRODUCED

BY RECTANGULAR AILERONS

(Correlating Technical Reports Nos. 298, 343, and 370)

By R. H. Heald  
Bureau of Standards

**FILE COPY**

to be returned to  
the files of the Langley  
Memorial Aeronautical  
Laboratory.

Washington  
January, 1933



3 1176 01425 5906

## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 441

ROLLING, YAWING, AND HINGE MOMENTS PRODUCED  
BY RECTANGULAR AILERONS

(Correlating Technical Reports Nos. 298, 343, and 370)

By R. H. Heald

## SUMMARY

In this report the studies of the characteristics of rectangular ailerons described in references 1, 2, and 3 are summarized in the form of empirical equations which relate the aileron dimensions and displacements to the rolling, yawing, and hinge moments for pitch angles of  $0^\circ$  and  $12^\circ$  degrees, corresponding to angles of attack of the wings of  $4^\circ$  and  $16^\circ$  degrees, respectively. The report also includes a comparison of the results obtained by computation from measurements on a single aileron, with those obtained by using two ailerons mounted on opposite wings and displaced simultaneously. Satisfactory agreement is shown between these methods. Rectangular airfoils (10-inch chord by 60-inch span) having the Clark Y and U.S.A. 27 profiles and mounted on a model fuselage were used throughout the investigation.

The work was conducted in the 10-foot wind tunnel of the Bureau of Standards with the cooperation of the Aeronautics Branch of the Department of Commerce and the National Advisory Committee for Aeronautics.

## INTRODUCTION

The rolling and yawing moments arising as a result of aileron displacement for a given attitude of an airplane are dependent primarily on three factors: (1) the magnitude of the displacement, (2) the size and shape of the ailerons, and (3) the lateral position of the ailerons with respect to the axis of the airplane. Further, it appears from a study of the available data that the corresponding coefficients are approximately proportional to the aileron angle and to simple functions of the aileron chord and span. These relationships suggested the possi-

bility of expressing the moments in the form of empirical equations suitable for purposes of preliminary design and this has been done using data taken from references 1, 2, and 3.

After the completion of the experimental work described in these references, attention was called to the work of Hartshorn, who pointed out from theoretical considerations (reference 4), that the method of images, which utilizes the half-span of the airfoil and a reflecting plane, or any method which involves measurement on a single aileron, may lead to incorrect conclusions when used to determine the effects of the displacement of two ailerons on opposite wings. The error is due to the mutual interference between the ailerons, which is absent in the test of a single aileron. In the investigation described in references 1 and 2, the measurements were made on one aileron mounted on an airfoil of full span, and it seemed advisable to make an experimental comparison of the one-aileron and two-aileron methods. The results of the comparison are given in this report.

#### Comparison of One-Aileron and Two-Aileron Methods

The 20 by 2.5 inch and the 20 by 3 inch ailerons mounted on the Clark Y wing were used in the comparison, the method of measurement being the same as described in references 1 and 2, in which photographs and dimensions of the model also are given.

It was found early in the analysis of the results that disturbing effects were introduced by a lack of symmetry in the model or an unsymmetrical air flow in the tunnel which resulted in different angles of attack of the wing at the two tips. Because of this factor, the addition of the net moments for equal up and down displacements of the aileron on one wing did not agree well with results obtained when both ailerons were displaced, and the observed results for right-aileron-up, left-aileron-down were not the same as for right-aileron-down, left-aileron-up. To avoid the effects of this disturbing factor in the comparison, the following procedure was adopted.

The rolling and yawing moment coefficients arising from the displacement of each aileron in both directions were first determined. Call the value obtained at a given

aileron setting, as measured with the right aileron down,  $M_{rd}$ ; with right aileron up,  $M_{ru}$ ; with left aileron down  $M_{ld}$ ; with left aileron up,  $M_{lu}$ . The total or combined coefficients were then obtained indirectly by adding  $\frac{M_{ru} + M_{lu}}{2}$  to  $\frac{M_{rd} + M_{ld}}{2}$ , i.e., using the mean values for the two ailerons for a given direction of displacement.

