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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 444

WORKING CHARTS FOR THE STRESS ANALYSIS OF ELLIPTIC RINGS

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By Walter F. Burke

SUMMARY

This report presents charts which reduce the stress analysis of circular and elliptic rings of uniform cross section subjected to balanced systems of concentrated loads from a statically indeterminate problem to a statically determinate one. The charts are constructed for two simple loading conditions into which most of the loading conditions encountered in the design of the main frames of a monocoque fuselage may be resolved.

The theoretical calculations required for the construction of the charts employed the "minimum energy method" but are omitted in the report.

To demonstrate the use of the charts in the stress analysis of elliptic rings an illustrative problem is included.

INTRODUCTION

The design of main frames for monocoque fuselages has been discussed by Roy A. Miller in reference 1, where he also developed equations for the shear, thrust, and moment in a circular ring of uniform cross section subjected to balanced systems of concentrated loads symmetrical with respect to a diameter. As the monocoque fuselage is perhaps more often elliptical than circular in section, the author has extended the solutions given in reference 1 to include the elliptic ring of uniform cross section subjected to balanced systems of concentrated loads symmetrical with respect to the major axis.

From structural theory and dimensional reasoning it developed that for any loading condition the shear, thrust, and moment at any point in a ring could be reduced to coefficient form and presented conveniently in charts, and that with a series of these charts the stress analysis of

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the ring is reduced from a statically indeterminate problem to a statically determinate one. The object of this report is to present a group of these charts for circular and elliptic rings which will be useful in the stress analysis of the main frames in monocoque fuselages.

The calculations required to evaluate the coefficients plotted in the charts employed the "minimum energy method" and were made during a study of "Elliptic Integrals" under Mr. D. K. Kazarinoff at the University of Michigan, whose valuable assistance and suggestions are gratefully acknowledged. Because the calculations were long and contained nothing new in principle, they have been omitted in the report. However, all the calculations were checked in detail by the National Advisory Committee for Aeronautics, so that the published charts may be considered accurate within the limits of the usual assumptions in structural theory.

To demonstrate the use of the charts in the stress analysis of elliptic rings subjected to balanced systems of concentrated loads symmetrical with respect to the major axis of the ellipse, an illustrative problem is included.

CHARTS

From structural theory and dimensional reasoning, it follows that the shear, thrust, and moment, respectively, at any point in a ring subjected to any system of external loads may be given by the following equations:

$$V = K W \tag{1}$$

$$P = K' W \tag{2}$$

$$M = K'' a W \tag{3}$$

where a is a dimension of the ring, K, K', and K'' are constants depending upon the shape of the ring and the distribution or position of the loads, and W is the load on the ring. For a circular or elliptic ring subjected to concentrated loads W, balanced and symmetrical as shown in Figure 1, K is zero at the ends of the axis of symmetry. The values of K' and K'' at one end of the axis of symmetry are given in Figures 2, 3, 4, and 5 for the two cases shown in Figure 1. These figures are self-ex-

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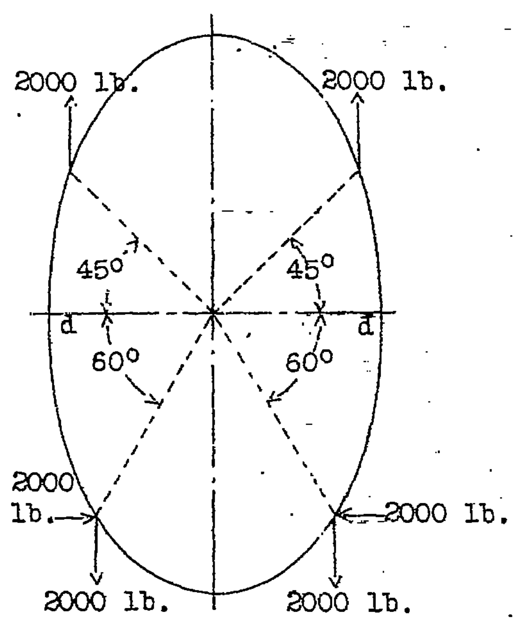
planatory and require no comment except that in these charts the position of the loads, W, is given by Φ, values of which are plotted against θ for the various a/b ratios in Figure 6.

