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LARGE-DEFLECTION THEORY OF CURVED SHEET

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SUMMARY

The equations are given for the elastic behavior of initially curved sheet in which the deflections are not small in comparison with the thickness but at the same time small enough to justify the use of simplified formulas for curvature. These equations are solved for the case of a sheet with circular cylindrical shape simply supported along two edges parallel to the axis of the generating cylinder.

Numerical results are given for three values of the curvature and for three ratios of buckle length to buckle width. The computations are carried to buckle deflections of about twice the sheet thickness.

It is concluded from the results that initial curvature may cause an appreciable increase in the buckling load but that, for edge strains which are several times the buckling strain, the initial curvature causes a negligibly small change in the effective width.

INTRODUCTION

The strength of curved sheet in axial compression plays an important role in determining the strength of the wings and fuselage of modern metal airplanes. Such sheet is usually reinforced by stringers. The portion of sheet between any pair of these stringers can, for purposes of computation, be considered as simply supported at the stringers. A study of the limiting case of flat sheet has already been made (reference 1). The present paper is an extension of this work to include the effects of initial curvature.

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FUNDAMENTAL EQUATIONS

NOTATION

An initially curved plate of uniform thickness will be considered. The following notation is used:

w_o initial displacement of points of the middle surface normal and relative to a plane through the corners of the plate

w additional displacement of points of the middle surface due to the applied loads

x, y coordinate axes

F stress function

q normal pressure

E Young's modulus

h plate thickness

μ Poisson's ratio

$D = \dfrac{Eh^3}{12(1-\mu^2)}$, flexural rigidity of the plate

σ_x, σ_y, τ_{xy} extreme-fiber stresses

σ_x', σ_y', τ_{xy}' median-fiber stresses

σ_x'', σ_y'', τ_{xy}'' extreme-fiber bending stresses

ϵ_x', ϵ_y', γ_{xy}' median-fiber strains

$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right)$, bending moment about y-axis

$w_{m,n}$ coefficient in expansion for displacement normal to the reference plane due to load

b width of cylindrical plate strip

$2a$ wave length of buckles

m, n, r, s subscripts representing integers

a_n coefficient in expansion for initial displacement normal to the reference plane

$\overline{p}_x, \overline{p}_y$ average compressive stresses

$b_{m,n}$ coefficient in expansion for F

$B_1, B_2, \ldots B_{12}$ coefficients in expansion for $b_{m,n}$

$q_{m,n}$ coefficient in expansion for q

$Q_1, Q_2, \ldots Q_{12}$ coefficients in expansion for $q_{m,n}$

R radius of curvature of plate

ϵ edge strain

ϵ_{cr} edge strain for buckling

Equations for the Deformation of Curved Plates

The fundamental equations governing the deformation of thin curved plates were developed by Donnell in reference 2 and by Marguerre in reference 3. These equations are also given by von Kármán and Tsien in reference 4. For the special case where the deflections are not small in comparison with the thickness, but at the same time small enough to justify the application of simplified formulas for curvatures of the plate, Marguerre (equations (4.3_3) and (4.5_1) of reference 3) gives these equations in essentially the following form:

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w_o}{\partial x \partial y} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_o}{\partial y^2} - \frac{\partial^2 w_o}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \quad (1)$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} + \frac{h}{D}\left[\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 (w + w_o)}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 (w + w_o)}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 (w + w_o)}{\partial x \partial y}\right] \quad (2)$$

Equation (2) is also given by Timoshenko (p. 319, equation (206) of reference 5).

The median-fiber stresses are

$$\sigma_x' = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y' = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy}' = -\frac{\partial^2 F}{\partial x \partial y} \quad (3)$$

and the median-fiber strains are

$$\left.\begin{aligned} \epsilon_x' &= \frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \mu\frac{\partial^2 F}{\partial x^2}\right) \\ \epsilon_y' &= \frac{1}{E}\left(\frac{\partial^2 F}{\partial x^2} - \mu\frac{\partial^2 F}{\partial y^2}\right) \\ \gamma_{xy}' &= -\frac{2(1 + \mu)}{E}\frac{\partial^2 F}{\partial x \partial y} \end{aligned}\right\} \quad (4)$$

The extreme-fiber bending and shearing stresses are

$$\left.\begin{aligned}
\sigma_x'' &= -\frac{Eh}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \mu\,\frac{\partial^2 w}{\partial y^2}\right)\\
\sigma_y'' &= -\frac{Eh}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \mu\,\frac{\partial^2 w}{\partial x^2}\right)\\
\tau_{xy}'' &= -\frac{Eh}{2(1+\mu)}\,\frac{\partial^2 w}{\partial x \partial y}
\end{aligned}\right\} \qquad (5)$$

Plate Strip with Cylindrical Initial Shape

If we limit the following discussion to the case of a plate strip of infinite length of initial cylindrical shape having deflections w_0 small enough to allow the use of simplified formulas for curvature and let the generators be parallel to the x-axis, then

$$\frac{\partial^2 w_0}{\partial x^2} = 0\,; \qquad \frac{\partial^2 w_0}{\partial x \partial y} = 0 \qquad (6)$$

Substituting equations (6) in equations (1) and (2) gives

$$\frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w_0}{\partial y^2}\right] \qquad (7)$$

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

$$= \frac{q}{D} + \frac{h}{D}\left(\frac{\partial^2 F}{\partial y^2}\,\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - 2\,\frac{\partial^2 F}{\partial x \partial y}\,\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\,\frac{\partial^2 w_0}{\partial y^2}\right) \qquad (8)$$

Simply Supported Cylindrical Plate Strip

General Solution

A solution of equations (7) and (8) for a simply supported cylindrical plate strip of infinite length must satisfy the following boundary conditions. The deflection and the edge bending moment per unit length are zero at the edges of the strip; that is, when $y = 0$ or $y = b$

$$w = 0$$

and

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right) = 0$$

These conditions are satisfied by the Fourier series

$$w = \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=1,2,3\ldots}^{\infty} w_{m,n} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{9}$$

The initial displacement must be a function of y alone since the elements of the cylindrical surface are parallel to the x-axis. The xy-plane can be so chosen (see fig. 1) that the initial deflection is zero along the edges $y = 0$ and $y = b$. These conditions are satisfied by the Fourier series

$$w_o = \sum_{n=1,2,\ldots}^{\infty} a_n \sin\frac{n\pi y}{b} \tag{10}$$

By substitution equation (7) is found to be satisfied if

$$F = -\frac{\overline{p}_x y^2}{2} - \frac{\overline{p}_y x^2}{2}$$

$$+ \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=0,1,2\ldots}^{\infty} b_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{11}$$

where $\bar{p}_x$ and $\bar{p}_y$ are equal to the average compressive stress in the x and y directions, respectively, and where

$$b_{m,n} = \frac{E}{4\left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)^2} (B_1 + B_2 + \ldots + B_{12}) \qquad (11a)$$

and

$$B_1 = \sum_{r=0}^{\infty} \sum_{s=1}^{n-1} \left[(r+m)(n-s)rs + (r+m)^2 s^2\right] w_{(r+m),(n-s)} w_{r,s}$$

$$B_2 = \sum_{r=0}^{\infty} \sum_{s=1}^{n-1} \left[r(n-s)(r+m)s + r^2 s^2\right] w_{r,(n-s)} w_{(r+m),s}$$

if $m \neq 0$

$B_2 = 0$, if $m = 0$

$$B_3 = \sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[(r+m)(s+n)rs - (r+m)^2 s^2\right] w_{(r+m),(s+n)} w_{r,s}$$

$$B_4 = \sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[(r+m)sr(s+n) - (r+m)^2 (s+n)^2\right] w_{(r+m),s} w_{r,(s+n)}$$

if $n \neq 0$

$B_4 = 0$, if $n = 0$

$$B_5 = \sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[r(s+n)(r+m)s - r^2 s^2\right] w_{r,(s+n)} w_{(r+m),s}$$

if $m \neq 0$

$B_5 = 0$, if $m = 0$

$$B_6 = \sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[rs(r+m)(s+n) - r^2(s+n)^2 \right] w_{r,s} w_{(r+m),(s+n)}$$

if $m \neq 0$ and $n \neq 0$

$B_6 = 0$, if $m = 0$ or $n = 0$

$$B_7 = \sum_{r=0}^{m} \sum_{s=1}^{n-1} \left[-rs(m-r)(n-s) + r^2(n-s)^2 \right] w_{r,s} w_{(m-r),(n-s)}$$

$$B_8 = \sum_{r=0}^{m} \sum_{s=1}^{\infty} \left[-r(s+n)(m-r)s - r^2 s^2 \right] w_{r,(s+n)} w_{(m-r),s}$$

$$B_9 = \sum_{r=0}^{m} \sum_{s=1}^{\infty} \left[-rs(m-r)(s+n) - r^2(s+n)^2 \right] w_{r,s} w_{(m-r),(s+n)}$$

if $n \neq 0$

$B_9 = 0$, if $n = 0$

$$B_{10} = \sum_{s=1}^{\infty} \left[-2m^2 s^2 \right] w_{m,(s+n)} a_s, \quad \text{if } n \neq 0$$

