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STRENGTH TESTS OF THIN-WALLED ELLIPTIC

DURALUMIN CYLINDERS IN PURE BENDING

AND IN COMBINED PURE BENDING AND TORSION

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SUMMARY

An analysis is presented of the results of tests
made by the Massachusetts Institute of Technology and by
the National Advisory Committee for Aeronautics on an
investigation of the strength of thin-walled circular and
elliptic cylinders in pure bending and in combined torsion
and bending. In each of the loading conditions, the bend-
ing moments were applied in the plane of the major axis of
the ellipse.

INTRODUCTION

The strength of thin-walled circular and elliptic
cylinders has been under investigation by the NACA for a
number of years. Papers have been published giving the
results of tests of thin-walled cylinders in torsion,
compression, pure bending, and combined transverse shear
and bending. The Massachusetts Institute of Technology
completed tests made under contract with the NACA of
cylinders under combined torsion and bending in the plane
of the major axis. The data were recorded in a thesis
written at M.I.T. in 1937 by W. E. Sangster and entitled,
"Strength Tests of Duralumin Cylinders in Pure Bending
and Bending Combined with Torsion." A report based on
the data was made by Prof. Joseph S. Newell to the NACA.

The M.I.T. data have now been combined with the NACA
data on thin-walled duralumin cylinders contained in refer-
ences 1, 2, and 3 and the enlarged group of data has been
analyzed.

MATERIAL

The material used in the M.I.T. tests was 17S-T aluminum alloy in sheet form with nominal thicknesses of 0.011 and 0.022 inch. The only important physical property of the material required for this paper is the modulus of elasticity E. The modulus of elasticity was determined by cutting a specimen from the undamaged portion of each cylinder after test and milling to the dimensions recommended by the American Society for Testing Materials for tensile tests on thin sheet. Extensometer tests with these specimens resulted in a value of 10.3×10^6 pounds per square inch for the modulus of elasticity.

The material used in the comparable NACA tests was 17S-T aluminum alloy in sheet form. The dimensions and properties of the sheet have been described in reference 1. In the tests of elliptic cylinders reported in reference 2, the nominal sheet thicknesses were identical with those of the sheets used in the M.I.T. tests, and the value of the modulus of elasticity E was 10.4×10^6 pounds per square inch. Thus, the material used in the two groups of tests was essentially the same.

SPECIMENS

The specimens used in the M.I.T. tests consisted of 48 thin-walled unstiffened cylinders, to which combined bending and torsional moments were applied, and of 32 similar cylinders, to which only pure bending moments were applied. Some of the cylinders were circular and others elliptical in cross section. The diameter of the circular cylinders was 15 inches; the major axis of all elliptical cylinders was 15 inches, and the minor axes of the elliptical cylinders were 12, 9, and 6 inches. The ratios of semiminor axis to semimajor axis b/a were thus 1.0, 0.8, 0.6, and 0.4. The lengths varied from 3.75 to 22.5 inches.

In the construction of the specimens, the sheet was cut approximately 1/16 inch shorter than the perimeter of the bulkheads. A butt strap 1 inch wide, having the same nominal thickness at the specimen, was used to complete the cylinder. A clearance of 1/16 inch between the butt strap and the bulkheads was allowed at each end. Care was taken to avoid injuring the specimens during their construction.

For b/a = 1.0, 0.8, and 0.6, the bulkheads used to
transmit the loads to the cylinders were those previously
used in NACA tests (references 1, 2, and 3). For those
specimens with b/a = 0.4 a new set of similar bulkheads
was made at M.I.T.

METHOD OF TESTING

In all cases the loads were applied to the bulkhead,
which in turn transmitted the load to the cylindrical
shell. The fact that the M.I.T. and the NACA test data
were in agreement where overlapping occurred indicates
that small differences in the method of applying the load
to the bulkheads were unimportant. In both the M.I.T. and
the NACA tests, loads were applied in increments of from
1 to 2 percent of the estimated load at failure.