LIMITATION OF THE CHARTS

The charts are derived for thin rings, rings in which the dimensions of the cross section are small as compared to the dimensions of the ellipse, but not so small that deformation under load becomes appreciable and changes in curvature cannot be closely approximated by the second derivative. Consequently, the charts must be considered as approximate only. For rings of the proportions usually encountered in the design of main frames for monocoque fuselages the approximation should be close.

ILLUSTRATIVE PROBLEM



Given: An elliptic ring a = 25 inches; b = 14.7 inches; $\frac{a}{b} = 1.70$ loaded as shown.

Required: To find the shear, thrust, and moment (V_d, P_d, and M_d) at the ends of the minor axis.

Solution: The procedure required is to separate the loads into a series of simple cases, A and B, for which the charts are constructed. The shear, thrust, and moment are then calculated for each of these simple cases and added algebraically. For this problem three simple cases are required as follows:

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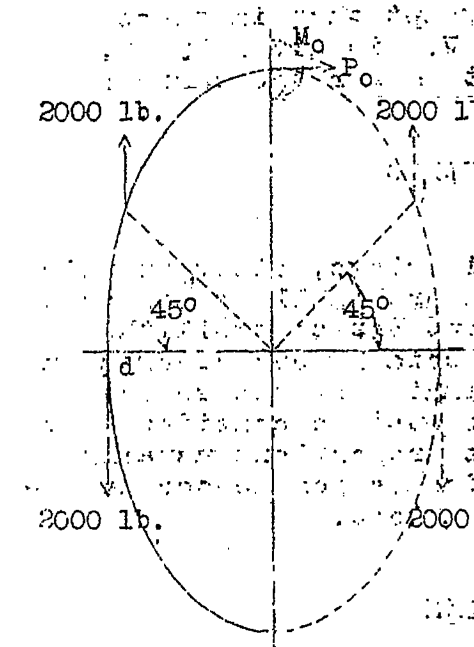


Case 1

From Figure 6

$\Phi = 30.5°$

From Figure 2

$P_o = 0.0495 \times 2,000 = 99$ pounds

From Figure 3

$M_o = 0.002 \times 25 \times 2,000 = 100$ pound-inches

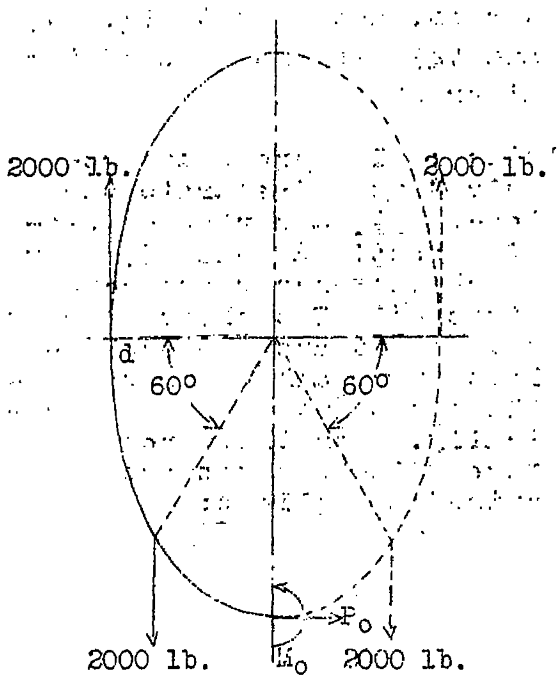
Then:

By inspection

$V_d = 99$ pounds

$P_d = 0$ pound

$M_d = M_o + P_o a - 2,000\ (b - b \cos \Phi) = 100 + 99 \times 25 - 2,000\ (14.7 - 14.7 \times 0.862) = -1,490$ pound-inches



Case 2

From Figure 6

$\Phi = 45.8°$

From Figure 2

$P_o = 0.096 \times 2,000 = 192$ pounds

From Figure 3

$M_o = 0.029 \times 25 \times 2,000 = 1,450$ pound-inches

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Then:

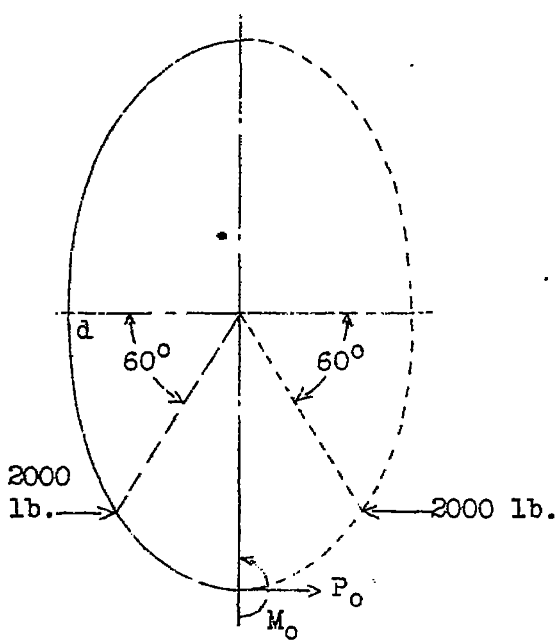
By inspection

$V_d = -192$ pounds

$P_d = 2{,}000$ pounds

$M_d = M_o + P_o a - 2{,}000\ (b - b \cos \Phi) = 1{,}450 + 192 \times 25 - 2{,}000\ (14.7 - 14.7 \times 0.697) = -2{,}650$ pound-inches

Case 3



From Figure 6

$\Phi = 45.8^\circ$

From Figure 4

$P_o = -0.92 \times 2{,}000 = -1{,}840$ pounds

From Figure 5

$M_o = 0.168 \times 25 \times 2{,}000 = 8{,}400$ pound-inches

Then:

By inspection

$V_d = -(-1{,}840) - 2{,}000 = -160$ pounds

$P_d = 0$ pound

$M_d = M_o + P_o a + 2{,}000\ (a \sin \Phi) = 8{,}400 - 1{,}840 \times 25 + 2{,}000\ (25 \times 0.717) = -1{,}760$ pound-inches

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Therefore the shear, thrust, and moment at the ends of the minor axis for the assumed problem are:

	Case 1		Case 2		Case 3		Total
V_d =	(99)	+	(-192)	+	(-160)	=	-253 pounds
P_d =	(0)	+	(2,000)	+	(0)	=	2,000 pounds
M_d =	(-1,490)	+	(-2,650)	+	(-1,760)	=	-5,900 pound-inches

In a similar manner the shear, thrust, and moment can be calculated at any point in the ring. By making a series of such calculations for a number of points, the maximum stress in the ring may be determined.

University of Michigan,

Ann Arbor, Mich., December 8, 1932.