$B_{10} = 0$, if $n = 0$

$$B_{11} = \sum_{s=1}^{\infty} \left[-2m^2(s+n)^2 \right] w_{m,s} \, a_{(s+n)}$$

$$B_{12} = \sum_{s=1}^{n-1} \left[2m^2(n-s)^2 \right] w_{m,s} \, a_{(n-s)}$$

By substitution of equations (9), (10), and (11) in equation (8), equation (8) is found to be satisfied if

$$q = \sum_{m=0,1,2,\ldots}^{\infty} \quad \sum_{n=1,2,\ldots}^{\infty} q_{m,n} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad (12)$$

where

$$q_{m,n} = D\, w_{m,n} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 - \overline{p}_x h\, w_{m,n}\, m^2 \frac{\pi^2}{a^2}$$

$$- \overline{p}_y\, h\, w_{m,n}\, n^2 \frac{\pi^2}{b^2} - \frac{h\pi^4}{4a^2b^2}\left(Q_1 + Q_2 + \ldots + Q_{12}\right) \qquad (12a)$$

and

$$Q_1 = \sum_{r=0}^{m} \sum_{s=1}^{n} \left[r(n - s) - s(m - r)\right]^2 w_{r,s}\, b_{(m-r),(n-s)}$$

$$Q_2 = \sum_{r=0}^{m} \sum_{s=1}^{\infty} \left[rs + (s + n)(m - r)\right]^2 w_{r,(s+n)}\, b_{(m-r),s}$$

$$Q_3 = -\sum_{r=0}^{m} \sum_{s=1}^{\infty} \left[r(s + n) + s(m - r)\right]^2 w_{r,s}\, b_{(m-r),(s+n)}$$

$$Q_4 = \sum_{r=0}^{\infty} \sum_{s=1}^{n} \left[(r + m)(n - s) + sr\right]^2 w_{(r+m),s}\, b_{r,(n-s)}$$

$$Q_5 = \sum_{r=0}^{\infty} \sum_{s=1}^{n} \left[r(n - s) + s(r + m)\right]^2 w_{r,s}\, b_{(r+m),(n-s)}$$

if $m \neq 0$

$Q_5 = 0$, if $m = 0$

$$Q_6 = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \left[(r + m)s - (s + n)r\right]^2 w_{(r+m),(s+n)}\, b_{r,s}$$

$$Q_7 = -\sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[(r + m)(s + n) - sr\right]^2 w_{(r+m),s}\, b_{r,(s+n)}$$

$$Q_8 = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \left[rs - (s+n)(r+m)\right]^2 w_{r,(s+n)}\, b_{(r+m),s}$$

if $m \neq 0$

$$Q_8 = 0, \quad \text{if} \quad m = 0$$

$$Q_9 = -\sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[r(s+n) - s(r+m)\right]^2 w_{r,s}\, b_{(r+m),(s+n)}$$

if $m \neq 0$

$$Q_9 = 0, \quad \text{if} \quad m = 0$$

$$Q_{10} = \sum_{s=1}^{n} \left[2m^2 s^2\right] a_s\, b_{m,(n-s)}$$

$$Q_{11} = \sum_{s=0}^{\infty} \left[2m^2 (s+n)^2\right] a_{(s+n)}\, b_{m,s}$$

$$Q_{12} = -\sum_{s=1}^{\infty} \left[2m^2 s^2\right] a_s\, b_{m,(n+s)}$$

Equation (12) represents a doubly infinite family of equations. In each equation the coefficients $b_{m,n}$ may be replaced by their values as given by equation (11a). The resulting equations will involve the known normal pressure coefficients $q_{m,n}$, the known average membrane pressures in the x- and y-directions $\overline{p}_x$ and $\overline{p}_y$, respectively, the known initial deflection coefficients a_n, and the unknown deflection coefficients $w_{m,n}$. The number of these equations is equal to the number of unknown deflection coefficients $w_{m,n}$.

In the following solutions the number of equations of the family of equation (12) was restricted by setting all deflection coefficients $w_{m,n}$ equal to zero except a selected number including the most important coefficients. This simplification introduces an error into the solution. The magnitude of the error will be estimated from the convergence as additional coefficients are used in the solution.

The resultant load must be constant in the x- and in the y-direction and the boundaries of the plate must remain straight. The first condition follows from the substitution of equations (3) and (11) in the following expressions for the total load:

$$\left.\begin{aligned}\text{Load in x-direction} &= \int_0^b h\sigma_x' \, dy = -\bar{p}_x bh \\ \text{Load in y-direction} &= \int_{-a}^{+a} h\sigma_y' \, dx = -2\bar{p}_y ah\end{aligned}\right\} \quad (13)$$

The second condition was checked by the substitution of equations (4), (9), and (11) in the following equations:

Displacement of edges in x-direction over a full wave-

$$\text{length} = \int_{-a}^{a} \left[\epsilon_x' - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] dx$$

$$= -\frac{2\bar{p}_x a}{E} + \mu \frac{2\bar{p}_y a}{E} - \frac{\pi^2}{4a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 w_{m,n}^2 \qquad (14)$$

Displacement of edges in y-direction =

$$\int_0^b \left[\epsilon_y' - \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial w_o}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w_o}{\partial y}\right)^2\right] dy = -\frac{\bar{p}_y b}{E} + \mu \frac{\bar{p}_x b}{E}$$

$$-\frac{\pi^2}{4b} \sum_{n=1}^{\infty} n^2 w_{o,n}^2 - \frac{\pi^2}{8b} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} n^2 w_{m,n}^2 - \frac{\pi^2}{2b} \sum_{n=1}^{\infty} n^2 w_{o,n} a_n \qquad (15)$$

Equations (13) to (15) are independent of x and y, thus showing that the conditions of constant load and constant edge displacement are satisfied by equations (9) to (12).

For a plate having a constant radius of curvature R between $y = 0$ and $y = b$ the coefficients a_n in equation (10) must have the values

$$a_n = \frac{4b^2}{R\pi^3 n^3} \tag{16}$$

Edge Compression of Plate Strip with Circular Cylindrical Shape ($\mu = 0.316$; Only $w_{0,1}$ and $w_{1,1}$ Are Assumed Different from Zero)

The following results apply to plates loaded in edge compression in the x-direction as shown in figure 1.

The normal pressure q and the edge compression in the y-direction $\bar{p}_y ah$ are zero. The initial displacement w_o is that of a circular cylinder of radius R.

If only the deflection coefficients $w_{0,1}$ and $w_{1,1}$ are taken different from zero we get from equations (11) and (16) the following equations for the stress coefficients:

$$\left.\begin{aligned}
b_{1,0} &= \frac{a^2 E}{4b^2}\left(-2w_{0,1}\,w_{1,1} - \frac{8}{31}\,\frac{b^2}{R}\,w_{1,1}\right) \\
b_{2,0} &= -\frac{a^2 E}{32b^2}\,w_{1,1}^2 \\
b_{0,2} &= \frac{a^2 E}{32b^2}\,w_{1,1}^2 \\
b_{1,2} &= \frac{Ea^2 b^2}{4(b^2 + 4a^2)^2}\left(2w_{0,1}\,w_{1,1} + \frac{16}{93}\,\frac{b^2}{R}\,w_{1,1}\right)
\end{aligned}\right\} \tag{17}$$