COMPUTATION OF STRESSES

Bending.- The stress f_b at failure on the extreme
fiber of the cylinder due to the application of a bending
moment is assumed to be given by the formula

$$f_b = \frac{Ma}{I} \tag{1}$$

where

M   bending moment at failure

a   semimajor axis of ellipse

I   moment of inertia of cross section about minor axis

If the wall thickness t is small compared with the semi-
major and semiminor axes a and b, the moment of inertia
I for the thin-walled elliptic cross section is given by
the equation

$$I = \int y^2 t\,ds \tag{2}$$

where

y   coordinate along major axis

ds   elemental length of perimeter of elliptic shell

The result of the evaluation of this elliptic integral can be given in the form

$$I = k_b a\, t \qquad\qquad\qquad (3)$$

Values of k_b for specimens considered in this report are:

b/a	k_b
1.0	π
.8	2.68
.6	2.24
.4	1.82

Values of k_b for other ratios of b/a may be read from the graph in figure 7 of reference 2.

Torsion.- The shearing stress S at failure in the plane of the skin due to the application of a twisting moment is assumed to be given by the formula

$$S = \frac{T}{2At} \qquad\qquad\qquad (4)$$

where

T   twisting moment at failure

A   enclosed area of ellipse, πab

PURE BENDING TESTS

The original NACA data for pure bending are given in references 2 and 3. These data have been combined with the M.I.T. data and the combination has been analyzed to show the effect of the three ratios: L/a, b/a, and

a/t; these symbols have been defined previously except
for L, which is the length of the cylinder.

In the analysis for the effects of L/a and b/a,
the test data have been divided into groups according
to the nominal sheet thickness of the material used in
the construction of the specimens. Consequently, in
the figures where the data have been so grouped, part of
the scatter of test points is caused by small variations
in the sheet thickness from the nominal thickness for
that group. It is believed, however, that the improve-
ment in clearness of the presentation justifies this
grouping of the data.

Effect of the ratio L/a.- In order to show the
effect of the ratio L/a of elliptic cylinders upon f_b
under pure bending, figure 1 has been prepared. Inspec-
tion of this figure shows that, for the cylinders tested,
L/a is an unimportant factor in the strength of thin-
walled elliptic cylinders subjected to pure bending in
the plane of the major axis. Consequently, in the follow-
ing analysis of the bending strength of thin-walled cir-
cular and elliptic cylinders, the effect of L/a, and
hence of length, is not considered.

Effect of the ratio b/a.- In order to show how
f_b for an elliptic cylinder in pure bending is related
to f_b for a circumscribed circular cylinder of the same
wall thickness in pure bending, figure 2 has been prepared.
The fact that the M.I.T. and NACA data are in agreement
in this figure signifies that both sets of data are com-
parable in quality and represent similar care in the
execution of the tests.

The experimental points for each nominal value of
a/t in figure 2 show a wide scatter that is characteris-
tic of the strength of unstiffened shells in compression
or bending. The curves drawn in figure 2, which give
the lower limit of the test data, are assumed to estab-
lish f_b for pure bending. These values are used later
in this report in the analysis of the test data on com-
bined bending and torsion.

The curves of figure 2 show that, for any value of
a/t, the value of f_b for pure bending increases with
decrease in the ratio b/a. This increase in strength
with eccentricity of the elliptical shape evidently
arises from the increased stability of the more highly
curved portions of the cylinder wall at the ends of the

major axis where the bending stresses are highest. The
increase in f_b for pure bending with decrease in b/a
cannot be maintained indefinitely for, when $b/a = 0$,
the cylinder is a deep narrow beam that is laterally
unstable. The increase in f_b for pure bending as b/a
decreases from 0.6 to 0.4 is very nearly zero, indicating
the presence of a maximum value for f_b in this neighbor-
hood.

Effect of the ratio a/t.- In figures 1 and 2 the
values of f_b for pure bending are plotted without regard
to the small effects of sheet thickness. This procedure
was adopted in order that the data could be handled con-
veniently in groups, showing the effect of the ratio L/a
in figure 1 and the effect of the ratio b/a in figure 2.