REFERENCES

1. Miller, Roy A.: A Solution of the Circular Ring. Airway Age, May, 1931.

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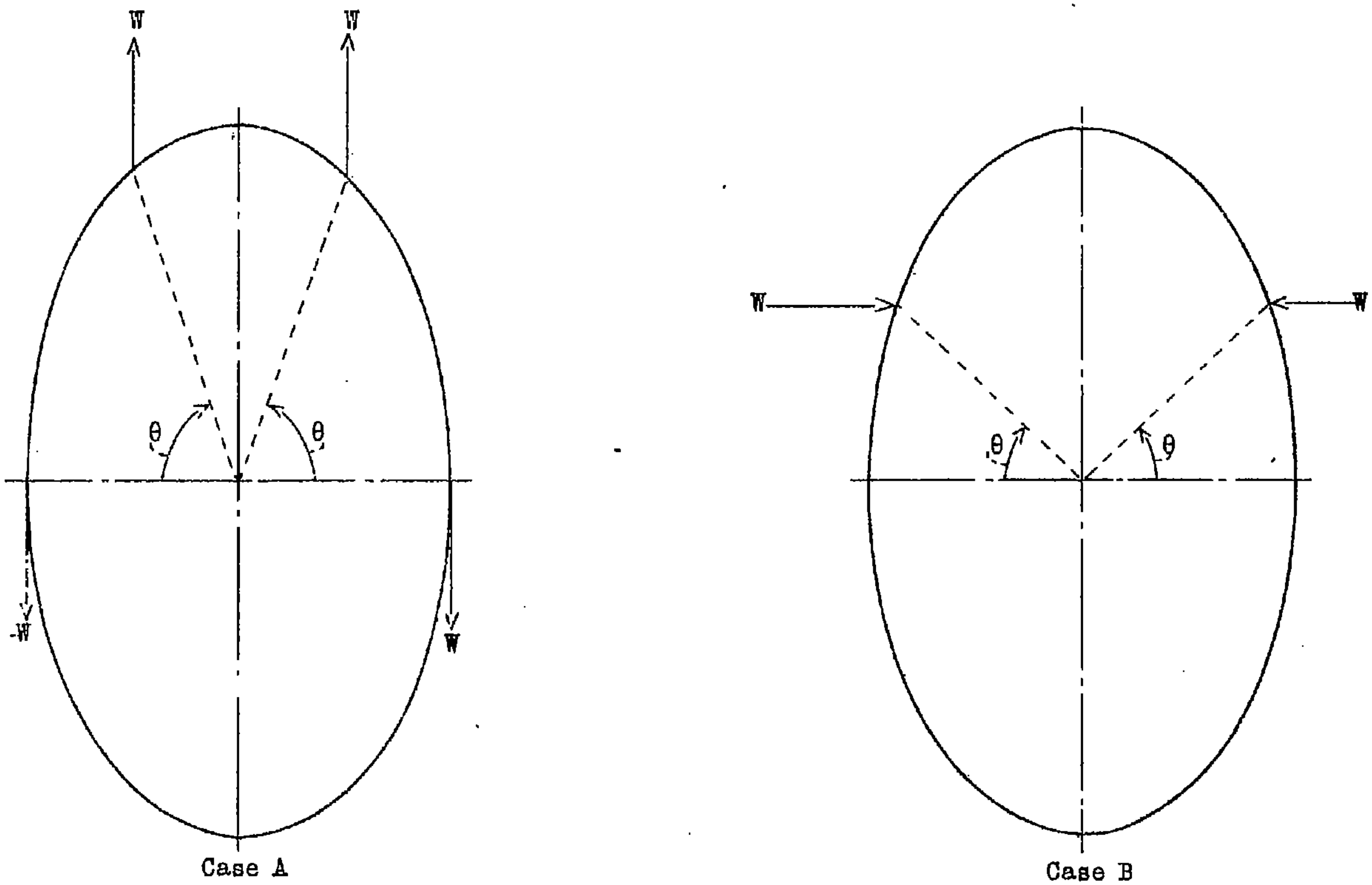

Figure 1.-The two simple loading conditions for which the working charts for the stress analysis of elliptic rings are constructed.

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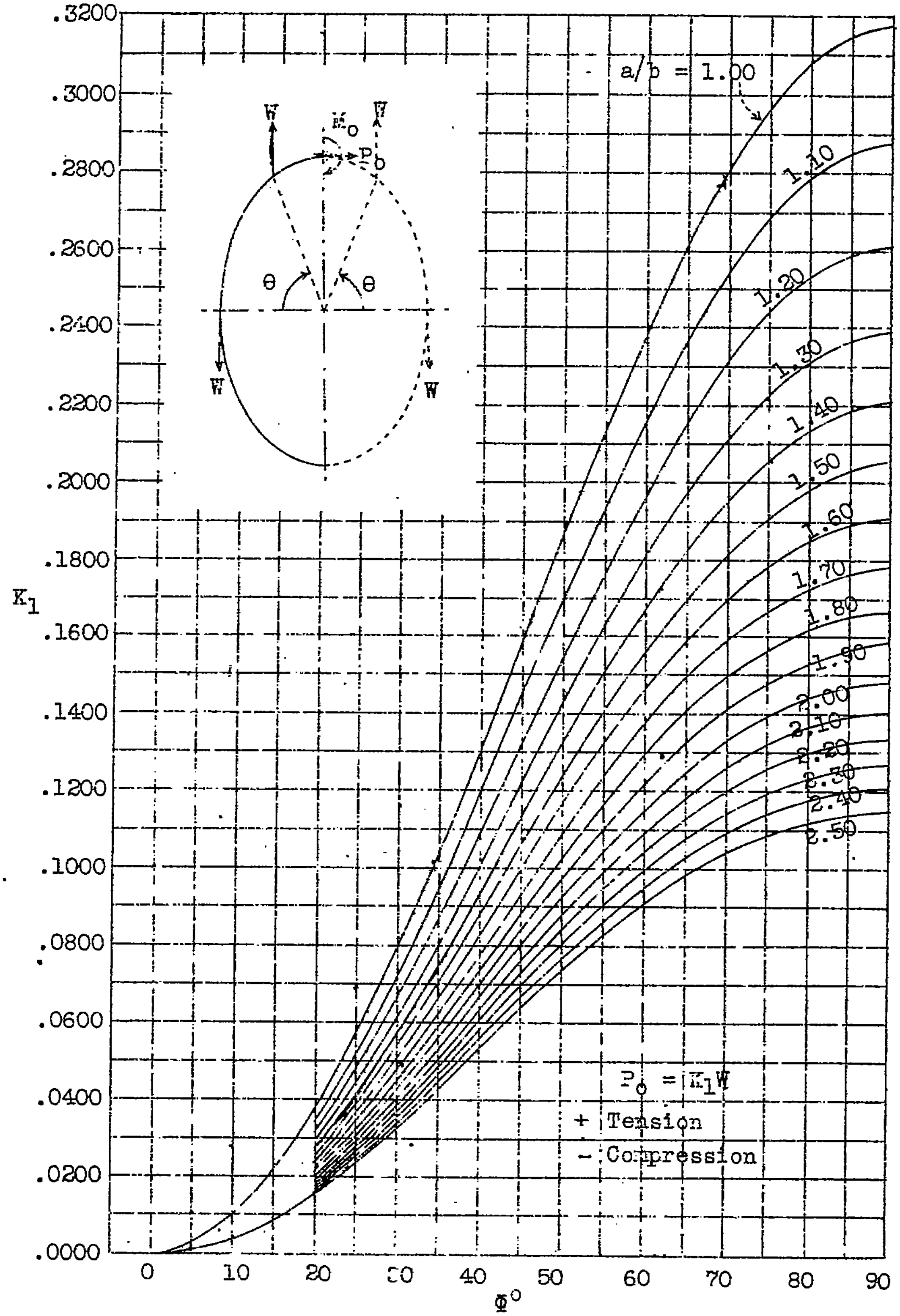