We also get from equations (12), (16), and (17) by equating $q_{0,1}$ and $q_{1,1}$ to zero the equations:

$$\frac{w_{0,1}}{h} = -\frac{w_{1,1}^2}{h^2}\frac{b^2}{Rh}\frac{4}{31}\frac{1 + \dfrac{1}{3(1 + 4a^2/b^2)^2}}{\dfrac{1}{2.7} + \dfrac{w_{1,1}^2}{h^2}\left[1 + \dfrac{1}{2(1 + 4a^2/b^2)^2}\right]} \qquad (18)$$

$$\frac{\overline{p}_x b^2 h}{\pi^2 D} = \left(\frac{b}{a} + \frac{a}{b}\right)^2 + 2.7\left\{\frac{w_{1,1}^2}{4h^2}\left(\frac{b^2}{a^2} + \frac{a^2}{b^2}\right)\right.$$

$$+ \frac{w_{0,1}^2}{h^2}\left[\frac{2a^2}{b^2} + \frac{1}{\left(\frac{b}{a} + \frac{4a}{b}\right)^2}\right] + \frac{w_{0,1}}{h}\frac{b^2}{Rh}\left[\frac{16a^2}{31b^2} + \frac{16}{93}\frac{1}{\left(\frac{b}{a} + \frac{4a}{b}\right)^2}\right]$$

$$\left. + \frac{b^4}{R^2h^2}\left[\frac{32a^2}{961b^2} + \frac{64}{8649\left(\frac{b}{a} + \frac{4a}{b}\right)^2}\right]\right\} \qquad (19)$$

Equations (18) and (19) were solved simultaneously by assuming values of buckle deflection $w_{1,1}/h$ and solving for the load ratio $\overline{p}_x b^2 h/\pi^2 D$. The results are plotted in figures 2 to 4 for values of the ratio of length to width of buckle a/b of 1/2, 3/4, and 1 and for values of the curvature ratio b^2/Rh of 0, 5, and 10. Values of $a/b > 1$ were not computed since the results for $a/b =$ 1/2, 3/4, and 1 indicate that the load required to maintain a buckle having $a/b > 1$ is greater than the load required to maintain a buckle having $a/b < 1$. Buckles for which $a/b > 1$ would therefore not occur. It is evident from these figures that as the buckle depth becomes comparable with the sheet thickness, $w_{1,1} = h$, the effect of the initial curvature on the load becomes negligible. For small deflections, however, an increase of the curvature ratio b^2/Rh from 0 to 10 causes an increase in the load ratio $\dfrac{\overline{p}_x b^2 h}{\pi^2 D}$ from 4.00 to 13.06 for a square buckle $a = b$.

The ratio of the effective width to the initial width (defined as the ratio of the actual load carried by the plate to the load the plate would have carried if the stress had been uniform and equal to the Young's modulus times the average edge strain) was computed from equations (13), (14), (18), and (19) with the results given in figures 5 to 7. Near the edge strain corresponding to buckling, curvature has a large effect on the effective width ratio; however, when the edge strain is several times the buckling strain of the corresponding flat panel the effect of curvature on the effective width ratio is negligible.

Edge Compression of Plate Strip with Circular Cylindrical Shape (μ = 0.316; Only $w_{0,1}$, $w_{1,1}$, $w_{2,1}$, $w_{3,1}$, $w_{1,3}$, and $w_{3,3}$ Are Assumed Different from Zero; Square Buckles a/b = 1)

In the solution of the previous section only the first two deflection coefficients $w_{0,1}$ and $w_{1,1}$ were used. The omission of higher terms in the deflection introduces an error in the results. In the present section the order of magnitude of this error will be determined for the special case of a square buckle by obtaining a more exact solution including the first six deflection coefficients $w_{0,1}$, $w_{1,1}$, $w_{2,1}$, $w_{3,1}$, $w_{1,3}$, and $w_{3,3}$.

The results again apply to plates loaded in edge compression in the x-direction as shown in figure 1. The normal pressure q and the edge compression in the y-direction $\bar{p}_y ah$ are zero. The initial displacement w_0 is that of a circular cylinder of radius R. The buckle is square, $a = b$.

Equations for the stress coefficients $b_{m,n}$ were obtained from equations (11) and (16) on the assumption that only $w_{0,1}$, $w_{1,1}$, $w_{2,1}$, $w_{3,1}$, $w_{1,3}$, and $w_{3,3}$ were not zero. The results are given in table I. Substituting these stress coefficients in equation (12) and equating the pressure coefficients $q_{m,n}$ and the stress coefficient $\bar{p}_y$ to zero gives the equations in table II. These equations relate the average membrane pressure in the x-direction $\overline{p}_x$, the initial radius of curvature R, and the deflection coefficients $w_{0,1}$, $w_{1,1}$, $w_{2,1}$, $w_{3,1}$, $w_{1,3}$, and $w_{3,3}$.

The equations in table II were solved by successive approximation for the case where $\overline{p}_x = 8.24\ Eh^2/b^2$ and $b^2/Rh = 10$. The resulting values of the deflection coefficients were $w_{0,1}/h = -1.017$, $w_{1,1}/h = 1.929$, $w_{2,1}/h = -0.243$, $w_{3,1}/h = -0.075$, $w_{1,3}/h = 0.036$, and $w_{3,3}/h = -0.006$. Using these values the effective width ratio was computed from equations (13) and (14). At an edge strain ratio of 3.61 times the critical edge strain ratio for flat sheet the effective width ratio was 0.625. The corresponding value using the solution with only two deflection coefficients was 0.642. (See fig. 5.) This indicates that the difference between the results using only two deflection coefficients and the results using six deflection coefficients is small for deflections up to about twice the sheet thickness and for values of b^2/Rh up to about 10.

Effective Width

The effective width curves of figures 5, 6, and 7 are replotted in figures 8, 9, and 10 using as abscissa the dimensionless edge strain ratio $\epsilon b^2/h^2$. An envelope curve is drawn which is tangent to the effective width curves for different ratios of length to width. This envelope curve corresponds to the effective width of a long plate in which the buckle length is adjusted to give the lowest possible effective width. In an actual plate of finite length, however, it is probable that after a buckle pattern has become established, a momentary increase in load would be required during the transition to the new buckle pattern having lower effective width.

It is interesting to observe that even in the case of a flat sheet (fig. 8) the buckle length corresponding to a minimum effective width decreases as the edge strain increases. For minimum effective width the buckle shape would be square initially, change to $a/b = 3/4$ for an edge strain ratio $\epsilon b^2/h^2 = 11$, and change to $a/b = 1/2$ for $\epsilon b^2/h^2 = 38$. Similar effects are evident for curved sheet (figs. 9 and 10).

The minimum effective width, corresponding to the envelope curve, is 0.500 when $\epsilon b^2/h^2 = 21.0$ for flat sheet as well as sheet with curvature ratios b^2/Rh equal to 5 and 10. It is evident from this and from figures 8, 9, and 10 that the effective width is nearly independent of initial curvature when the critical edge strain has been several times exceeded.

Comparison with Results by Other Authors

Experimental and theoretical investigations of the load carried by curved sheet after buckling have been made by von Kármán and Tsien (references 4 and 6), Cox and Clenshaw (reference 7), Newell (reference 8), Ebner (reference 9) and Wenzek (reference 10).

von Kármán and Tsien (references 4 and 6) consider a sheet curved to form a closed circular cylinder without longitudinal reinforcements. Such a sheet buckles into diamond-shape buckles (see, for example, fig. 242, p. 461, reference 5). These buckles do not satisfy the condition assumed in the present paper that the sheet is simply supported along the edges and that the initial displacements are small enough to justify the use of simplified formulas for curvature. This case is therefore outside the scope of the present work.

Cox and Clenshaw (reference 7) give experimental results for a large number of plates having clamped longitudinal edges but no longitudinal reinforcements. Although this edge condition is different from that of simple support assumed in the present paper, the effect of curvature should be similar in the two cases. They observe that the initial buckling load of curved sheet might considerably exceed that for flat sheet and that the effective width of curved sheet after the buckling load has been exceeded several times is nearly the same as that for flat sheet. These experimental results are in agreement with the theoretical curves in figures 5 to 7 of the present paper.

Newell (reference 8) gives design charts based on the results of tests to failure of curved sheet supported between v-grooves. These results indicate a considerable increase in the failing load as the curvature is increased. It is probable that the failing load with this type of support was the buckling load of the curved sheet.

The results are then in agreement with the increase in buckling load with increasing curvature shown in figures 2, 3, and 4 of the present paper.

Ebner (fig. 40, reference 9) gives the results of tests on curved panels with longitudinal stiffeners in which he shows that the failing stress of a given type of panel is nearly independent of the initial curvature. The edge restraint of the sheet of these panels by the longitudinal stiffeners probably enabled them to support stresses well above the buckling stress. His result is in agreement with figures 8, 9, and 10 which show only a slight change in effective width with curvature when the buckling stress is exceeded several times.