Although the ratio L/a has no important effect
upon f_b for pure bending, the effect of the ratio b/a
is large and cannot be neglected. It therefore becomes
necessary to consider the effect of thickness upon the
strength of the elliptic cylinders at a given value of
b/a. The circular cylinder, for which $b/a = 1$, has
been extensively studied and is therefore the most logical
value of b/a for study of the effect of the ratio a/t.
Thus, figure 3 has been prepared from the combined M.I.T.
and NACA data. In this figure f_b/E for pure bending
is plotted against the ratio a/t.

If sufficient data were available, other figures
similar to figure 3 could be prepared for other values
of b/a. In the absence of such data it is necessary
to regard figure 3 as establishing a relationship between
the ratio a/t and the value of f_b for pure bending
of a circular cylinder. The increments by which these
values of f_b should be increased in order to obtain
values of f_b for pure bending of elliptic cylinders can
then be established by referring to a figure such as
figure 2, which correlates the bending strength of an
elliptic cylinder with the bending strength of a circular
cylinder of the same a/t ratio.

The curves A, B, and C in figure 3 are the curves
given in figures 7 and 9 of reference 4, which presents
the NACA study of circular cylinders in compression.
Curves A and B, respectively, are the graphs of the
equations

$$\frac{f_b}{E} = 0.605 \frac{t}{a} \qquad \text{and} \qquad \frac{f_b}{E} = 0.363 \frac{t}{a}$$

The plotted points in figures 7 and 9 of reference 4
representing the compressive strength of carefully con-
structed cylinders scattered between curves B and C.

In figure 3 the plotted points for cylinders in pure
bending fall between curves B and C at large values of
a/t and above curve B at small values of a/t. This
trend toward higher values of the stress modulus ratio
for pure bending as compared with compression must be
ascribed to effects caused by changes in the thickness,
because the radius a was held constant in the tests
at 7.5 inches. Slippage of the thickest walled cylinders
on the bulkhead during test has been suggested as a
probable explanation of this trend. Inspection of the
plotted data in figure 3 shows that the experimental
points for the M.I.T. and the NACA data are in agreement.
Consequently, if any slippage occurred in one set of
tests, it must also have occurred in the other set in
such a manner as to give substantially identical strengths.
The experimental points in figure 3 actually represent
the true bending strength of the cylinders tested. Whether
or not a different method for bringing the loads into the
skin from the bulkheads would give the same values of
f_b/E is a matter for future research.

TORSION TESTS

The strength of thin-walled circular and elliptic
cylinders in torsion is well established by the tests
reported in references 1, 2, and 5. In reference 2 it
was concluded that the shearing stress at failure for a
thin-walled elliptic cylinder is the same as the shearing
stress at failure for a circumscribed circular cylinder
of the same length and wall thickness, when values of
b/a lie between 0.6 and 1.0. It is reasonable to assume
that this conclusion also holds for elliptic cylinders
with b/a = 0.4 in the absence of torsion tests on such
cylinders.

In reference 2 it was recommended that, for general
design purposes, the strength of elliptic cylinders in
torsion should be calculated on the basis of a circum-
scribed circular cylinder of the same length and wall
thickness by use of Donnell's design formula of refer-
ence 5. This formula is, for the cylinders of this

report,

$$S = \frac{0.6}{1-\mu^2} E\left(\frac{t}{L}\right)^2 \left[4.6 + \sqrt{7.8 + 1.67\left(\sqrt{1-\mu^2}\ \frac{L^2}{2at}\right)^{3/2}}\right] \qquad (5)$$

where μ is Poisson's ratio for the material.

COMBINED PURE BENDING AND TORSION TESTS

In order to correlate the data obtained from the M.I.T. tests on thin-walled elliptic cylinders under a combination of torsion and pure bending in the plane of the major axis with the data on pure bending alone and torsion alone, the following stress ratios have been computed:

R_b ratio of bending stress at failure in combined pure bending and torsion to bending stress at failure in pure bending alone

R_s ratio of shearing stress at failure in combined pure bending and torsion to shearing stress at failure in torsion alone

In the calculation of these stress ratios for the cylinders tested, the values of the bending stress at failure for pure bending alone are assumed to be given by the curves of figure 2 and the values of the shearing stress at failure for torsion alone are assumed to be given by equation (5).