The coefficients were then determined from measurements with the right aileron up and the left aileron down, giving a value  $M_{ru,ld}$  and also with the right aileron down and the left aileron up, giving a value  $M_{rd,lu}$ . In this method, the combined coefficients were measured directly, and the mean value  $\frac{M_{ru,ld} + M_{rd,lu}}{2}$  was used for comparison with  $\frac{M_{rd} + M_{ld}}{2} + \frac{M_{ru} + M_{lu}}{2}$  as determined by the indirect method. In both methods, the effect of any constructional dissymmetry between the right and left ailerons or wing tips as well as any effect due to air spin in the tunnel is thus balanced out. Approximately as good agreement was obtained by comparing  $M_{rd} + M_{lu}$  with  $M_{rd,lu}$  or  $M_{ru} + M_{ld}$  with  $M_{ru,ld}$ .

The results of the comparison of the direct and indirect methods, using the mean values, are given in Table I and Figure 1. It will be noted that in general the values of the combined rolling moment coefficient determined by using the direct method are somewhat greater than the combined values which were obtained indirectly, the mean of all the ratios of the directly observed to the indirectly determined coefficients being 1.03. In the case of the yawing moment coefficients the corresponding mean is 1.01. However, the range of the ratios is considerably greater in the latter case due to the fact that the precision of the yawing moment observations is less. Based on this investigation, the conclusion appears to be valid, that the indirect method may be expected to give results which are in substantial agreement with those obtained by use of the direct method.

When the indirect method (with one aileron) is used, the results apply to the angle of attack of the wing at the aileron. Since the aileron moments depend greatly on the angle of attack, care must be taken to know this angle

when applying the results. For example, in the results published in references 1 and 2 on the effect of aileron dimensions, some of the ailerons were located on the right wing and some on the left wing to simplify the construction of the model. The results for these two groups of ailerons indicated a difference between the angles of attack at the two wing tips of about one degree.

EMPIRICAL EQUATIONS

The possibility of expressing the rolling, yawing, and hinge moment coefficients, given in references 1, 2, and 3, in the form of empirical equations, was pointed out in the introduction. The monoplane model which was used in the experiments consisted only of a fuselage and wing, the tail assembly, landing gear, etc. being omitted. The dimensions of the ailerons are given in Table II.

TABLE II. Dimensions of Ailerons  
 U.S.A.27 and Clark Y Wings

ba by ca (inches)	Position
20 by 1.5 .....	right wing tip
20 " 2.0 .....	" " "
20 " 3.0 .....	" " "
20 " 2.5 .....	" " "
10 " 2.5 .....	left wing tip
15 " 2.5 .....	" " "
20 " 2.5 .....	" " "

Rolling and yawing moments were determined simultaneously by observing the changes in tension of small wires running from the wing tip and tail of the model, respectively, to the roll and yaw balances, the model being supported by a mast projecting from the wall of the tunnel into the air stream and carrying at its extremity a universal joint housed within the fuselage. (References 1 and 2.) The hinge moments were determined using the same mounting, the universal joint being locked at the desired angles. The tension wire connecting the model and the balance in this case was attached to the trailing edge of the aileron at its midspan. (Reference 3.)

The measurements of rolling and yawing moments were made with the variable-chord group of ailerons mounted on the right wing and the variable-span group mounted on the left wing. This arrangement was modified in the hinge-moment measurements so that both groups were mounted on the right wing, in order to minimize the effects of air spin in the tunnel and warp of the model.

The coefficients in references 1, 2, and 3 were based on the dimensions of the wing. To introduce the aileron dimensions, it was assumed as a trial that the coefficients were proportional to powers of the chord, span, and angular displacement of the aileron. In order to determine the values of the exponents, the coefficients were plotted logarithmically against the chord, span, and displacement of the aileron in turn. The mean values of the exponents were found from these plots and the nearest half power was used in the equations. Having determined the exponents, the wing dimensions and the distance from midspan of the wing to midspan of the aileron were introduced to make the equations dimensionless. The equations for rolling and yawing moment coefficients refer to body axes, those for hinge moment refer to an axis along the leading edge of the aileron, midway between the upper and lower surfaces.