Wenzek (reference 10) gives the results of tests on brass sheets clamped along the longitudinal edges to a stiffener having a closed section. His results indicate that an increase in the curvature is accompanied by an increase in the buckling load but that this increase in load-carrying capacity drops off with increase in the edge strain. These general conclusions are in agreement with the results of the present paper. Some of Wenzek's experimental results (fig. 7, cyl. 7, 8, and 10, reference 10) and theoretical results (fig. 3, reference 10) show values of the ratio of effective width to initial width which are nearly zero. These results seem extreme and are not confirmed by the results in the present paper.

CONCLUSIONS

It may be concluded that for small deflections the initial curvature has a large effect on the load carried in axial compression and may increase the buckling load several hundred percent. When the buckle depth becomes comparable with the sheet thickness, however, the effect of the initial curvature on the load carried in axial compression becomes negligible.

In terms of effective width this may be expressed as follows: The effective width ratio is increased considerably by an increase in curvature for loads near the buckling load. When the edge strain is several times the critical buckling strain of the corresponding flat sheet, however, the effect of curvature on the effective width ratio is negligible.

National Bureau of Standards,
Washington, D. C., January 8, 1943.

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10. Wenzek, W. A.: The Effective Width of Curved Sheet after Buckling. T.M. No. 880, NACA, 1938.

TABLE I.- STRESS COEFFICIENTS FOR a = b, INITIAL RADIUS OF CURVATURE R.

$$b_{1,0}/E = -.5000\, w_{0,1} w_{1,1} - .2500\, w_{1,1} w_{2,1} - .2500\, w_{2,1} w_{3,1} - .06450\, \frac{b^2}{R} w_{1,1} - .02150\, \frac{b^2}{R} w_{1,3}$$

$$4\, b_{2,0}/E = -.5000\, w_{0,1} w_{2,1} - .1250\, w_{1,1}^2 - .2500\, w_{1,1} w_{3,1} - 2.250\, w_{1,3} w_{3,3} - 1.125\, w_{1,3}^2 - .06450\, \frac{b^2}{R} w_{2,1}$$

$$9\, b_{3,0}/E = -.2500\, w_{1,1} w_{2,1} - .5000\, w_{0,1} w_{3,1} - .06450\, \frac{b^2}{R} w_{3,1} - .02150\, \frac{b^2}{R} w_{3,3}$$

$$16\, b_{4,0}/E = -.2500\, w_{1,1} w_{3,1} - .1250\, w_{2,1}^2 - 2.250\, w_{1,3} w_{3,3}$$

$$25\, b_{5,0}/E = -.2500\, w_{2,1} w_{3,1}$$

$$36\, b_{6,0}/E = -.1250\, w_{3,1}^2 - 1.125\, w_{3,3}^2$$

$$4\, b_{0,2}/E = +0.1250\, w_{1,1}^2 + .5000\, w_{2,1}^2 + 1.125\, w_{3,1}^2 - .2500\, w_{1,1} w_{1,3} - 2.250\, w_{3,1} w_{3,3}$$

$$4\, b_{1,2}/E = +.0800\, w_{0,1} w_{1,1} + .3600\, w_{1,1} w_{2,1} + 1.000\, w_{2,1} w_{3,1} - .0800\, w_{0,1} w_{1,3} - 1.000\, w_{2,1} w_{1,3} - .3600\, w_{2,1} w_{3,3} + .00688\, \frac{b^2}{R} w_{1,1} - .01238\, \frac{b^2}{R} w_{1,3}$$

$$4\, b_{2,2}/E = +.1250\, w_{0,1} w_{2,1} + .2500\, w_{1,1} w_{3,1} - .2500\, w_{1,1} w_{1,3} - w_{1,3} w_{3,1} + .01094\, \frac{b^2}{R} w_{2,1}$$

$$9\, b_{3,2}/E = +.01332\, w_{1,1} w_{2,1} + .2398\, w_{0,1} w_{3,1} - .2398\, w_{0,1} w_{3,3} - .6526\, w_{2,1} w_{1,3} + .02061\, \frac{b^2}{R} w_{3,1} - .03709\, \frac{b^2}{R} w_{3,3}$$

$$16\, b_{4,2}/E = +.0400\, w_{1,1} w_{3,1} - w_{1,3} w_{3,1} - .3600\, w_{1,1} w_{3,3}$$

$$25\, b_{5,2}/E = .00743\, w_{2,1} w_{3,1} - .6020\, w_{2,1} w_{3,3}$$

TABLE I.- (CONTINUED)

$$36\,b_{6,2}/E = -.8100\,w_{3,1}w_{3,3}$$

$$16\,b_{0,4}/E = +.2500\,w_{1,1}w_{1,3} + 2.250\,w_{3,1}w_{3,3}$$

$$16\,b_{1,4}/E = +.02769\,w_{0,1}w_{1,3} + .6785\,w_{1,3}w_{2,1} + 1.121\,w_{2,1}w_{3,3} + .0004761\,\frac{b^2}{R}w_{1,1} + .003061\,\frac{b^2}{R}w_{1,3}$$

$$16\,b_{2,4}/E = +.3600\,w_{1,1}w_{3,3} + w_{1,3}w_{3,1} + .040\,w_{1,1}w_{1,3} + .001376\,\frac{b^2}{R}w_{2,1}$$

$$16\,b_{3,4}/E = .1152\,w_{0,1}w_{3,3} + .1600\,w_{2,1}w_{1,3} + .001982\,\frac{b^2}{R}w_{3,1} + .01274\,\frac{b^2}{R}w_{3,3}$$

$$16\,b_{4,4}/E = +.2500\,w_{1,3}w_{3,1}$$

$$25\,b_{5,4}/E = +.03348\,w_{2,1}w_{3,3}$$

$$36\,b_{6,4}/E = .1198\,w_{3,1}w_{3,3}$$

$$36\,b_{0,6}/E = +.125\,w_{1,3}^2 + 1.125\,w_{3,3}^2$$

$$36\,b_{1,6}/E = +.0000969\,\frac{b^2}{R}w_{1,1} + .0003770\,\frac{b^2}{R}w_{1,3}$$

$$36\,b_{2,6}/E = +.8100\,w_{1,3}w_{3,3} + .0003318\,\frac{b^2}{R}w_{2,1}$$

$$36\,b_{3,6}/E = +.0005895\,\frac{b^2}{R}w_{3,1} + .002294\,\frac{b^2}{R}w_{3,3}$$

$$36\,b_{4,6}/E = .1198\,w_{1,3}w_{3,3}$$

$$b_{5,6}/E = 0$$

$$b_{6,6}/E = 0$$

TABLE II.- EQUATIONS RELATING DEFLECTION COEFFICIENTS $w_{0,1}$, $w_{1,1}$, $w_{2,1}$, $w_{3,1}$, $w_{1,3}$, AND $w_{3,3}$, THE RADIUS OF CURVATURE R, AND THE AVERAGE MEMBRANE PRESSURE IN THE X-DIRECTION $\bar{p}_x$ FOR A SQUARE BUCKLE, a = b. POISSON'S RATIO μ = 0.316. LATERAL PRESSURE q AND MEMBRANE PRESSURE $\bar{p}_y$ ARE ZERO.