Figure 2.- Thrust coefficients for case A.

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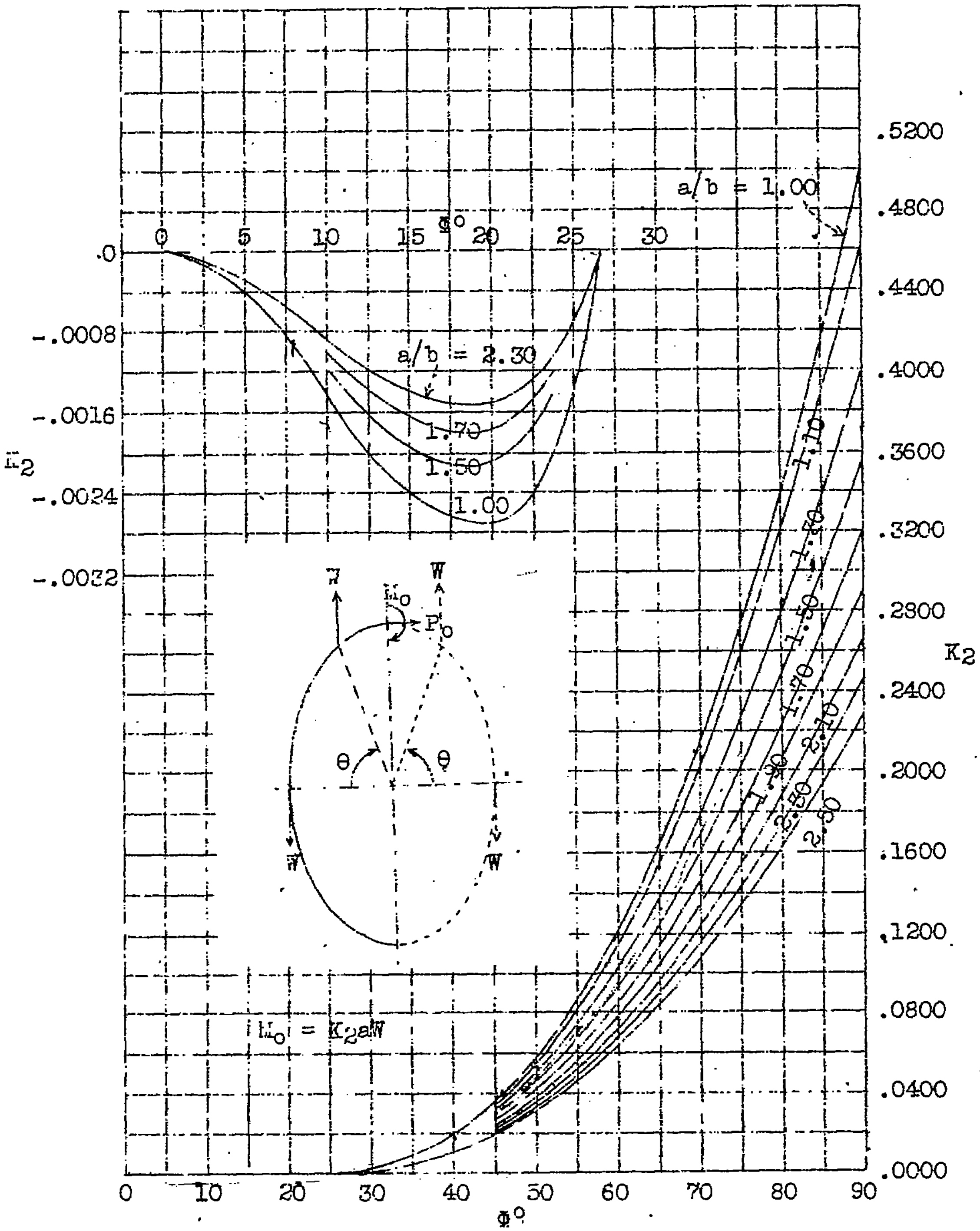