NOTATION

- b, wing span (ft.)
- c, wing chord (ft.)
- b<sub>A</sub>, aileron span (ft.)
- c<sub>A</sub>, aileron chord (ft.)
- f, distance from center of rotation to end of fuselage (ft.)
- V, air speed (ft./sec.)
- q,  $\frac{1}{2} \rho V^2 = 0.001189 V^2$
- L, rolling moment (lb-ft.)
- N, yawing moment (lb-ft.)

H, hinge moment (lb-ft.)

$\delta_A$ , aileron displacement (degrees)

$$C_L = \frac{L}{q b^2 c}$$

$$C_N = \frac{N}{q f b c}$$

$$C_H = \frac{H}{q b c^2}$$

$$C_{l_1} = \frac{L}{q b_A c_A \left( \frac{b}{2} - \frac{b_A}{2} \right)}$$

$$C_{n_1} = \frac{N}{q b_A c_A \left( \frac{b}{2} - \frac{b_A}{2} \right)}$$

$$C_{h_1} = \frac{H}{q b_A c_A^2}$$

Equations for rolling moment.- The equations for the determination of rolling moment for the conventional arrangement of equal up and down displacements of opposite ailerons apply to the ailerons on the Clark Y and U.S.A.27 wings as mounted on the fuselage, the pitch angle being either 0 or 12° (angle of attack 4 or 16°) as stated. The ordinates given in Figures 2a and 3a were obtained by multiplying the rolling moment coefficients given in references 1 and 2 by the expression:

$$\frac{b^2 \sqrt{c}}{b_A \sqrt{c_A} \left( \frac{b}{2} - \frac{b_A}{2} \right)}$$

It can be shown that this is the same as multiplying the rolling moment L, by

$$\frac{\sqrt{\frac{c_A}{c}}}{q b_A c_A \left( \frac{b}{2} - \frac{b_A}{2} \right)}$$

Calling

$$\frac{L}{q b_A c_A \left( \frac{b}{2} - \frac{b_A}{2} \right)} = C_{l_1}$$

we have:

$$\frac{L \sqrt{\frac{c_A}{c}}}{q b_A c_A \left(\frac{b}{2} - \frac{b_A}{2}\right)} = C_{l_1} \sqrt{\frac{c_A}{c}}$$

Introduction of the square-root relationship for aileron displacement results in bringing the higher values of  $C_{l_1} \sqrt{\frac{c_A}{c}}$  more nearly into line. Straight lines were drawn through the plotted points so that the percentage departures of the extreme points were approximately the same throughout the range of aileron angles. Actually these lines must pass through the origin somewhat as indicated by the dotted lines in the figures. No measurements were made for aileron angles below 4 degrees.

Using this method the following equations were obtained:

$$C_{l_1} \sqrt{\frac{c_A}{c}} = 0.55 (\sqrt{\delta_A} - 1.0) \quad \text{for the Clark Y, } 0^\circ \text{ pitch (4}^\circ \text{ angle of attack)} \quad (1)$$

$$C_{l_1} \sqrt{\frac{c_A}{c}} = 0.25 (\sqrt{\delta_A} - 1.0) \quad \text{for the Clark Y, } 12^\circ \text{ pitch (16}^\circ \text{ angle of attack)} \quad (2)$$

$$C_{l_1} \sqrt{\frac{c_A}{c}} = 0.50 (\sqrt{\delta_A} - 1.0) \quad \text{for the U.S.A.27, } 0^\circ \text{ pitch (4}^\circ \text{ angle of attack)} \quad (3)$$

$$C_{l_1} \sqrt{\frac{c_A}{c}} = 0.28 (\sqrt{\delta_A} - 1.0) \quad \text{for the U.S.A.27, } 12^\circ \text{ pitch (16}^\circ \text{ angle of attack)} \quad (4)$$