The results of all the combined pure bending and torsion tests, as well as the results of the tests with pure bending alone and with torsion alone, are plotted in figure 4 with R_s as abscissa and R_b as ordinate. This method of plotting data on combined loading was used by Bridget, Jerome, and Vosseller in reference 6 and by Shanley and Ryder in reference 7.

Inspection of figure 4 shows that the data for combined torsion and bending scatter as widely as the data for torsion alone plotted along the axis of abscissa and for pure bending alone plotted along the axis of ordinates. The entire group of data therefore establishes a broad band that is characteristic of unstiffened shells in most loading conditions.

The data plotted in figure 4 lie almost entirely
within the band established by two concentric circles
about the origin as center with radii of 1.0 and 1.6.
The circle with a radius 1.0 establishes the allowable
stresses for circular and elliptic thin-walled cylinders
in combined torsion and pure bending in the plane of the
major axis. The equation of this circle is

$$R_s^2 + R_b^2 = 1 \tag{6}$$

This relation was also found by Shanley and Ryder to
apply for combined torsion and bending of round steel
tubing. (See fig. 2 of reference 7.) This same formula
has also been recently confirmed by the National Bureau
of Standards for combined torsion and bending of round
tubing.

CONCLUSIONS

1. The ratio of length to semimajor axis of the
ellipse is an unimportant factor in the strength of
thin-walled elliptic cylinders subjected to pure bending
in the plane of the major axis.

2. For any ratio of semimajor axis to wall thick-
ness, the bending stress on the extreme fiber at failure
for pure bending of thin-walled elliptic cylinders in
the plane of the major axis increases with the eccen-
tricity of the elliptical shape. This increase in
strength with eccentricity evidently arises from the
increased stability of the more highly curved portions
of the cylinder wall at the ends of the major axis where
the bending stresses are highest. As the ellipse becomes
more and more eccentric the increase in strength cannot
be maintained indefinitely, for in the limit the cylinder
becomes a deep narrow beam that is laterally unstable.
Data indicate that the strength in pure bending is a
maximum when the ratio of minor to major axis is in the
neighborhood of 0.4 to 0.6.

3. For the case of circular cylinders, the relation
between the ratio of the bending stress at failure to
the modulus of elasticity and the ratio of radius to
thickness is established by a chart of f_b/E against a/t.

4.   It was found that thin-walled elliptic cylinders subjected to combined torsion and pure bending in the plane of the major axis will fail when the stress ratio satisfy the equation

$$R_s{}^2 + R_b{}^2 = 1$$

where

R_b ratio of bending stress at failure in combined pure
        bending and torsion to bending stress at failure
        in pure bending alone

R_s ratio of shearing stress at failure in combined
        pure bending and torsion to shearing stress at
        failure in pure torsion alone