$$
\begin{aligned}
0.3704\, h^2 w_{0,1} = {} & 0.04\, w_{0,1} w_{1,1} w_{1,3} + 0.4796\, w_{0,1} w_{3,1} w_{3,3} \\
& - 2.0133\, w_{1,1} w_{2,1} w_{3,1} + 0.59\, w_{1,1} w_{2,1} w_{1,3} \\
& + 0.10332\, w_{1,1} w_{2,1} w_{3,3} + 1.9026\, w_{2,1} w_{3,1} w_{1,3} \\
& - 5.4026\, w_{2,1} w_{1,3} w_{3,3} - 1.02\, w_{1,1}^2 w_{0,1} - 0.84\, w_{1,1}^2 w_{2,1} \\
& - 1.125\, w_{2,1}^2 w_{0,1} - 1.2398\, w_{3,1}^2 w_{0,1} - 0.02173\, w_{1,3}^2 w_{0,1} \\
& - 2.5424\, w_{1,3}^2 w_{2,1} - 0.3046\, w_{3,3}^2 w_{0,1} - 0.03821\, w_{1,1} w_{1,3} \frac{b^2}{R} \\
& + 0.01359\, w_{3,1} w_{3,3} \frac{b^2}{R} - 0.1307\, w_{1,1}^2 \frac{b^2}{R} - 0.1397\, w_{2,1}^2 \frac{b^2}{R} \\
& - 0.1496\, w_{3,1}^2 \frac{b^2}{R} - 0.003286\, w_{1,3}^2 \frac{b^2}{R} - 0.04426\, w_{3,3}^2 \frac{b^2}{R}
\end{aligned}
$$

$$
\begin{aligned}
1.4815\, h^2 w_{1,1} = {} & 0.4053\, b^2 \bar{p}_x w_{1,1}/E - 3.36\, w_{0,1} w_{1,1} w_{2,1} \\
& - 4.0267\, w_{0,1} w_{2,1} w_{3,1} + 1.18\, w_{0,1} w_{2,1} w_{1,3} \\
& + 0.20665\, w_{0,1} w_{2,1} w_{3,3} + 2\, w_{1,1} w_{1,3} w_{3,1} \\
& + 4.68\, w_{1,1} w_{3,1} w_{3,3} - 4.68\, w_{1,1} w_{1,3} w_{3,3} \\
& - 22.5\, w_{1,3} w_{3,1} w_{3,3} - 2.04\, w_{0,1}^2 w_{1,1} + 0.04\, w_{0,1}^2 w_{1,3} \\
& - 0.75\, w_{1,1}^2 w_{3,1} + 0.75\, w_{1,1}^2 w_{1,3} - 2.8115\, w_{2,1}^2 w_{1,1} \\
& - 3\, w_{2,1}^2 w_{3,1} + 3.3225\, w_{2,1}^2 w_{1,3} + 0.81\, w_{2,1}^2 w_{3,3} \\
& - 4.26\, w_{3,1}^2 w_{1,1} + 6.5\, w_{3,1}^2 w_{1,3} - 4.26\, w_{1,3}^2 w_{1,1} \\
& - 6.5\, w_{1,3}^2 w_{3,1} - 1.62\, w_{3,3}^2 w_{1,1} - 0.5\, w_{1,1}^3 \\
& - 0.5229\, w_{0,1} w_{1,1} \frac{b^2}{R} - 0.07643\, w_{0,1} w_{1,3} \frac{b^2}{R} \\
& - 0.418\, w_{1,1} w_{2,1} \frac{b^2}{R} - 0.4753\, w_{2,1} w_{3,1} \frac{b^2}{R} \\
& + 0.06904\, w_{1,3} w_{2,1}\, b^2/R - 0.02891\, w_{2,1} w_{3,3}\, b^2/R \\
& - 0.03359\, w_{1,1} \left(b^2/R\right)^2 - 0.01057\, w_{1,3} \left(b^2/R\right)^2
\end{aligned}
$$

TABLE II.- (CONTINUED)

$$
\begin{aligned}
9.259\,h^2 w_{2,1} = {} & 1.621\,b^2 \bar{p}_x w_{2,1}/E - 4.0267\,w_{0,1} w_{1,1} w_{3,1} \\
& + 1.18\,w_{0,1} w_{1,1} w_{1,3} + 0.20665\,w_{0,1} w_{1,1} w_{3,3} \\
& + 3.805\,w_{0,1} w_{1,3} w_{3,1} - 10.805\,w_{0,1} w_{1,3} w_{3,3} \\
& - 6\,w_{1,1} w_{2,1} w_{3,1} + 6.6451\,w_{1,1} w_{2,1} w_{1,3} + 1.62\,w_{1,1} w_{2,1} w_{3,3} \\
& + 12.5\,w_{2,1} w_{1,3} w_{3,1} + 22.548\,w_{2,1} w_{3,1} w_{3,3} - 15.869\,w_{2,1} w_{1,3} w_{3,3} \\
& - 2.25\,w_{0,1}^2 w_{2,1} - 1.68\,w_{1,1}^2 w_{0,1} - 2.8115\,w_{1,1}^2 w_{2,1} \\
& - 16.25\,w_{3,1}^2 w_{2,1} - 5.085\,w_{1,3}^2 w_{0,1} - 12.131\,w_{1,3}^2 w_{2,1} \\
& - 8.447\,w_{3,3}^2 w_{2,1} - 4.25\,w_{2,1}^3 - 0.5590\,w_{0,1} w_{2,1}\,b^2/R \\
& - 0.4753\,w_{1,1} w_{3,1}\,b^2/R + 0.06907\,w_{1,1} w_{1,3}\,b^2/R \\
& - 0.02891\,w_{1,1} w_{3,3}\,b^2/R + 0.3070\,w_{1,3} w_{3,1}\,b^2/R \\
& - 1.4276\,w_{1,3} w_{3,3}\,b^2/R - 0.03515\,w_{2,1} (b^2/R)^2 \\
& - 0.2090\,w_{1,1}^2\,b^2/R - 0.6613\,w_{1,3}^2\,b^2/R
\end{aligned}
$$

$$
\begin{aligned}
37.04\,h^2 w_{3,1} = {} & 3.647\,b^2 \bar{p}_x w_{3,1}/E - 4.0267\,w_{0,1} w_{1,1} w_{2,1} \\
& + 3.805\,w_{0,1} w_{2,1} w_{1,3} + 13\,w_{1,1} w_{1,3} w_{3,1} \\
& - 22.5\,w_{1,1} w_{1,3} w_{3,3} - 2.4796\,w_{0,1}^2 w_{3,1} \\
& + 0.4796\,w_{0,1}^2 w_{3,3} - 4.26\,w_{1,1}^2 w_{3,1} + w_{1,1}^2 w_{1,3} \\
& + 2.34\,w_{1,1}^2 w_{3,3} - 3\,w_{2,1}^2 w_{1,1} - 16.25\,w_{2,1}^2 w_{3,1} \\
& + 6.25\,w_{2,1}^2 w_{1,3} + 11.274\,w_{2,1}^2 w_{3,3} + 60.75\,w_{3,1}^2 w_{3,3} \\
& - 6.5\,w_{1,3}^2 w_{1,1} - 29.5\,w_{1,3}^2 w_{3,1} - 86.61\,w_{3,3}^2 w_{3,1} \\
& - 0.25\,w_{1,1}^3 - 20.5\,w_{3,1}^3 - 0.5984\,w_{0,1} w_{3,1}\,b^2/R \\
& + 0.02714\,w_{0,1} w_{3,3}\,b^2/R - 0.4753\,w_{1,1} w_{2,1}\,b^2/R \\
& + 0.3068\,w_{2,1} w_{1,3}\,b^2/R - 0.03689\,w_{3,1} (b^2/R)^2 \\
& - 0.00497\,w_{3,3} (b^2/R)^2 .
\end{aligned}
$$

TABLE II.- (CONCLUDED)

$$\begin{aligned}
37.04\,h^2 w_{1,3} = {} & 0.4053\, b^2 \bar{p}_x w_{1,3}/E + 1.18\, w_{0,1} w_{1,1} w_{2,1} \\
& + 3.806\, w_{0,1} w_{2,1} w_{3,1} - 10.17\, w_{0,1} w_{2,1} w_{1,3} \\
& - 10.806\, w_{0,1} w_{2,1} w_{3,3} - 13\, w_{1,1} w_{1,3} w_{3,1} \\
& - 22.5\, w_{1,1} w_{3,1} w_{3,3} + 0.04\, w_{0,1}^2 w_{1,1} \\
& - 0.04346\, w_{0,1}^2 w_{1,3} + w_{1,1}^2 w_{3,1} - 4.26\, w_{1,1}^2 w_{1,3} \\
& - 2.34\, w_{1,1}^2 w_{3,3} + 3.3226\, w_{3,1}^2 w_{1,1} + 6.25\, w_{2,1}^2 w_{3,1} \\
& - 12.13\, w_{2,1}^2 w_{1,3} - 7.933\, w_{2,1}^2 w_{3,3} + 6.5\, w_{3,1}^2 w_{1,1} \\
& - 29.5\, w_{3,1}^2 w_{1,3} - 60.75\, w_{1,3}^2 w_{3,3} - 86.61\, w_{3,3}^2 w_{1,3} \\
& + 0.25\, w_{1,1}^3 - 20.5\, w_{1,3}^3 - 0.07643\, w_{0,1} w_{1,1}\, b^2/R \\
& - 0.01315\, w_{0,1} w_{1,3}\, b^2/R + 0.06906\, w_{1,1} w_{2,1}\, b^2/R \\
& + 0.3068\, w_{2,1} w_{3,1}\, b^2/R - 1.335\, w_{2,1} w_{1,3}\, b^2/R \\
& - 1.428\, w_{2,1} w_{3,3}\, b^2/R - 0.01056\, w_{1,1} (b^2/R)^2 \\
& - 0.0047\, w_{1,3} (b^2/R)^2
\end{aligned}$$