Equations for yawing moment.— The equations for yawing moment were obtained in a manner similar to that used in the case of rolling moments. In this case, however, the yawing moment coefficients (references 1 and 2) were

multiplied by the factor  $\frac{f b \sqrt{c}}{b_A \sqrt{c_A} \left(\frac{b}{2} - \frac{b_A}{2}\right)}$ . The values

corresponding to the resulting expression:



$$\frac{N}{q b_A c_A \left( \frac{b}{2} - \frac{b_A}{2} \right)} \sqrt{\frac{c_A}{c}} \quad \text{or} \quad C_{n_1} \sqrt{\frac{c_A}{c}}$$

are shown plotted against  $\sqrt{\delta_A}$  in Figures 2b and 3b for the case of the Clark Y wing. Here, again, introduction of the square-root relationship for the aileron displacement brings the higher values into better alignment. Equations were determined for both the Clark Y and U.S.A.27 wings as follows:

$$C_{n_1} \sqrt{\frac{c_A}{c}} = 0.055 (\sqrt{\delta_A} - 1.0) \quad \text{for Clark Y, } 0^\circ \text{ pitch (} 4^\circ \text{ angle of attack)} \quad (5)$$

$$C_{n_1} \sqrt{\frac{c_A}{c}} = 0.085 (\sqrt{\delta_A} - 1.0) \quad \text{for Clark Y, } 12^\circ \text{ pitch (} 16^\circ \text{ angle of attack)} \quad (6)$$

$$C_{n_1} \sqrt{\frac{c_A}{c}} = 0.035 (\sqrt{\delta_A} - 1.0) \quad \text{for U.S.A.27, } 0^\circ \text{ pitch (} 4^\circ \text{ angle of attack)} \quad (7)$$

$$C_{n_1} \sqrt{\frac{c_A}{c}} = 0.075 (\sqrt{\delta_A} - 1.0) \quad \text{for U.S.A.27, } 12^\circ \text{ pitch (} 16^\circ \text{ angle of attack)} \quad (8)$$

Equations for hinge moment.— The equations given below were obtained by multiplying the hinge moment coefficients given in reference 3 by  $\frac{b c^2}{b_A c_A^2}$ , the resulting expression having the form

$$C_{h_1} = \frac{H}{q b_A c_A^2}$$

The values of  $C_{h_1}$  were plotted directly against aileron displacement. (Figures 2c and 3c.) Straight lines drawn through these points are represented by the equations:

$$C_{h_1} = 0.022 \delta_A \quad \text{for the Clark Y, } 0^\circ \text{ pitch (} 4^\circ \text{ angle of attack)} \quad (9)$$

$$C_{h_1} = 0.020 \delta_A \quad \text{for the Clark Y, } 12^\circ \text{ pitch (} 16^\circ \text{ angle of attack)} \quad (10)$$

$$C_{h_1} = 0.019 \delta_A \quad \text{for the U.S.A.27, } 0^\circ \text{ pitch (} 4^\circ \text{ angle of attack)} \quad (11)$$

$$C_{h_1} = 0.018 \delta_A \quad \text{for the U.S.A.27, } 12^\circ \text{ pitch (} 16^\circ \text{ angle of attack)} \quad (12)$$

There appeared to be no advantage from the standpoint of precision in bringing the square root of the aileron displacement into the equations for hinge moment.

### CONCLUSION

The indirect method of determining the combined rolling and yawing moments of a model by adding the mean values for corresponding up and down displacements of each aileron, using data obtained by displacing right and left ailerons separately appears from the results of these tests to be of satisfactory accuracy. When one aileron is used, however, and the combined moments are obtained by adding the values for corresponding up and down displacements, care must be taken to avoid uncertainty in angle of attack due to warp of the model and spin of the tunnel air stream, since a difference of the order of  $1^\circ$  in angle of attack of the two wing tips may result in considerable difference between the moments arising from the same angular displacement of opposite ailerons.

Empirical equations are given, which illustrate the approximate relationships between the moments due to the displacement of rectangular ailerons when the pitch angle is increased from  $0^\circ$  to  $12^\circ$ . For example, the rolling moment is decreased approximately one-half as a result of this change. This decrease is accompanied by an increase in yawing moment of the order of 100 per cent, the hinge moment remaining substantially constant.