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    National Advisory Committee for Aeronautics
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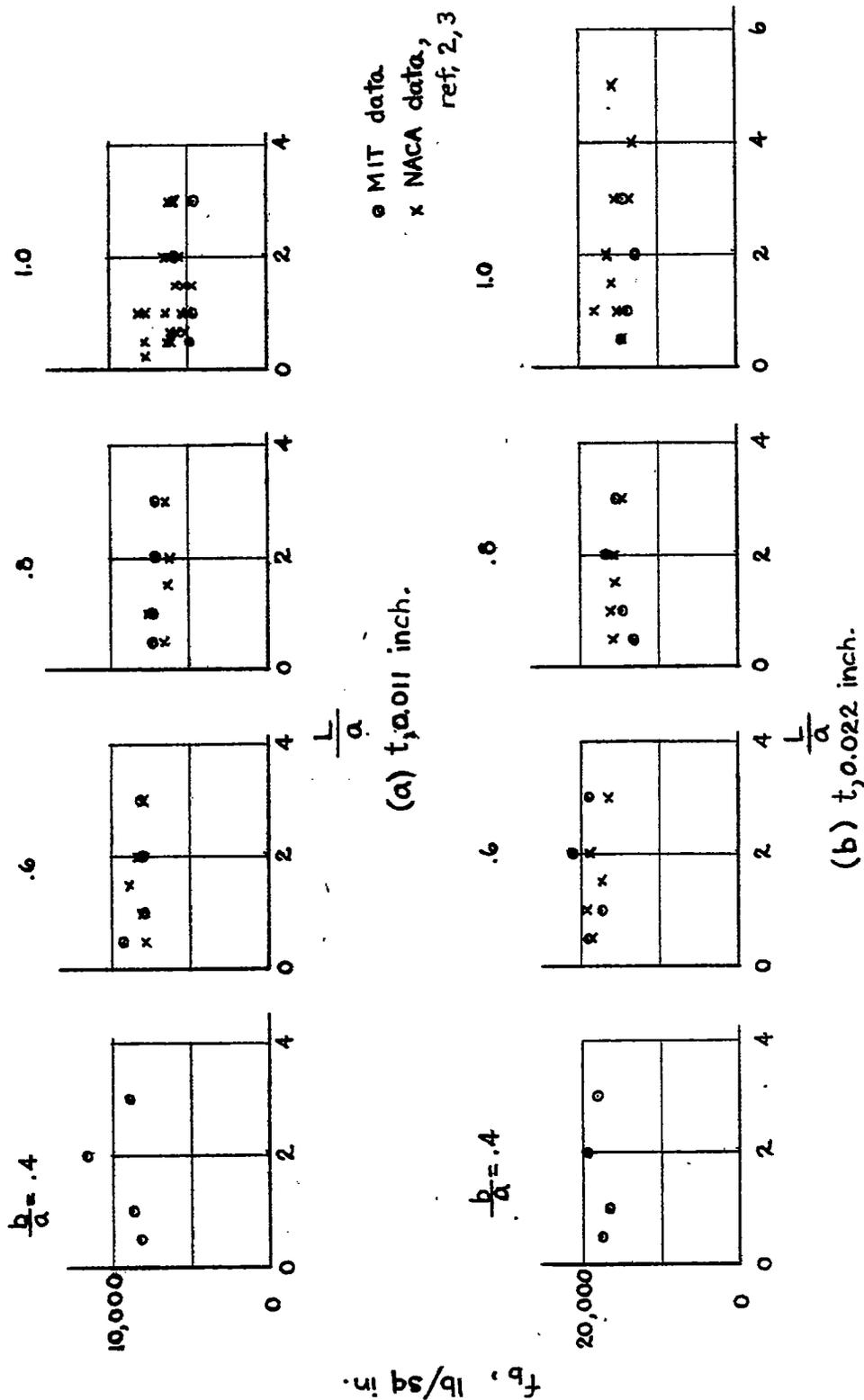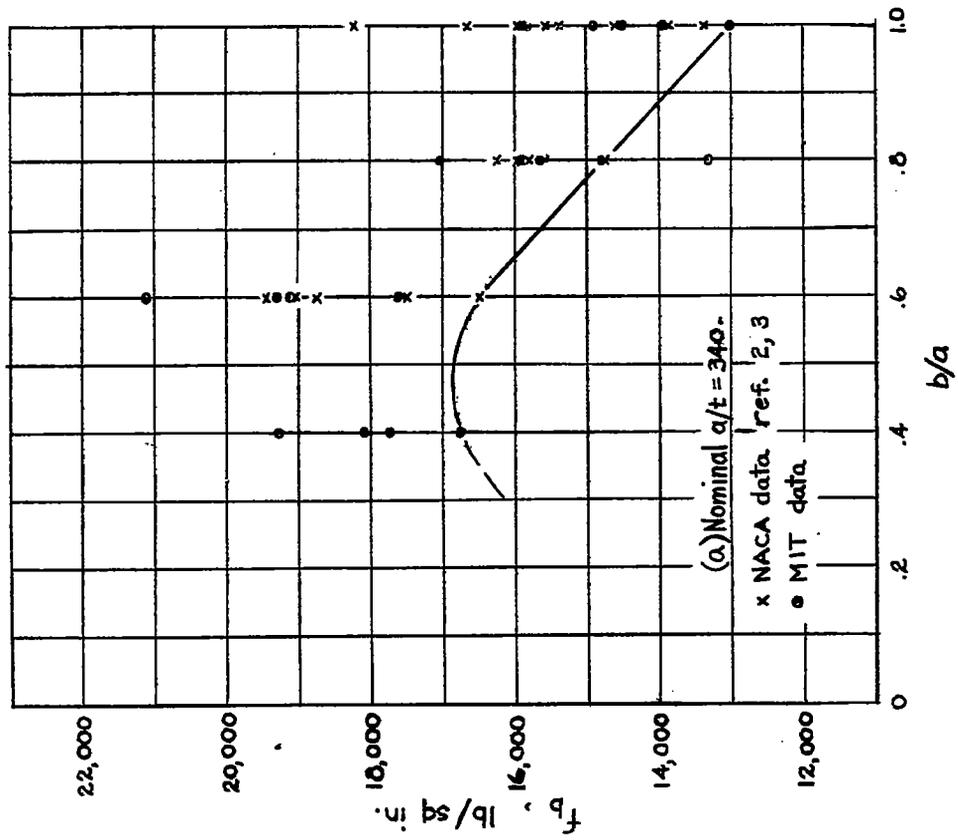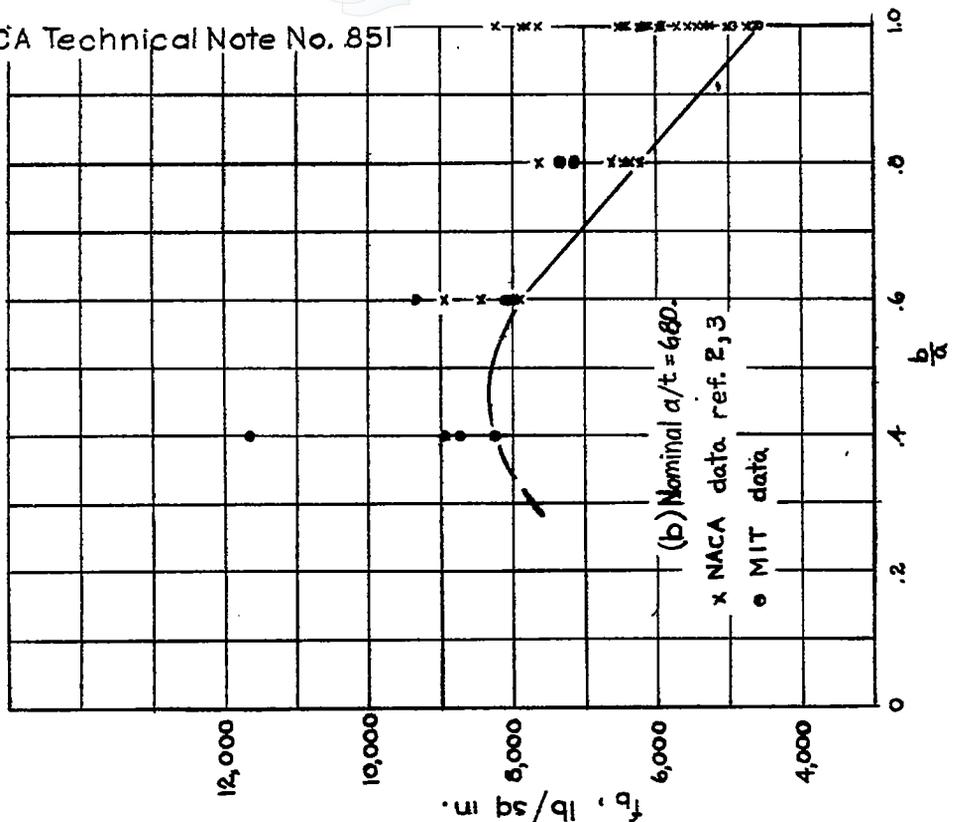   vol. 36, no. 6, June 1937, pp. 28, 29, 43, 66-70.