$$\begin{aligned}
120\,h^2 w_{3,3} = {} & 3.647\, b^2 \bar{p}_x w_{3,3}/E + 0.20664\, w_{0,1} w_{1,1} w_{2,1} \\
& - 10.805\, w_{0,1} w_{2,1} w_{1,3} - 22.5\, w_{1,1} w_{1,3} w_{3,1} \\
& + 0.4794\, w_{0,1}^2 w_{3,1} - 0.6090\, w_{0,1}^2 w_{3,3} \\
& + 2.34\, w_{1,1}^2 w_{3,1} - 2.34\, w_{1,1}^2 w_{1,3} - 1.62\, w_{1,1}^2 w_{3,3} \\
& + 0.81\, w_{2,1}^2 w_{1,1} - 7.938\, w_{2,1}^2 w_{1,3} + 11.274\, w_{2,1}^2 w_{3,1} \\
& - 8.448\, w_{2,1}^2 w_{3,3} - 86.61\, w_{3,1}^2 w_{3,3} - 86.61\, w_{1,3}^2 w_{3,3} \\
& - 20.25\, w_{1,3}^3 + 20.25\, w_{3,1}^3 - 40.5\, w_{3,3}^3 \\
& + 0.0272\, w_{0,1} w_{3,1}\, b^2/R - 0.17705\, w_{0,1} w_{3,3}\, b^2/R \\
& - 0.0289\, w_{1,1} w_{2,1}\, b^2/R - 1.4277\, w_{1,3} w_{2,1}\, b^2/R \\
& - 0.00496\, w_{3,1} (b^2/R)^2 - 0.0168\, w_{3,3} (b^2/R)^2
\end{aligned}$$

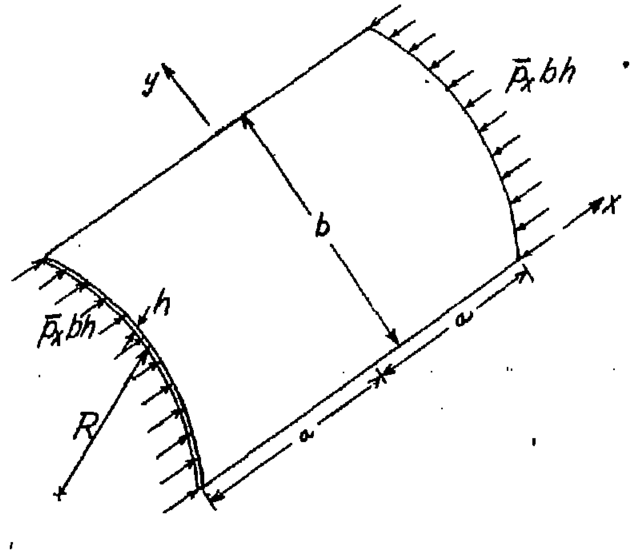

Figure 1.- Plate with circular cylindrical initial shape in edge compression under axial load $\bar{p}_x bh$.

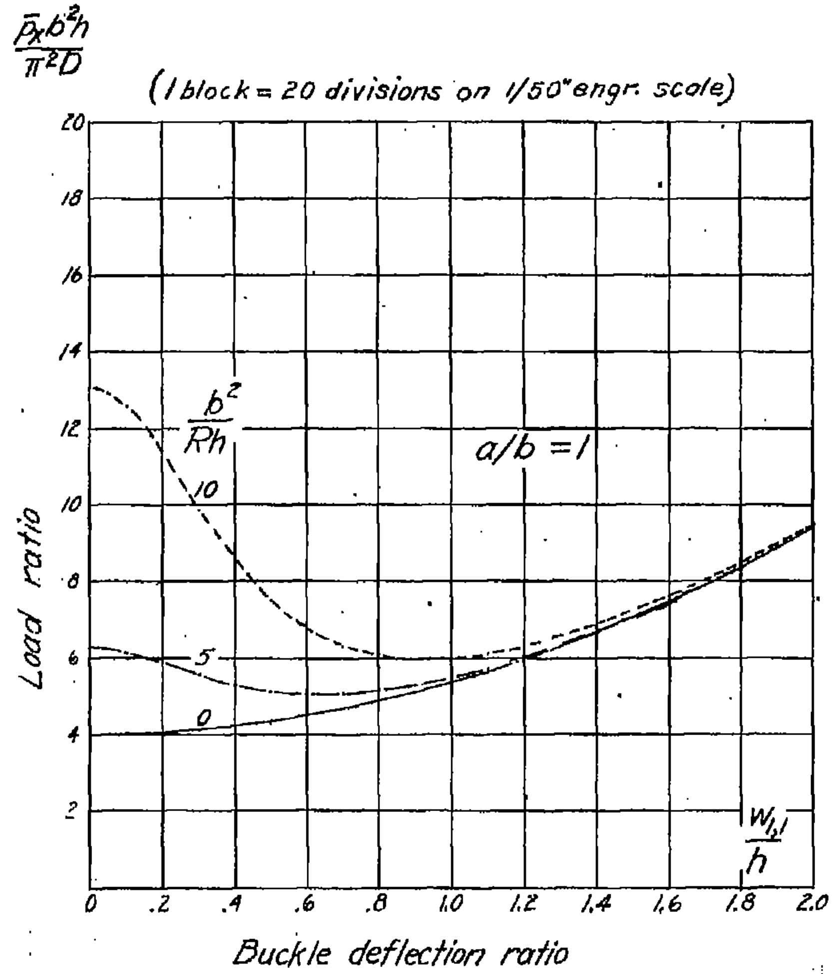

Figure 2.- Variation of load ratio with buckle deflection ratio for ratio of length to width of buckle a/b=1 and for different curvature ratios b^2/Rh.

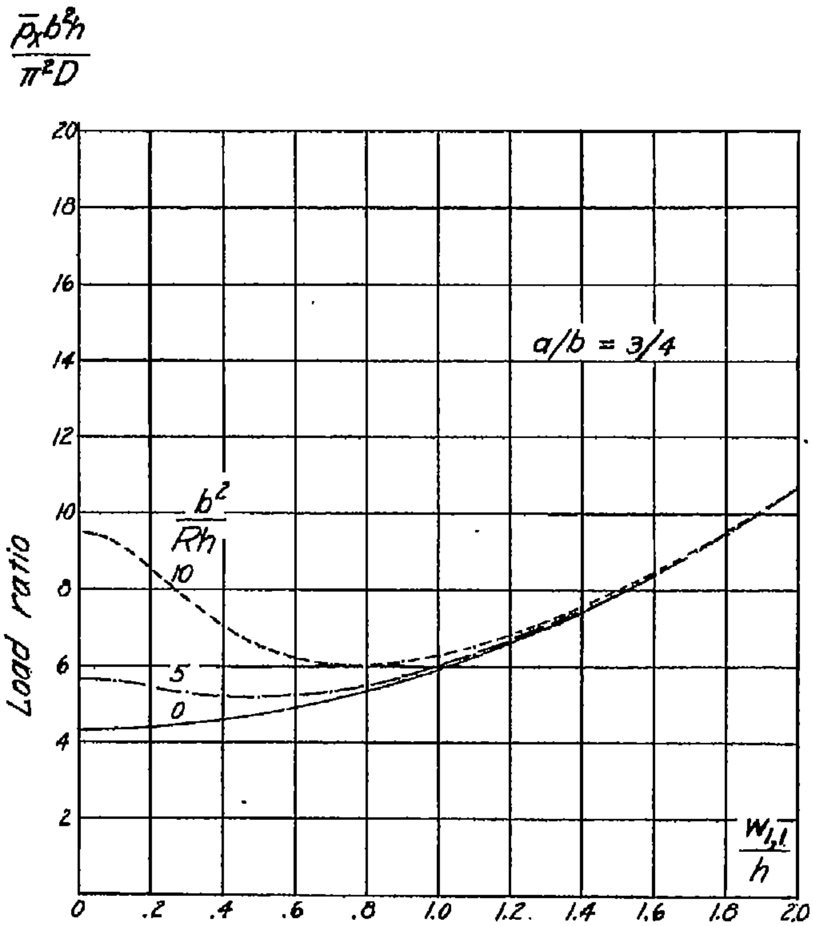

Figure 3.- Variation of load ratio with buckle deflection ratio for ratio of length to width a/b = 3/4 and for different curvature ratios b^2/Rh.