The precision of the equations with respect to the values of the coefficients given in references 1, 2, and 3 is of the order of  $\pm 15$  per cent. In the case of  $12^\circ$  pitch angle the greatest departures of the individual points from the straight lines are of the order of  $\pm 20$  per cent. In all cases the points show a decided departure from the linear relationship and increased dispersion when the aileron angle is greater than about  $24^\circ$ . It should be pointed out in this connection that while the equations are based on the results of a somewhat extended investigation they may be expected to differ considerably from those derived on a basis of other observations, obtained under what appear to be only slightly different conditions.

For example, it is known that an open gap between the aileron and wing equal in width to 0.8 per cent of the wing chord decreases the observed rolling moment by as much as 40 per cent compared with a completely sealed gap, such as was used in the investigation on which the equations are based. (Reference 5.) It is also known that the fuselage exerts a considerable effect on the lift of the wing (reference 6) and it is reasonable to suppose that a part of this effect extends to the ailerons. The equations, therefore, should not be expected to hold rigorously but it is believed they will prove useful for the estimation of aileron moments in preliminary design, particularly with regard to monoplanes.

Bureau of Standards,  
Washington, D. C., Sept. 2, 1932.

TABLE I. Comparison of Rolling and Yawing Moment Coefficients Determined by Experiments on One and Two Ailerons, Respectively, Clark Y Wing\*

Pitch Angle angle of (de- attack grees) (de- grees)	Aileron dis- place- ment (de- grees)	Aileron (de- grees)	Rolling moment coefficient		Yawing moment coefficient		
			Com- bined (by ad- dition from one- aileron tests)	Com- bined (by ob- serva- tion from two- aileron tests)	Com- bined (by ad- dition from one- aileron tests)	Combined (by ob- servation from two- aileron tests)	
0	4	20 by 2.5	8	+0.050	+0.053	-.012	-.013
			16	.091	.093	-.020	-.022
			24	.113	.110	-.029	-.026
			32	.135	.136	-.032	-.031
			44	.163	.164	-.033	-.033
8	12	20 by 3	8	+0.041	+0.044	-.016	-.015
			16	.079	.079	-.029	-.029
			24	.097	.103	-.030	-.032
			32	.116	.117	-.037	-.035
			44	.128	.135	-.034	-.030
12	16	20 by 3	8	+0.021	+0.024	-.026	-.025
			16	.052	.050	-.042	-.045
			24	.071	.069	-.046	-.051
			32	.076	.079	-.057	-.057
			44	.093	.079	-.059	-.059

\*The signs (+ and -) refer (respectively,) to the right aileron up, left down condition. These values differ somewhat from those published in references 1 and 2 because they represent averages of measurements on right and left ailerons.

*Moyle*

REFERENCES

1. Heald, R. H., and Strother, D. H.: Effect of Variation of Chord and Span of Ailerons on Rolling and Yawing Moments in Level Flight. T.R. No. 298, N.A.C.A., 1928.
2. Heald, R. H., Strother, D. H., and Monish, B. H.: Effect of Variation of Chord and Span of Ailerons on Rolling and Yawing Moments at Several Angles of Pitch. T.R. No. 343, N.A.C.A., 1930.
3. Monish, B. H.: Effect of Variation of Chord and Span of Ailerons on Hinge Moments at Several Angles of Pitch. T.R. No. 370, N.A.C.A., 1930.
4. Hartshorn, A. S.: Theoretical Relationship for a Wing with Unbalanced Ailerons. R. & M. No. 1259, British A.R.C., 1929.
5. Irving, H. B., Ower, E., and Hankins, G. A.: An Investigation of the Aerodynamic Properties of Wing Ailerons. Part I: The Effect of Variation of Plan Form of Wing Tip, and Span of Aileron. R. & M. No. 550, British A.C.A., 1918.
6. Ower, E., and Hutton, C. H.: Interference of a Streamline Nacelle on a Monoplane Wing. R. & M. No. 1395, British A.R.C., 1931.