Figure 3.- Moment coefficients for case A.

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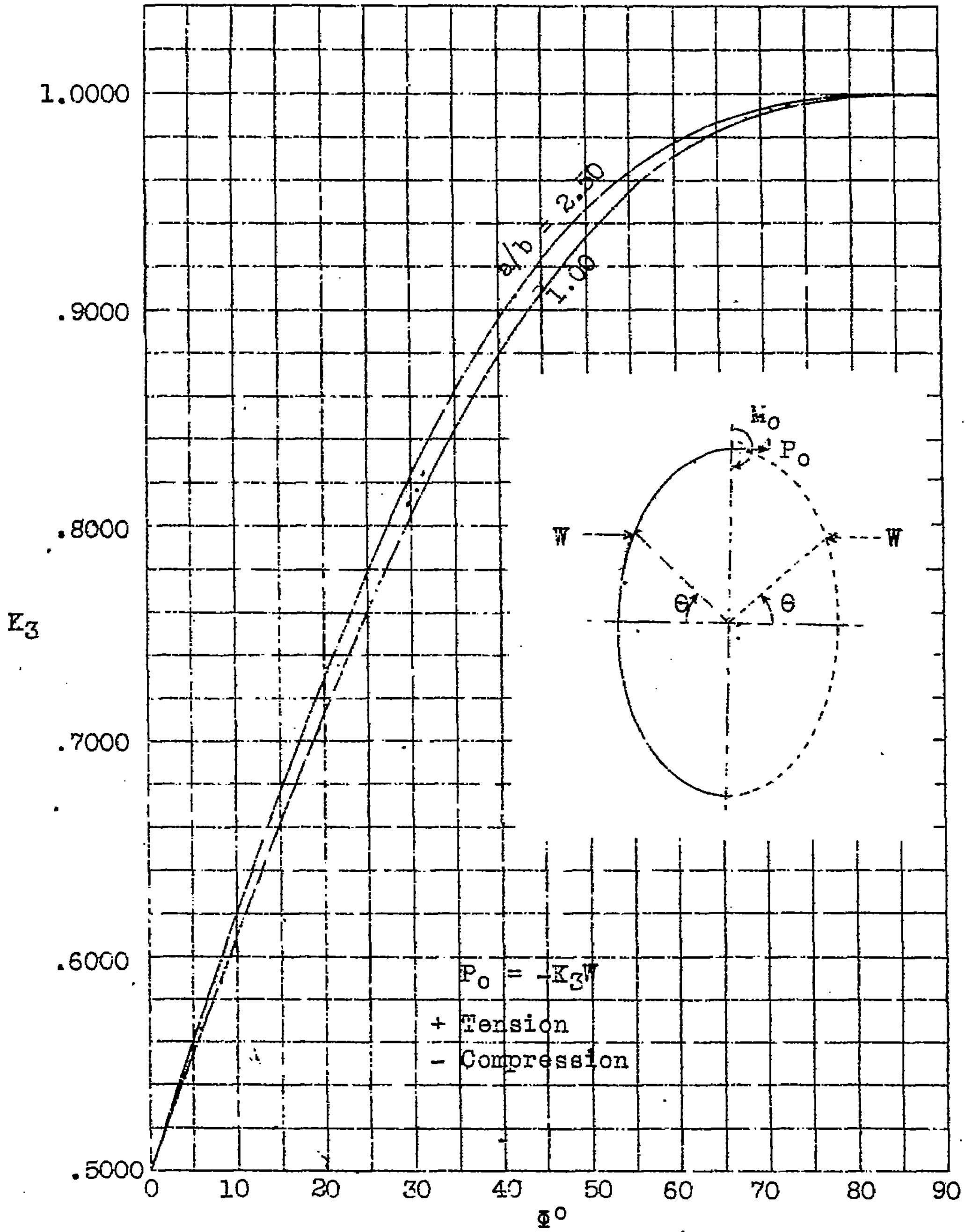