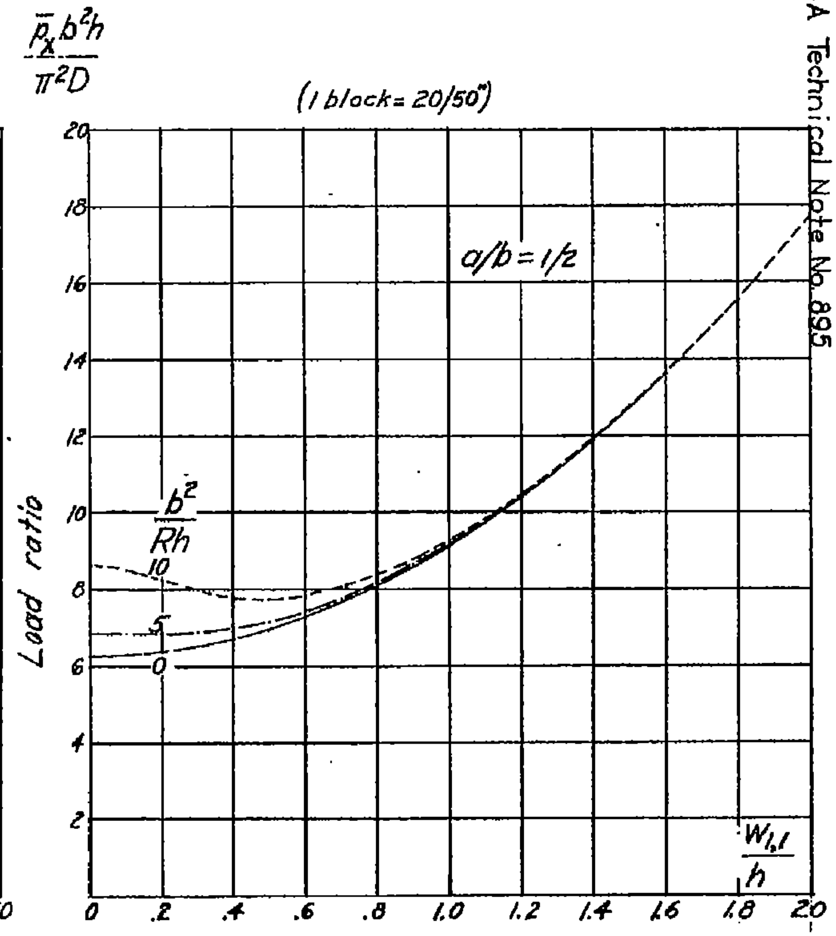

Figure 4.- Variation of load ratio with buckle deflection ratio for ratio of length to width a/b = 1/2 and for different curvature ratios $b^2 = Rh$.

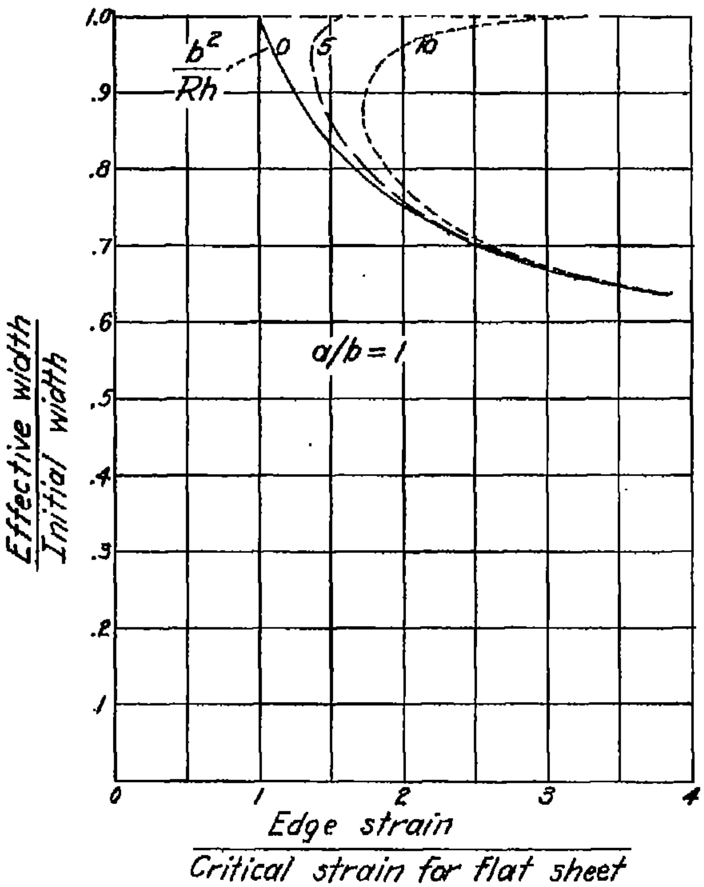

Figure 5.- Variation of effective width ratio with edge strain ratio for curvature ratios b^2/Rh of 0, 5 and 10, $a/b = 1$.

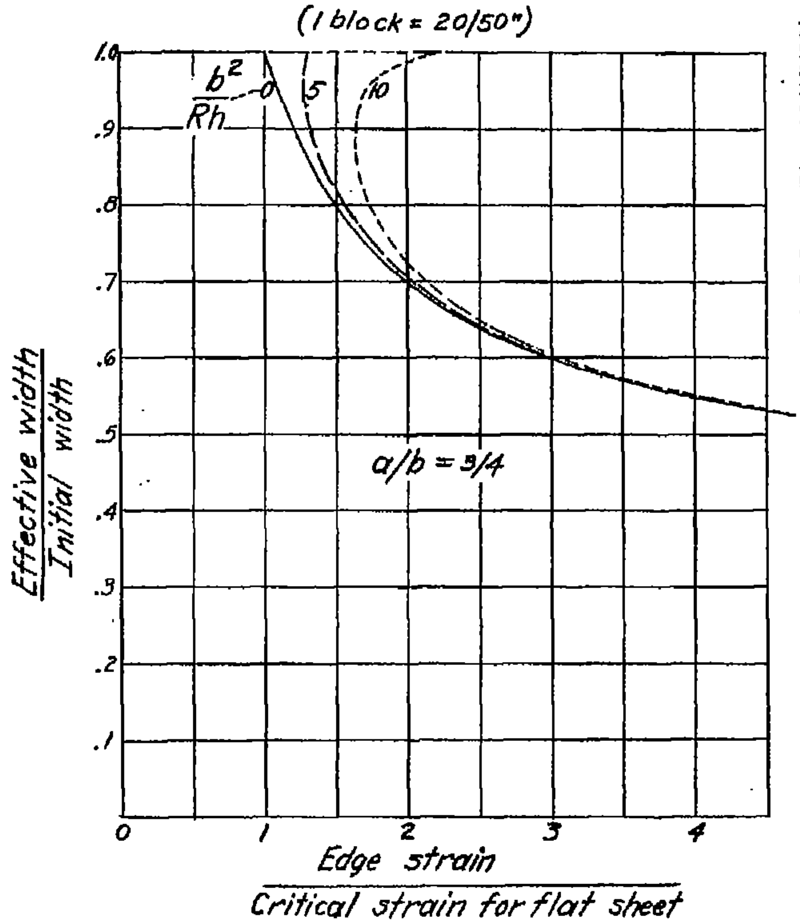

Figure 6.- Variation of effective width ratio with edge strain ratio for curvature ratios b^2/Rh of 0, 5 and 10, $a/b = 3/4$.