Figure 1. - Effect of the ratio L/a upon the bending stress f_b at failure of thin-walled elliptic cylinders under pure bending in the plane of the major axis.

Fig. 2

(b) Nominal $a/t = 680$.

x NACA data ref. 2,3
• MIT data

f_b, lb/sq in.

(a) Nominal $a/t = 340$.

x NACA data ref. 2,3
• MIT data

f_b, lb/sq in.

b/a

Figure 2.– Effect of the ratio b/a upon the bending stress at failure f_b of thin–walled elliptic cylinders under pure bending in the plane of the major axis.
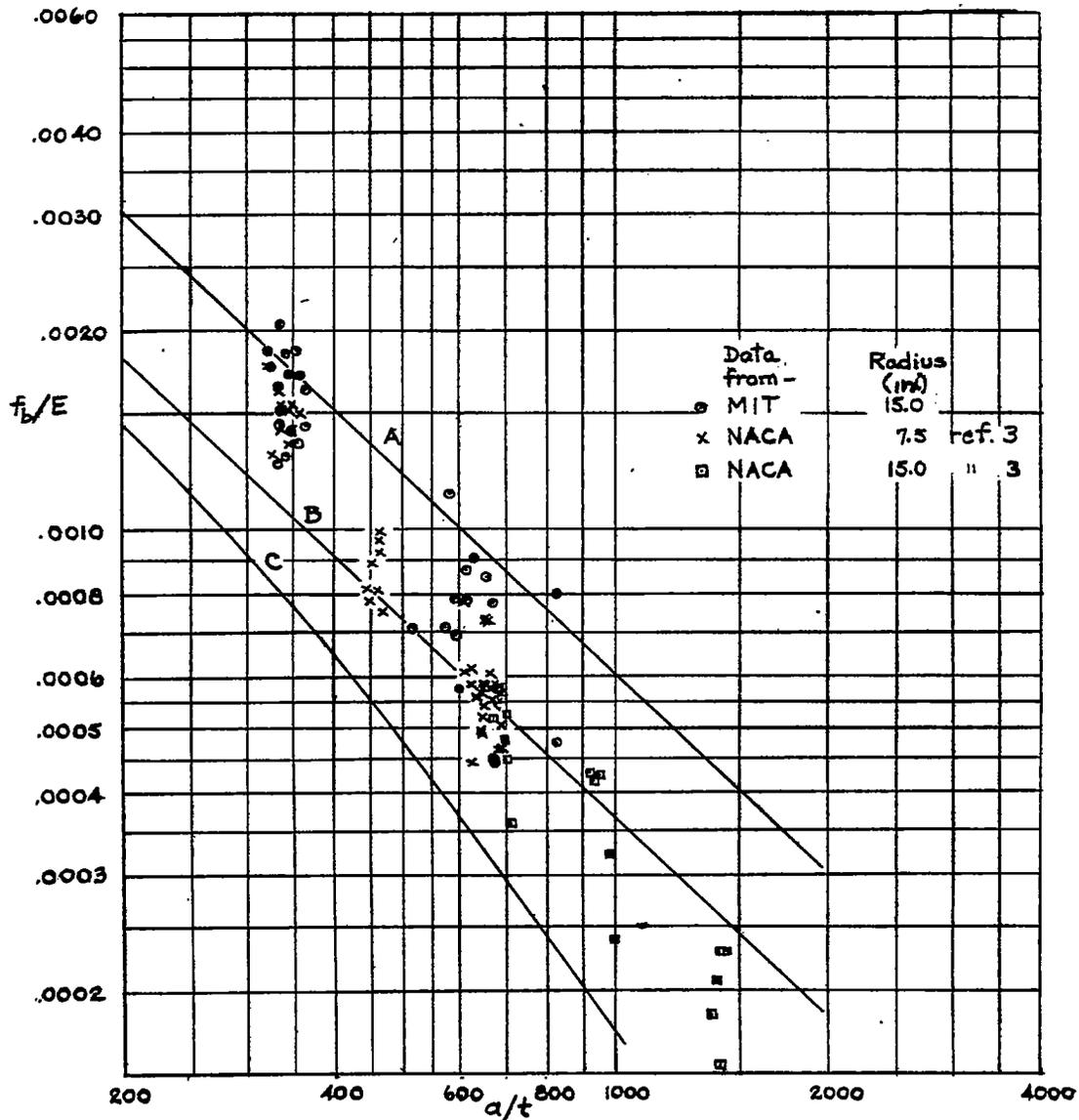
Figure 2.–Concluded.