Figure 4.- Thrust coefficients for case B.

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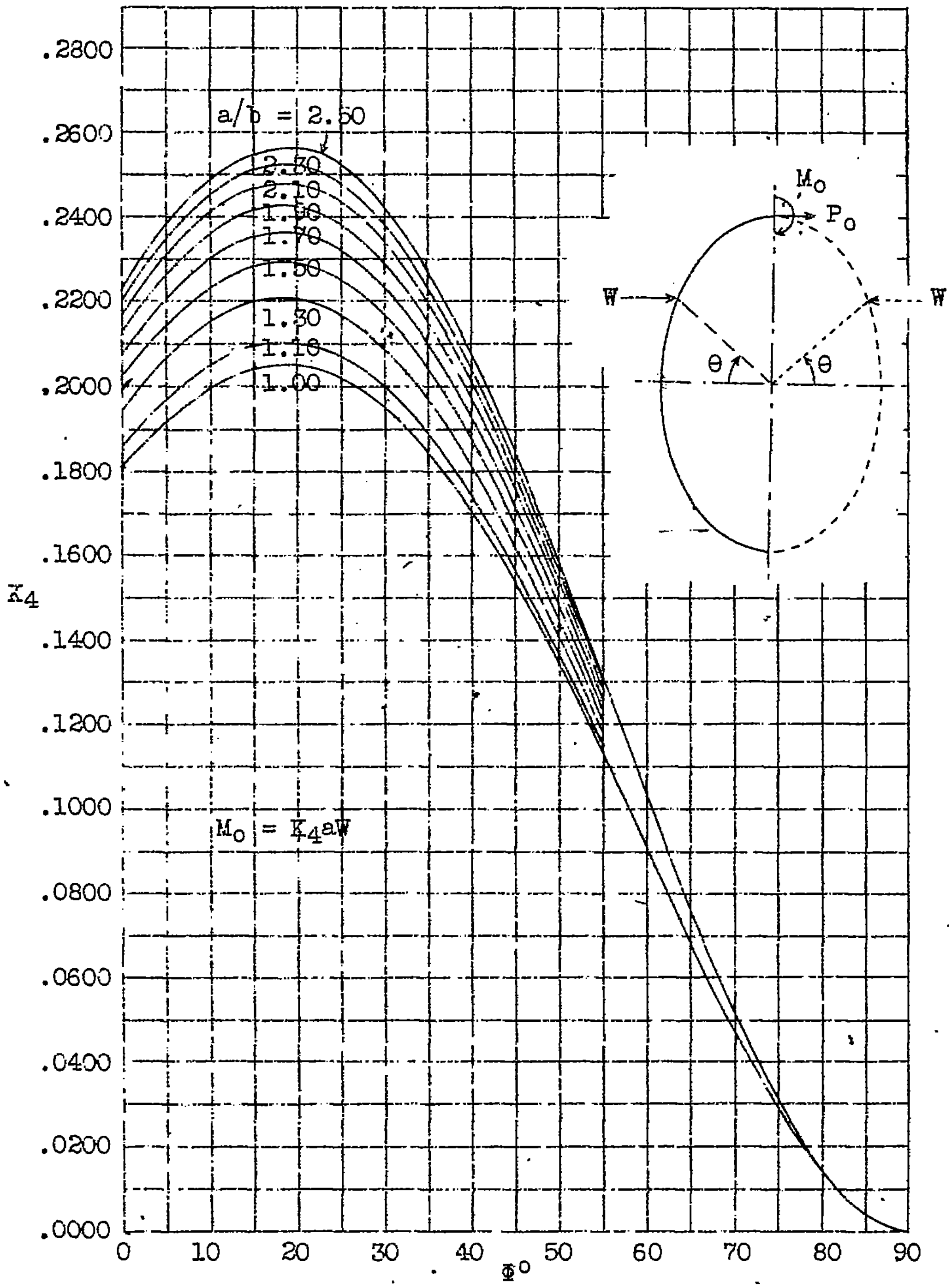