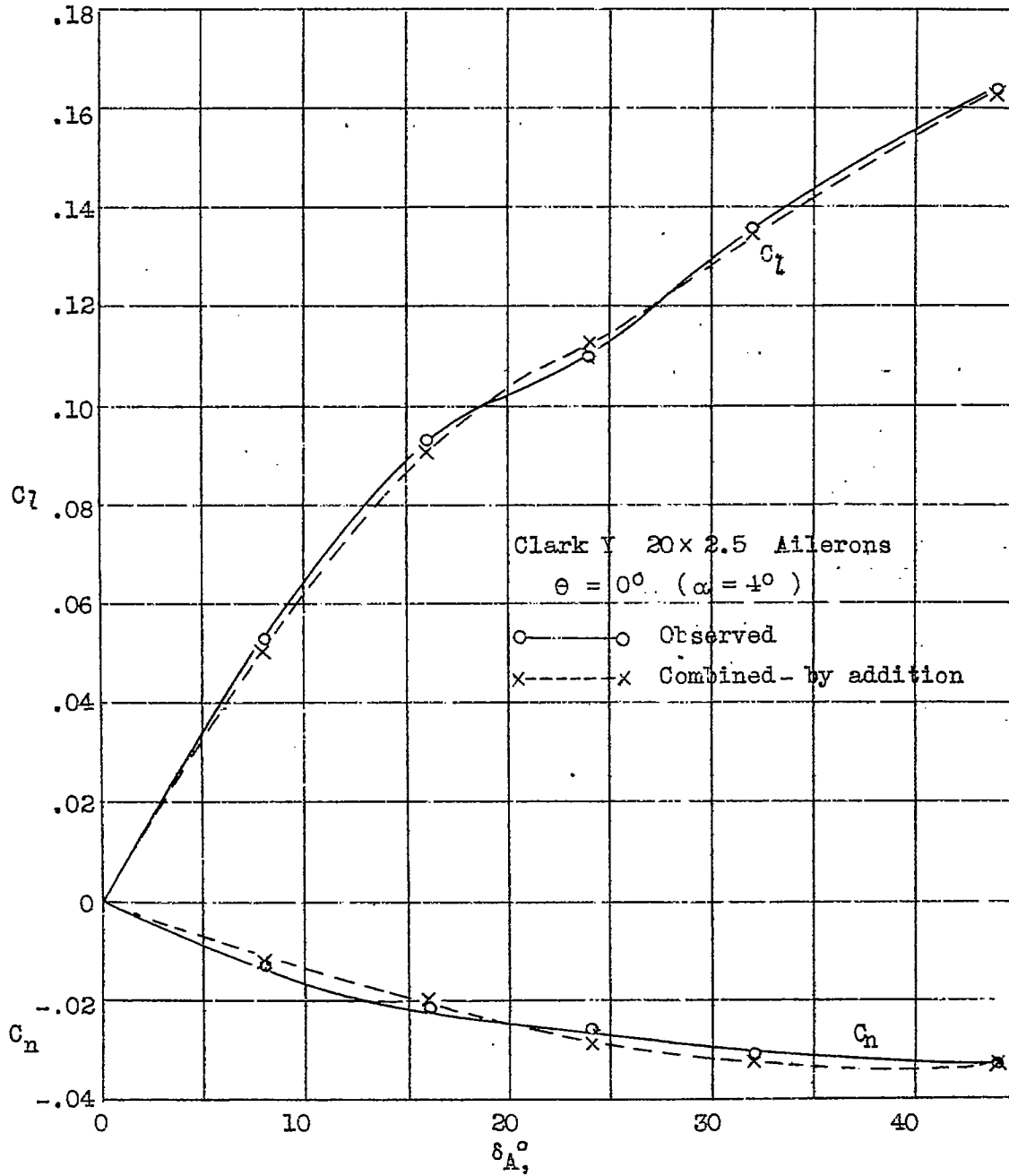


Figure 1. - Comparison of observed and combined rolling and yawing moment coefficients. (Using two ailerons). The signs refer to the right up left down condition. ( $+\delta_A = -\delta_A$ )