Figure 3. – Effect of the ratio a/t upon the value of f_b/E at failure
for thin-walled circular cylinders under pure bending.
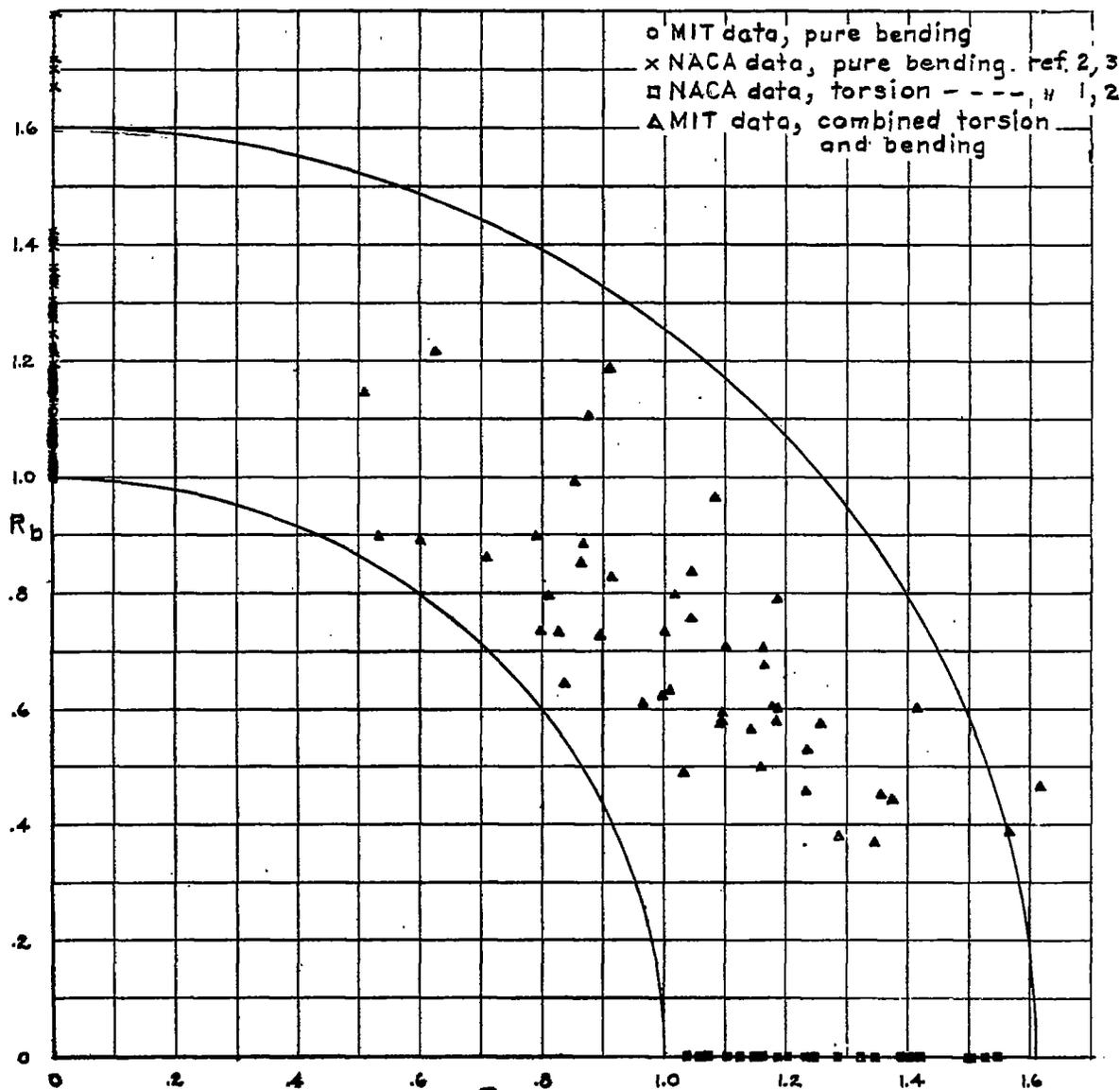(Curves A, B and C from figure 7, reference 3.)

Figure 4.—
Strength of thin-walled elliptic cylinders in combined torsion and pure bending in the plane of the major axis.