Figure 5.- Moment coefficients for case B.

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x = a sin ϕ
y = b cos ϕ

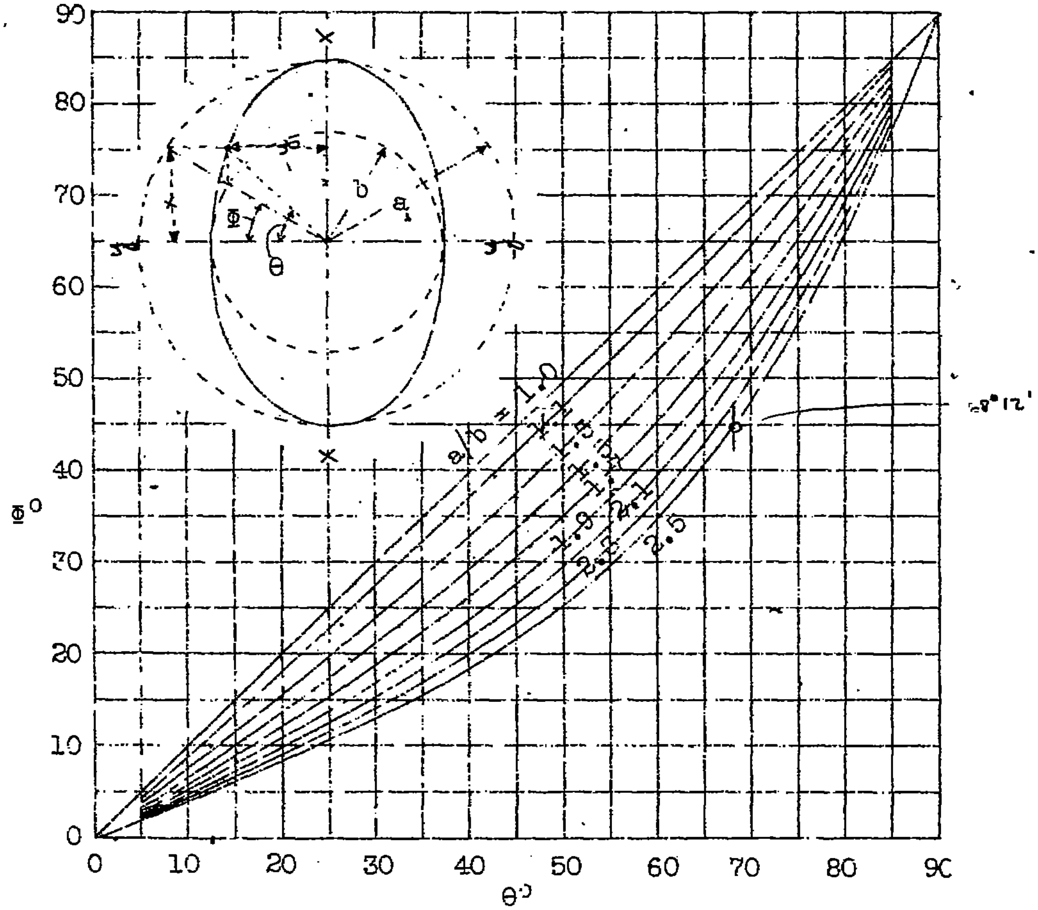

Figure 6.- Values of Φ.