*Handwritten notes:*  
 10-15  
 $\delta_A$

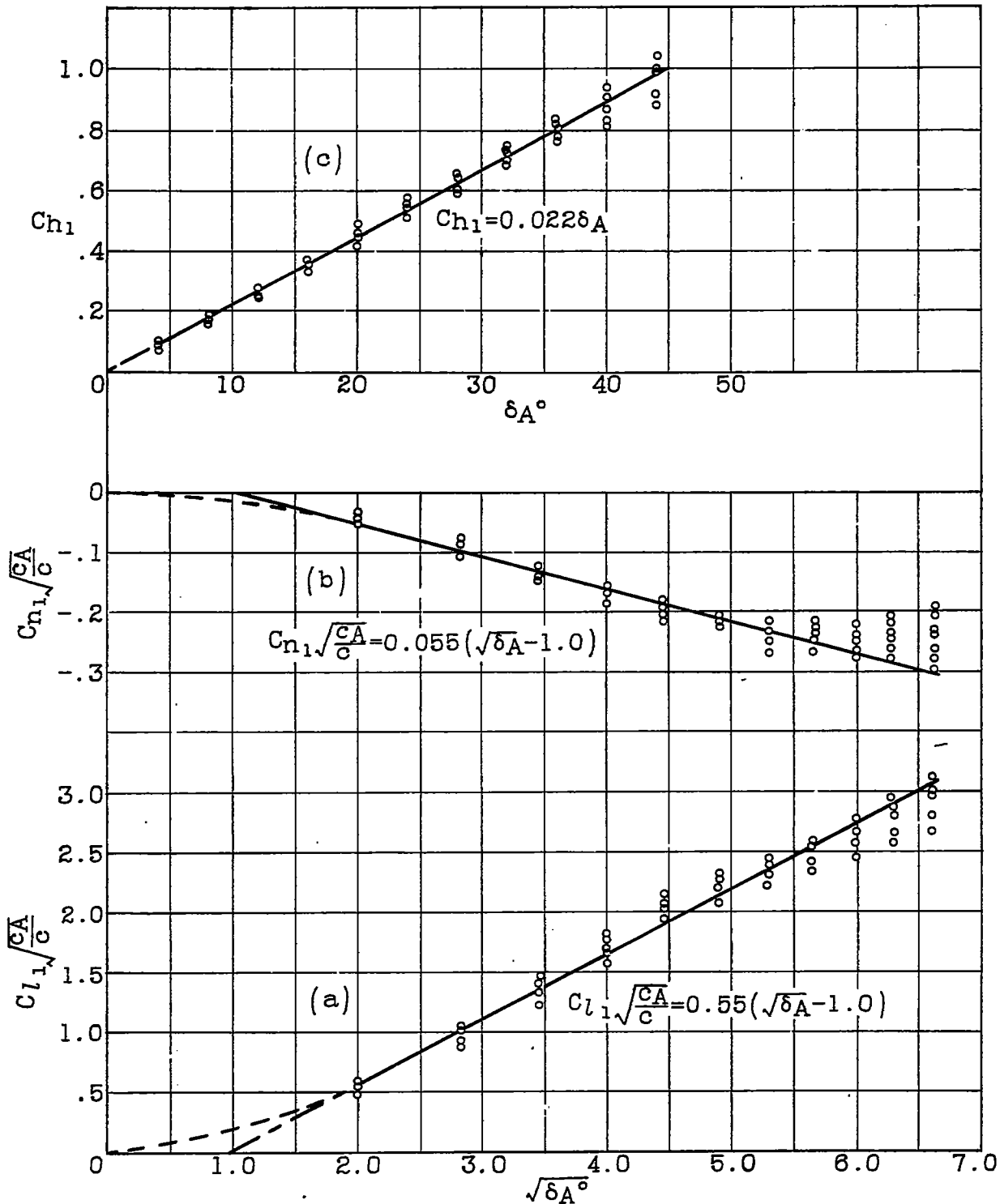


Figure 2.-Plots for determination of rolling, yawing and hinge moment equations. Clark Y wing on fuselage, 0 degrees pitch ( $\alpha=4^\circ$ ). The signs refer to the right aileron up, left down condition. ( $+\delta A = -\delta A$ )