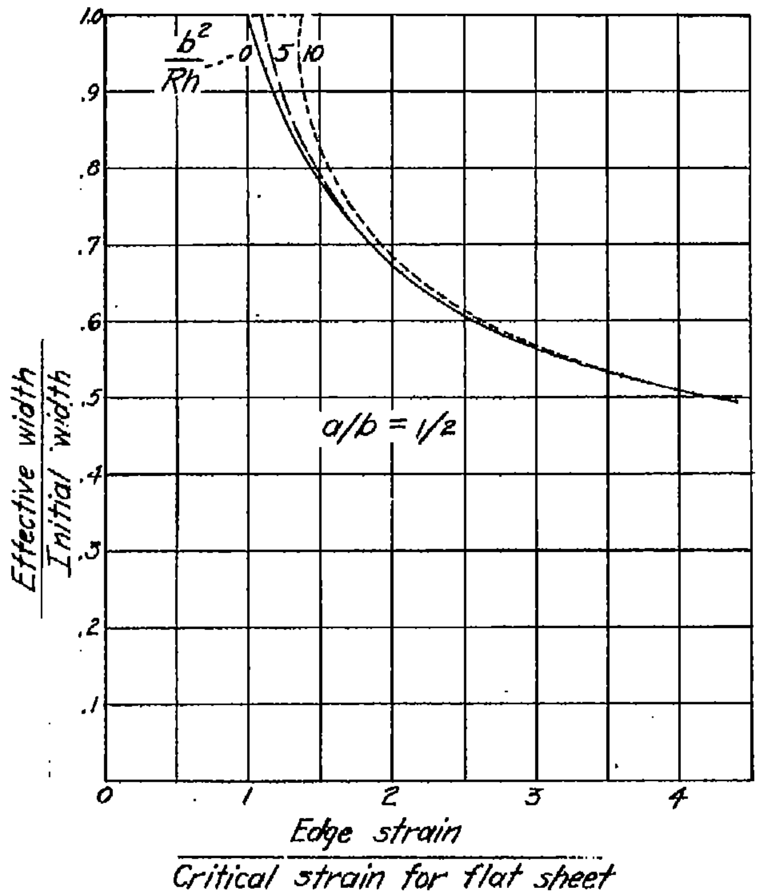

Figure 7.- Variation of effective width ratio with edge strain ratio for curvature ratios b^2/Rh of 0, 5 and 10, $a/b = 1/2$.

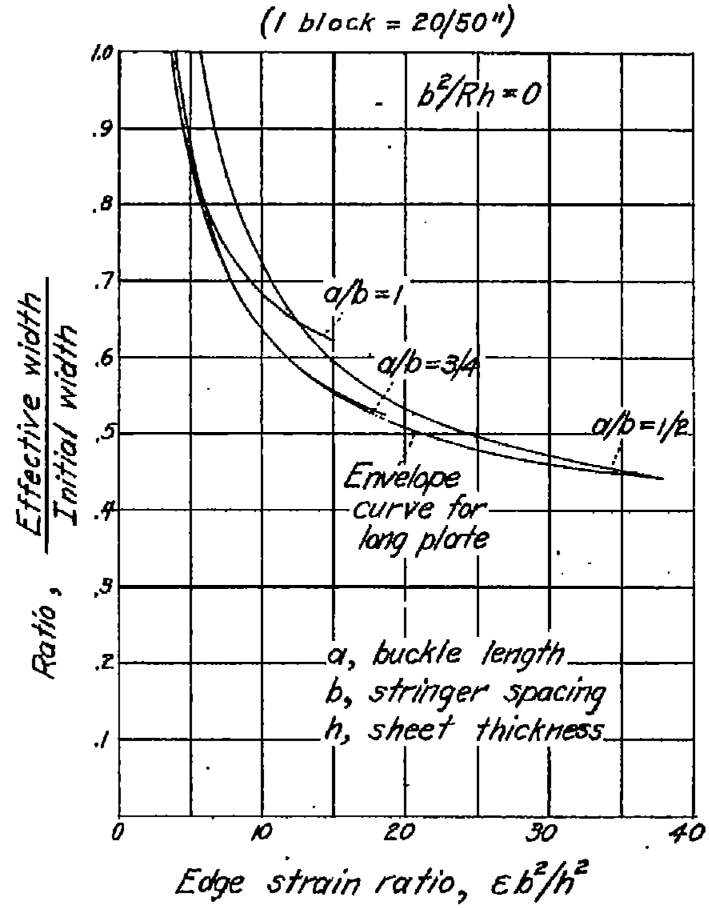

Figure 8.- Effective width ratio for flat sheet, $\mu = 0.316$.

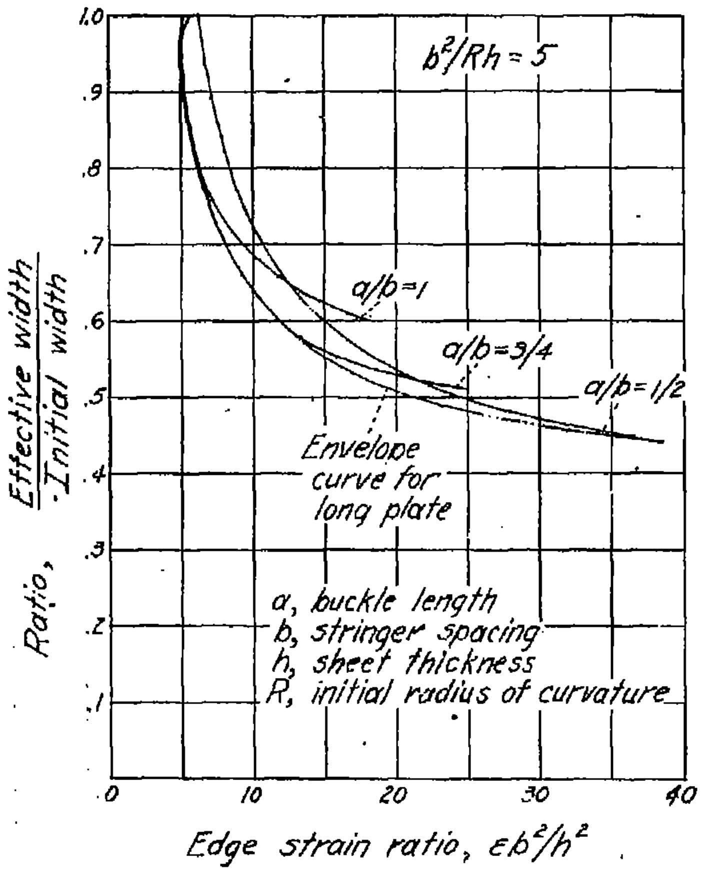

Figure 9.- Effective width ratio for curved plate $b^2/Rh = 5$, $\mu = 0.316$.

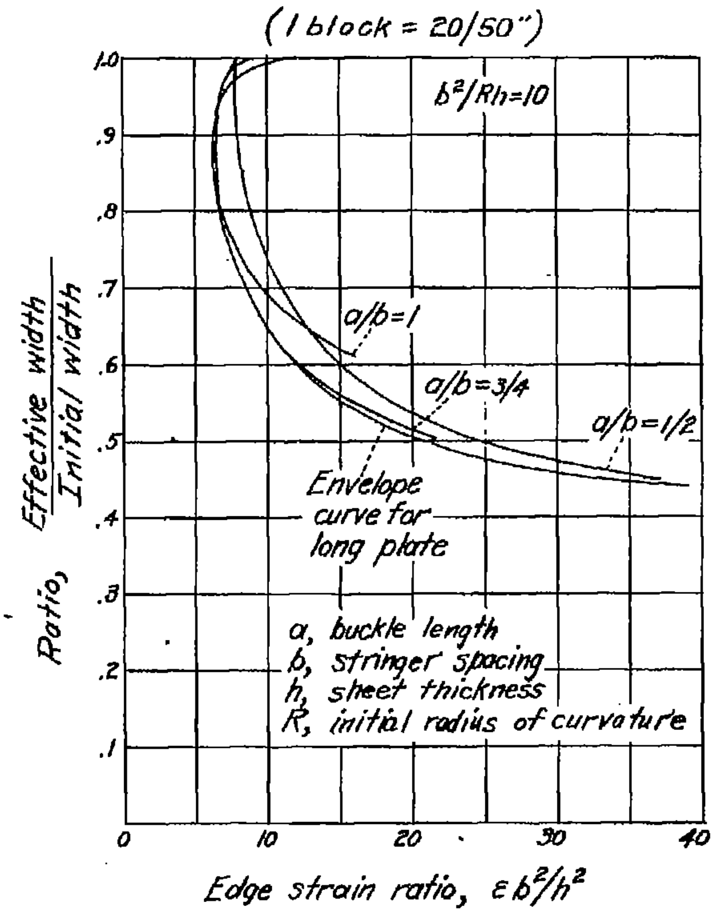

Figure 10.- Effective width ratio for curved plate $b^2/Rh = 10$, $\mu = 0.316$.