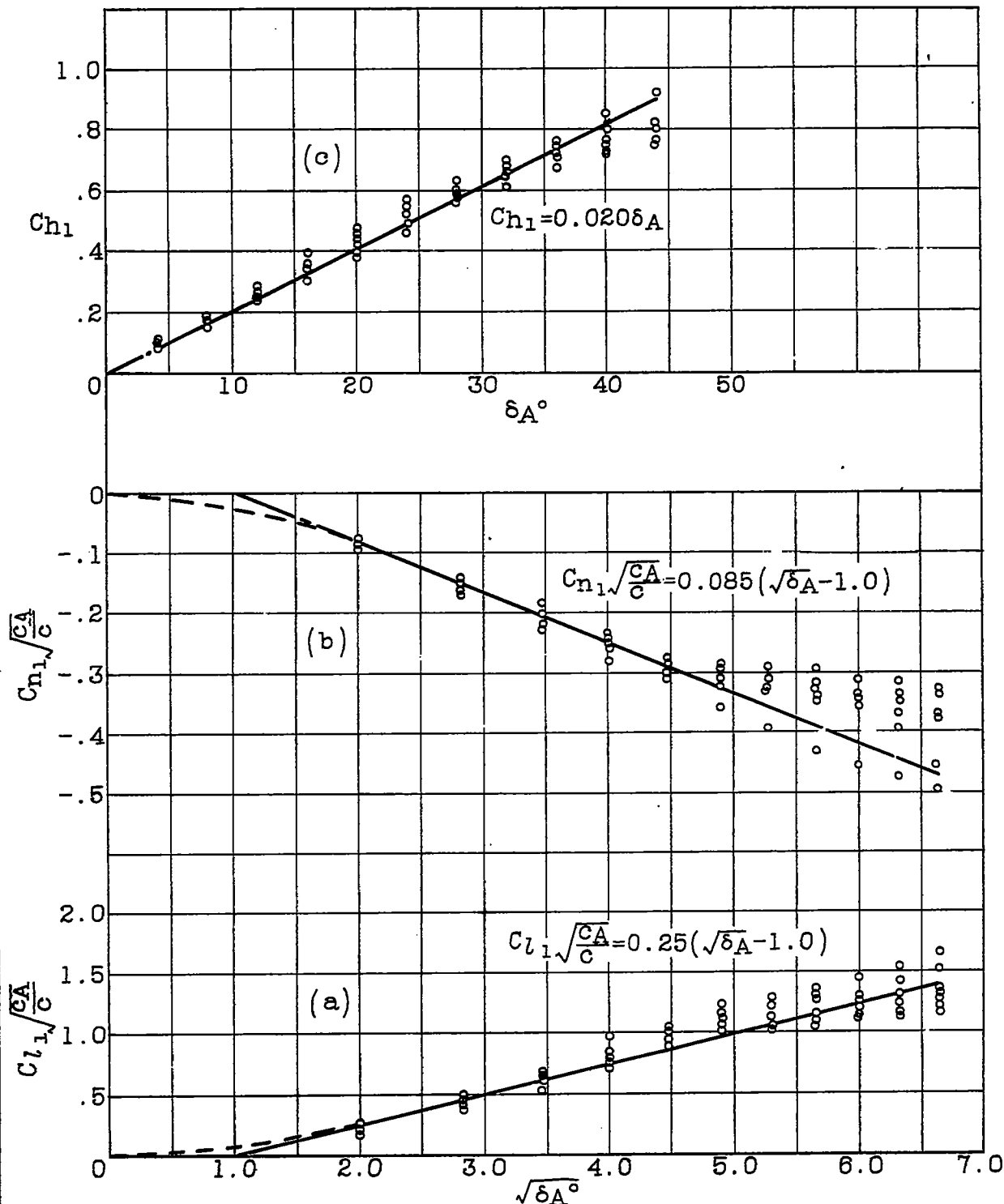


Figure 3.-Plots for determination of rolling, yawing and hinge moment equations. Clark Y wing on fuselage, 12 degrees pitch ( $\alpha=16^\circ$ ). The signs refer to the right aileron up, left down condition. ( $+\delta A=-\delta A$ )